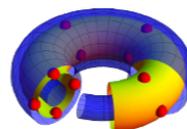


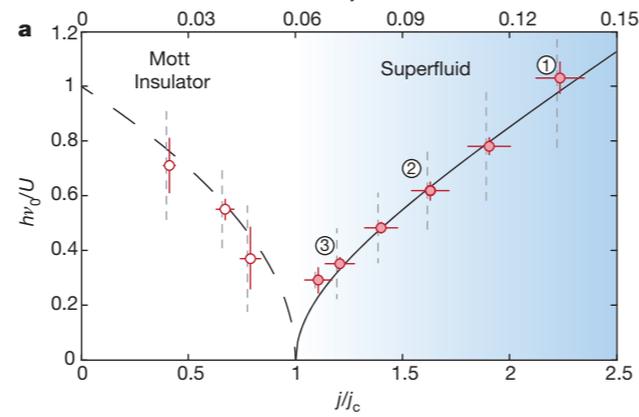


# Amplitude modes and cold atoms

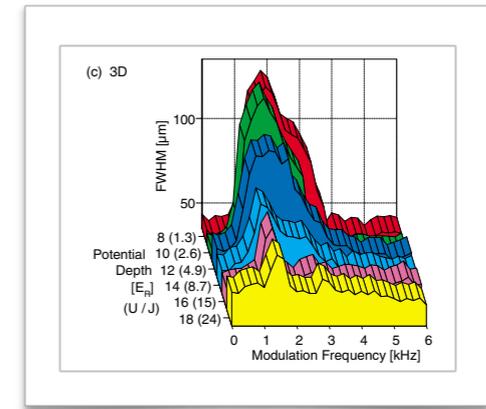
Sebastian Huber



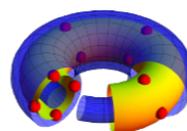
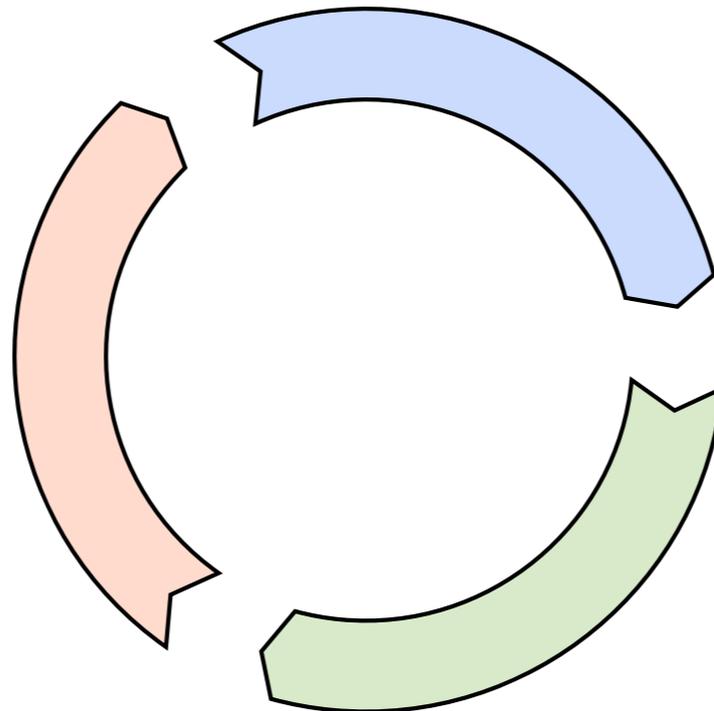
Endres Nature 2012



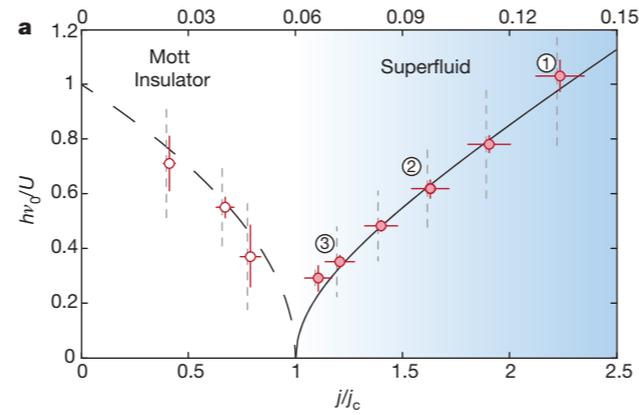
Experiments



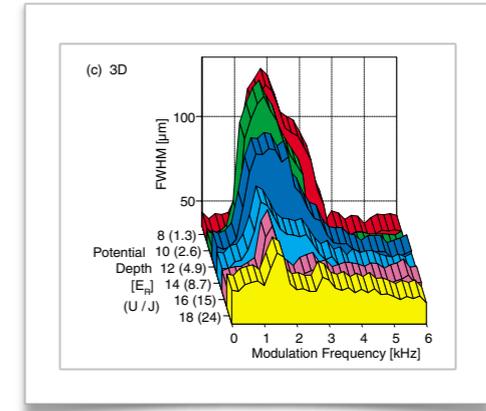
Stöferle PRL 2004



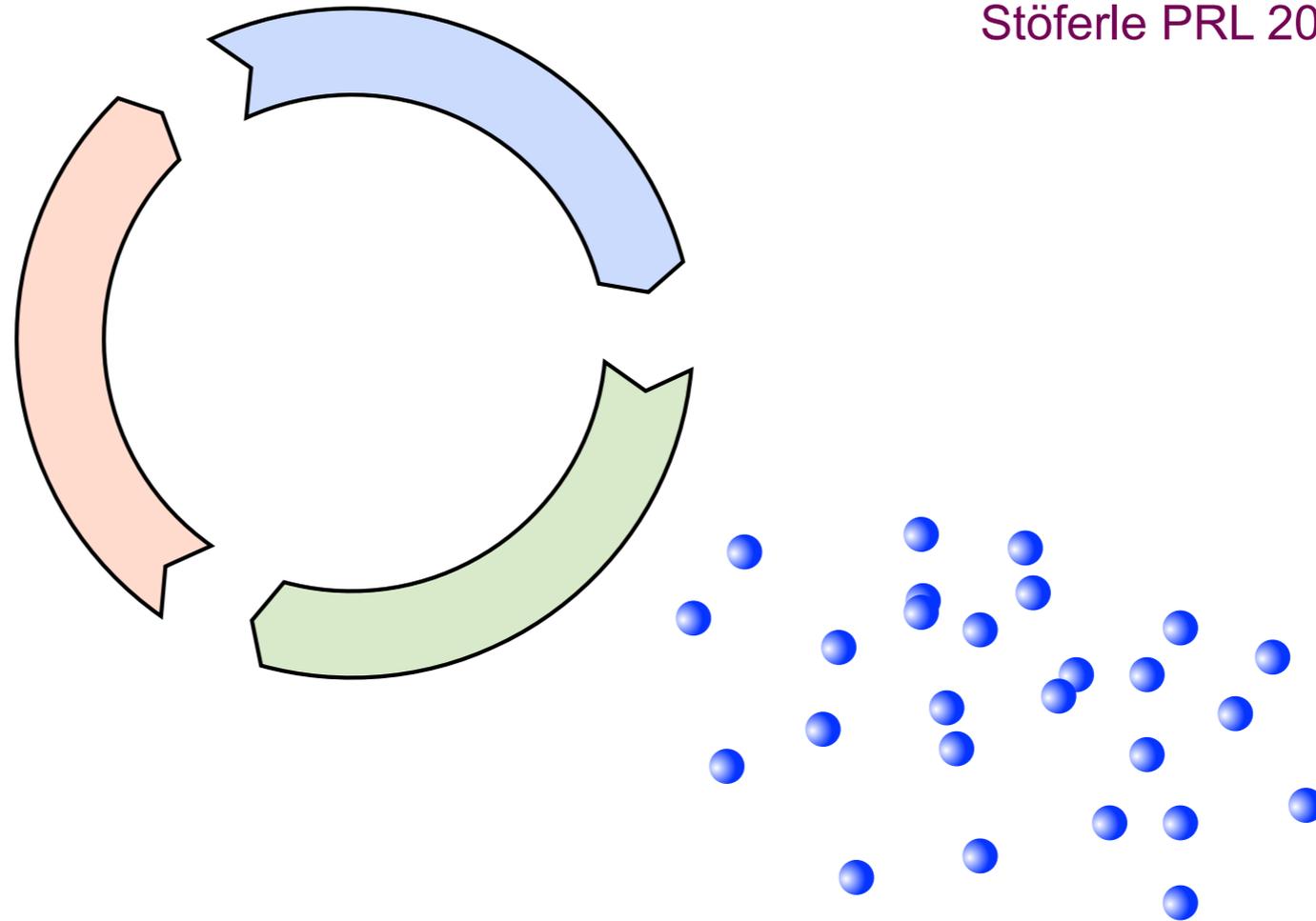
Endres Nature 2012



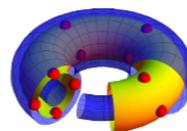
Experiments



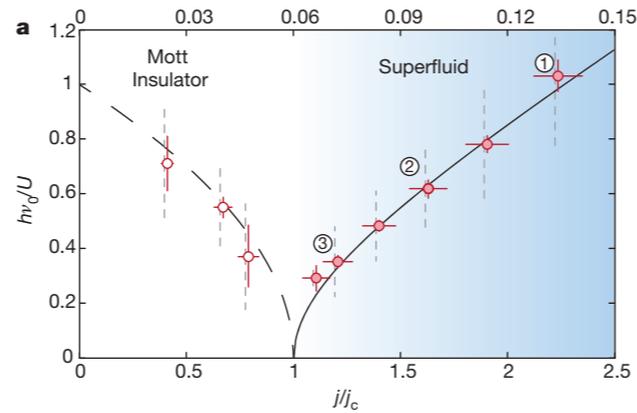
Stöferle PRL 2004



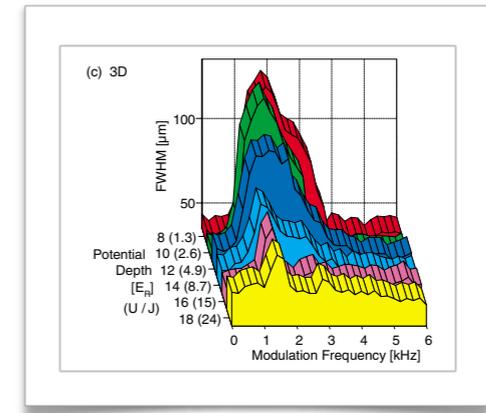
Microscopic description



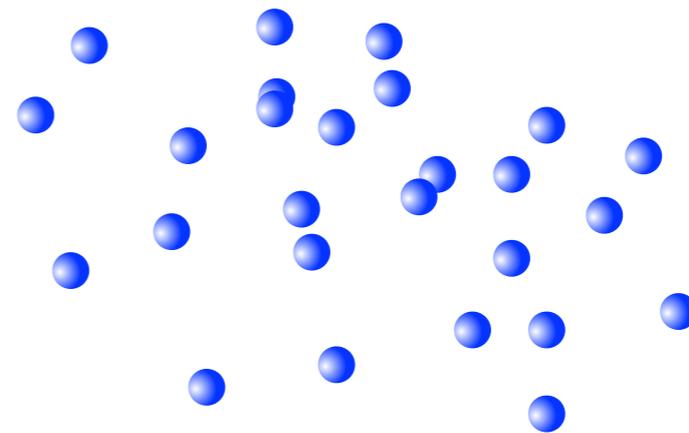
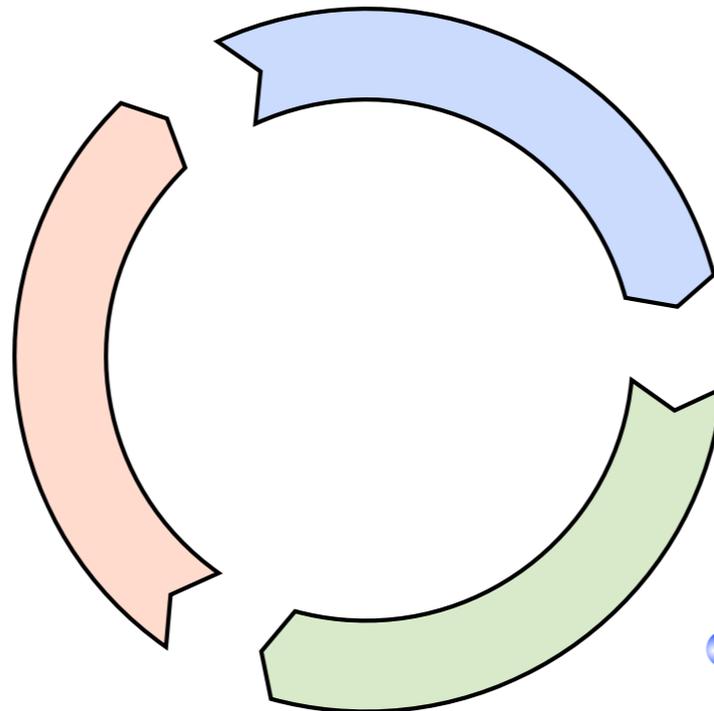
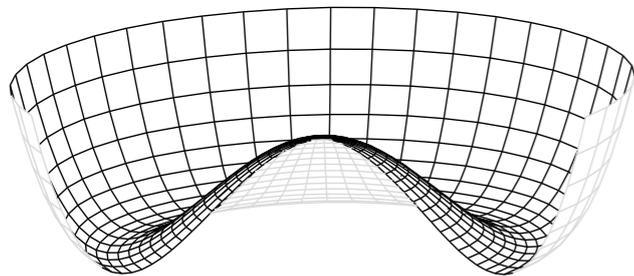
Endres Nature 2012



Experiments

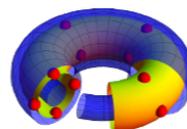


Stöferle PRL 2004

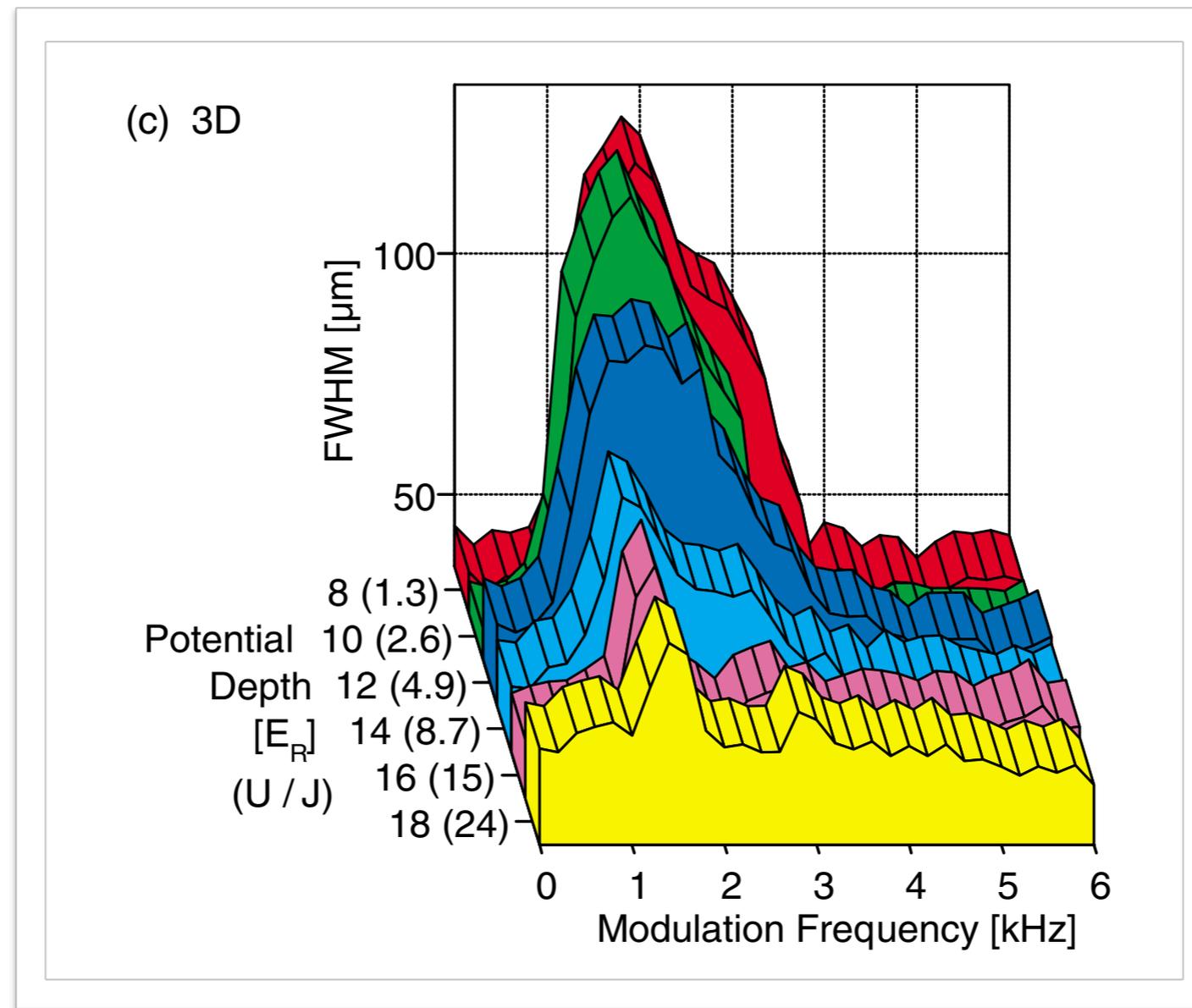


Microscopic description

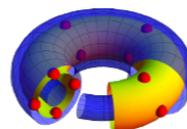
Interpretation in terms of a “Higgs” phenomena



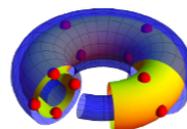
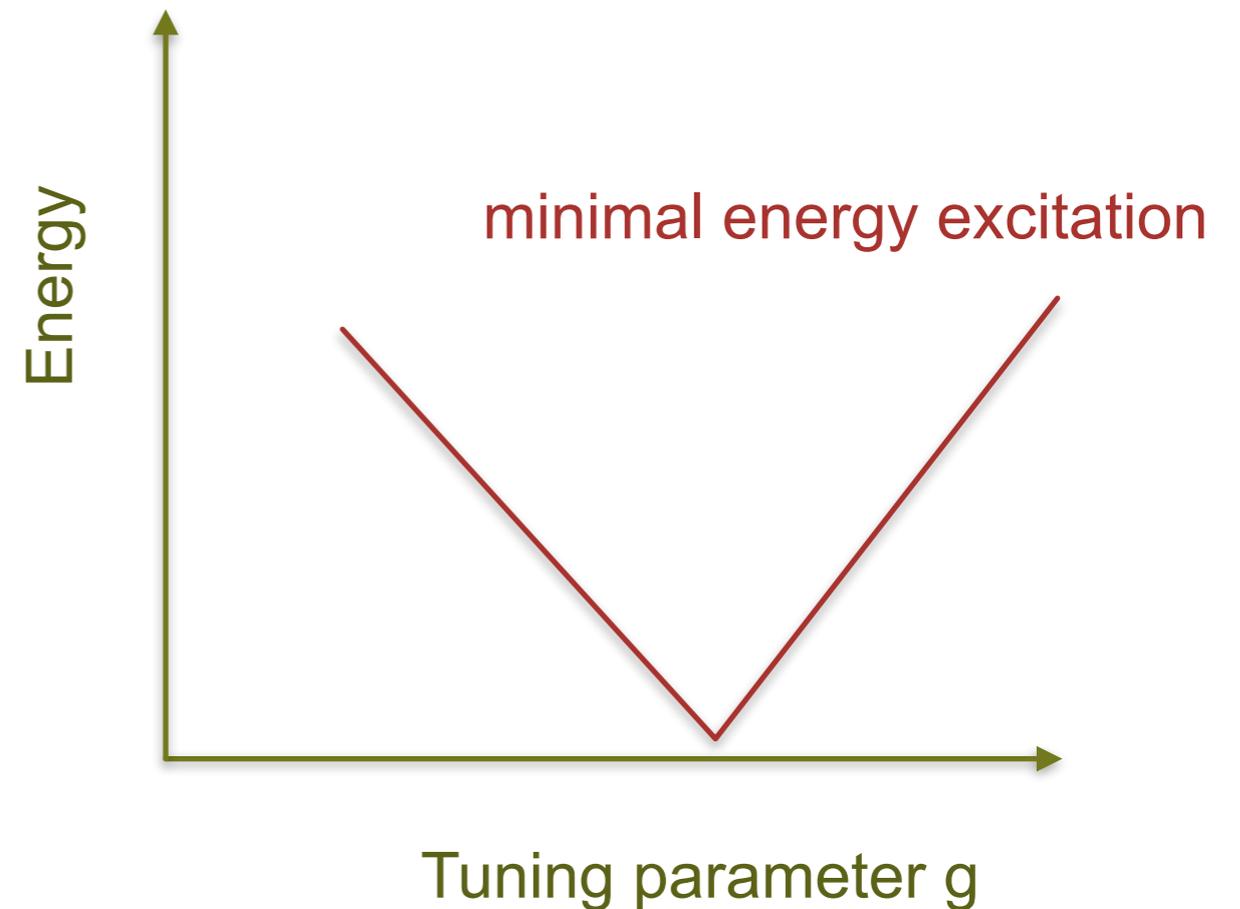
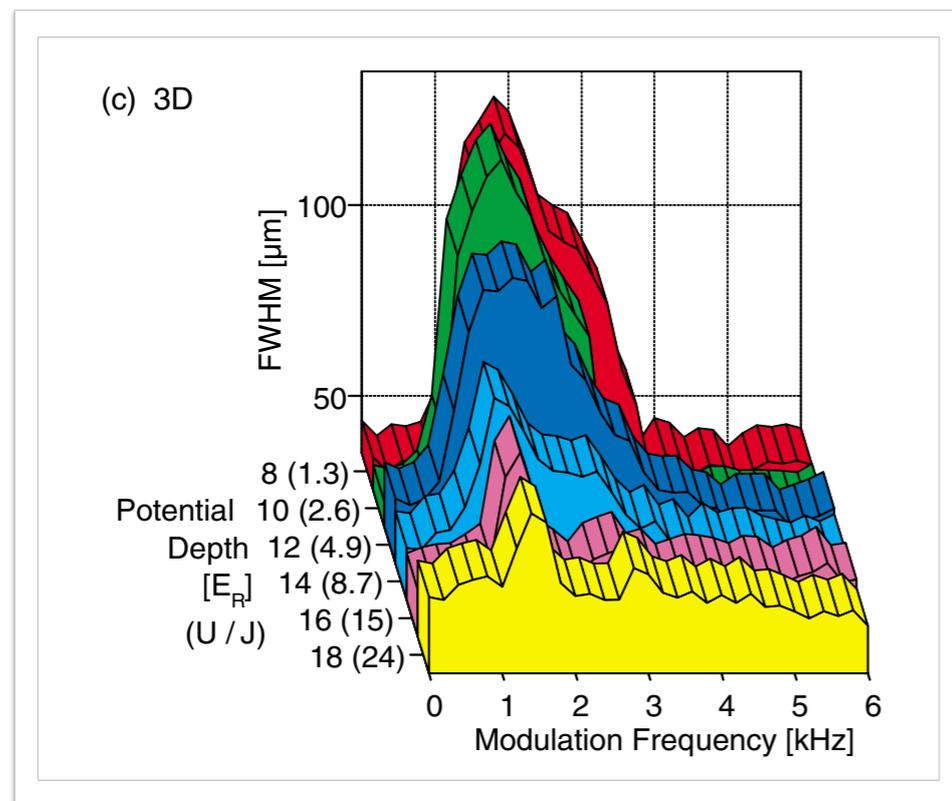
# The starting point



Stöferle PRL 2004



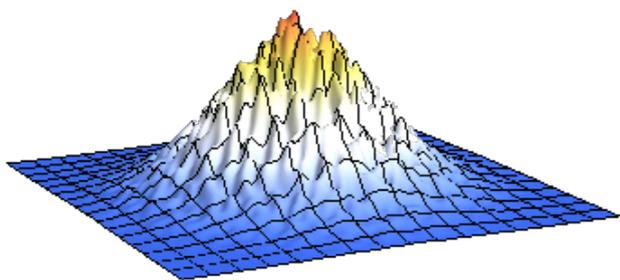
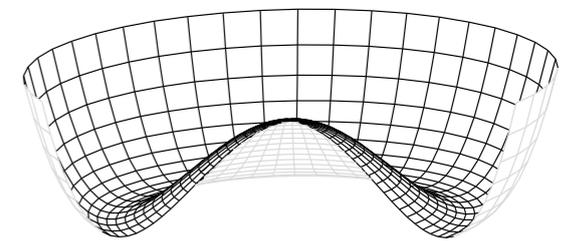
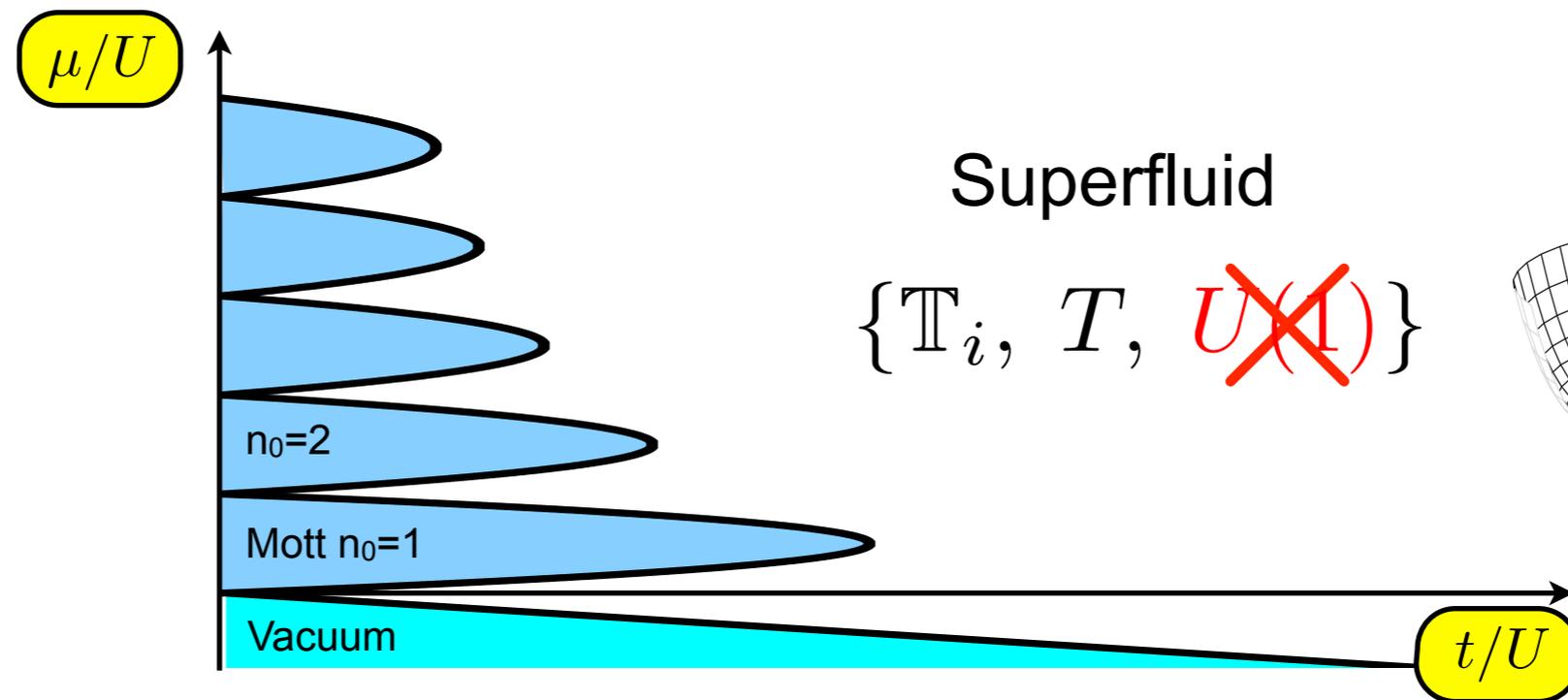
# The starting point



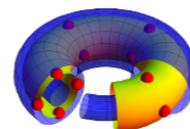
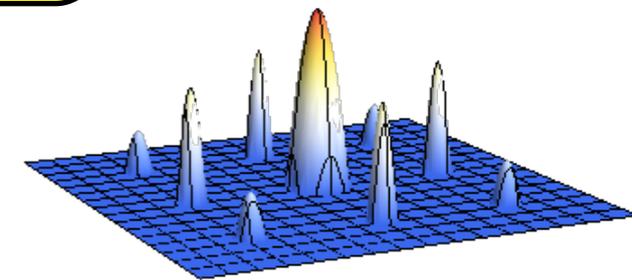
# The system: the Bose Hubbard model

$$H_{\text{BH}} = -t \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i a_i^\dagger a_i (a_i^\dagger a_i - 1) - \mu \sum_i a_i^\dagger a_i$$

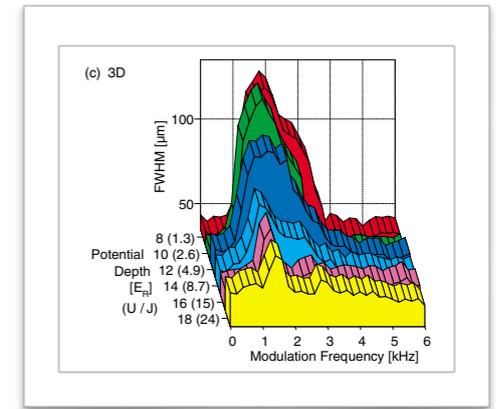
Symmetries:  $\{\mathbb{T}_i, T, U(1)\}$



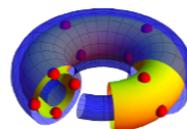
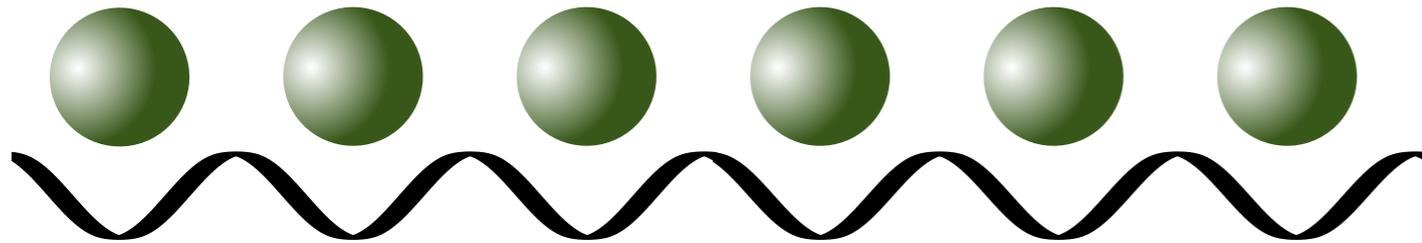
Fisher PRB 1989  
Greiner Nature 2002



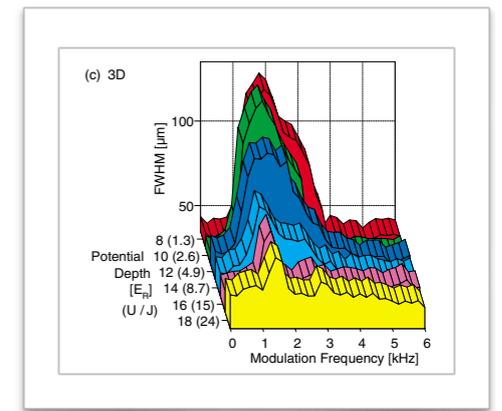
# Excitations: Microscopic description I



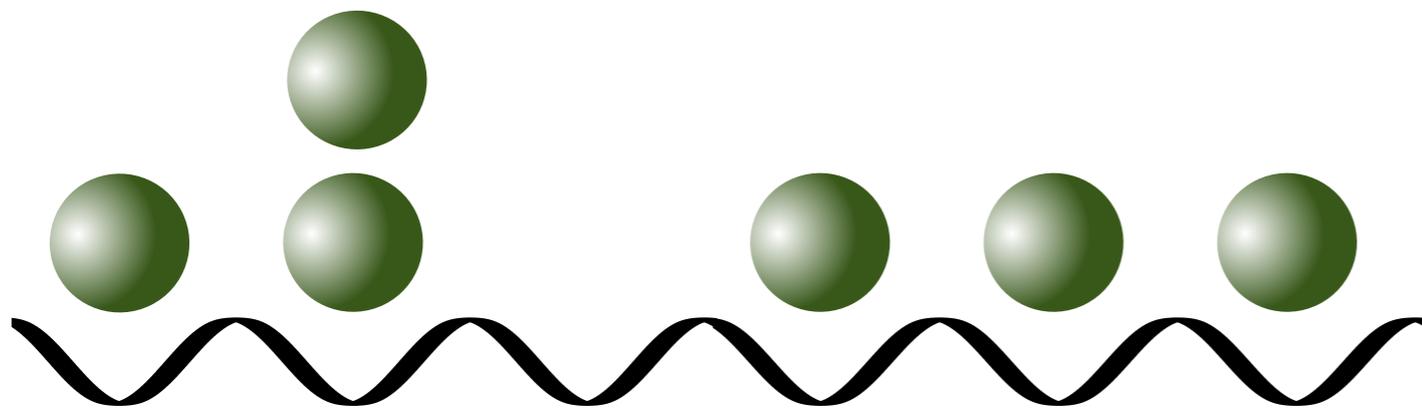
Stöferle PRL 2003



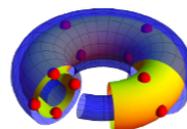
# Excitations: Microscopic description I



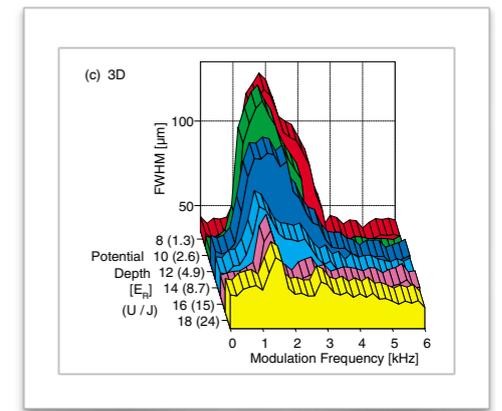
Stöferle PRL 2003



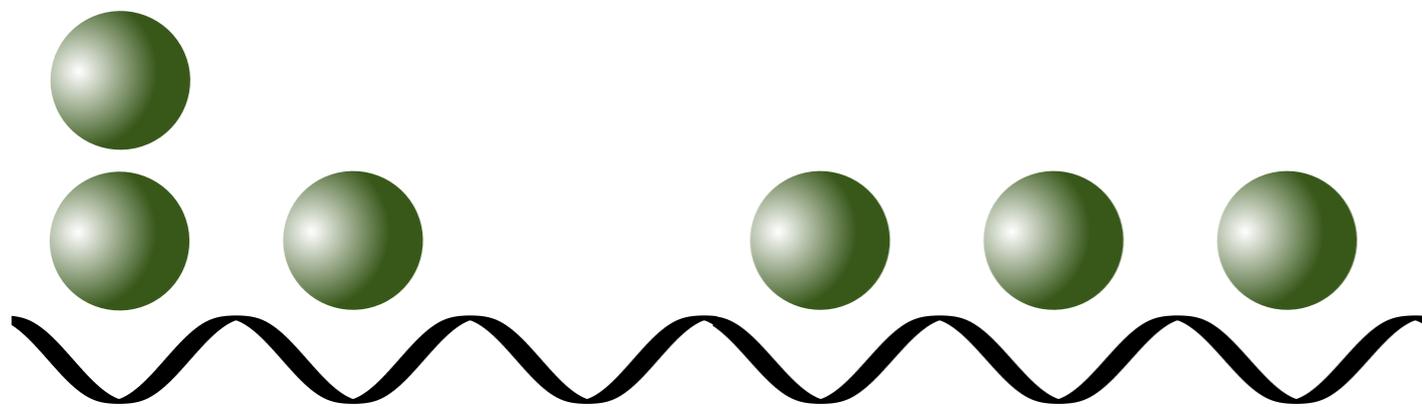
$$\hbar\omega(k) = \frac{U}{2}$$



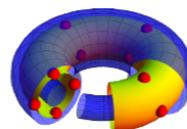
# Excitations: Microscopic description I



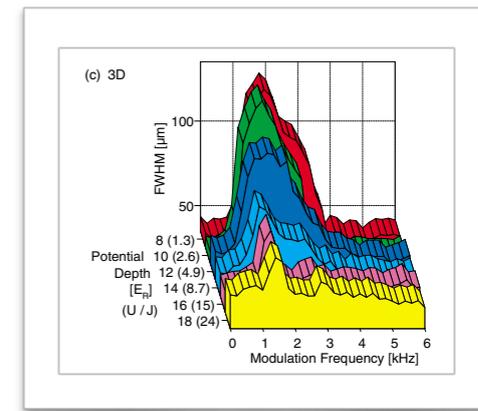
Stöferle PRL 2003



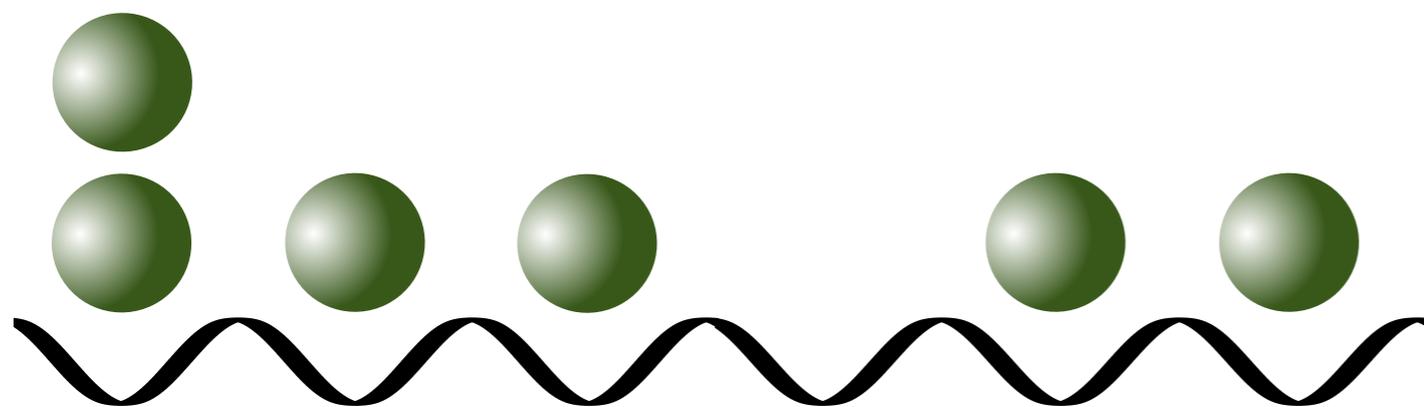
$$\hbar\omega(k) = \frac{U}{2} - 4t \cos(k)$$



# Excitations: Microscopic description I

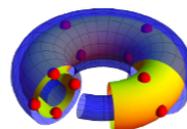


Stöferle PRL 2003

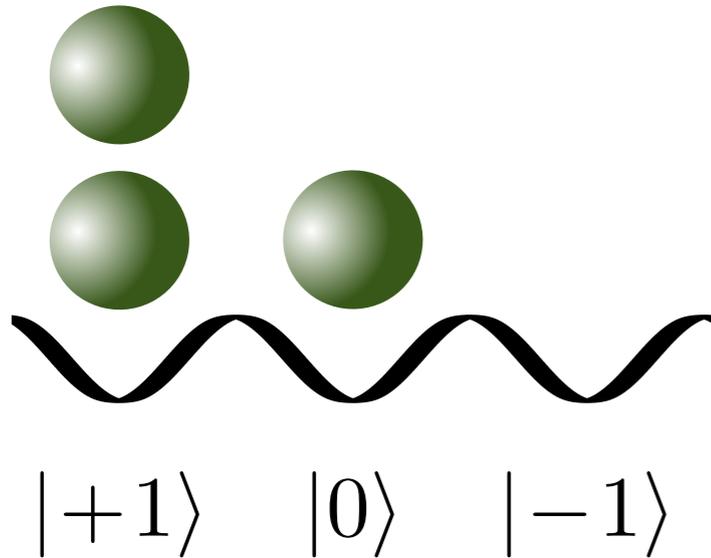


$$\hbar\omega(k) = \frac{U}{2} - 4t \cos(k)$$

$$\hbar\omega(k) = \frac{U}{2} - 2t \cos(k)$$



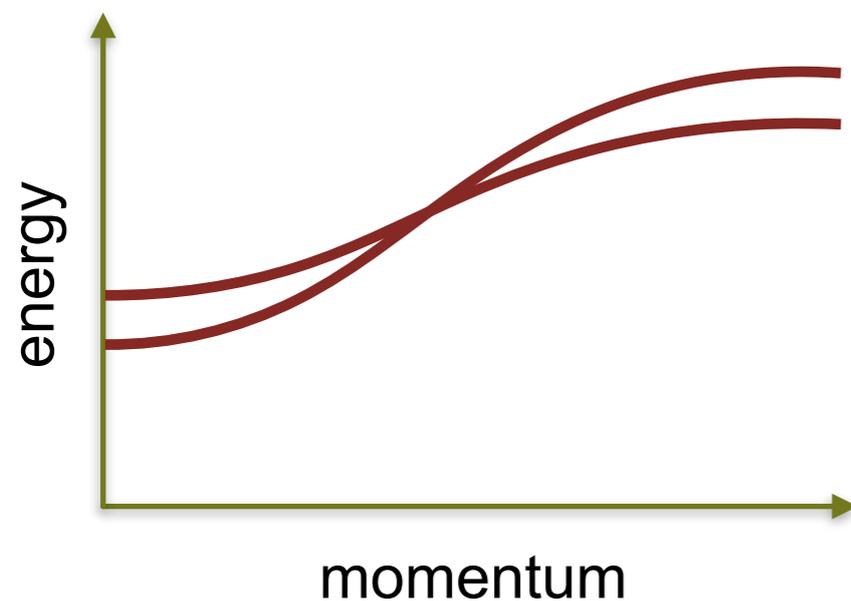
# Excitations: Microscopic description II



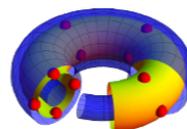
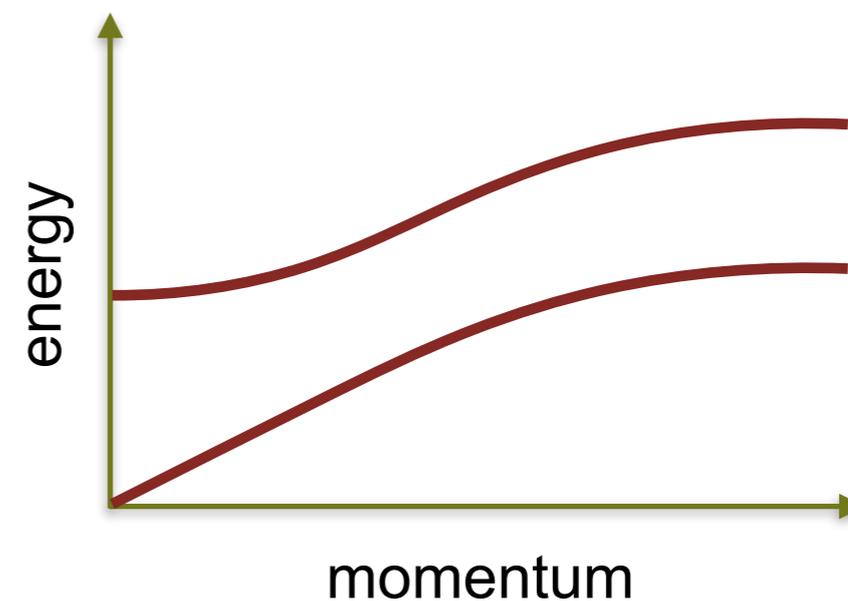
$$H_{\text{eff}} = -\frac{t}{2} \sum_{\langle i,j \rangle} S_i^+ S_j^- + \frac{U}{2} \sum_i (S_i^z)^2 - t\xi \sum_{\langle i,j \rangle} S_i^+ S_i S_j^- + \dots$$

Altman PRL 2002  
SDH PRB 2007

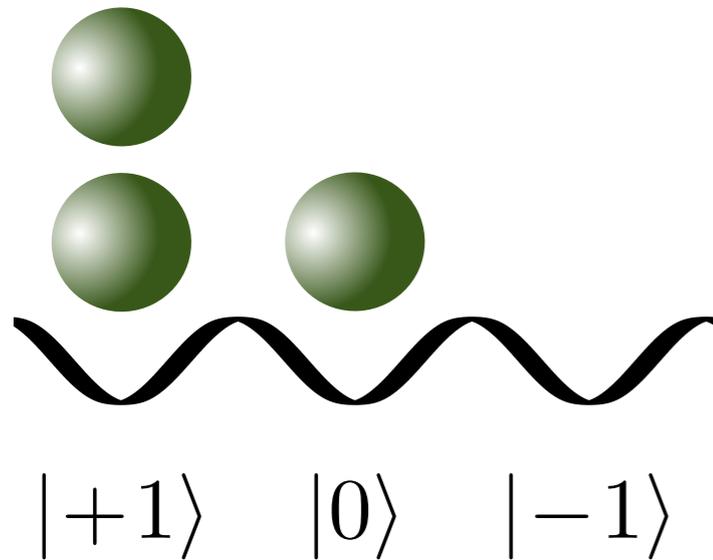
Mott insulator



Superfluid

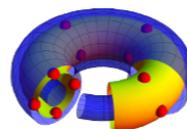
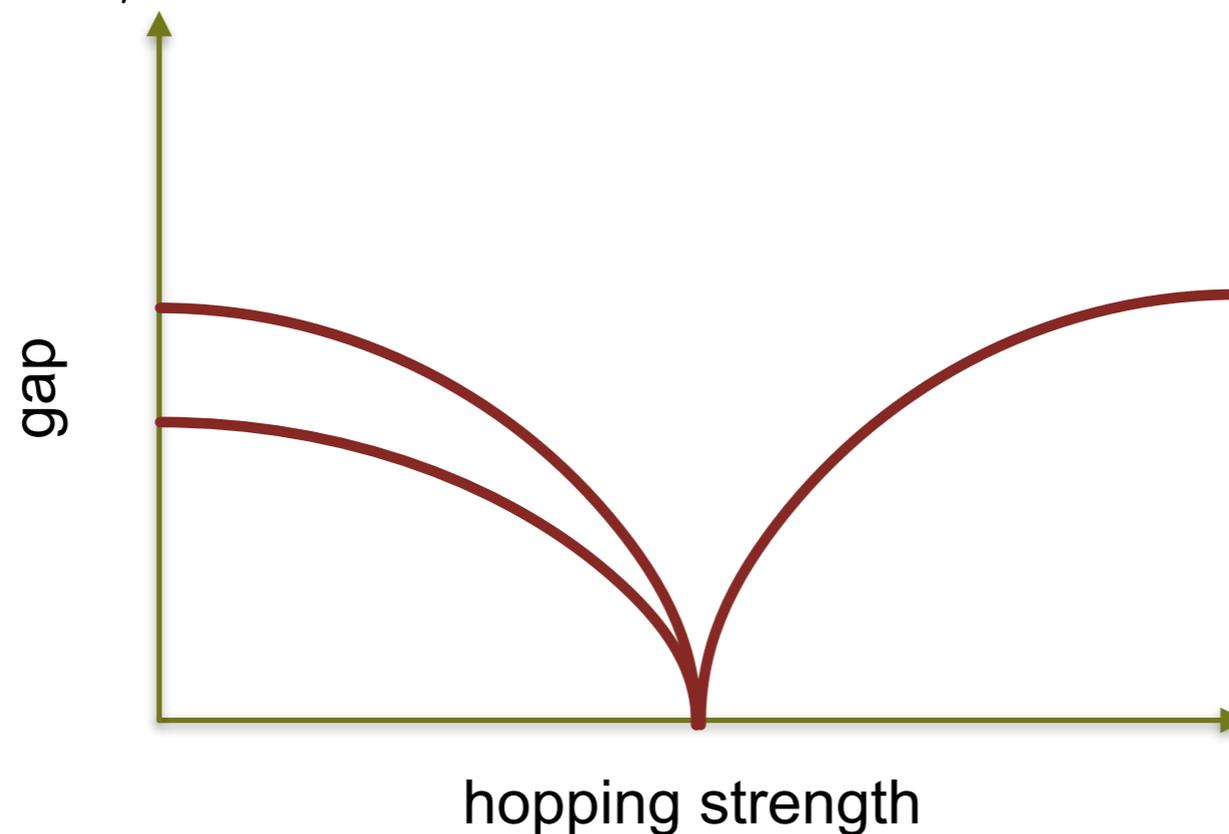


# Excitations: Microscopic description III

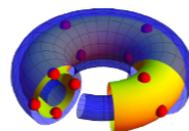
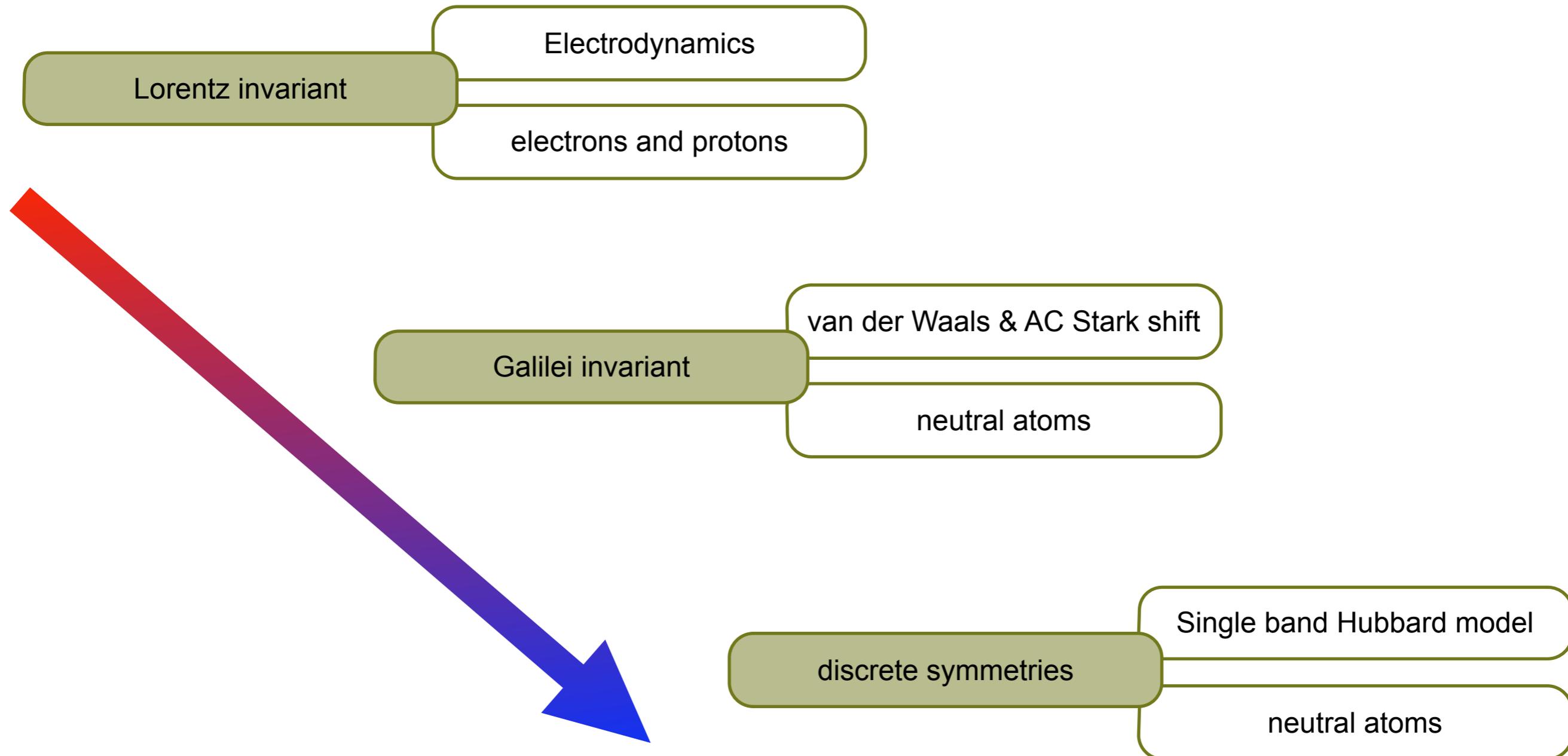


$$H_{\text{eff}} = -\frac{t}{2} \sum_{\langle i,j \rangle} S_i^+ S_j^- + \frac{U}{2} \sum_i (S_i^z)^2 - t\xi \sum_{\langle i,j \rangle} S_i^+ S_i S_j^- + \dots$$

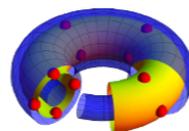
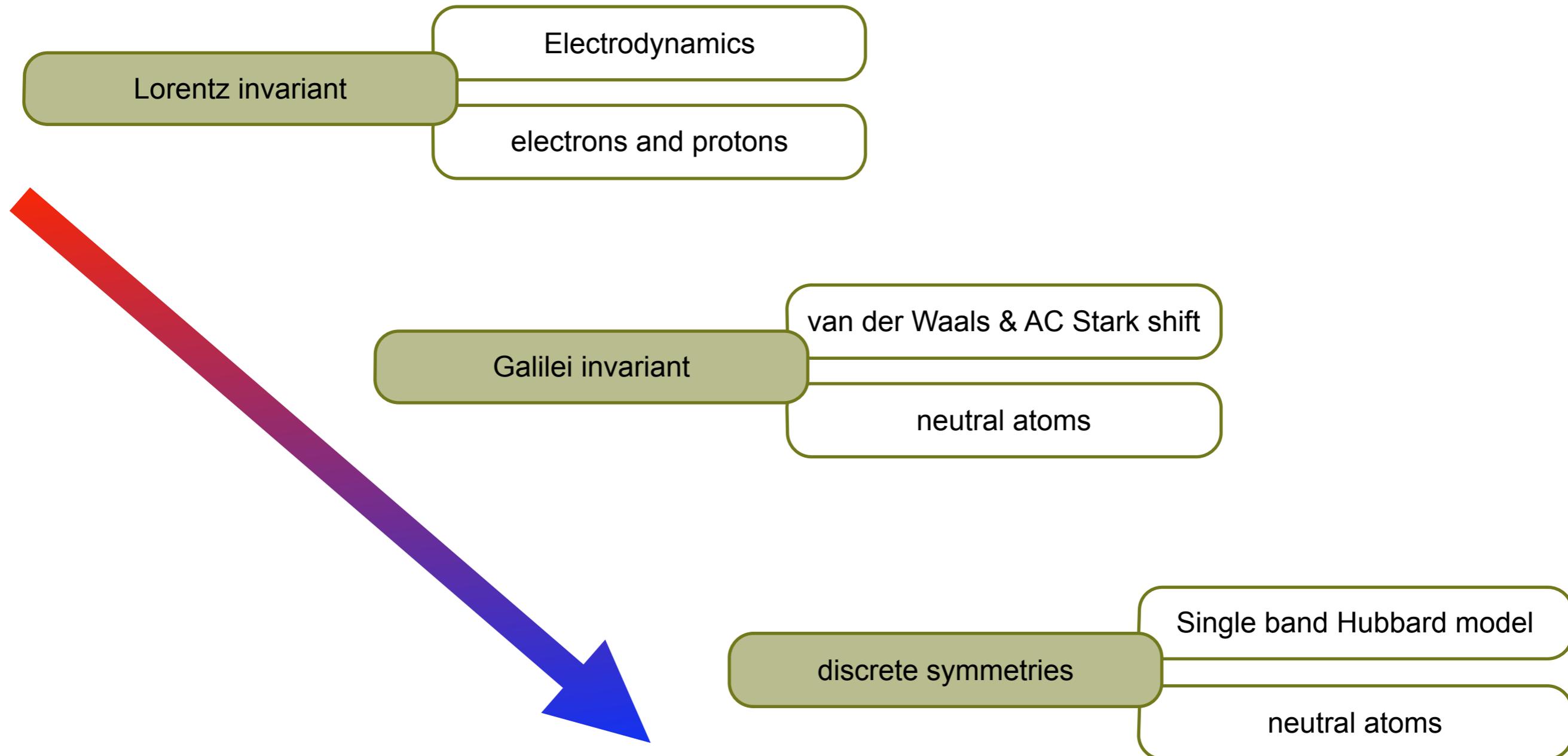
SDH PRB 2007  
SDH PRL 2008



# Symmetries and energy scales



# Symmetries and energy scales

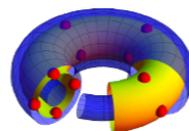
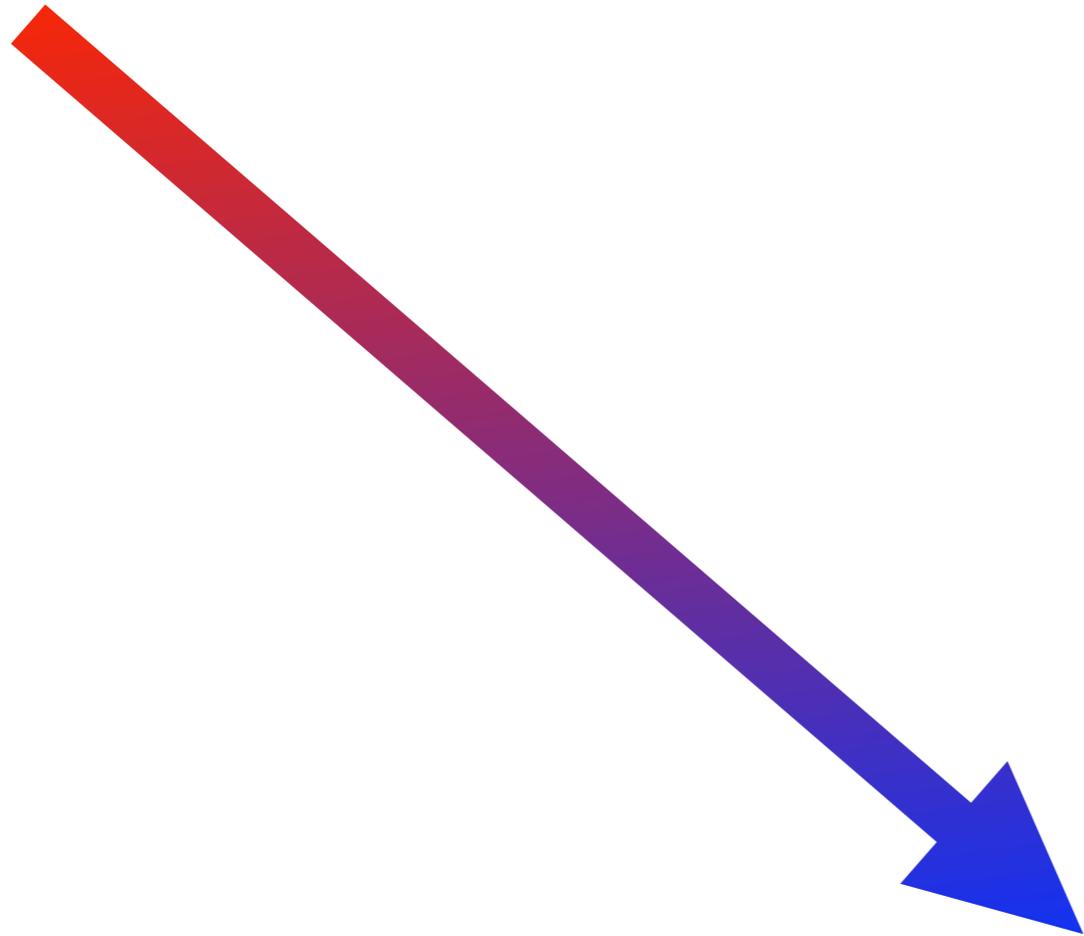


# Symmetries and energy scales

Single band Hubbard model

discrete symmetries

neutral atoms

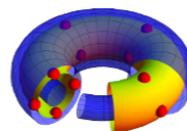
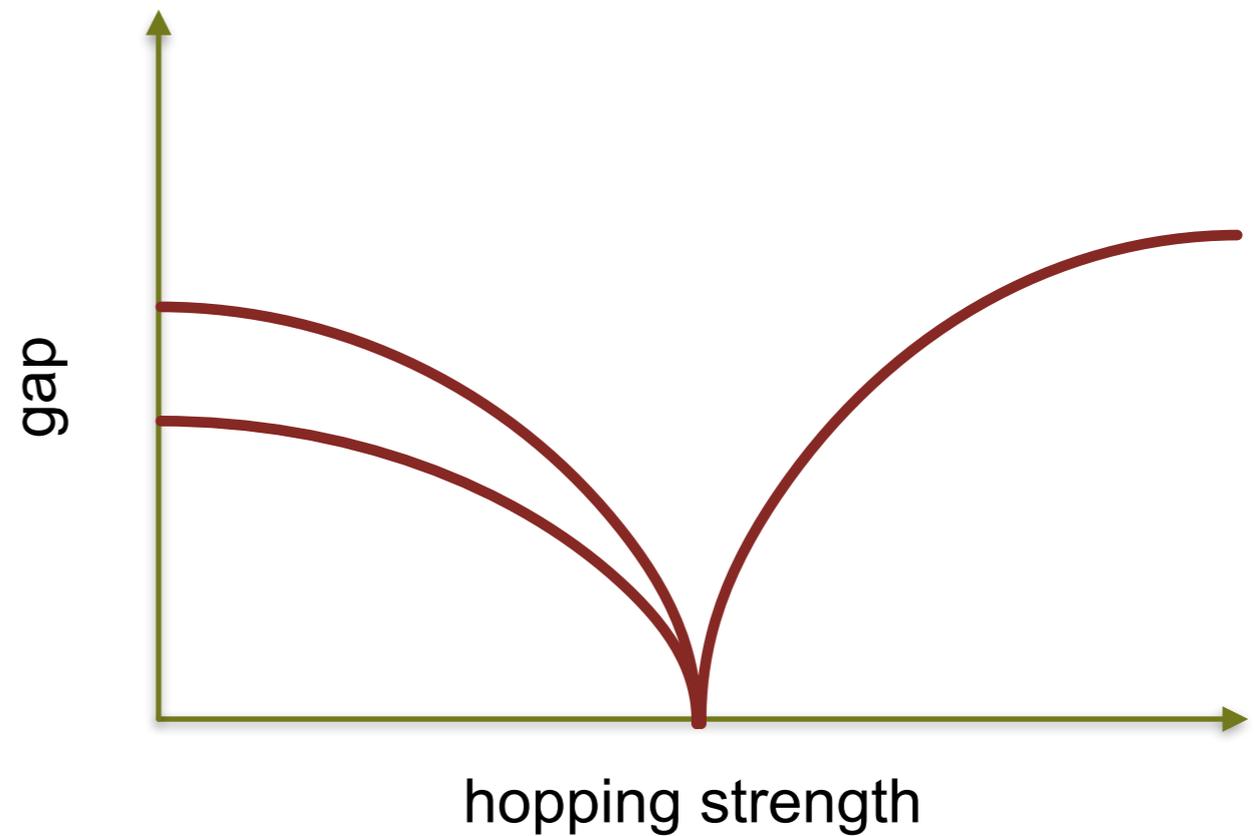


# Symmetries and energy scales

discrete symmetries

Single band Hubbard model

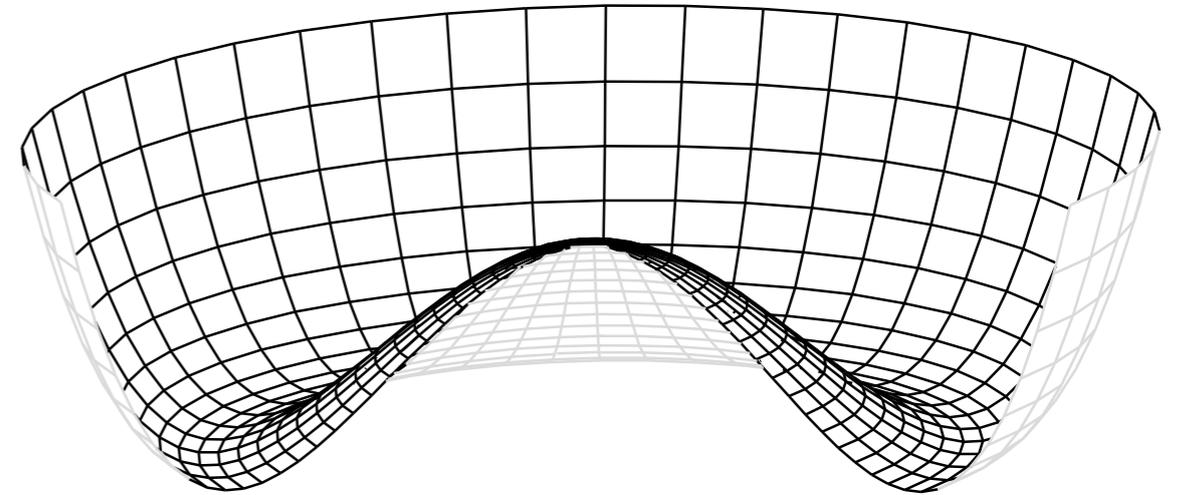
neutral atoms



# Excitation: Long-wavelength theory

$$\psi_i = \langle a_i \rangle$$

$$V(\psi_i) = r\psi_i^2 + \frac{u}{2}\psi_i^4$$

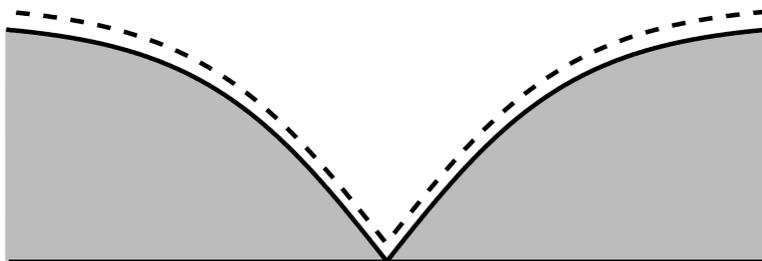


weakly interacting bosons: Gross-Pitaevskii

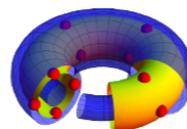
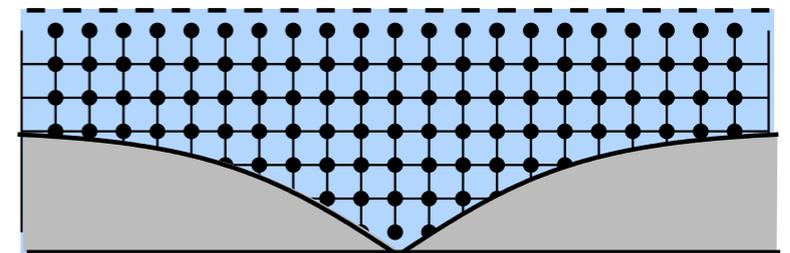
$$i\psi^* \partial_t \psi + (\nabla \psi)^2$$

Mott transition: relativistic critical theory

$$(\partial_t \psi)^2 + (\nabla \psi)^2$$

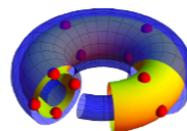
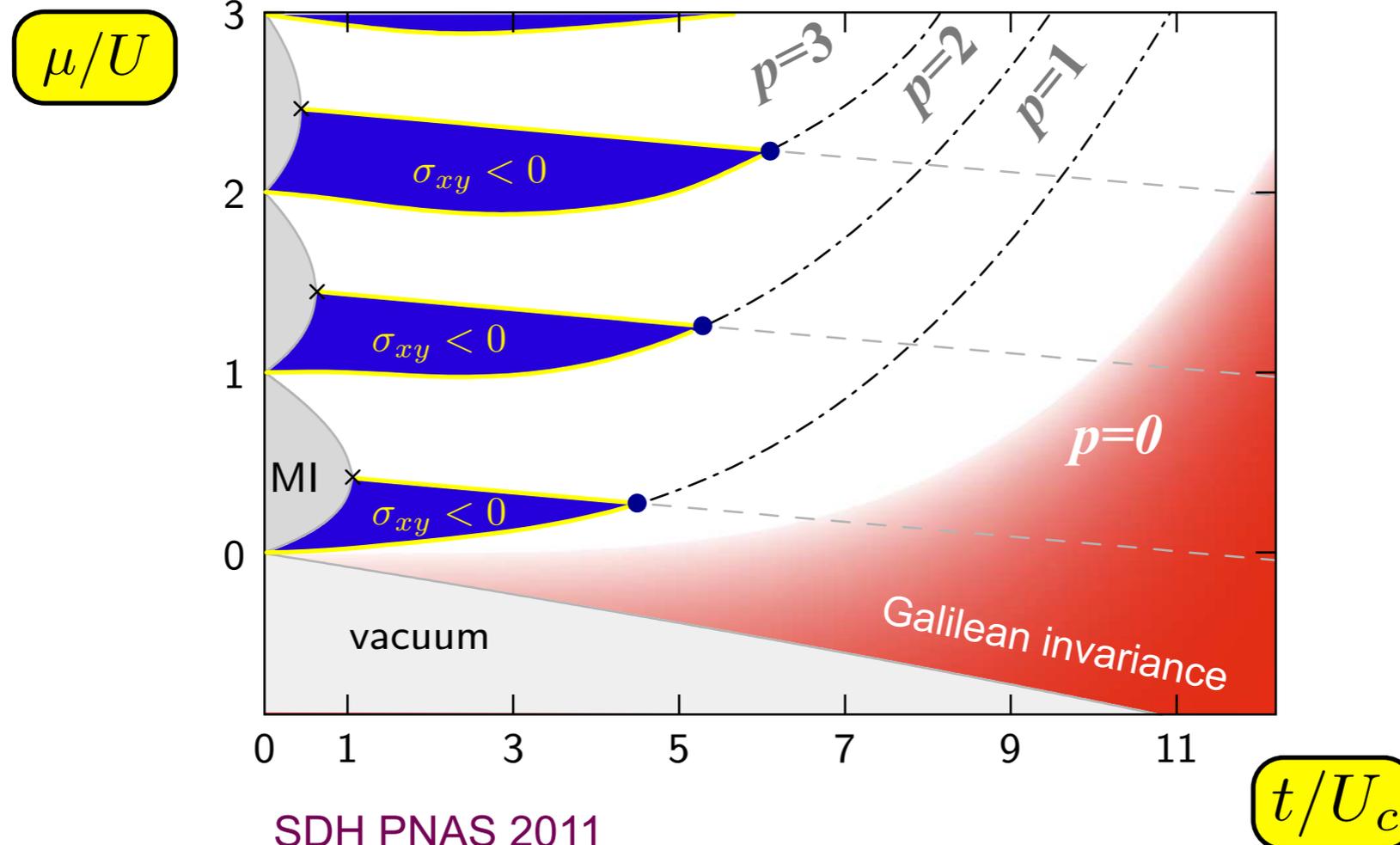


SDH PRL 2008  
Polkovnikov PRA 2005  
Varma J. Low Temp. Phys. 2002



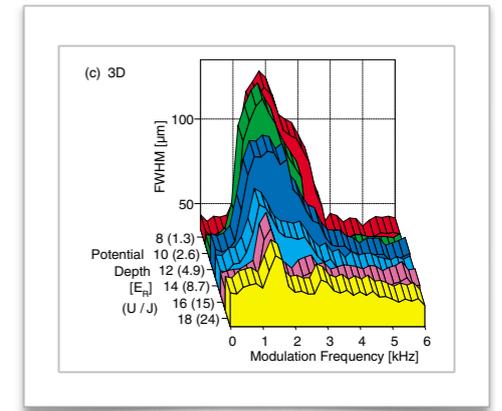
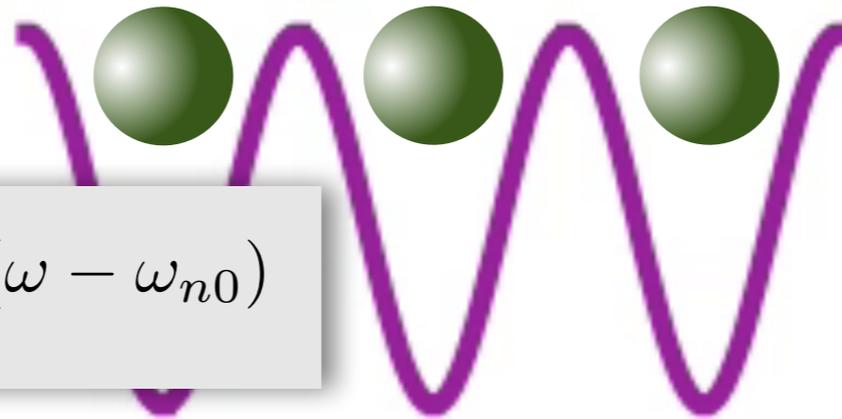
# Where is the emergent physics relativistic?

$$\sigma_{xy} = \rho + p$$

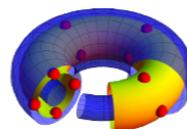
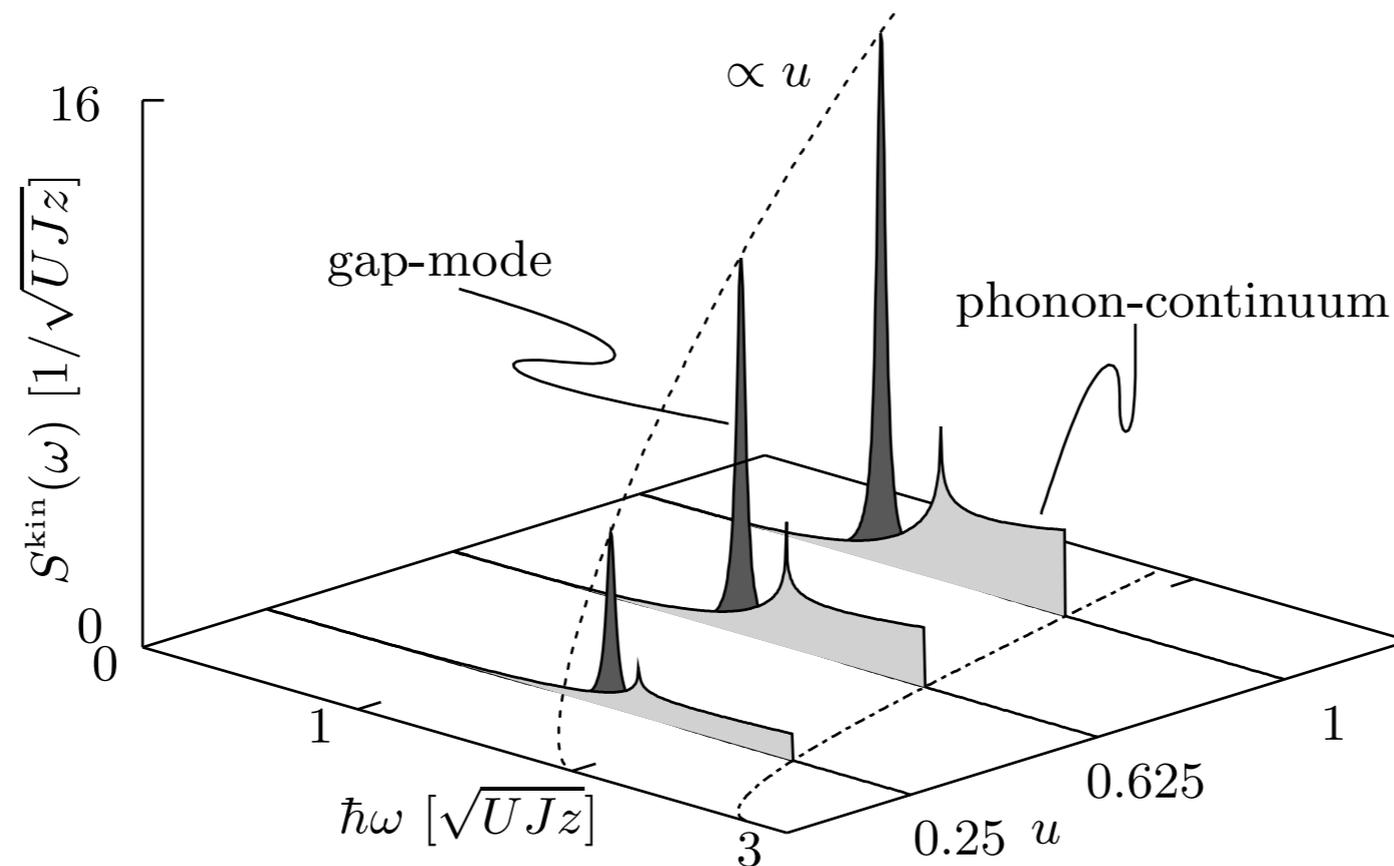


# Experiments?

$$S_{(x)}^{\text{kin}}(\omega) = \sum_n |\langle n | H_{(x)}^{\text{kin}} | 0 \rangle|^2 \delta(\omega - \omega_{n0})$$

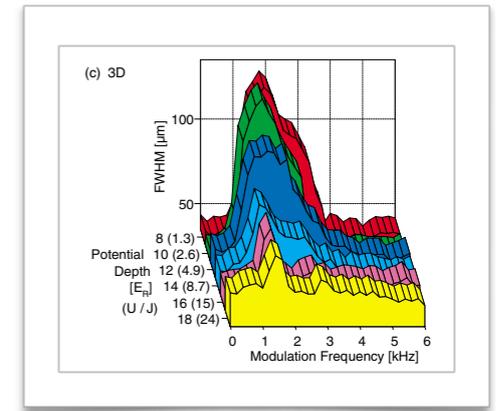
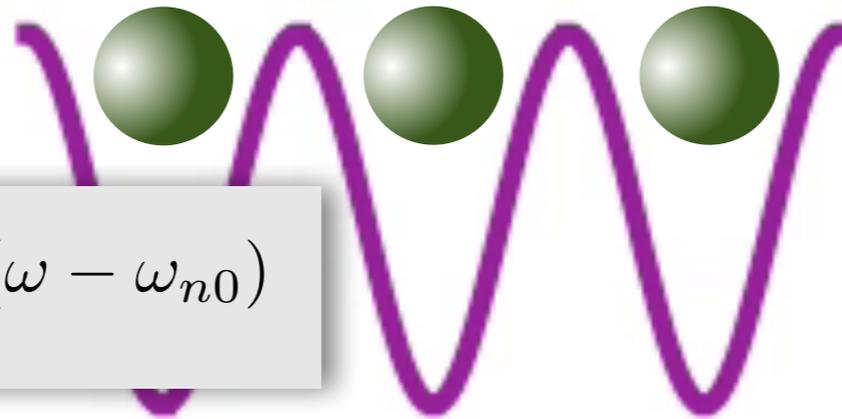


SDH PRB 2007  
SDH PRL 2008

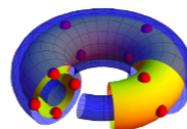
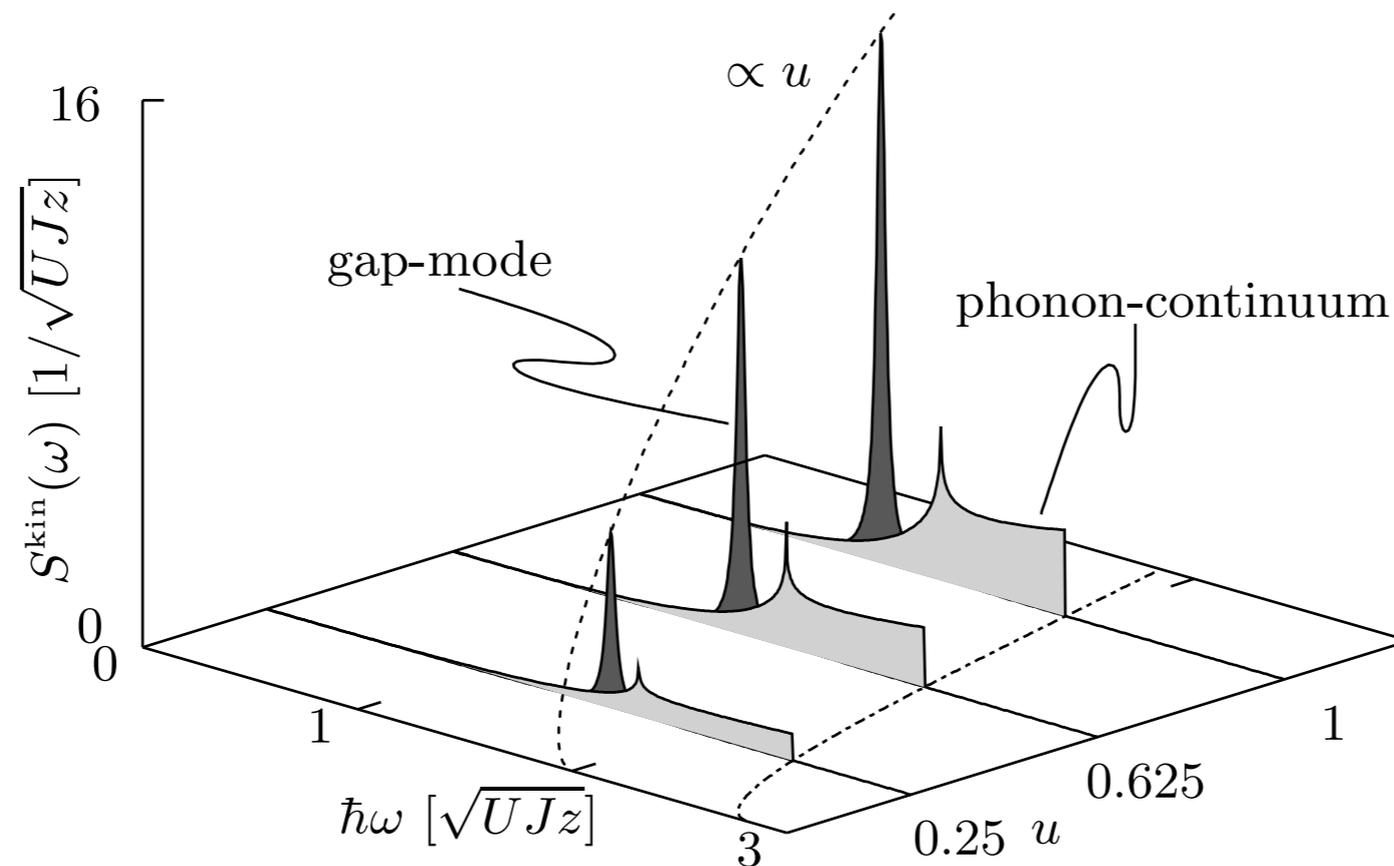


# Experiments?

$$S_{(x)}^{\text{kin}}(\omega) = \sum_n |\langle n | H_{(x)}^{\text{kin}} | 0 \rangle|^2 \delta(\omega - \omega_{n0})$$



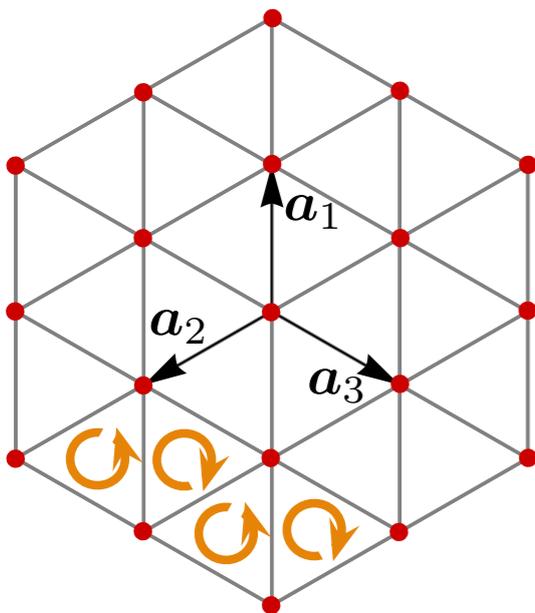
SDH PRB 2007  
SDH PRL 2008



# More models: more amplitude modes

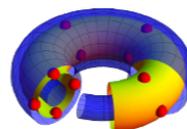
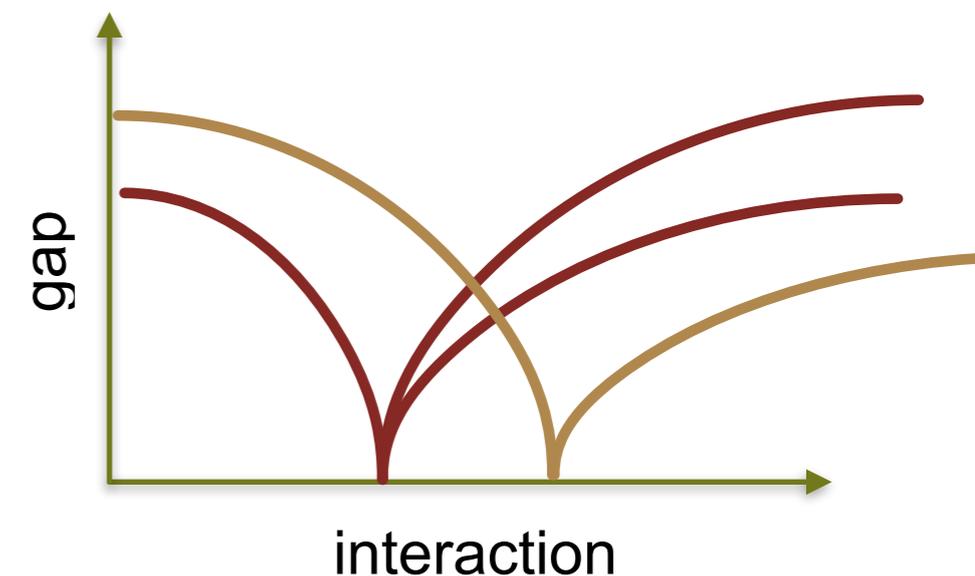
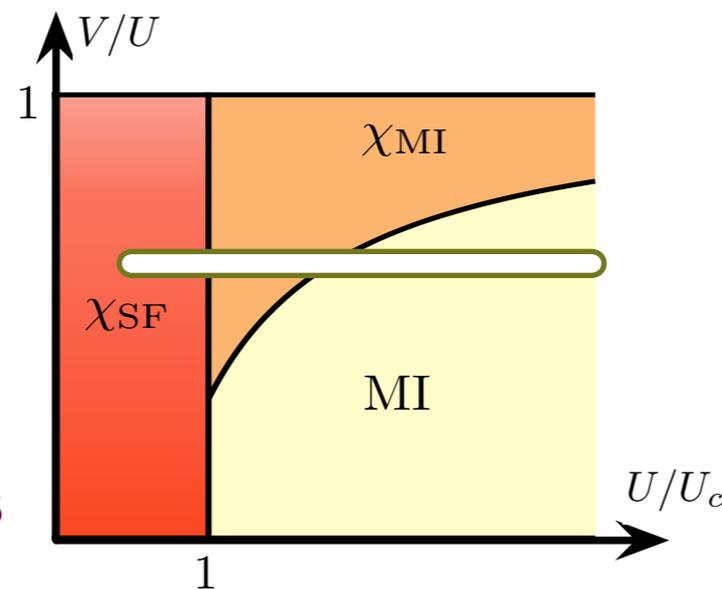
$$\{\mathbb{T}_i, T, U(1)\} \Rightarrow \{\mathbb{T}_i, T, \cancel{U(1)}\}$$

$$\{\mathbb{T}_i, T, U(1)\} \Rightarrow \{\mathbb{T}_i, \cancel{T}, U(1)\} \Rightarrow \{\mathbb{T}_i, \cancel{T}, \cancel{U(1)}\}$$



Rüegg arXiv 2013

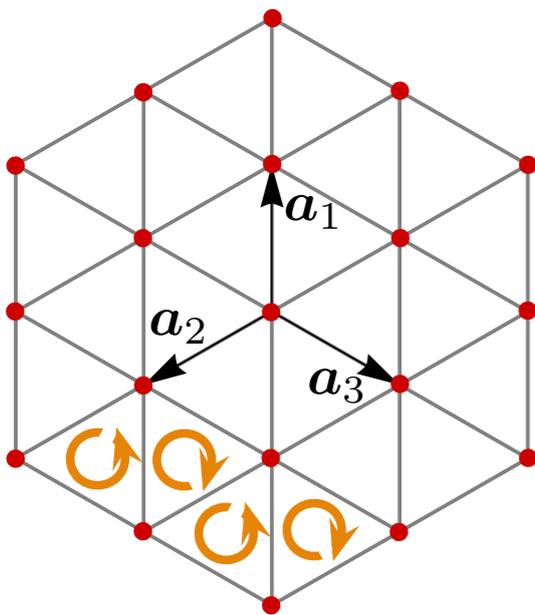
$$H = |t| \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i (\delta n_i)^2 + V \sum_{\langle i,j \rangle} \delta n_i \delta n_j$$



# More models: more amplitude modes

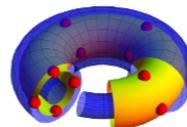
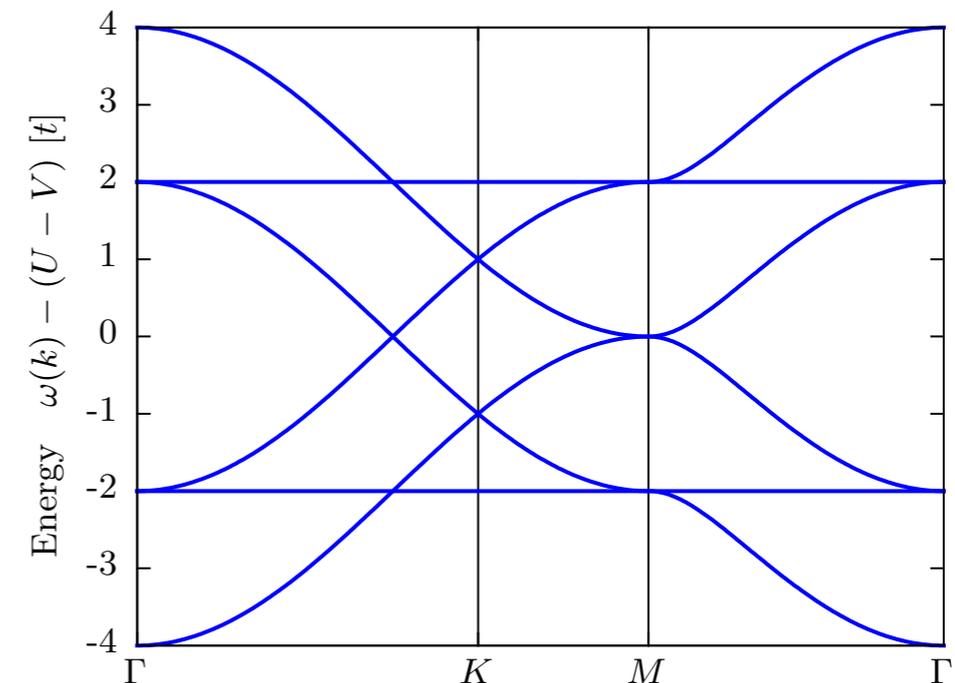
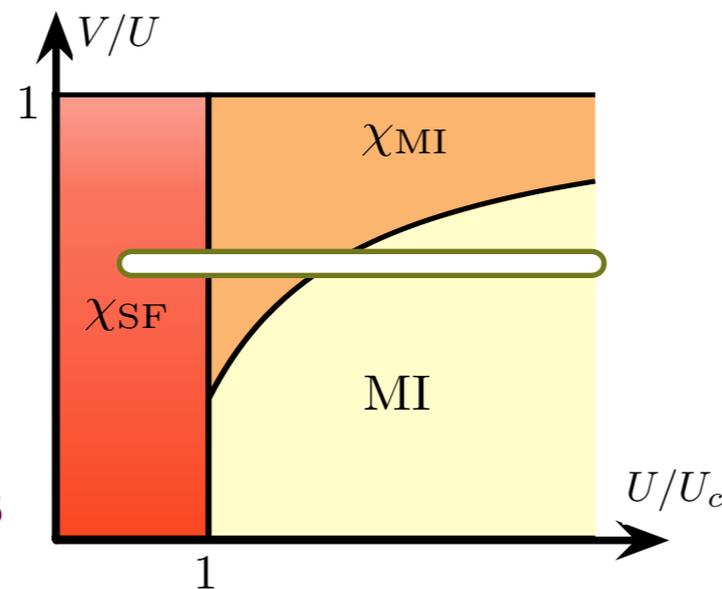
$$\{\mathbb{T}_i, T, U(1)\} \Rightarrow \{\mathbb{T}_i, T, \cancel{U(1)}\}$$

$$\{\mathbb{T}_i, T, U(1)\} \Rightarrow \{\mathbb{T}_i, \cancel{T}, U(1)\} \Rightarrow \{\mathbb{T}_i, \cancel{T}, \cancel{U(1)}\}$$



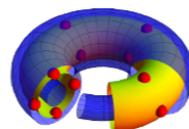
Rüegg arXiv 2013

$$H = |t| \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i (\delta n_i)^2 + V \sum_{\langle i,j \rangle} \delta n_i \delta n_j$$



# Conclusions

- Cold atoms host and **amplitude mode** close to the **superfluid to Mott transition**.
- One can understand this mode in the framework of an **emergent “Higgs” particle**.
- By (time-) modulating the distance to the critical point one should be able to **excite this mode**.
- The **emergent Lorentz invariance** has a profound impact on the low energy behaviour of the system.



## (Once) Open questions

- **Stability** of the cold-atoms Higgs mode
- Can one **measure** this mode unambiguously
- Hall conductivity in the presence of more broken symmetries: super-solids
- **Chiral Mott insulator**: good microscopic understanding of the mode softening

