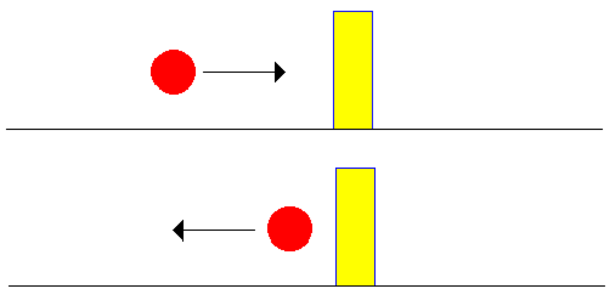
Transmission and reflection properties of Nambu-Goldstone modes

Yusuke Kato (Dept. Basic Science, The University of Tokyo)

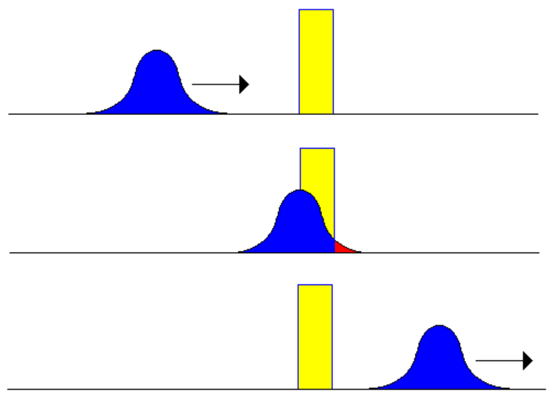
collaborator: Shohei Watabe（Dept. Physics, Tokyo University of Science)

Yoji Ohashi（Dept. Physics, Keio University）

Perfect reflection？



Perfect transmission ？



Nambu-Goldstone modes in condensed matter physics

From "Basic notions of condensed matter physics "(P. W. Anderson)

TABLE I: MASTER TABLE OF BROKEN-SYMMETRY PHENOMENA

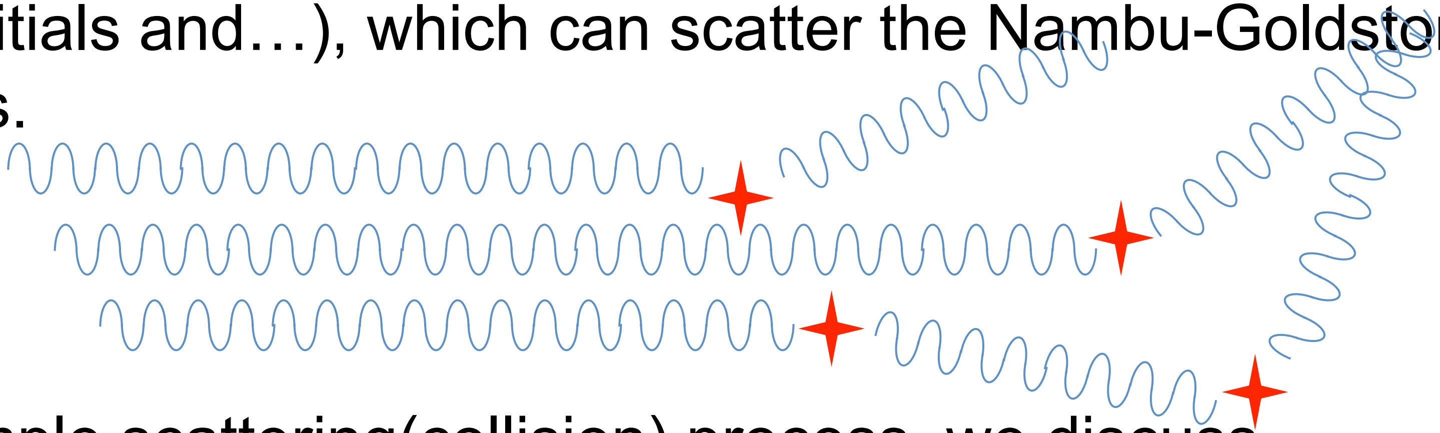
Phenomenon	High Phase	Low Phase	Order Parameter	(constant of motion)	Order-Parameter Dimensionality $T\to 0$	T_c	Common Transition Type (Can Always Be First Order)	"Goldstone Bosons" (or "Higgs" Bosons)	Fluctuations	Collective Hydrodynamic Modes	Generalized Rigidity Phenomenon	Long-Range Forces Due to General . Rigidity	Singularities
Ferroelectricity (Pyroelectricity)	Non-Polar crystal	Polar crystal	$\vec{P}$	(no)	1	1 or 3	2nd or 1st nearly 2nd	no (optical phonons)	Soft Modes	No	Ferroelectric hysteresis	No	Domain walls (thin)
Ferromagnetism	Paramagnet	Ferromagnet	$\vec{M}$	(yes)	1, often ≈ 3	1 or 3	2nd (time reversal)	Spin waves, one branch $\omega\propto k^2$ (+const)	Spin waves	No ?	Permanent magnetism: hysteresis	Suhl-Nakamura	Domain walls (very mobile, ≈ 3 dim.)
Antiferromagnetism	Paramagnet	Antiferromagnet	$\vec{M}_{sublattice}$	(no)	1, often ≈ 3	1 or 3	2nd (or first)	Spin waves, 2 branches $\omega\propto\sqrt{k^2+\text{const}}$ ("Fermions" in metal case?)	Spin waves (diffusion?)	No?	subtle effects in A.F. resonance	Suhl-Nakamura	Domain walls
Superconductivity	Normal Metal	Superconductor	$\langle\psi_\sigma^*\psi_{-\sigma}^*\rangle = Fe^{i\varphi}$	(no)	2	2	2nd (Gauge; no 3rd order terms)	no (plasmons)	diffusive fluctuations of gap	Mostly not	super conductivity	No: Penetration depth	Flux lines (or normal domains in Type I)
He II	Normal liquid	Superfluid	$\langle\psi\rangle$	(no)	2	2	2nd (Gauge; no 3rd order terms)	phonons (1 branch)	diffusive fluctuations of $\langle\psi\rangle$	2nd sound, especially	superfluidity	Yes, vortex lines unscreened	vortex lines
Crystal	liquid	solid	ρ_G, all G on recip. lattice	(no)	3(2 orient, 1 phase) at least	3	1st	yes: 3 kinds 2 transverse 1 longitudinal	phonons	2nd sound in solid, etc.	rigidity	Yes: elasticity effects	dislocations, grain boundaries, points (vacancies, interstitials)

Recent progress in Nambu-Goldstone modes

- Counting rule, dispersion in non-relativistic case:
  Nambu(04)
  Watanabe-Brauner(11),
  Watanabe-Murayama(12), Hidaka(12),
- Generalization (in the presence of topological defects): Nitta-Kobayashi-Takahashi (13,14)

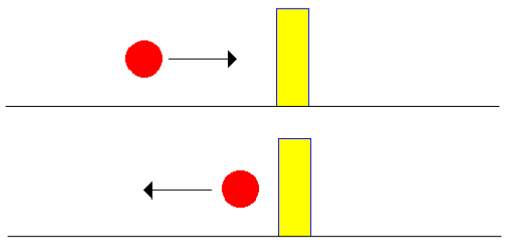
Tunneling phenomena

In real materials, there are *extrinsic* objects (impurities, interstitials and…), which can scatter the Nambu-Goldstone modes.

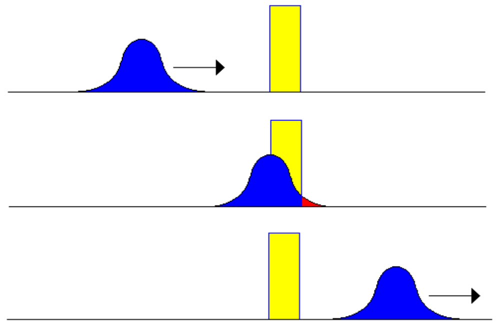


As a simple scattering(collision) process, we discuss transmission-reflection property of collective modes against a potential barrier in superfluid, Heisenberg magnets and phonon.

Classical mechanics



Quantum mechanics



http://abyss.uoregon.edu/~js/cosmo/lectures/lec08.html

Cf. Tunneling of Tomonaga-Luttinger liquids (whose excitation has *linear* dispersion)

Kane-Fisher (1992), Furusaki-Nagaosa(1992)

Transmission-reflection property of Bogoliubov mode in weakly interacting Bosons

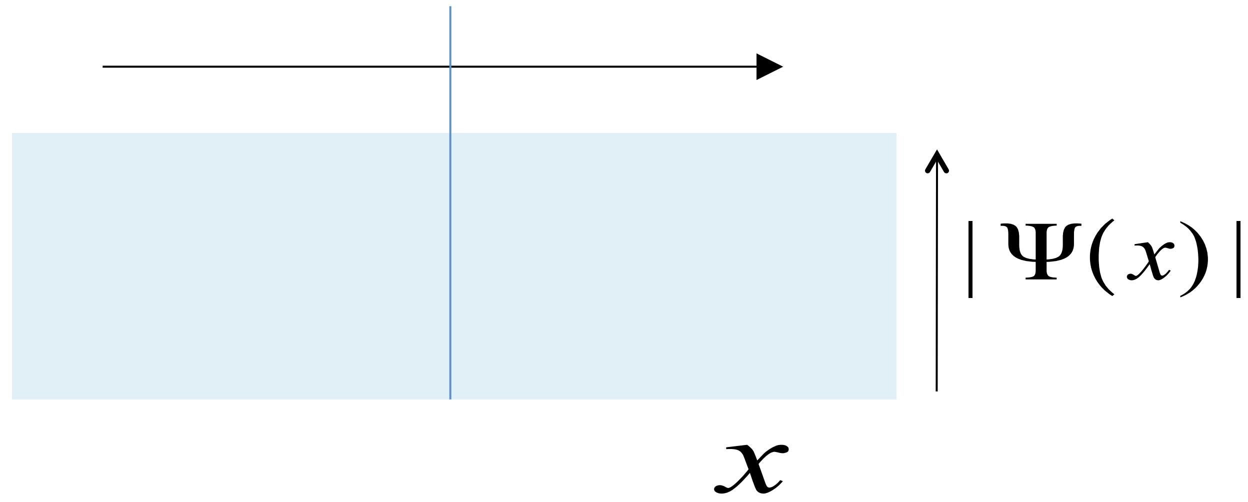
Kovrhizhin2001, Kagan et al 2003

Gross-Pitaevskii equation for condensate wave function Ψ

$$(H_0 + g|\Psi|^2)\Psi = 0, \quad H_0 = -\frac{\hbar^2\nabla^2}{2m} - \mu$$

Bogoliubov equation for wave functions of excitations u, v

$$\begin{pmatrix} H_0 + 2g|\Psi|^2 & -g(\Psi)^2 \\ g(\Psi^*)^2 & -H_0 - 2g|\Psi|^2 \end{pmatrix} \begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = \epsilon \begin{pmatrix} u(x) \\ v(x) \end{pmatrix}$$



Transmission-reflection property of Bogoliubov mode in weakly interacting Bosons

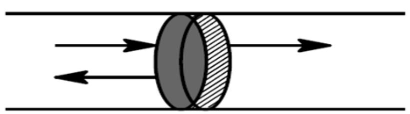
Kovrhizhin2001,
Kagan et al 2003
Danshita et al. 05, 06
Danshita-Tsuchiya. 07
Kato et al. 08
Ohashi-Tsuchiya 08
Tsuchiya-Ohashi 08
Takahashi-Kato 09
Watabe et al. 11, 12

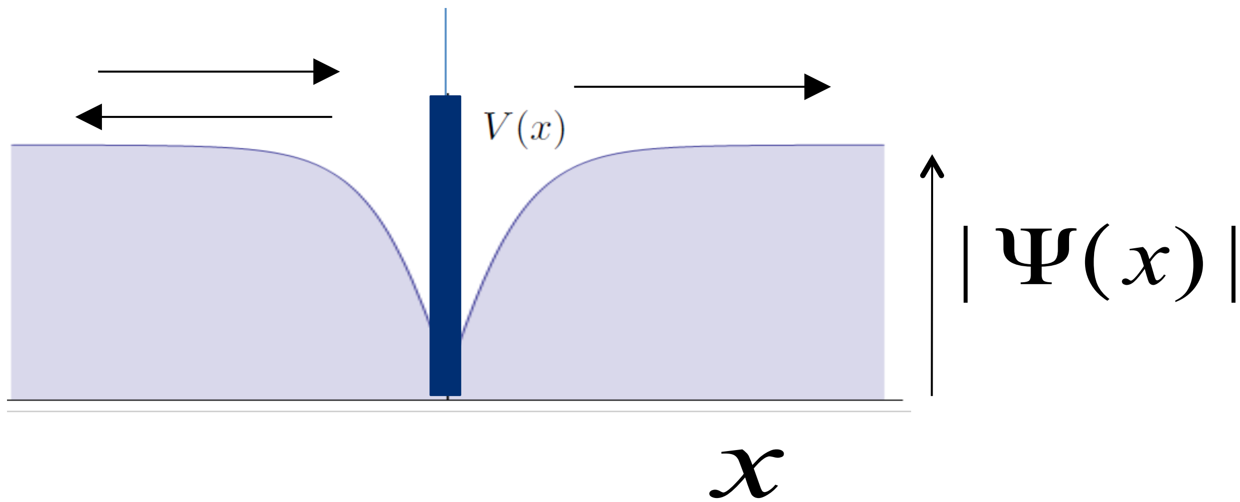
Gross-Pitaevskii equation for condensate wave function Ψ

$$(H_0 + g|\Psi(x)|^2 + V(x))\Psi(x) = 0,\ H_0 = -\frac{\hbar^2\nabla^2}{2m} - \mu$$

Bogoliubov equation for wave function of excitations u, v

$$\begin{pmatrix} H_0 + 2g|\Psi(x)|^2 + V(x) & -g(\Psi(x))^2 \\ g(\Psi^*(x))^2 & -H_0 - 2g|\Psi(x)|^2 - V(x) \end{pmatrix} \begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = \epsilon \begin{pmatrix} u(x) \\ v(x) \end{pmatrix}$$



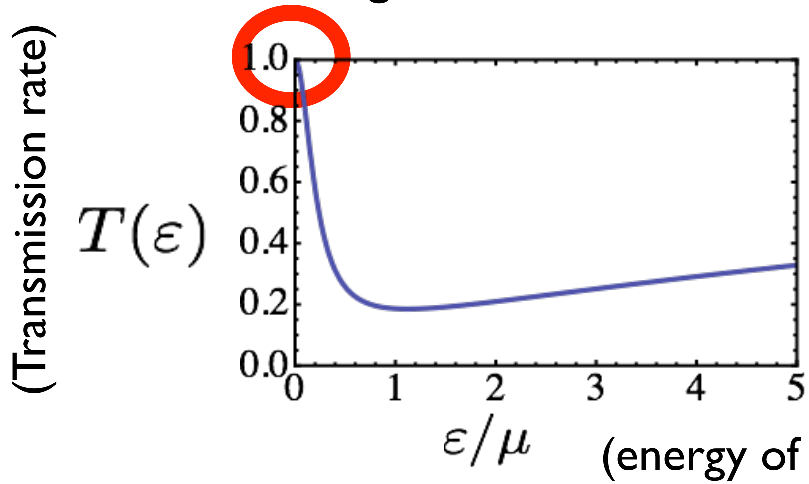


Anomalous Tunneling effect (Kovrizhin 2001,Kagan et al 2003)

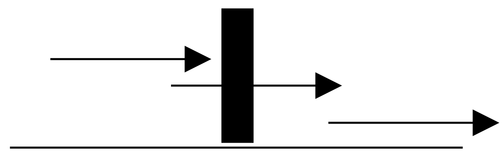
= Perfect transmission in low energy limit

Danshita et al. 05, 06
Danshita-Tsuchiya. 07
Kato et al. 08
Ohashi-Tsuchiya 08
Tsuchiya-Ohashi 08
Takahashi-Kato 09
Watabe et al. 11, 12

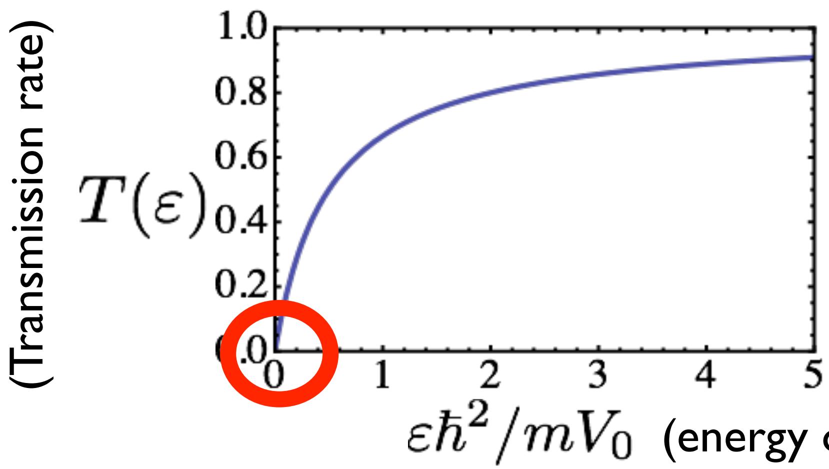
Bogoliubov exicitation



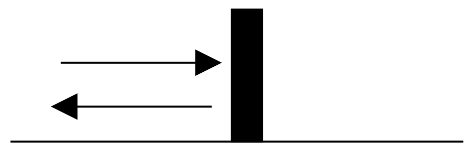
$(V(x) = V_0\delta(x))$



Single particle in quantum mechanics



$(V(x) = V_0\delta(x))$



Key to proof "**Fetter's solution**" (Fetter1972)

When $\varepsilon \to 0$ $\begin{pmatrix} u(\boldsymbol{r}) \\ v(\boldsymbol{r}) \end{pmatrix} = \begin{pmatrix} \Psi(\boldsymbol{r}) \\ \Psi^*(\boldsymbol{r}) \end{pmatrix}$ yields a solution of Bogoliubov eq.

"u,v→Ψ" scenario

$$\lim_{\epsilon \to 0} \begin{pmatrix} u \\ v \end{pmatrix} \propto \begin{pmatrix} \Psi \\ \Psi^* \end{pmatrix}$$

⟺

Perfect transmission in low energy limit

$$\{H_0(\mathbf{r}) + g|\Psi(\mathbf{r})|^2\}\Psi(\mathbf{r}) = 0 \qquad H_0(\mathbf{r}) = -\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{ext}}(\mathbf{r}) - \mu$$

Gross-Pitaevskii equation

$$\begin{pmatrix} H_0(\mathbf{r}) + 2g|\Psi(\mathbf{r})|^2 & -g[\Psi(\mathbf{r})]^2 \\ g[\Psi^*(\mathbf{r})]^2 & -H_0(\mathbf{r}) - 2g|\Psi(\mathbf{r})|^2 \end{pmatrix} \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} = \varepsilon_i \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix}$$

Bogoliubov equation for $u_i(\mathbf{r}), v_i(\mathbf{r})$

Q.

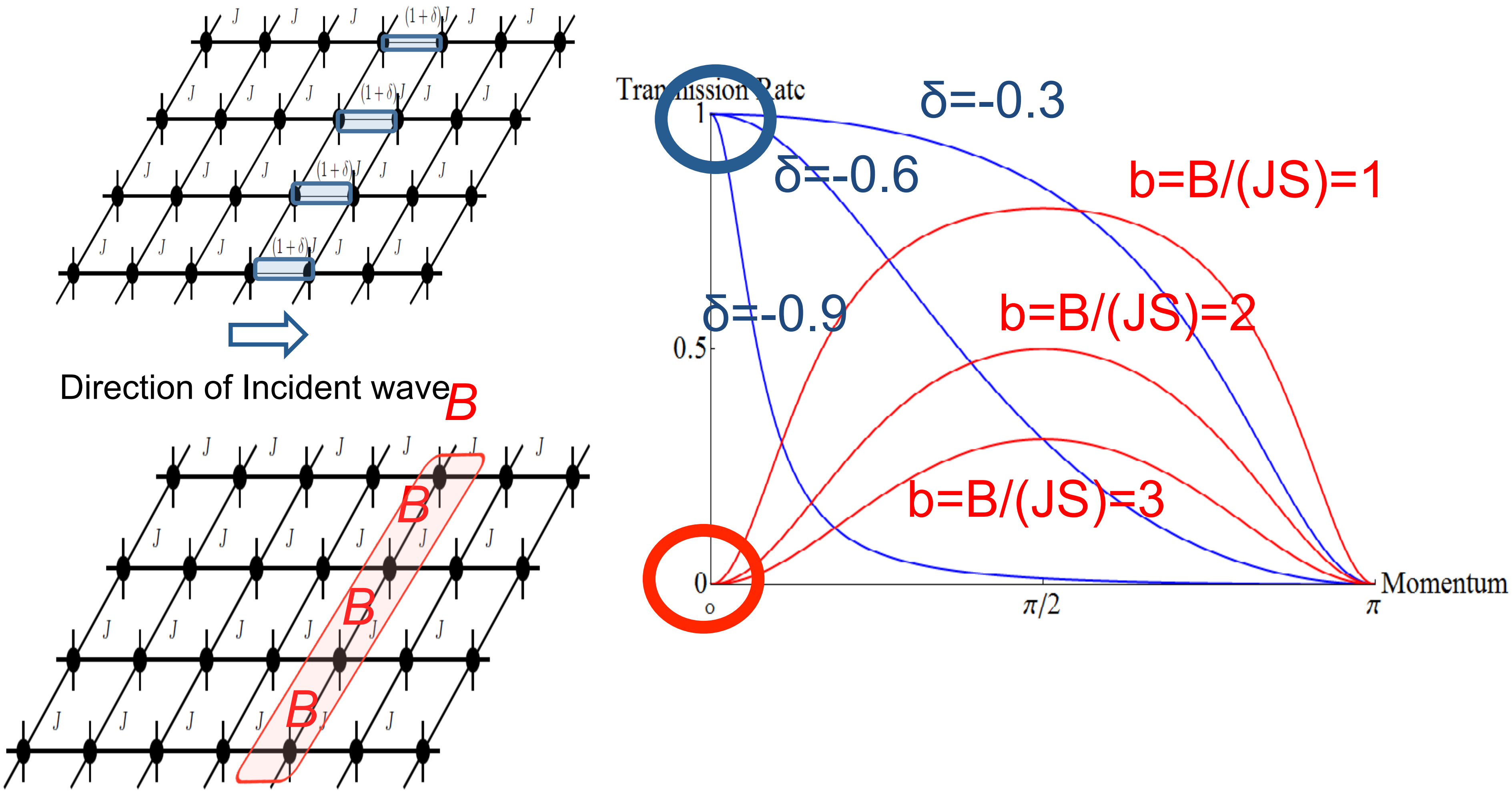
- Is anomalous tunneling *specific* to Bogoliubov mode in *superfluids*?

A.

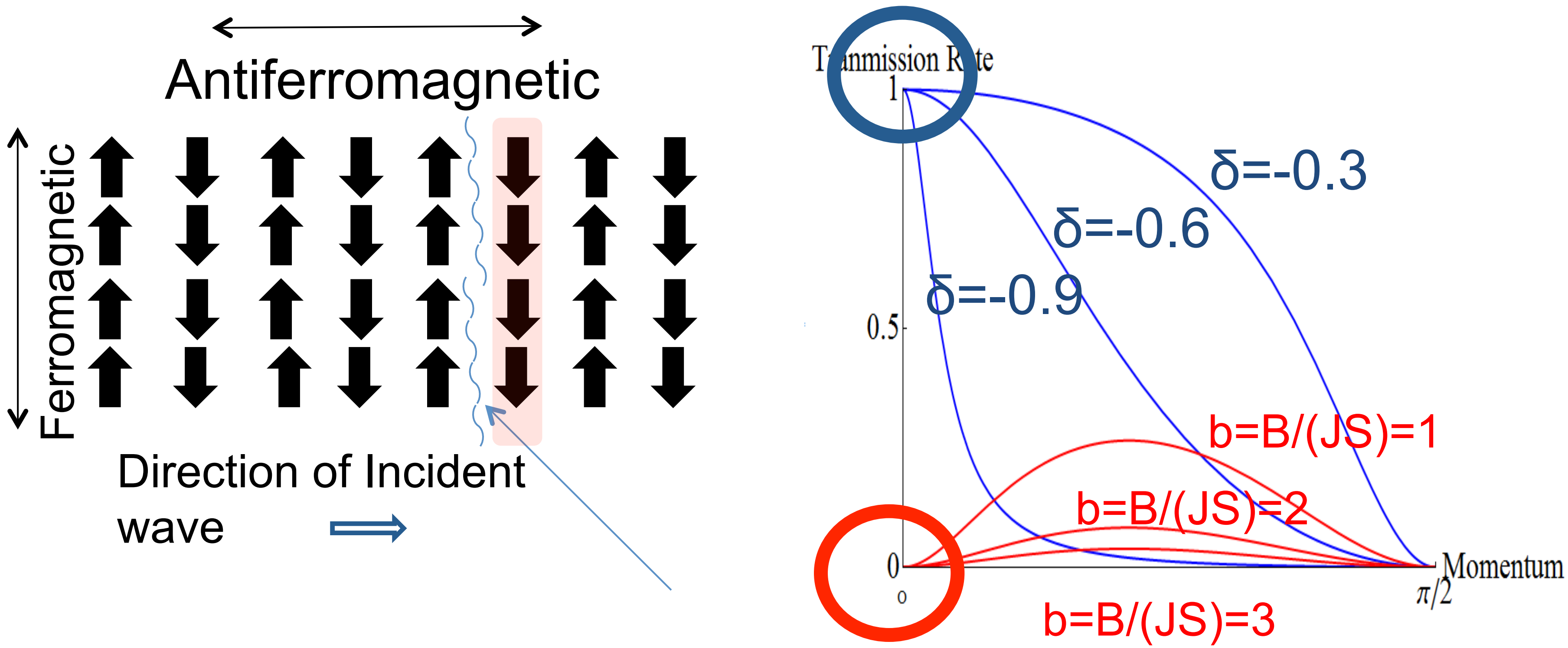
- Our conclusion: Anomalous tunneling is a *common* property of Nambu-Goldstone modes under potentials *preserving a continuous symmetry*.

Anomalous Tunneling in Spin wave in Heisenberg models

Transmission of spin wave in classical Heisenberg ferromagnet (3D)

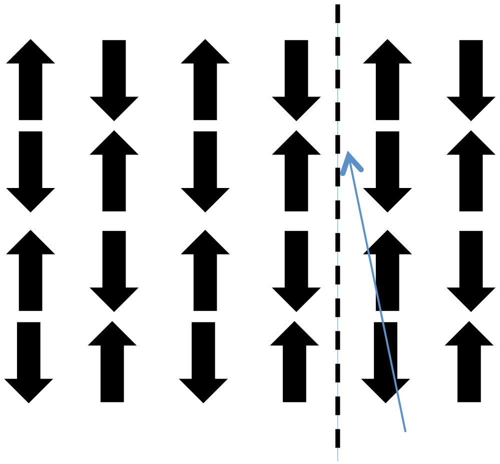


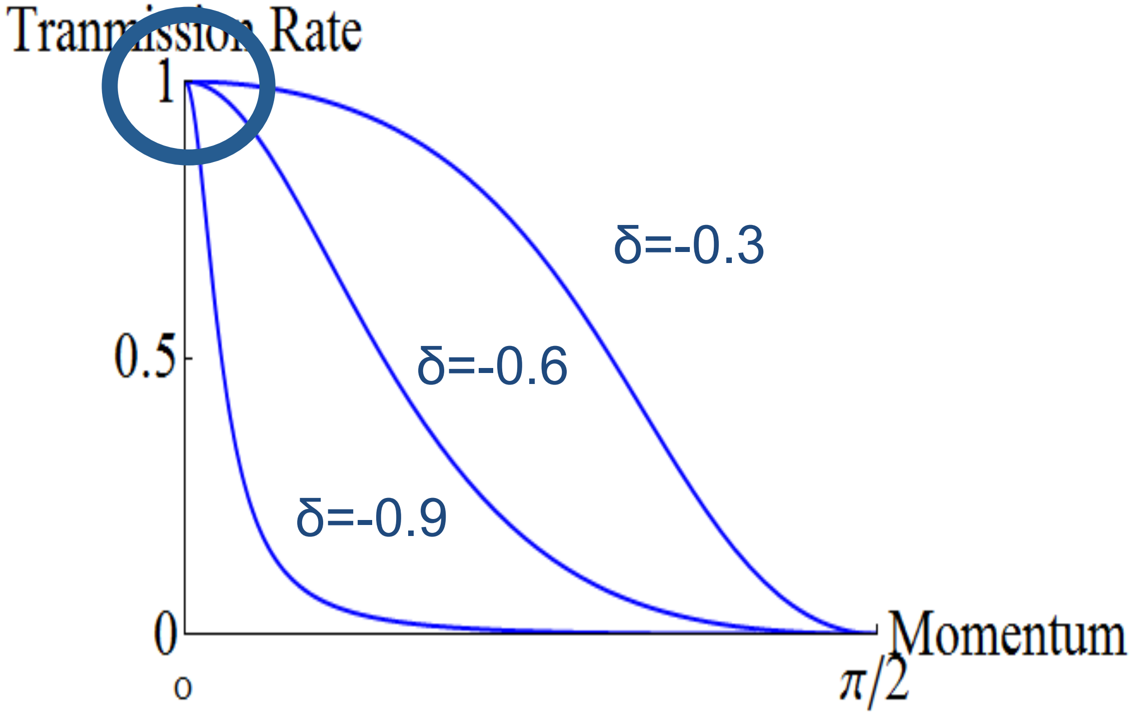
Transmission of spin wave in classical Heisenberg antiferromagnet(A-type:3D)



Location of weak bond
or
local magnetic field

Transmission of spin wave in classical Heisenberg antiferromagnet(G-type:3D)



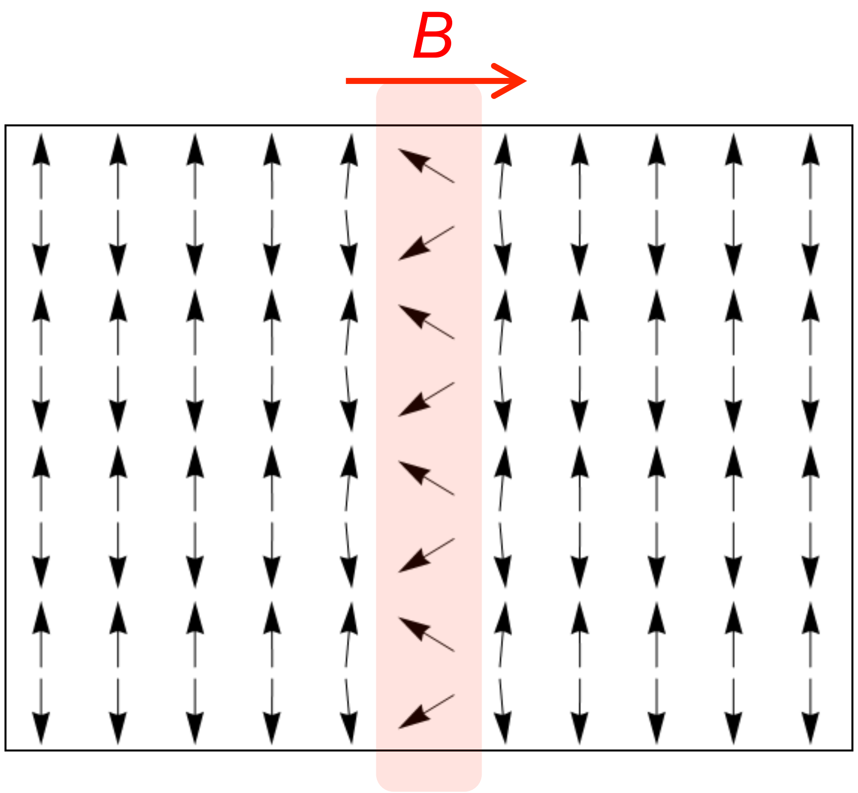


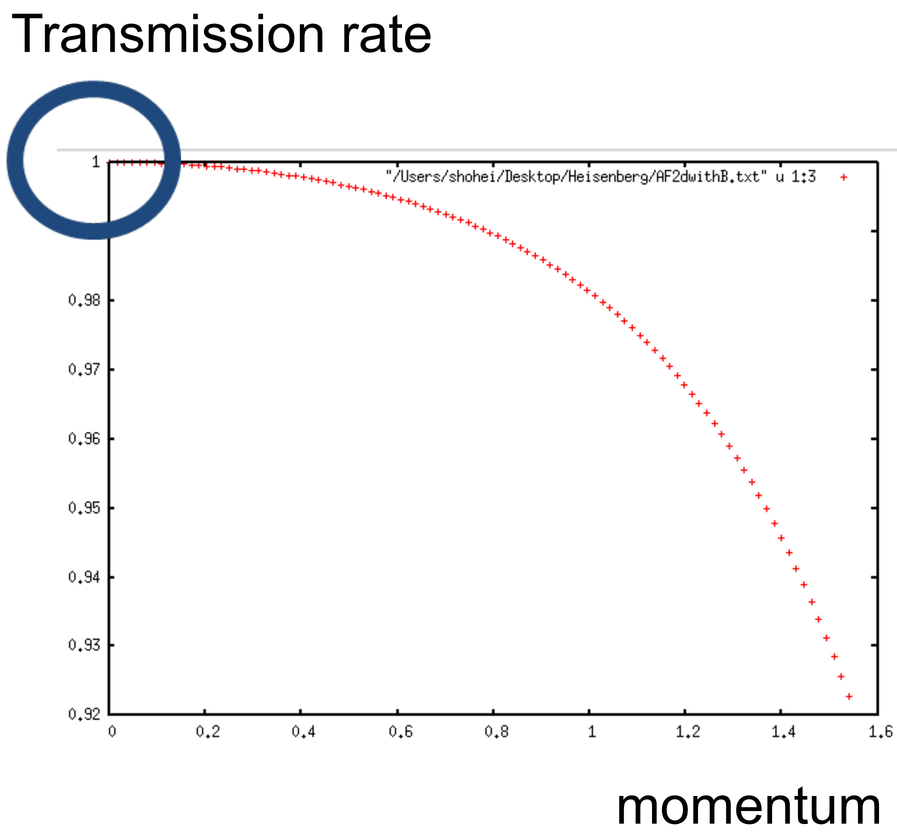
Location of weak bonds

⇨

Direction of Incident wave

Transmission of spin wave in classical Heisenberg antiferromagnet(G'-type; 2D)





Short Discussion I

- In the presence of weak bonds,
  spin symmetries of Hamiltonian and ground state are the same as those in bulk.

$\Longrightarrow$ Nambu-Goldstone modes exist.

Perfect transmission occurs in long-wavelength limit.

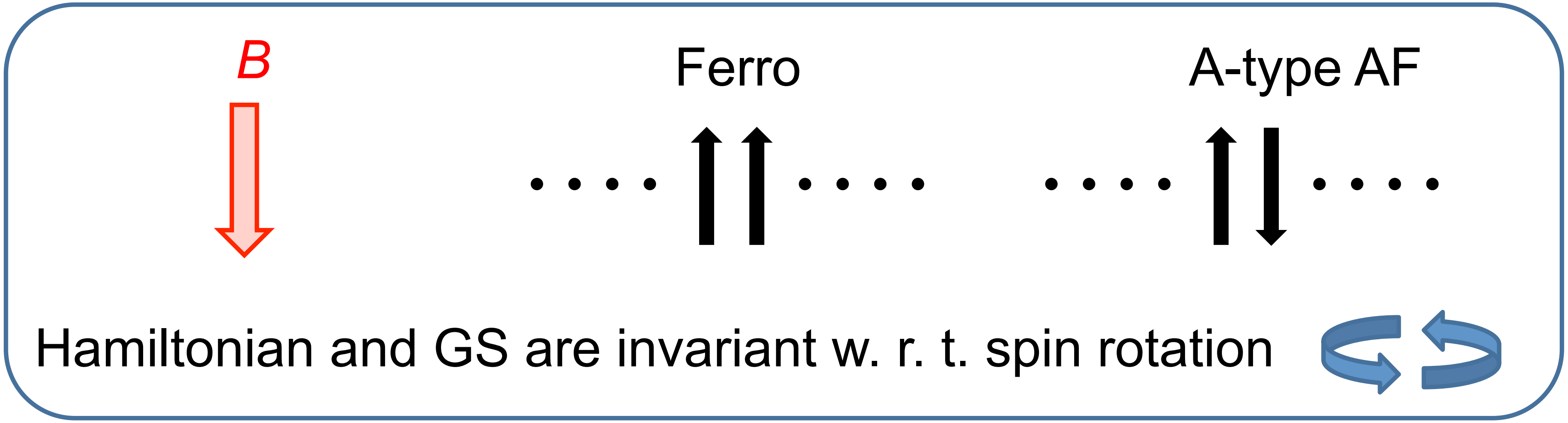
Short Discussion II

(perfect reflection in Ferro and A-type Antiferro in the presence of local ***B***)

- spin symmetry of Hamiltonian *G* reduces from SO(3) to SO(2) (spin rotation around the axis parallel to local ***B***)

Spin symmetries *H* of ground states in Ferromagnet and A-type antiferromagnet are the same as those of Hamiltonian (i.e. *G*=*H*)

⇒ No spontaneous symmetry breaking,
No Nambu-Goldstone modes



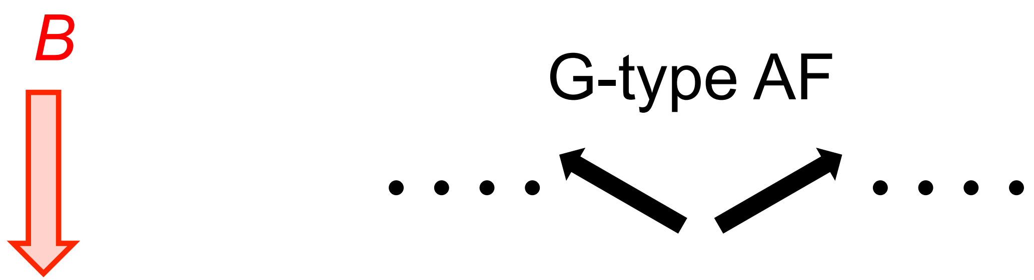
Short Discussion III

(perfect transmission in G-type Antiferro in the presence of local ***B***)

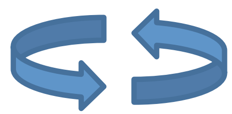
- spin symmetry of Hamiltonian *G* is SO(2) (spin rotation around the axis parallel to local ***B***)

GS in G-type antiferro is not invariant w. r. t. the spin rotation (i.e. *G/H*=SO(2))

⇒SO(2) symmetry broken spontaneously, a Nambu-Goldstone mode exists

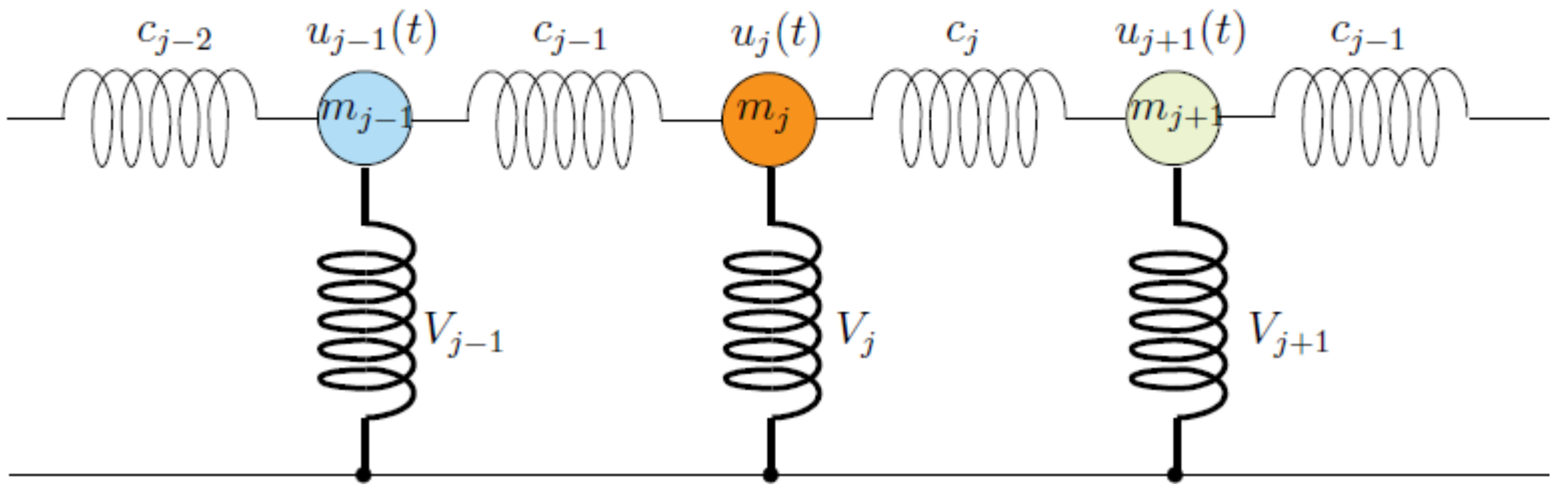


Hamiltonian is invariant but GS is not w. r. t. rotation



More Discussion in terms of conservation law

Simplest case: phonon



Lagrangian:

$$\mathcal{L} = \frac{m(x)}{2}\left(\frac{\partial u(x,t)}{\partial t}\right)^2 - \frac{c(x)}{2}\left(\frac{\partial u(x,t)}{\partial x}\right)^2 - \frac{V(x)u(x,t)^2}{2}$$

Note that the spatial dependent $m(x)$ and $c(x)$ do not break the translational invariance of $\mathcal{L}$ with $u(x) \to u(x) + a$ while the presence of $V(x)$ does.

Eq. of Motion:

$)u(x).$

$$\underbrace{\frac{\partial}{\partial t}\left(m(x)\frac{\partial u(x,t)}{\partial t}\right)}_{\text{"charge"}} + \underbrace{\frac{\partial}{\partial x}\left(-\left(c(x)\frac{\partial u(x,t)}{\partial x}\right)\right)}_{\text{"current"}} = \underbrace{-V(x)u(x)}_{\text{"source"}}$$

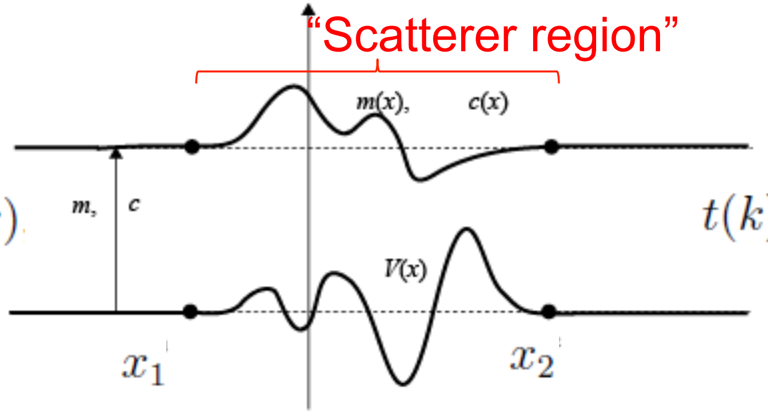
which expresses the conservation of Noether current when $V(x) = 0$.

Setting $u(x,t) = u_k(x)\exp(-i\omega t)$,

$$\underbrace{\frac{d}{dx}\left(c(x)\frac{du_k(x)}{dx}\right)}_{j(x)} = (-\omega^2 m(x) + V(x))u_k(x), \text{ with } \omega = (c/m)^{\frac{1}{2}}k,\ k > 0$$

$$\frac{\partial j(x)}{\partial x} = \begin{cases} O(\omega^2) = O(k^2), & V(x) = 0 \\ O(1), & V(x) \neq 0 \end{cases}$$

$\exp(ikx) + r(k)\exp(-ikx)$



$t(k)\exp(ikx)$

$$u_k'(x_1) = u_k'(x_2) + \frac{1}{c}\int_{x_2}^{x_1}(-\omega^2 m(x') + V(x'))u_k(x')dx'$$

$$u_k(x_1) = u_k(x_2) + I_c u_k'(x_2)$$

$$+\int_{x_2}^{x_1}\left(\frac{1}{c(x')}\int_{x_2}^{x'}(-\omega^2 m(x'') + V(x''))u_k(x'')dx''\right)dx'$$

$$I_c = \left(\int_{x_2}^{x_1}\frac{c\,dx'}{c(x')}\right)$$

When $V(x) = 0$,

$$u_k(x_1) = u_k(x_2) + O(k)$$
$$u'_k(x_1) = u'_k(x_2) + O(k^2)$$

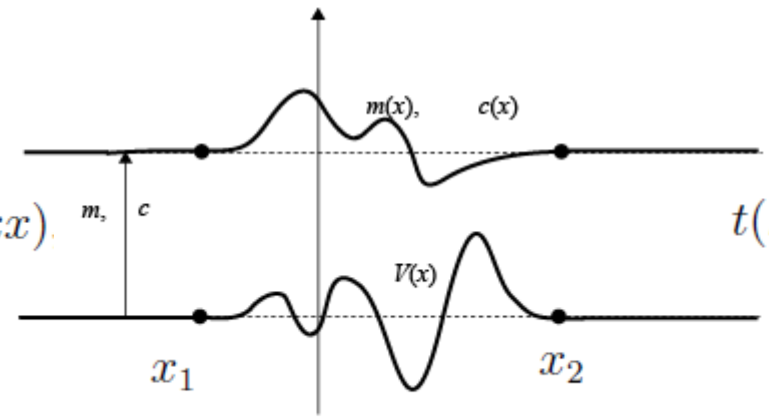
$\Rightarrow$ $t(k \to 0) = 1$

When $V(x) \neq 0$,

$$u_k(x_1) = u_k(x_2) + O(1)$$
$$u'_k(x_1) = u'_k(x_2) + O(1)$$

$\Rightarrow$ $t(k \to 0) = 0$

$\exp(\mathrm{i}kx) + r(k)\exp(-\mathrm{i}kx)$



$t(k)\exp(\mathrm{i}kx)$

Short Summary (perfect transmission of phonon)

- Existence/absence of Noether current (Conservation law)
- Long-wavelength(small k)

⇓

$$\frac{\partial j(x)}{\partial x} = \begin{cases} O(\omega^2) = O(k^2), & V(x) = 0 \\ O(1), & V(x) \neq 0 \end{cases}$$

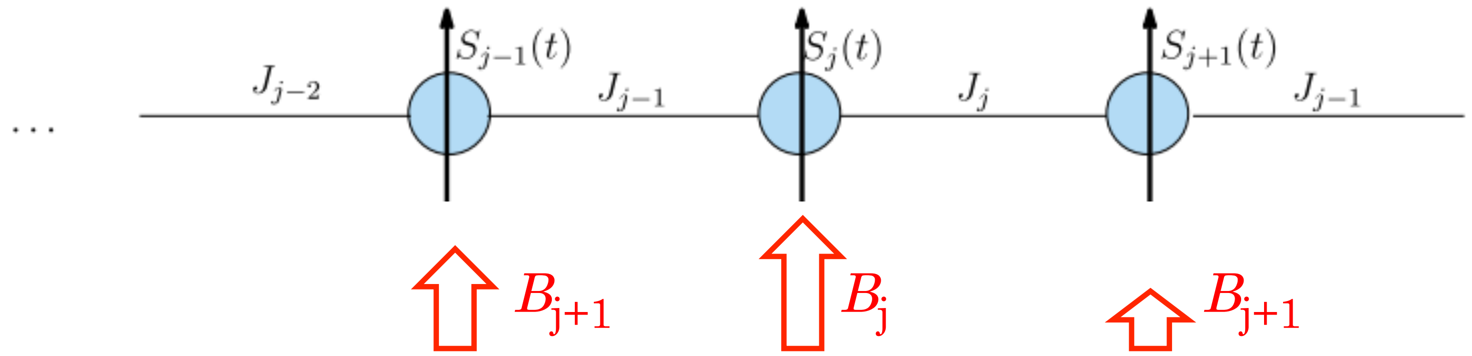
⇓

$$t(k \to 0) = 1 \qquad V(x) = 0$$

$$t(k \to 0) = 0 \qquad V(x) \neq 0$$

Note: Constancy of Noether current is crucial for perfect transmission

Ferro spin wave1



Eq. of Motion:

$$\frac{\partial \boldsymbol{S}(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left(a^2 J(x)\frac{\partial}{\partial x}\boldsymbol{S}(x,t)\right) \times \boldsymbol{S}(x,t) + \boldsymbol{B}(x) \times \boldsymbol{S}(x,t)$$

Linearization (spin wave approximation) $\left(\frac{\partial \boldsymbol{S}(\boldsymbol{r},t)}{\partial t} = \boldsymbol{S}(\boldsymbol{r},t) \times \nabla^2 \boldsymbol{S}(\boldsymbol{r},t) \right)$

Landau-Lifshitz eq.

$$\boldsymbol{S}_j(t) = \sqrt{S^2 - (\delta \boldsymbol{S}_j(t))^2}\boldsymbol{e}_z + \delta \boldsymbol{S}_j(t) \sim S\boldsymbol{e}_z + \delta \boldsymbol{S}_j(t)$$

Eq. of Motion of spin wave

$$\frac{\partial}{\partial t}\left(\mathrm{i}\delta S^+(x,t)\right) + \frac{\partial}{\partial x}\left(a^2 S J(x)\frac{\partial}{\partial x}\delta S^+(x,t)\right) = B(x)\delta S^+(x,t)$$

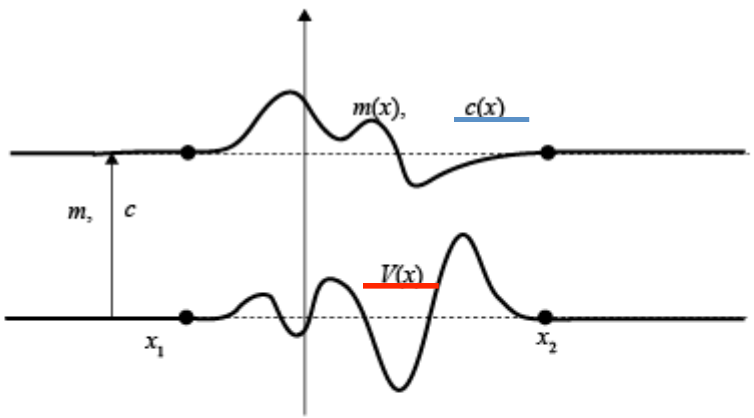
"charge" "current" "source "

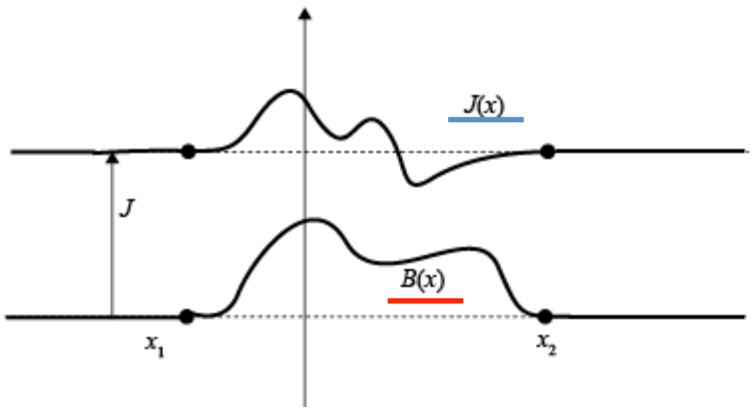
has the form of conservation law when B=0

Ferro spin wave2

phonon	ferromagnetic spin wave
	Setting $\delta S^+(x,t) = S^+(x,\omega)\exp(-i\omega t)$
$\frac{d}{dx}\left(c(x)\frac{du_k(x)}{dx}\right) = (V(x) - \omega^2 m(x))u_k(x)$ $= (V(x) - O(k^2))u_k(x)$	$\frac{\partial}{\partial x}\left(a^2 S J(x)\frac{\partial}{\partial x}S^+(x,\omega)\right) = (B(x)-\omega)S^+(x,\omega)$ $= \left(B(x) - O(k^2)\right) S^+(x;\omega)$
$\omega = (c/m)^{\frac{1}{2}}\lvert k \rvert$ linear dispersion in bulk	$\omega = JSk^2$ quadratic dispersion in bulk
Order parameter; not conserved	Order parameter; conserved quantity

$= \Big(B(x$





Conclusion:

- The results on BECs and Heisenberg model strongly suggest that anomalous tunneling inherent to Nambu-Goldstone modes in symmetry broken states.

- Perfect transmission of Nambu-Goldstone mode resulting from the spontaneously breaking of symmetry G occurs when the potential of scatterer preserves a continuous symmetry.

- Perfect transmission can be attributed to constancy of Noether currents at low energy limit.