Higgs, Nambu-Goldstone: Summary

Hitoshi Murayama (Berkeley, Kavli IPMU)

Yukawa Institute for Theoretical Physics, Kyoto University

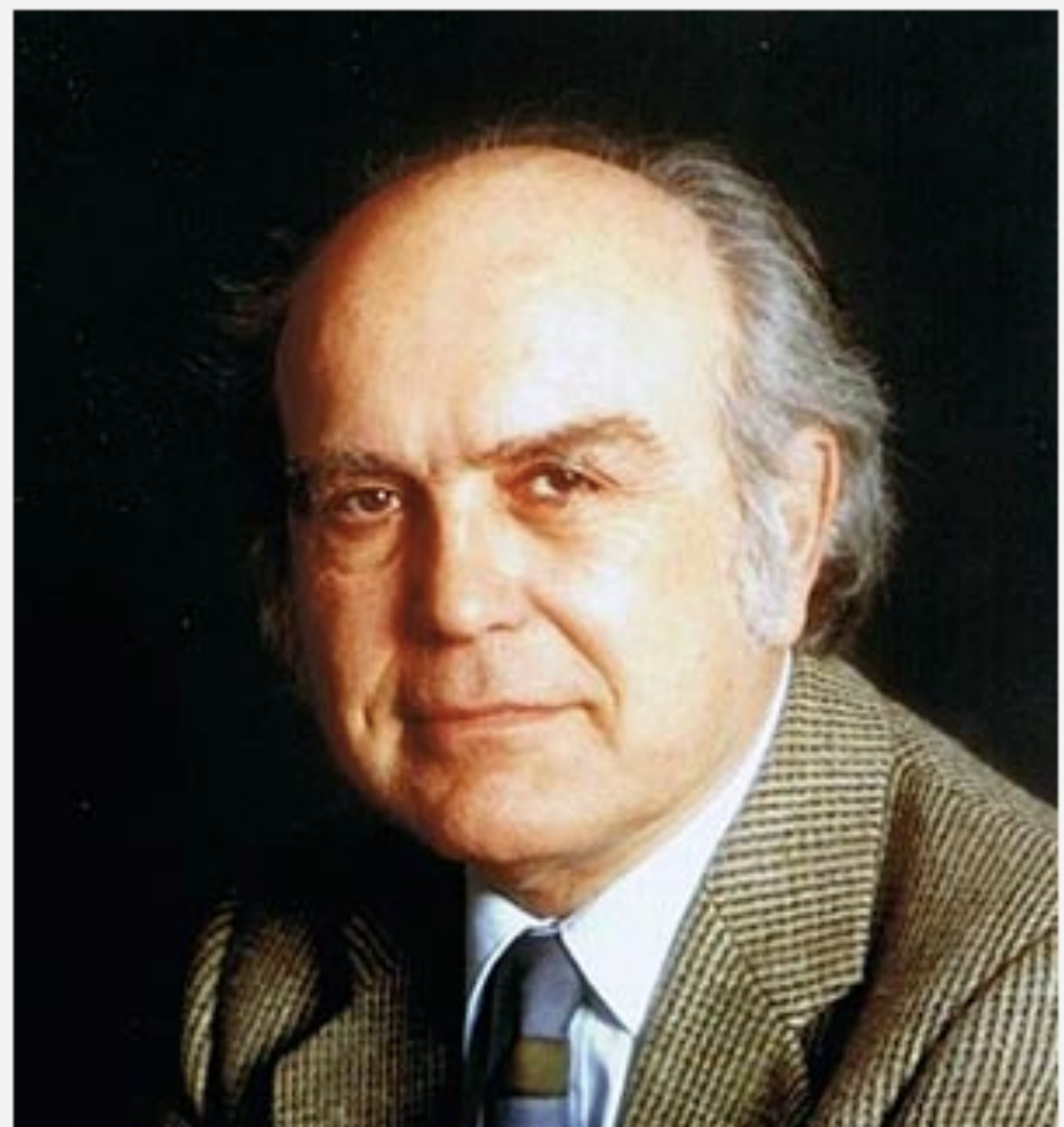
Higgs Modes in Condensed Matter and Quantum Gases

2014-06-23 — 2014-06-25

Bruno Zumino, an architect of supersymmetry, dies at 91

By Robert Sanders, Media Relations | June 24, 2014

BERKELEY — Bruno Zumino, a professor emeritus of physics at the University of California, Berkeley, who was best known for developing supersymmetry, a theory now considered as a leading candidate for explaining the fundamental forces of nature, died Sunday, June 22, at his home in Berkeley, Calif. He was 91.



Supersymmetry or SUSY, developed in the early 1970s at the European Center for Nuclear Research (CERN) in Geneva, Switzerland, by Zumino and Julius Wess, was conceived to explain particle interactions involving three of the four main forces in nature – the strong, electromagnetic and weak forces. One consequence of the theory is that every particle we see today has a supersymmetric partner – the quark has an associated squark, for example, while the electron has a selectron. Zumino and Stanley Deser later extended the so-called "Wess-Zumino model" of supersymmetry to include gravity, creating a theory called supergravity.

To date, none of these superpartners has been detected, though CERN's Large Hadron Collider, which in 2012 produced evidence for the Higgs boson, a particle that endows the rest of matter with mass, is now looking for heavier particles that would be evidence of supersymmetry. Scientists even hold out hope that one of the superpartners

PHYSICAL REVIEW VOLUME 177, NUMBER 5 25 JANUARY 1969

Structure of Phenomenological Lagrangians. I*

S. COLEMAN
Harvard University, Cambridge, Massachusetts 02138

AND

J. WESS† AND BRUNO ZUMINO
New York University, New York, New York 10003
(Received 13 June 1968)

The general structure of phenomenological Lagrangian theories is investigated, and the possible transformation laws of the phenomenological fields under a group are discussed. The manifold spanned by the phenomenological fields has a special point, called the origin. Allowed changes in the field variables, which do not change the on-shell S matrix, must leave the origin fixed. By a suitable choice of fields, the transformations induced by the group on the manifold of the phenomenological fields can be made to have standard forms, which are described in detail. The mathematical problem is equivalent to that of finding all (nonlinear) realizations of a (compact, connected, semisimple) Lie group which become linear when restricted to a given subgroup. The relation between linear representations and nonlinear realizations is discussed. The important special case of the chiral groups $SU(2)\times SU(2)$ and $SU(3)\times SU(3)$ is considered in detail.

coset space G/H

PHYSICAL REVIEW VOLUME 177, NUMBER 5 25 JANUARY 1969

Structure of Phenomenological Lagrangians. II*

CURTIS G. CALLAN, JR. AND SIDNEY COLEMAN
Harvard University, Cambridge, Massachusetts 02138

AND

J. WESS† AND BRUNO ZUMINO
New York University, New York, New York 10003
(Received 13 June 1968)

The general method for constructing invariant phenomenological Lagrangians is described. The fields are assumed to transform according to (nonlinear) realizations of an internal symmetry group, given in standard form. The construction proceeds through the introduction of covariant derivatives, which are standard forms for the field gradients. The case of gauge fields is also discussed.

Bruno Zumino

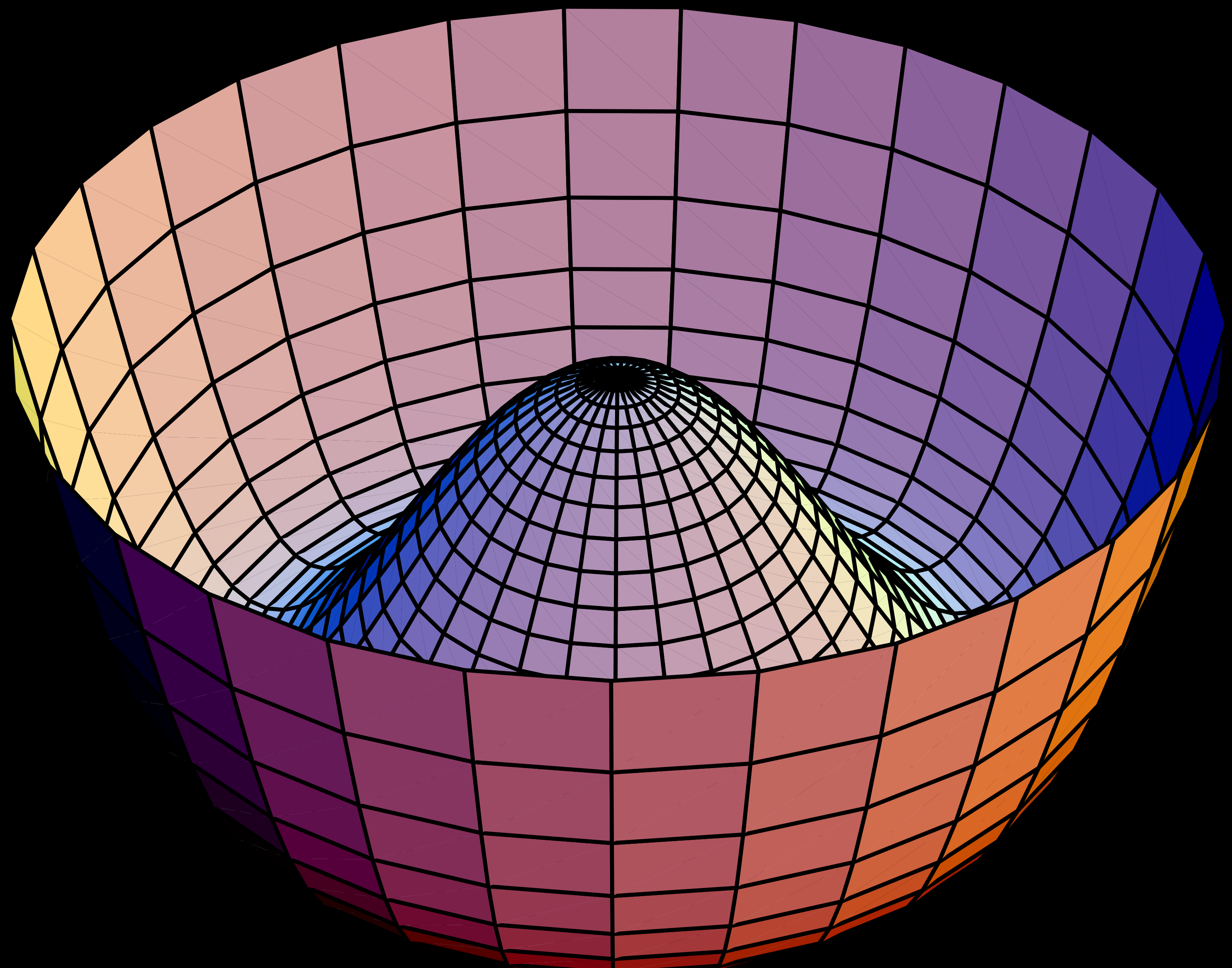
Higgs, Nambu-Goldstone: Summary

Hitoshi Murayama (Berkeley, Kavli IPMU)

Yukawa Institute for Theoretical Physics, Kyoto University

Higgs Modes in Condensed Matter and Quantum Gases
2014-06-23 — 2014-06-25

Wow!




Bardeen


Cooper


Schrieffer

Nambu-Goldstone boson

Nambu

Goldstone

For every broken symmetry generator,
there is one massless boson
assuming Lorentz invariance

IL NUOVO CIMENTO VOL. XIX, N. 1 1° Gennaio 1961

Field Theories with «Superconductor» Solutions.

J. GOLDSTONE

CERN - Geneva

(ricevuto l'8 Settembre 1960)

Summary. — The conditions for the existence of non-perturbative type « superconductor » solutions of field theories are examined. A non-covariant canonical transformation method is used to find such solutions for a theory of a fermion interacting with a pseudoscalar boson. A covariant renormalisable method using Feynman integrals is then given. A « superconductor » solution is found whenever in the normal perturbative-type solution the boson mass squared is negative and the coupling constants satisfy certain inequalities. The symmetry properties of such solutions are examined with the aid of a simple model of self-interacting boson fields. The solutions have lower symmetry than the Lagrangian, and contain mass zero bosons.

Anderson

The Goldstone zero-mass difficulty is not a serious one, because we can probably cancel it off against an equal Yang-Mills zero-mass problem.

Englert

Brout

Higgs

Nobel Prizes and Laureates

Physics Prizes ‹ 2013 ›

- ▶ About the Nobel Prize in Physics 2013
- ▶ François Englert
- ▼ Peter Higgs
  - Facts
  - Nobel Lecture
  - Banquet Speech
  - Interview
  - Nobel Diploma
  - Photo Gallery
  - Prize Presentation
  - Other Resources

The Nobel Prize in Physics 2013
François Englert, Peter Higgs

Share this: 20

Peter Higgs - Nobel Lecture

Evading the Goldstone Theorem

Peter Higgs delivered his Nobel Lecture on 8 December 2013, at Aula Magna, Stockholm University. He was introduced by Professor Lars Brink, Chairman of the Nobel Committee for Physics.

See a Video of the Nobel Lecture
Presentation
39 sec.
→ Play

Most particle theorists at the time did not pay much attention to the ideas of Nambu and Goldstone. Quantum field theory was out of fashion, despite its successes in quantum electrodynamics; it was failing to describe either the strong or the weak interactions.

Besides, condensed matter physics was commonly viewed as another country. At a Cornell seminar in 1960 Victor Weisskopf remarked (as recalled by Robert Brout)

"Particle physicists are so desperate these days that they have to borrow from the new things coming up in many body physics – like BCS. Perhaps something will come of it."

Goldstone's theorem

Scanned at the American
Institute of Physics

For every broken symmetry generator,
there is one massless boson
assuming Lorentz invariance

TODIAS 東京大学国際高等研究所 TODAI INSTITUTES FOR ADVANCED STUDY

wpi World Premier International Research Center Initiative

BERKELEY CENTER FOR THEORETICAL PH

What's wrong with Goldstone?

Hitoshi Murayama

+ Haruki Watanabe, Tomáš Brauner

Stanford Institute for Theoretical Physics Seminar

April Fool's Day 2013

arXiv:1203.0609, 1302.4800, 1303.1527

Generalized theorem applies to all systems!

PRL **108,** 251602 (2012) PHYSICAL REVIEW LETTERS week ending 22 JUNE 2012

Unified Description of Nambu-Goldstone Bosons without Lorentz Invariance

Haruki Watanabe[1,2,*] and Hitoshi Murayama[1,3,4,†]

[1]*Department of Physics, University of California, Berkeley, California 94720, USA*
[2]*Department of Physics, University of Tokyo, Hongo, Tokyo 113-0033, Japan*
[3]*Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
[4]*Kavli Institute for the Physics and Mathematics of the Universe (WPI), Todai Institutes for Advanced Study, University of Tokyo, Kashiwa 277-8583, Japan*

(Received 3 March 2012; published 21 June 2012)

Using the effective Lagrangian approach, we clarify general issues about Nambu-Goldstone bosons without Lorentz invariance. We show how to count their number and study their dispersion relations. Their number is less than the number of broken generators when some of them form canonically conjugate pairs. The pairing occurs when the generators have a nonzero expectation value of their commutator. For non-semi-simple algebras, central extensions are possible. The underlying geometry of the coset space in general is partially symplectic.

SUGGESTION

Physics
spotlighting exceptional research

also Yoshimasa Hidaka

Nuclear Physics B106 (1976) 292–340

A PHENOMENOLOGICAL PROFILE OF THE HIGGS BOSON

John ELLIS, Mary K. GAILLARD * and D.V. NANOPOULOS **
CERN, Geneva

Received 7 November 1975

A discussion is given of the production, decay and observability of the scalar Higgs boson H expected in gauge theories of the weak and electromagnetic interactions such as the Weinberg-Salam model. After reviewing previous experimental limits on the mass of the Higgs boson, we give a speculative cosmological argument for a small mass. If its mass is similar to that of the pion, the Higgs boson may be visible in the reactions $\pi^- p \to$ Hn or $\gamma p \to$ Hp near threshold. If its mass is $\lesssim 300$ MeV, the Higgs boson may be present in the decays of kaons with a branching ratio $O(10^{-7})$, or in the decays of one of the new particles: $3.7 \to 3.1 + H$ with a branching ratio $O(10^{-4})$. If its mass is $\leqslant 4$ GeV, the Higgs boson may be visible in the reaction $pp \to H + X$, $H \to \mu^+\mu^-$. If the Higgs boson has a mass $\leqslant 2m_\mu$, the decays $H \to e^+e^-$ and $H \to \gamma\gamma$ dominate, and the lifetime is $O(6 \times 10^{-4}$ to $2 \times 10^{-12})$ seconds. As thresholds for heavier particles (pions, strange particles, new particles) are crossed, decays into them become dominant, and the lifetime decreases rapidly to $O(10^{-20})$ sec for a Higgs boson of mass 10 GeV. Decay branching ratios in principle enable the quark masses to be determined.

We should perhaps finish with an apology and a caution. We apologize to experimentalists for having no idea what is the mass of the Higgs boson, unlike the case with charm [3,4] and for not being sure of its couplings to other particles, except that they are probably all very small. For these reasons we do not want to encourage big experimental searches for the Higgs boson, but we do feel that people performing experiments vulnerable to the Higgs boson should know how it may turn up.

We would like to thank B.W. Lee, J. Prentki, B. and F. Schrempp, G. Segré and B. Zumino for valuable remarks, comments and suggestions.

Note added in proof

Since writing our paper we have learnt of some more considerations [55–57] about the mass of the Higgs boson. Also, we have been encouraged [58] to calculate its production in neutrino collisions. We also make here some further remarks about the model dependence of our previous results.

In two papers [55,56], Sato and Sato have given astrophysical arguments against very light Higgs bosons. They argue that present understanding of the cosmic background radiation excludes 0.1 eV $< m_H <$ 100 eV [55], and that stellar evolution would be drastically affected if $m_H < 0.1 \times m_e$ [56].

Most recently, Linde and Weinberg have derived [57] an approximate lower comparable with the effects of virtual $W^{\pm}$ and Z^0 exchanges, and impossible to disentangle from hadronic contributions in standard QED. If $m_H \gg m_\mu$, then $(\Delta g_\mu)_H$

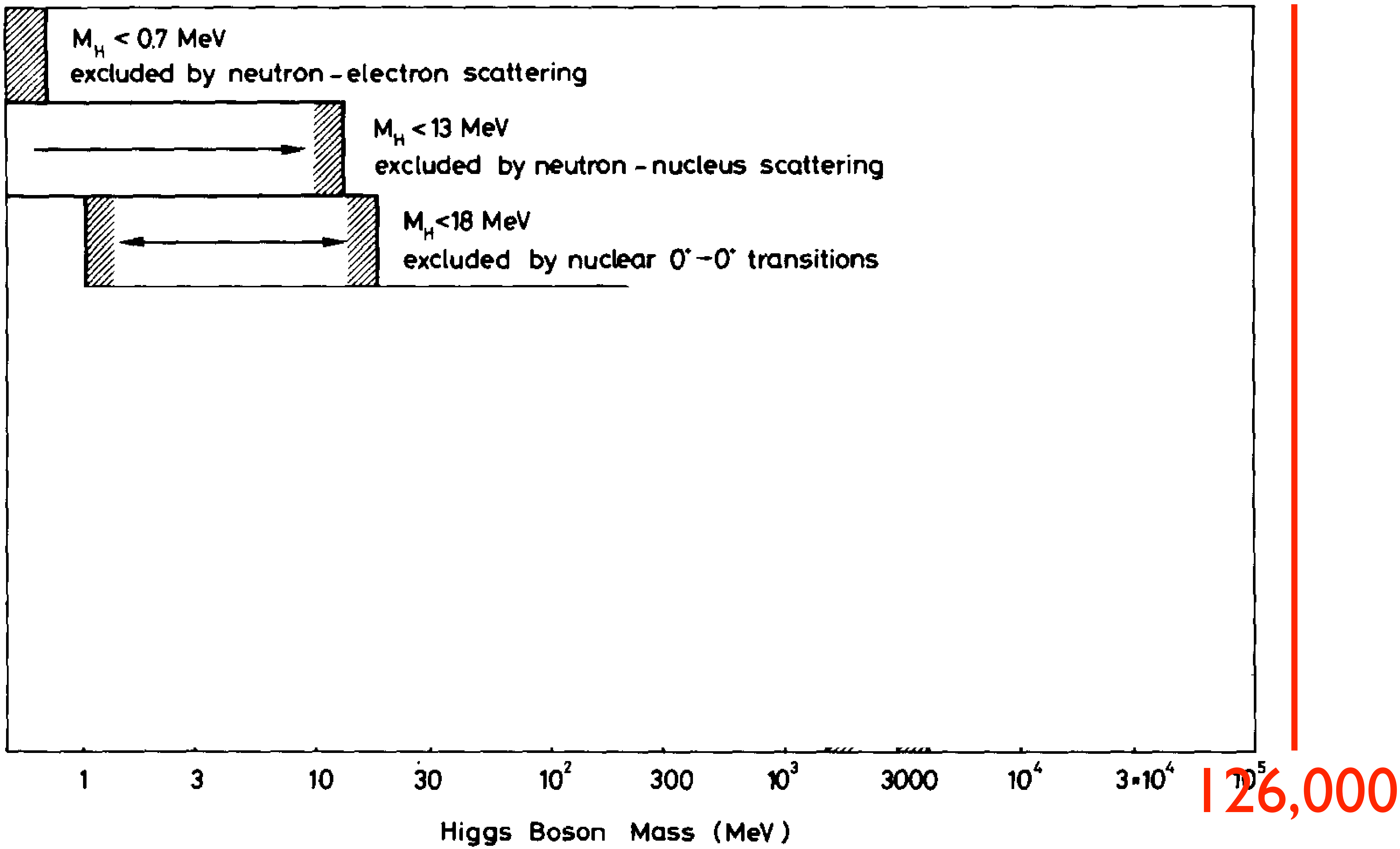
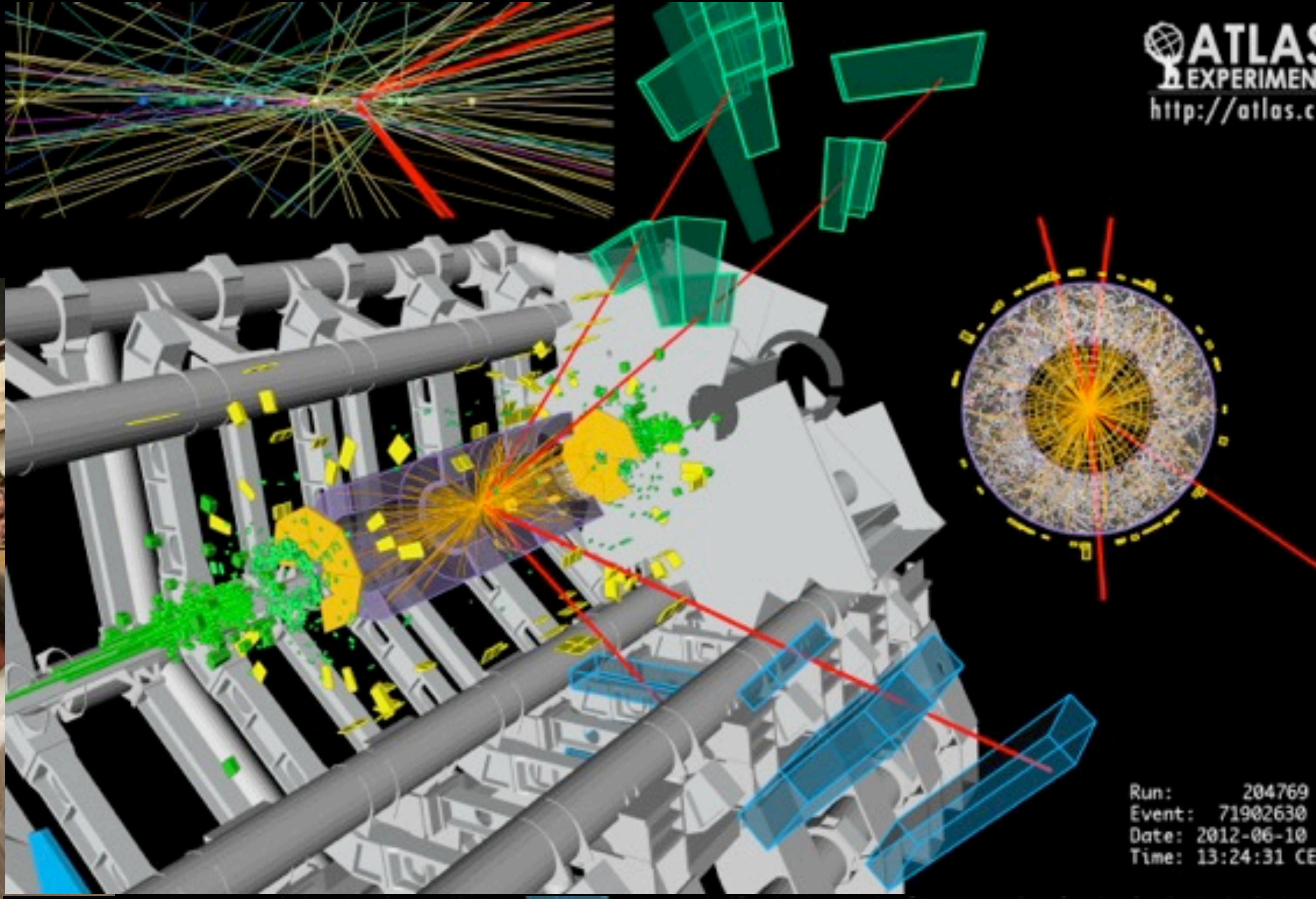
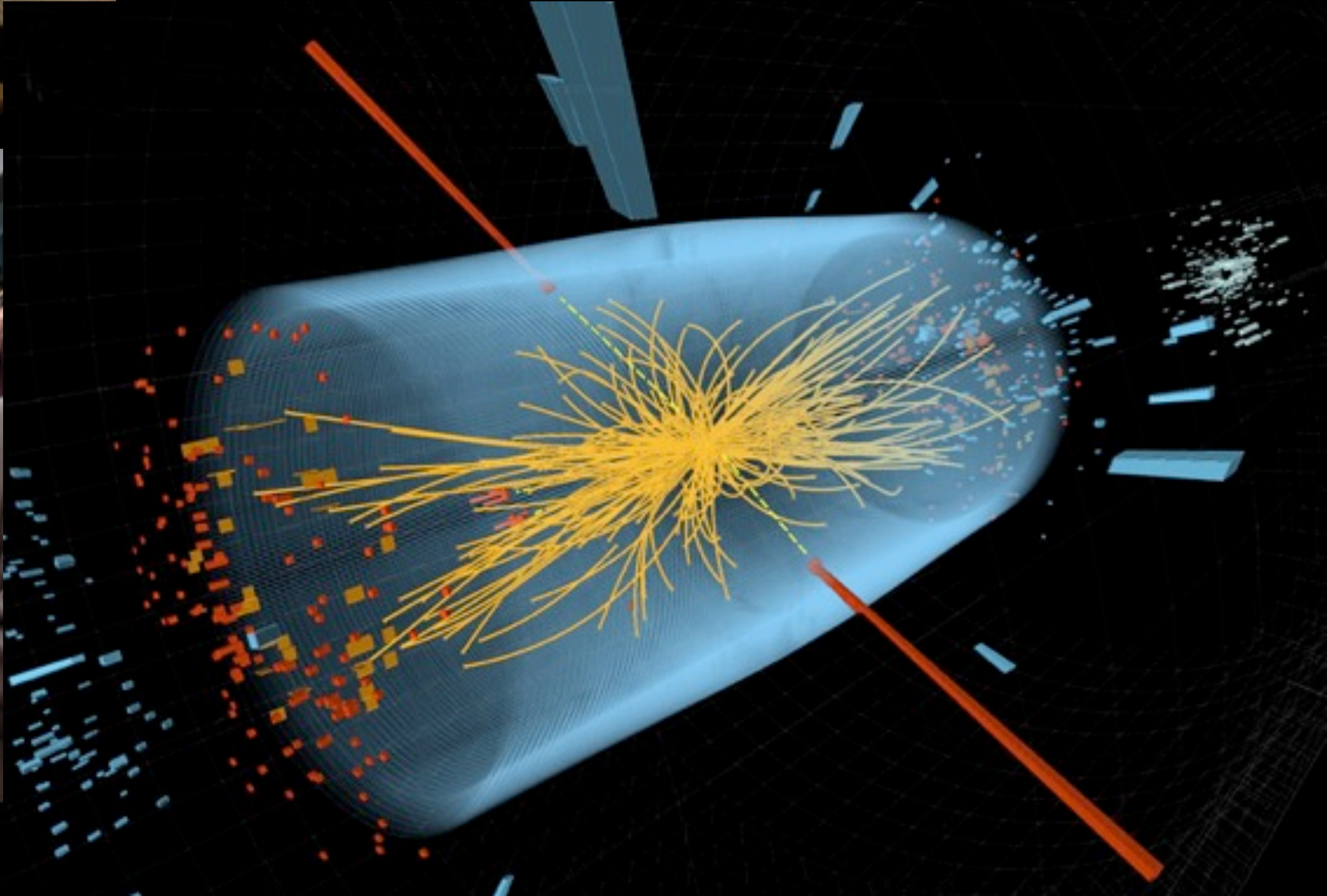


Fig. 3. Present and possible future limits on the Higgs boson mass.

2012.7.4

discovery of Higgs boson



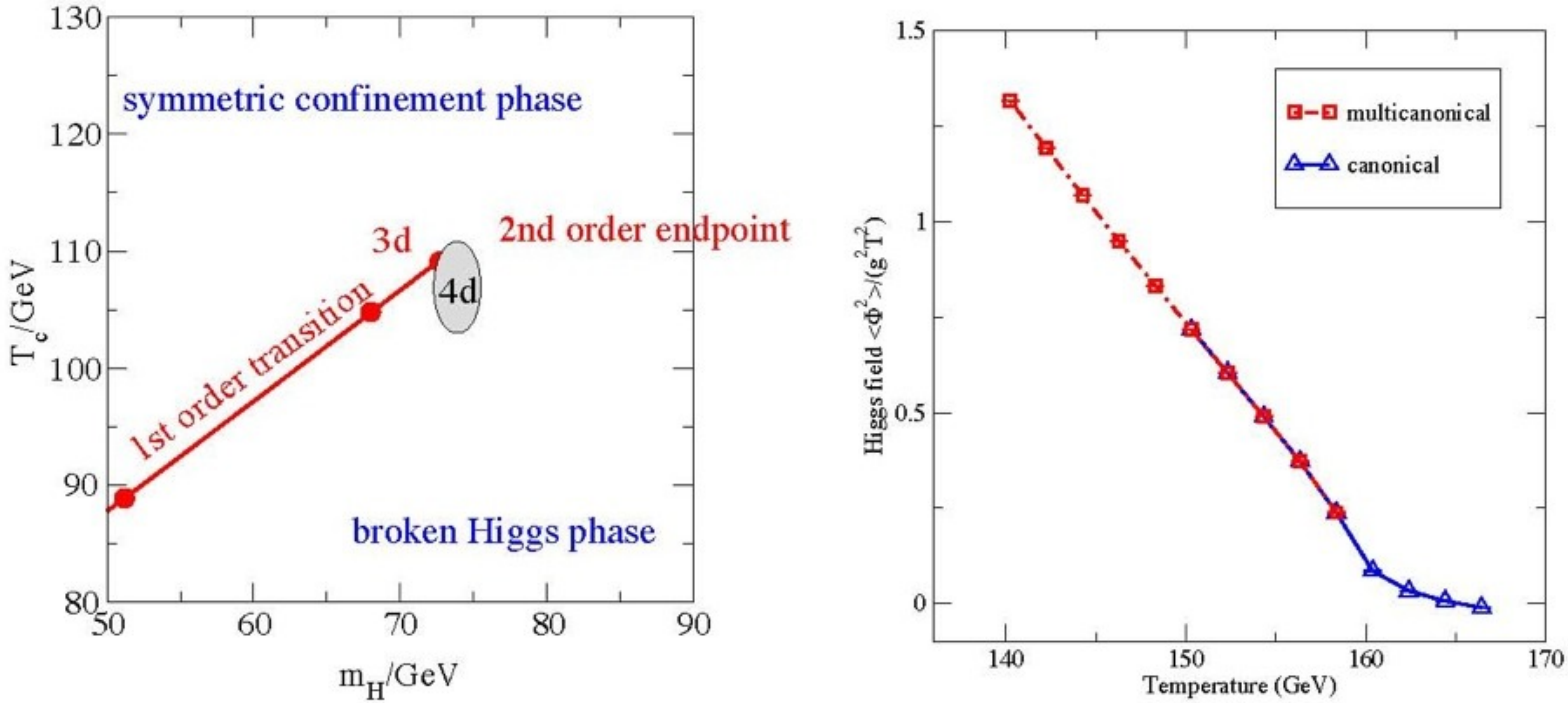


No topological defects

- $G=SU(2)_L \times U(1)_Y$
- $H=U(1)_{EM}$
- $\pi_1(G/H)=0$ no Abrikosov vortices
- $\pi_2(G/H)=0$ no magnetic monopoles
- $\pi_3(G/H)=Z$ but no θ-vacuum because of the trivial topology of $U(1)_Y$

Mikko Laine (Bern)

Phase diagram for the Standard Model:



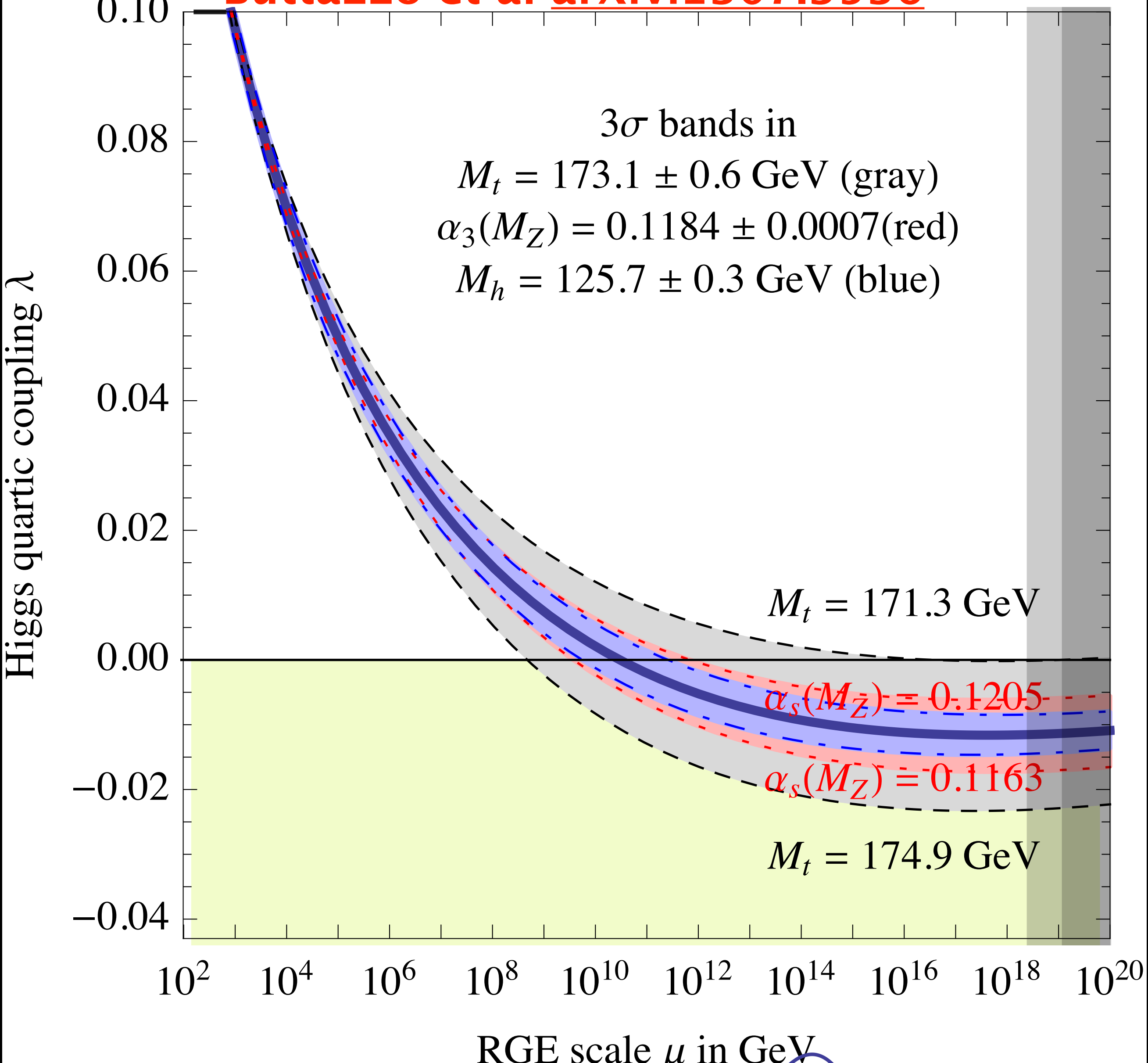
<H>=0 from gauge invariance (Elitzur)

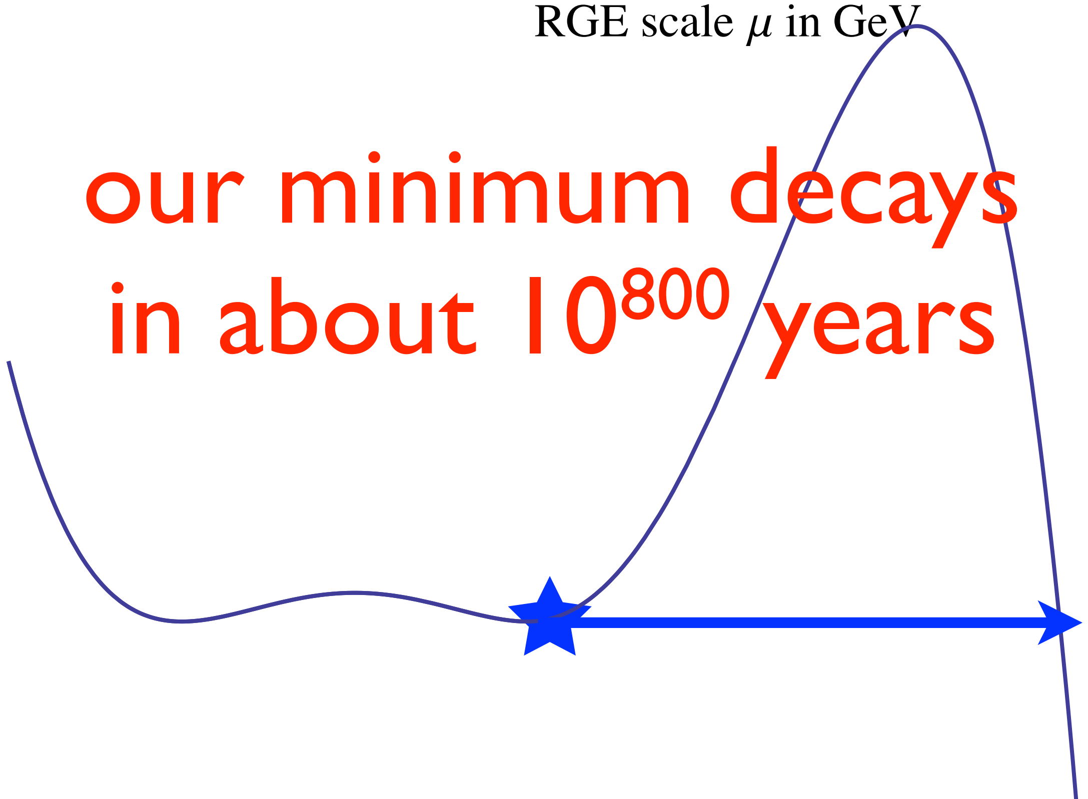
$<H^\dagger H>$ is not an order parameter

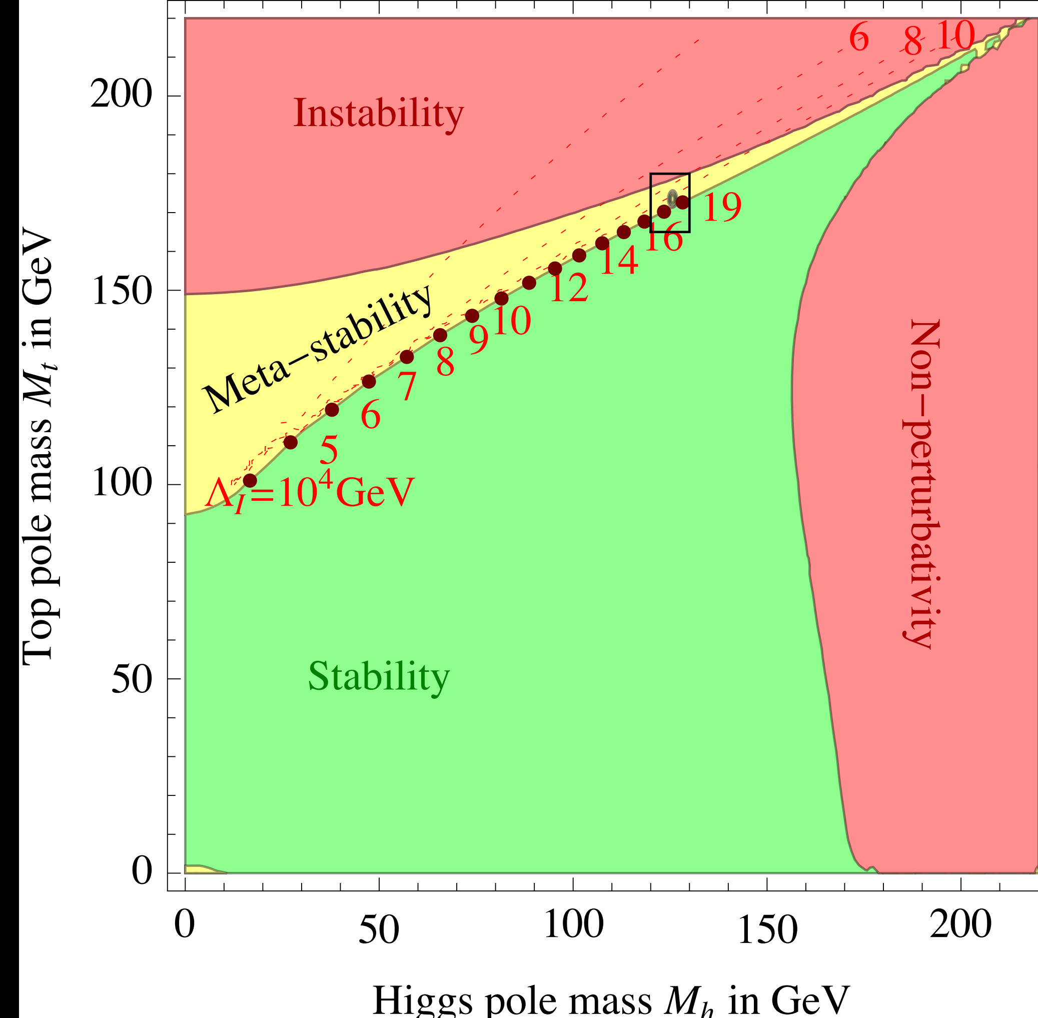
for m_h=126GeV, it is crossover

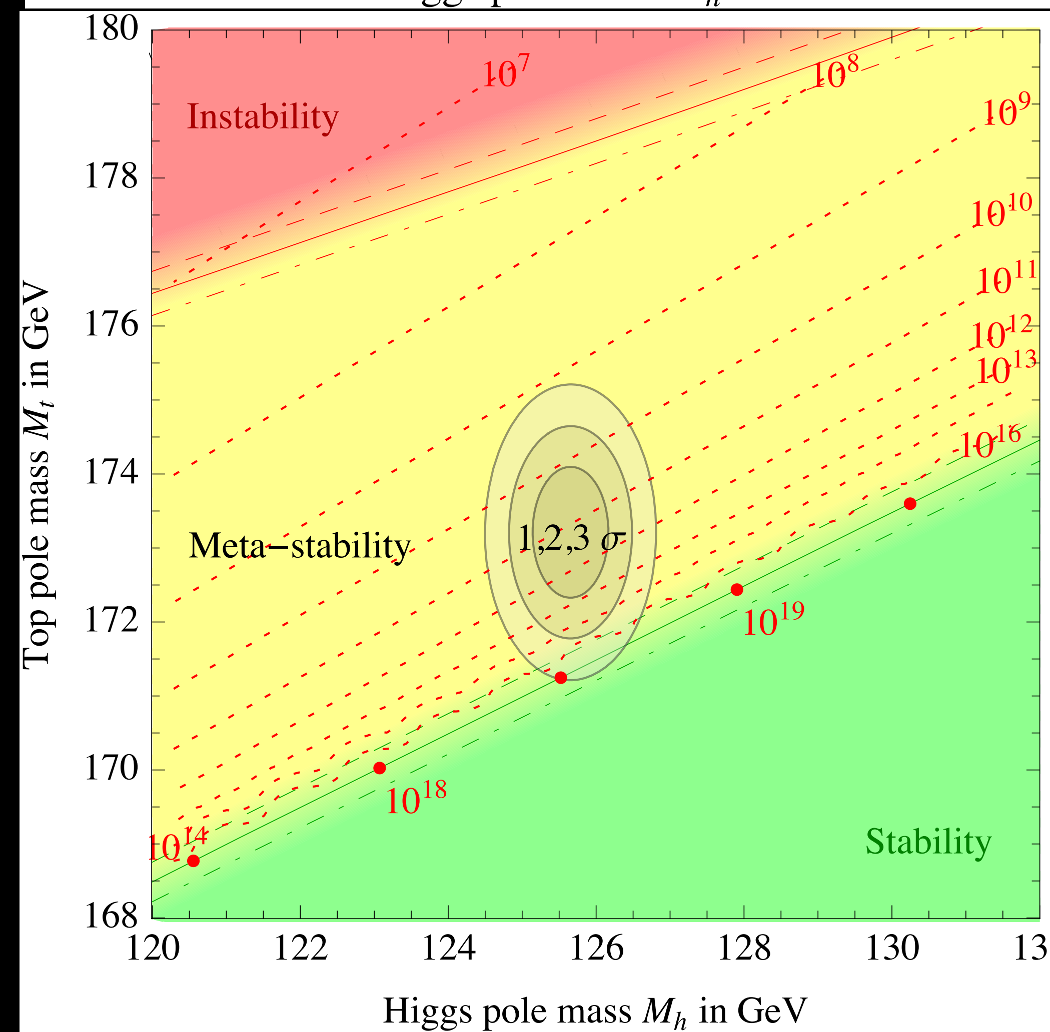
No phase transition in the Minimal Standard Model

Buttazzo et al arXiv:1307.3536







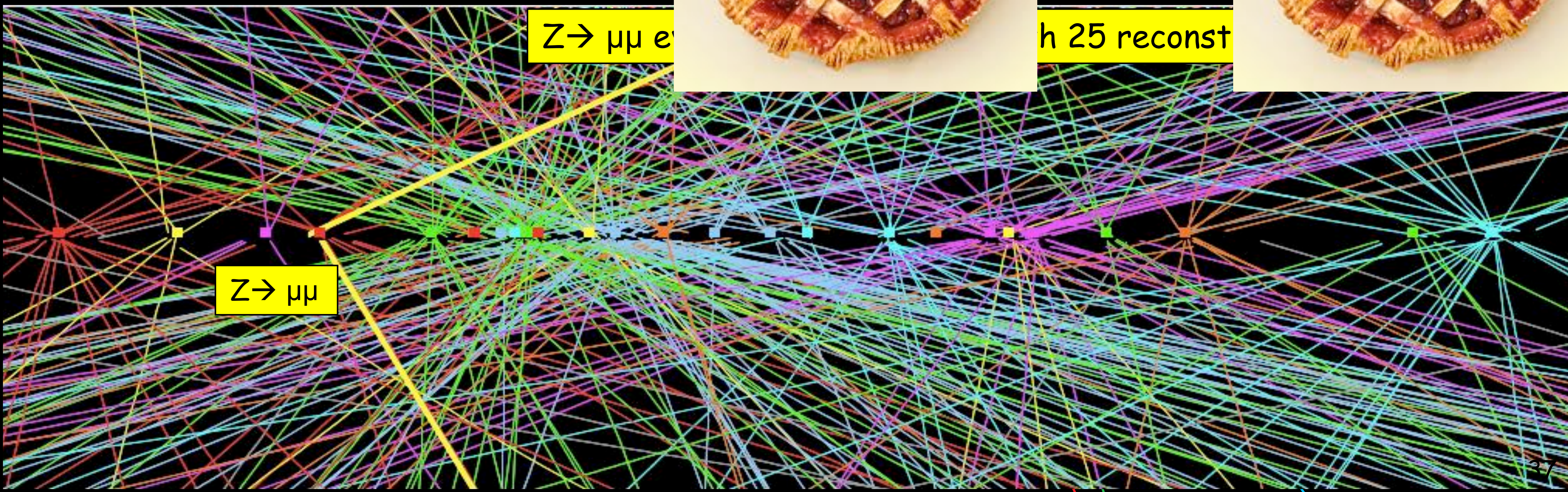


our minimum decays in about 10^{800} years

Spin

- every elementary particles spin forever
- electrons, photons, quarks, ...
- only Higgs boson doesn't spin
- Faceless! A *spooky particle*
- I had proposed "Higgsless theories"
- *Is it the only one?*
- *does it have siblings? relatives?*
- *Maybe it's spinning in extra dimensions?*
- *maybe composite?*
- *why did it freeze in?*

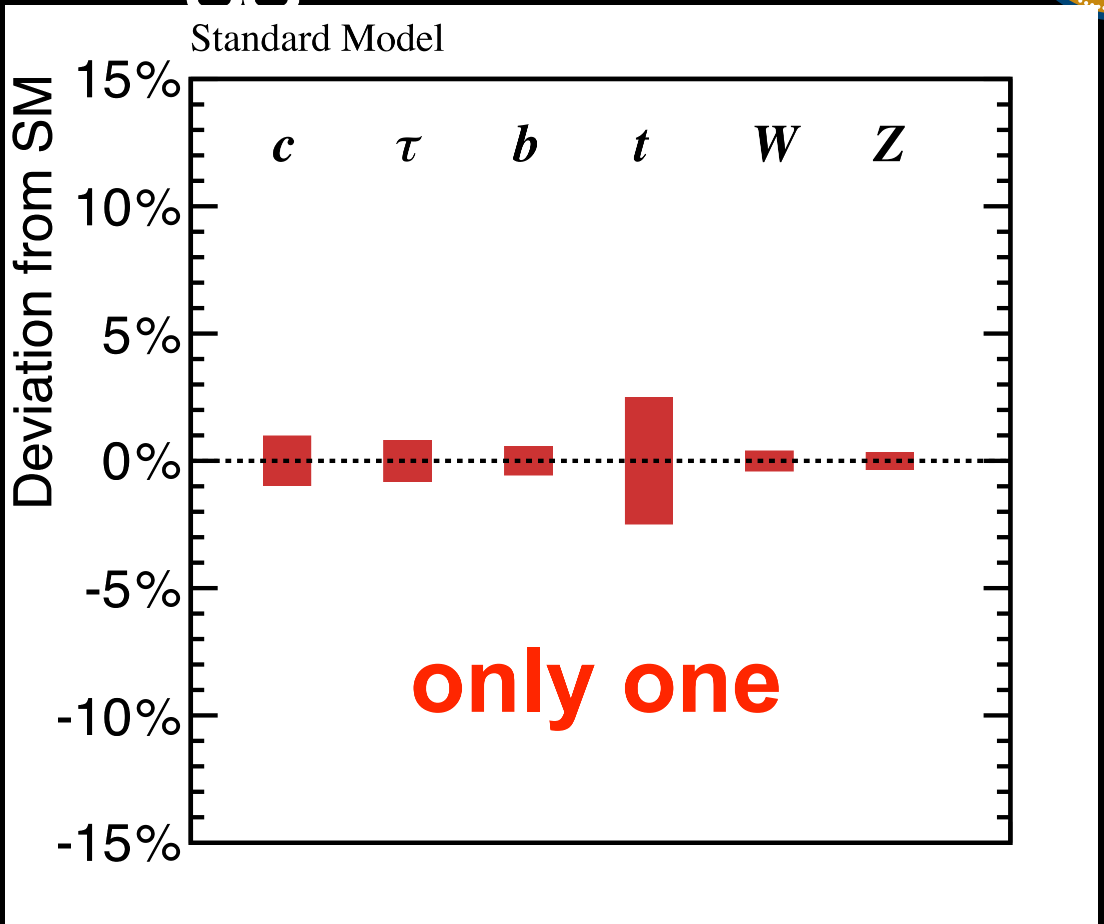
Intl Line llide



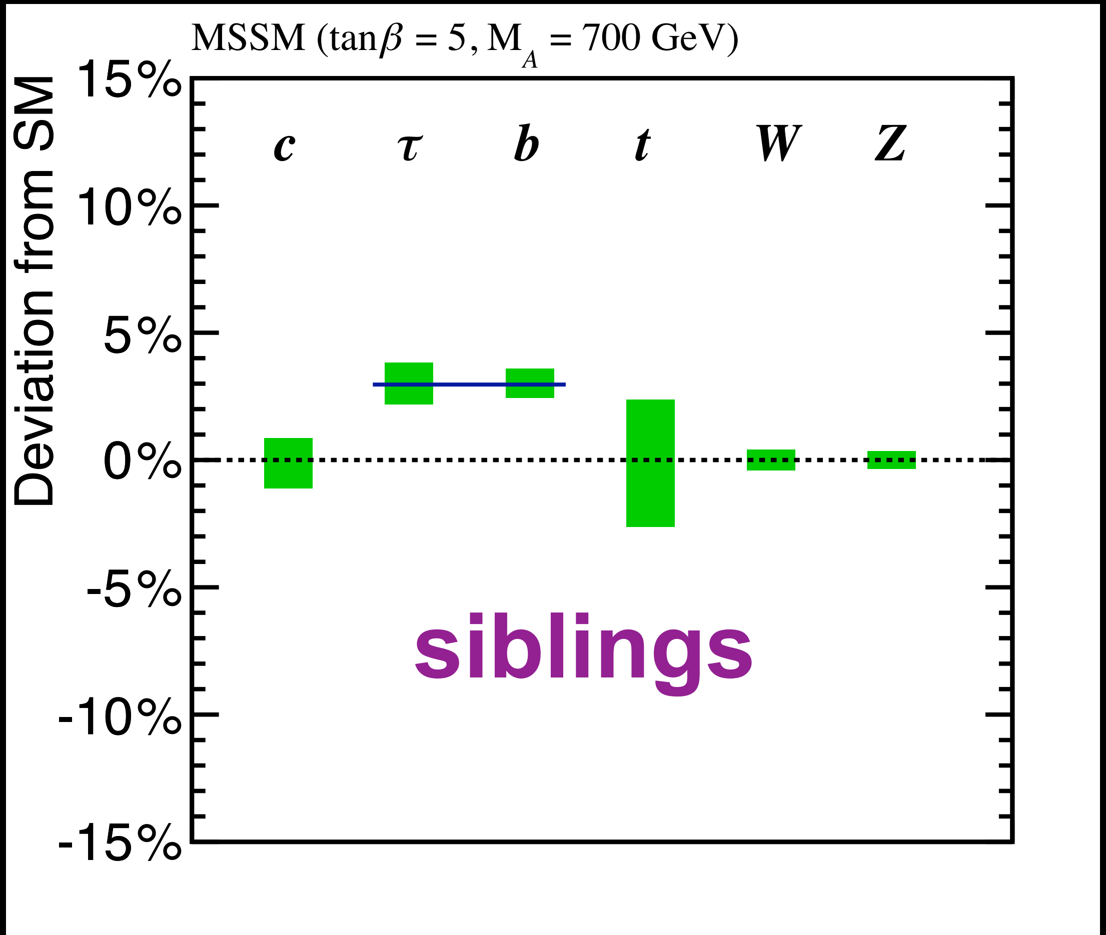
- e^+, e^- are elementary particles

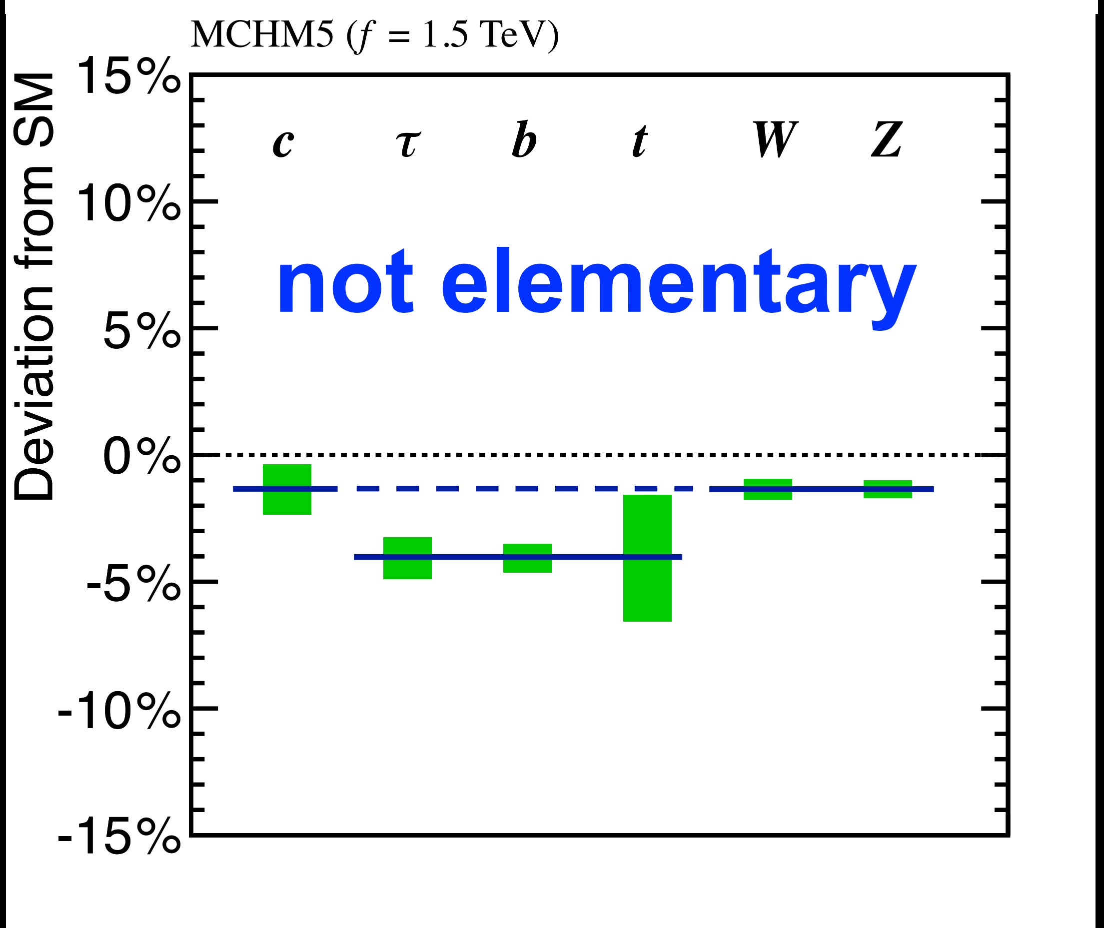
What exactly is the Higgs boson?

Only one scalar boson?
Siblings and relatives?
Maybe not elementary?



Lumi 1920 fb-1, sqrt(s) = 250 GeV
Lumi 2670 fb-1, sqrt(s) = 500 GeV

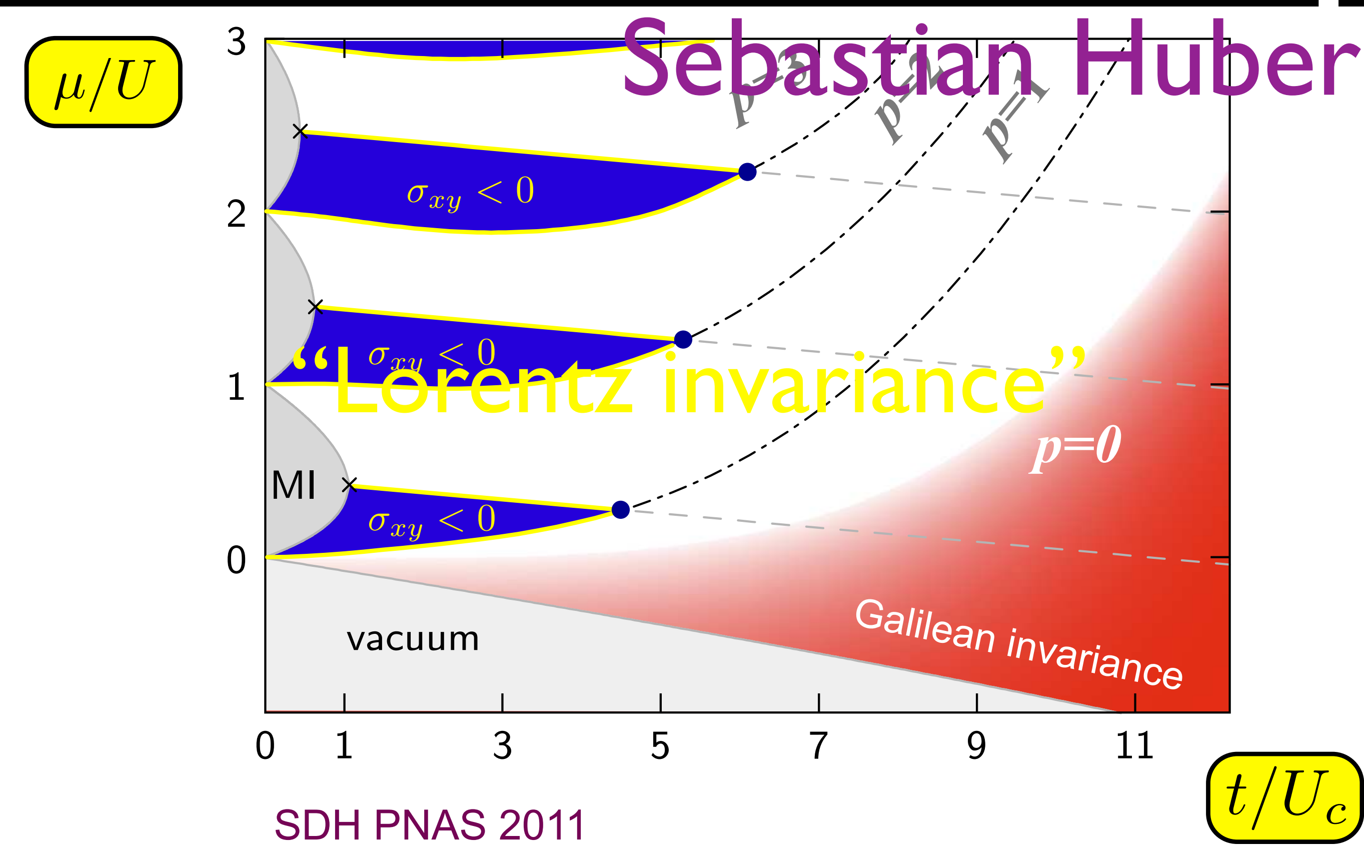


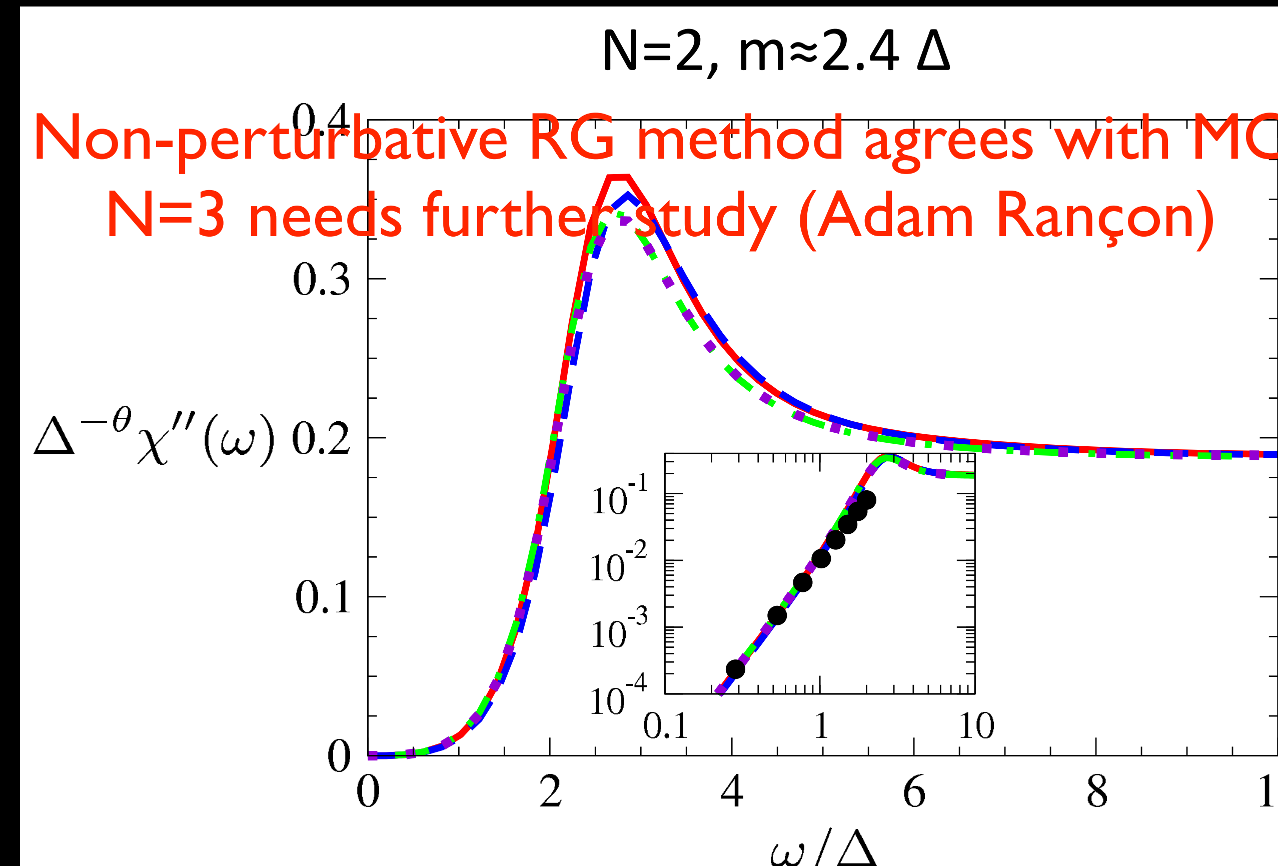


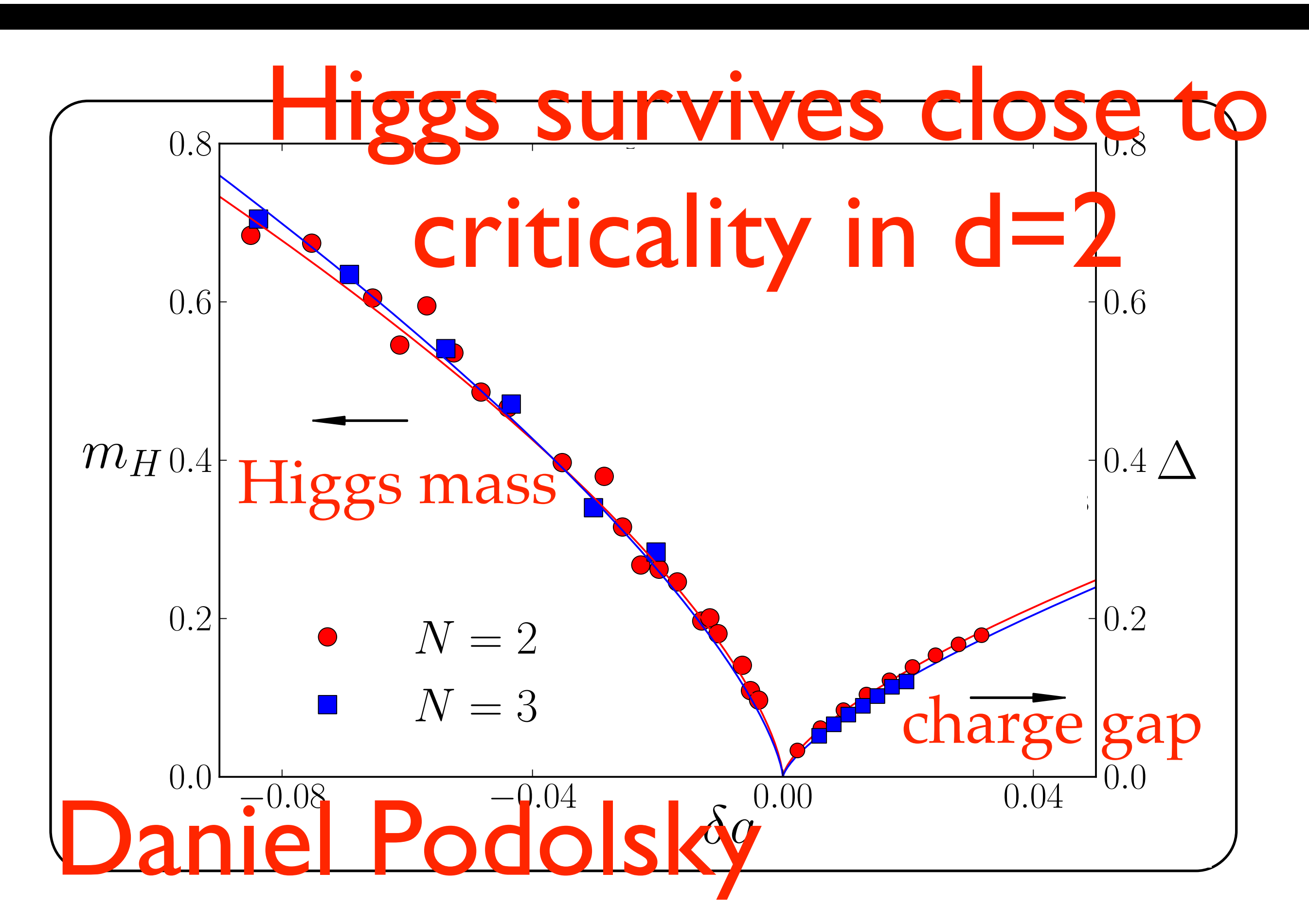
Highlights

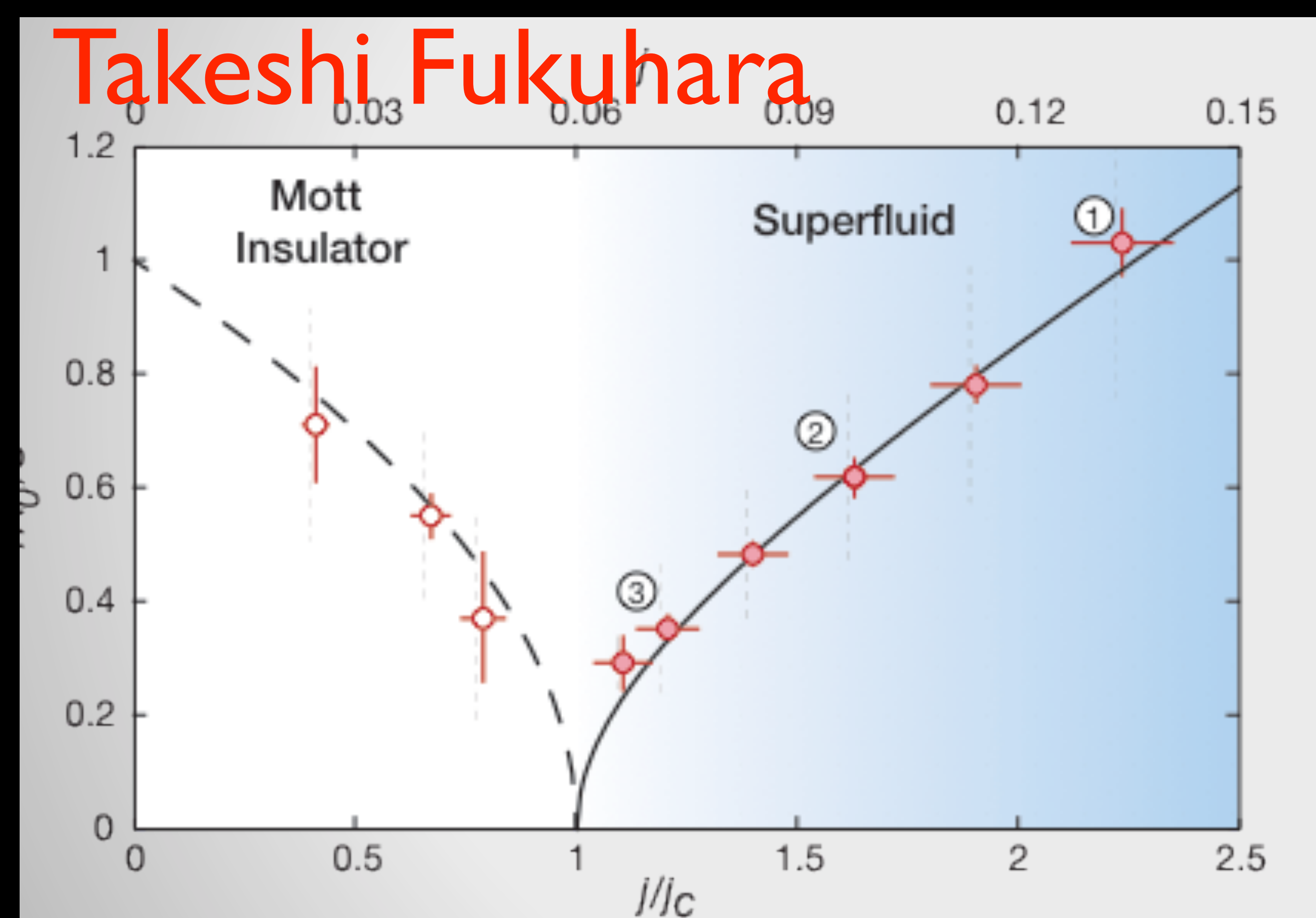
bosons in optical lattices

+Kun Chen

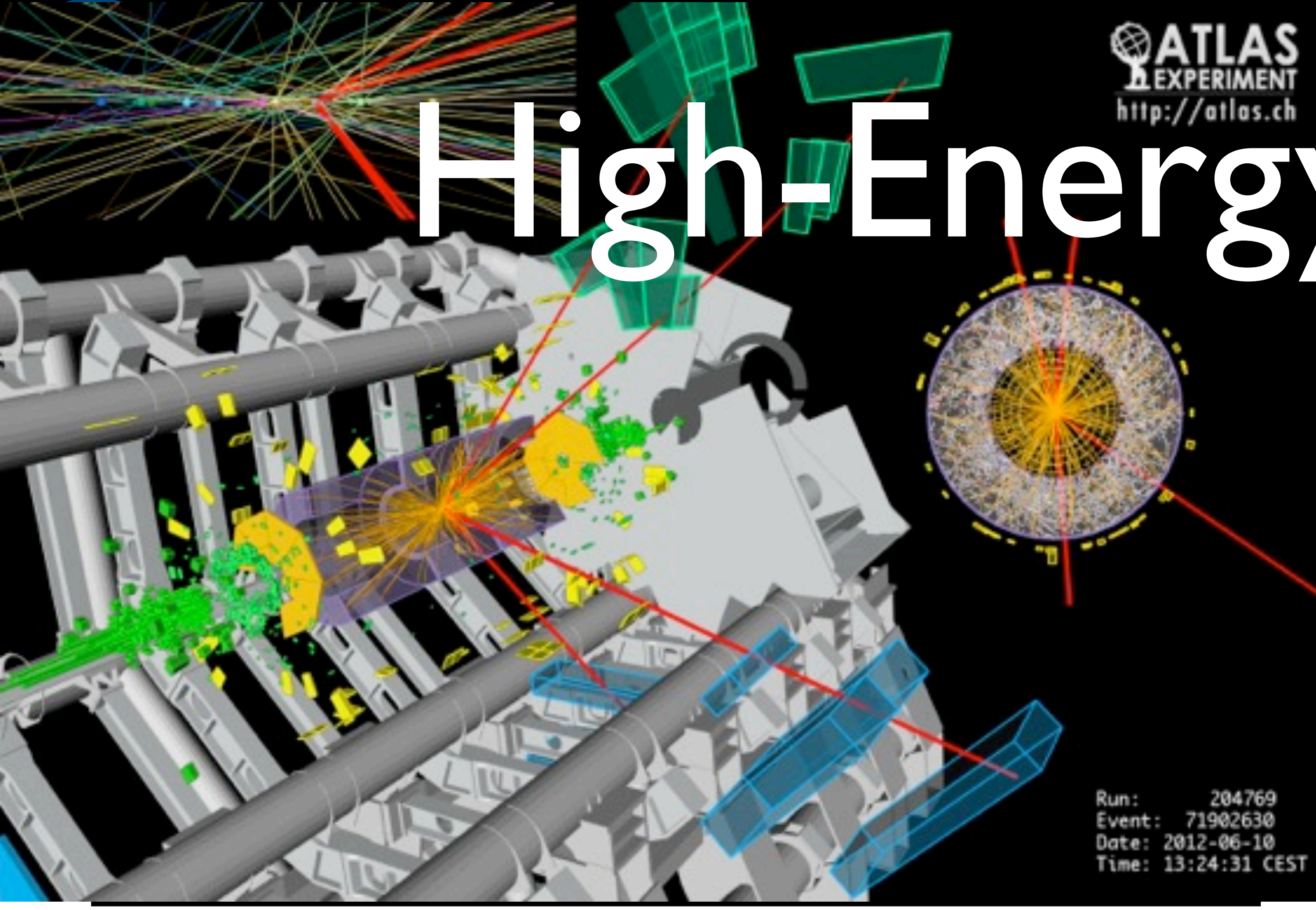


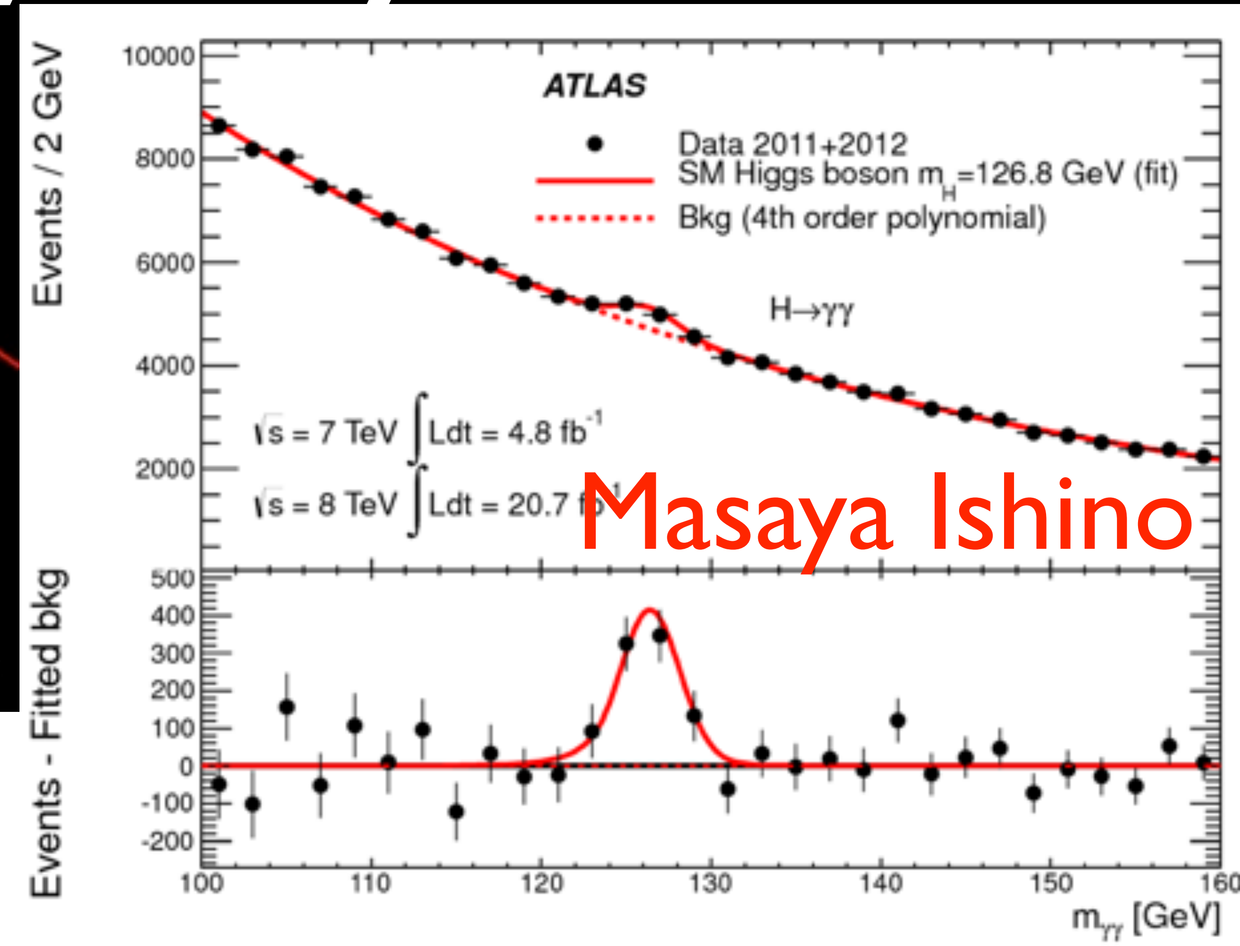






High-Energy Physics





Masaya Ishino

Fine-tuning worse than 1% seems unavoidable in MSSM.

(MSSM =Minimal SUSY Standard Model)

What does it imply ??

1. **No SUSY ?**

Koichi Hamaguchi

2. (It's anyway fine-tuned, then....)

   Very heavy SUSY ? (10~100 TeV, or even higher...)

3. (still.....)

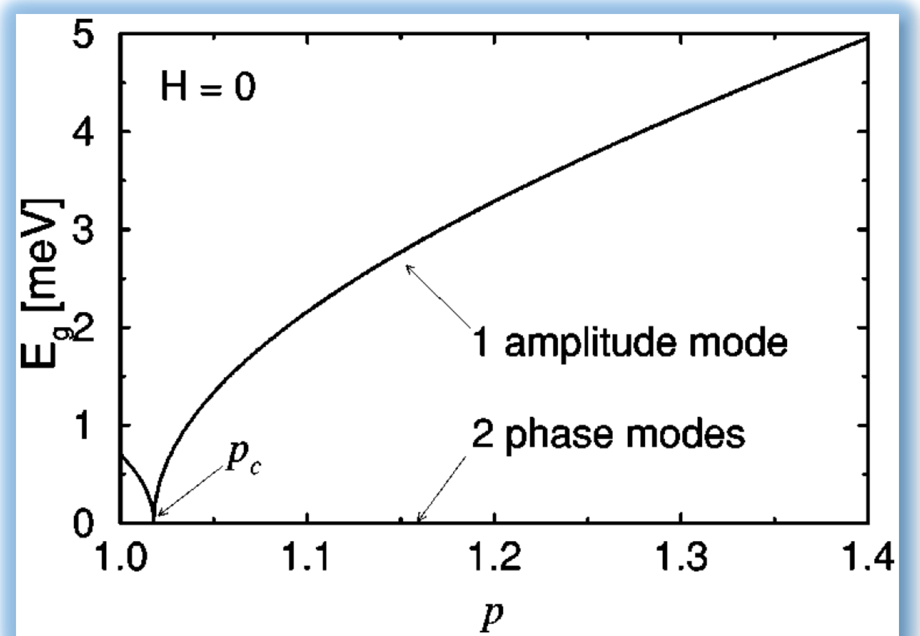
   (0.1-1) TeV SUSY ? (fine-tuned, but less than 2 and 3...)

composite?

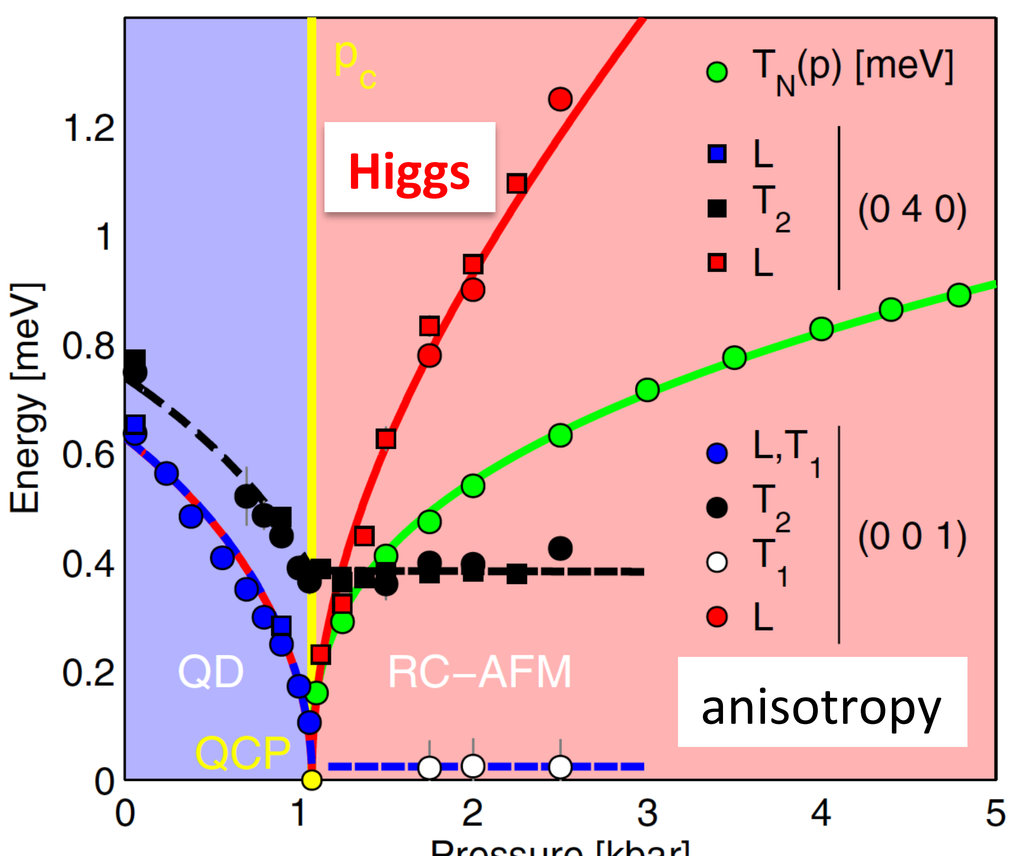
Mikhail Zubkov

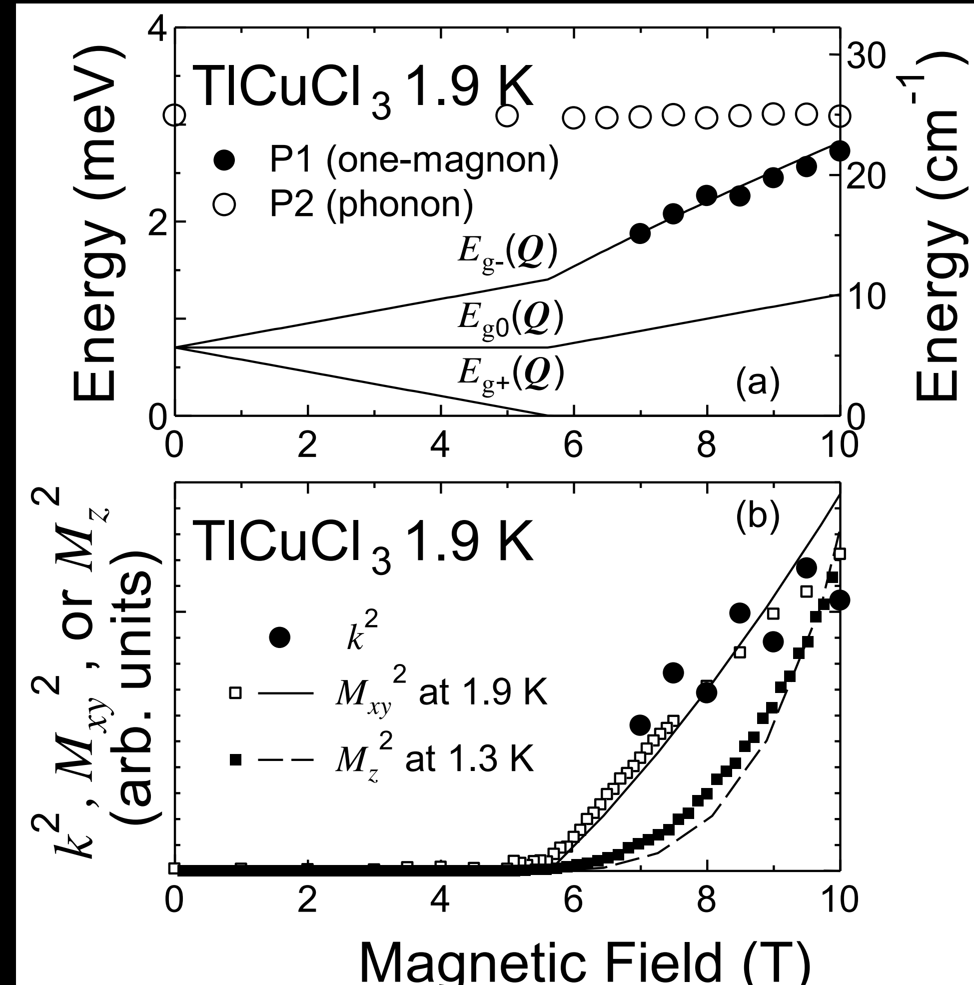
quantum magnets

Masashige Matsumoto



Ch. Rüegg et al., PRL (2008)





Magnetic field dependences of P1 energy (upper panel) and integrated intensity (lower panel).

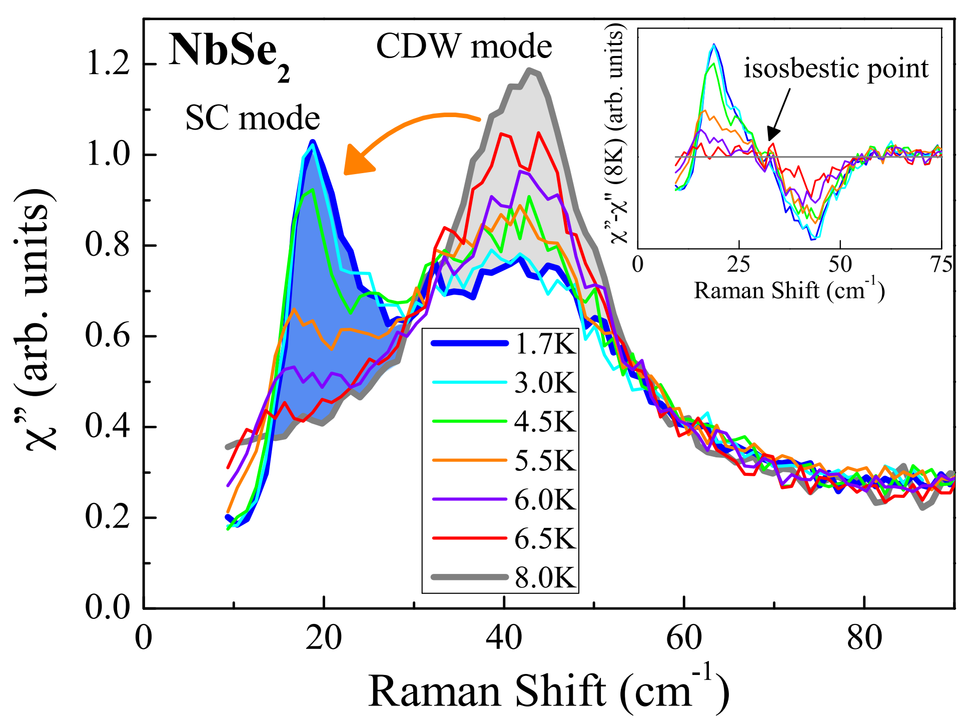
2nd order magnetic Raman process
Haruhiko Kuroe

fermionic superfluids, superconductors

True test of theory: Conservation of Weight.

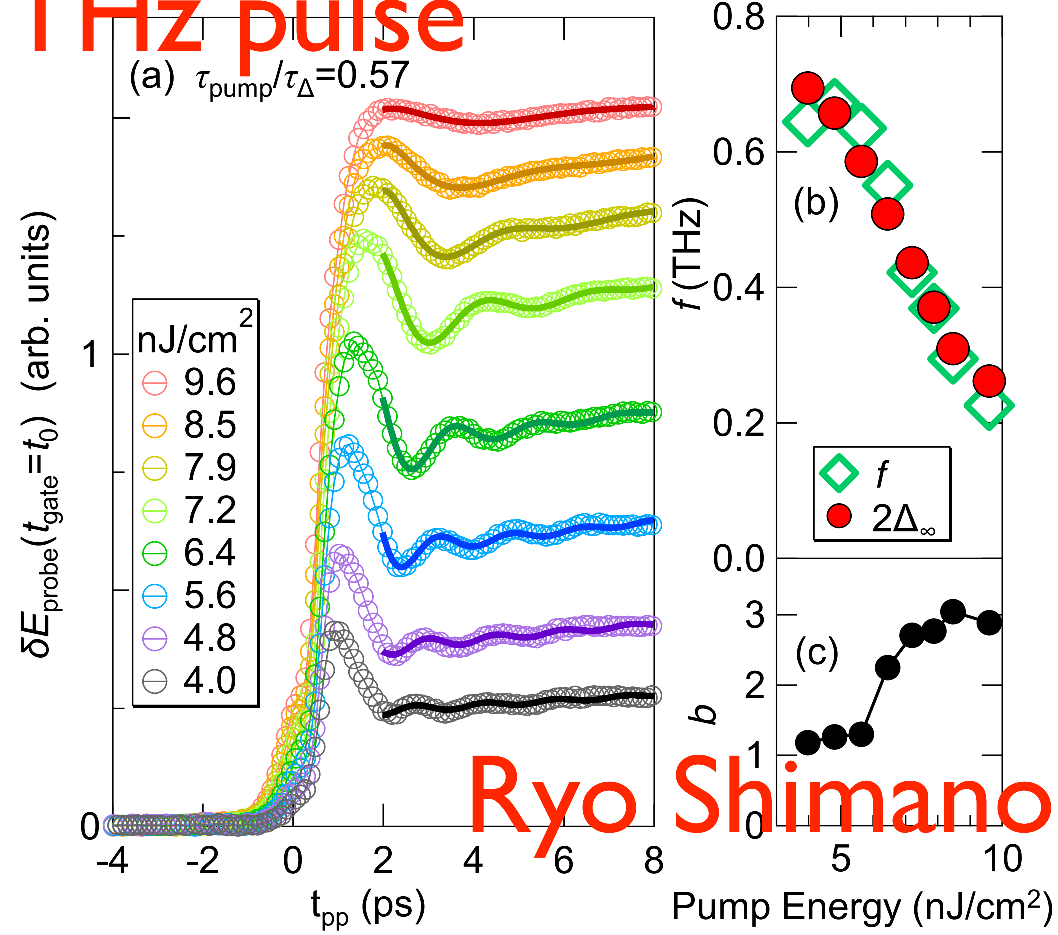
New Experiments: (M-A. Méasson, A. Sacuto, Paris)

Phys. Rev. B 2013 (and preprint)

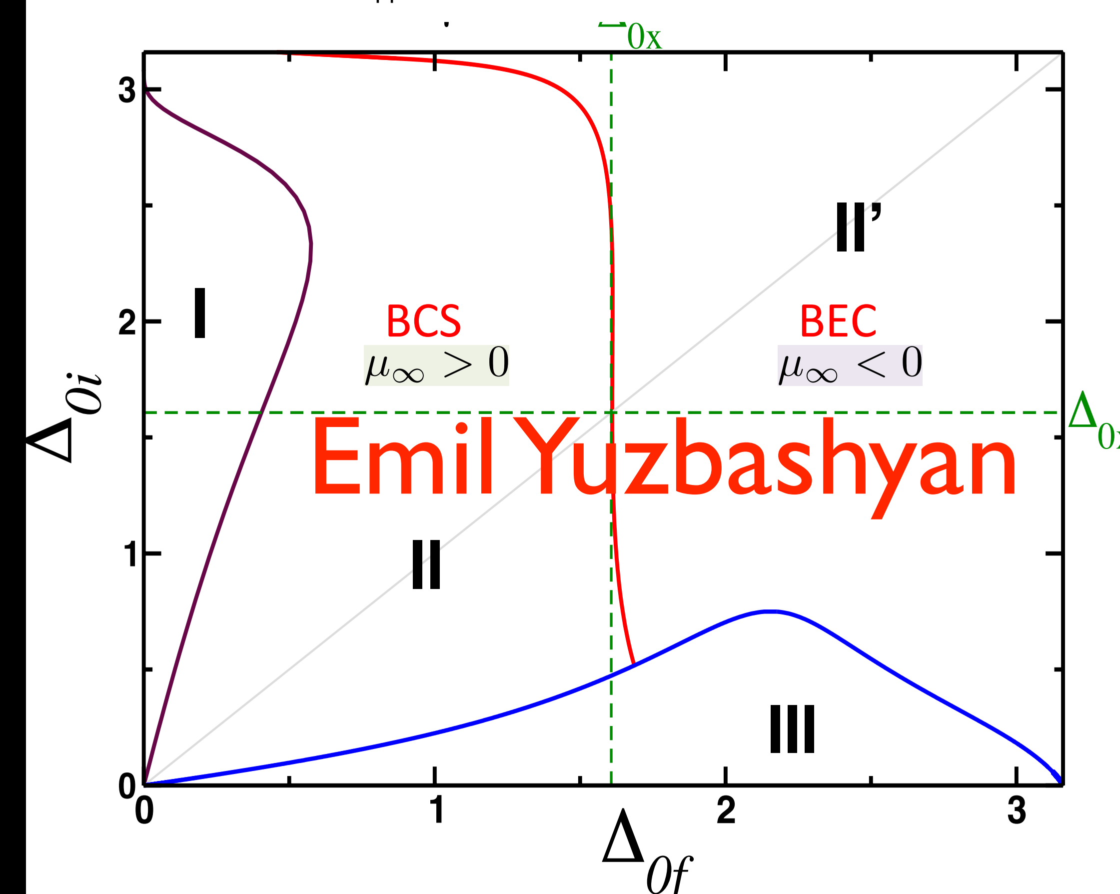


Chandra Varma

THz pulse



Ryo Shimano

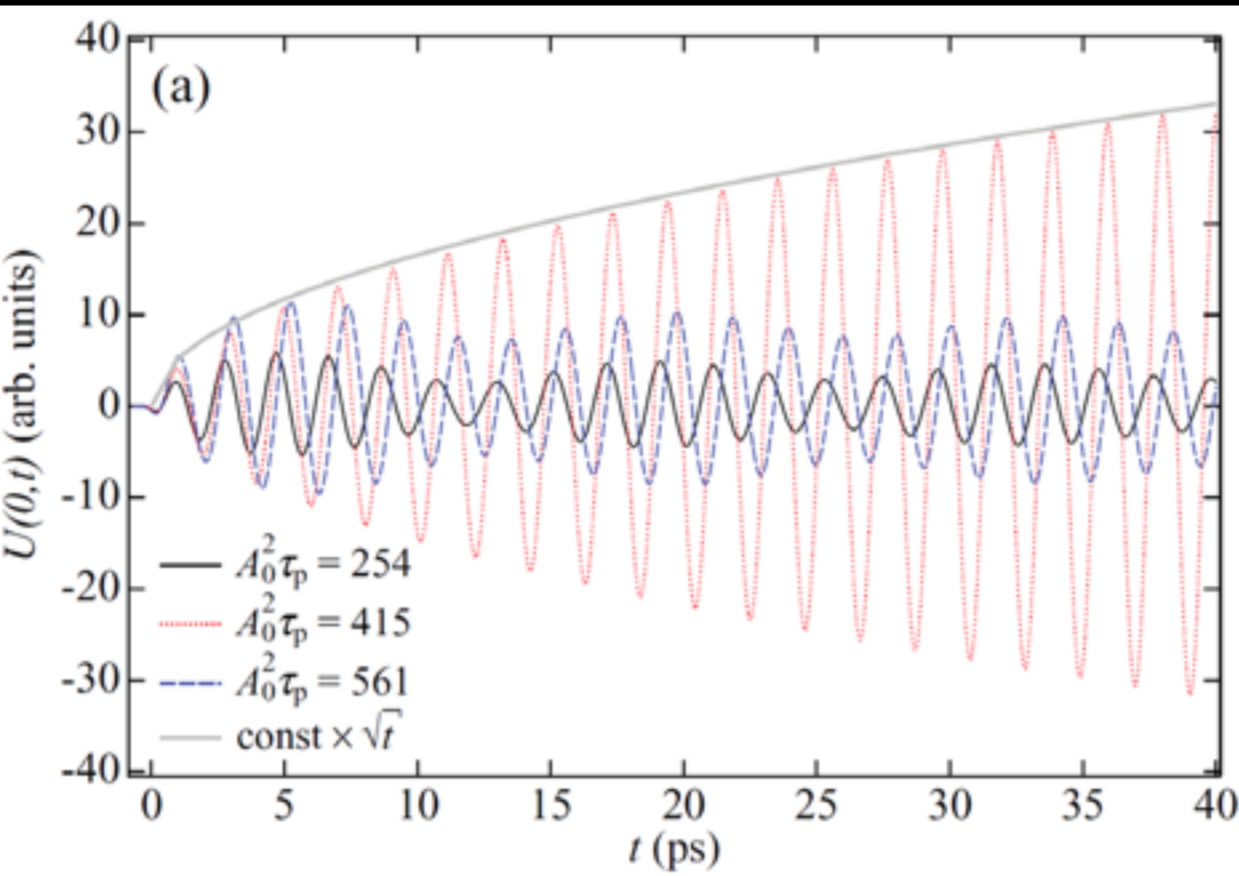


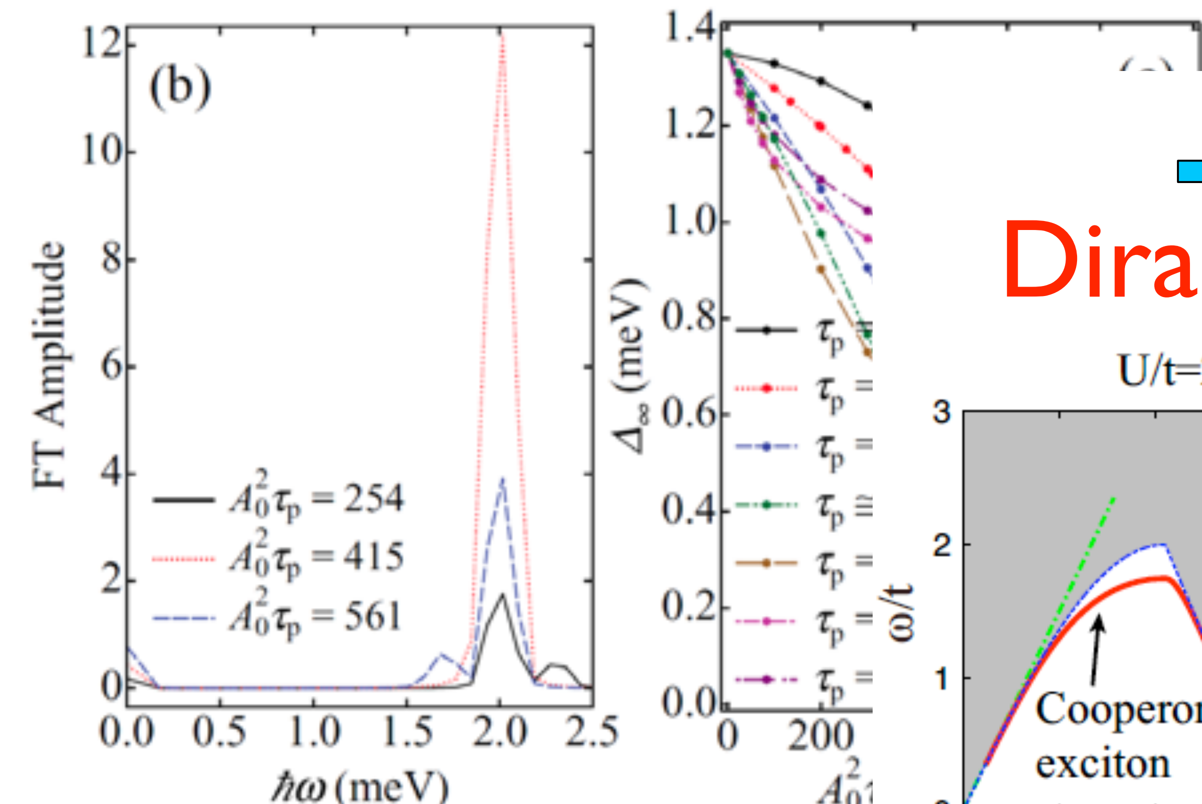
Emil Yuzbashyan

Steady states of BCS dynamics

Δ_{0i}, Δ_{0f} – *ground state* gaps for g_i, g_f

Dirk Manske





pseudospin resonance in superconductors

Analytical solution:

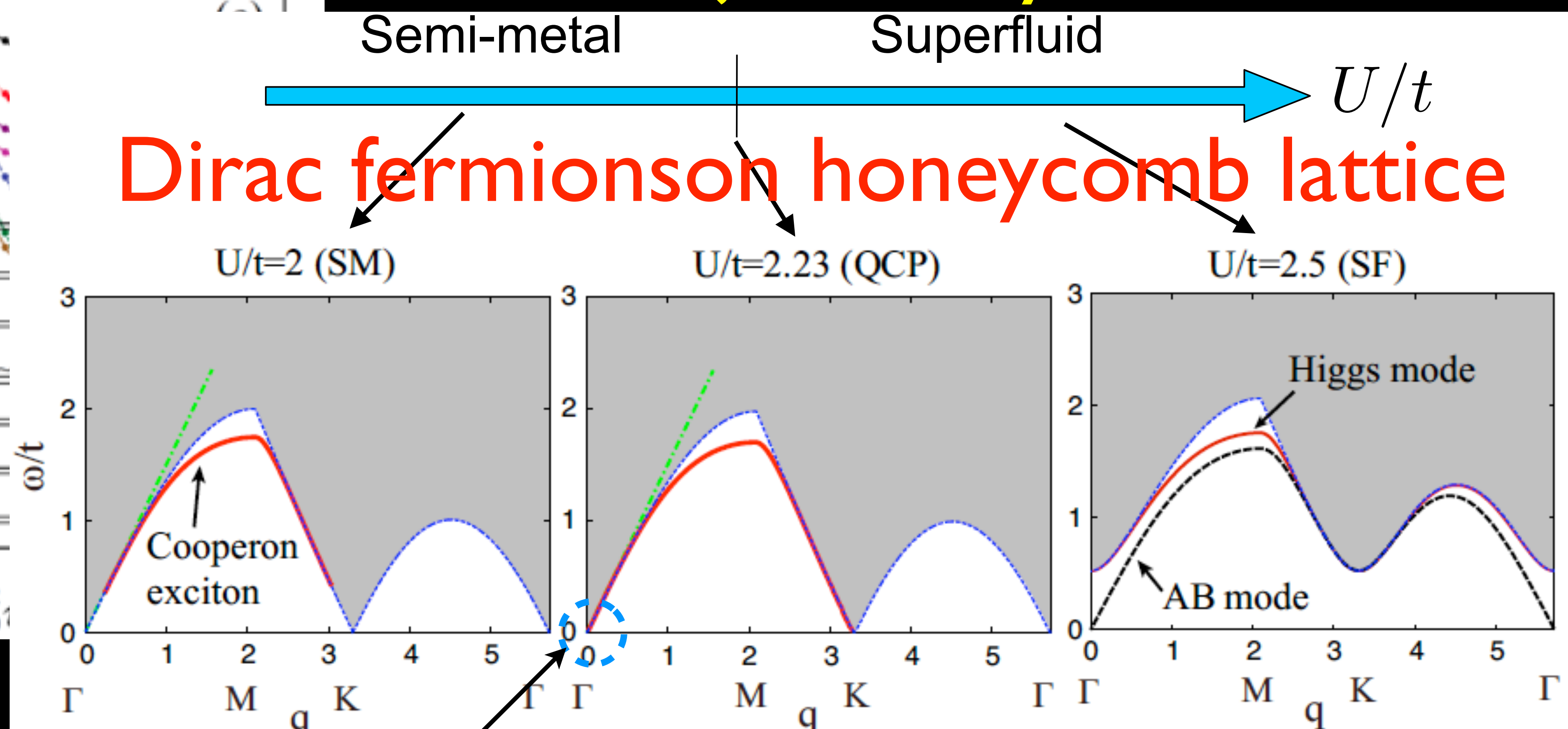
For U quench, see Yuzbashyan, Dzero (2006); Barankov, Levitov (2006)

Naoto Tsuji

$$\frac{\delta\Delta(t)}{\alpha_1 e^2 A^2 \Delta} \sim \frac{1}{4\lambda}\left[\frac{2}{\pi^{3/2}}\frac{\Omega^2}{\Omega^2-\Delta^2}\frac{1}{\sqrt{\Delta t}}\cos\left(2\Delta t+\frac{\pi}{4}\right)-1\right] + \frac{1-\cos 2\Omega t}{4} + \boxed{\frac{1}{4\lambda}\times\begin{cases}\dfrac{\Omega}{\sqrt{\Delta^2-\Omega^2}}\dfrac{\cos 2\Omega t}{\sin^{-1}\left(\frac{\Omega}{\Delta}\right)} & \Omega<\Delta \\ \dfrac{\Omega}{\sqrt{\Omega^2-\Delta^2}}\dfrac{\cos(2\Omega t-\varphi)}{\sqrt{\left[\cosh^{-1}\left(\frac{\Omega}{\Delta}\right)\right]^2+\left(\frac{\pi}{2}\right)^2}} & \Omega>\Delta\end{cases}}$$

Shunji Tsuchiya

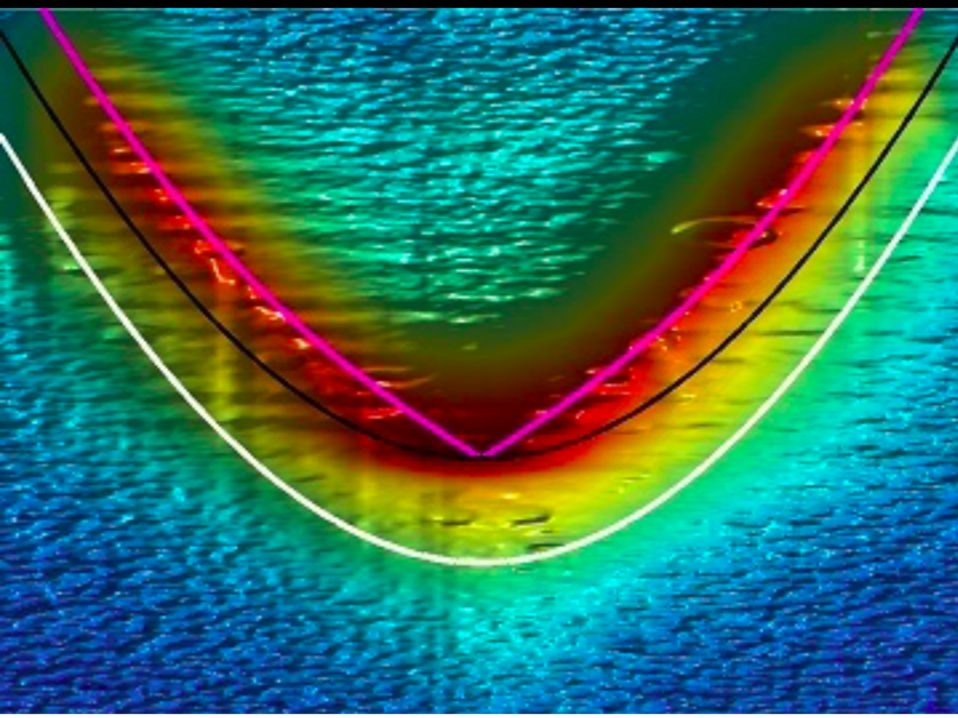
Dirac fermionson honeycomb lattice

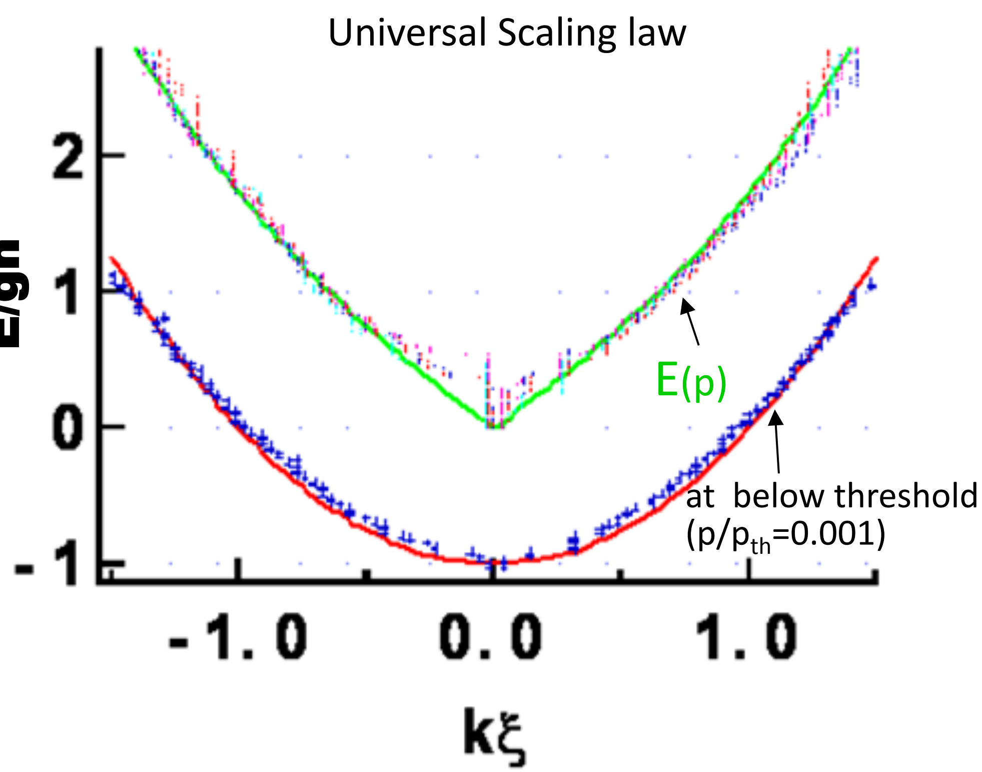


exiton-polariton, cavity photon

Peter Littlewood

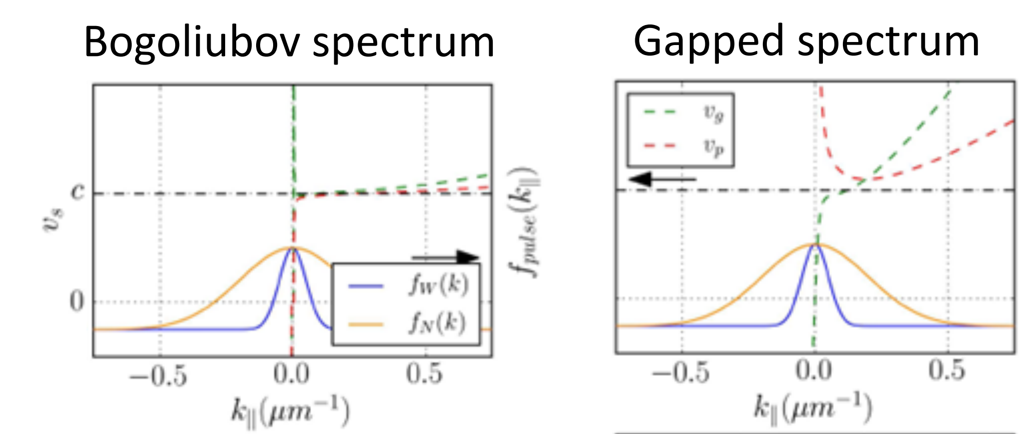
Wuming Liu



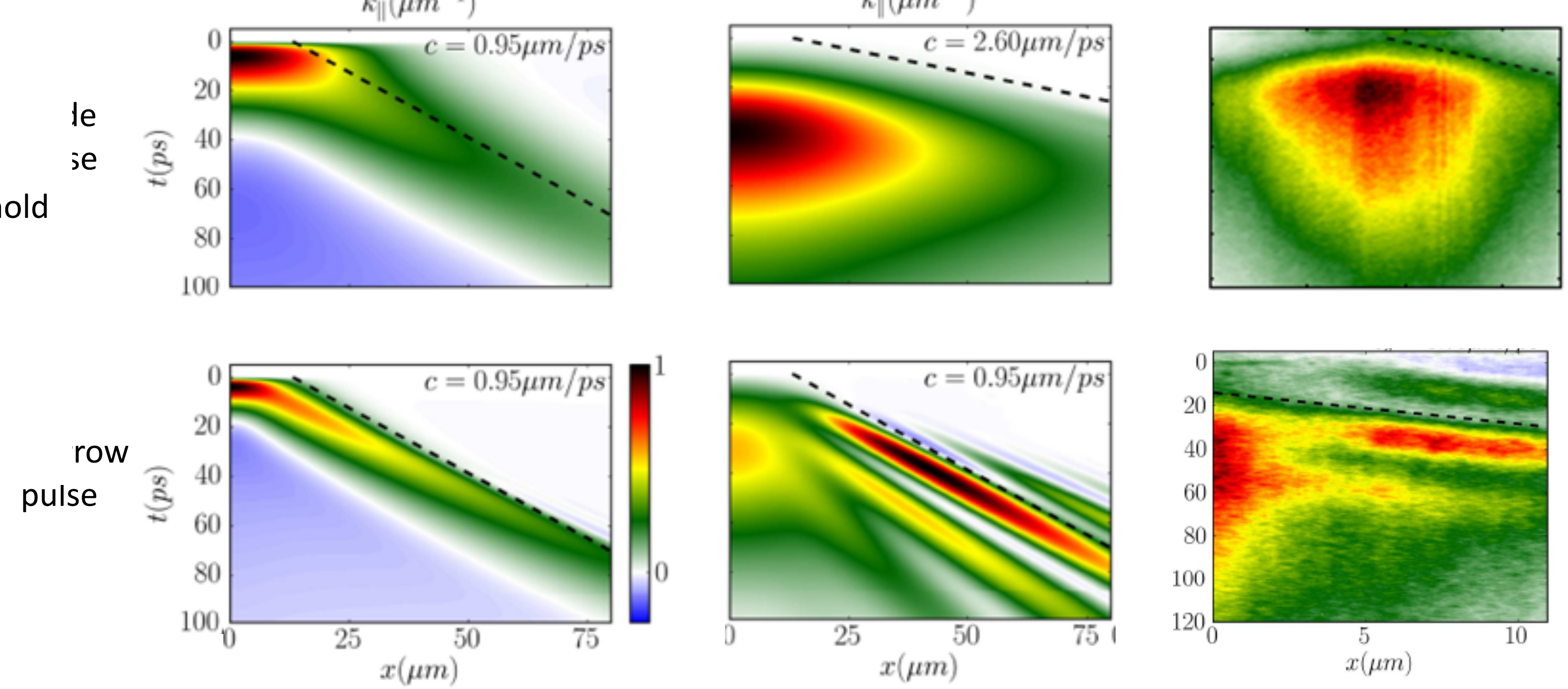


Yoshihisa Yamamoto

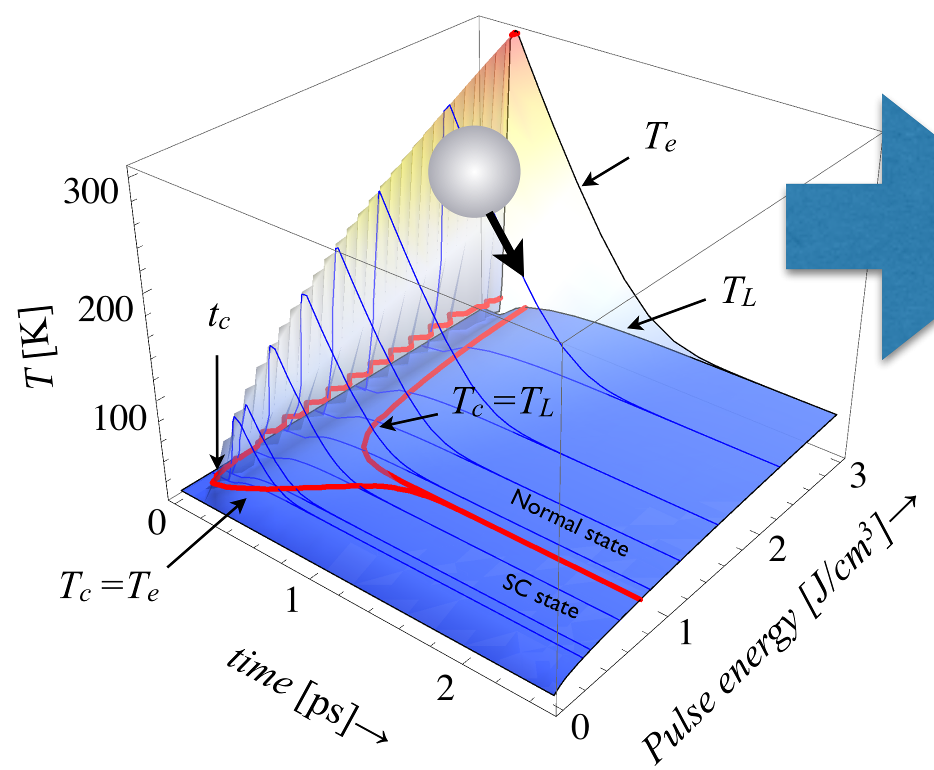
Dispersive Shock Wave and Dark Solution

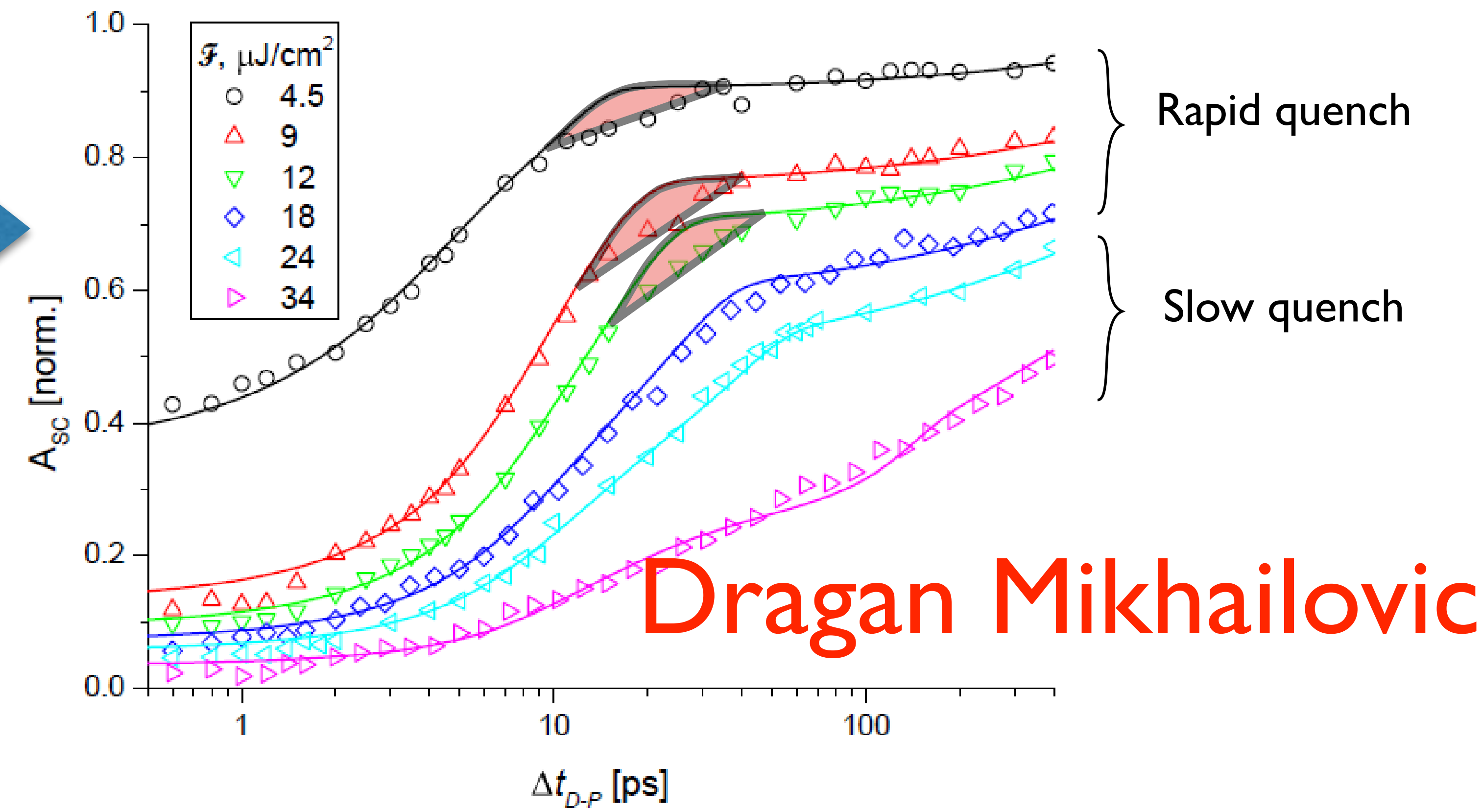


Experimental results suggest a gapped spectrum

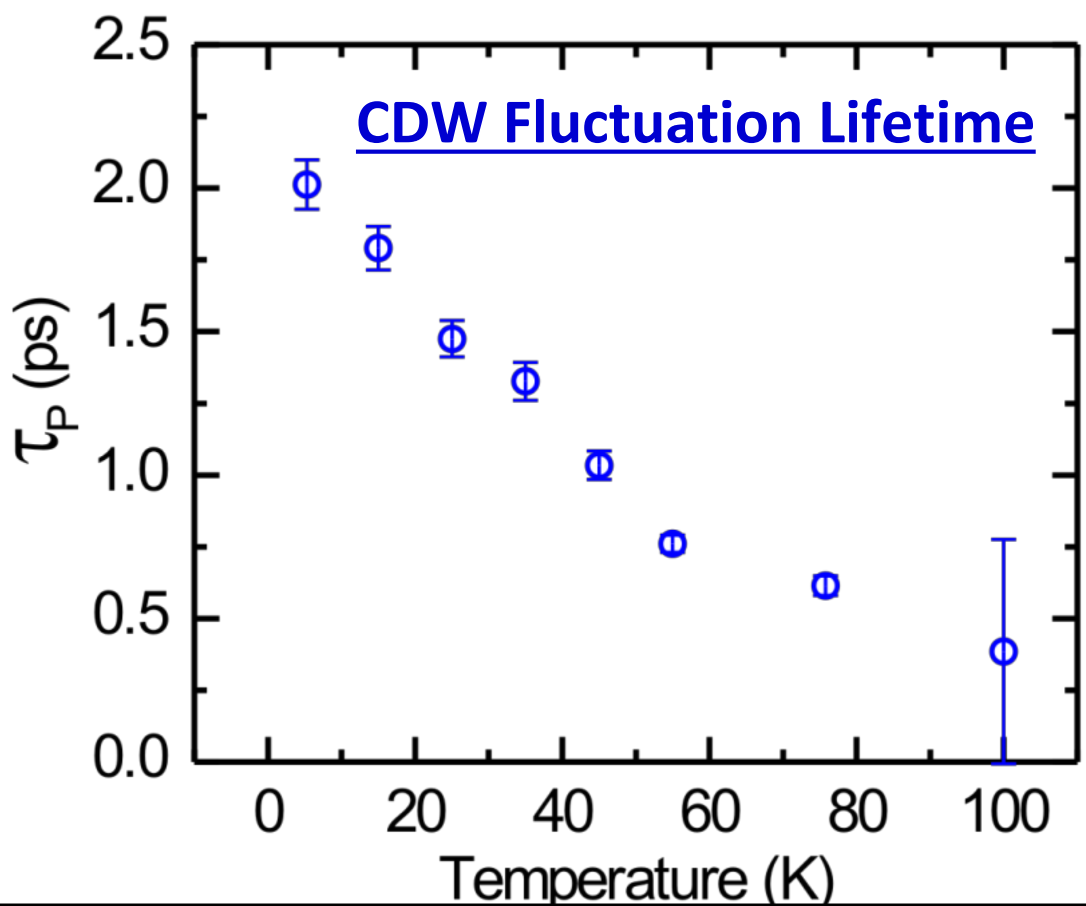


charge density waves





Dragan Mikhailovic



Source of damping?

- **If intrinsic damping then $\Gamma \sim T^2$ or $\Gamma \sim T^5$**
- **CDW fluctuations**

Phason lifetime

↓

Measure of CDW lifetime

Fahad Mahmood

Nambu-Goldstone mode

Summary

For SSB of internal symmetries

- Independent elastic variable=N_{BS}
- $N_{\text{type-B}} = \frac{1}{2}\text{rank}\langle[iQ_a, Q_b]\rangle$
- $N_{\text{type-A}} = N_{\text{BS}} - N_{\text{type-B}}$
- $N_{\text{gapped}} = \frac{1}{2}\left(\text{rank}\langle[iQ_a, \phi_i]\rangle - N_{\text{type-A}}\right)$

Yoshimasa Hidaka

The second derivative term in the effective Lagrangian
Karasawa, Gongyo('14)

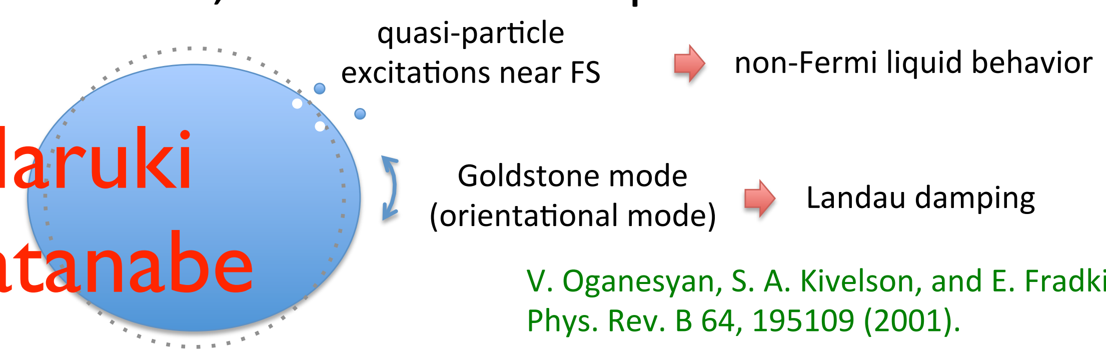
Type-A (Type-I): $\omega = ak - ibk^2$
Type-B (Type-II): $\omega = a'k^2 - ib'k^4$

Non-Fermi liquid through NGBs

- Usually, interaction between NGBs with other fields are derivative coupling $\psi^\dagger \vec{\nabla}\psi \cdot \vec{\nabla}\theta$

interaction vanishes in the low-energy, long wavelenghth limit

- However, there is an exception



Haruki Watanabe

V. Oganesyan, S. A. Kivelson, and E. Fradkin, Phys. Rev. B 64, 195109 (2001).

- I pinned down the condition for NFL:

$$[Q, \vec{P}] = 0$$

HW and Ashvin Vishwanath, arXiv:1404.3728

Muneto Nitta
internal and spacetime symmetries may not commute via central extension
NGBs become "type-B"

Yusuke Kato
anomalous tunneling is common to NGBs, for $k \to 0$ due to constancy of Noether currents

Higgs and Nambu-Goldstone bosons with or without Lorentz invaraiance

Hitoshi Murayama (Berkeley, Kavli IPMU)
+ Haruki Watanabe (Berkeley)
+ Tomáš Brauner (TU Wien)

arXiv:1203.0609, 1302.4800, 1303.1527, 1401.8139, 1402.7066, 1403.3365, 1405.0997

Applications

$$n_{NGB} = n_{BG} - \frac{1}{2}\text{rank}\rho$$

example	coset space	BG	NGB	rank ρ	theorem
anti-ferromagnet	O(3)/O(2)	2	2	0	2=2-0
ferromagnet	O(3)/O(2)	2	1	2	1=2-1
superfluid ^{4}He	U(1)	1	1	0	1=1-0
superfluid ^{3}He B phase	O(3)xO(3)xU(1)/O(2)	4	4	0	4=4-0
(in magnetic field)	O(2)xO(3)xU(1)/O(2)	4	3	2	3=4-1
BEC (F=0)	U(1)	1	1	0	1=1-0
BEC (F=1) polar	O(3)xU(1)/U(1)	3	3	0	3=3-0
BEC (F=1) ferro	O(3)xU(1)/SO(2)	3	2	2	2=3-1
3-comp. Fermi liquid	U(3)/U(2)	5	3	4	3=5-2
neutron star	U(1)	1	1	0	1=1-0
kaon cond. (μ=0)	U(2)/U(1)	3	3	0	3=3-0
kaon cond. (μ≠0)	U(2)/U(1)	3	2	2	2=3-1
crystal	$\mathbb{R}^3/\mathbb{Z}^3$	3	3	0	3=3-0
(in magnetic field)	$\mathbb{R}^3/\mathbb{Z}^3$	3	2	2	2=3-1

Low-*E* Effective L

- consider $\pi^a(x)$ fields: $\mathbb{R}^{3,1} \to G/H$ ("*pions*")
- Write *action* $S=\int d^4x\, L(\pi,\partial\pi)$
  which is *G*-invariant
- expand in powers of derivative, keep low orders (often up to the second order)

$$\mathcal{L}_{\rm eff} = g_{ab}(\pi)\partial_\mu\pi^a\partial^\mu\pi^b$$

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

Leutwyler

General formula

$$\mathcal{L}_{\text{eff}} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- Define a commutator among broken generators

$$\rho_{ab} = \frac{-i}{V}\langle 0|[Q^a, Q^b]|0\rangle \qquad c_a\dot{\pi}^a \approx \frac{1}{2}\rho_{ab}\pi^b\dot{\pi}^a$$

- n_B = 1/2 rank ρ counts the number of canonically conjugate pairs (Type-B) generically $\Rightarrow$ $E \propto p^2$
  - each pair describes one d.o.f.
- the remainder $n_A = n_{BG} - 2n_B$
  - stand-alone NGB d.o.f. (Type-A) generically $\Rightarrow$ $E \propto p$

$$n_{NGB} = n_A + n_B = n_{BG} - \frac{1}{2}\text{rank}\rho$$

conjectured by Watanabe and Brauner, agrees with Hidaka

Presymplectic Geometry

closed G-inv
$d\,c = \pi^*\omega_2$

$$\begin{tikzcd} G/H \arrow[d, "\pi"] & F \arrow[l] \\ \omega_2 \; G/U & \end{tikzcd}$$

symplectic
homogeneous

Type A
$E \propto p$

Type B
$E \propto p^2$

$$\omega_2 = \frac{1}{2}\rho_{ab}\, d\pi^a \wedge d\pi^b + O(\pi)^3 \qquad \rho_{ab} = \frac{-i}{V}\langle 0|[Q^a, Q^b]|0\rangle$$

NGBs for generators *a* and *b* are symplectic pairs and describe a single degree of freedom

$$\dim G - \dim H = n_A + 2n_B$$

allows for complete classification of possibilities

n_A	n_B	U
30	0	.
20	5	$SU(5) \times U(1)$
14	8	$SU(4) \times SU(2) \times U(1)$
12	9	$SU(4) \times U(1)^2$
12	9	$SU(3)^2 \times U(1)$
8	11	$SU(3) \times SU(2) \times U(1)^2$
6	12	$SU(3) \times U(1)^3$
6	12	$SU(2)^3 \times U(1)^2$
4	13	$SU(2)^2 \times U(1)^3$
2	14	$SU(2) \times U(1)^4$
0	15	$U(1)^5$

TABLE III. Possible number of type-A and type-B NGBs for $SU(6)/U(1)^5$.

n_A	n_B	U
40	0	.
24	8	$SO(8) \times U(1)$
20	10	$U(5)$
14	13	$SO(6) \times U(2)$
12	14	$SO(6) \times U(1)^2$
12	14	$U(4) \times U(1)$
10	15	$SO(4) \times U(3)$
8	16	$U(3) \times U(2)$
6	17	$SO(4) \times U(2) \times U(1)$
6	17	$U(3) \times U(1)^2$
4	18	$SO(4) \times U(1)^3$
4	18	$U(2)^2 \times U(1)$
2	19	$U(2) \times U(1)^3$
0	30	$U(1)^5$

TABLE IV. Possible number of type-A and type-B NGBs for $SO(10)/U(1)^5$.

n_A	n_B	$U \subset SO(11)$	$U \subset Sp(5)$
50	0	.	.
32	9	$SO(9) \times U(1)$	$Sp(4) \times U(1)$
20	15	$SO(7) \times U(2)$	$Sp(3) \times U(2)$
20	15	$U(5)$	$U(5)$
18	16	$SO(7) \times U(1)^2$	$Sp(3) \times U(1)^2$
14	18	$SO(5) \times U(3)$	$Sp(2) \times U(3)$
14	18	$SO(3) \times U(4)$	$Sp(1) \times U(4)$
12	19	$U(4) \times U(1)$	$U(4) \times U(1)$
10	20	$SO(5) \times U(2) \times U(1)$	$Sp(2) \times U(2) \times U(1)$
8	21	$SO(5) \times U(1)^3$	$Sp(2) \times U(1)^3$
8	21	$SO(3) \times U(3) \times U(1)$	$Sp(1) \times U(3) \times U(1)$
8	21	$U(3) \times U(2)$	$U(3) \times U(2)$
6	22	$SO(3) \times U(2)^2$	$Sp(1) \times U(2)^2$
6	22	$U(3) \times U(1)^2$	$U(3) \times U(1)^2$
4	23	$SO(3) \times U(2) \times U(1)^2$	$Sp(1) \times U(2) \times U(1)^2$
4	23	$U(2)^2 \times U(1)$	$U(2)^2 \times U(1)$
2	24	$SO(3) \times U(1)^4$	$Sp(1) \times U(1)^4$
2	24	$U(2) \times U(1)^3$	$U(2) \times U(1)^3$
0	25	$U(1)^5$	$U(1)^5$

TABLE V. Possible number of type-A and type-B NGBs for $SO(11)/U(1)^5$ and $Sp(5)/U(1)^5$.

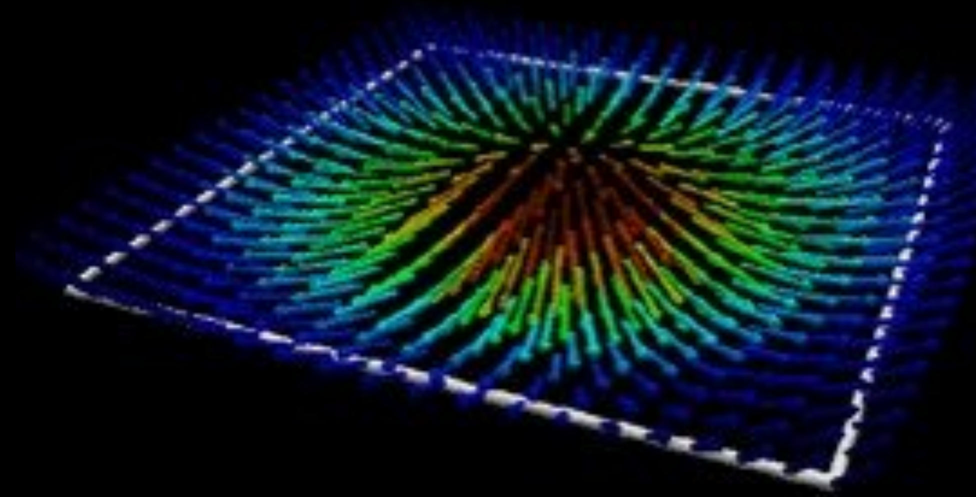
List of possible *U* for *G* with rank=5

Topological solitons

$$[P_x, P_y] \propto N_{\text{topological}}$$

H. Watanabe and HM, arXiv:1401.8139

skyrmion



- Consider a Heisenberg terromagnet
- On a two-dimensional plane, non-trivial maps $\mathbb{R}^2 \to S^2$ classified by $\pi_2(S^2) = \mathbb{Z}$
- skyrmion has moduli:
  - translations in *x* and *y* directions
  - dilation
  - rotation
- derive effective Lagrangian for moduli
- momenta don't commute!

$$[P_x, P_y] = i\hbar\, 4\pi s N_{\text{skyrmion}}$$

Derivation

- Effective Lagrangian
- All spins up at infinity
- canonical commutator
- Noether charges for translations
- commutator has a surface term that we normally ignore
- it is precisely the winding number!
- similarly for vortices in superfluid

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\frac{n_y \dot{n}_x - n_x \dot{n}_y}{1+n_z} - c_s^2 \frac{1}{2}\vec{\nabla} n_i \vec{\nabla} n_i$$

$$\mathcal{L}_{\rm eff} = s(1+\cos\theta)\dot{\phi} - f^2((\vec{\nabla}\theta)^2 + \sin^2\theta(\vec{\nabla}\phi)^2)$$

$$[s\cos\theta(x), \phi(y)] = -i\hbar\delta^2(x-y)$$

$$P_i = \int d^2x\ s(1+\cos\theta)\nabla_i\phi$$

$$[s\cos\theta(x),\phi(y)] = -i\hbar\delta^2(x-y)$$

$$P_i = \int d^2x\ s(1+\cos\theta)\nabla_i\phi$$

$$\begin{aligned}
[P_1,P_2] &= \int d^2x d^2y\ [s(1+\cos\theta)\nabla_1\phi(x), s(1+\cos\theta)\nabla_2\phi(y)] \\
&= -i\hbar\int d^2x d^2y\ [\nabla_2^y\delta(x-y)\nabla_1^x\phi(x)s(1+\cos\theta)(y) \\
&\qquad - \nabla_2^x\delta(x-y)\nabla_2^y\phi(y)s(1+\cos\theta)(x)] \\
&= i\hbar s\int d^2x d^2y\ [\nabla_1^x\phi(x)\nabla_2^y\cos\theta(y) \\
&\qquad - \nabla_2^y\phi(y)\nabla_2^x\cos\theta(x)]\delta(x-y) \\
&= i\hbar s\int d^2x\ [\nabla_1\phi\nabla_2\cos\theta - \nabla_2\phi\nabla_1\cos\theta] \\
&= i\hbar\ 4\pi s N_{\text{winding}}
\end{aligned}$$

consequence

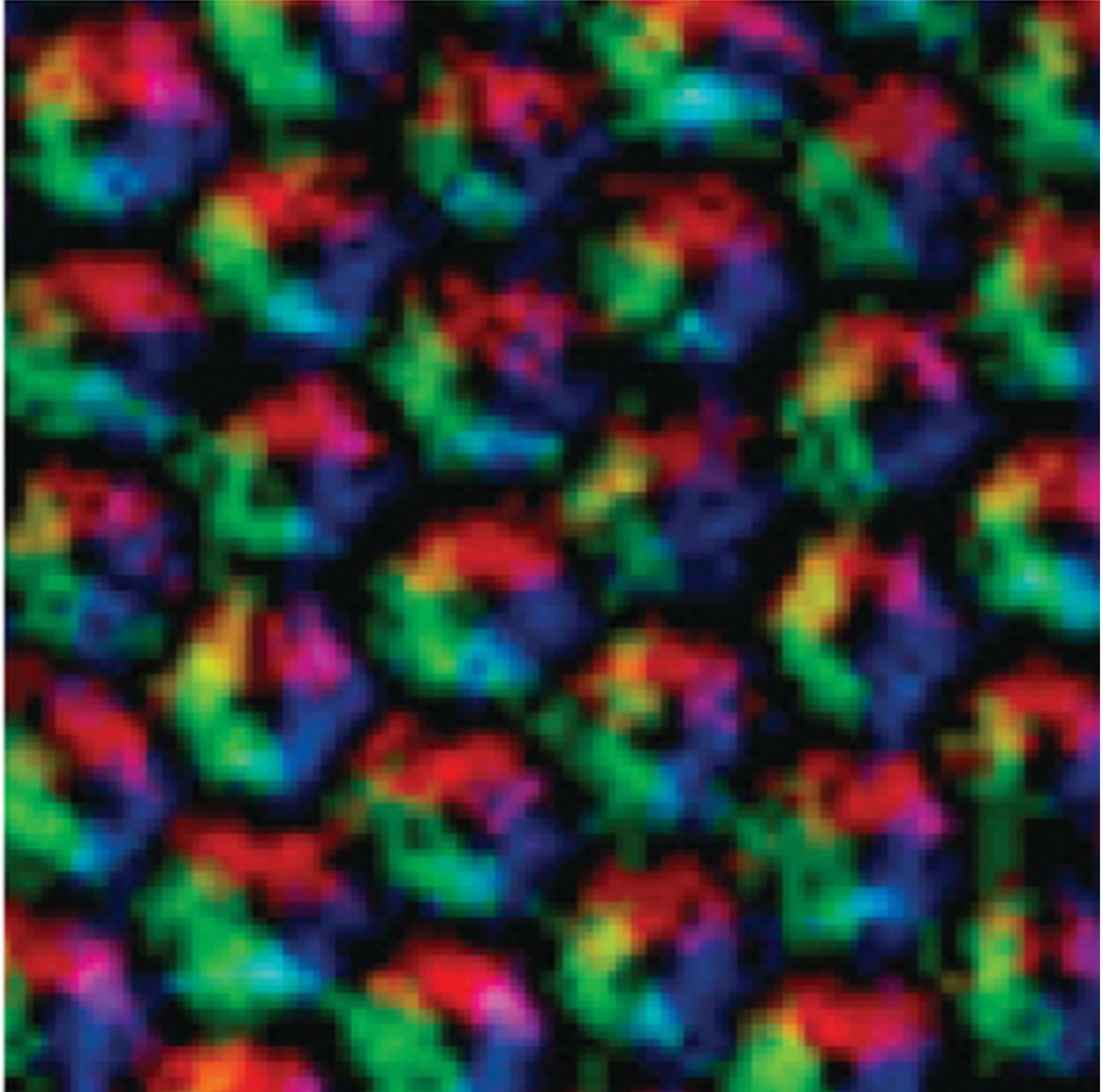
- If you push a skyrmion, it moves *sideways* called Magnus force

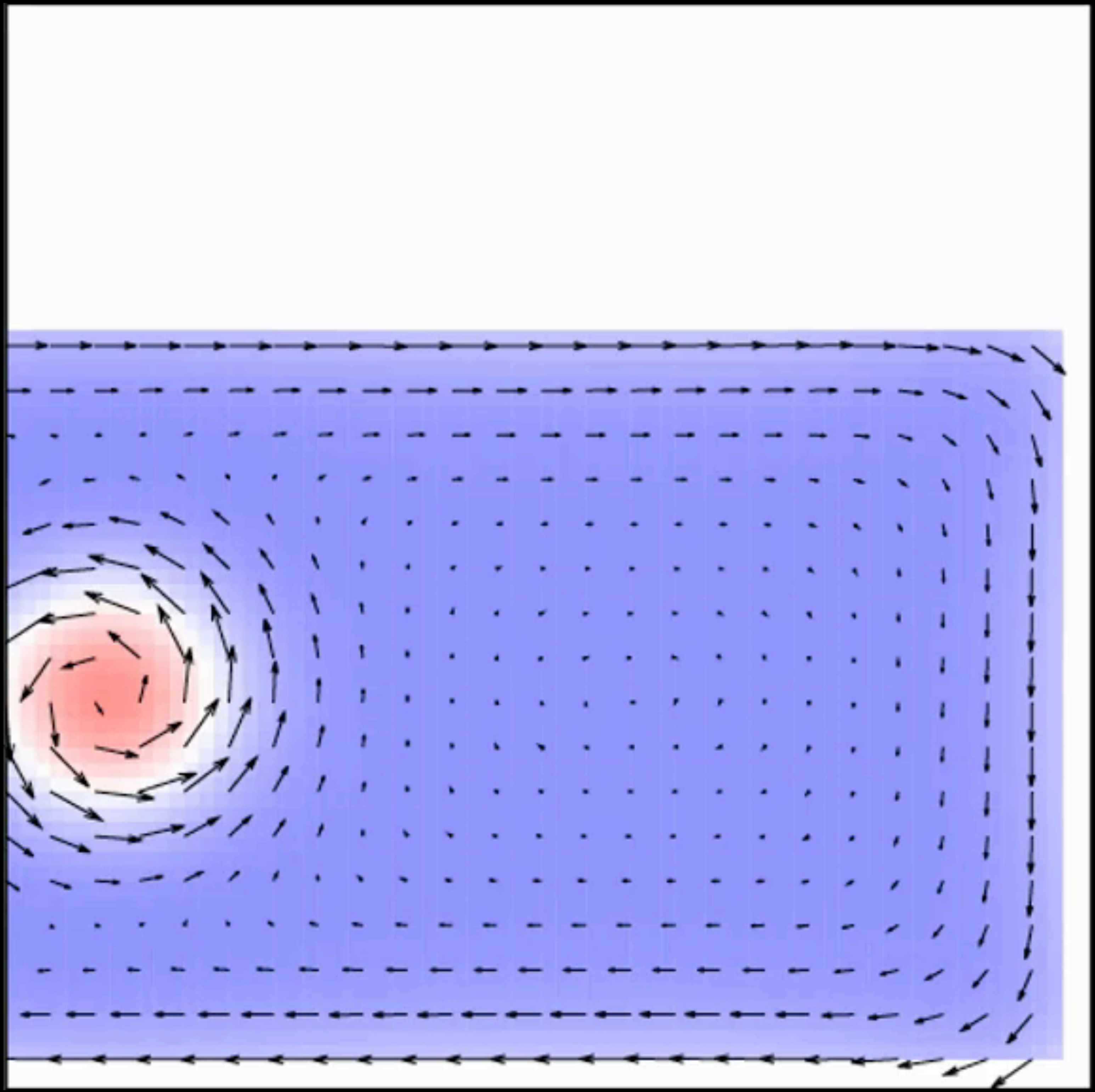
$$L = \frac{1}{2}(x\dot{y} - y\dot{x}) - Fx$$

$$\dot{y} - F = 0$$

- skyrmion lives in a "magnetic field" without external fields
- the same happens to vortices in superfluids

c





Iwasaki, Mochizuki, Nagaosa, Nature Nanotech 8, 742 (2013)

General

- Consider any compact Kähler manifold *K* as the target space
- allows for a topological soliton $H_2(K) \neq 0$
- holomorphic maps $\mathbb{C} \to K$ solve EoM
- Use symplectic structure on *K* for Type-B NGBs
- consider moduli for translations for *x* & *y*
- They don't commute!
- very similar to *central extension for extended supersymmetry by magnetic charge*

$$\{Q_i^\alpha, Q_j^\beta\} = \epsilon_{ij}\epsilon^{\alpha\beta} Z$$

massive NGB

H. Watanabe, T. Brauner, and HM, arXiv:1303.1527

massive NGB

- normally, we can say few things about gapped modes based on symmetries alone
- But exact gap predicted for $\tilde{H} = H - \mu Q$ (à la BPS)

$$n_{mNGB} = \frac{1}{2}(\mathrm{rank}\rho - \mathrm{rank}\tilde{\rho})$$

$$\rho_{ab} = \frac{-i}{V}\langle 0|[Q_a, Q_b]|0\rangle \qquad [Q_a, H] = 0$$

$$\tilde{\rho}_{ab} = \frac{-i}{V}\langle 0|[\tilde{Q}_a, \tilde{Q}_b]|0\rangle \qquad [\tilde{Q}_a, \tilde{H}] = [\tilde{Q}_a, H - \mu Q] = 0$$

$$\tilde{H}(E_\alpha|0\rangle) = \mu\alpha(E_\alpha|0\rangle)$$

H. Watanabe, T. Brauner, and HM, arXiv:1303.1527

Englert-Brout-Higgs mechanism

H. Watanabe and HM, arXiv:1405.0997

Anderson-Englert-Brout-Higgs-Guralnik-Hagen-Kibble

- massless vector boson in *d* dimension has *d*-1 dof
- *massive* vector boson has *d* dof
- it needs to *eat* massless NGB

mismatch

- Heisenberg model breaks SO(3) to SO(2)
- two broken generators, one type-B NGB
- If SO(3) gauged, two broken gauge bosons
- but only one NGB to be “eaten”
- what happens?
- NB type-B NGB comes with charge density

charge neutrality

- we only need to make sure the current density is cancelled by a "background"

$$\mathcal{L} \supset -j_\mu A^\mu$$

- "charge neutrality constraint" (Kapusta)

- fine for U(1) but non-abelian???

$$\mathcal{L} \supset \cancel{-j^a_\mu A^{\mu a}}$$

two possibilities

- If there is a charge density, the gauge bosons acquire VEVs and compensate it
  - it necessarily breaks rotational invariance (Gusynin, Miransky, Shovkovy)
- Or, we bring in another sector that cancels the charge density
- either way, smooth $e \to 0$ limit with spectrum varying continuously

cf. P06 Shintaro Karasawa

turning on gauge field

- Yang-Mills equation $D_\mu F^{\mu\nu} = ej^\nu$

$$\nabla \cdot \vec{E} = ie[\vec{A}, \vec{E}] + ej^0$$

- thermodynamics limit requires r.h.s.=0
- Non-zero charge density j^0 forces A^0 to acquire VEV
- Space-time constant solution requires

$$e[A_i, [A_i, A^0] = j^0$$

- Spatial components of gauge field also acquire VEVs

charge density

- Noether current for a global gauge transformation: $j^\mu = i[A_\nu, F^{\mu\nu}] + j^\mu_{\rm matter}$
- matter contribution cancelled by the gauge contribution
- NGBs for global gauge transformation from gauge bosons provide additional dof to be eaten, become type-A
- additional NGBs from gauge bosons for broken rotation

S^2=SU(2)/U(1)

- Heisenberg model with spin density // z
- need a VEV for a spatial component
- other A^1 are NGBs for rotation symmetry together with a massless type-A NGB
- A^2 and A^3 acquire mass eating two type-A NGBs

$$D_\mu F^{\mu\nu} = j^\nu$$

$$[A_i, [A_i, A^0]] = j^0$$

$$\langle eA_\mu \rangle = \begin{array}{c c} \begin{array}{c c c} 1 & 2 & 3 \end{array} & \\ \left(\begin{array}{c c c} 0 & 0 & O(e) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ O(\sqrt{e}) & 0 & 0 \end{array} \right) & \begin{array}{c} t \\ x \\ y \\ z \end{array} \end{array}$$

second prescription

- bring two systems with same type-B spectrum $\mathcal{L} = \frac{1}{2}e_z(\pi_x\dot{\pi}_y - \pi_y\dot{\pi}_x) - \frac{1}{2}g(\vec{\nabla}\pi_a)^2$

$$- \frac{1}{2}e_z(\Pi_x\dot{\Pi}_y - \Pi_y\dot{\Pi}_x) - \frac{1}{2}G(\vec{\nabla}\Pi_a)^2$$

- gauge the diagonal subgroup
- 1 type-B NGB for two generators from each system
- correct EBH mechanism
- don't need second power in time-derivative

can be tested?

- multi-layer graphene?
- spin-orbit coupling?
- frustrated spin liquid?
- high-density QCD?

redundancies

H. Watanabe and HM, arXiv:1302.4800

spacetime symmetries

- so far all discussions are internal symmetries
- but there are situations when n_{NGB} is further reduced for spacetime symmetries
- spontaneously broken scale and conformal symmetries lead to only one NGB (dilaton) (Salam-Strathdee)
- crystal breaks both translations (P_i) and rotations (J_i), but only phonons for P_i

Noether constraints

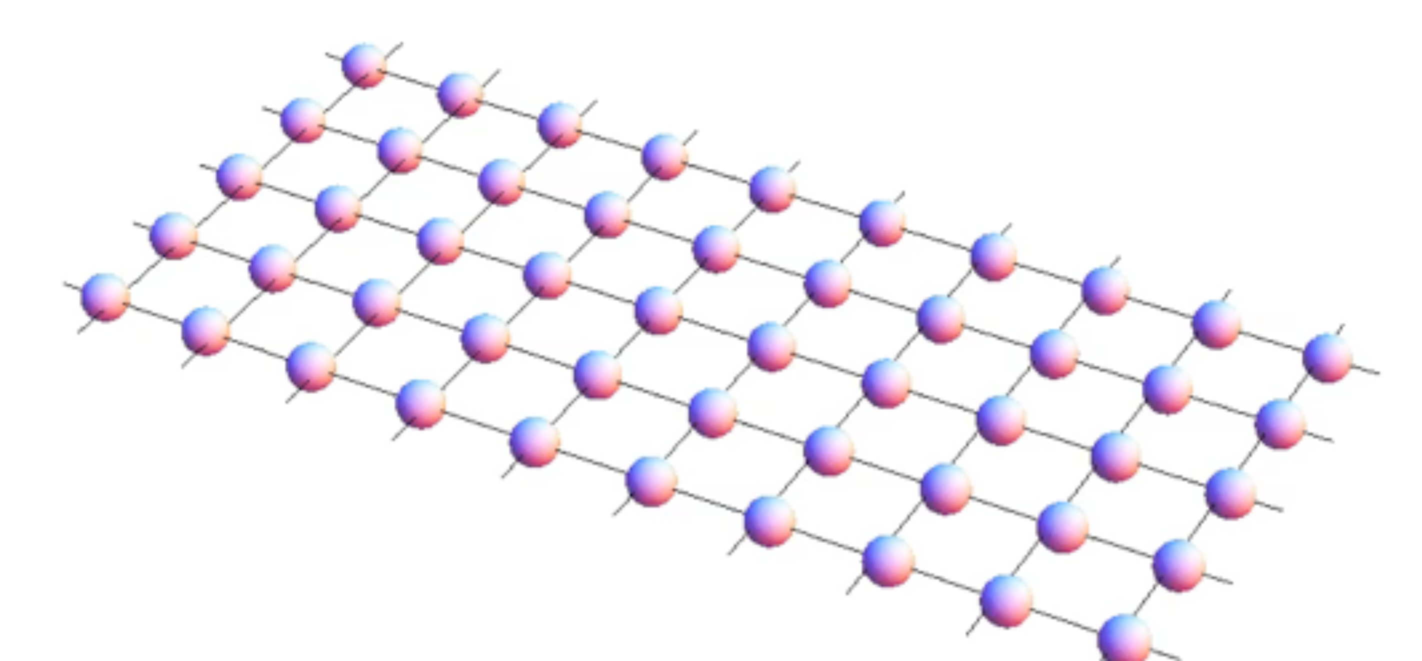
- They can be understood as a consequence of *Noether constraints* $\int d^d x \sum_a c_a(x) j_a^0(x)|0\rangle = 0$
- For broken symmetries, we have $\langle \pi_b | j_a^0(x)|0\rangle \neq 0$
- then they are linearly *redundant*

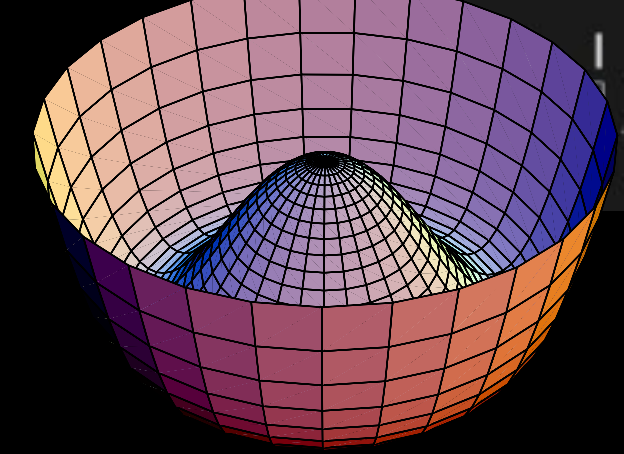
$$0 = \sum_b |\pi_b\rangle\langle\pi_b| \int d^d x \sum_a c_a(x) j_a^0(x)|0\rangle$$

$$= \sum_b |\pi_b\rangle \int d^d x c_a(x) \langle \pi_b | j_b^0(x)|0\rangle$$

Examples

- crystal: translations and rotations are both spontaneously broken
- they are both generated by the energy-momentum tensor $R^{0i} = \epsilon_{ijk} x^j T^{0k}$
- would-be NGBs for rotations are the same excitations as those for translations (phonons)





Examples

- Ginzburg-Landau theory

$$V = -\mu\psi^*\psi + \lambda(\psi^*\psi)^2$$

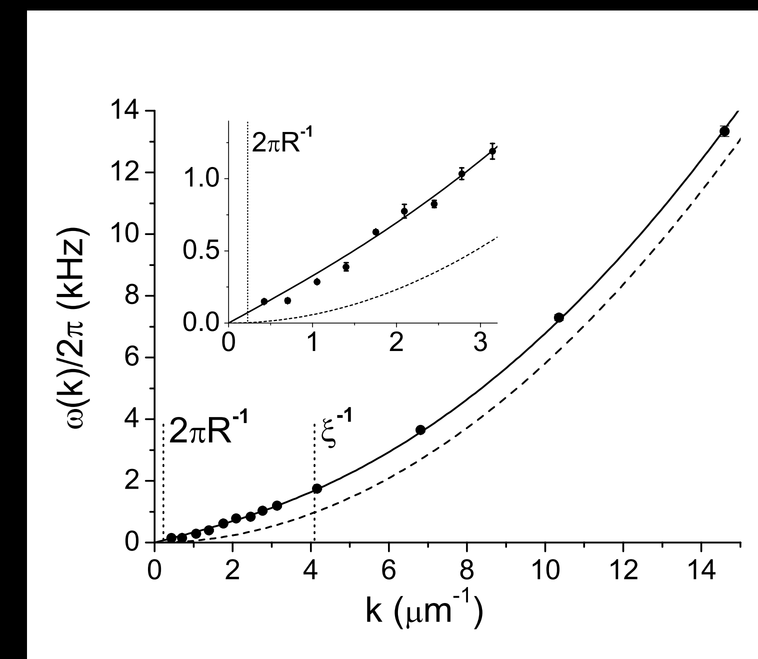
- G=U(1), H=0
- ^{4}He superfluid
- scalar BEC $\langle 0|\psi|0\rangle \neq 0$
- U(1) $\psi(\vec{x},t) \to e^{i\theta}\psi(\vec{x},t)$
- Galilean boost

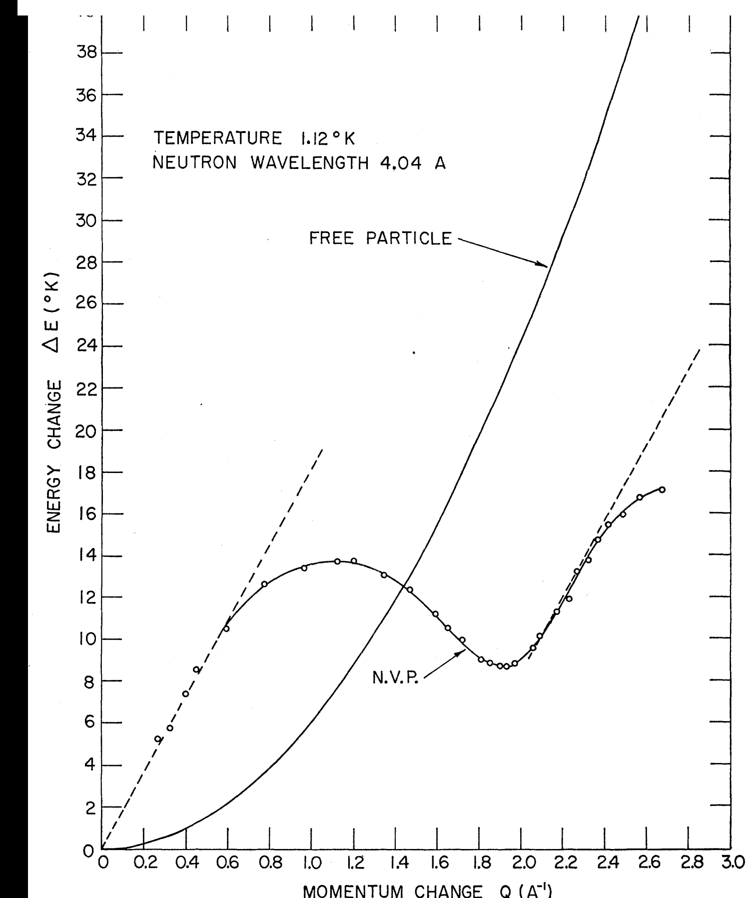
$$\psi(\vec{x},t) \to e^{i(m\vec{x}\cdot\vec{x}-\frac{1}{2}m\vec{v}^2t)}\psi(\vec{x}-\vec{v}t,t)$$

- both broken n_{BG}=1+3=4

$$B^{i\mu} = tT^{i\mu} - mx^i j^\mu$$

⇒ no separate NGBs for Galilean boosts

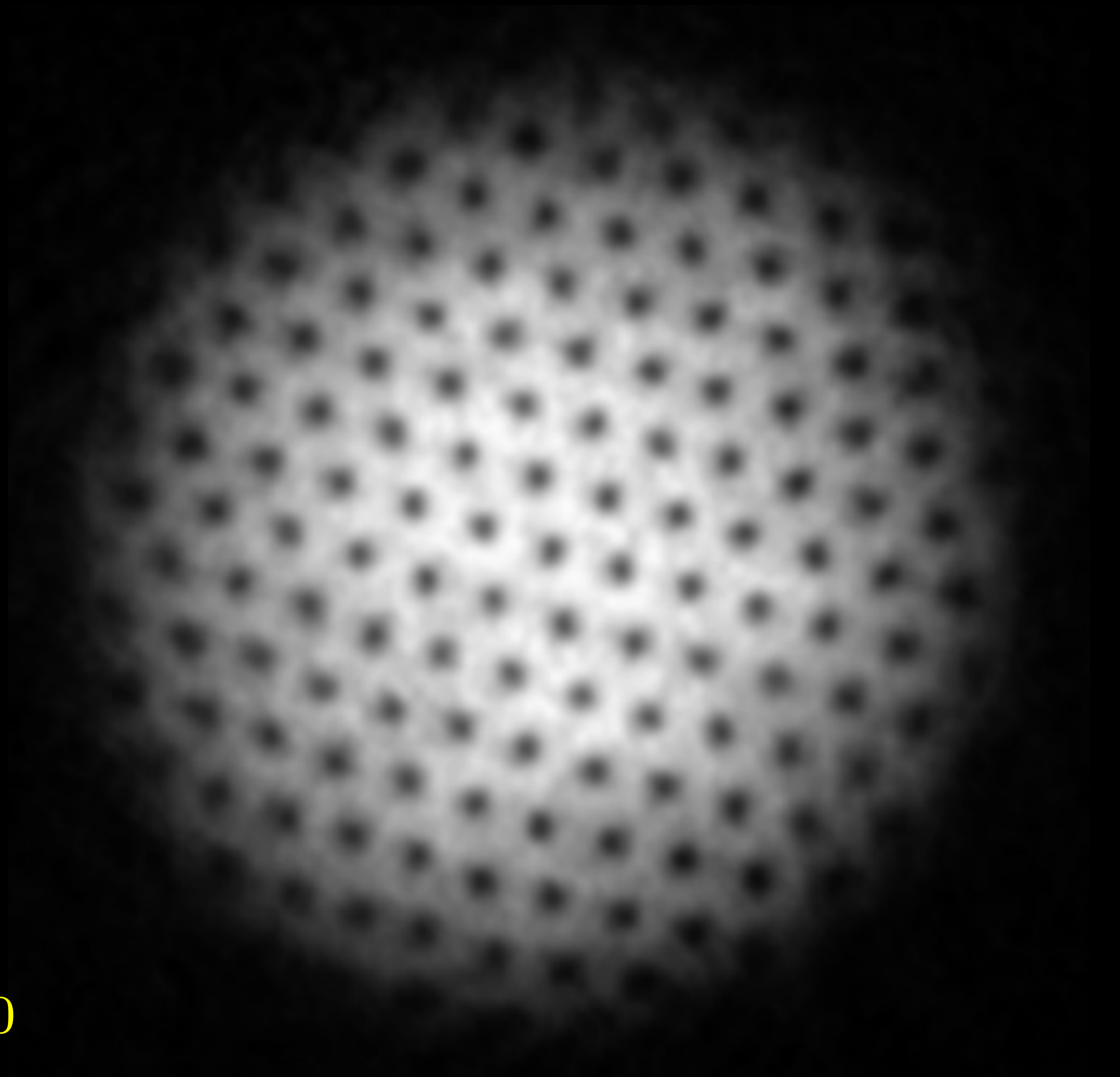




vortex lattice

- rotate a (2d) BEC
- vortices form a triangular lattice
- broken: U(1), $P_{x,y}$, J_z
- only one Type-A NGB with $E \propto p^2$
- called Tkachenko mode

$$T^{0i} = mj^i - 2m\Omega\epsilon^{ij}x^j j^0$$



we have a precise effective Lagrangian for this

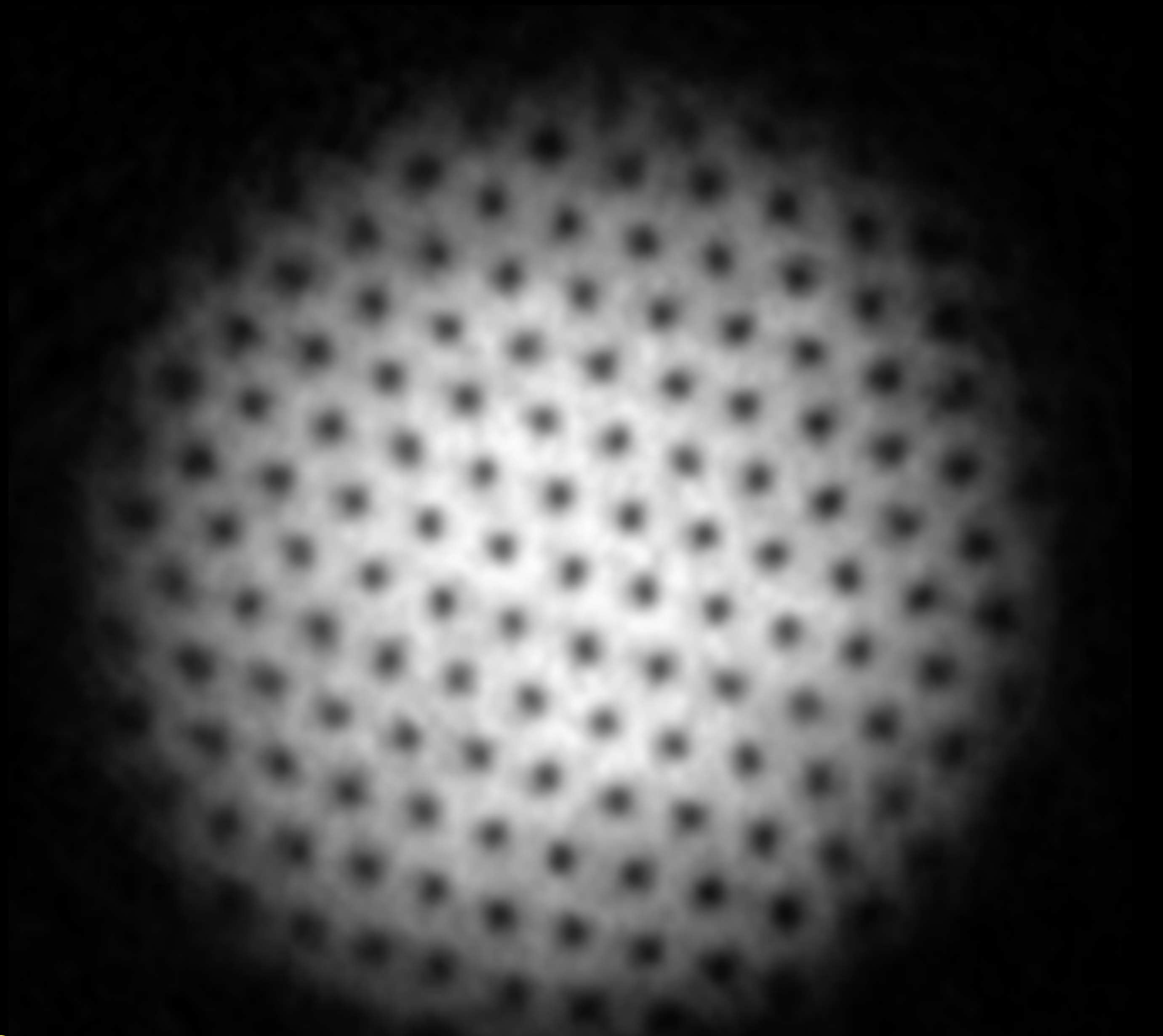
vortex lattice

- translation of the lattice causes the phase shift

$$\theta \to \theta + 2m\vec{a} \cdot \vec{\Omega} \times \vec{x}$$

$$\mathcal{L}_{\text{eff}} \sim \dot{\theta}^2 - (\vec{\nabla}^2\theta)^2$$

- Type A but $E \propto p^2$



$$T^{0i} = mj^i - 2m\Omega\epsilon^{ij}x^j j^0$$

we have a precise effective Lagrangian for this

More excitements to come!