Nambu-Goldstone modes localized around vortices and solitons

June 25/2014 **Higgs modes in condensed matter and quantum gases @ YITP**

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Topological Quantum Phenomena in Condensed Matter with Broken Symmetries

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[1] M.Kobayashi & MN, PTEP:021B01,2014 [arXiv:1307.6632]
[2] M.Kobayashi & MN, arXiv:1402.6826 [hep-th],
[3] M.Kobayashi & MN, Phys.Rev.D arXiv:1403.4031 [hep-th]
[4] D.A.Takahashi & MN, arXiv:1404.7696[cond-mat.quant-gas]
[5] MN, S.Uchino & W.Vinci, arXiv:1311.5408 [hep-th]
[6] D.A.Takahashi,M.Kobayashi &MN, in preparation

Number of Nambu-Goldstone(NG) modes

type-I (A) $\omega \sim k^{\ }$ $\#=N_{\mathrm{I}}$

type-II (B) $\omega \sim k^{2}$ $\#=N_{\mathrm{II}}$

Only type I in relativistic theories

non-relativistic theories

Nielsen-Chadha inequality

Nielsen-Chadha('76),Nambu('04)

$$N_{\mathrm{I}}+2N_{\mathrm{II}} \geq N_{\mathrm{BG}} = \#\,\text{broken generators}$$

$$N_{\mathrm{II}} = N_{\mathrm{BG}} - N_{\mathrm{NG}} = \frac{1}{2}\,\mathrm{rank}\,\rho$$

$$\rho_{ij} = \langle \mathrm{GS} | [T_i, T_j] | \mathrm{GS} \rangle$$

WB matrix

Watanabe-Brauner(WB) relation

Nambu ('01,'04)

Watanabe-Brauner('11), Watanabe-Murayama('12), Hidaka('12)

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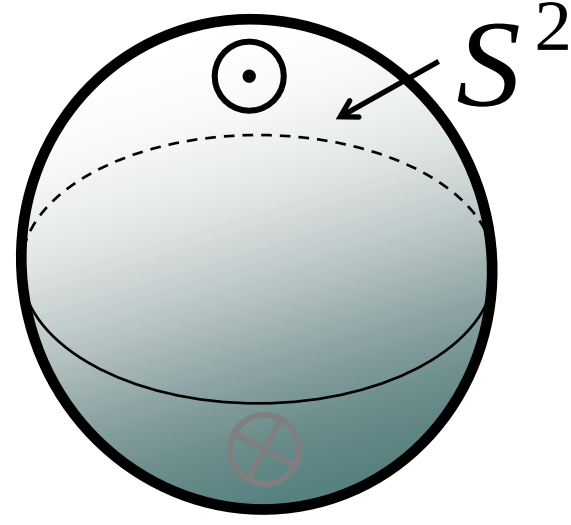
type-I (A) $\omega \sim k \quad \# = N_{\mathrm{I}}$

type-II (B) $\omega \sim k^2 \quad \# = N_{\mathrm{II}}$



example **Heisenberg magnets**

$$SO(3)/SO(2) = S^2 \quad N_{\mathrm{BG}} = 2$$

$$\langle \mathrm{GS} | [S_x, S_y] | \mathrm{GS} \rangle = i \langle \mathrm{GS} | S_z | \mathrm{GS} \rangle$$

= 0 **Anti-ferro** ↑↓↑↓↑↓↑↓ $N_{\mathrm{I}} = 2, \; N_{\mathrm{II}} = 0 \quad N_{\mathrm{NG}} = 2$

≠ 0 **Ferro** ↑↑↑↑↑↑↑↑ $N_{\mathrm{I}} = 0, \; N_{\mathrm{II}} = 1 \quad N_{\mathrm{NG}} = 1$

Classification of NG modes completed for internal symmetry

NG modes localized around vortices,solitons

but not yet for space-time symmetry

objects	systems
Quantized vortex	Superfluid He, BEC
Domain wall	Anisotropic ferromagnets 2 component BEC
Skyrmion lines	Isotropic ferromagnets
Non-Abelian vortices	Multicomponent BEC

(1)Finite R	Broken sym	NG type	Dispersion for finite R	
Vortex line in superfluid	X, Y	II	$e \sim \log R \ \ k^2$ kelvon	e.g. Kobayashi & MN ('13.07)
Skyrmion line	X, Y	II	$e \sim k^2$ kelvon	Well-known
(scale inv)	D, ϑ	II	$e \sim k^2$ dilaton-magnon	Kobayashi & MN ('14.03)
(scale violated)	~~D~~, ϑ	I	$e \sim k$	
Domain wall in ferromagnet	X, ϑ	II	$e \sim k^2$ ripplon-magnon	Kobayashi & MN ('14.02)
Domain wall in 2comp BEC	X, ϑ	II	$e \sim R^{1/2} k^2$ ripplon-magnon	Takeuchi & Kasamatsu('13.09) well-known for ripplon

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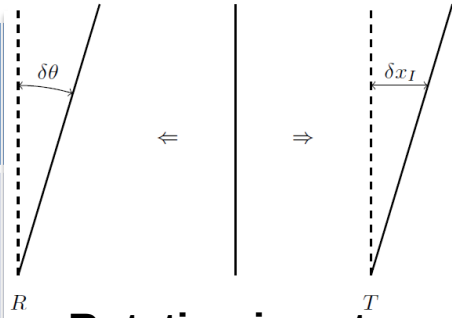
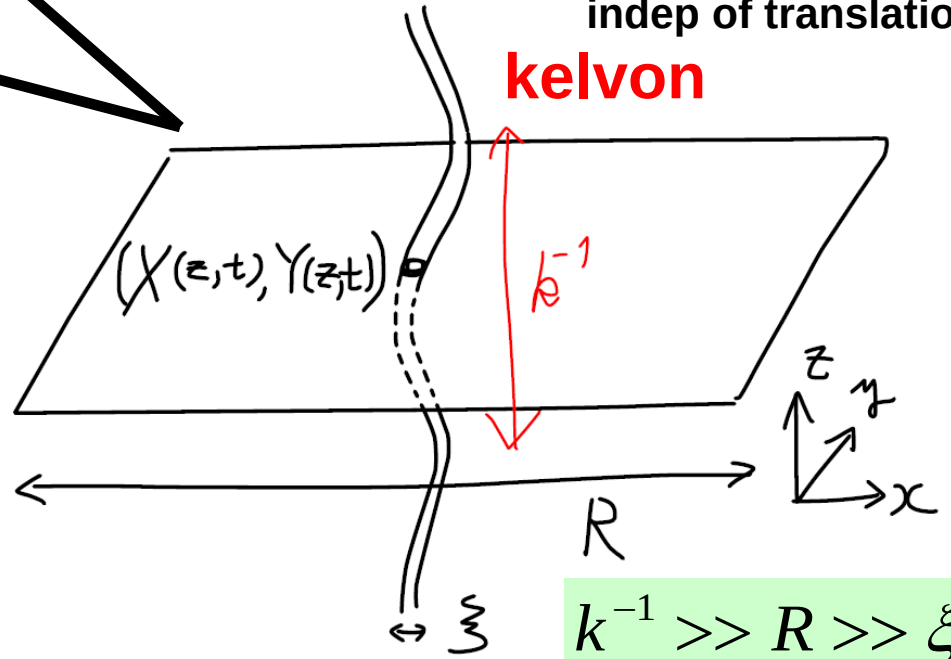


Rotation is not indep of translation



$$k^{-1} >> R >> \xi$$

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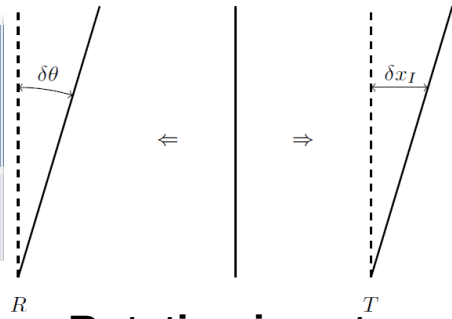

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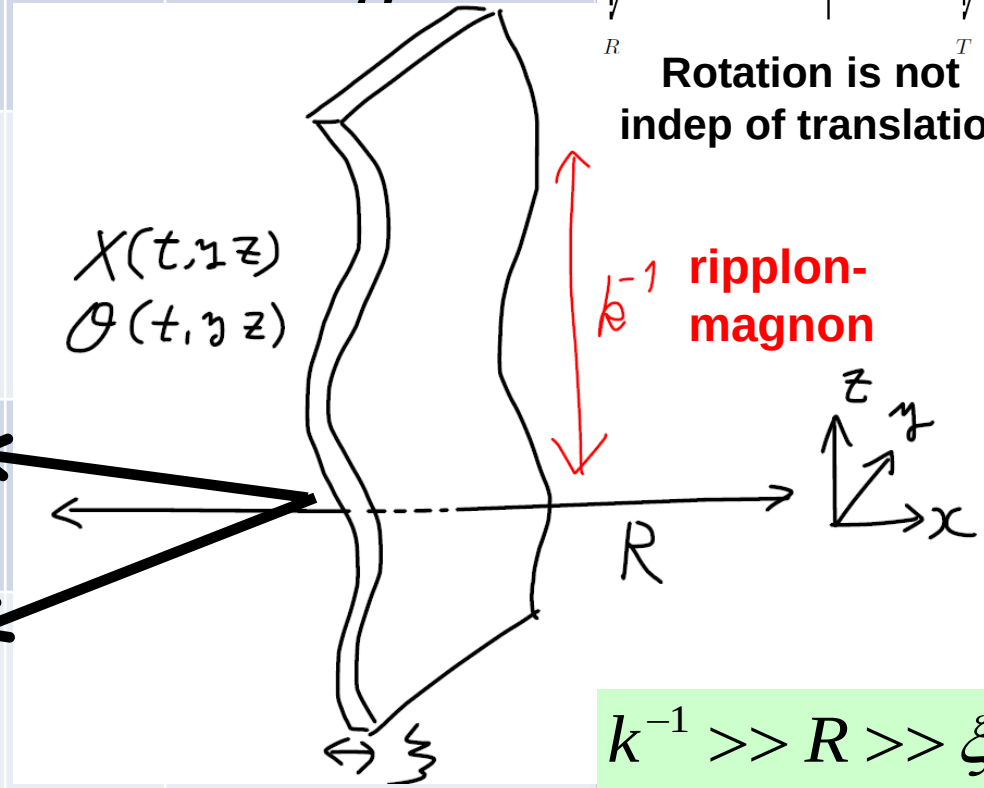

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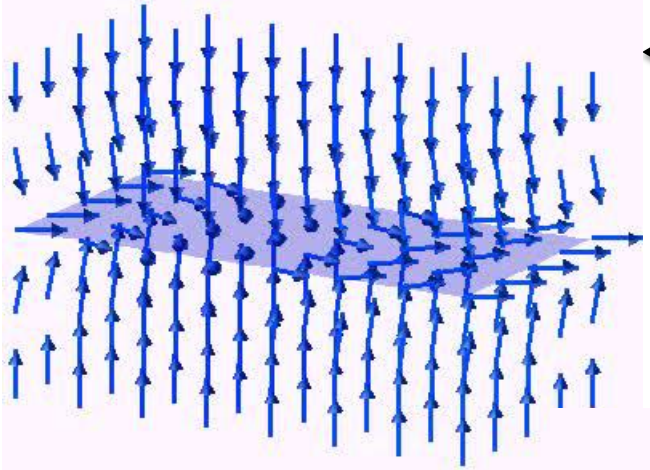
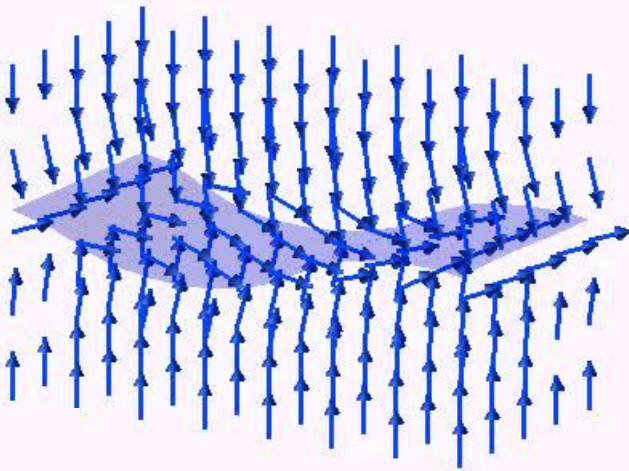
Relativistic

← **U(1) phase**
(magnon)

translation →
(ripplon)

type-I NG

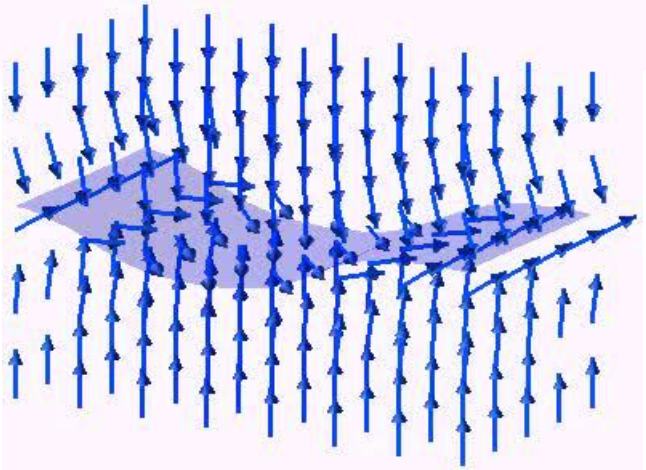


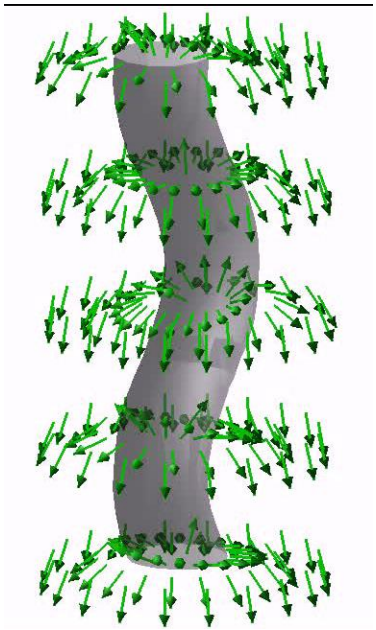


Non-relativistic

coupled
magnon-ripplon

type-II NG



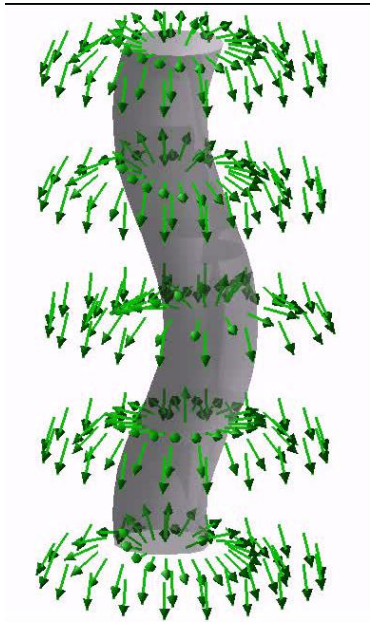


Relativistic

← translation X

translation Y →

type-I NG

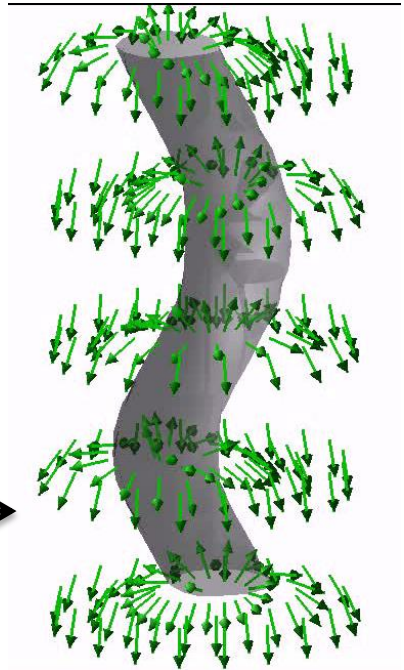


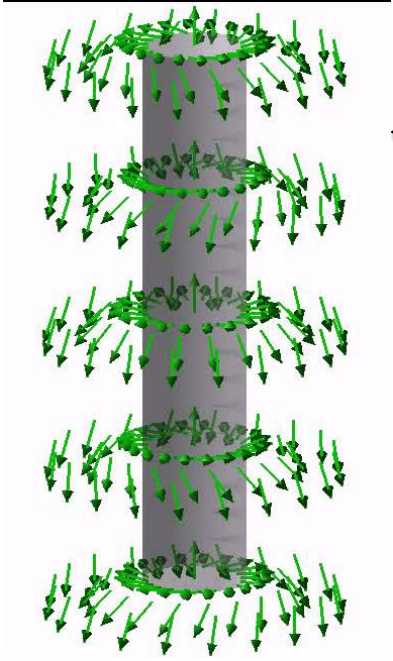
Non-relativistic

coupled translation

(Kelvon) (X, Y) →

type-II NG



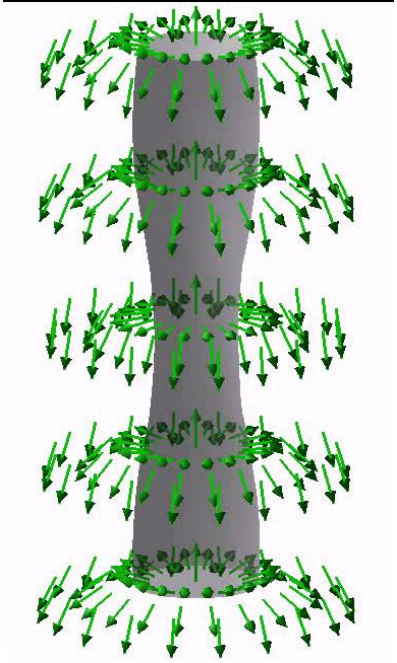


Relativistic

← **U(1) phase**
(magnon) θ

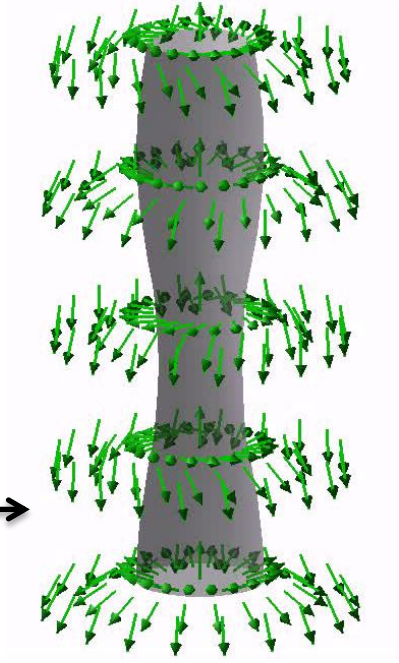
size (dilaton) R →

type-I NG



Non-relativistic

coupled
magnon-dilaton →
(R, θ) **type-II NG**



(2)Symmetry	Broken sym
Vortex line in superfluid	X, Y
Skyrmion line (scale inv) (scale violated)	X, Y D, ϑ ~~D~~, ϑ
Domain wall in ferromagnet	X, ϑ
Domain wall in 2comp BEC	X, ϑ

Watanabe-Brauner relation

<[broken,broken]>
=/=0
for type-II

(2)Symmetry	Broken sym	Watanabe-Brauner relation
Vortex line in superfluid	X, Y	$[P_x, P_y] \sim \text{vortex charge}$ Watanabe&Murayama('14.01)
Skyrmion line (scale inv) (scale violated)	X, Y D, ϑ ~~D~~, ϑ	$[P_x, P_y] \sim \text{skyrmion charge}$ Watanabe&Murayama('04.01) $[D, \Theta] \sim r^2$ (skyrmion charge Kobayashi&MN('14.03)
Domain wall in ferromagnet	X, ϑ	$[P_x, \Theta] \sim \text{wall charge}$ Kobayashi&MN('14.02)
Domain wall in 2comp BEC	X, ϑ	$[P_x, \Theta] \sim \text{wall charge}$ Kobayashi&MN('14.02) Watanabe&Murayama('14.03)

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Vortex line in superfluid	X, Y	$[P_x, P_y] \sim \text{vortex charge}$ Watanabe&Murayama('14.01)
Skyrmion line (scale inv)	X, Y D, ϑ	$[P_x, P_y] \sim \text{skyrmion charge}$ Watanabe&Murayama('04.01)
		$[D, \Theta] \sim r^2$ (skyrmion charge Kobayashi&MN('14.03)
in ferromagnet		$[P_x, \Theta] \sim \text{wall charge}$ Kobayashi&MN('14.02)
Domain wall in 2comp BEC	X, ϑ	$[P_x, \Theta] \sim \text{wall charge}$ Kobayashi&MN('14.02) Watanabe&Murayama('14.03)

Central extension of algebra

(2)Symmetry	Broken sym
Vortex line	X, Y
Skyrmion line	X, Y
(scale inv)	D, ϑ
(scale violated)	~~D~~, ϑ
Domain wall	X, ϑ

well-known (Magnus force)

[space-time,internal] =/=0

Cf) Coleman&Mandula('67) for relativistic case

Watanabe-Brauner relation

$[P_x, P_y] \sim \text{vortex charge}$

Watanabe&Murayama('14.01)

$[P_x, P_y] \sim \text{skyrmion charge}$

Watanabe&Murayama('04.01)

$[D, \Theta] \sim r^2$ (skyrmion charge

Kobayashi&MN('14.03)

$[P_x, \Theta] \sim \text{wall charge}$

Kobayashi&MN('14.02)

$[P_x, \Theta] \sim \text{wall charge}$

Kobayashi&MN('14.02)
Watanabe&Murayama('14.03)

(3)Infinite R	Broken sym	NG type	Dispersion for finite R	Dispersion for infinite R
Vortex line in superfluid	X, Y	II	$e \sim \log R\, k^2$	$e \sim -k^2 \log k$
Skyrmion line (scale inv)	X, Y	II	$e \sim k^2$	$e \sim k^2$
(scale violated)	D, ϑ	II	$e \sim k^2$	$e \sim k^2$
			$e \sim k$	$e \sim k$
Domain wall in ferromagnet	X, ϑ	II	$e \sim k^2$	$e \sim k^2$
Domain wall in 2comp BEC	X, ϑ	II	$e \sim R^{1/2} k^2$	$e \sim k^{3/2}$

non-normalizable

$R \to k^{-1}$

well-known

$R >> k^{-1} >> \xi$

normalizable

Takahashi&MN('14.04)

non-normalizable

$R \to k^{-1}$

Takeuchi & Kasamatsu ('13.09)

well-known for ripplon

Summary

(1) **Dispersion** relations in **finite systems**

type-I NG: $e \sim k$, type-II NG: $e \sim k^2$

(2) **Symmetry** (commutation relation)

Watanabe-Brauner relation

<[X,Y]>=/=0: 1 type-II <[space-time,internal]>=/=0

<[X,Y]> = 0: 2 type-I

(3) **Dispersion** relations in **infinite systems**

normalizable : the same with finite system

non-normalizable $e \sim f(\mathrm{R}) k^n$: $R \rightarrow k^{-1}$,non-integer power

(Kelvon&ripplon were *not* recognized as NG thus far)

What I didn't talk about:

1) So far, mean field,
   Beyond mean field: Coleman-Mermin-Wargner
   type-II NG seem to be stable at quantum level
   …can be proved by Bethe ansatz (for some case)
2) Bogoliubov theory approach: Gram matrix

Disscussion:

1) Proof for general cases (finite & infinite *R*)
2) Localized Higgs modes?
3) NG fermions (SSB of fermionic symmetry)
   - edge mode of topological insulator/superconductor?