Higgs mode and universal dynamics near quantum criticality

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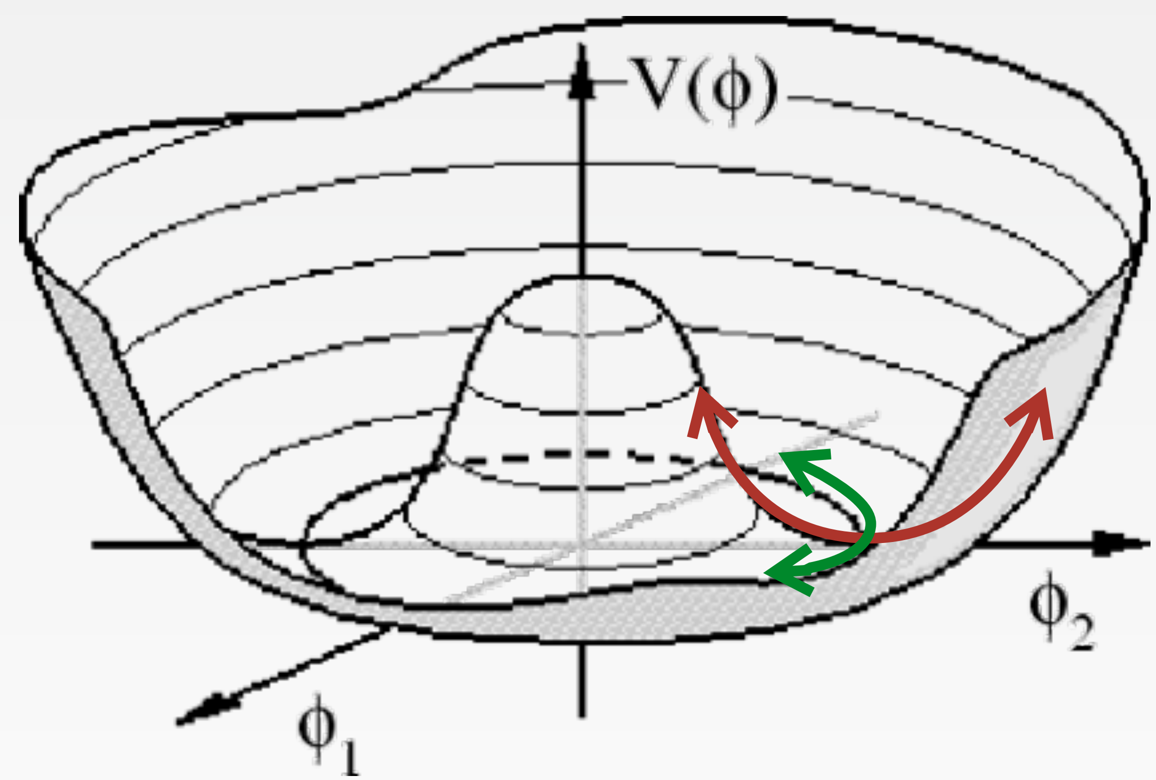
June 2014



Technion Israel Institute of Technology

Podolsky, Auerbach, Arovas, PRB 84, 174522 (2011)
Podolsky and Sachdev, PRB 86, 054508 (2012)
Gazit, Podolsky, Auerbach, PRL 110, 140401 (2013)
Gazit, Podolsky, Auerbach, Arovas PRB 88, 235108 (2013)
Tenenbaum Katan and Podolsky, to appear (2014)
Gazit, Podolsky, Auerbach, to appear (2014)

Outline

- The Higgs mode in condensed matter

- How to see the Higgs mode in 2d?

- Universal dynamics near quantum criticality

- Can the Higgs be seen through conductivity measurements?

- Amplitude mode in solid Helium 4?

Spontaneous Symmetry Breaking

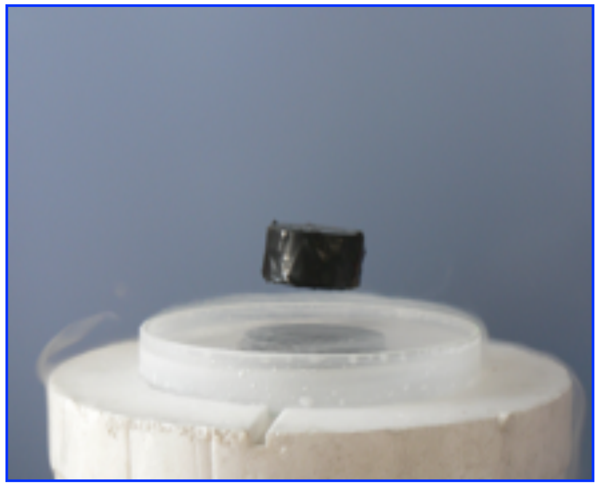
SSB - Ground state has less symmetry than the Hamiltonian

Ubiquitous phenomenon in physics. Order parameter $\phi(x,t)$ gets an expectation value. Usually results in massless Goldstone bosons.

physical system	broken symmetry	Goldstone bosons
ferromagnets	spin rotational invariance	spin waves
crystals, density waves	translational invariance	phonons and phasons
superfluids	phase rotation invariance	phonons

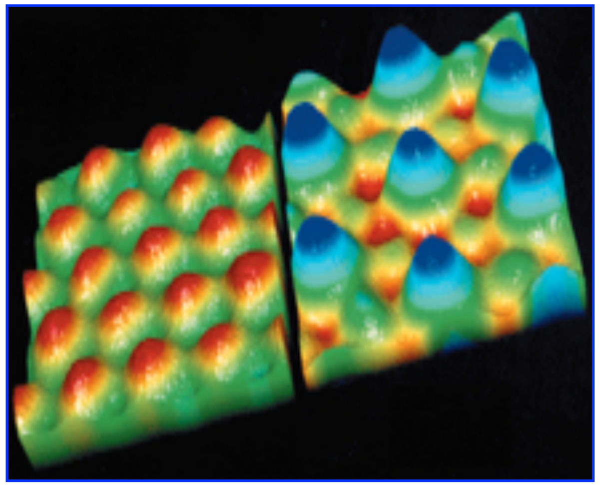
MAGNETS

magnetization M



SUPERCONDUCTORS

pair field $\Psi = |\Psi|\, e^{i\phi}$



DENSITY WAVES

Fourier mode $|\varrho_G|\, e^{i\phi_G}$

THE UNIVERSE

Higgs field $\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

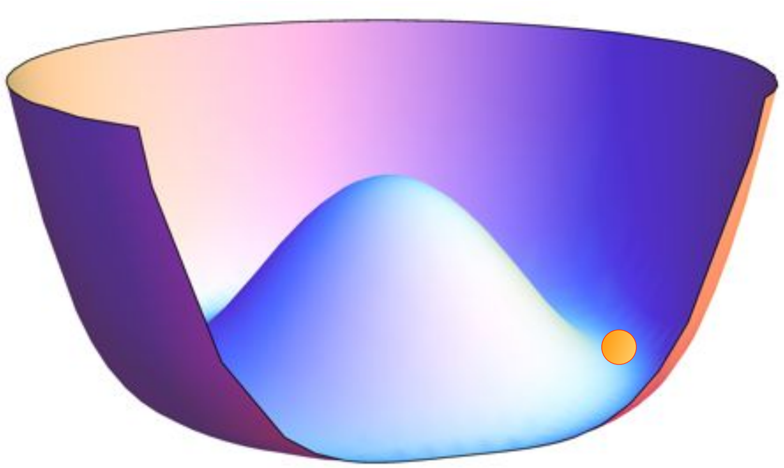
Spontaneous Symmetry Breaking

N-component order parameter:

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \dots \\ \phi_N \end{pmatrix}$$

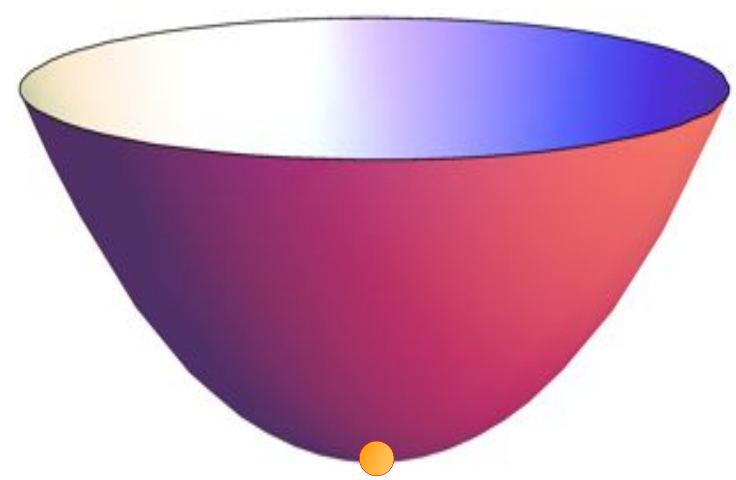
$$V(\phi) = g\phi^2 + u\left(\phi^2\right)^2$$

$g < 0$

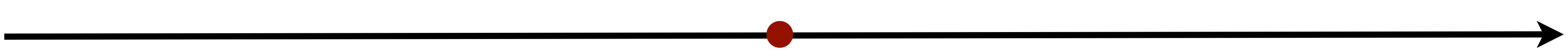


$\langle \phi \rangle \neq 0$

$g > 0$



$\langle \phi \rangle = 0$



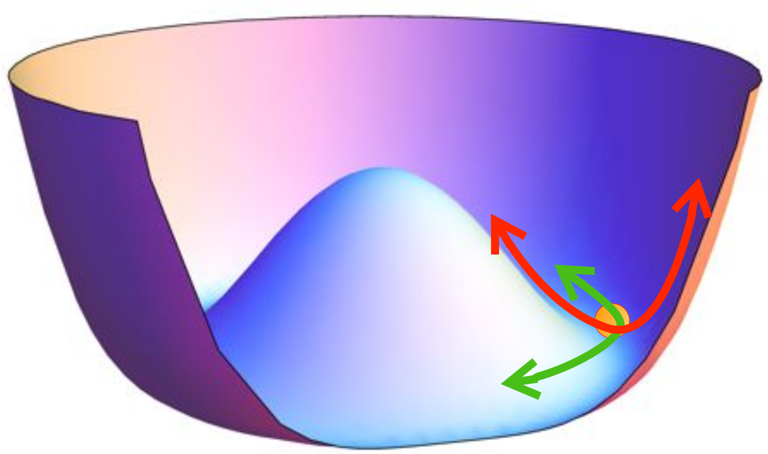
$g_c = 0$

g

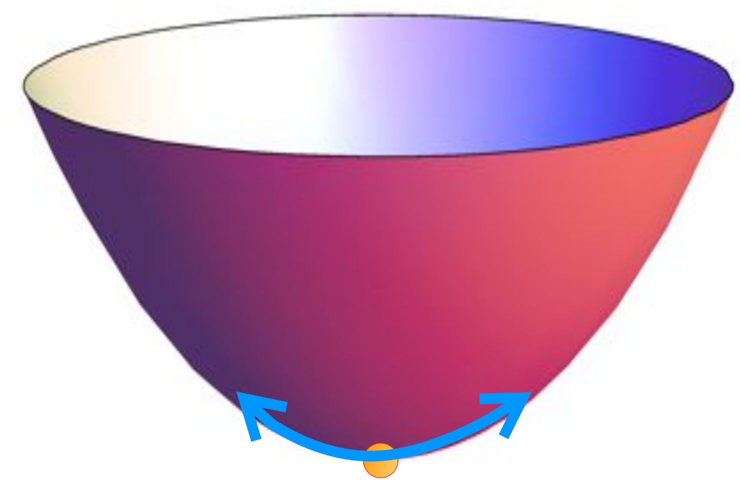
Collective excitations

Action:

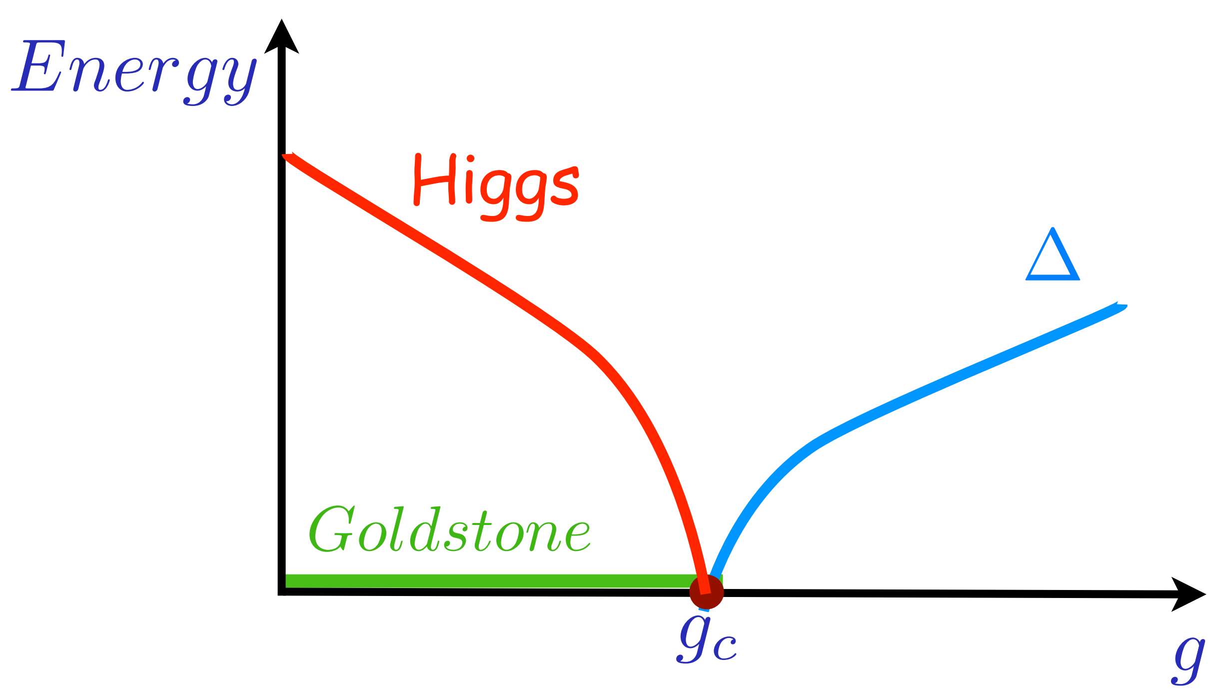
$$\mathcal{S} = \int d^d x \, dt \, \left[(\partial_t \phi)^2 - (\nabla \phi)^2 - g\phi^2 - u(\phi^2)^2 \right]$$



$\langle \boldsymbol{\phi} \rangle \neq 0$

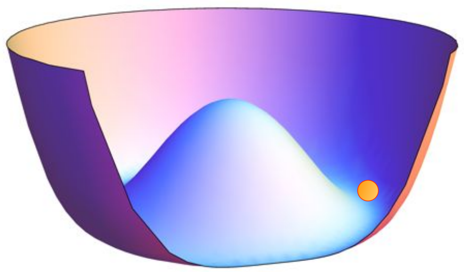


$\langle \boldsymbol{\phi} \rangle = 0$



Higgs mechanism vs Higgs boson

Gauged O(2) theory :



$$\mathcal{L} = \frac{1}{2}\left|(\partial_\mu + ieA_\mu)\,\psi\right|^2 - m^2\left(\psi^*\psi - 1\right)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Amplitude and phase: $\psi = (1+\eta)\,e^{i\xi}$

Gauge invariance: $\psi \to e^{-i\xi}\,\psi\,,\qquad \widetilde{A}_\mu \equiv A_\mu - e^{-1}\partial_\mu \xi$

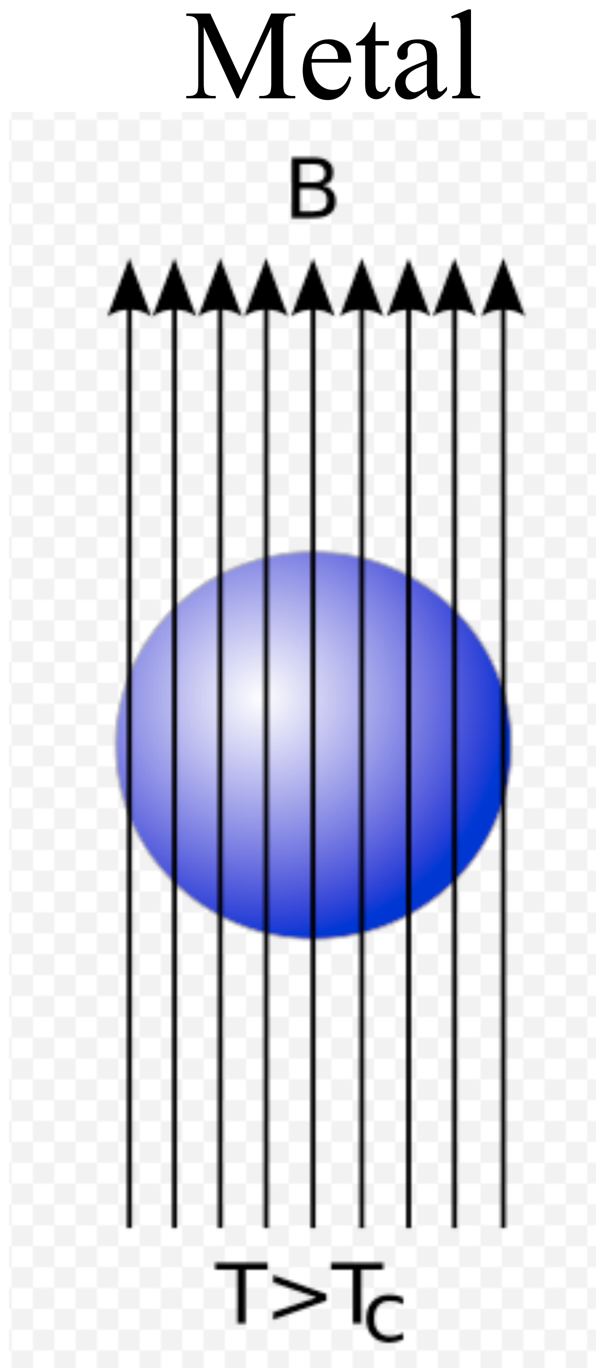
Then:

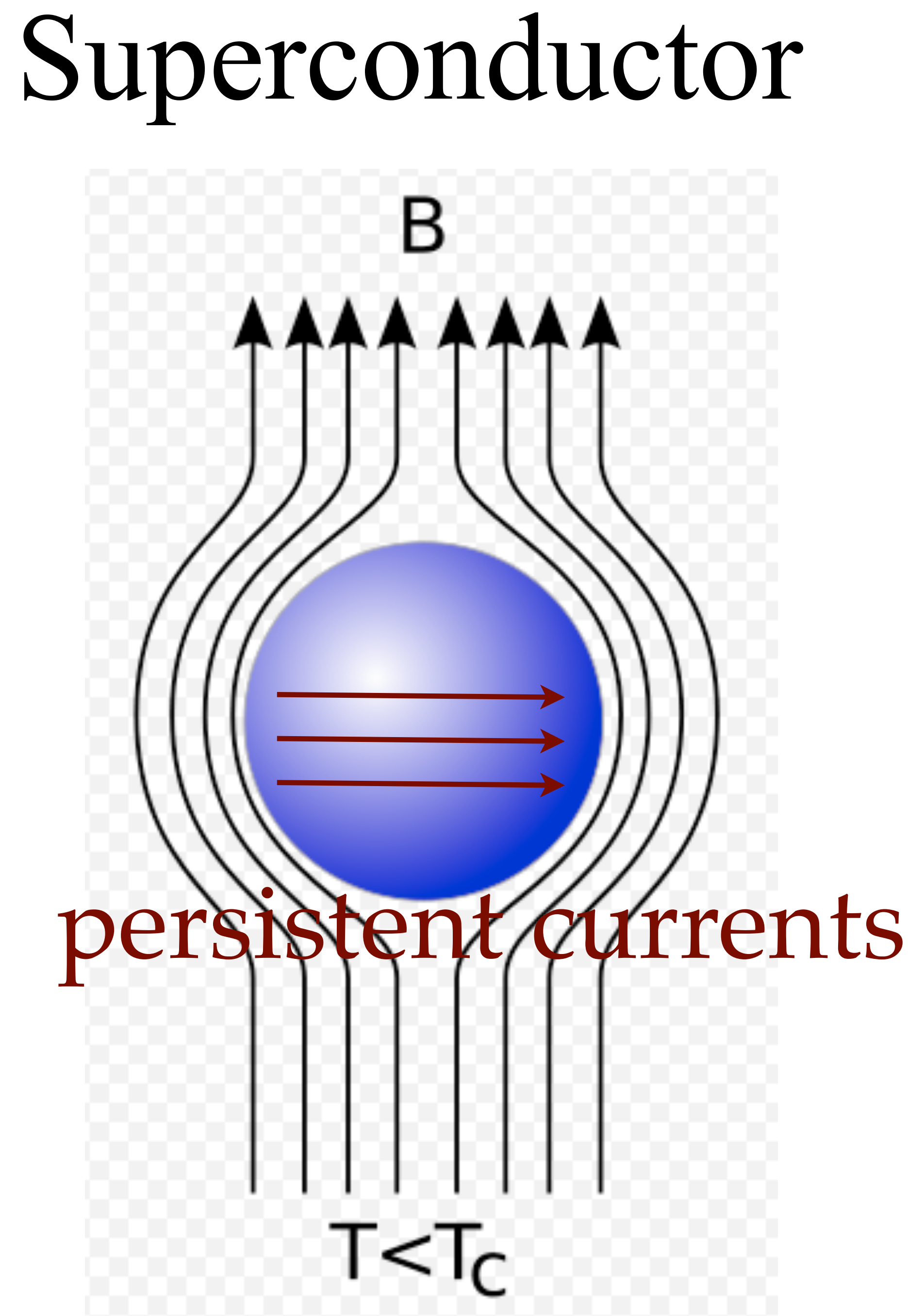
$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \eta\right)^2 + m^2\left(\eta^2 + 2\eta^3 + 2\eta^4\right) + e^2(1+\eta)^2\tilde{A}_\mu \tilde{A}^\mu - \frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}$$

- ✶ There is no ξ particle !
- ✶ Photon becomes massive triplet $\widetilde{A}_\mu$
- ✶ The Higgs boson (η) has mass, $m_H = \sqrt{2}m$, independent of e

Higgs mechanism in condensed matter

Meissner Effect, 1933

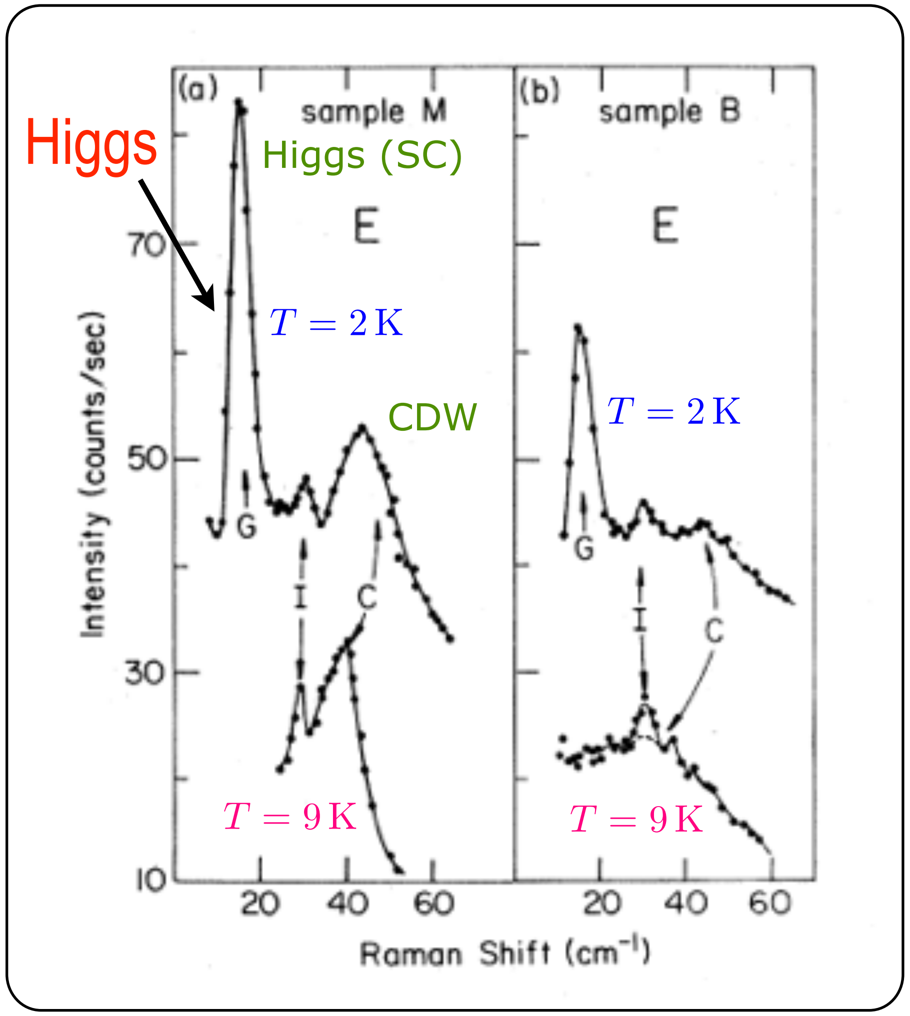




Higgs mechanism, but where is the "Higgs boson"?

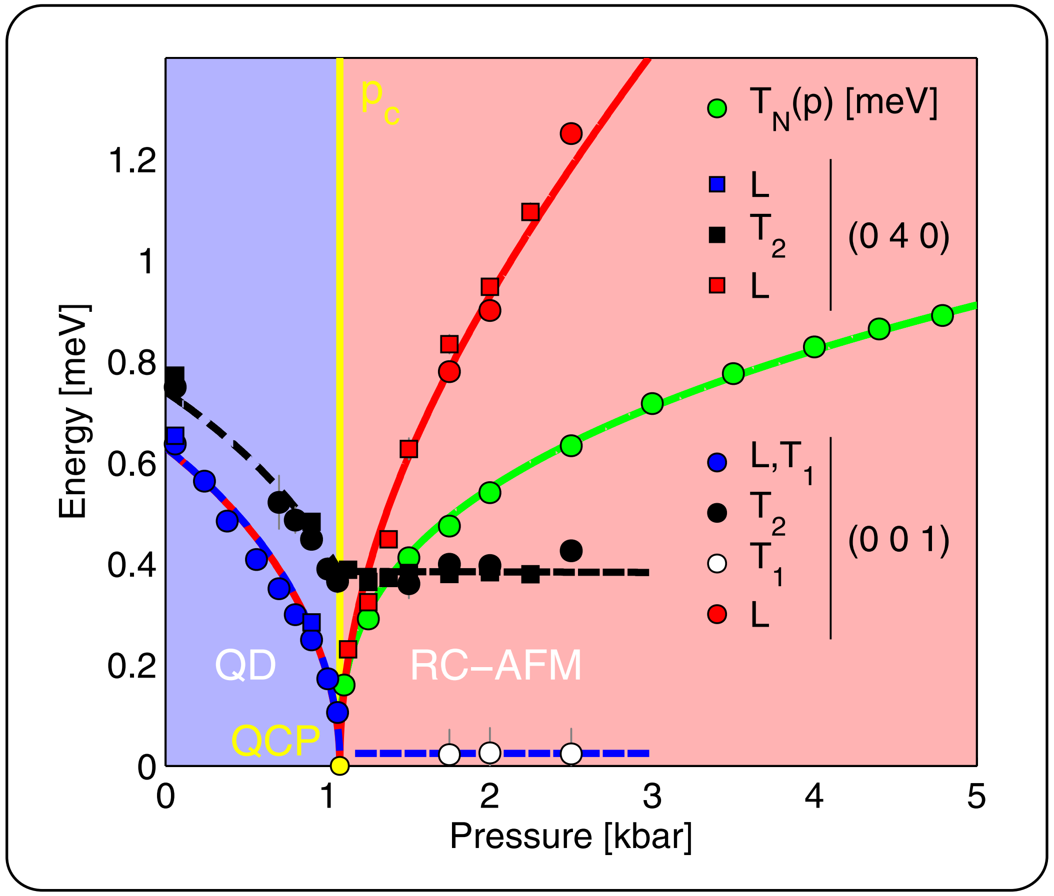
Some experimental candidates

Superconducting 2H-$NbSe_2$



R. Sooryakumar et al., PRB (1981)

Antiferromagnetic $TlCuCl_3$



Ch. Rüegg et al., PRL (2008)

First observation of the Higgs mode?

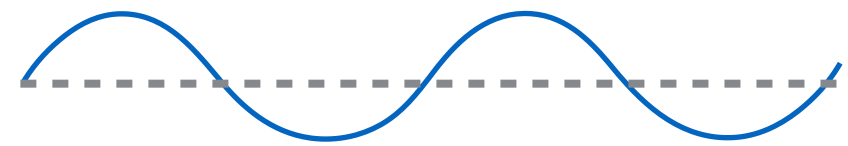
P. B. Littlewood and C. M. Varma, PRB (1982)

Higgs mode softens at quantum phase transition

Charge density waves

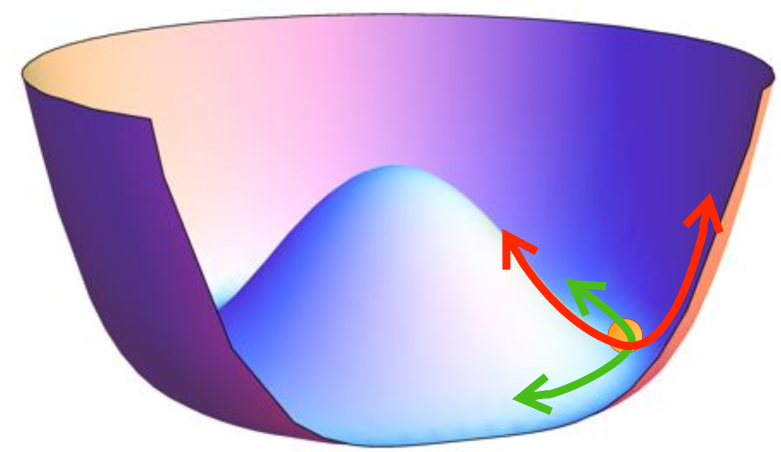
Density modulation:

$$\delta\rho(x) = \mathrm{Re}\left[\psi\, e^{iQx}\right] = |\psi| \cos(Qx + \varphi)$$

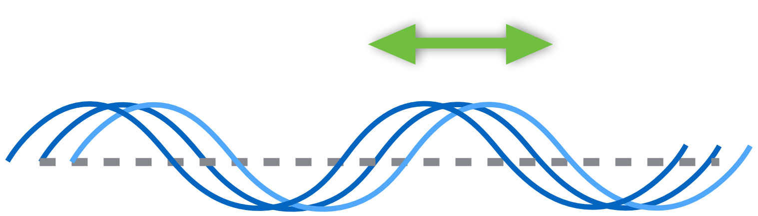


Complex order parameter:

$$\psi = |\psi|\, e^{i\varphi}$$

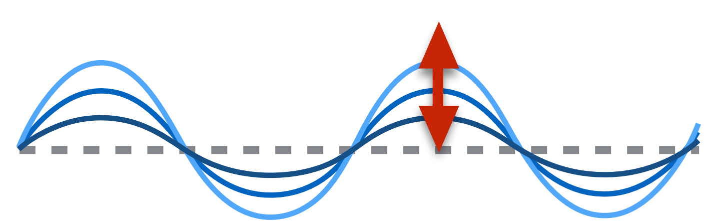


"phason"



$$\omega \sim cq$$

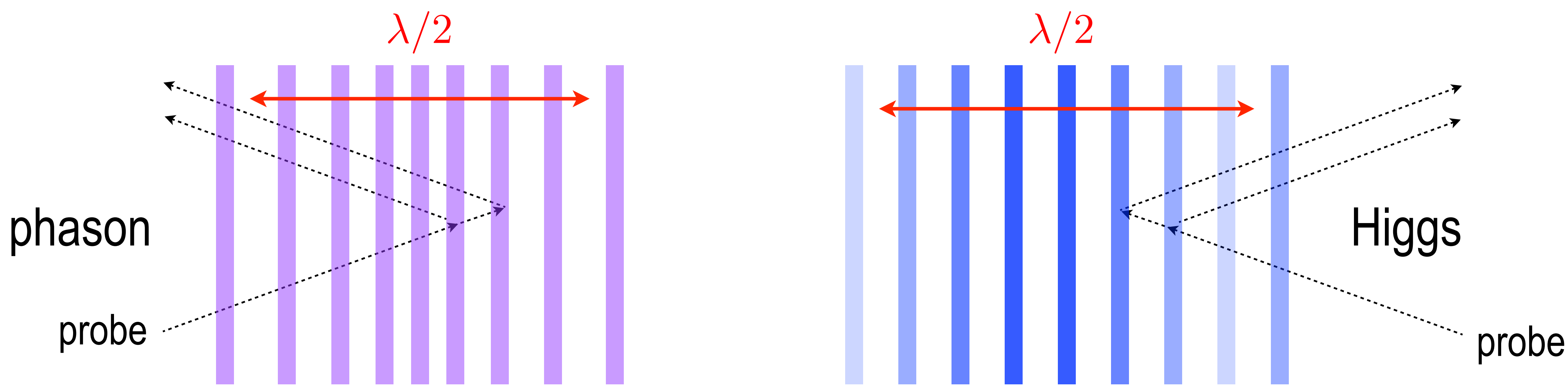
"amplitudon" (Higgs)

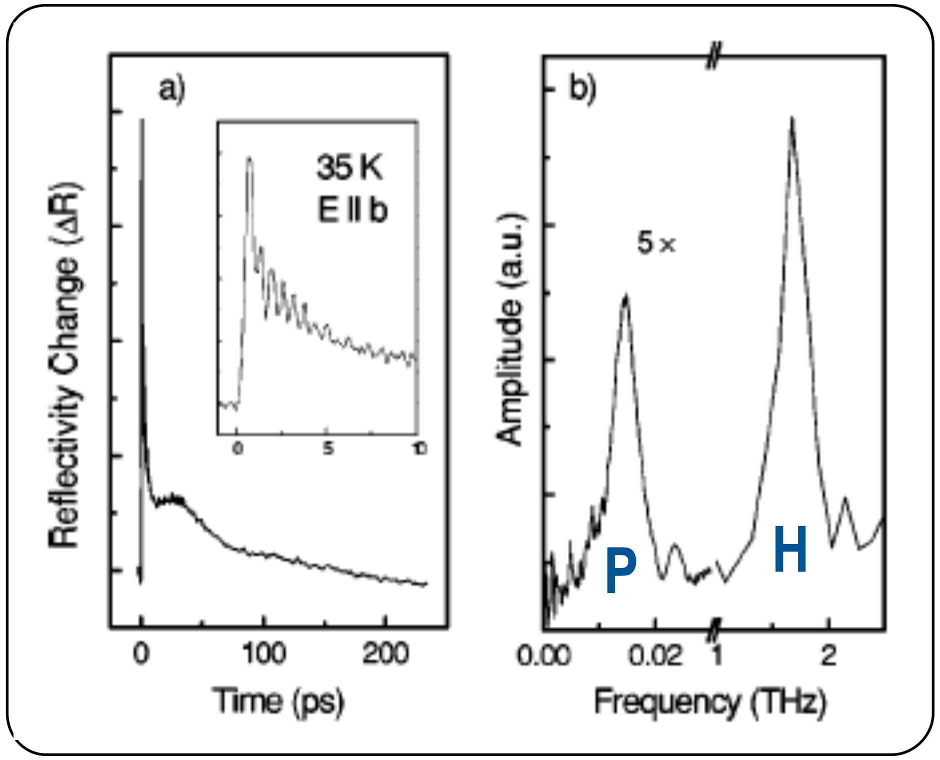


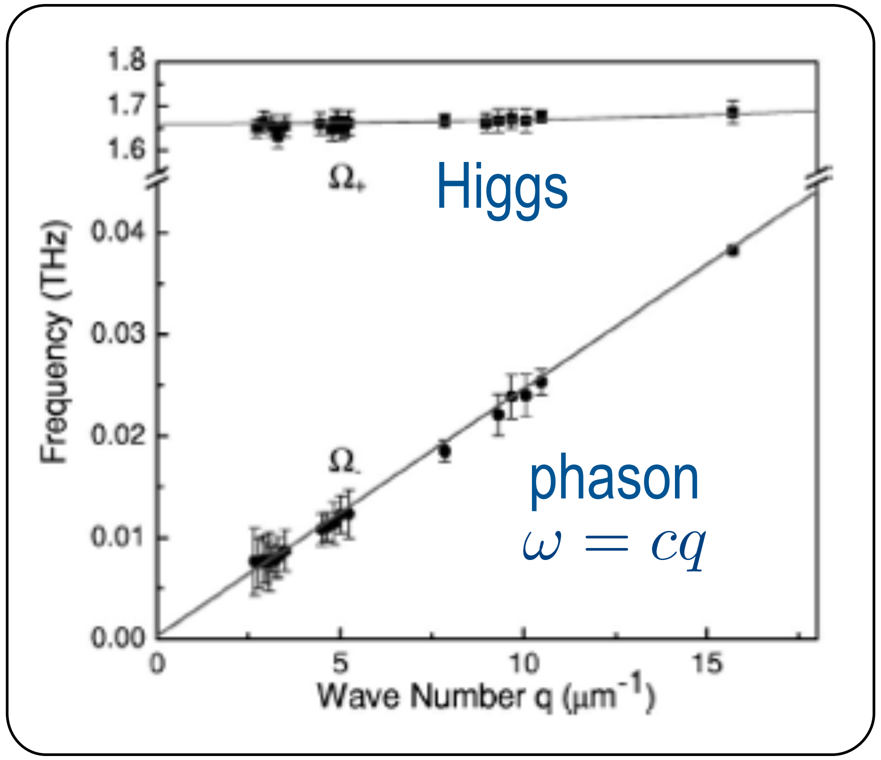
$$\omega \sim \sqrt{m^2 + c^2 q^2}$$

FS pump-probe spectroscopy

One-dimensional CDW conductor $K_{0.3}MoO_3$:





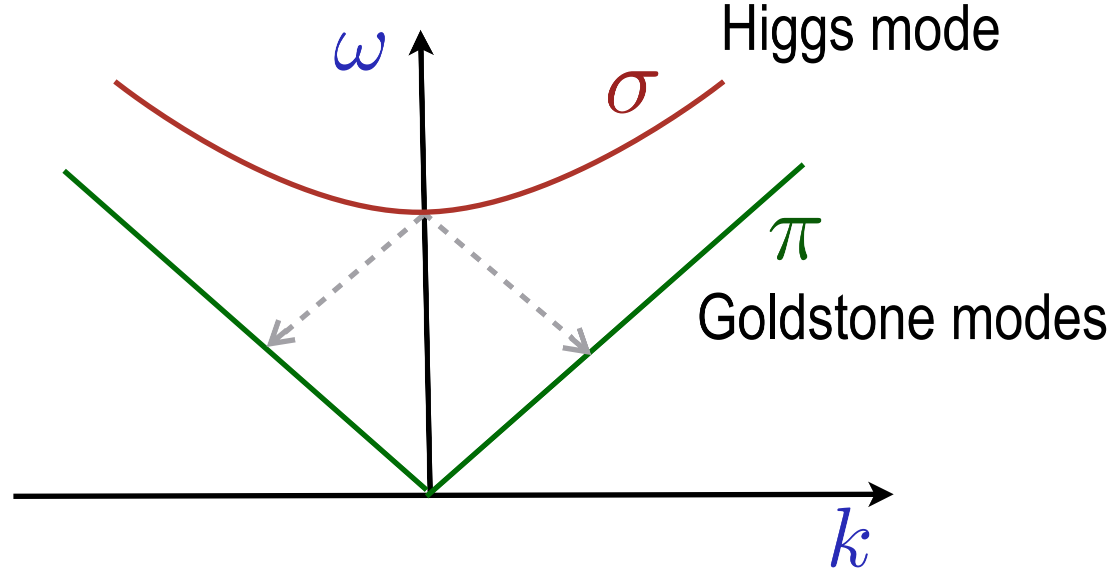


Y. Ren, Z. Xu, and G. Lüpke, J. Chem. Phys. **120**, 4755 (2004)

How to see the Higgs mode in 2d?

The Higgs decay

The Higgs mode can decay into a pair of Goldstone bosons:



d=3 Higgs decay rate is finite. As $g \to g_c$, Higgs mode becomes sharper and sharper

Affleck and Wellman (1992)

d=2 Longitudinal response diverges at low frequency, even at weak coupling!

(Nepomnyaschii)^2 (1978)
Sachdev (1999), Zwerger (2004)

Behavior of different response functions

longitudinal susceptibility

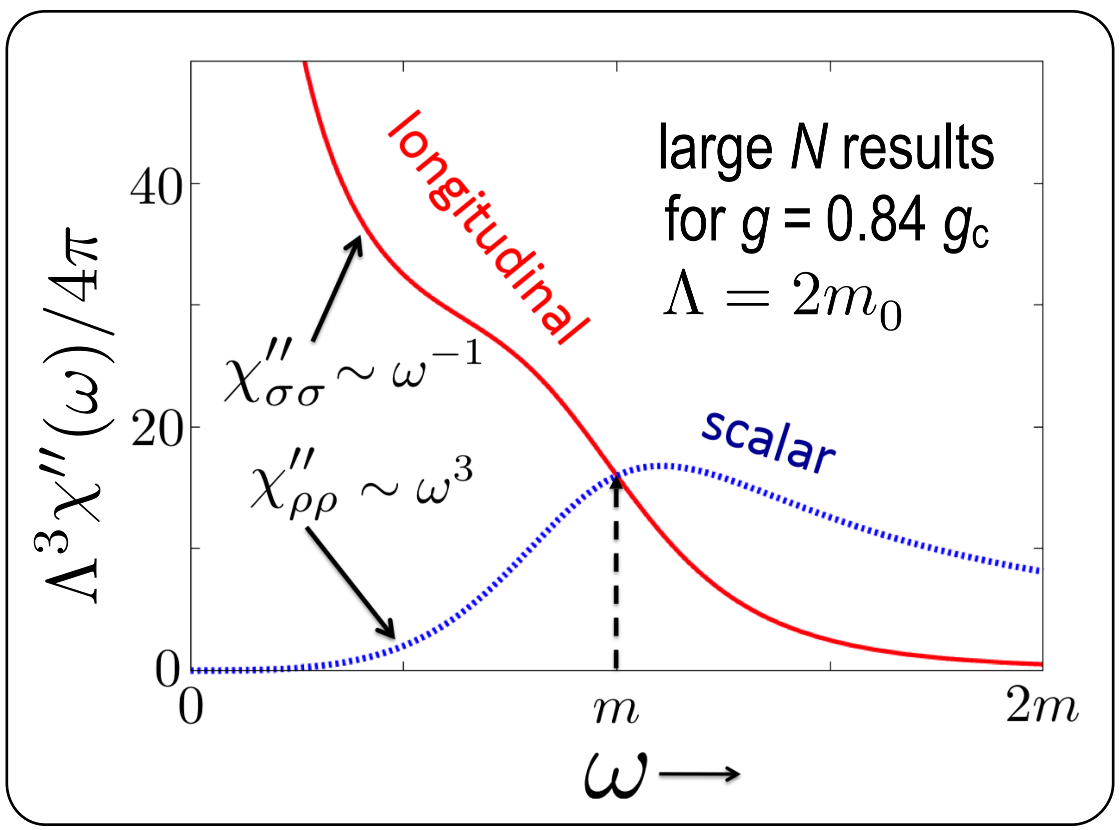
$$\chi_{\rm long}(\omega) = \langle \phi_1(\omega)\phi_1(-\omega)\rangle \sim \omega^{-1}$$

infrared divergent in d=2

(Nepomnyaschii)[2] (1978)
Sachdev (1999), Zwerger (2004)

scalar susceptibility

$$\chi_{\rm scalar}(\omega) = \langle |\vec{\phi}|^2(\omega)|\vec{\phi}|^2(-\omega)\rangle \sim \omega^3$$



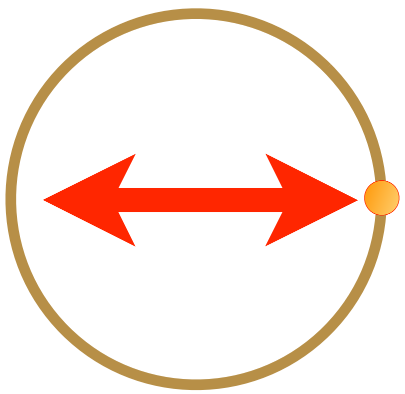
infrared regular in d=2

Podolsky, Auerbach and Arovas, PRB (2011)

Longitudinal versus scalar measurements

Longitudinal: couples to order parameter as a vector

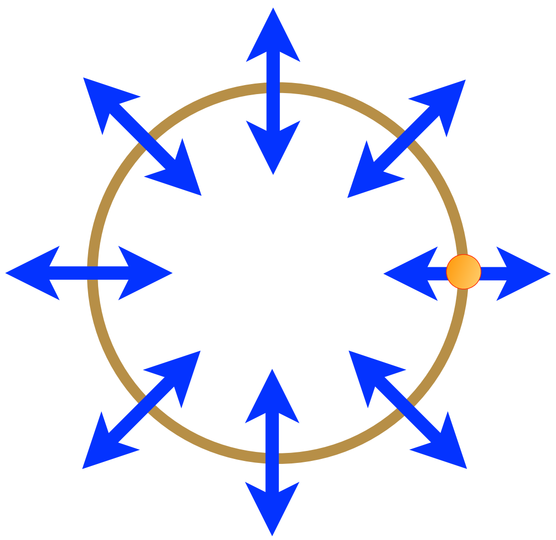
$$\mathcal{H}_{\text{probe}} = \vec{h}_{\text{ext}} \cdot \vec{\phi}$$



Example: neutron scattering in an antiferromagnet.

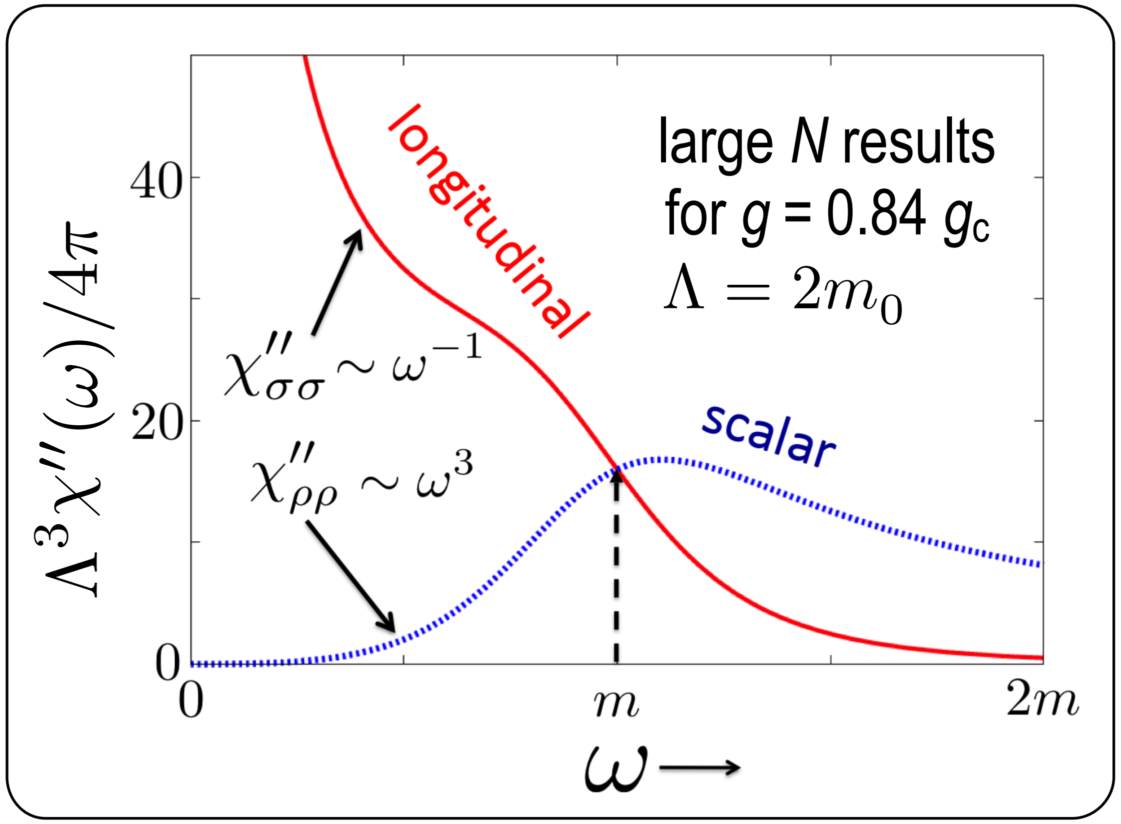
Scalar: couples to the magnitude of the order parameter

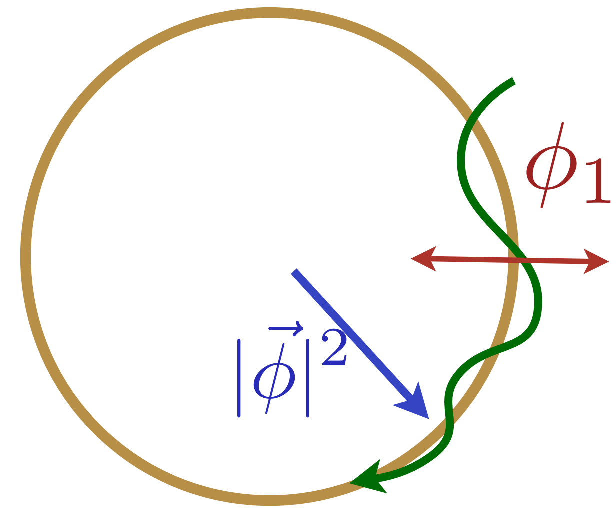
$$\mathcal{H}_{\text{probe}} = u_{\text{ext}} \, |\vec{\phi}|^2$$



Example: lattice depth modulation of bosons

Why is the scalar response sharper?





Radial motion is less damped since it is not effected by azimuthal meandering.

Bosons in an optical lattice

Bose-Hubbard model $H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i^2 - \mu \sum_i n_i$

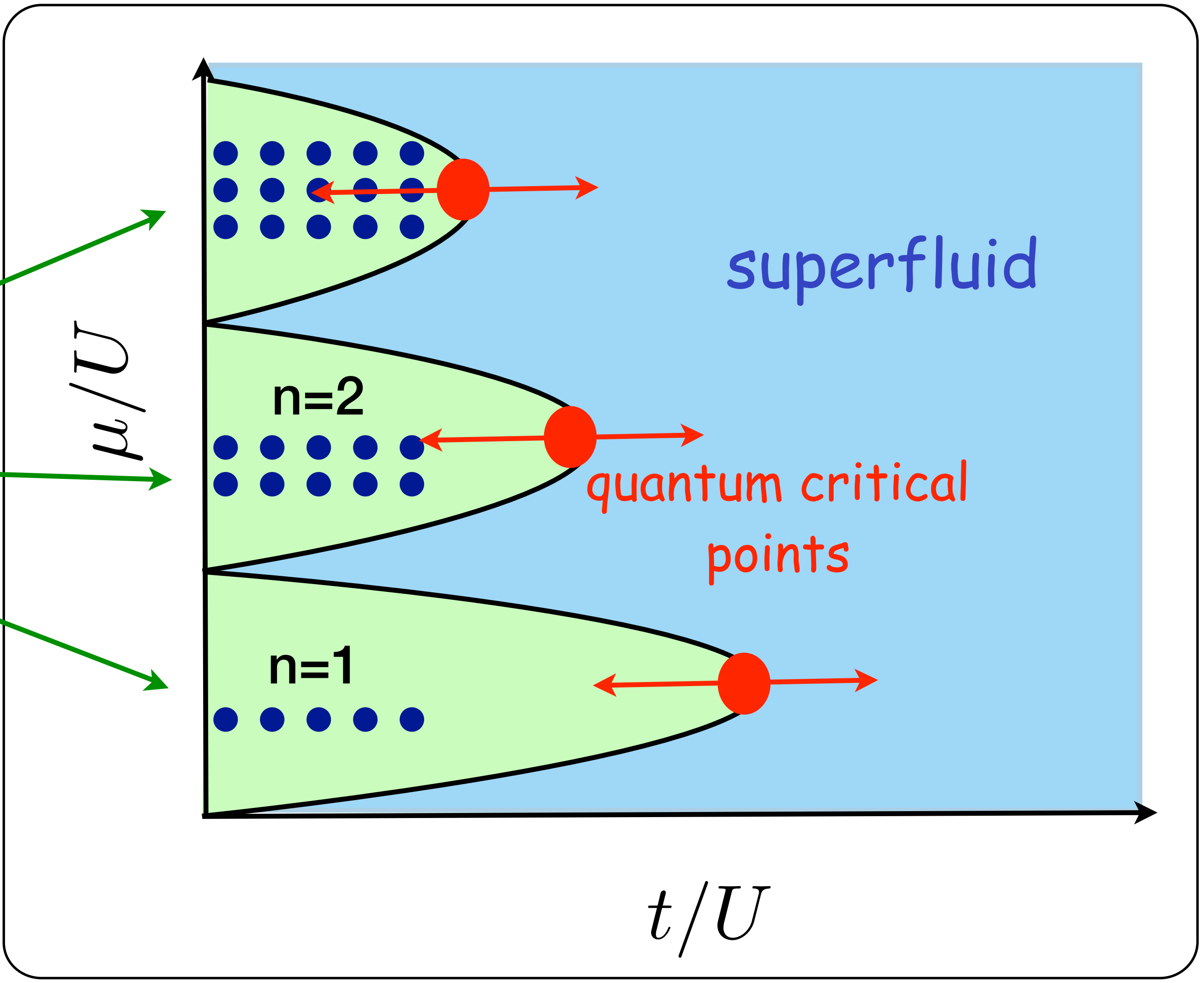
t>>U : superfluid (Bose condensate)

U>>t: Mott insulator (gapped charge fluctuations)

Mott insulator
incompressible

$$\langle b \rangle = 0$$

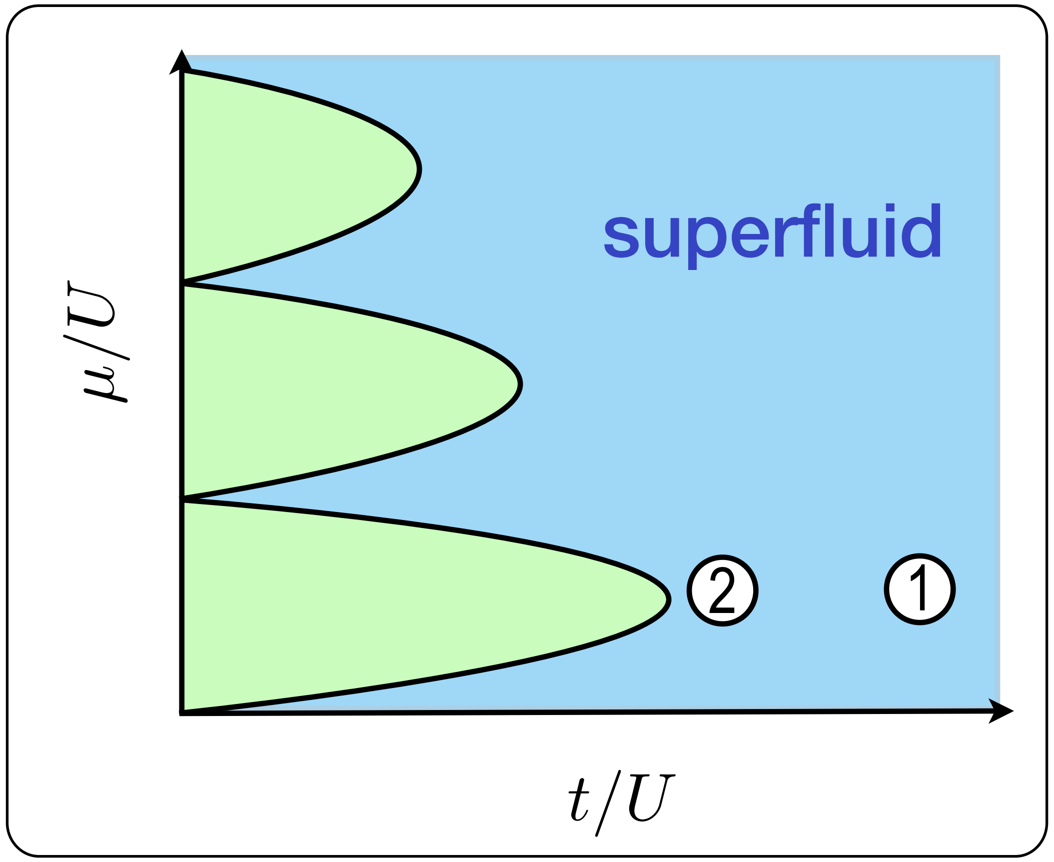
$$\frac{\partial \langle n_i \rangle}{\partial \mu} = 0$$



Bosons in an optical lattice

Bose-Hubbard model $$H = -t\sum_{\langle ij\rangle} b_i^\dagger b_j + U\sum_i n_i^2 - \mu\sum_i n_i$$

Dynamics:



1) Far from Mott lobe, Gross-Pitaevskii model,

$$\mathcal{L} = -i\psi^*\partial_t\psi - \frac{\hbar^2}{2m^*}|\nabla\psi|^2 + \mu|\psi|^2 - g|\psi|^4$$

Gapless Goldstone mode, but no Higgs.

Varma, J. Low Temp. Phys. (2002)
Huber et al, PRB (2008)
Huber and Lindner, PNAS (2011)

2) Close to Mott lobe, relativistic model,

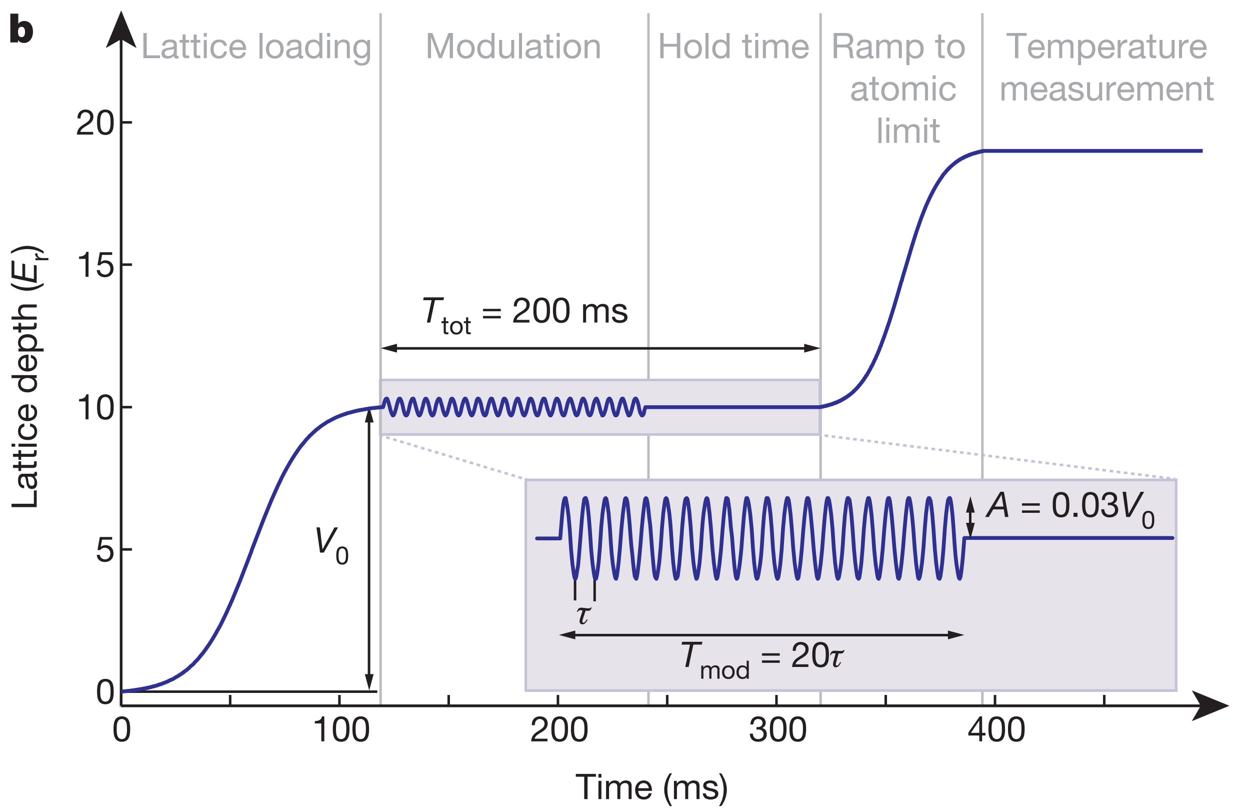
$$\mathcal{L} = |\partial_t\psi|^2 - c^2|\nabla\psi|^2 + r|\psi|^2 - u|\psi|^4$$

Goldstone and Higgs.

LETTER
doi:10.1038/nature11255

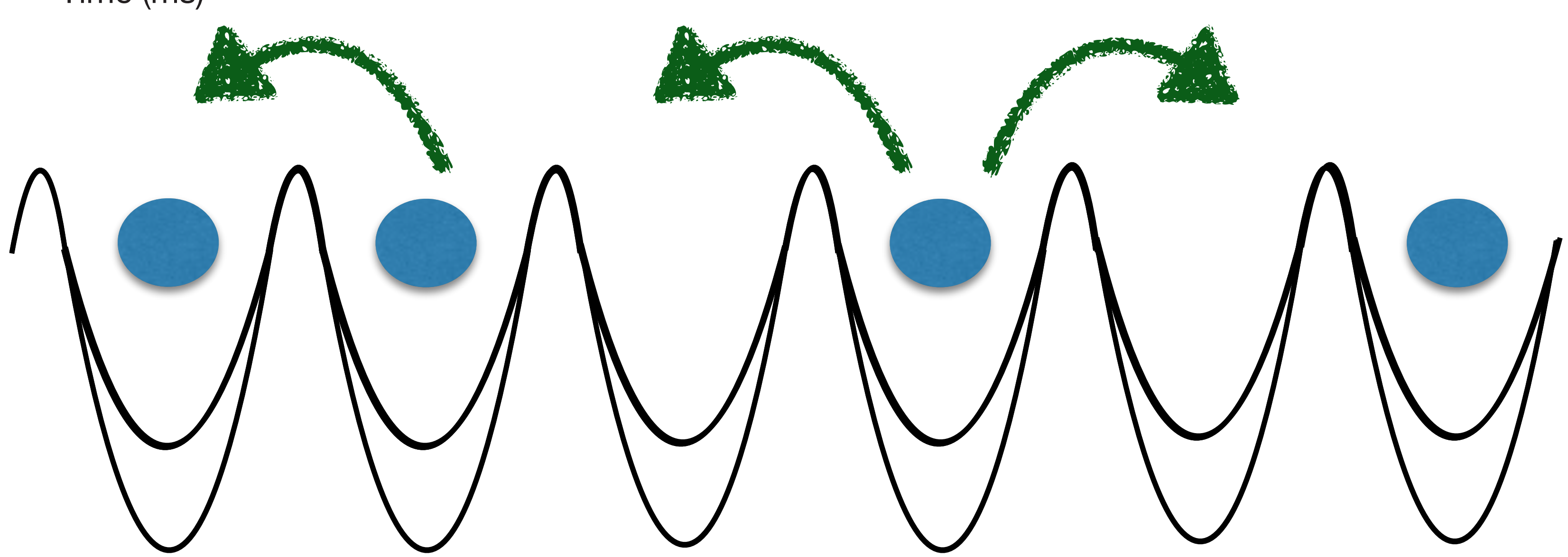
The ‘Higgs’ amplitude mode at the two-dimensional superfluid/Mott insulator transition

Manuel Endres[1], Takeshi Fukuhara[1], David Pekker[2], Marc Cheneau[1], Peter Schauß[1], Christian Gross[1], Eugene Demler[3], Stefan Kuhr[1,4] & Immanuel Bloch[1,5]



Energy absorption rate of periodically modulated lattice

$$\propto \omega \chi''_{\mathrm{scalar}}(\omega)$$



LETTER

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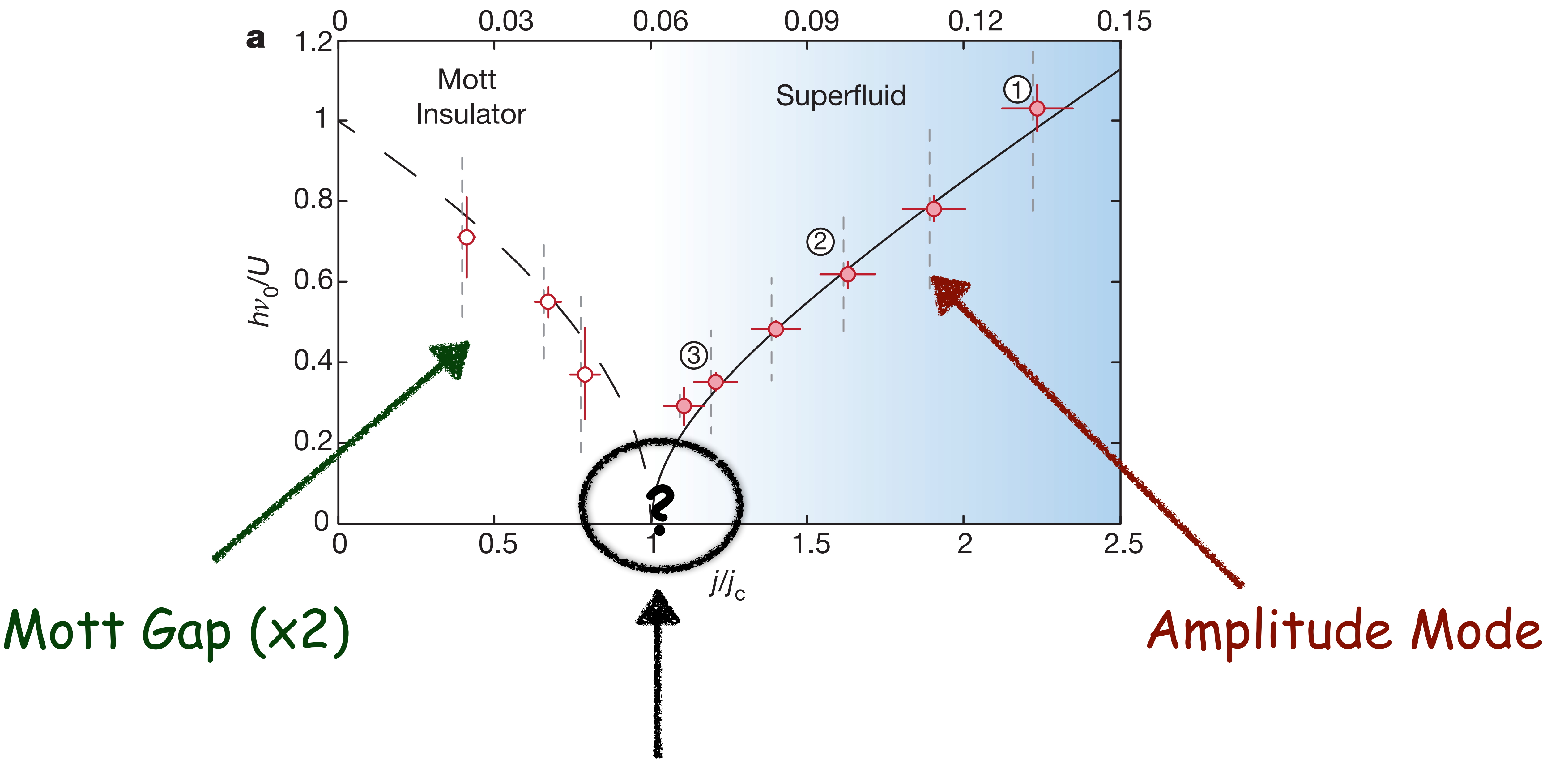


What happens near the quantum critical point???

Does the Higgs survive near quantum criticality?

Scaling near criticality

gap: $\Delta \sim |g - g_c|^{\nu}$ $\nu = 0.6717(1) \quad (N = 2)$

$$\chi_{scalar}(\omega) = \Delta^{3-2/\nu} \Phi_s \left(\frac{\omega}{\Delta} \right) + \dots$$

universal function

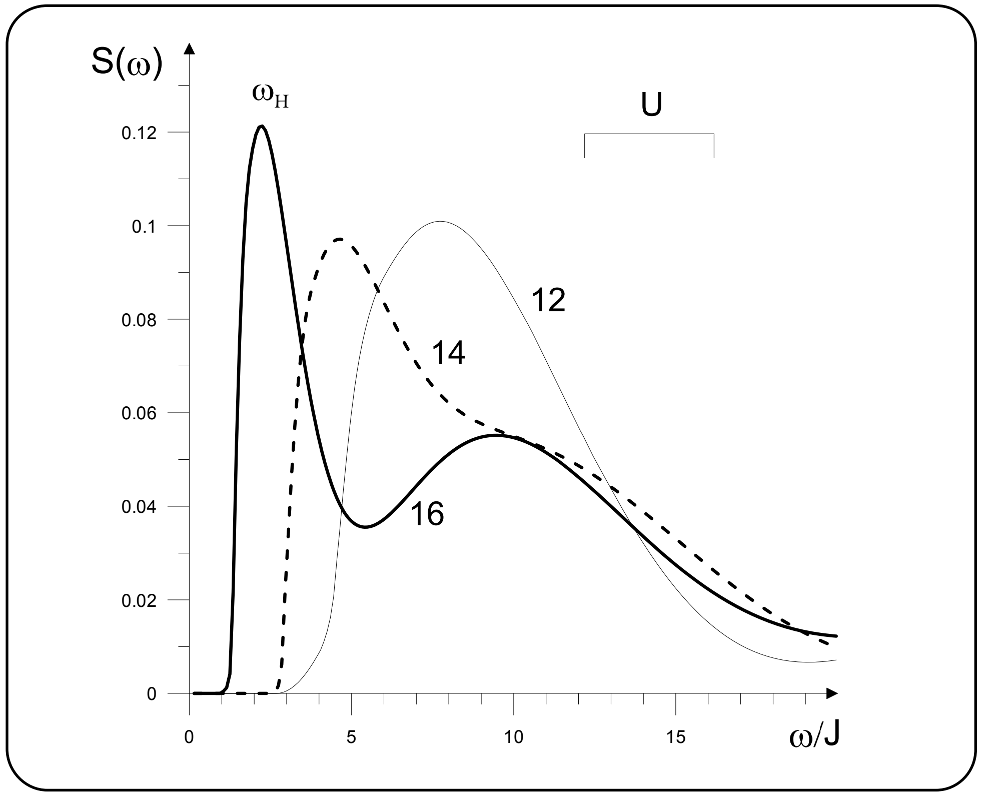
Podolsky and Sachdev, PRB (2012)

Does it have a peak?

First indications

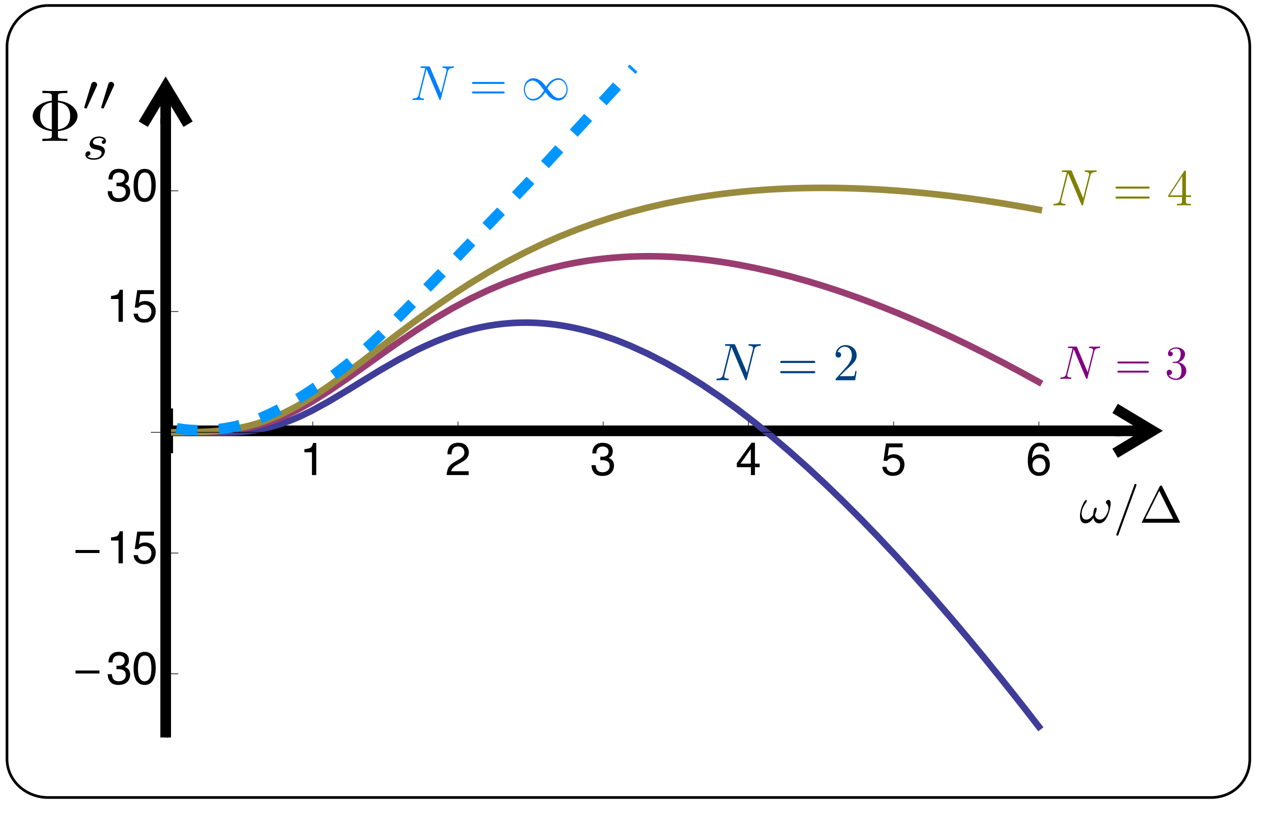
Numerics on Bose-Hubbard model

L. Pollet and N. Prokof'ev, PRL (2012)



Scaling function to $O(1/N)$

Podolsky and Sachdev, PRB (2012)

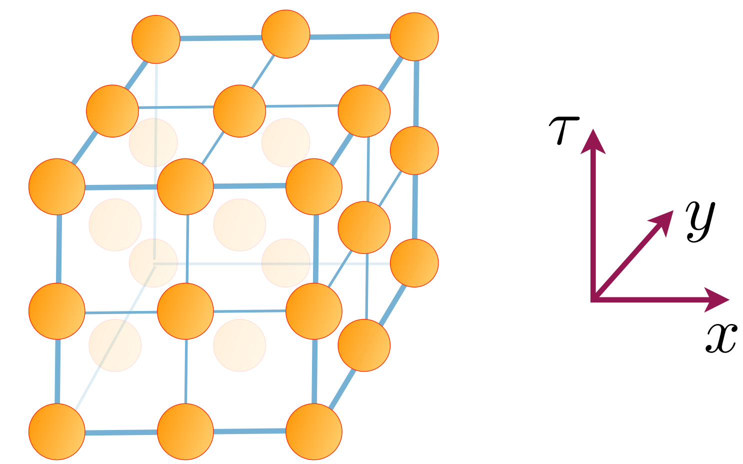


Monte Carlo Simulations

Discrete model:

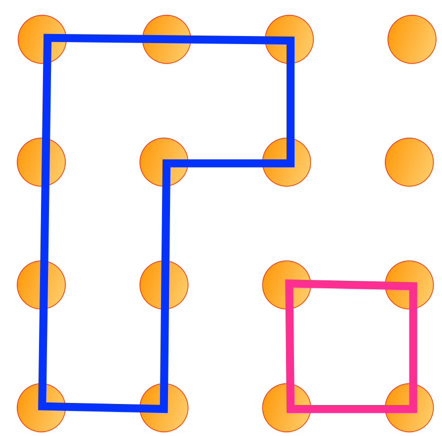
$$\mathcal{Z} = \int \mathcal{D}\vec{\phi}\, e^{-S[\vec{\phi}]}$$

$$S = -\sum_{\langle ij \rangle} \vec{\phi}_i \cdot \vec{\phi}_j + \mu \sum_i |\vec{\phi}_i|^2 + g \sum_i |\vec{\phi}_i|^4$$



Worm algorithm:

Dual loop model
with N flavors:

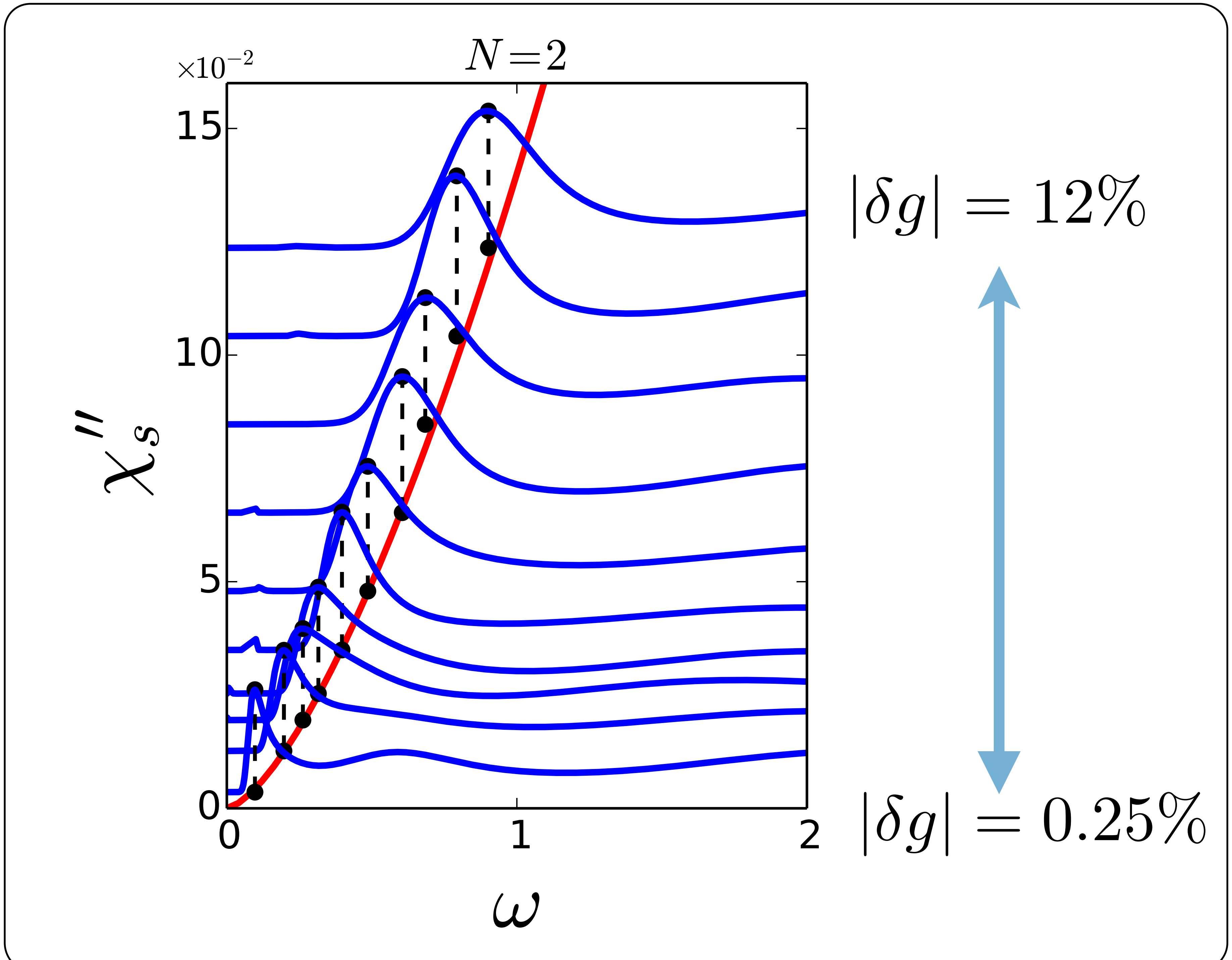


System size: $1 \ll \xi \ll L \qquad (1 \ll 30 \ll 200)$

Numerical analytical continuation from Matsubara to real frequencies

Tracking the Higgs peak

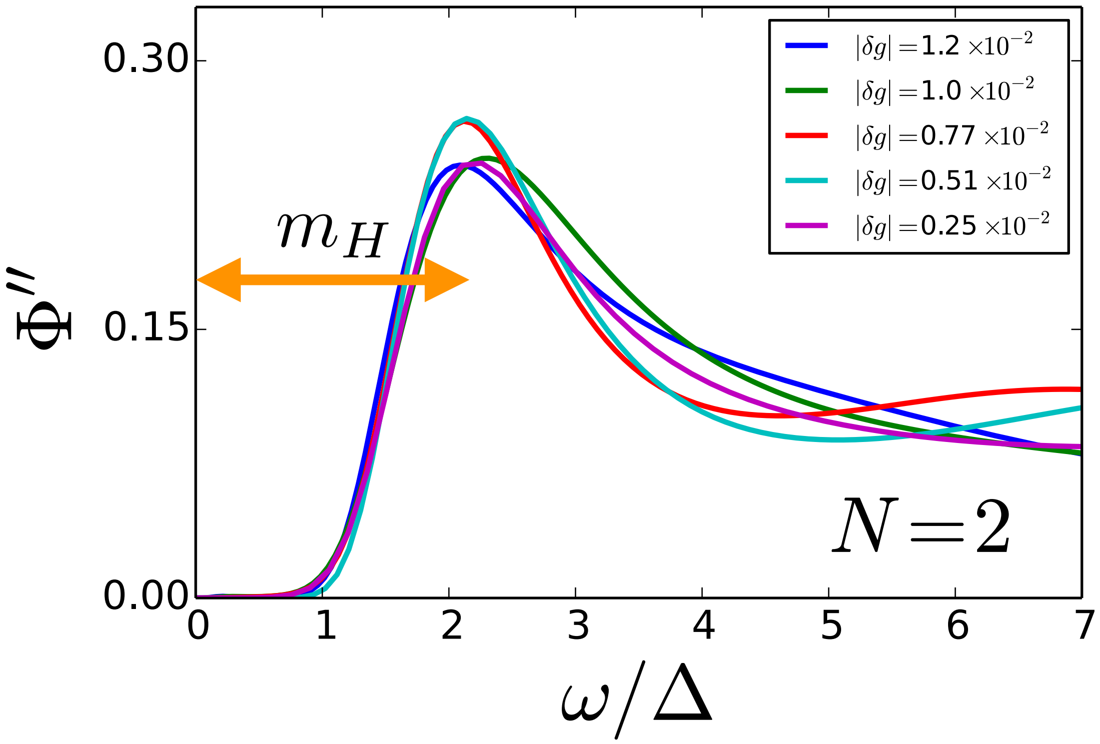
Scalar susceptibility in ordered phase: $\chi_s(\omega) = \langle \phi^2(\omega)\phi^2(-\omega) \rangle$

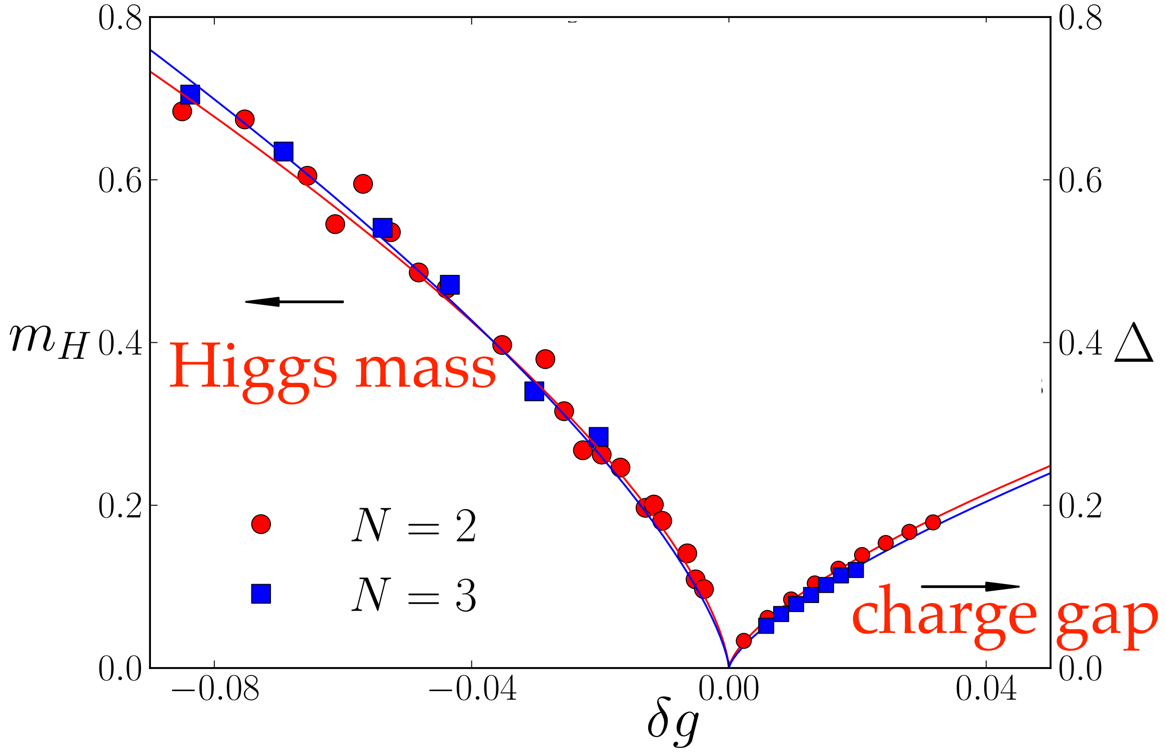


$m_H = B|\delta g|^{\nu}$

Gazit, Podolsky, Auerbach, PRL (2013)

Spectral function at the QCP





$$\frac{m_H}{\Delta} = 2.1(3)$$

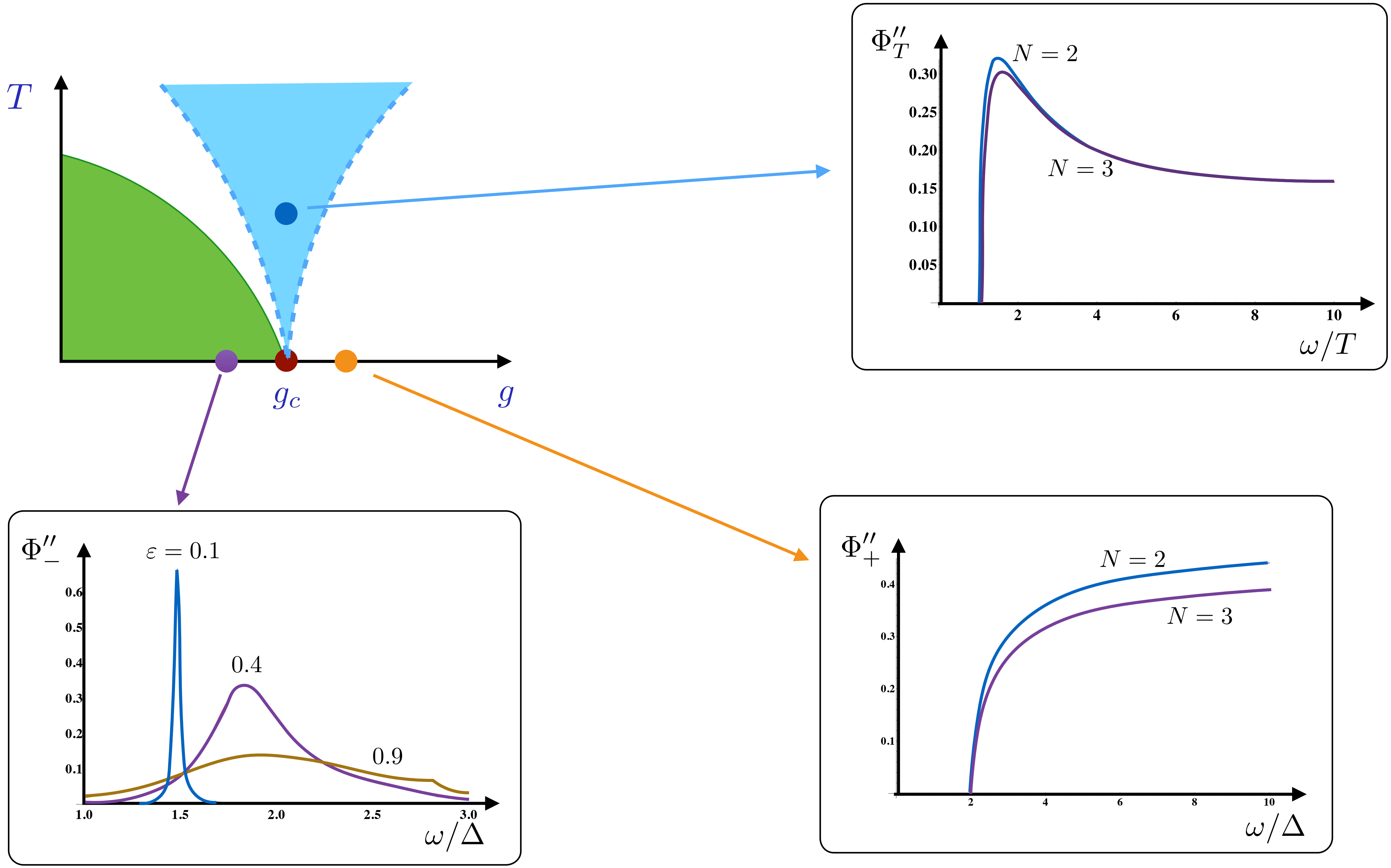
Mean field: $$\frac{m_H}{\Delta} = \sqrt{2}$$

Conclusion: Higgs resonance **survives** close to criticality in d=2

Chen et al, Bose-Hubbard Model (2013): $$\frac{m_H}{\Delta} = 3.3(8)$$

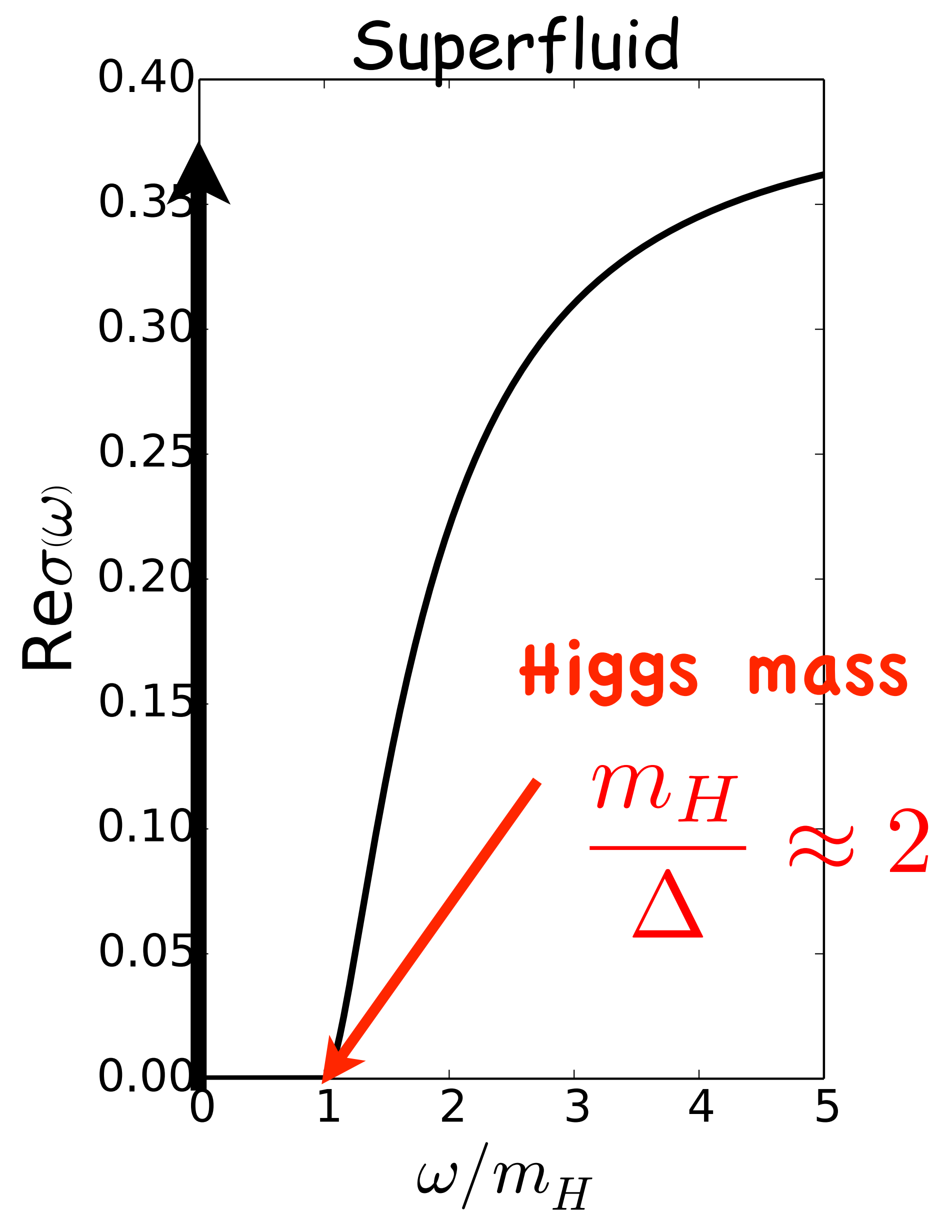
Rancon and Dupuis (2014): $$\frac{m_H}{\Delta} = 2.4$$

Scaling functions in D= 4- ε dimensions



Tenenbaum Katan and Podolsky, unpublished

Higgs in optical conductivity



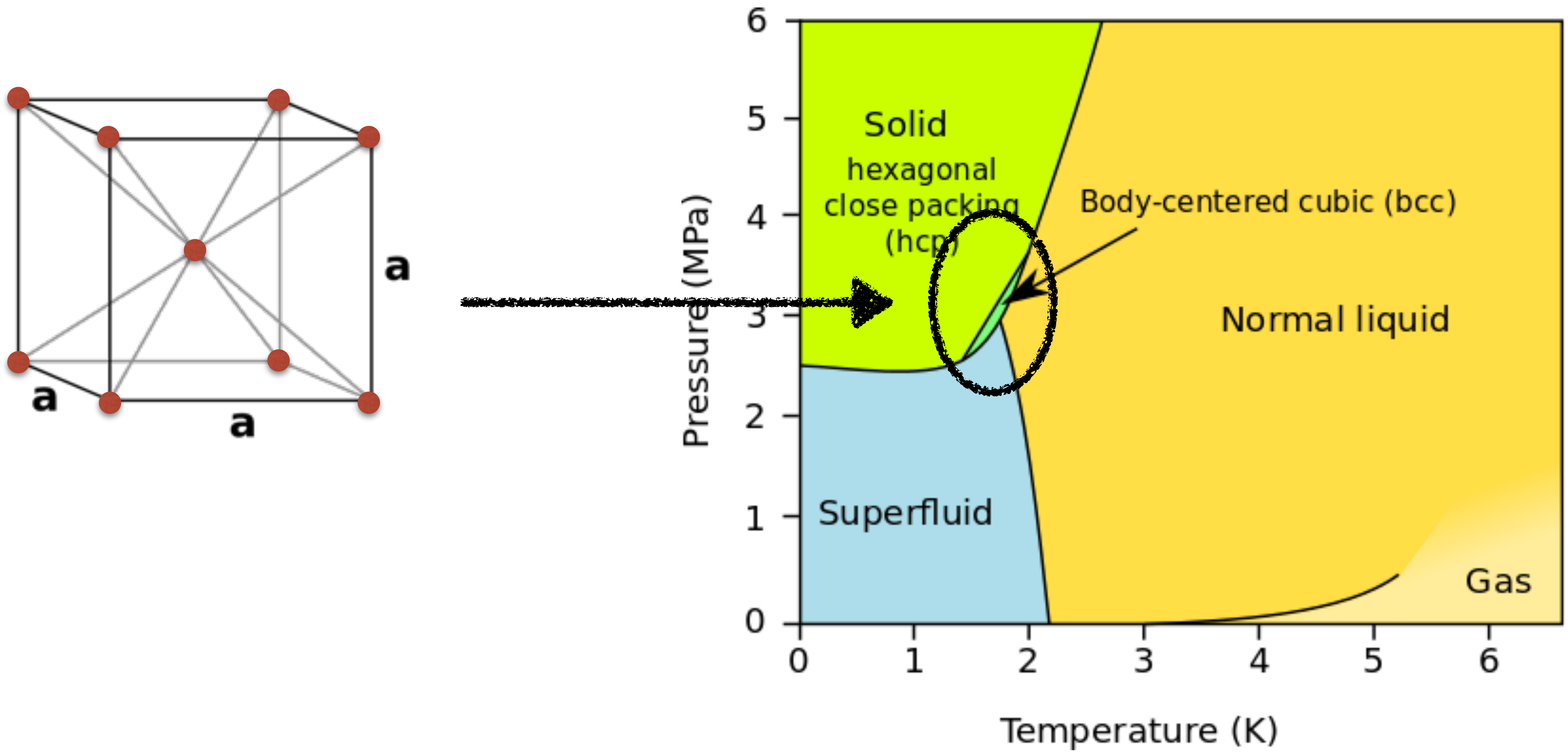
$$\sigma_{\rm SF}(\omega) = 2\pi\sigma_Q \left(\frac{\omega^2 - m_H^2}{4\omega^2}\right)^2 \Theta(\omega - m_H)$$

Lindner, Auerbach (2010)

Podolsky, Auerbach, Arovas (2011)

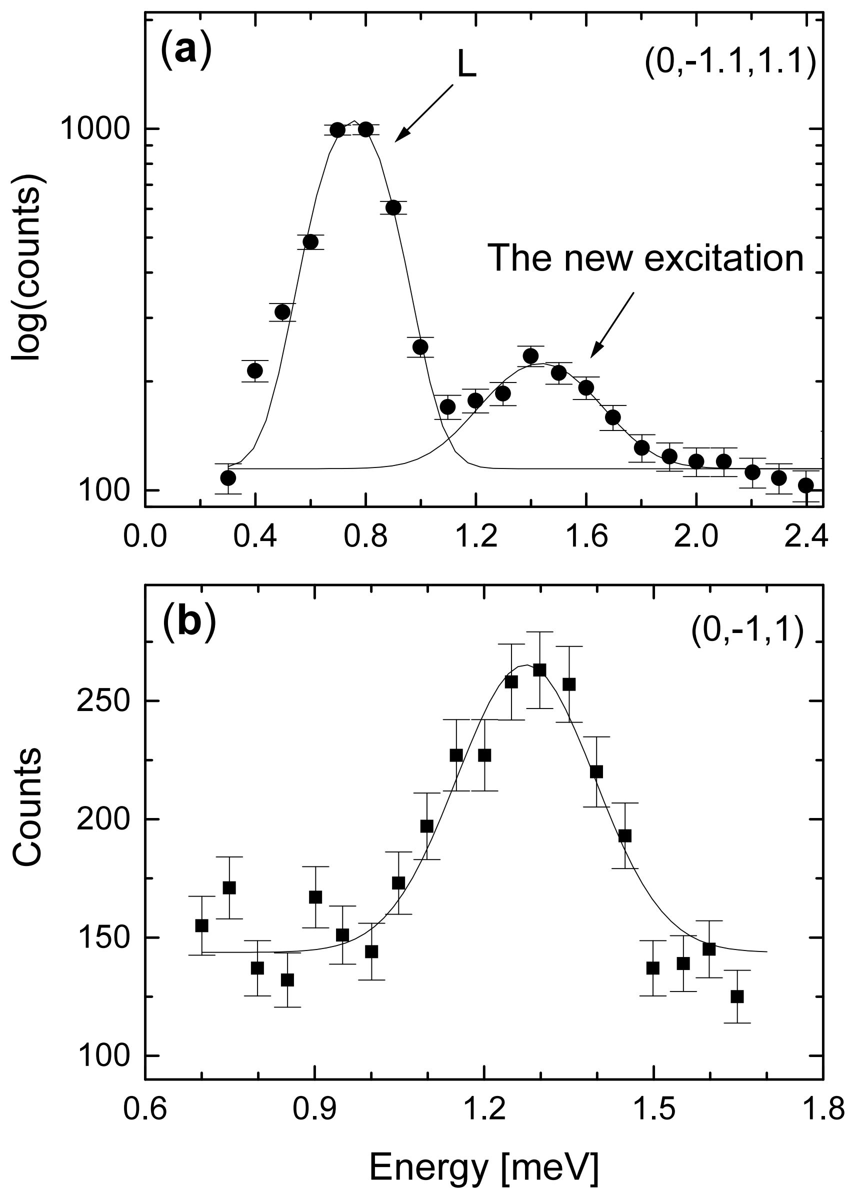
Gapped modes of a quantum solid

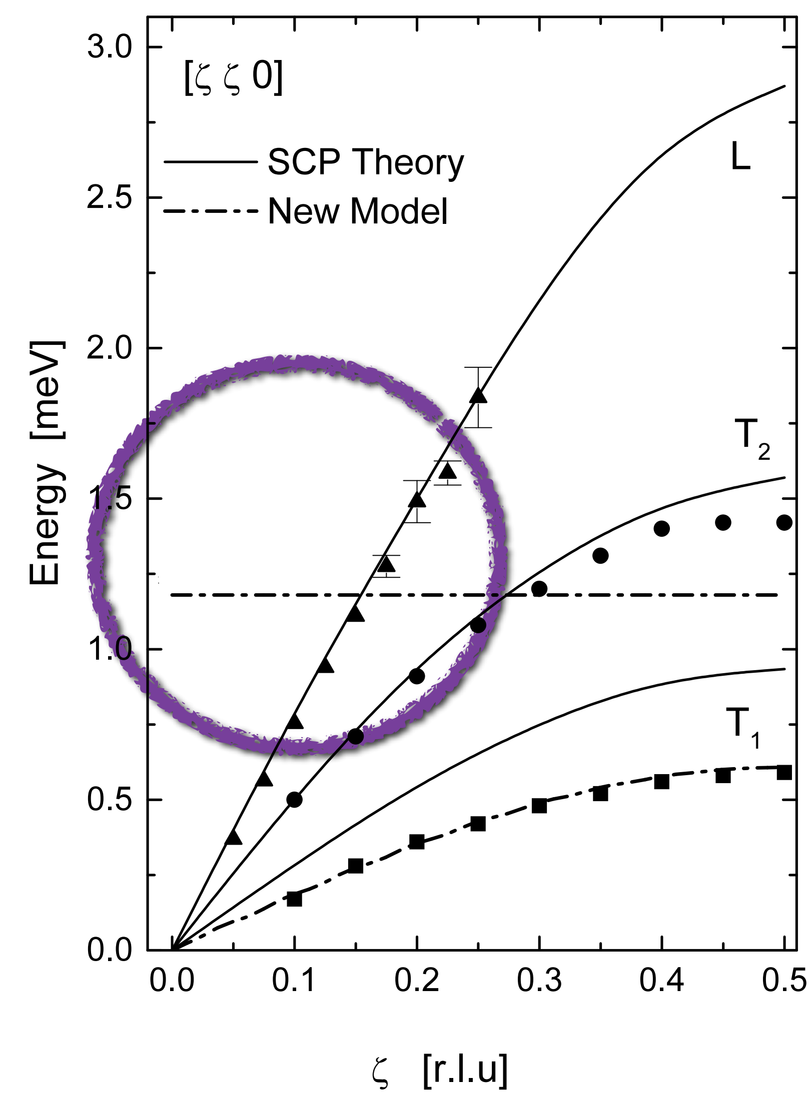
Helium 4 - Phase diagram



Inelastic neutron scattering

Optical mode observed!

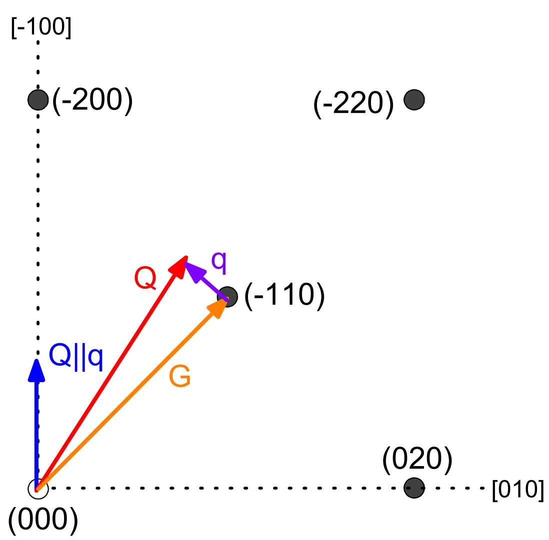




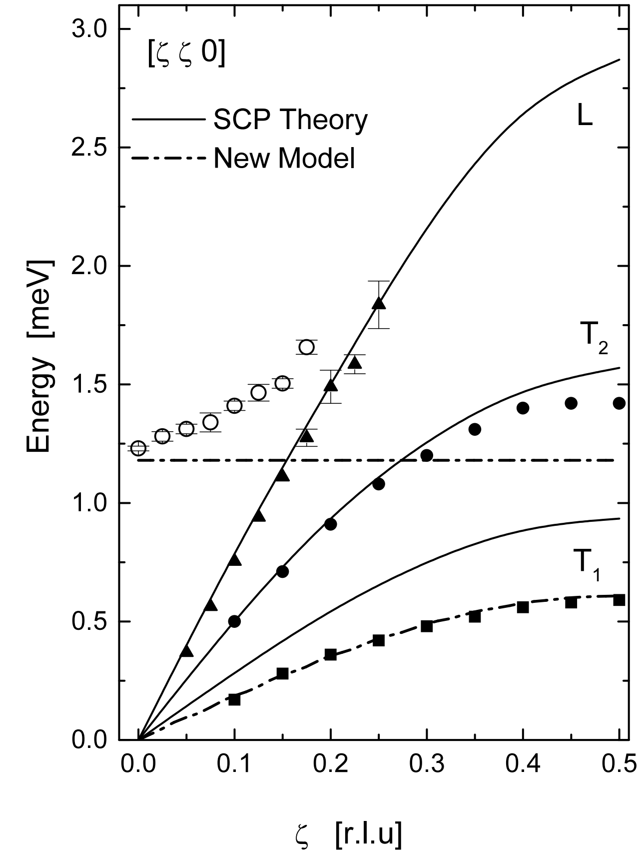
Markovic et al., PRL **88**, 195301 ('02)

Multiple optical modes?

Look in different directions & polarizations

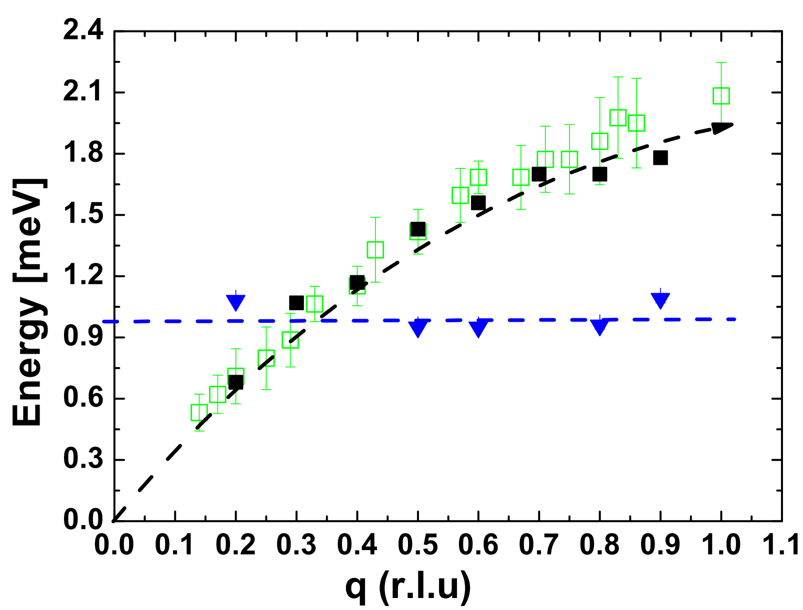


(110)



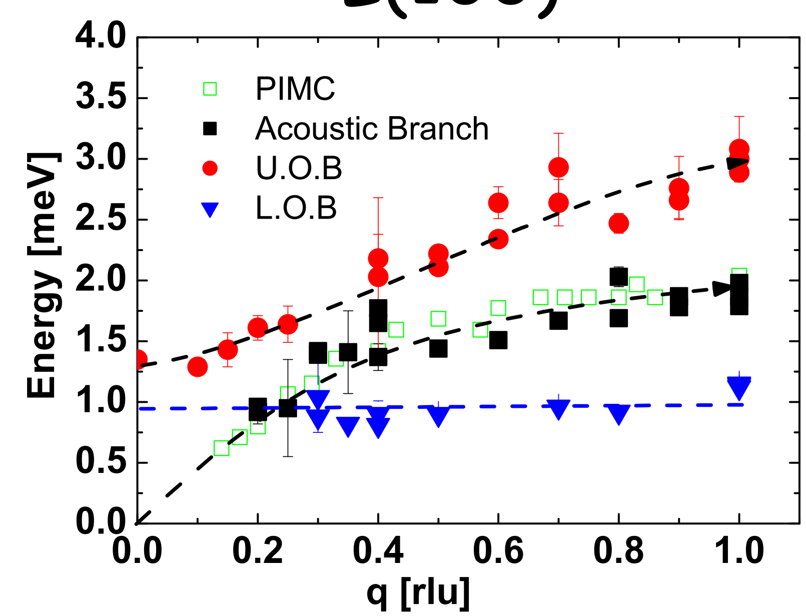
Markovic et al., PRL **88**, 195301 ('02)

T(100)



Pelleg et al, PRB **73**, 180301R ('06)

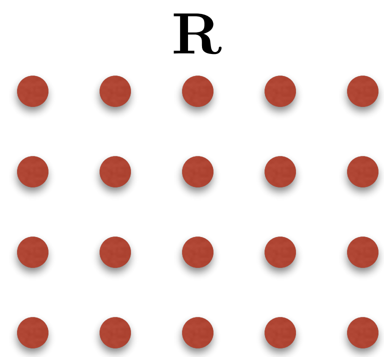
L(100)



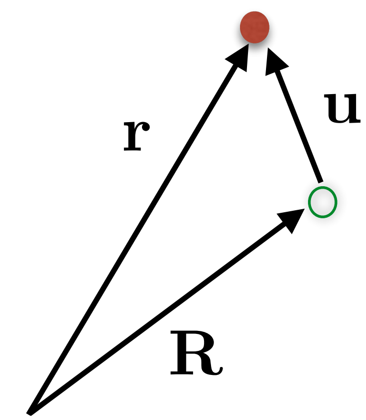
Pelleg et al, JLTP **151**, 1164 ('08)

Harmonic theory of solids

Equilibrium:



Fluctuations:



$$\mathbf{r}(\mathbf{R}) = \mathbf{R} + \mathbf{u}(\mathbf{R})$$

Small fluctuations $\sqrt{\langle \mathbf{u}^2 \rangle} \ll \Delta R$

$$U_{\text{harm}} = \frac{1}{2} \sum_{\mathbf{R}\mathbf{R}'} \sum_{\mu\nu} u_\mu(\mathbf{R}) D_{\mu\nu}(\mathbf{R} - \mathbf{R}') u_\nu(\mathbf{R}')$$

Monatomic Bravais lattice ⇨ acoustic phonons only

Corrections to harmonic theory: $U_{\text{anh}} \sim u^3 + u^4 + \ldots$

Lindemann criterion: $\sqrt{\langle \mathbf{u}^2 \rangle} = 0.1\,\Delta R$ ⇨ melting

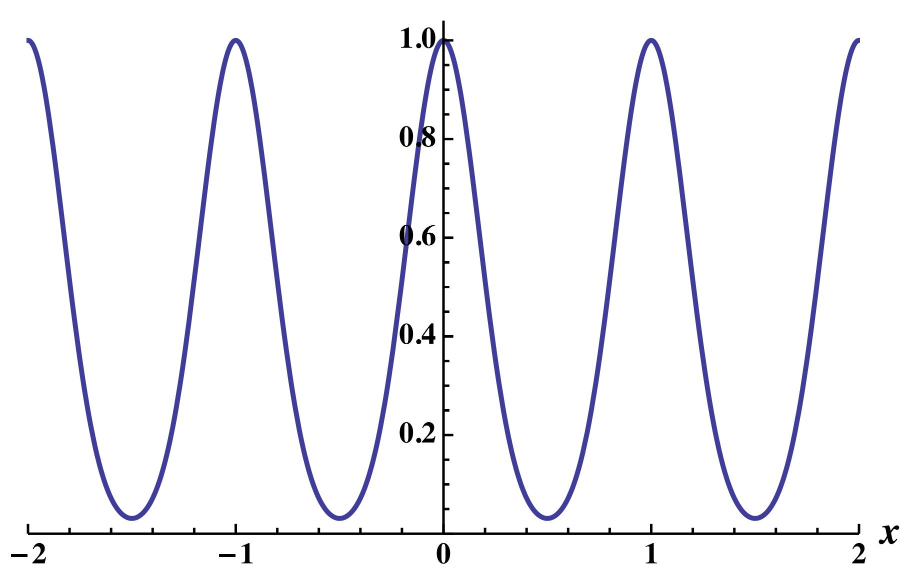
Helium – A quantum solid

Zero point motion

H. Glyde, "Helium, Solid"

Rare-gas crystal	Debye temperature θ_D (K)	Melting temperature T_M(K)	Debye zero point energy $E_{ZD} = \frac{9}{8}\theta_D$	Lindemann parameter $\delta = \langle u^2 \rangle^{1/2}/R$
^{3}He(bcc)	19	0.65	21	0.368
^{4}He(bcc)	25	1.6	28	0.292
Ne	66	24.6	74	0.091
Ar	84	83.8	95	0.048
Kr	64	161.4	72	0.036
Xe	55	202.0	62	0.028

Can we think of solid He-4 as a charge density wave (CDW)?



Collective modes of 3d CDW

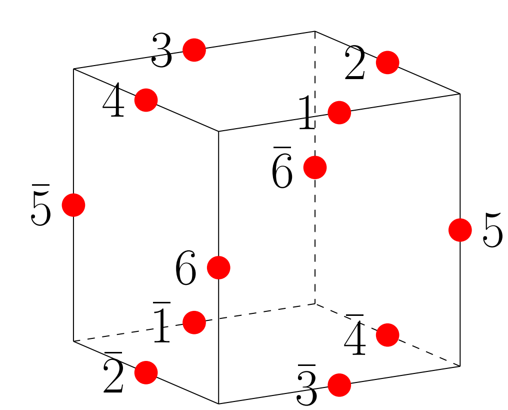
Apply Ginzburg-Landau analysis to 3d solid   Alexander and McTague, PRL (1978)

Assume weakly first order transition ⇨ $\rho(\mathbf{r}) \equiv n(\mathbf{r}) - n_0$ is small

Dynamical Ginzburg-Landau:
$$L = \frac{1}{2} \int d^3 r \left(\frac{\partial \rho}{\partial t} \right)^2 - F_{\mathrm{GL}}$$

Fluctuations about mean-field:
$$\rho(\mathbf{r}, t) = \sum_i (\bar{\rho}_i + \psi_i(\mathbf{r}, t)) e^{i \mathbf{G}_i \cdot \mathbf{r}}$$
$$\psi_i(\mathbf{r}, t) = \psi^*_{-i}(\mathbf{r}, t)$$

Solve linearized Euler-Lagrange equations



6 pairs of reciprocal lattice vectors ⇨ 12 modes!

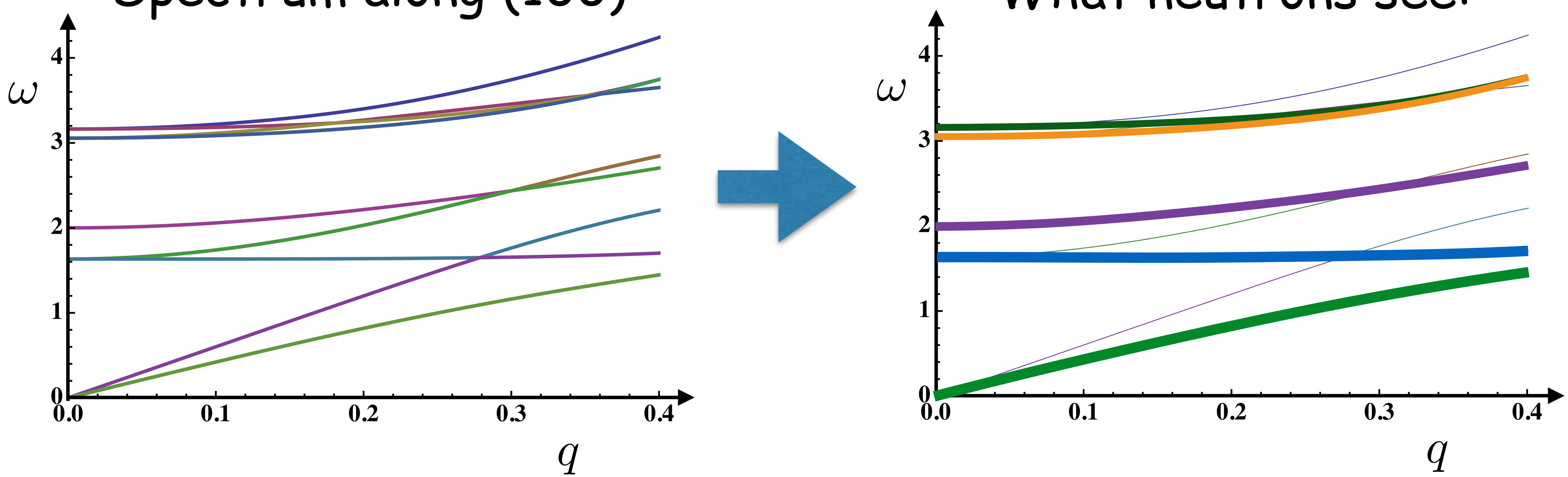
Spectrum

12 modes = 3 acoustic + 9 optical

T(100)

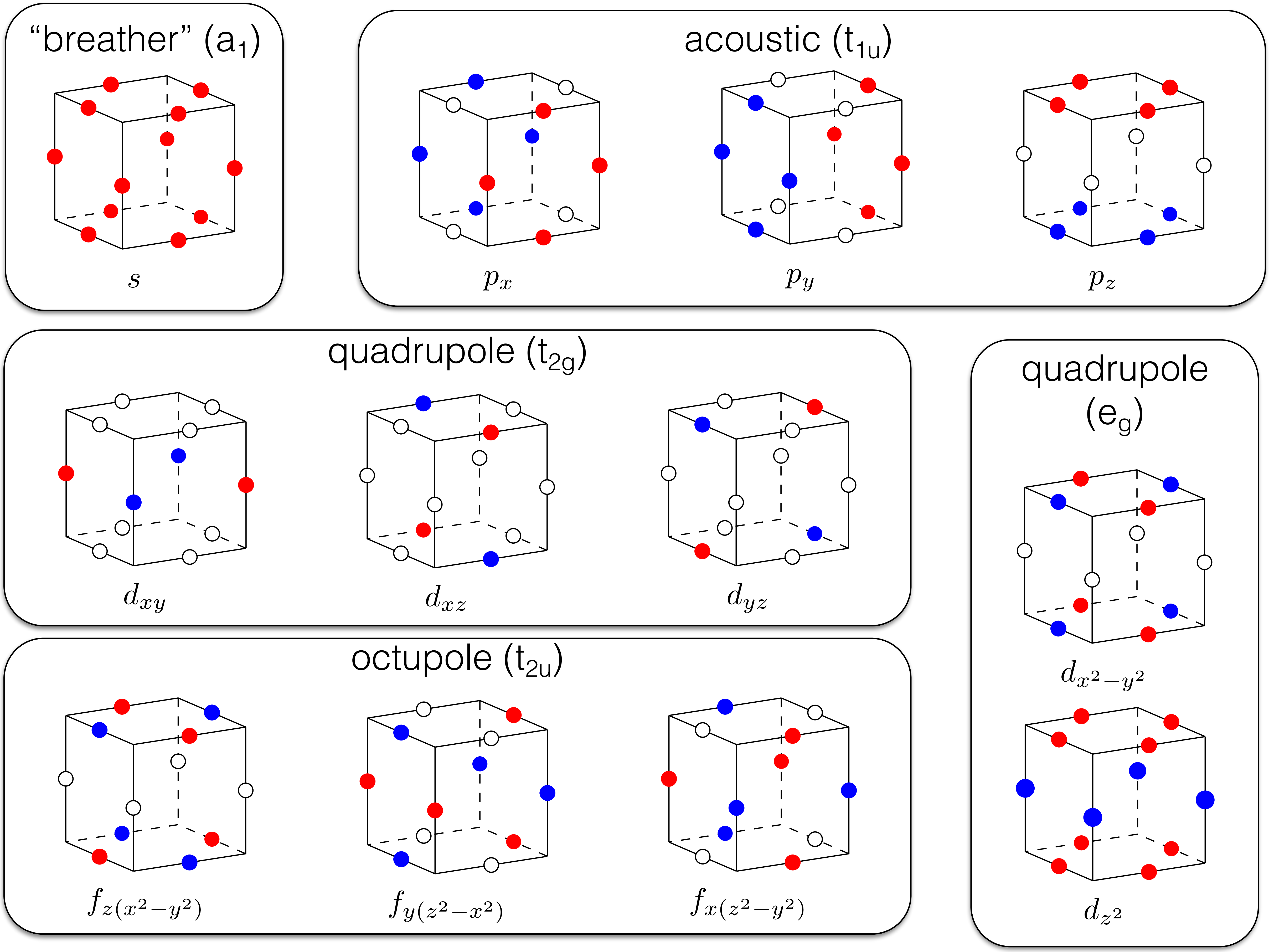
Spectrum along (100)

What neutrons see:

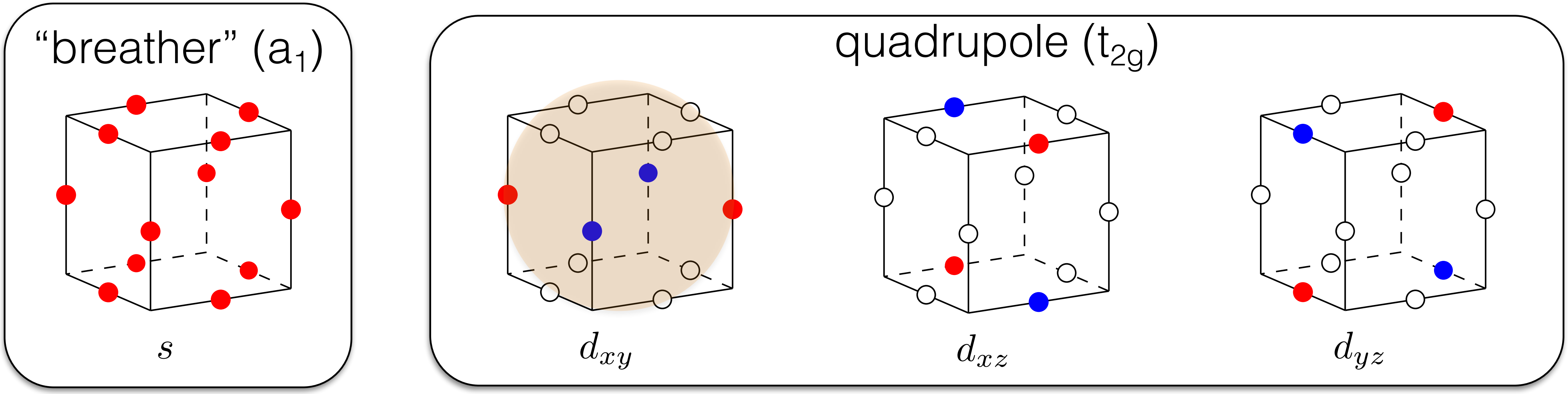


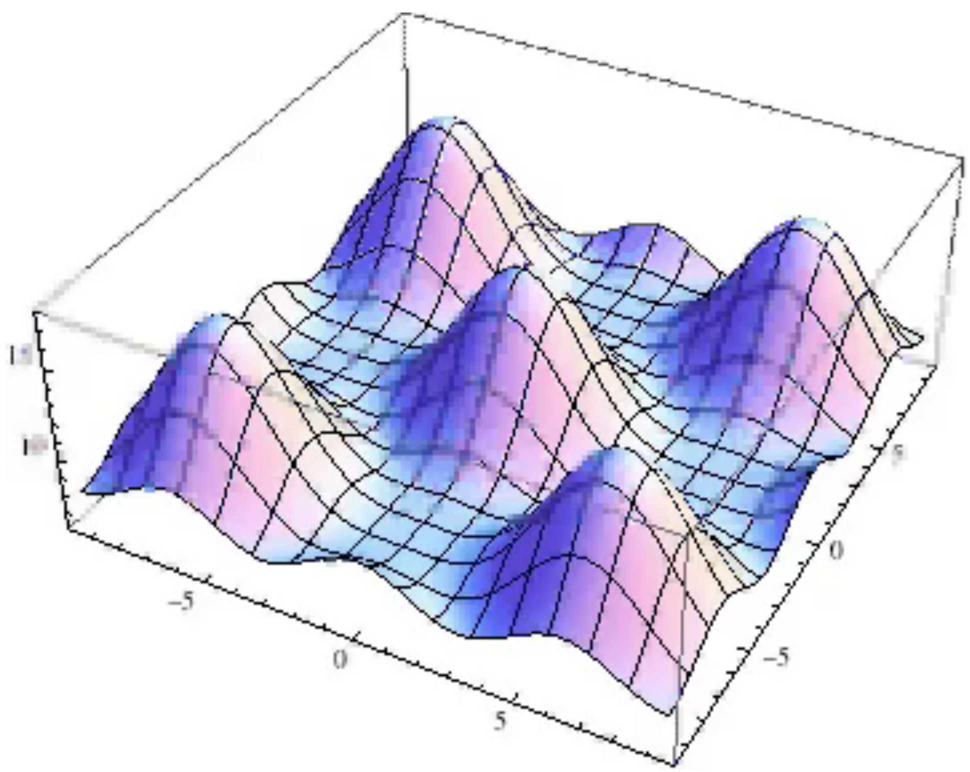
Structure factor: $S(\mathbf{q}, \omega) = \langle \rho(\mathbf{q}, \omega) \rho(-\mathbf{q}, -\omega) \rangle$

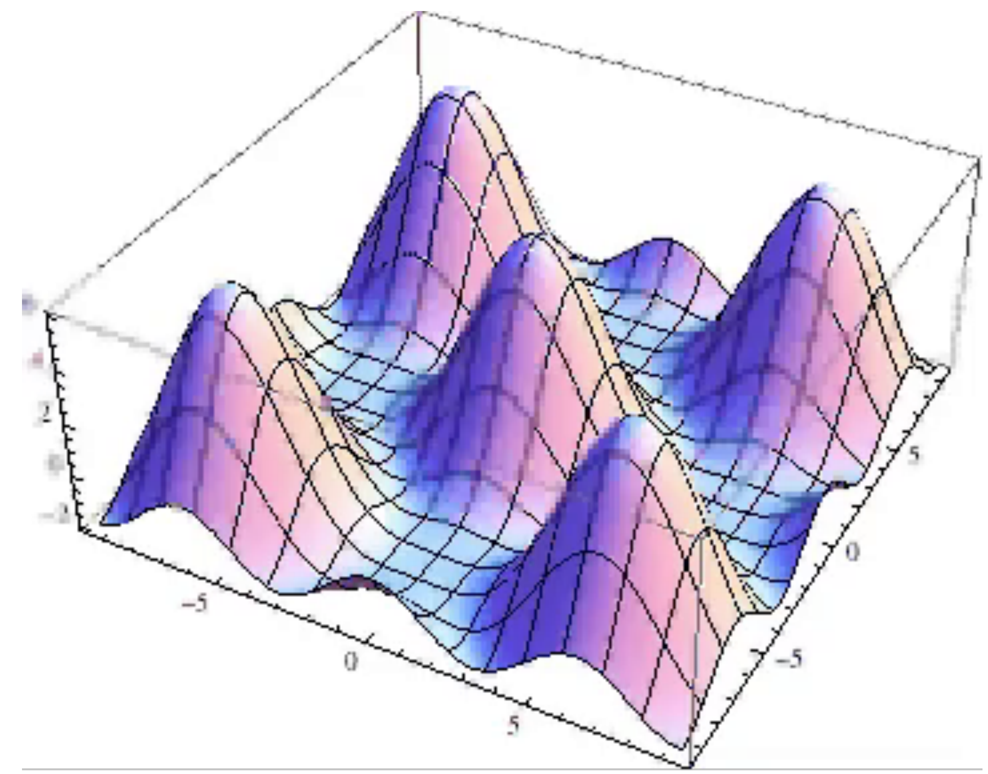
Symmetry of the excitations



Visualizing the optical modes



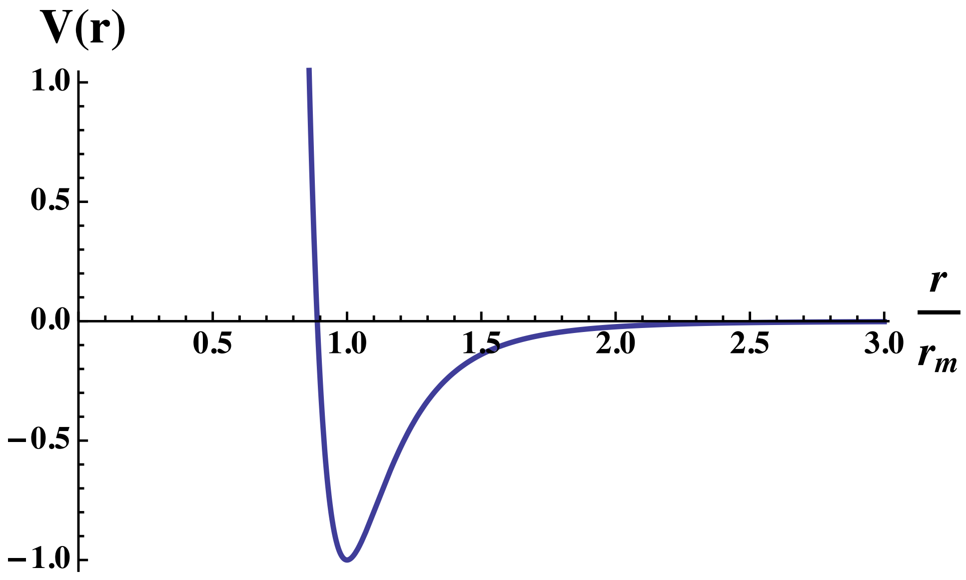




d_{xy} “quadrupolon” has vanishing z-axis spring constant ⇨ flat band

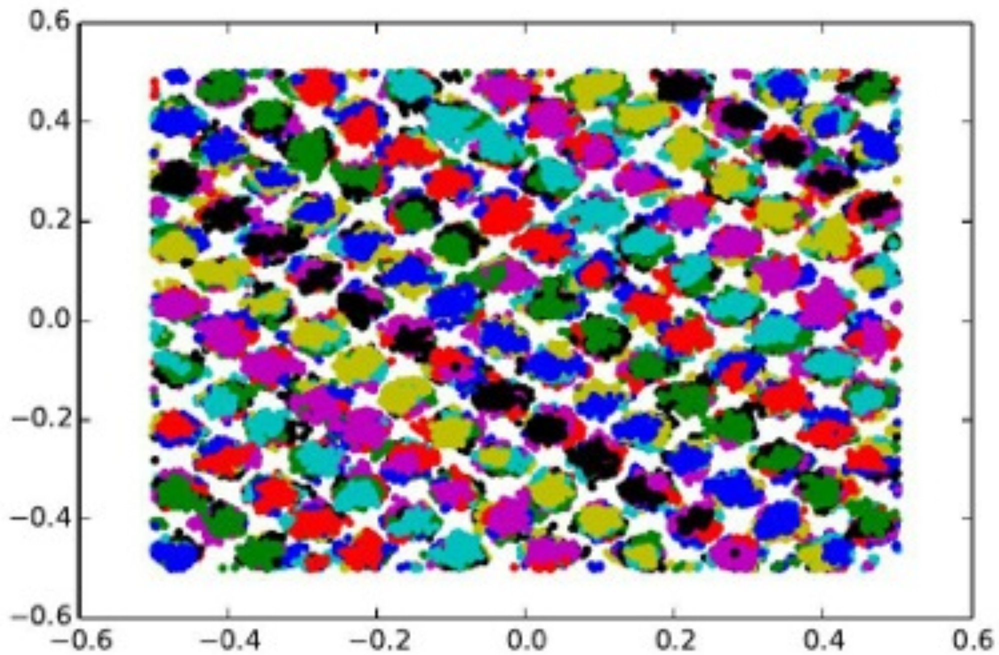
Quantum Monte Carlo

AB-inito simulations (Aziz potential)



2000 He4 Atoms

Continuous space path integral QMC



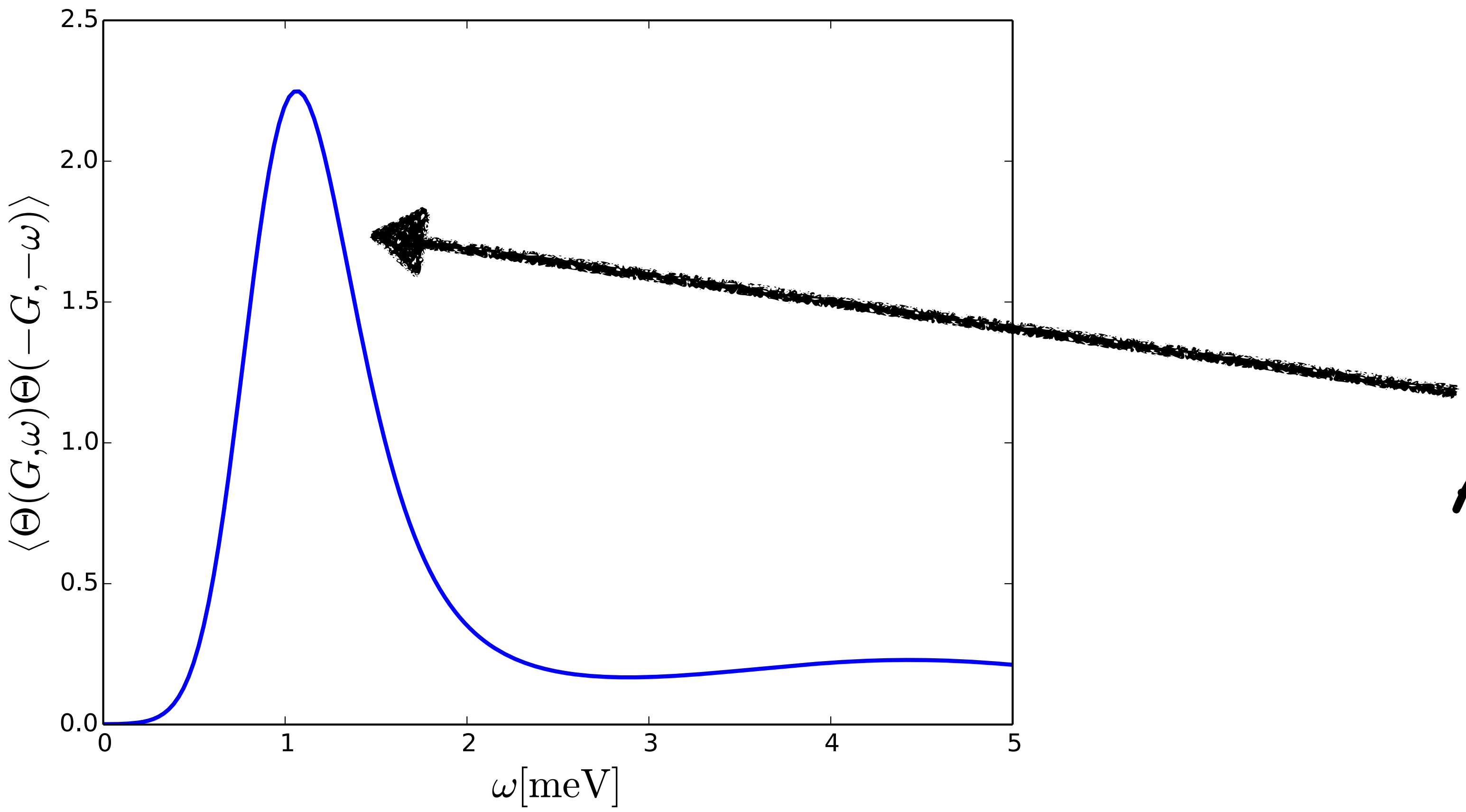
QMC results

Structure factor:

$$S(\mathbf{q},\omega) = \langle \rho(\mathbf{q},\omega)\rho(-\mathbf{q},-\omega)\rangle \qquad \rho(\mathbf{q},t) = \sum_n e^{i\mathbf{q}\cdot\mathbf{r}_n(t)}$$

"Scalar susceptibility":

$$S_\mathrm{s}(\mathbf{q},\omega) = \langle \Theta(\mathbf{q},\omega)\Theta(-\mathbf{q},-\omega)\rangle \qquad \Theta(\mathbf{q},t) = \left|\sum_n e^{i\mathbf{q}\cdot\mathbf{r}_n(t)}\right|^2$$

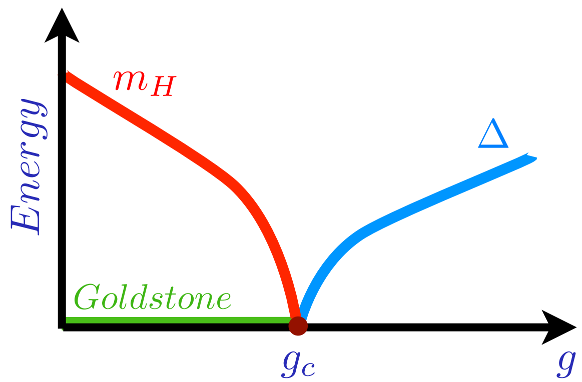


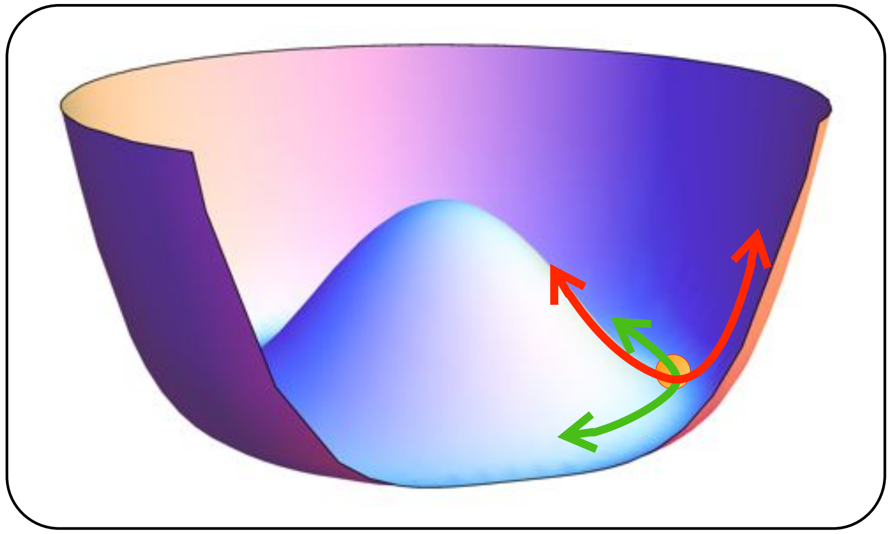
A clear peak at

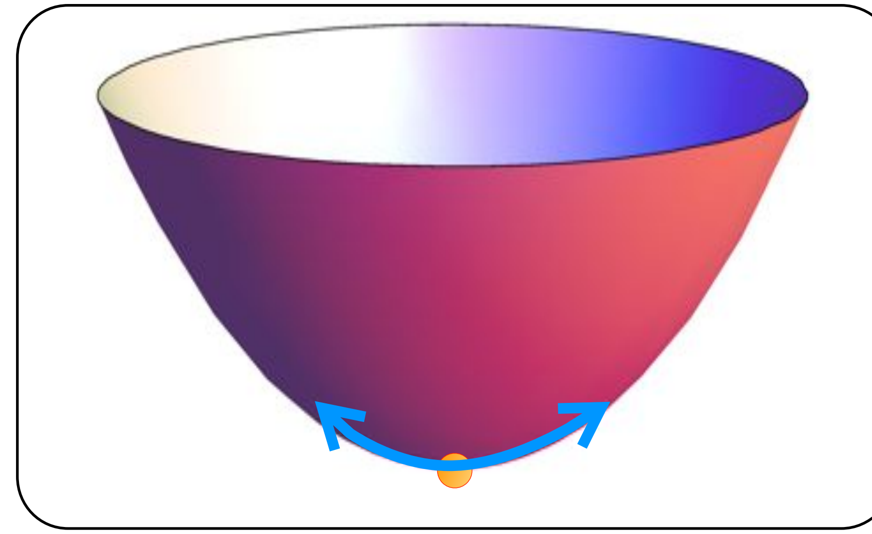
$$\omega_H \approx 1meV$$

Summary

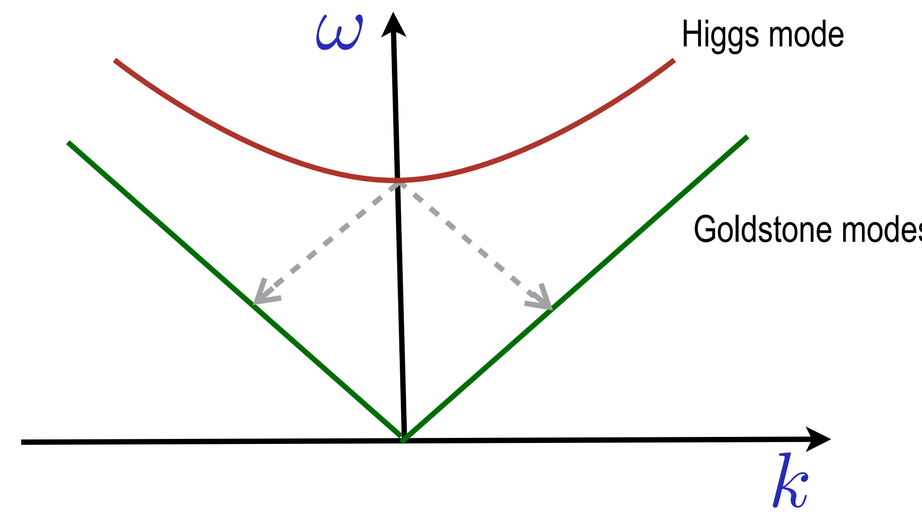
O(N) model

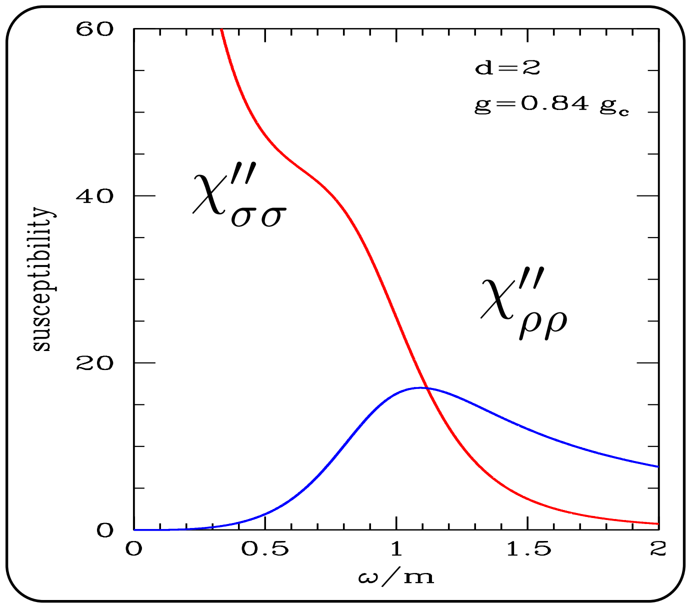






Higgs decays into Goldstone bosons





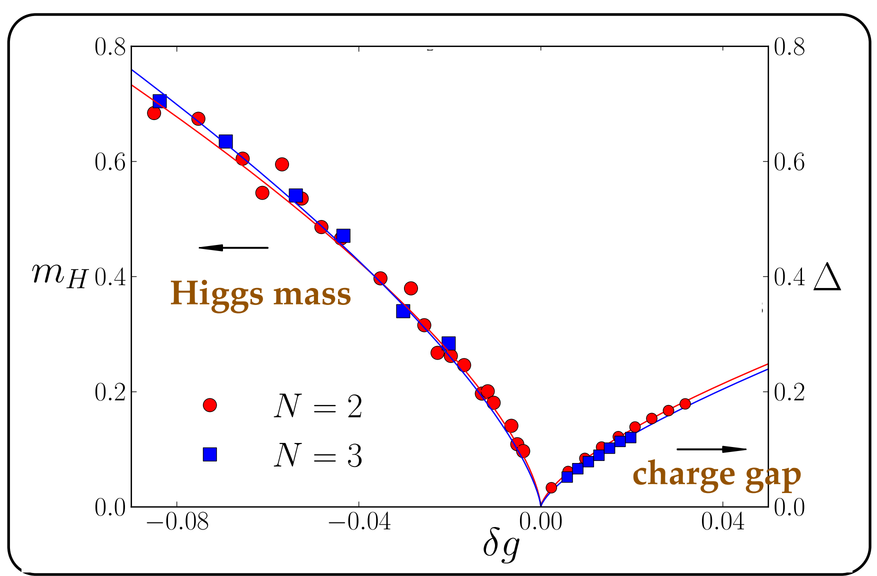
scalar $\chi''_{\rho\rho}$ is sharper than longitudinal $\chi''_{\sigma\sigma}$

O($1/N$), scaling and numerical simulations

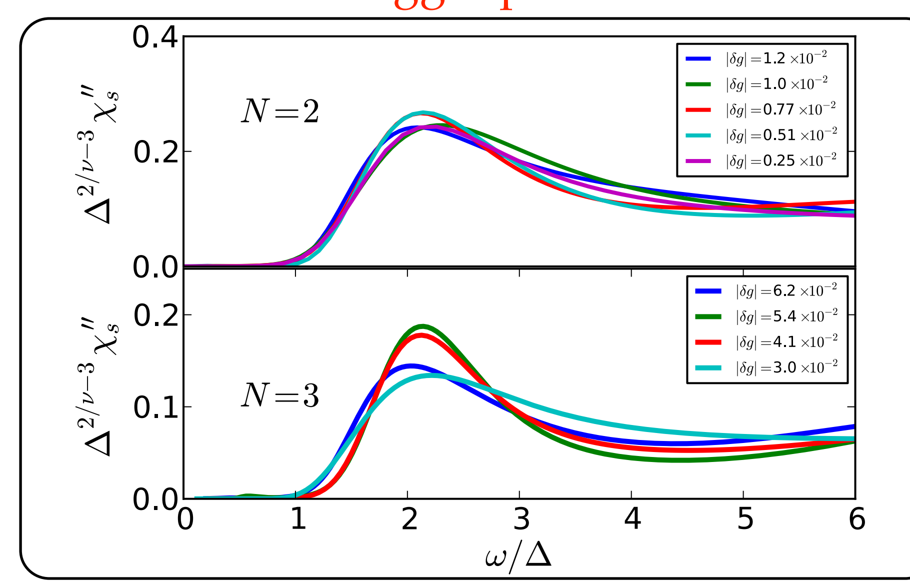
$$\chi_{\rho\rho}(\omega) = \Delta^{3-2/\nu}\Phi_\rho\left(\frac{\omega}{\Delta}\right)$$

$$\frac{m_H}{\Delta} = 2.1(3)$$

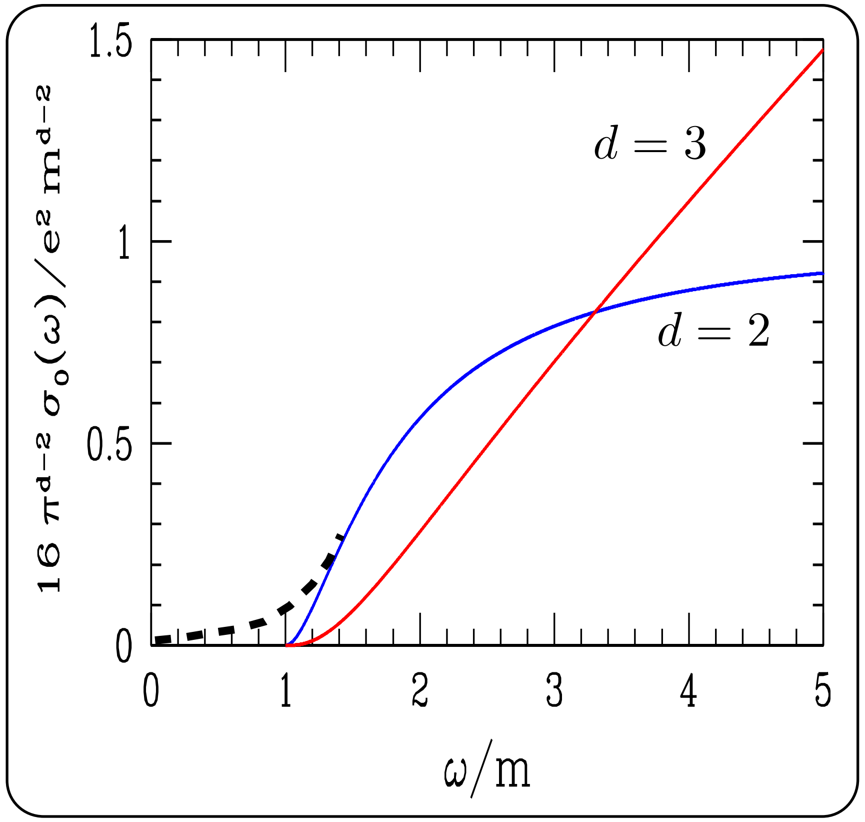
N=2 and N=3 critical behavior

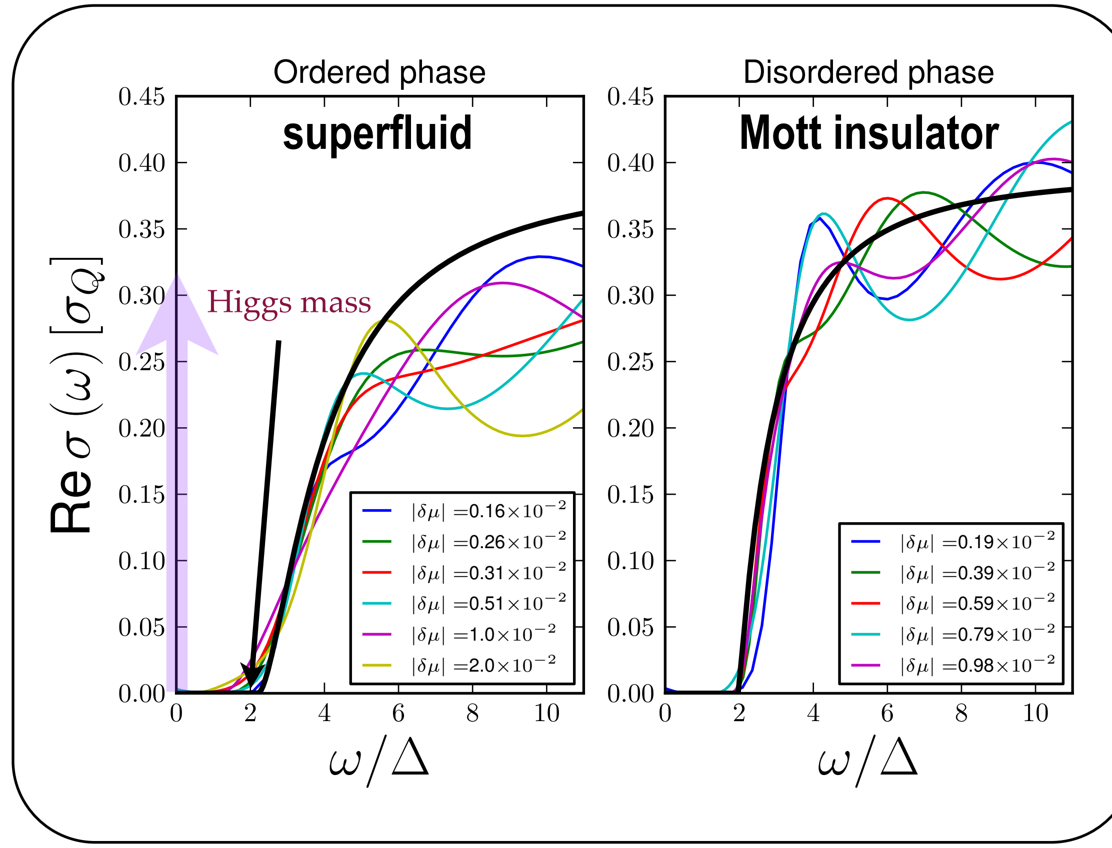


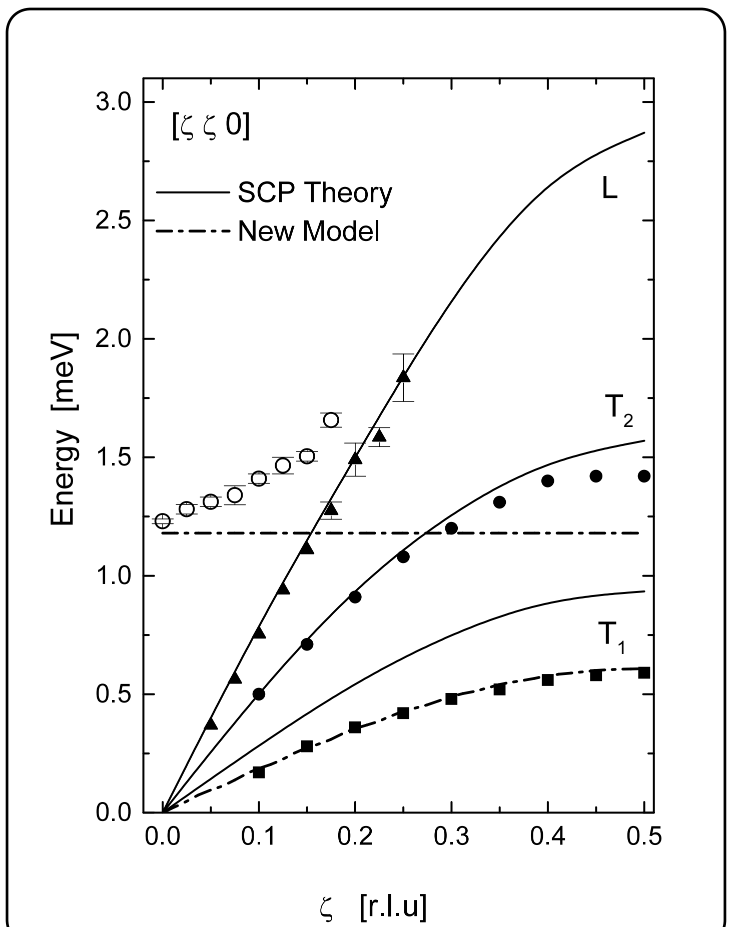
universal Higgs spectral function



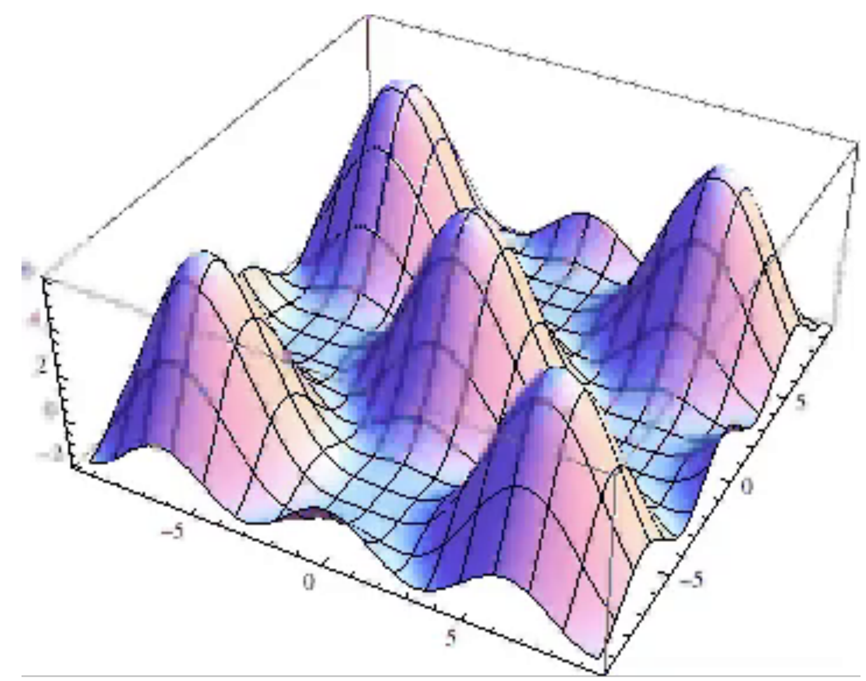
conductivity
pseudogap





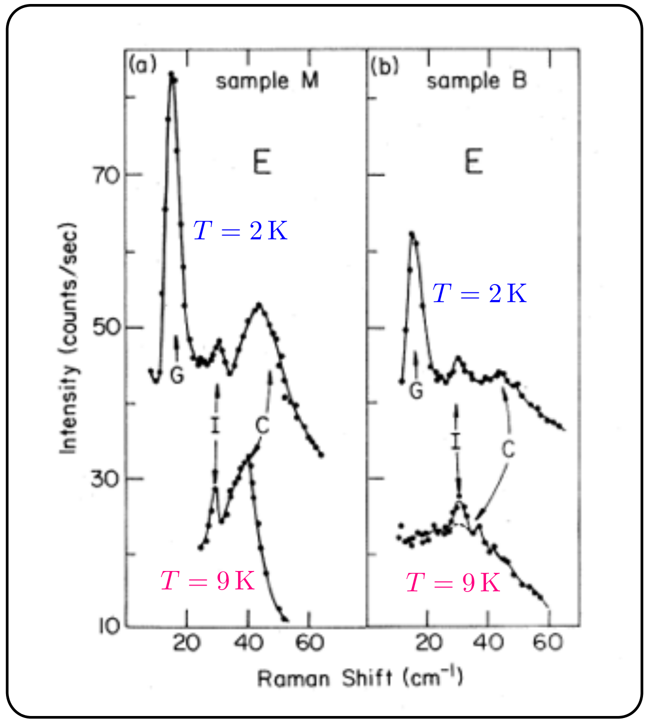


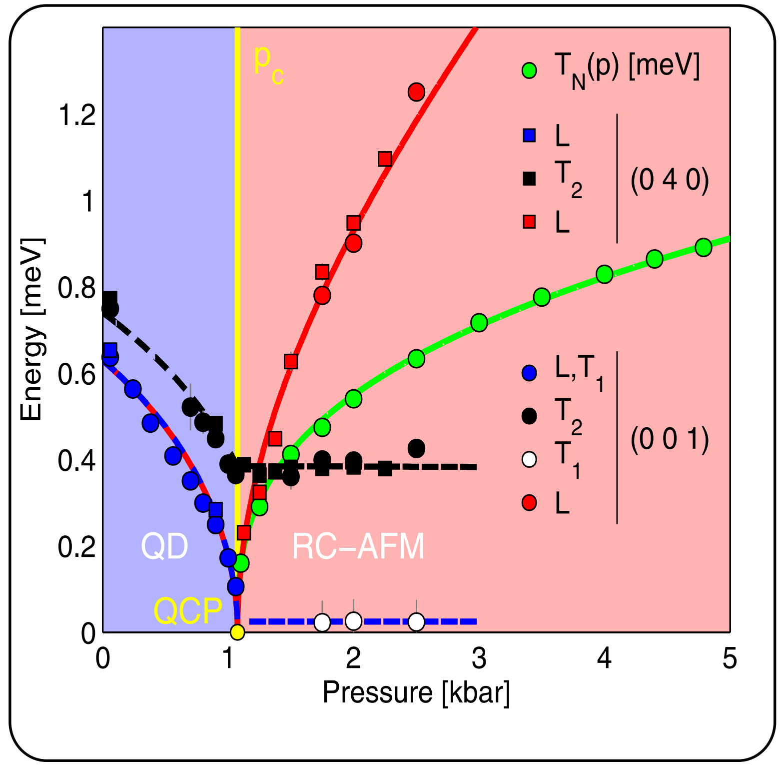
Amplitude mode
in bcc ^{4}He?

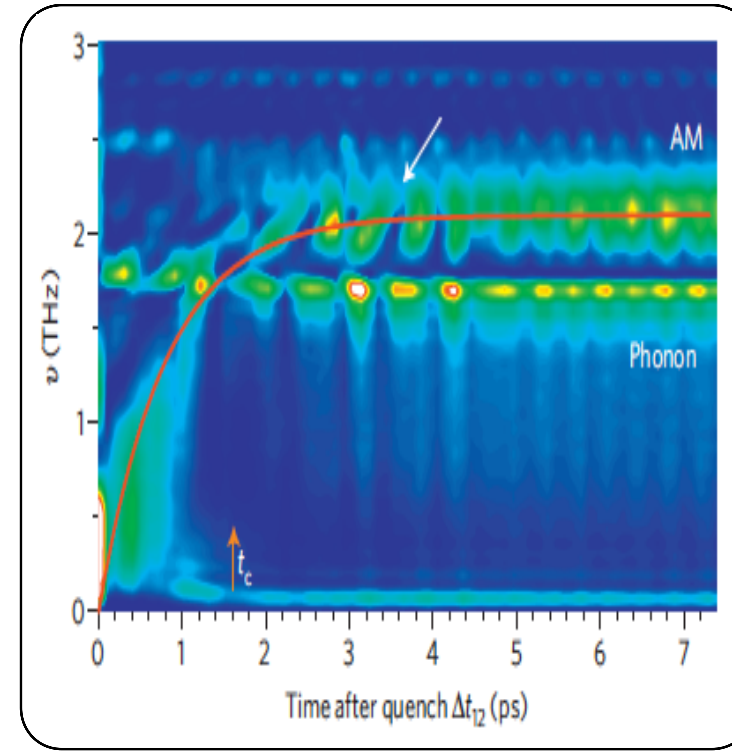


Experimental systems

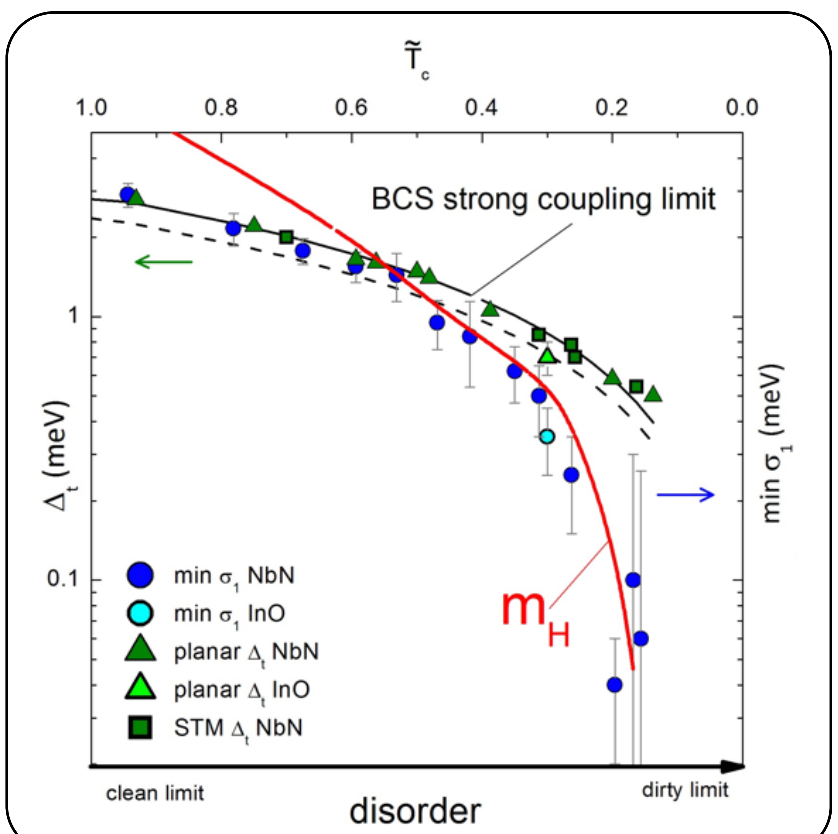
superconductors | antiferromagnets | cold atomic gases | CDWs



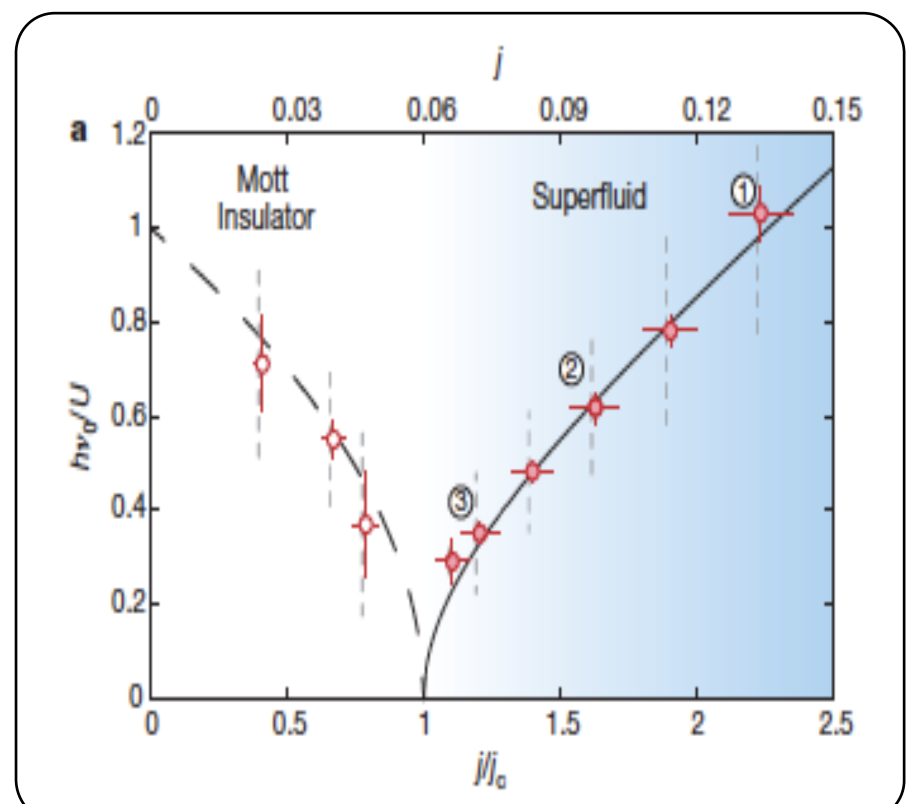




d=3



d=2



Higgs Hunters

Dan Arovas
circa 1981

Assa Auerbach

Snir Gazit

Heloise Nonne

Subir Sachdev

Yaniv Tenenbaum Katan

Thank you!