Higgs amplitude mode in the vicinity of a (2+1)-dimensional quantum critical point

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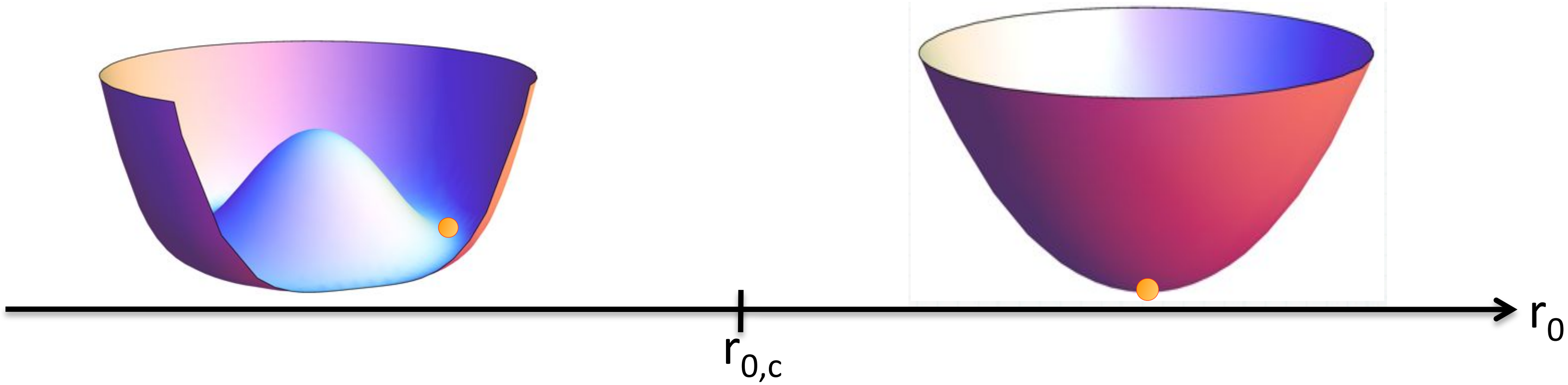
Outline

- Introduction
- Non-Perturbative Renormalization Group
- Fate of the amplitude mode close to the QCP
- Conclusion

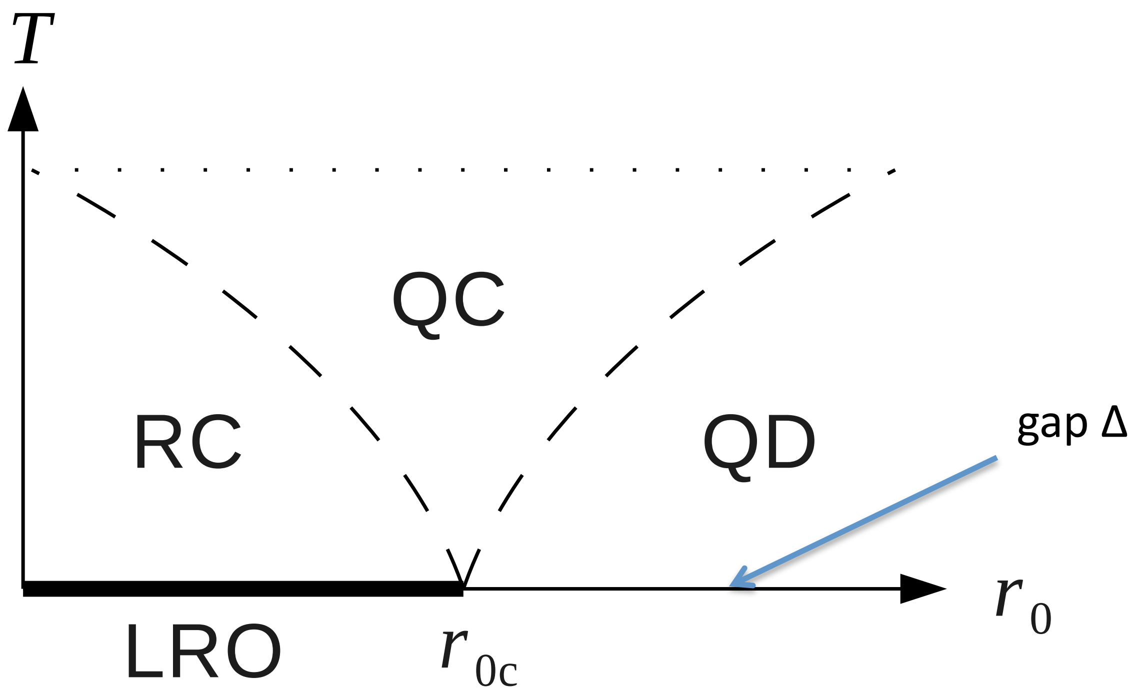
Quantum O(N) model

$$S = \int_0^\beta d\tau \int d^d x \left\{ \frac{1}{2}\psi_i(-c^{-2}\partial_\tau^2 - \nabla^2 + r_0)\psi_i + \frac{u_0}{4!N}(\psi_i^2)^2 \right\}$$



- Generalization of classical O(N) model
- at T=0, Lorentz symmetry
- Describes critical regime of a number of systems :
  - bosons in optical lattices
  - antiferromagnets
  - Josephson junction arrays
  - granular superconductors, …

Typical phase diagram in 2D

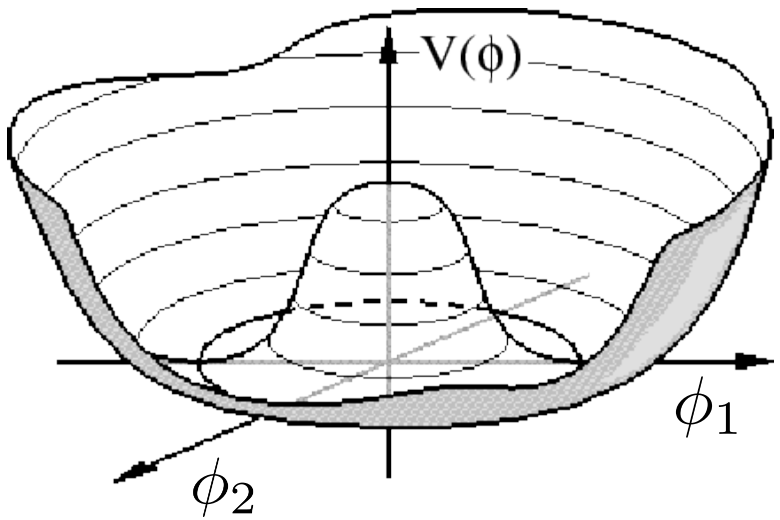


Quantum critical point described by the classical O(N) model in 2+1 D

Non trivial critical exponent : $\eta,\ \nu$

One (quantum) exponent : dynamical exponent z = 1 (Lorentz symmetry)

Amplitude mode : Mean Field



Broken symmetry phase $\langle \phi_i \rangle = \delta_{i1}\phi_0$

Mean-field picture : - N-1 transverse (massless) Goldstone modes $G_\pi(p) = \frac{1}{p^2}$

- 1 gapped longitudinal mode $G_\sigma(p) = \frac{1}{p^2 + 2\Delta^2}$

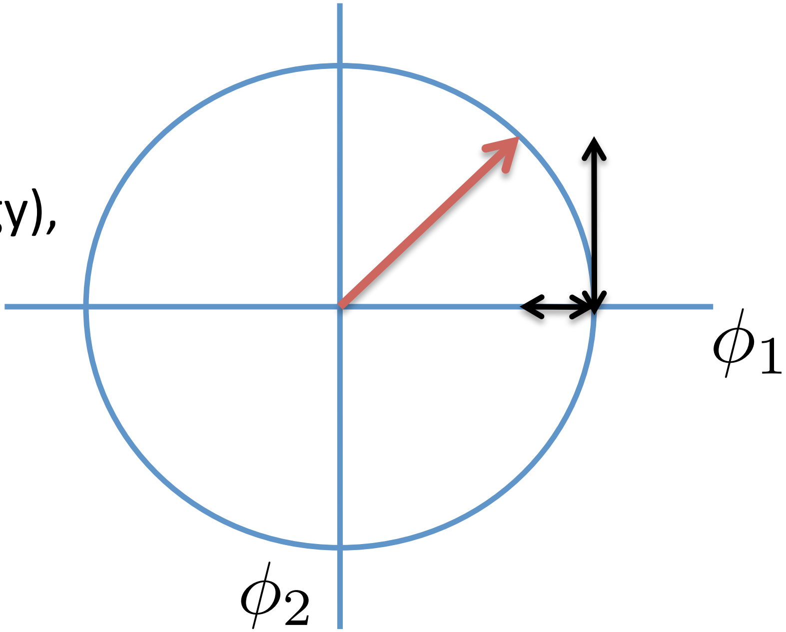
$$p^2 = \omega^2 + c^2\mathbf{q}^2$$

MF spectral function of one gapped mode arbitrarily close to the QCP ($\Delta \to 0$)

$$\chi_\sigma(\omega) \propto \delta(\omega - \sqrt{2}|\Delta|)$$

Amplitude mode : fluctuations

At low energy (smaller than Ginzburg energy), amplitude $\langle|\phi|\rangle$ constant.



Strong coupling between longitudinal and transverse fluctuations :

$$G_\sigma(p) \propto \frac{1}{|p|}$$ at low energy

Amplitude fluctuations seem not to be a well defined mode close to criticality: the 1/p divergence might the resonance as Δ goes to zero.

Zwerger 2004, Dupuis 2011

Amplitude mode : scalar fluctuation

Podolsky et al. 2011 : it might depend on the correlation function !

$$\phi = \sqrt{\rho}\mathbf{n} \qquad\qquad G_\rho = \langle \rho\rho \rangle$$

magnitude

Not the same correlation function !
Example for antiferromagnets: - longitudinal = neutron scattering
- scalar (amplitude) = Raman scattering

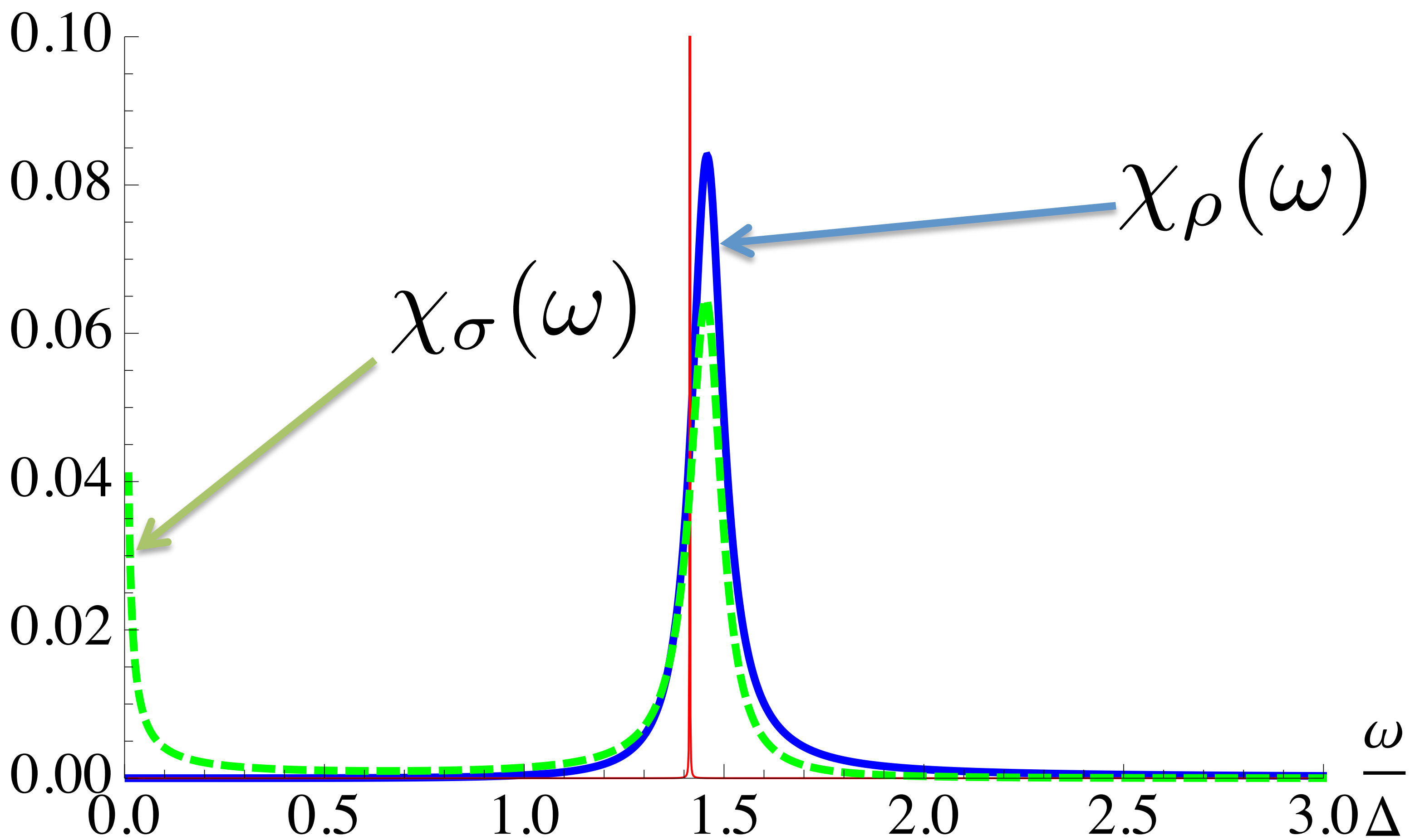
At low energy, coupling for decay of an amplitude fluctuation into two Goldstone is small (derivative coupling – proportional to the energy square).

We expect the spectral function to be of order ω^3 at small ω (no divergence), so the resonance might still be visible close to the QCP.

But is the amplitude mode still well defined close to the quantum critical point ?

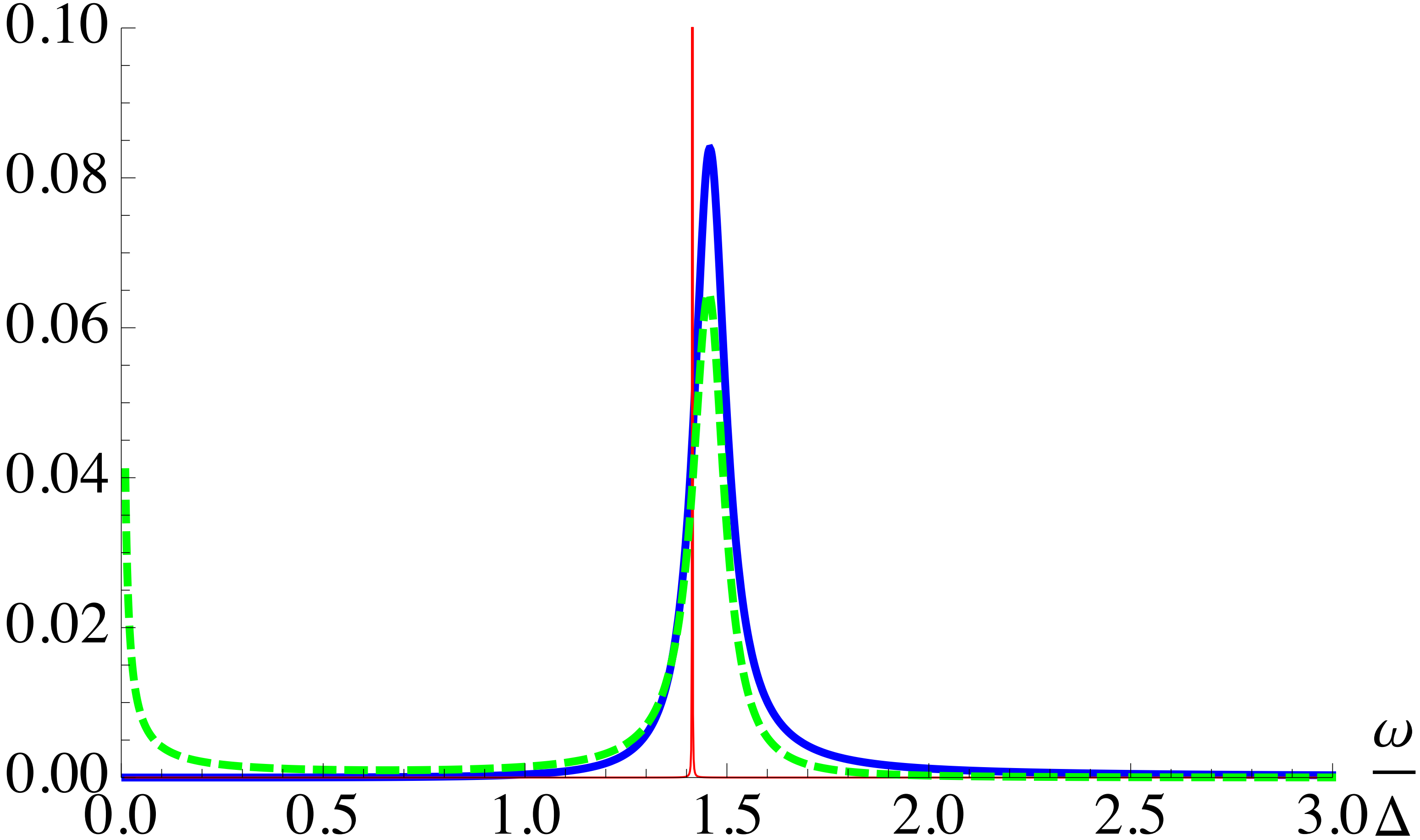
Amplitude mode : Large N

Spectral functions : far from criticality



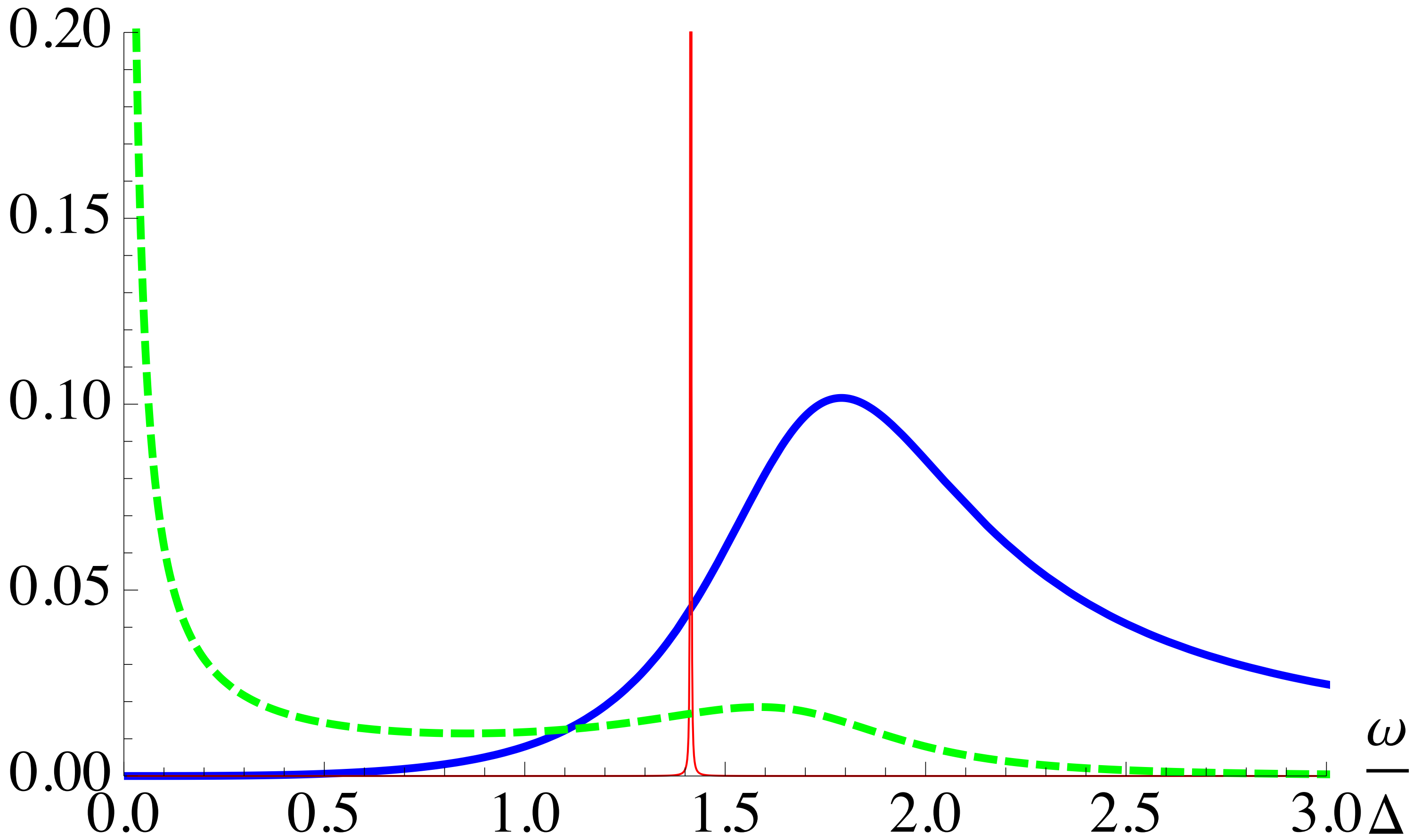
Amplitude mode : Large N

Spectral functions : far from criticality



Amplitude mode : Large N

Spectral functions : closer to criticality



Amplitude mode : Large N

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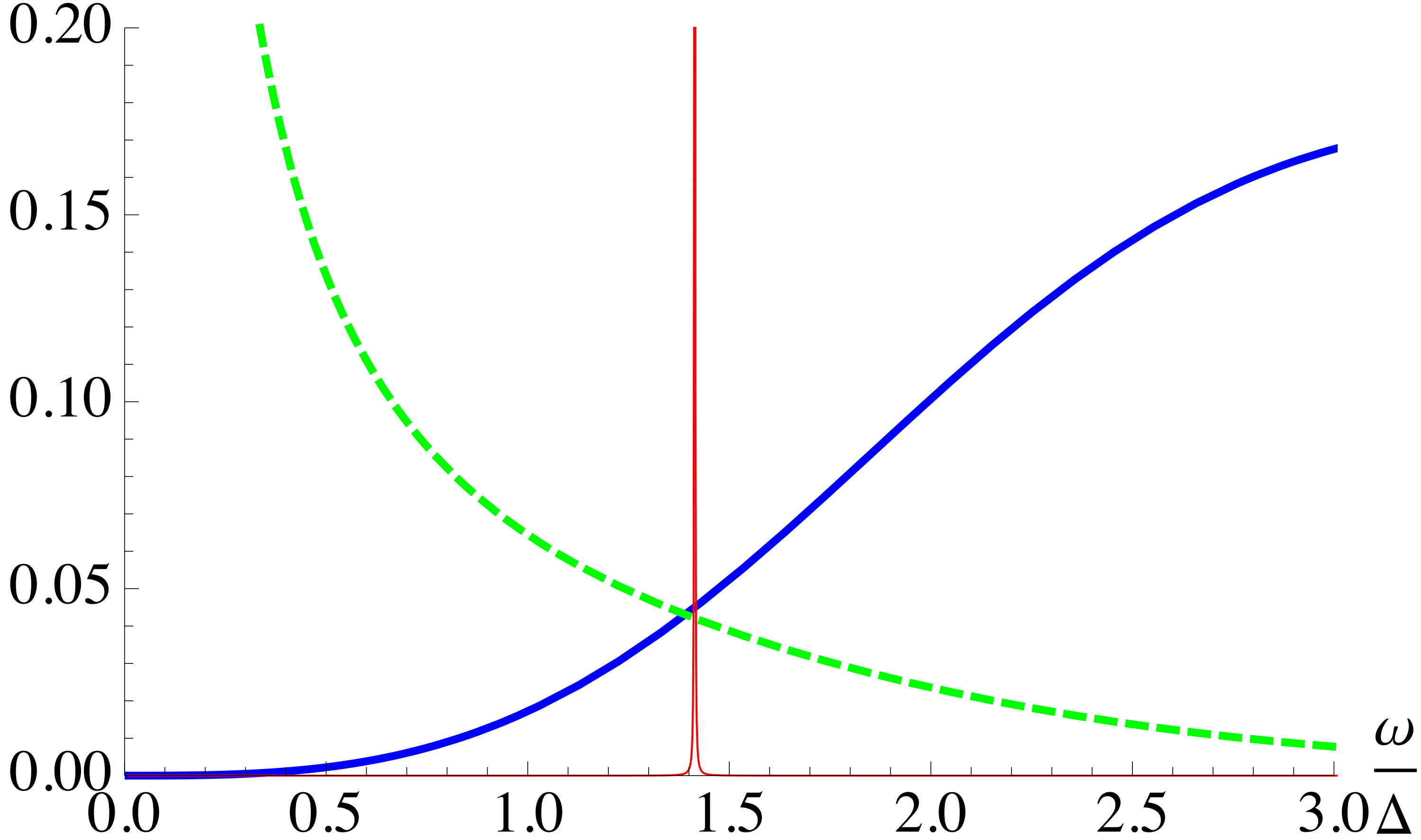
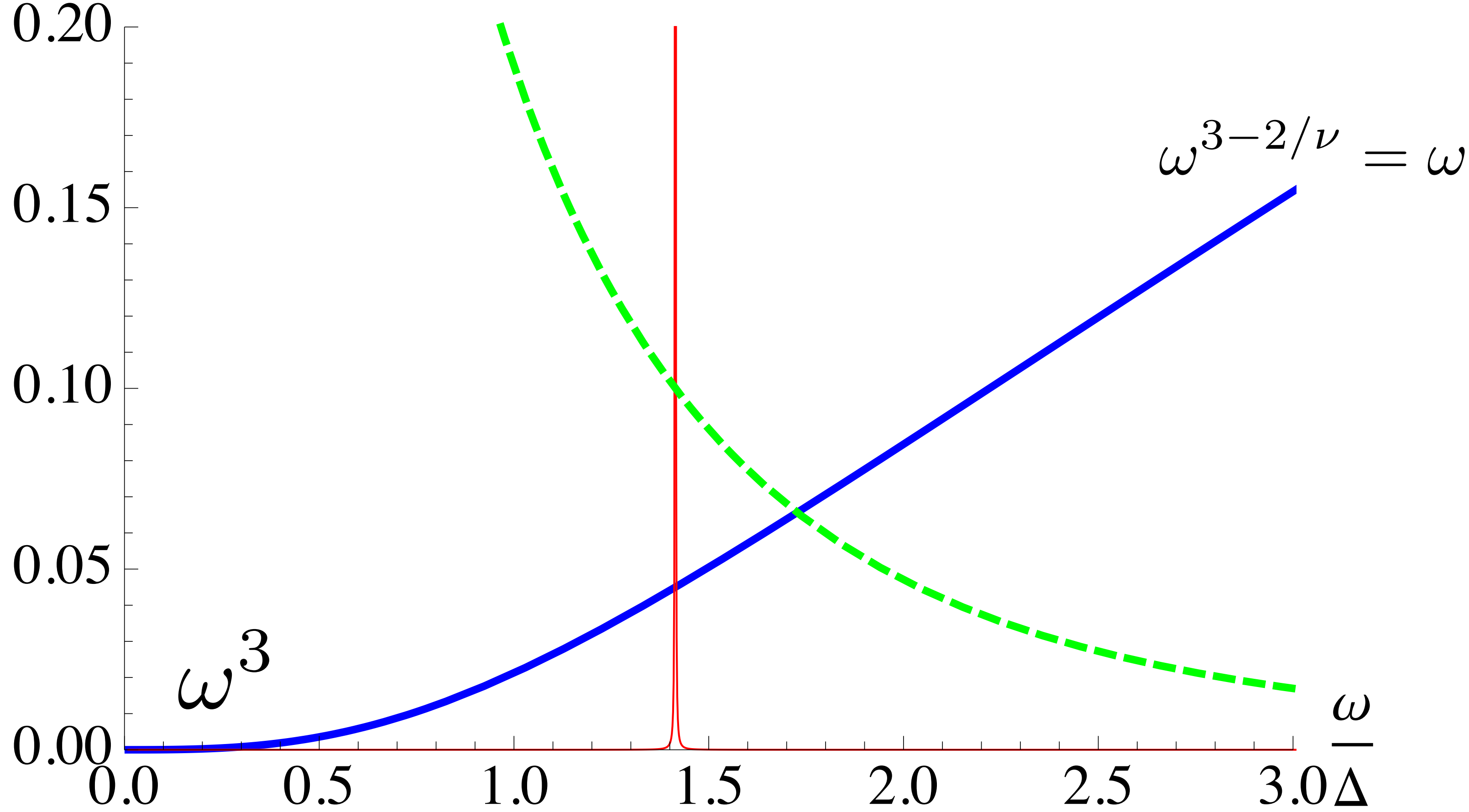


Amplitude mode : Large N

Spectral functions : critical regime – no amplitude mode

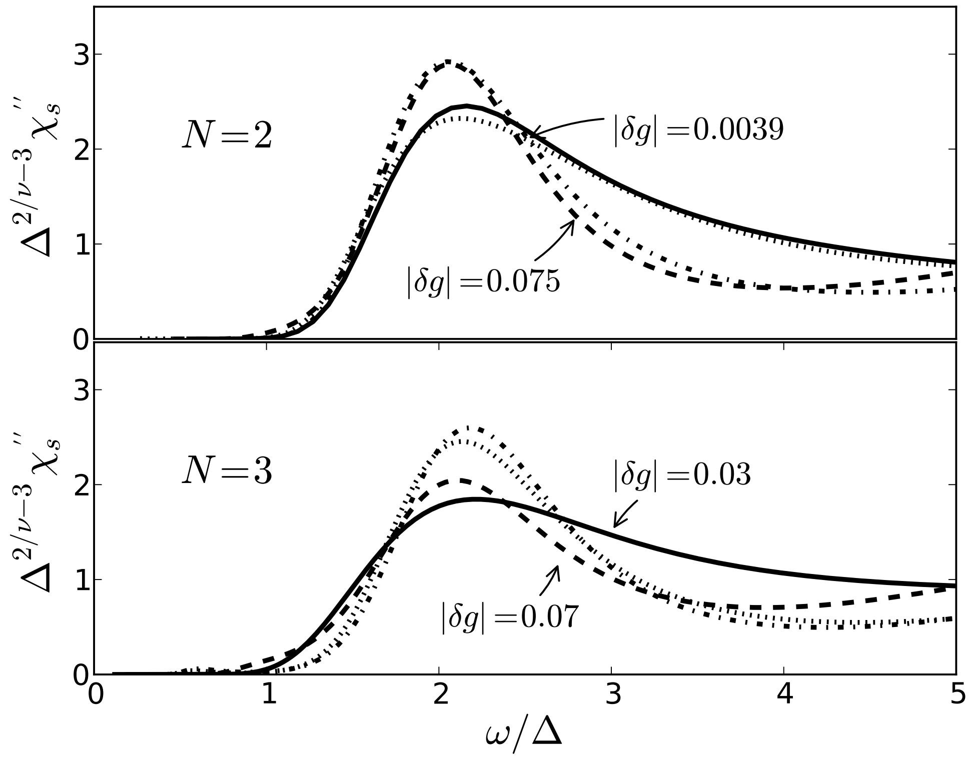


Amplitude mode : Small N Monte Carlo

Recent Monte Carlo simulations : Polet et al. (2012), Gazit et al. (2013), Chen et al. (2013) for N=2 and 3
Also measured in cold atoms experiments : ``Higgs" mode (I. Bloch group)

resonance between 2.1Δ and 3.3Δ

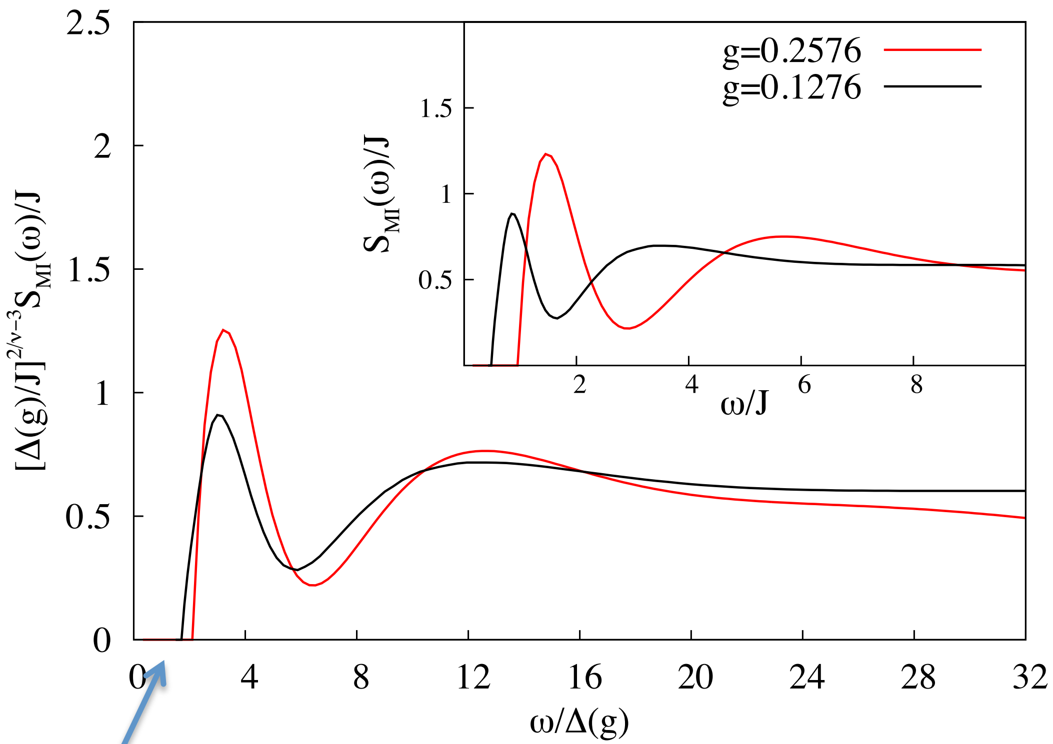
≈2.2Δ



Amplitude mode defined close to criticality,
but how does it disappear as N increases ?

Amplitude mode : disordered phase ?

Kun Chen et al. : amplitude mode in the symmetric (disordered) phase !?



Gapped system : threshold at ω=2Δ (ρ -> 2 massive bosons)

Gave an RG argument, but is it true ?

Non-Pertubative Renormalization Group

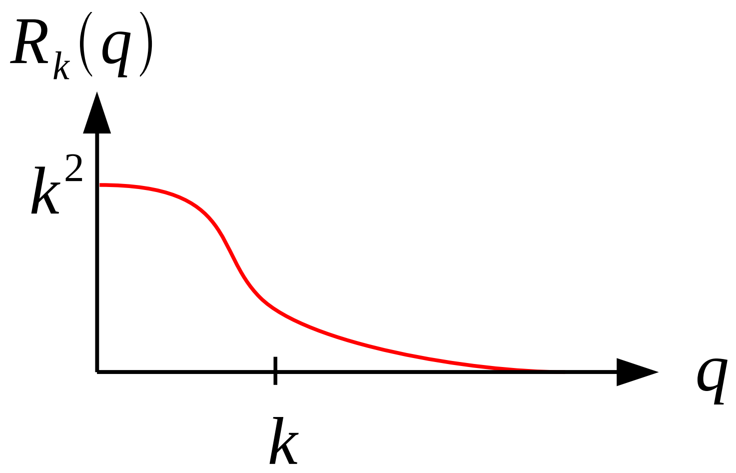
Family of actions indexed by momentum scale k

$S+\Delta S_\Lambda$ → $S+\Delta S_k$ → $S+\Delta S_{k=0}=S$

Reference system — System of interest

RG equation: $\partial_k \Gamma_k[\phi]$

$$\Delta S_k[\psi] = \sum_i \int_q R_k(q)\psi_i(q)\psi_i(-q)$$



Exact Flow equation (Wetterich `93) : $\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\left\{ \partial_k R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1} \right\}$

Average Effective Action
("Gibbs" free energy)

Bare action :

$$S=\int_0^\beta d\tau\int d^dx\left\{\frac{1}{2}\psi_i(-c^{-2}\partial_\tau^2-\nabla^2+r_0)\psi_i+\frac{u_0}{4!N}(\psi_i^2)^2\right\}$$

"Functional free energy" gives access to all the physics :

$$W_k[J,h]=\ln\int D\psi_i e^{-S-\Delta S_k+J_i\psi_i+h\rho}$$

modified Legendre transform $\quad \phi_i(\tau)=\langle\psi_i(\tau)\rangle$

$$\Gamma_k[\phi,h]$$

for h=0 in constant field $\rho=\sum_i\phi_i^2$

functional derivatives

$$V_k(\rho)$$

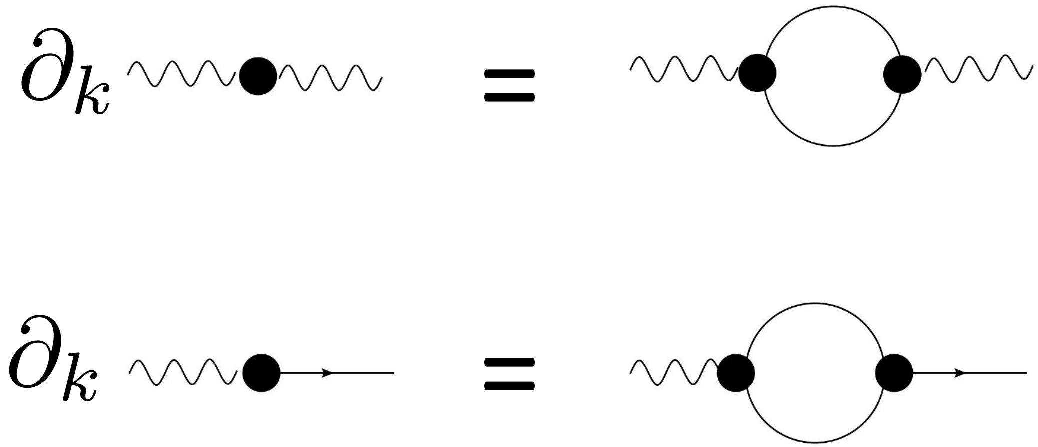
thermodynamics :
pressure, order parameter, *etc.*

$$G_k(i\omega,\mathbf{p})$$

propagator, scattering amplitudes, *etc.*

Approximations

Write down an Ansatz for $\Gamma_k[\phi, h]$ (expansion in ϕ and h)

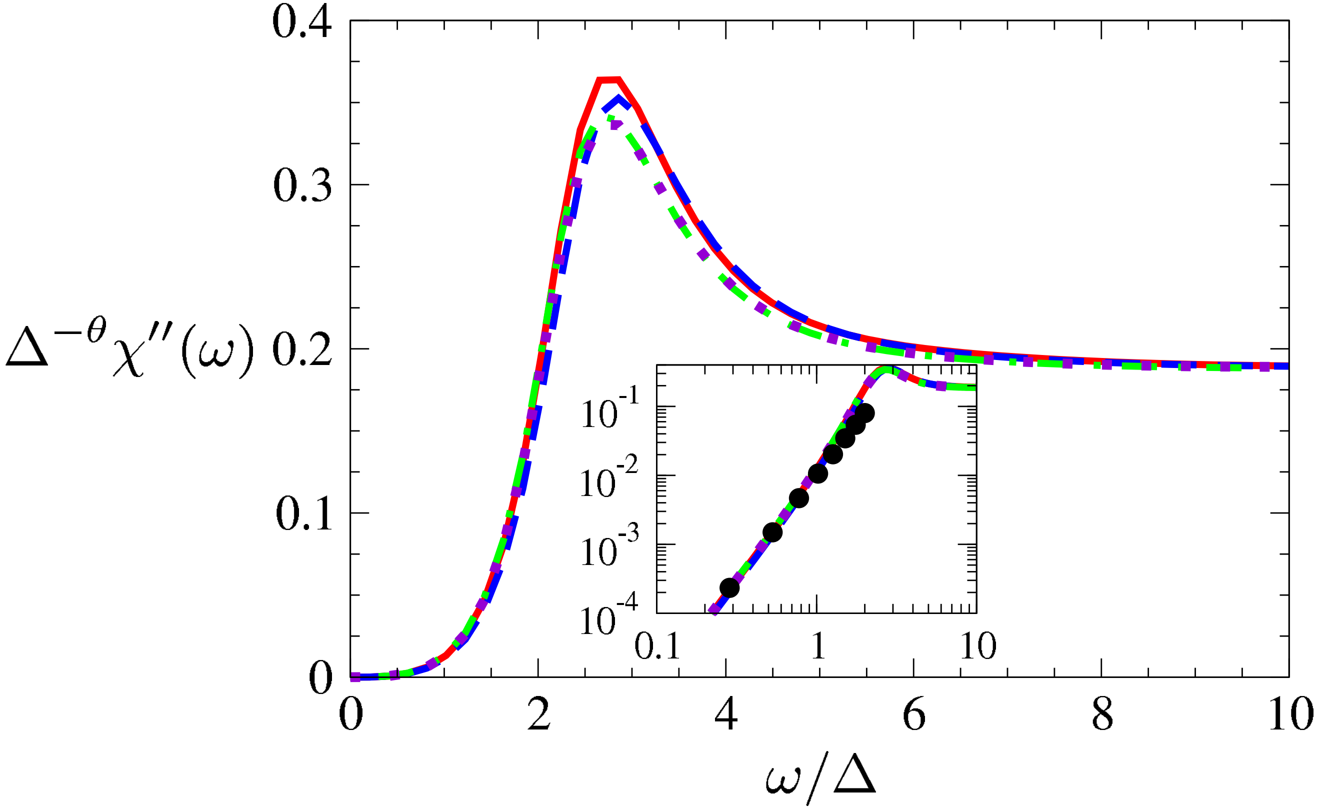


Propagator computed within a derivative expansion but keep the Matsubara frequency dependence. In the end, analytic continuation.

Technical details in AR and N. Dupuis, PRB 2014

Amplitude mode : N=2

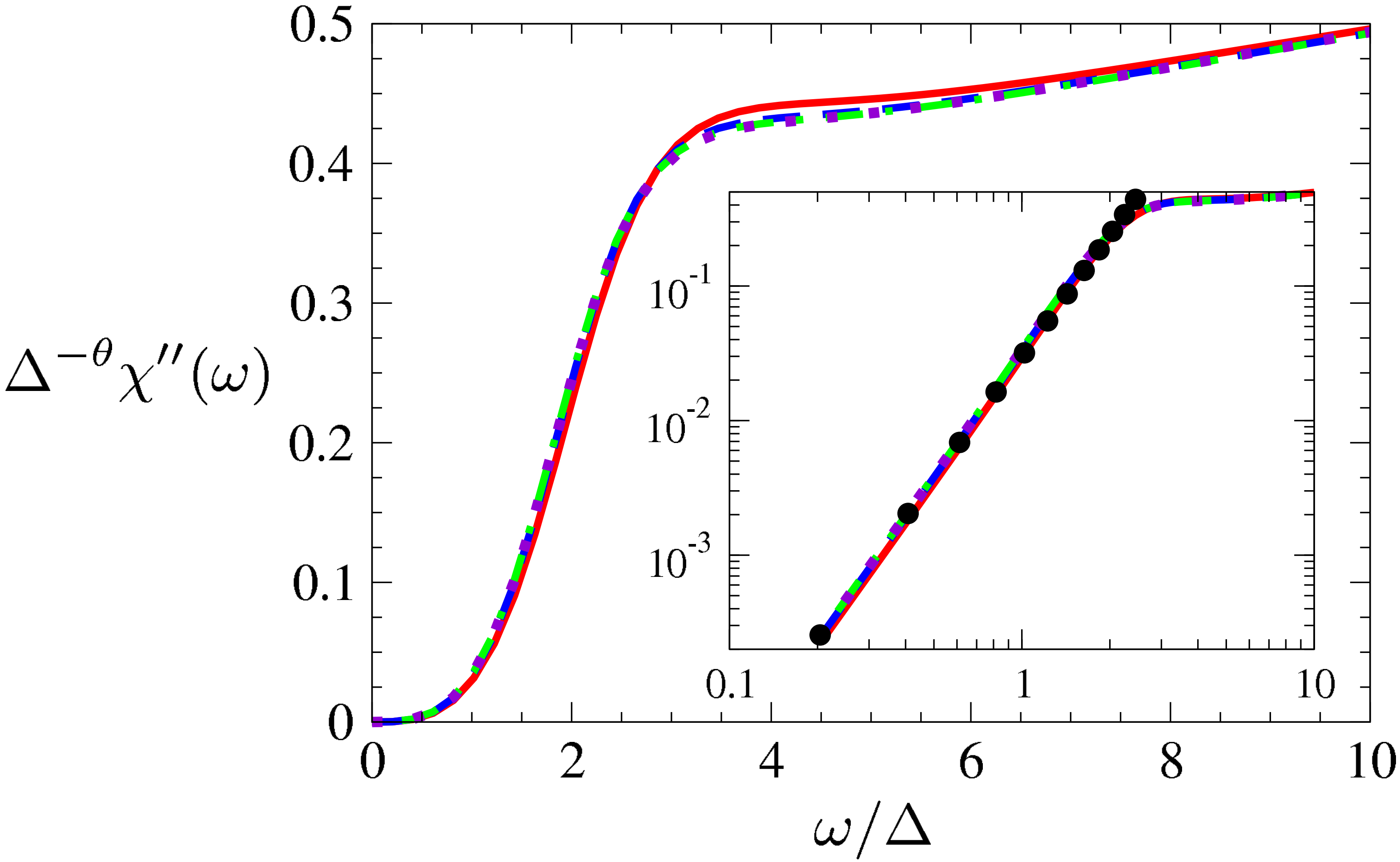
N=2, m≈2.4 Δ



$\theta \neq 3 - 2/\nu$ as it should.

AR & N. Dupuis, PRB 89 180501(R) (2014)

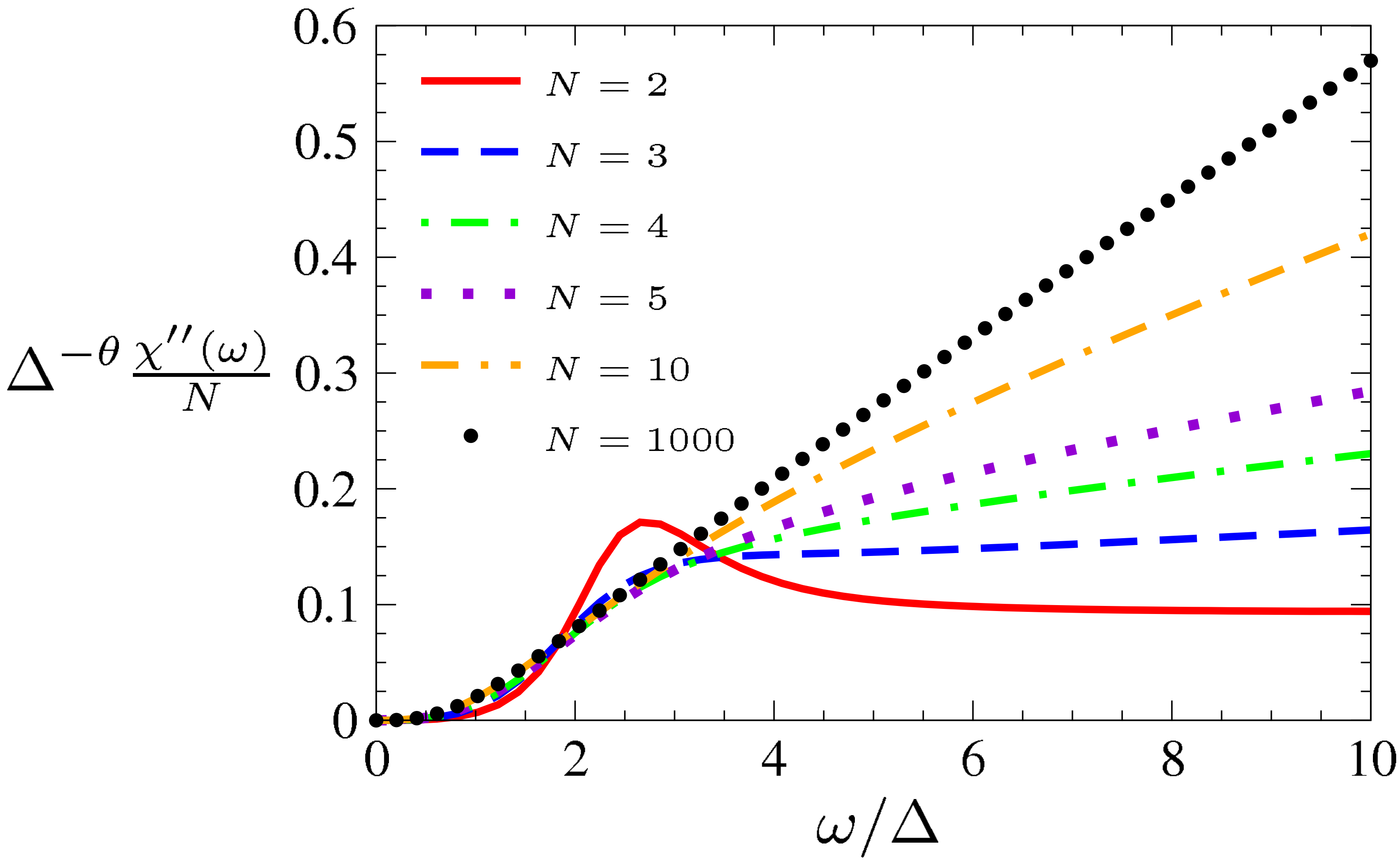
Amplitude mode : N=3



AR & N. Dupuis, PRB 89 180501(R) (2014)

Amplitude mode : all N

No resonance for N>2



AR & N. Dupuis, PRB 89 180501(R) (2014)

T=0 disordered phase - 1

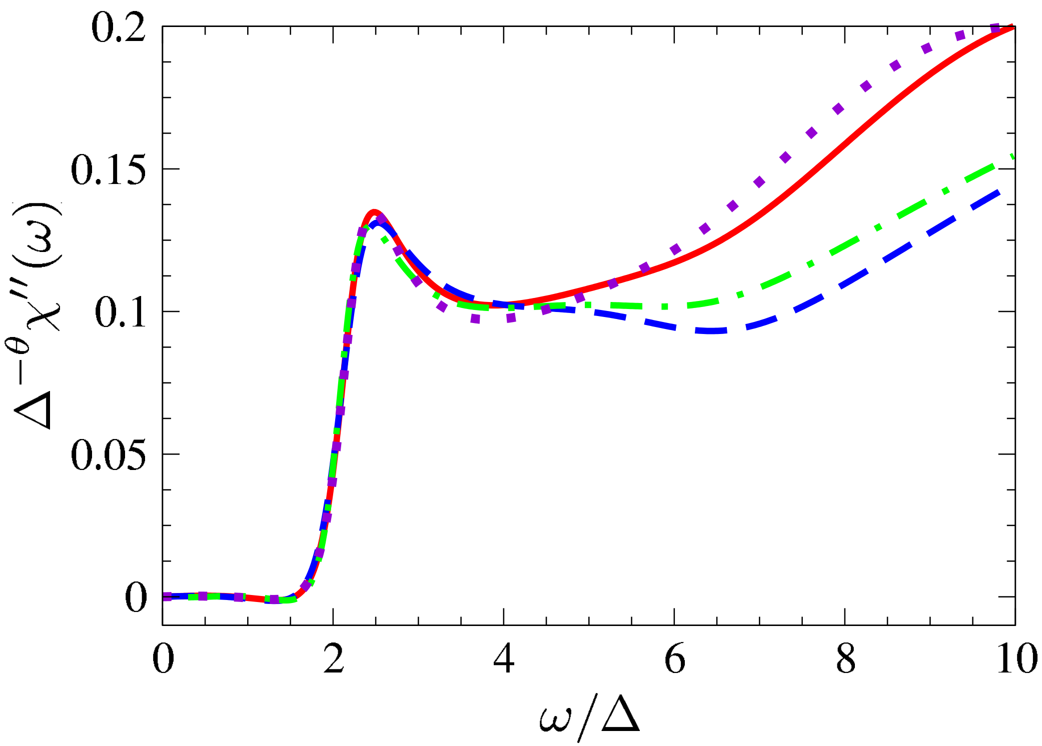
-"Resonance" close to gap at 2Δ : related to amplitude mode ?

-If so : resonance must appear in the critical regime (otherwise : not related)

-But in critical regime : scaling

$$\chi''_{\rm crit}(\omega) \propto \omega^{3-2/\nu}$$

-Implies no resonance in critical regime



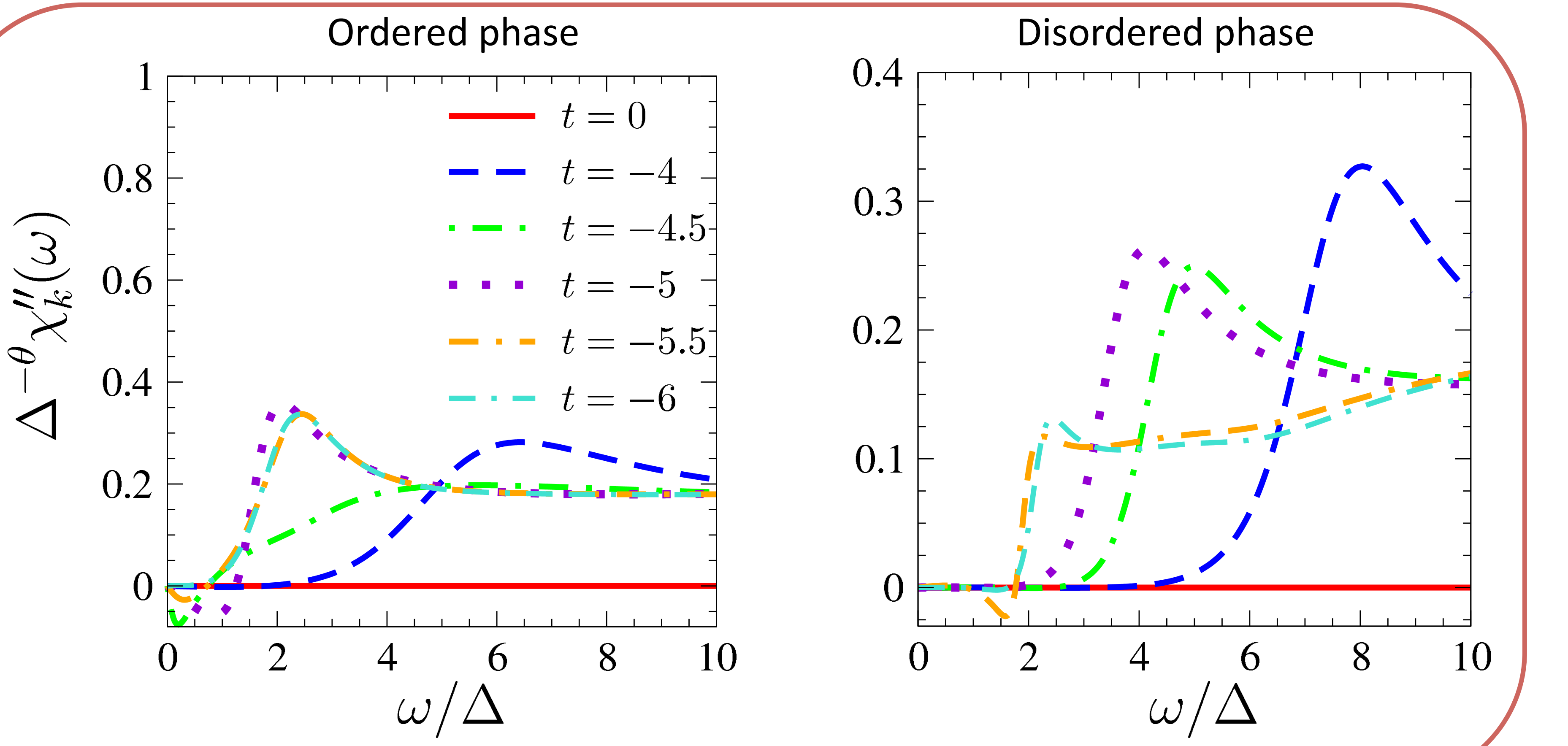
- Thus resonance in ordered/ disordered regime are not related – not the same physics

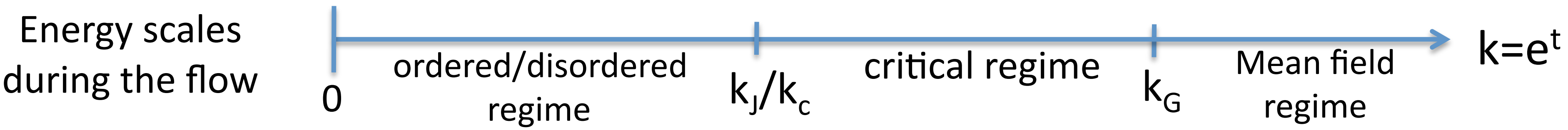
Energy scales

AR & N. Dupuis, PRB 89 180501(R) (2014)

Quantum disordered phase - 2

No resonance in the disordered phase : no amplitude mode precursor in the renormalization flow when the peak appears in the disordered phase.





Amplitude mode : Kosterlitz-Thouless phase

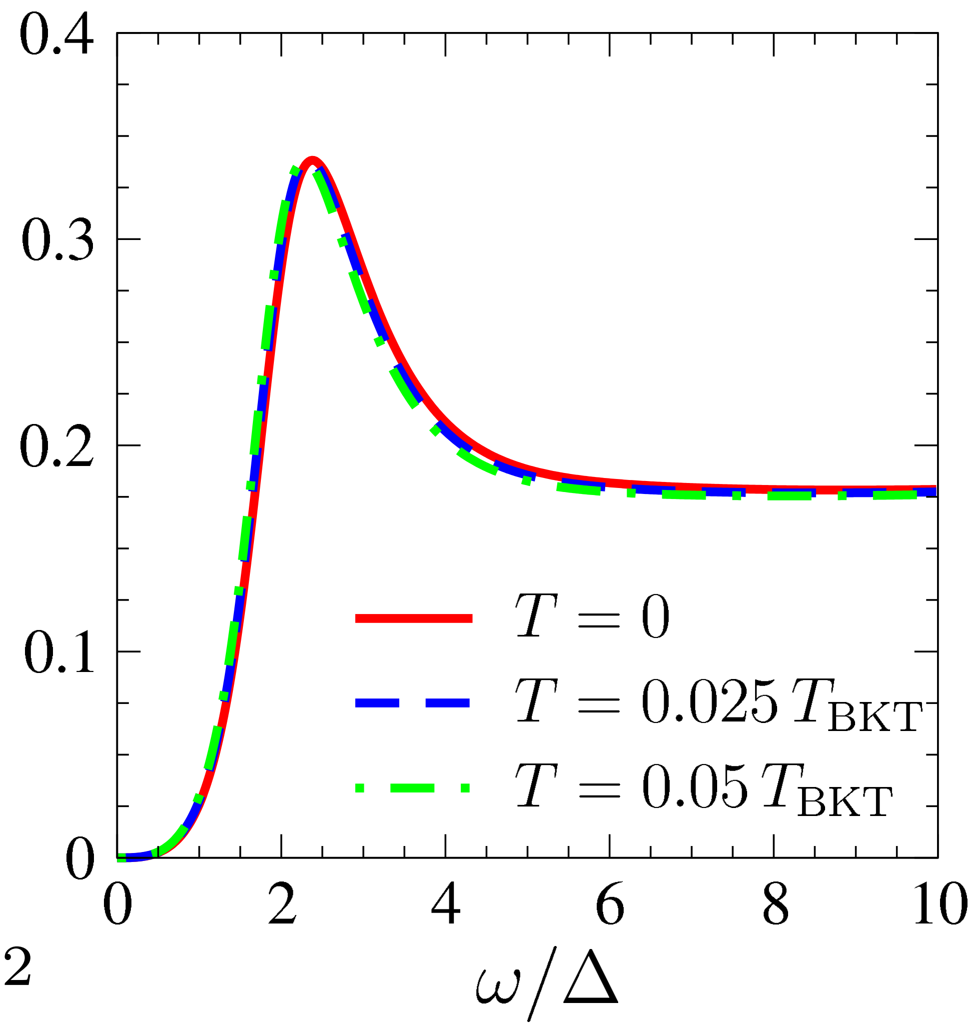
N=2, T>0: BKT phase
(quasi-long range order)

$T_{BKT} \simeq 0.42\Delta$

AR et al. PRE 2013

Numerical analytic continuation hard for ω < 2πT.
Pertubative calculation :

$$\chi''(\omega) \propto \omega^3 \coth\left(\frac{\omega}{2T}\right) \rightarrow T\omega^2$$



AR & N. Dupuis, PRB 89 180501(R) (2014)

Conclusion and Perspectives

- NPRG study of amplitude mode close to a non-trivial critical point. Available approaches : Monte-Carlo, NPRG, others ?
- Study of the amplitude mode for all N. Nice agreement with Monte-Carlo for N=2. Case N=3 needs further study.
- No amplitude mode resonance in the quantum disordered phase.
- Resonance exists in the BKT phase (vanishing order parameter !). Up to what temperature ? ($T_{BKT} \approx m_H/5$)

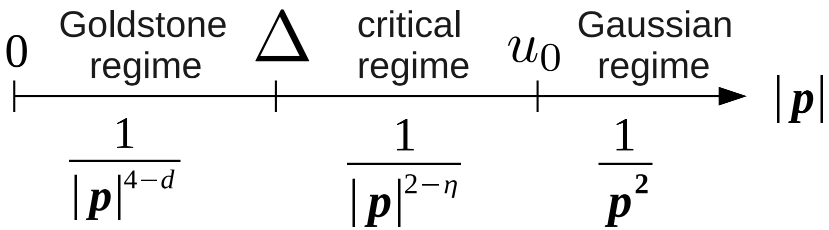
- Future calculation : Conductivity / viscosity within NPRG.

Typical energy scales

$$S=\int_0^\beta d\tau\int d^dx\left\{\frac{1}{2}\psi_i(-c^{-2}\partial_\tau^2-\nabla^2+r_0)\psi_i^2+\frac{u_0}{4!N}(\psi_i^2)^2\right\}$$

Four energy scales : T , Δ , u_0 , $\omega_c \propto \sqrt{r_0 - r_{0,c}}$ (not all independent)

(a) Critical regime: $\omega_c \ll u_0$



(b) Non-critical regime: $\omega_c \gg u_0$