Higgs mode in a superfluid of Dirac fermions

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TOKYO UNIVERSITY OF SCIENCE since 1881

Outline of talk:

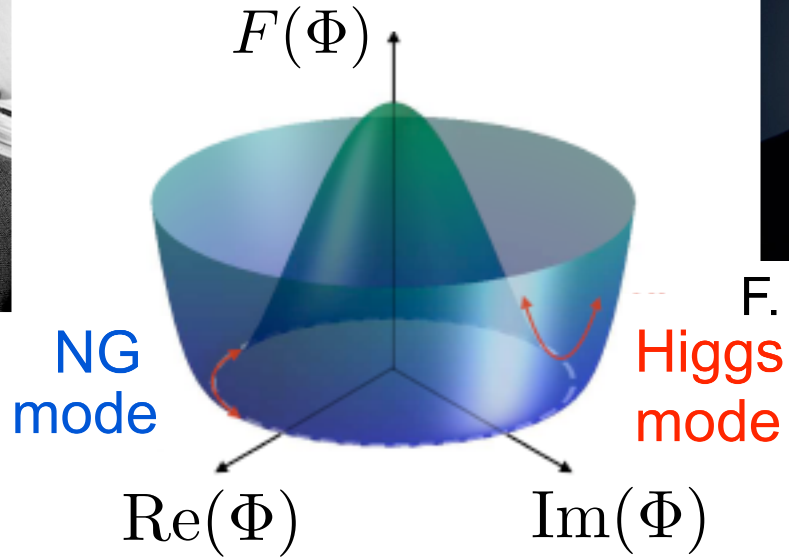
- Introduction : Higgs mode in superconductors
- Model and system: Dirac fermions on honeycomb lattice
- Quantum phase transition between semimetal and s-wave superfluid phases
- Excitations in semi-metal: Cooperons and excitons
- Collective modes in SF: Higgs mode and NG mode
  - evolution across quantum critical point
- Visibility of the Higgs mode
- Summary

Spontaneous Symmetry breaking and collective modes

Y. Nambu

J. Goldstone

“Mexican hat” potential



F. Englert P. Higgs

P. Anderson

- ■ Nambu-Goldstone mode - massless phase mode

Pions, magnons, phonons in crystals and atomic BECs etc.

- ■ Higgs mode - massive amplitude mode

SM, Bose gases, SCs, SFs, magnets, and CDW systems

- growing interest in Higgs mode in various cond. mat. systems!

Observation of Higgs mode in superconductors

- Raman scattering experiments in $NbSe_2$ in 1980
  - First observation of the Higgs mode!

$NbSe_2$: CDW transition at 40K and a SC transition at 7.2 K

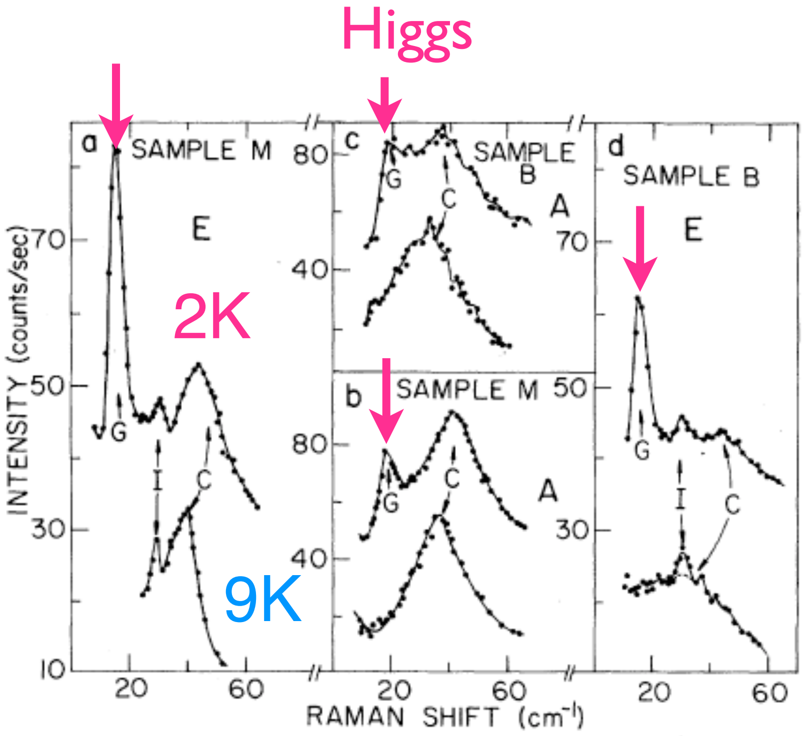


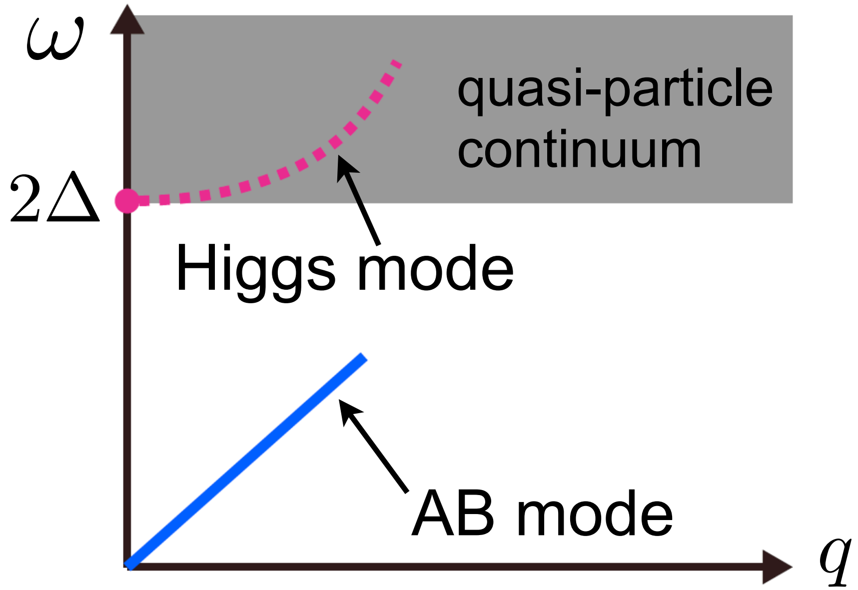
FIG. 1. Raman spectrum of samples M and B. The lower curve of each pair [(a)–(d)] is at 9 K and the upper at 2 K. Raman symmetries [polarizations] are E [(xy)] and A [$(xx)-(xy)$]. C labels CDW modes; G, gap excitations; and I, the interlayer mode characteristic of the $2H$ polytype. Incident laser beam at

PRL 45, 660 (1980)

- The new peak at 2Δ arises below SC Tc.
- Littlewood and Varma developed the microscopic theory for the Higgs mode by extending the BCS theory and found that this peak arises from amplitude oscillations of superconducting gap Δ.

PRL 47, 811 (1980)

Collective modes in superfluids/superconductors



- Anderson-Bogoliubov mode : NG mode (sound mode)

➡ plasma mode in SCs : Anderson-Higgs mechanism

- mass acquisition mechanism in particle physics

- Higgs mode : amplitude mode - not renormalized by Coulomb int.

gapped by 2Δ - enters quasiparticle continuum

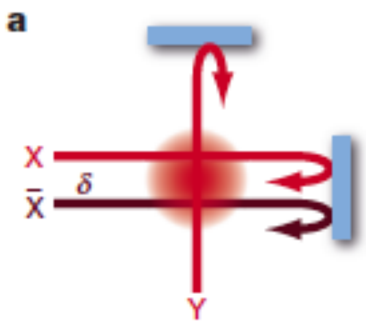
unstable to decay into pairs of quasiparticles

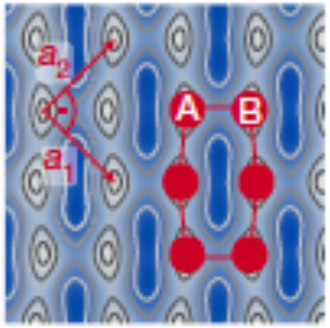
➡ difficult to observe weak damping at $q \simeq 0$: $NbSe_2$, $Nb_{1-x}Ti_xN$

Higgs mode in honeycomb lattice systems

- We propose the possibility of a *stable Higgs mode* against decay into quasi-particles in the *s-wave superfluid state of Dirac fermions* on the honeycomb lattice.

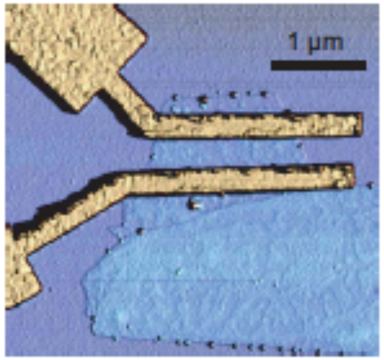
- Fermions loaded onto a honeycomb optical lattice

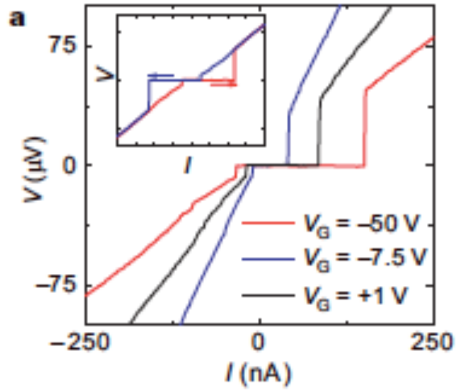




Tarruell et al., Nature, 483, 302 (2012)

- Induced superconductivity in graphene



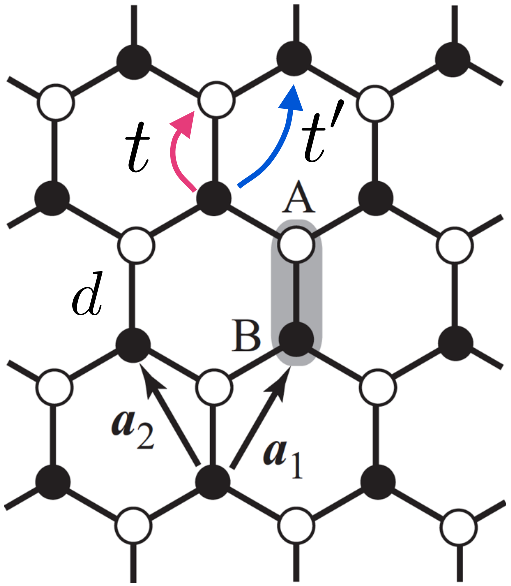


- Supercurrent in graphene

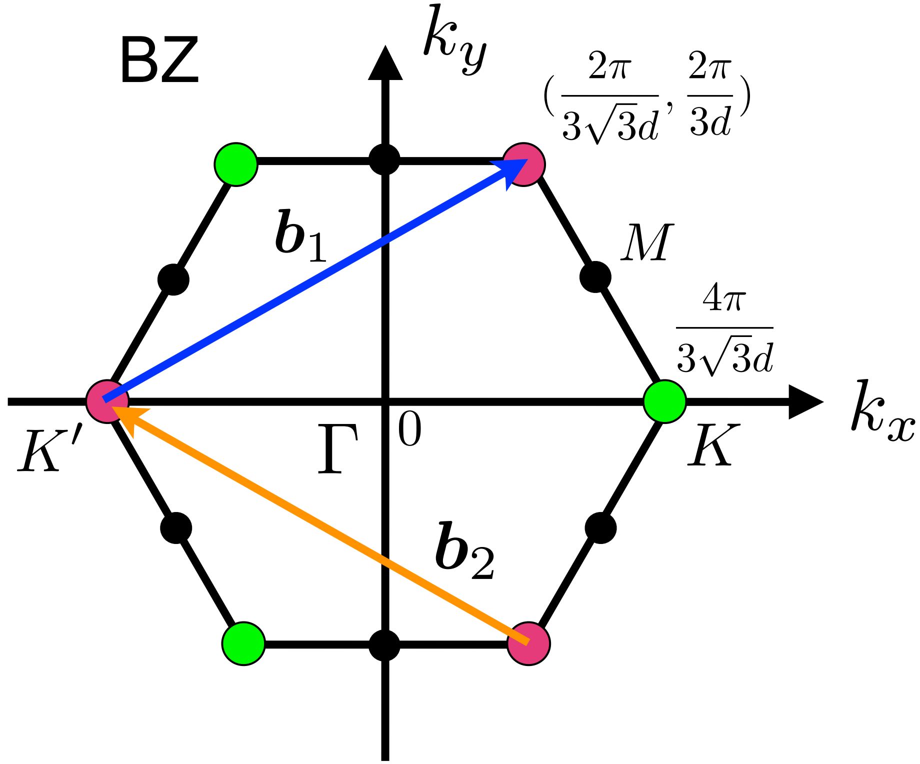
Heersche et al., Nature (2007)

Model and system

- Isotropic honeycomb lattice



unit cell forms a triangular lattice



$\boldsymbol{b}_i$: reciprocal lattice vector

- Attractive Hubbard model : $U > 0$ on-site attraction

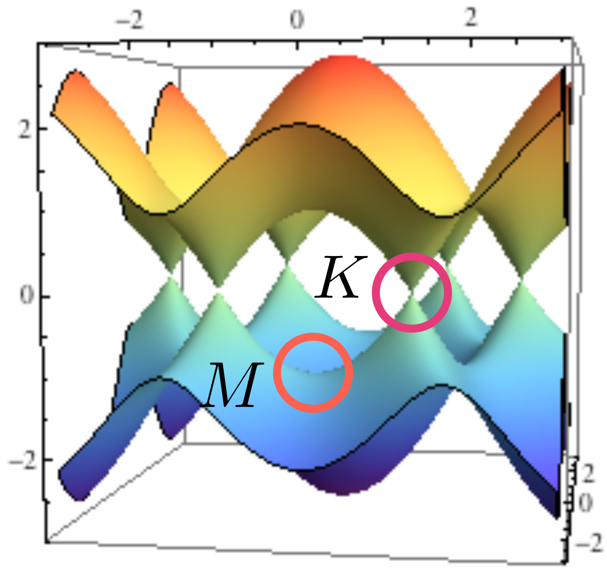
$$H = -\sum_{i,j,\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} - U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

Dirac fermions on honeycomb lattice

- Tight-binding hopping term on honeycomb lattice

$$H_t = \sum_{\boldsymbol{k},\sigma} \left(c^\dagger_{\boldsymbol{k}A\sigma}, c^\dagger_{\boldsymbol{k}B\sigma} \right) \begin{pmatrix} 0 & \gamma_{\boldsymbol{k}} \\ \gamma^*_{\boldsymbol{k}} & 0 \end{pmatrix} \begin{pmatrix} c_{\boldsymbol{k}A\sigma} \\ c_{\boldsymbol{k}B\sigma} \end{pmatrix}$$

$$= \sum_{\boldsymbol{k},\sigma} \left(c^\dagger_{\boldsymbol{k}+\sigma}, c^\dagger_{\boldsymbol{k}-\sigma} \right) \begin{pmatrix} \varepsilon_{\boldsymbol{k}} & 0 \\ 0 & -\varepsilon_{\boldsymbol{k}} \end{pmatrix} \begin{pmatrix} c_{\boldsymbol{k}+\sigma} \\ c_{\boldsymbol{k}-\sigma} \end{pmatrix}$$



$$\gamma_{\boldsymbol{k}} = -t(1 + e^{i\boldsymbol{k}\cdot\boldsymbol{a}_1} + e^{i\boldsymbol{k}\cdot\boldsymbol{a}_2})$$

$\varepsilon_{\boldsymbol{k}} = |\gamma_{\boldsymbol{k}}|$: conduction band

$-\varepsilon_{\boldsymbol{k}}$: valence band

: particle-hole symmetry

Note *t'* breaks p-h symmetry

K : Dirac point

M : saddle point

Dirac fermions on honeycomb lattice

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$$H_t = \sum_{\boldsymbol{k},\sigma} \left(c^\dagger_{\boldsymbol{k}A\sigma}, c^\dagger_{\boldsymbol{k}B\sigma} \right) \begin{pmatrix} 0 & \gamma_{\boldsymbol{k}} \\ \gamma^*_{\boldsymbol{k}} & 0 \end{pmatrix} \begin{pmatrix} c_{\boldsymbol{k}A\sigma} \\ c_{\boldsymbol{k}B\sigma} \end{pmatrix}$$

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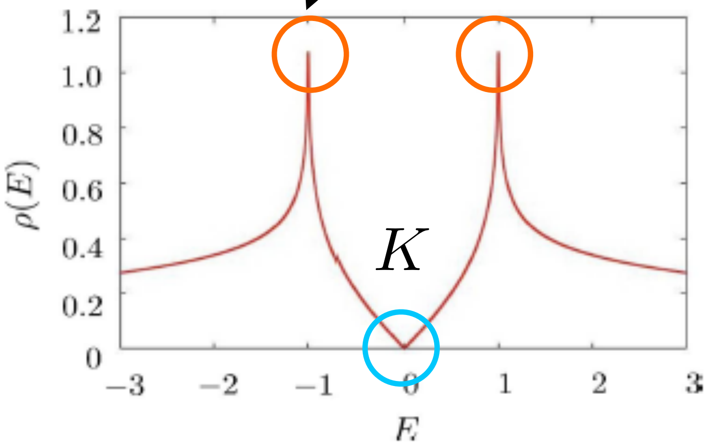
van Hove singularity

M DOS



$$\gamma_{\boldsymbol{k}} = -t(1 + e^{i\boldsymbol{k}\cdot\boldsymbol{a}_1} + e^{i\boldsymbol{k}\cdot\boldsymbol{a}_2})$$

$\varepsilon_{\boldsymbol{k}} = |\gamma_{\boldsymbol{k}}|$: conduction band

$-\varepsilon_{\boldsymbol{k}}$: valence band

: particle-hole symmetry

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Dirac fermions on honeycomb lattice

- Linear approximation around *K* and *K'* points

$$v_F = \sqrt{3}t/2 \quad \psi_{K\boldsymbol{p}} = (a_{\boldsymbol{p}}, b_{\boldsymbol{p}})^t$$

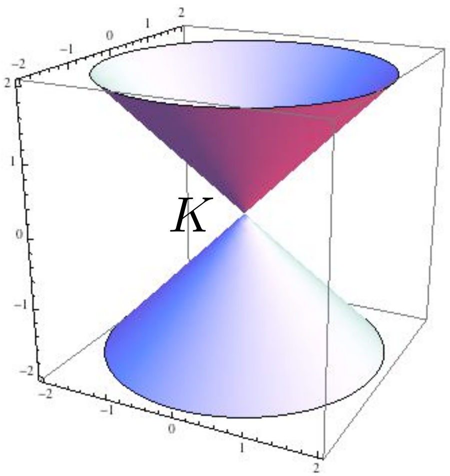
$$H_t = \sum_{\boldsymbol{p}} \left[\psi^{\dagger}_{K\boldsymbol{p}} (v_F \boldsymbol{\sigma} \cdot \boldsymbol{p}) \psi_{K\boldsymbol{p}} + \psi^{\dagger}_{K'\boldsymbol{p}} (-v_F \boldsymbol{\sigma}^* \cdot \boldsymbol{p}) \psi_{K'\boldsymbol{p}} \right]$$

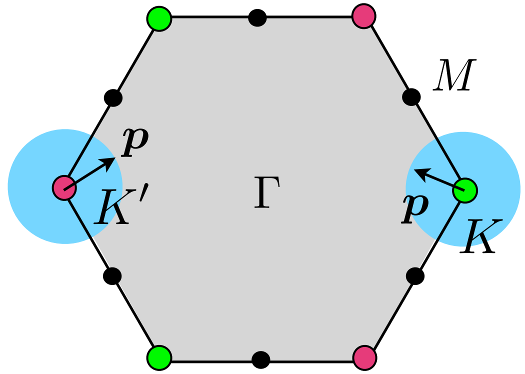
:Dirac Hamiltonian in 2D

$$\boldsymbol{\sigma} = (\sigma_x, \sigma_y)$$

describes gapless Dirac fermions with the dispersion

$$E_{\boldsymbol{p}} = \pm v_F p$$



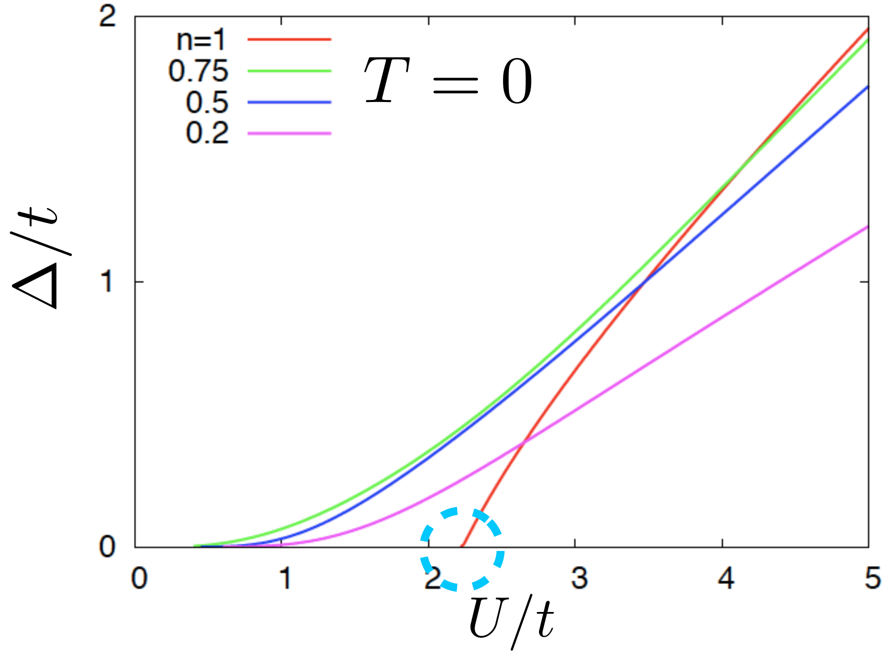


- half-filling (*n*=1)

μ at Dirac cone -absence of Fermi surface semi-metal

Quantum phase transition to SF phase

- *s*-wave SF : order parameter $\Delta = U\langle c_{i\downarrow} c_{i\uparrow}\rangle$



Zhao and Paramekanti (2006)

- BCS-BEC crossover away from half-filling

- Half-filling: quantum phase transition between Dirac semi-metal and superfluid phases at U/t~2.23 in MF theory

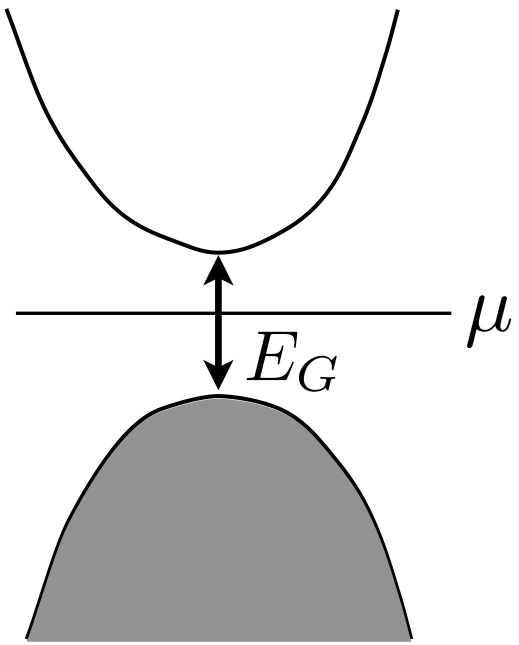
← zero density of states at Dirac cone

QMC: U/t~3.869 Sorella et al. (2012)

Superconductivity from insulator

- BCS theory Fermi surface is unstable to *infinitesimally* weak attractive interaction

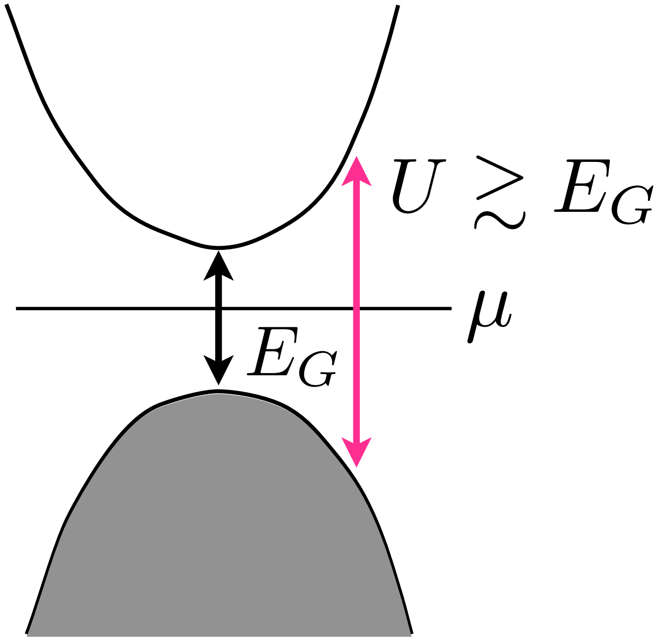
→ Cooper pair formation



Insulators (semiconductors) can become superconducting?

Superconductivity from insulator

- BCS theory — Fermi surface is unstable to *infinitesimally* weak attractive interaction
  → Cooper pair formation



Insulators (semiconductors) can become superconducting?

↓ Yes (theoretically...)

SC phase transition takes place, if attractive interaction is large enough to overcome the band gap.

Kohmoto and Takada JPSJ, 59, 1541 (1990)

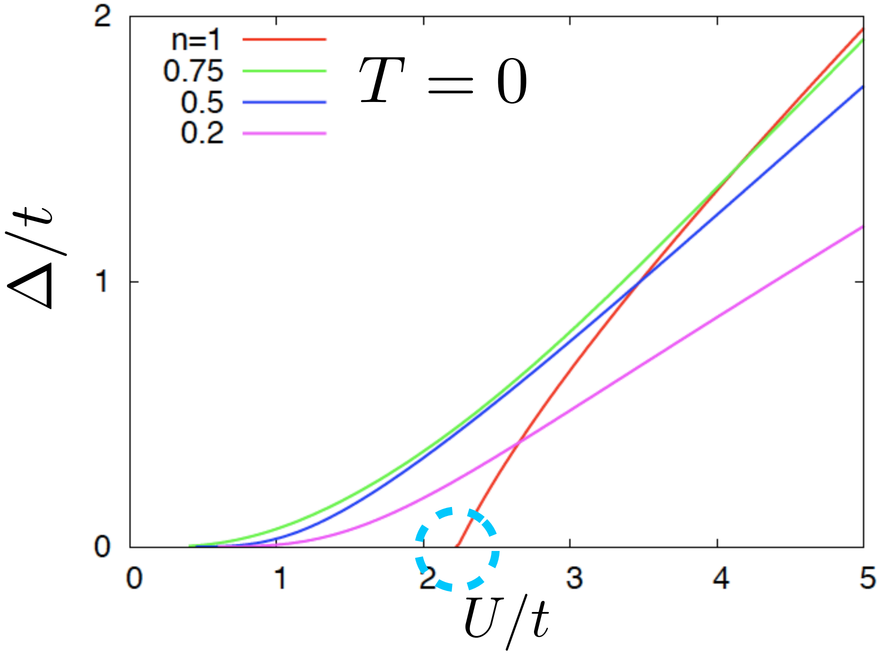
Nozieres and Pistolesi

Eur. Phys. J. B10, 649 (1999)

similar to excitonic insulators for Coulomb int.

Quantum phase transition to SF phase

- *s*-wave SF : order parameter $\Delta = U\langle c_{i\downarrow} c_{i\uparrow} \rangle$



We focus on collective modes in the vicinity of the quantum critical point.

- Half-filling: quantum phase transition between Dirac semi-metal and superfluid phases at U/t~2.23 in MF theory

⬅ zero density of states at Dirac cone

QMC: U/t~3.869 Sorella et al. (2012)

Generalized Random Phase Approximation

- Introduce *fictitious* external fields   Cote and Griffin (1993)

$$H = H_{\mathrm{Hubbard}} + \sum_i P_i(\tau) n_i + \sum_i [Q_i^*(\tau) c_{i\uparrow} c_{i\downarrow} + \mathrm{h.c.}]$$

- Single-particle Green's function

$$\hat{G}(1,2) = -\langle T\Psi(1)\Psi^\dagger(2)\rangle \quad \Psi(1) = \begin{pmatrix} c_\uparrow(1) \\ c_\downarrow^\dagger(1) \end{pmatrix}$$

- Susceptibility matrices

$$m(1) = c_\downarrow(1) c_\uparrow(1)$$

$$\hat{L}(1,2,3) = \frac{\delta \tilde{G}(1,2)}{\delta P(3)} \quad \hat{M}(1,2,3) = \frac{\delta \tilde{G}(1,2)}{\delta Q(3)} \quad \tilde{G}(1,2) = \hat{\tau}_3 \hat{G}(1,2)$$

$$\hat{L}(1,3) \equiv \hat{L}(1,1^+,3) = \begin{pmatrix} -\langle T\delta n_\uparrow(1)\delta n(3)\rangle & -\langle T\delta m(1)\delta n(3)\rangle \\ \langle T\delta m^\dagger(1)\delta n(3)\rangle & -\langle T\delta n_\downarrow(1)\delta n(3)\rangle \end{pmatrix}$$

:density response

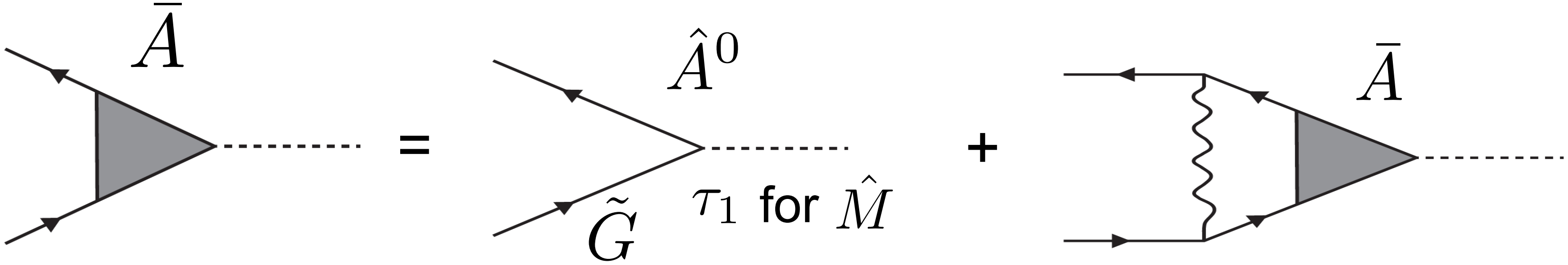
$$\hat{M}(1,3) \equiv \hat{M}(1,1^+,3) = \begin{pmatrix} -\langle T\delta n_\uparrow(1)\delta m^\dagger(3)\rangle & -\langle T\delta m(1)\delta m^\dagger(3)\rangle \\ \langle T\delta m^\dagger(1)\delta m^\dagger(3)\rangle & -\langle T\delta n_\downarrow(1)\delta m^\dagger(3)\rangle \end{pmatrix}$$

:pairing response

Single-particle and two-particle Green's functions are treated consistently

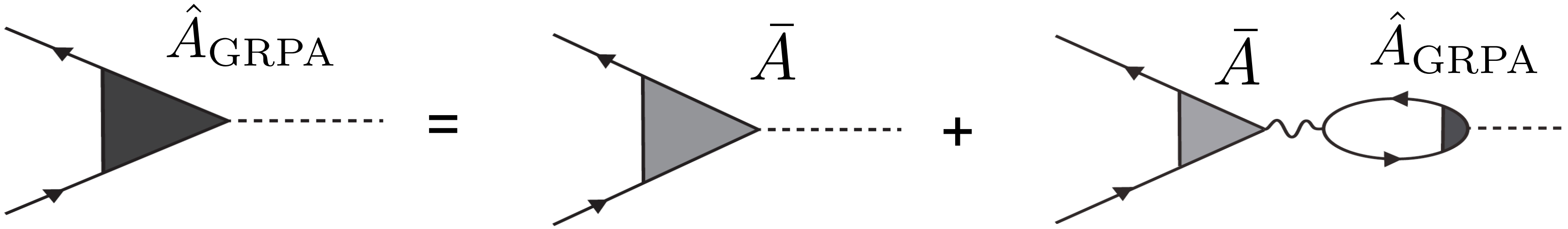
➡ Conserving approximation Kadanoff-Baym (1961)

- Hartree-Fock-Gor'kov app. for self-energy $\hat{\Sigma}$
- GRPA equations $\hat{A} = \hat{L}, \hat{M}$ $\bar{A}$: irreducible susceptibility



order parameter fluctuation

$$\bar{A}^{\nu_1\nu_2}(q) = \hat{A}^{0\nu_1\nu_2}(q) + \frac{2U}{\beta N}\sum_{\nu_3}\sum_{\boldsymbol{p},\omega_n}\tilde{G}^{\nu_1\nu_3}(p+q)\bar{A}^{\nu_3\nu_2}(q)\tilde{G}^{\nu_3\nu_1}(p)$$



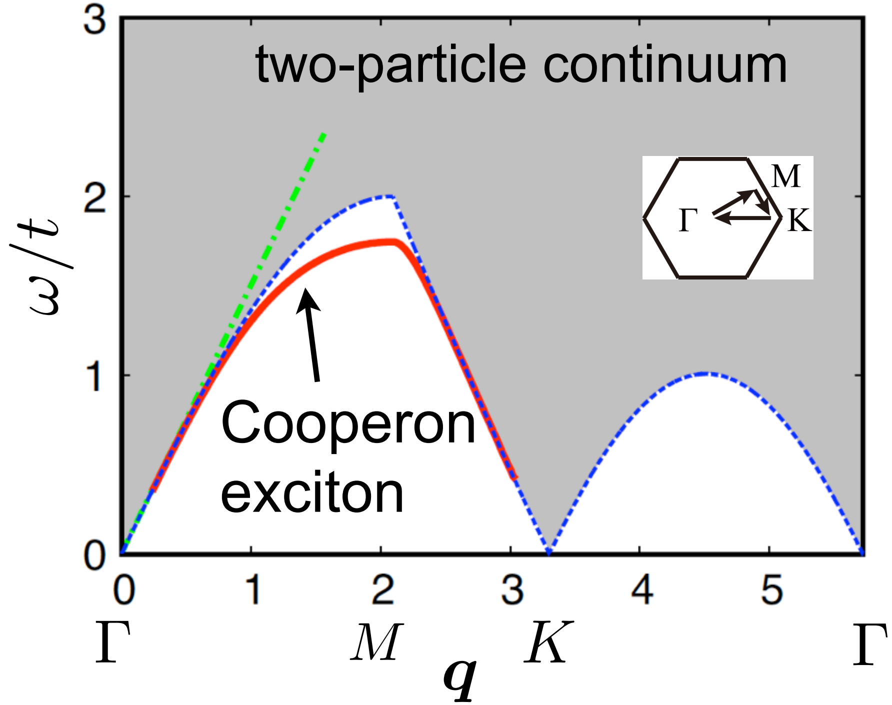
density fluctuation

$$\hat{A}^{\nu_1\nu_2}_{\mathrm{GRPA}}(q) = \bar{A}^{\nu_1\nu_2}(q) - U\sum_{\nu_3}\bar{L}^{\nu_1\nu_3}(q)\mathrm{Tr}\{\hat{A}^{\nu_3\nu_2}_{\mathrm{GRPA}}(q)\}$$

should be treated on equal footing for SU(2) pseudospin symmetry

Excitations in semi-metal phase

U/t=2, t'=0, half-filling



- "Cooperon" pole in pairing response $\chi_{mm^\dagger}$

$m = c_\downarrow c_\uparrow$

particle-particle bound state
- preformed Cooper pairs

- exciton pole in density response χ_{nn}

particle-hole bound state

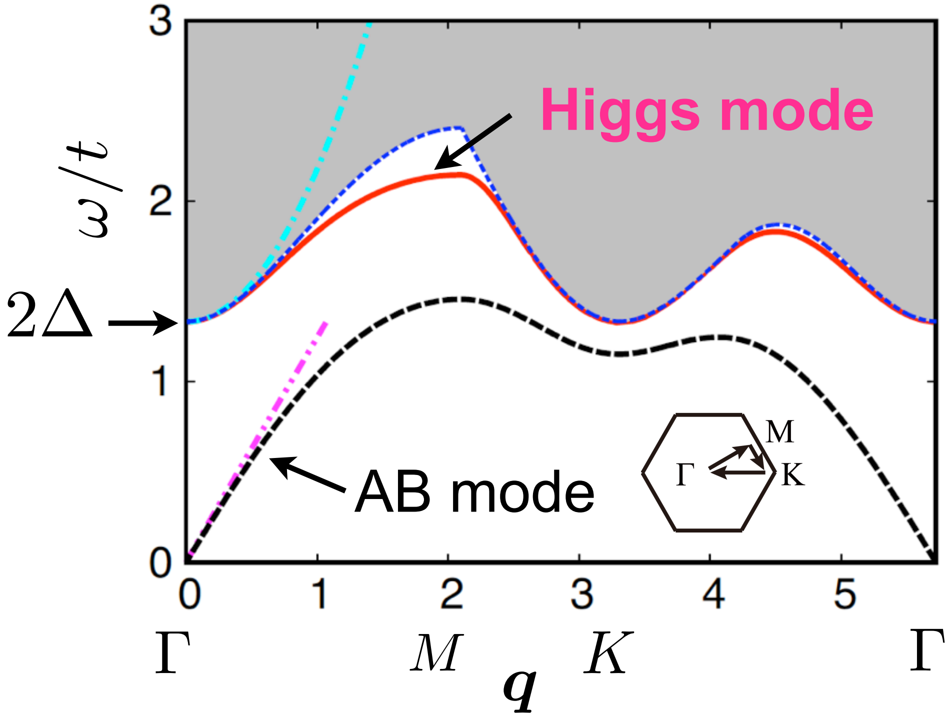
- *stable* Cooperon and exciton excitations inside the *window of continuum*

← linear dispersion of Dirac fermion

t'=0 -> degenerate Cooperon and exciton :particle-hole symmetry

Excitations in superfluid phase

U/t=3, t'=0, half-filling



- AB mode pole

$$\chi_{nn} \propto \chi_{\phi\phi}$$

phase/density mode

$$q \ll 1 \quad \omega_{\rm AB} = \lambda v_F q \quad (\lambda \leq 1)$$

- Higgs mode pole

$$\chi_{|\Delta||\Delta|}$$

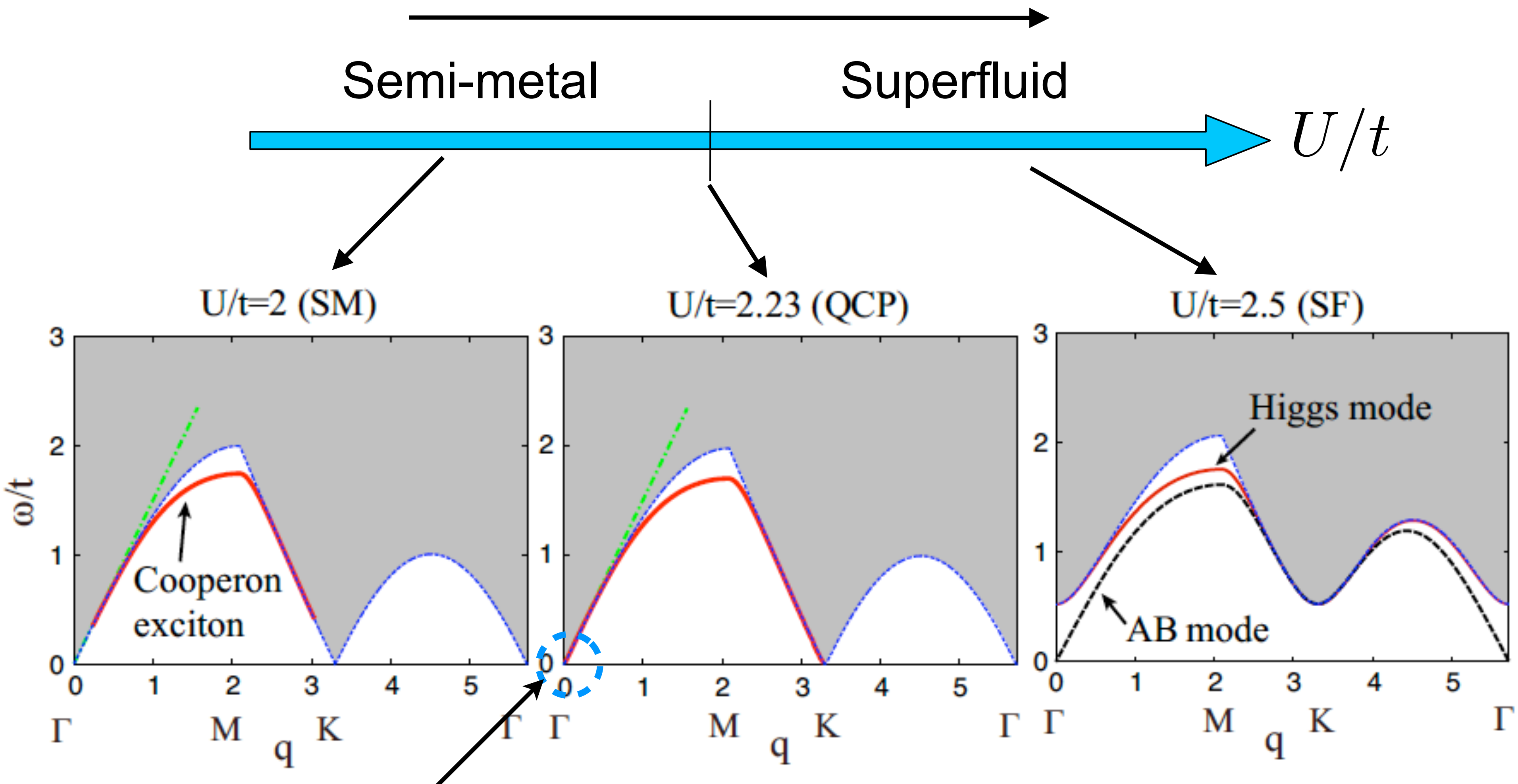
amplitude mode

- *Stable* Higgs mode inside the window of q.p. continuum

$$q \ll 1 \quad \omega_{\rm Higgs}^2 = 4\Delta^2 + v_F^2 q^2$$

well separated from the continuum edge close to the M point

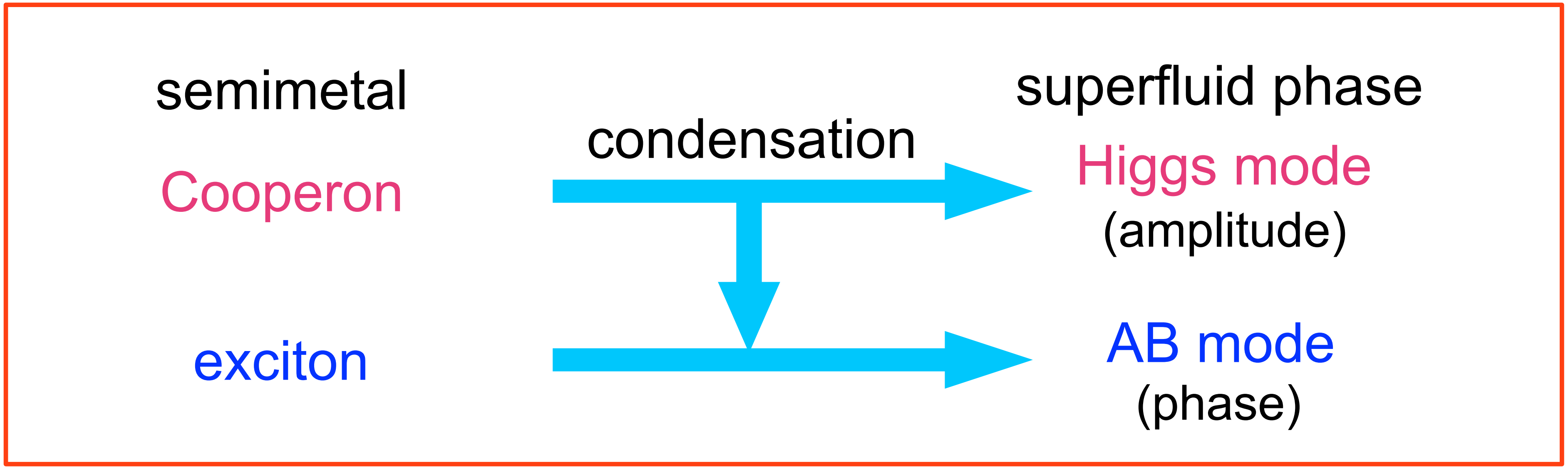
Evolution of collective modes



Cooperon condensation at QCP

- system undergoes *phase transition to the SF phase*

Relation of collective modes



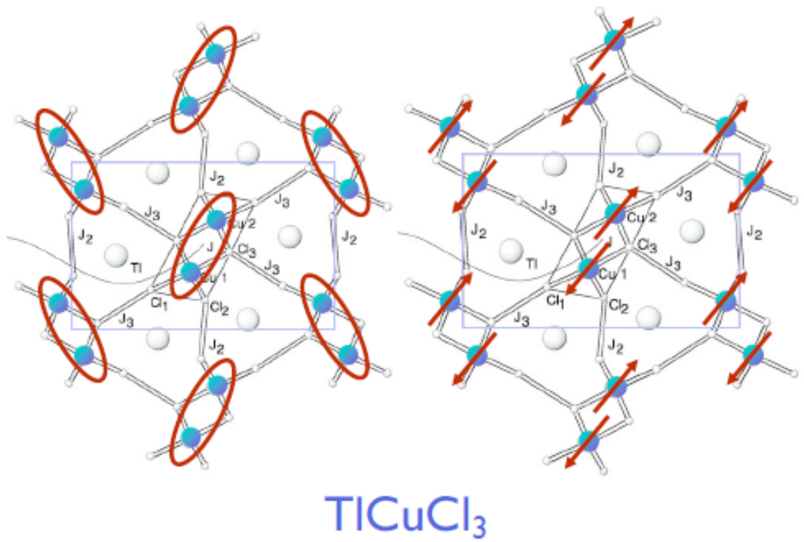
- Cooperon and exciton hybridize to become the Higgs and AB modes
- Cooperon component splits into amplitude and phase fluctuations of order parameter
- Exciton involved in the AB mode allows density fluctuations

PRL 100, 205701 (2008) PHYSICAL REVIEW LETTERS week ending 23 MAY 2008

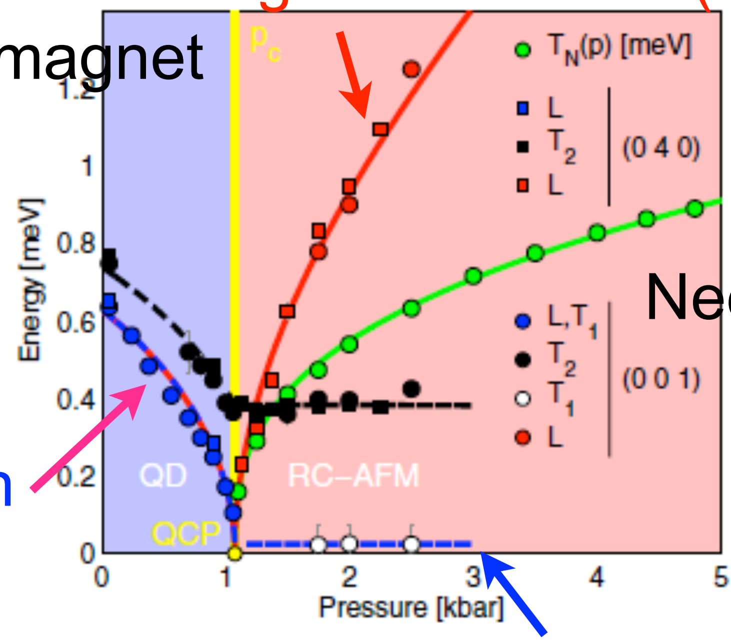
analogous to the spin dimer system

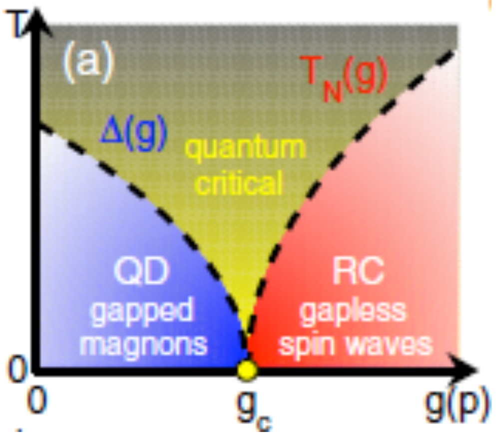
Quantum Magnets under Pressure: Controlling Elementary Excitations in $TlCuCl_3$

Ch. Rüegg,[1] B. Normand,[2,3] M. Matsumoto,[4] A. Furrer,[5] D. F. McMorrow,[1] K. W. Krämer,[6] H.-U. Güdel,[6] S. N. Gvasaliya,[5] H. Mutka,[7] and M. Boehm[7]



$$\mathcal{H} = \sum_i J(p)\mathbf{S}_{i,l}\cdot\mathbf{S}_{i,r} + \sum_{ij;m,m'=l,r} J_{ij}(p)\mathbf{S}_{i,m}\cdot\mathbf{S}_{j,m'}$$



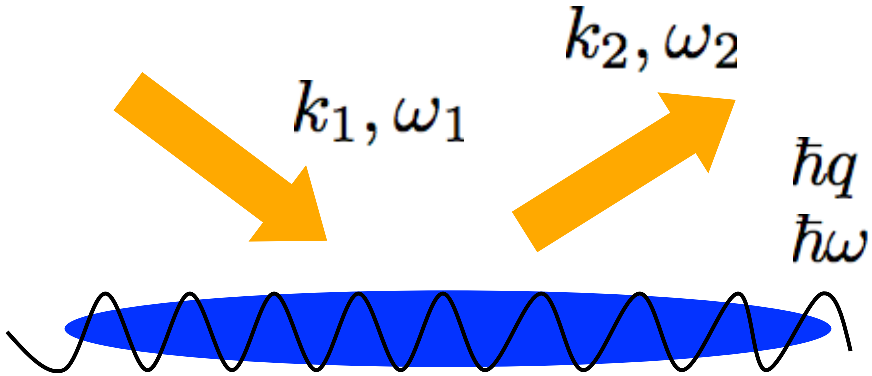


- triplon (broken valence bond excitations)

condensation → Higgs + NG mode

Observation of Higgs mode in cold atom systems

- Bragg scattering



$$\hbar q = \hbar k_1 - \hbar k_2$$
$$\hbar \omega = \hbar \omega_1 - \hbar \omega_2$$

- Direct measurement of <u>dynamic structure factor</u>

$$S(\boldsymbol{q}, \omega) = -\frac{1}{\pi} \mathrm{Im}[\chi_{nn}(\boldsymbol{q}, \omega)]$$

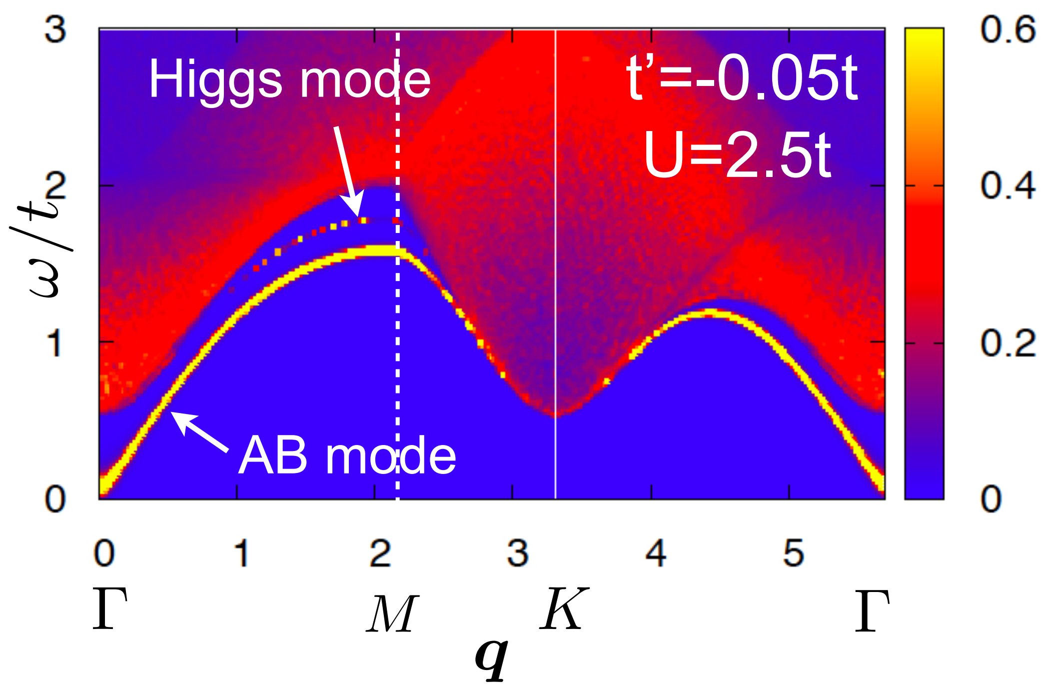
e.g. energy absorption rate: $\frac{dE}{dt} \propto \omega S(\boldsymbol{q}, \omega)$

Observation of Higgs mode

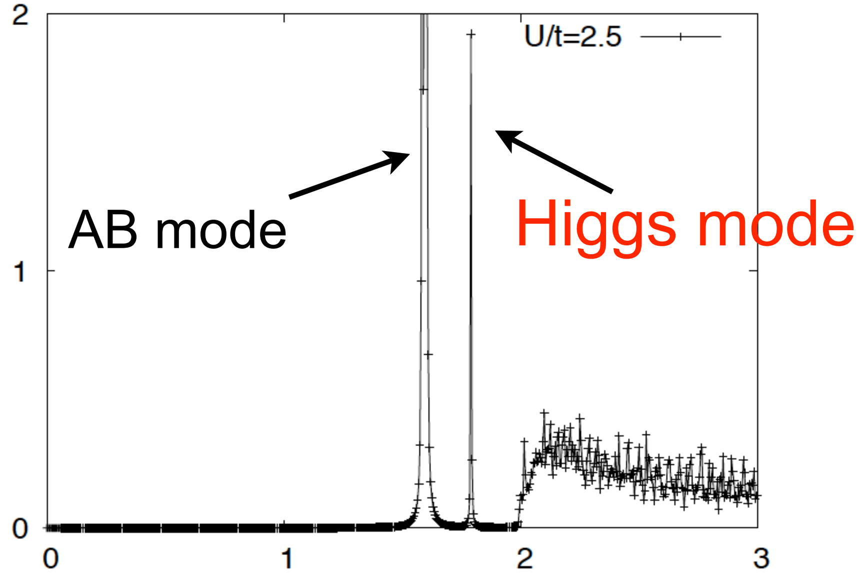
- Higgs mode has small density component if particle-hole symmetry is broken by, e.g., finite t'

dynamic structure factor

$$S(\boldsymbol{q},\omega) = -\mathrm{Im}[\chi_{nn}(\boldsymbol{q},\omega)]/\pi$$



$$\boldsymbol{q} = \boldsymbol{q}_M$$



Bragg spectroscopy is useful for Higgs hunting!

Summary

- Higgs amplitude mode in the *s*-wave superfluid on the honeycomb lattice in the vicinity of a phase transition between semi-metal and superfluid phases
- Higgs mode exists as a *stable* excitation inside the window of single-particle continuum that arises from the linear dispersion of underlying Dirac fermions.
- "Cooperon" and exciton excitations in the semi-metal phase evolve into the AB mode and Higgs mode.
- Higgs mode could be observed in cold atom experiments by using Bragg scattering technique.
- The same scenario for the repulsive Hubbard model due to the p-h mapping at half-filling : $c_{i\downarrow} \to (-1)^i c_{i\downarrow}^\dagger$