Higgs mode and Anderson pseudospin resonance in superconductors

Tsuji, Aoki, arXiv:1404.2711; in prep.

24 June 2014 @ YITP workshop
"*Higgs mode in condensed matter and quantum gases*"

Naoto Tsuji (Univ. of Tokyo)

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Raman Scattering by Superconducting-Gap Excitations and Their Coupling to Charge-Density Waves

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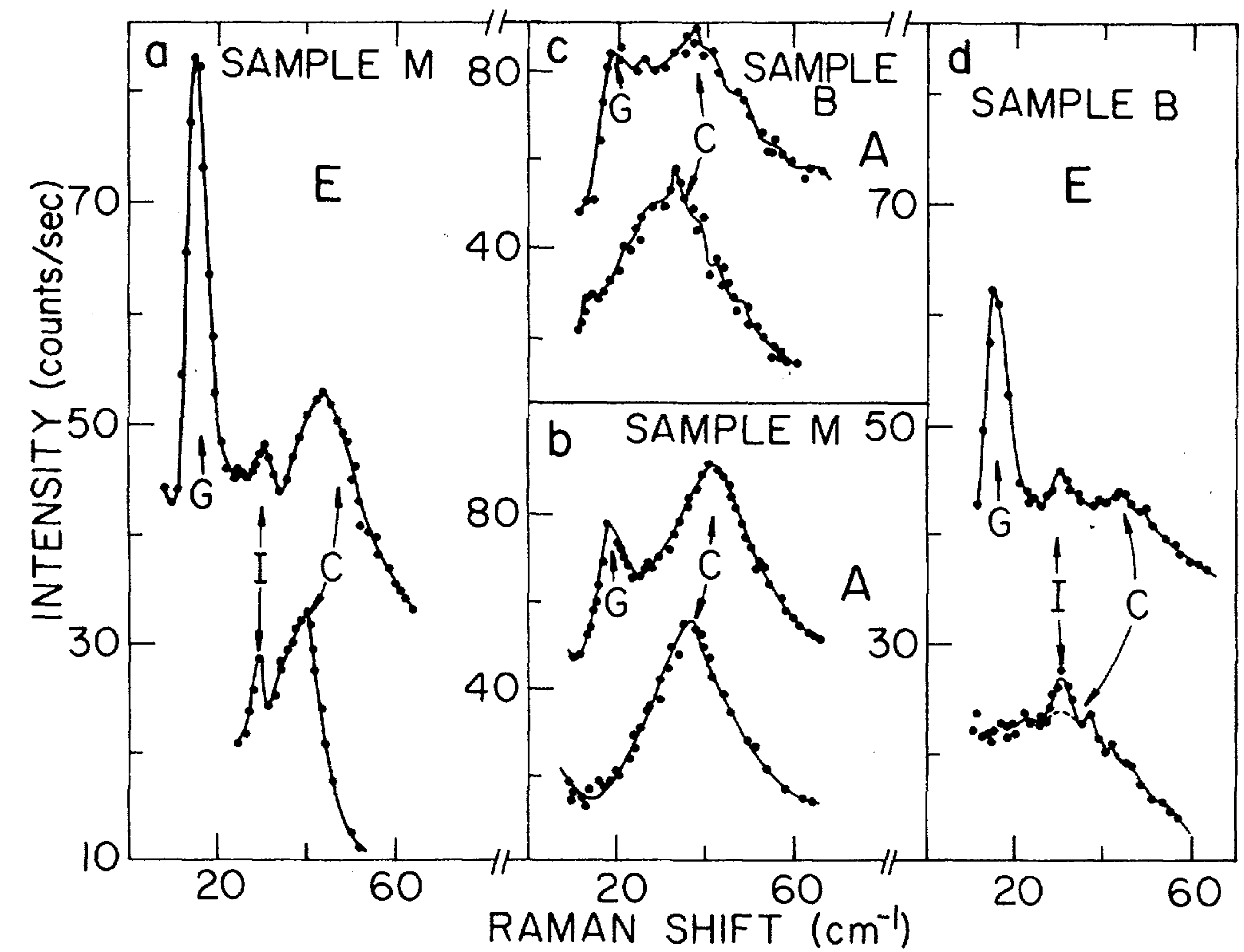

2H-NbSe2
(SC+CDW)

See also Littlewood, Varma, (1981, 1982); Measson et al. (2014).

Higgs Amplitude Mode in the BCS Superconductors $Nb_{1-x}Ti_xN$ Induced by Terahertz Pulse Excitation

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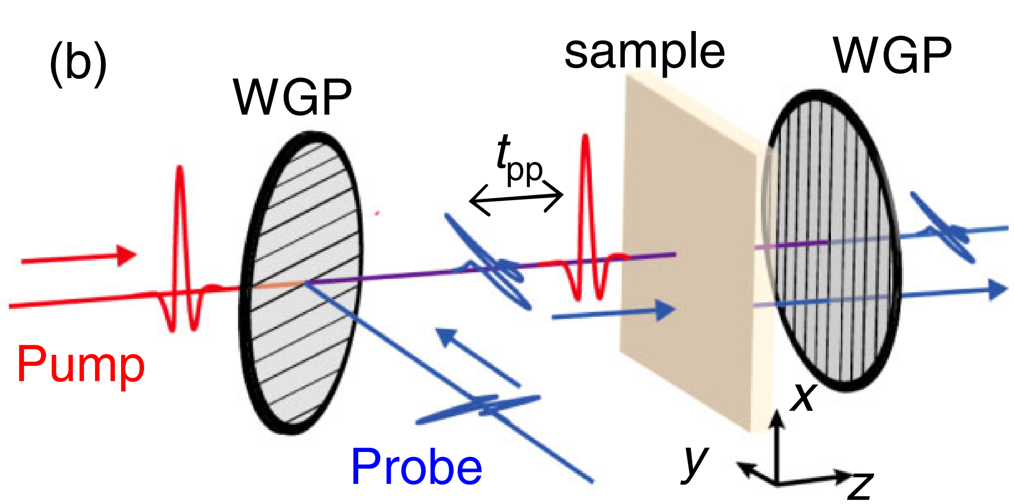
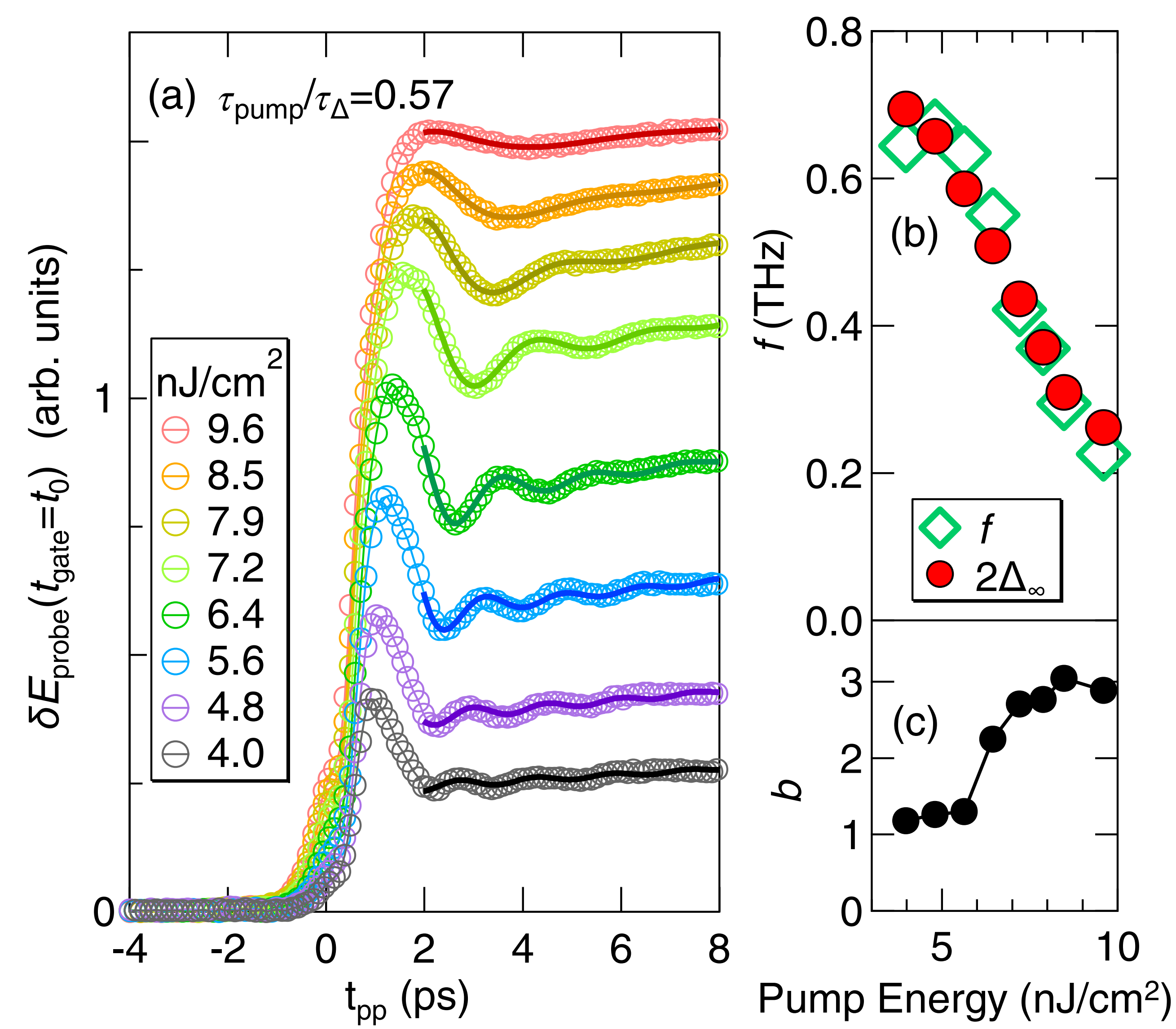




Plan of the talk

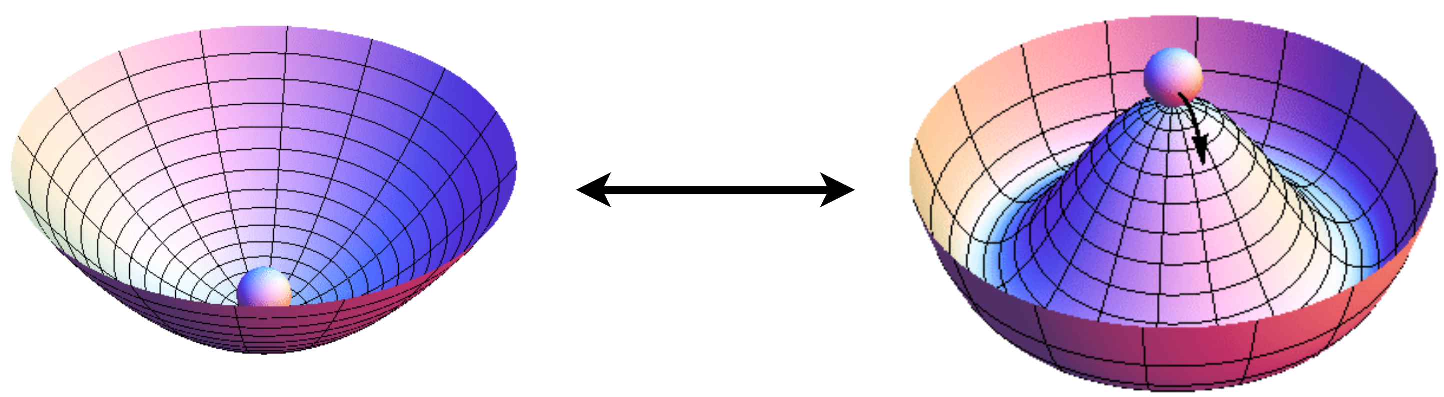
- Consider an s-wave superconducting state during irradiation of an ac electric field.
- Analytically solve the equation of motion within BCS theory.
- Reveal a phenomenon, "Anderson pseudospin resonance".
- Discuss effects of electron-electron scattering (nonequilibrium DMFT) and impurity scattering (Abrikosov-Gor'kov theory).

Tsuji, Aoki, arXiv:1404.2711; in prep.

Dynamics of superconductors

- Time-dependent Ginzburg-Landau equation

$$-\Gamma \frac{\partial \Delta}{\partial t} = \frac{\delta \mathcal{F}_{GL}}{\delta \Delta^*} = a\Delta + b|\Delta|^2\Delta - \frac{c}{2m}\nabla^2\Delta$$



- Microscopically justified
  - near the critical point (Ginzburg condition: $|T\text{-}T_c|>T_G$)
  - when (time scale of order parameter) >> (quasiparticle relaxation time)

Dynamics of superconductors

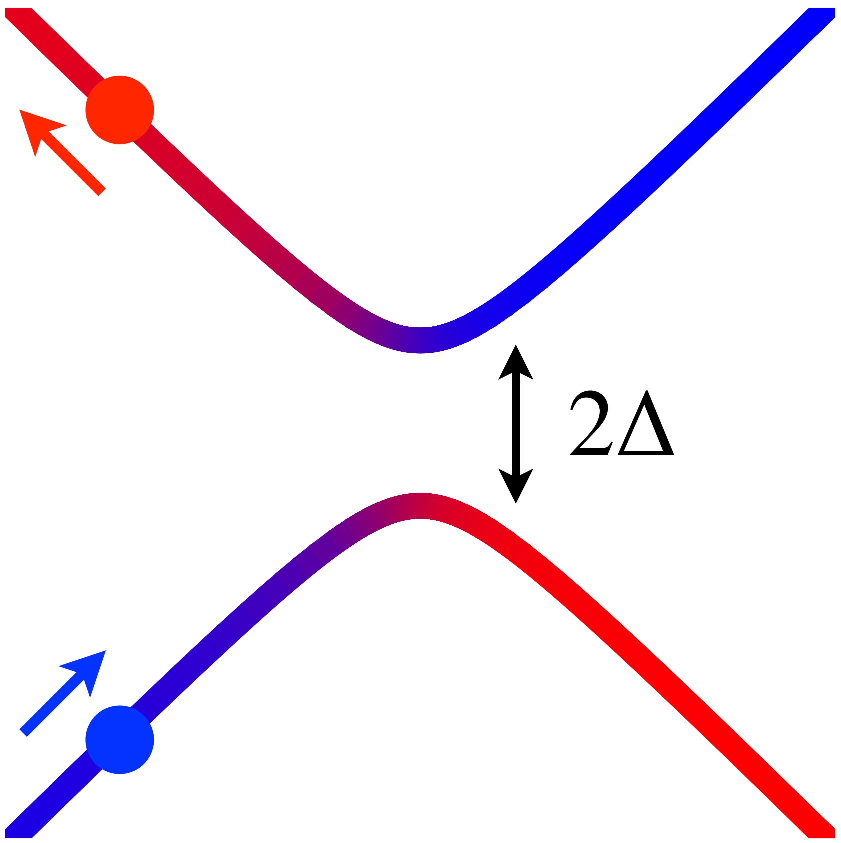
- Bogoliubov-de Gennes equation coupled to an electric field

electron

$$i\partial_t \Psi_{\boldsymbol{k}} = \begin{pmatrix} \epsilon_{\boldsymbol{k}-eA(t)} & -\Delta^* \\ -\Delta & -\epsilon_{\boldsymbol{k}+eA(t)} \end{pmatrix} \Psi_{\boldsymbol{k}}$$

hole

$$\Psi_{\boldsymbol{k}} = \begin{pmatrix} c_{\boldsymbol{k}\uparrow} \\ c^\dagger_{-\boldsymbol{k}\downarrow} \end{pmatrix}$$: Nambu spinor



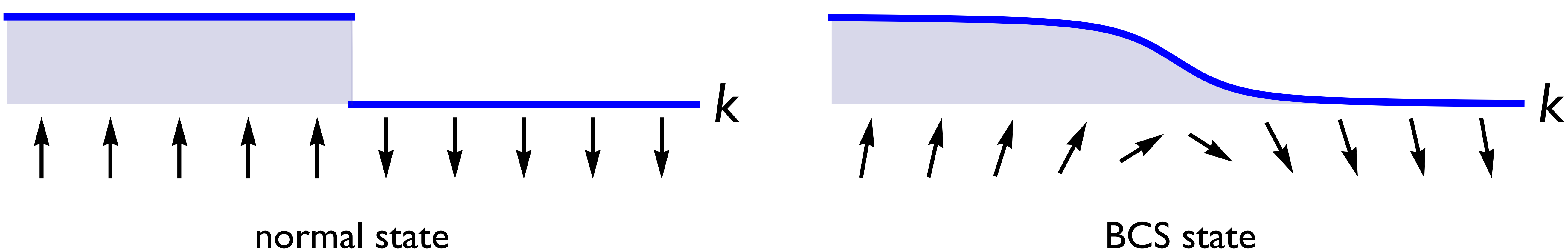
Anderson pseudospin

$$\boldsymbol{\sigma}_{\boldsymbol{k}} = \frac{1}{2}\Psi^{\dagger}_{\boldsymbol{k}} \cdot \boldsymbol{\tau} \cdot \Psi_{\boldsymbol{k}}$$

Anderson, Phys. Rev. 112, 1900 (1958)

$$\boxed{\partial_t \boldsymbol{\sigma}_{\boldsymbol{k}} = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}} \qquad \boldsymbol{b}_{\boldsymbol{k}} = \left(-\Delta', -\Delta'', \frac{\epsilon_{\boldsymbol{k}-e\boldsymbol{A}(t)} + \epsilon_{\boldsymbol{k}+e\boldsymbol{A}(t)}}{2}\right)}$$

Tsuji, Aoki, arXiv:1404.2711



- Particle-hole symmetric by construction.
- Linear response vanishes.

Light-pseudospin coupling

$$\boxed{\partial_t \boldsymbol{\sigma}_{\boldsymbol{k}} = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}} \qquad \boldsymbol{b}_{\boldsymbol{k}} = \left(-\Delta', -\Delta'', \frac{\epsilon_{\boldsymbol{k}-e\boldsymbol{A}(t)} + \epsilon_{\boldsymbol{k}+e\boldsymbol{A}(t)}}{2}\right)}$$

$$b^z_{\boldsymbol{k}} = \epsilon_{\boldsymbol{k}} + \frac{1}{2}\sum_{ij} \frac{\partial^2 \epsilon_{\boldsymbol{k}}}{\partial k_i \partial k_j} e^2 A_i(t) A_j(t) + O(A^4)$$

- Let *x* be the polarization direction of the electric field.

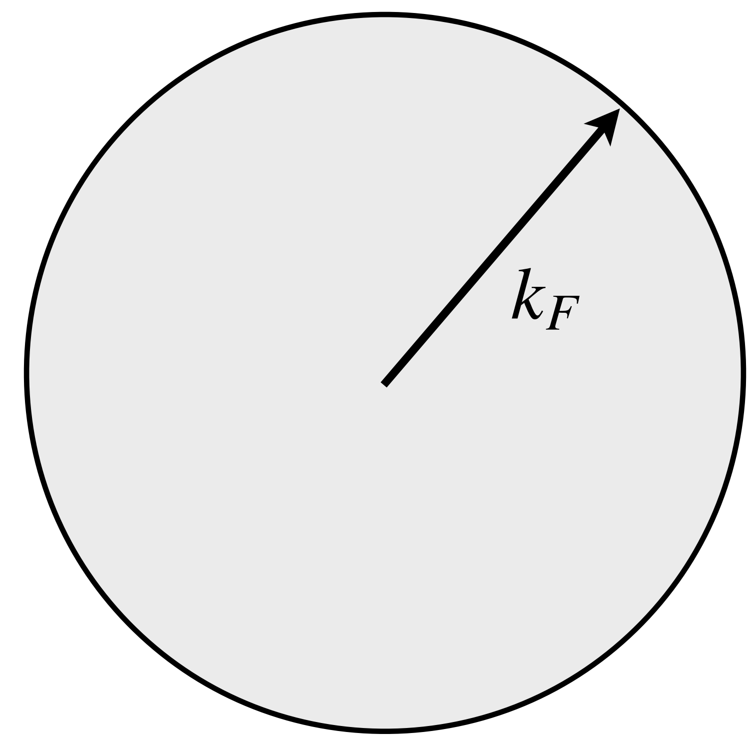
- When all the directions are equivalent, one can symmetrize

$$\frac{\partial^2 \epsilon_{\boldsymbol{k}}}{\partial k_x^2} \to \frac{1}{d} \nabla^2_{\boldsymbol{k}} \epsilon_{\boldsymbol{k}}$$

$$\boxed{\partial_t \boldsymbol{\sigma}_{\boldsymbol{k}} = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}} \quad \boldsymbol{b}_{\boldsymbol{k}} = \left(-\Delta', -\Delta'', \epsilon_{\boldsymbol{k}} + \frac{1}{2}e^2 A(t)^2 \cdot \frac{1}{d}\nabla^2_{\boldsymbol{k}}\epsilon_{\boldsymbol{k}}\right)}$$

- Consider an isotropic system [$\epsilon_{\boldsymbol{k}} = \epsilon(|\boldsymbol{k}|)$]
- Expand $\epsilon_{\boldsymbol{k}}$ near the Fermi surface:

$$\epsilon_{\boldsymbol{k}} = \sum_{n=1}^{\infty} c_n(|\boldsymbol{k}| - k_F)^n$$



- Let us define an expansion

$$\frac{1}{d}\nabla^2_{\boldsymbol{k}}\epsilon_{\boldsymbol{k}} = \alpha_0 + \alpha_1\epsilon_{\boldsymbol{k}} + \alpha_2\epsilon^2_{\boldsymbol{k}} + \cdots$$

with $\alpha_0 = 2c_2 d^{-1} + c_1(1 - d^{-1})k_F^{-1}$

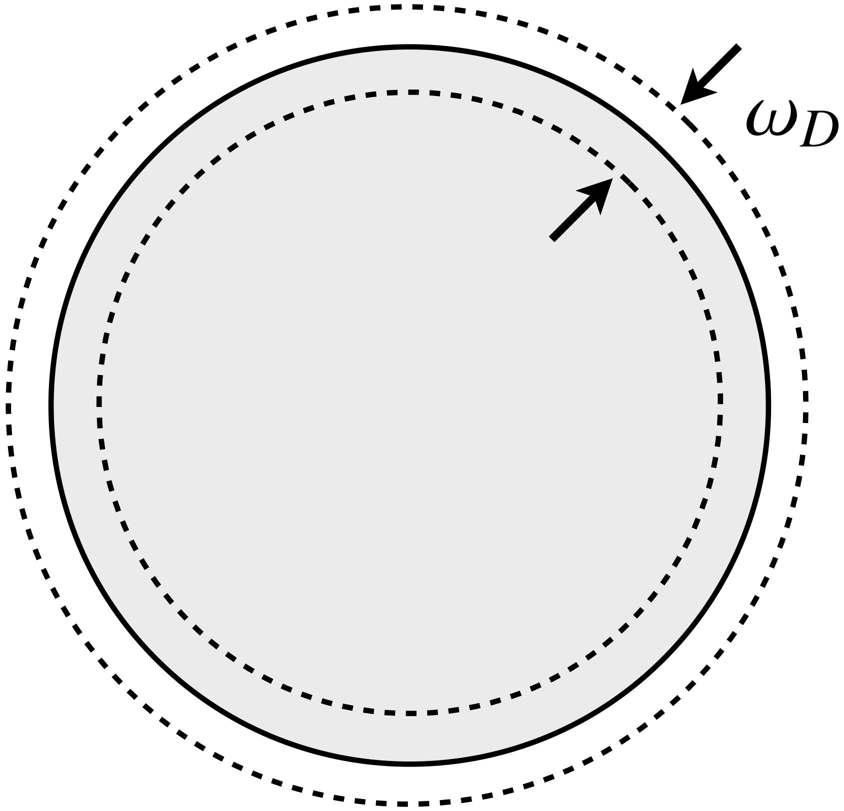
$\alpha_1 = c_1^{-1}[6c_3 d^{-1} + (1 - d^{-1})(2c_2 k_F^{-1} - c_1 k_F^{-2})]$, etc.

$$\boxed{\partial_t \boldsymbol{\sigma}_{\boldsymbol{k}} = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}} \quad \boldsymbol{b}_{\boldsymbol{k}} = \left(-\Delta', -\Delta'', \epsilon_{\boldsymbol{k}} + \frac{1}{2}e^2 A(t)^2 \cdot \frac{1}{d}\nabla^2_{\boldsymbol{k}}\epsilon_{\boldsymbol{k}}\right)}$$

$$\frac{1}{d}\nabla^2_{\boldsymbol{k}}\epsilon_{\boldsymbol{k}} = \alpha_0 + \alpha_1\epsilon_{\boldsymbol{k}} + \alpha_2\epsilon^2_{\boldsymbol{k}} + \cdots$$

Remarks:

- The α_n term has a contribution of order $(\omega_D/\epsilon_F)^n$.
- The first term (potential shift) can be gauged out.
- The α_1 term is the leading.
- For the ideal parabolic band $\epsilon_{\boldsymbol{k}} = \boldsymbol{k}^2/2m$, $\alpha_1 = 0$.



Anderson pseudospin precession

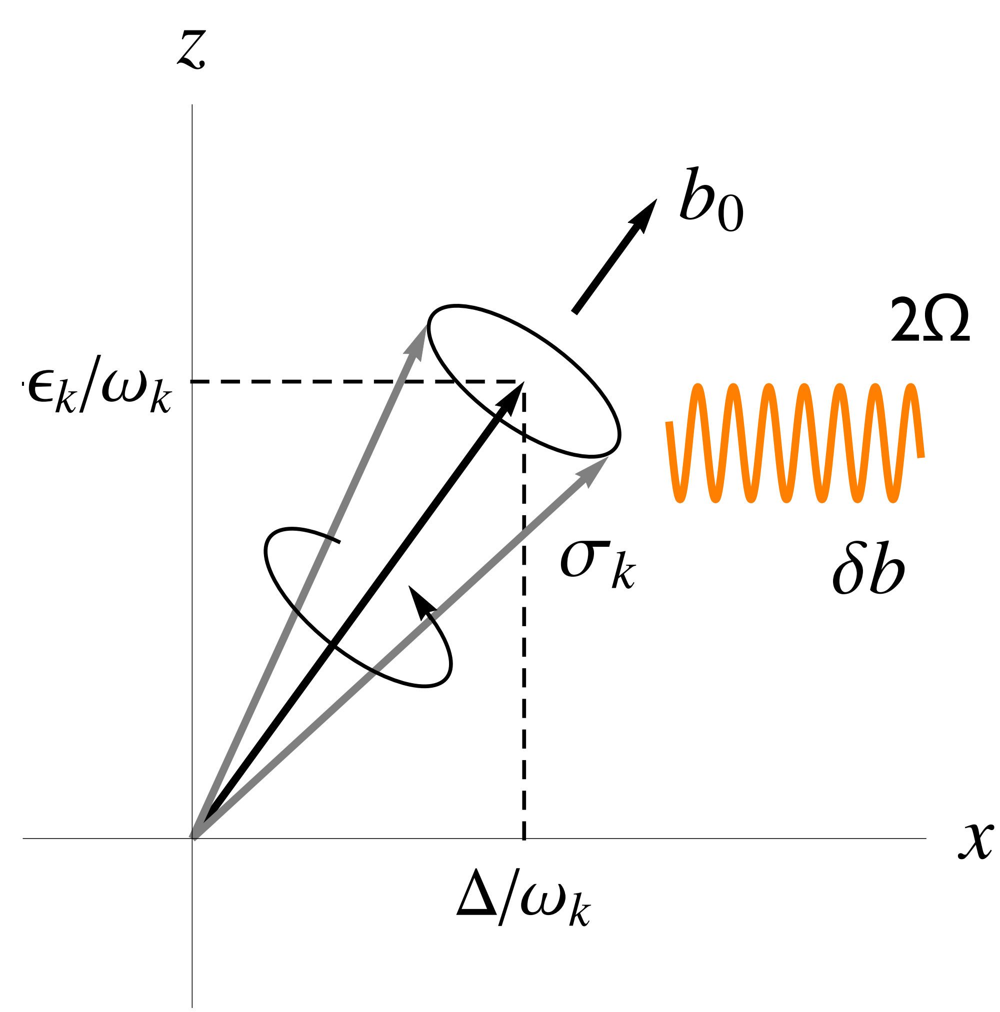
$$\boxed{\partial_t \boldsymbol{\sigma}_{\boldsymbol{k}} = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}} \quad \boldsymbol{b}_{\boldsymbol{k}} = \left(-\Delta', -\Delta'', \epsilon_{\boldsymbol{k}} + \frac{1}{2}\alpha_1 \epsilon_{\boldsymbol{k}}\, e^2 A(t)^2\right)}$$

- Let us first take Δ to be time independent (no self-consistency for Δ).

→ Usual spin resonance problem.

$$\delta\sigma_{\boldsymbol{k}}(t) \sim \frac{1}{(2\Omega)^2 - \omega_{\boldsymbol{k}}^2}\delta b_{\boldsymbol{k}}(t)$$

$$\omega_{\boldsymbol{k}} = 2\sqrt{\epsilon_{\boldsymbol{k}}^2 + \Delta^2}$$



$$\partial_t \boldsymbol{\sigma}_{\boldsymbol{k}} = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}} \qquad \boldsymbol{b}_{\boldsymbol{k}} = \left(-\Delta', -\Delta'', \epsilon_{\boldsymbol{k}} + \frac{1}{2}\alpha_1 \epsilon_{\boldsymbol{k}}\, e^2 A(t)^2\right)$$

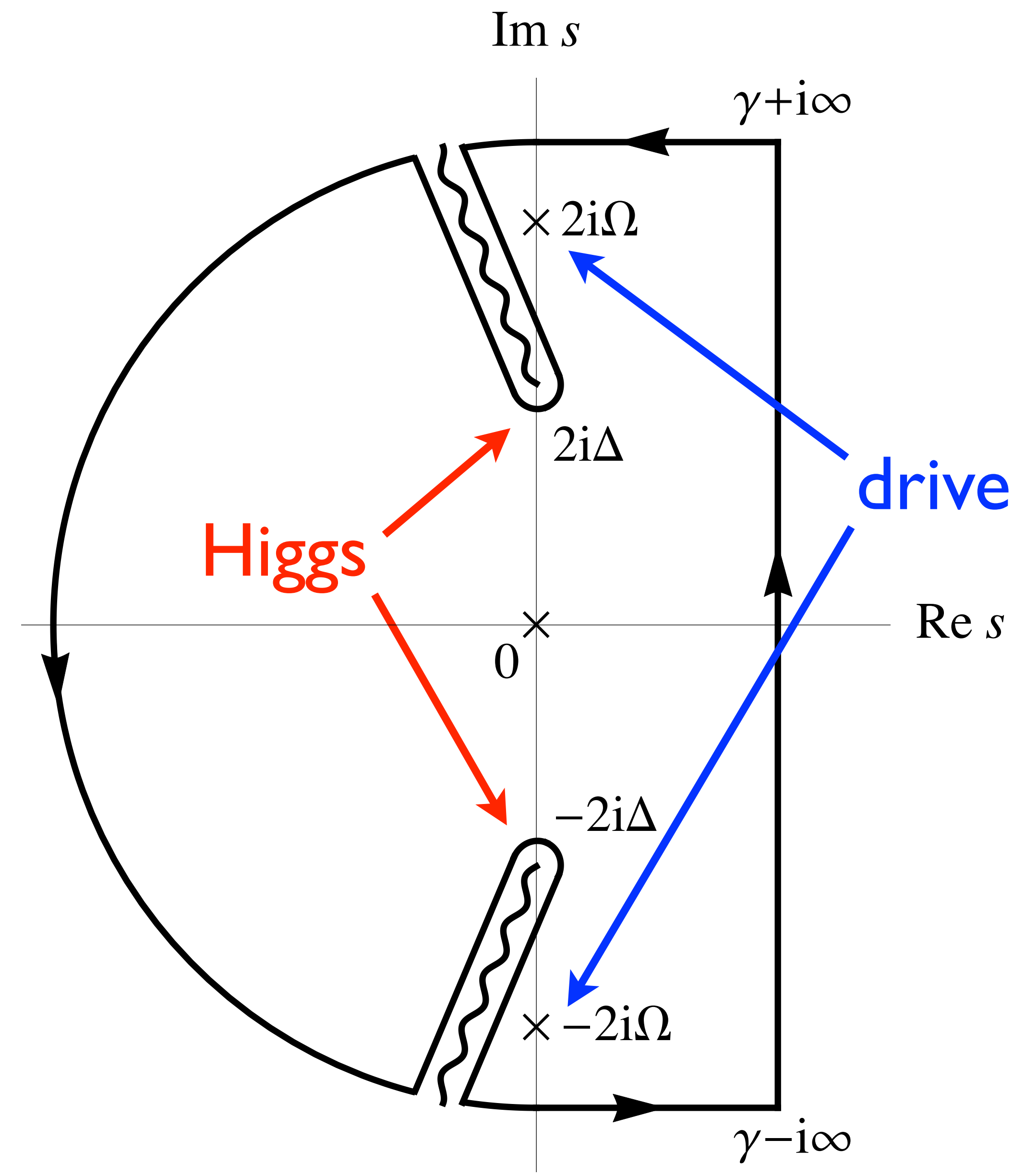
$$\Delta = U \sum_{\boldsymbol{k}} (\sigma^x_{\boldsymbol{k}} + i\sigma^y_{\boldsymbol{k}})$$ (self-consistency condition)

- Solve the equation of motion up to $O(A^2)$.
- Laplace transformation:

$$\frac{\delta\Delta(s)}{\alpha_1 e^2 A^2 \Delta} = \frac{\Omega^2}{s(s^2+4\Omega^2)}\left[1 - \frac{1}{\lambda(s^2+4\Delta^2)F(s)}\right]$$

$$F(s) = \frac{1}{s\sqrt{s^2+4\Delta^2}} \sinh^{-1}\left(\frac{s}{2\Delta}\right)$$

cf. Volkov, Kogan (1973)



$$
\partial_t \boldsymbol{\sigma}_{\boldsymbol{k}} = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}} \qquad \boldsymbol{b}_{\boldsymbol{k}} = \left(-\Delta', -\Delta'', \epsilon_{\boldsymbol{k}} + \frac{1}{2}\alpha_1 \epsilon_{\boldsymbol{k}}\, e^2 A(t)^2\right)
$$

$$
\Delta = U \sum_{\boldsymbol{k}} (\sigma^x_{\boldsymbol{k}} + i\sigma^y_{\boldsymbol{k}})
$$
(self-consistency condition)

Analytical solution:

For U quench, see Yuzbashyan, Dzero (2006); Barankov, Levitov (2006).

$$
\frac{\delta\Delta(t)}{\alpha_1 e^2 A^2 \Delta} \sim \frac{1}{4\lambda}\left[\frac{2}{\pi^{3/2}}\frac{\Omega^2}{\Omega^2-\Delta^2}\frac{1}{\sqrt{\Delta t}}\cos\left(2\Delta t + \frac{\pi}{4}\right) - 1\right]
$$

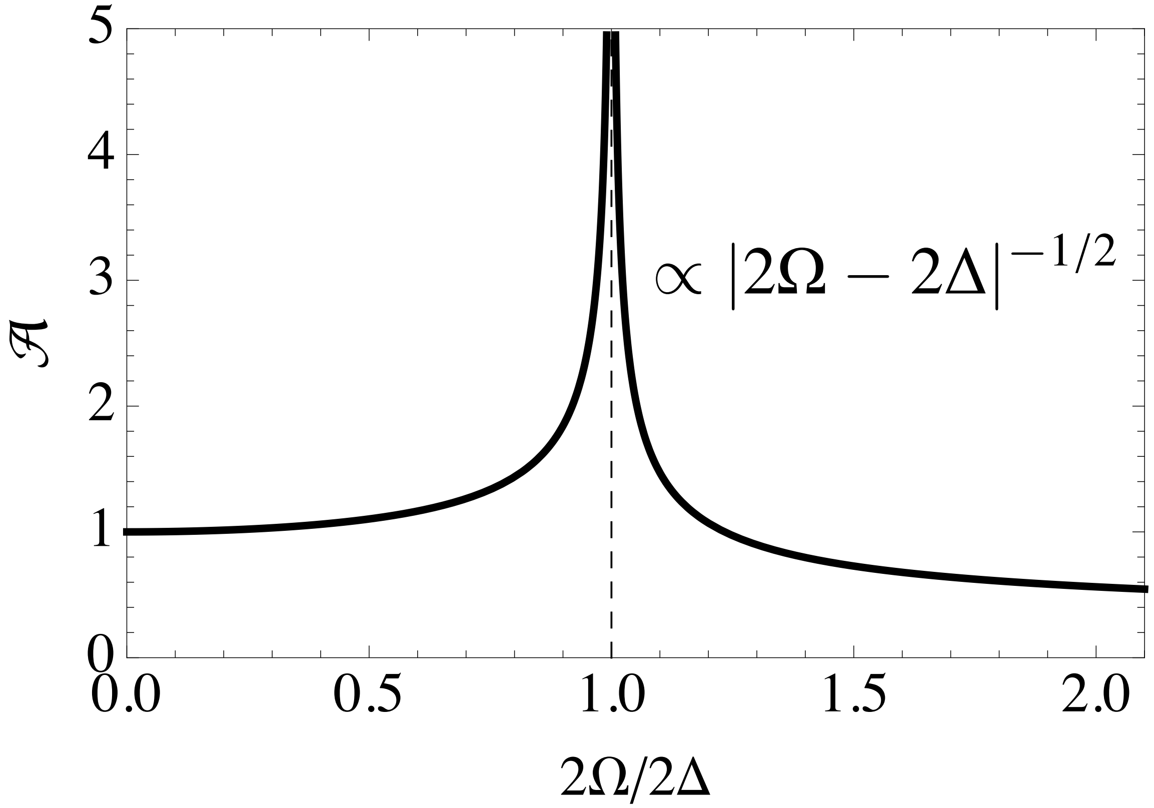
$$
+ \frac{1-\cos 2\Omega t}{4} + \boxed{\frac{1}{4\lambda} \times \begin{cases} \dfrac{\Omega}{\sqrt{\Delta^2-\Omega^2}} \dfrac{\cos 2\Omega t}{\sin^{-1}\left(\frac{\Omega}{\Delta}\right)} & \Omega < \Delta \\[2ex] \dfrac{\Omega}{\sqrt{\Omega^2-\Delta^2}} \dfrac{\cos(2\Omega t - \varphi)}{\sqrt{\left[\cosh^{-1}\left(\frac{\Omega}{\Delta}\right)\right]^2 + \left(\frac{\pi}{2}\right)^2}} & \Omega > \Delta \end{cases}}
$$

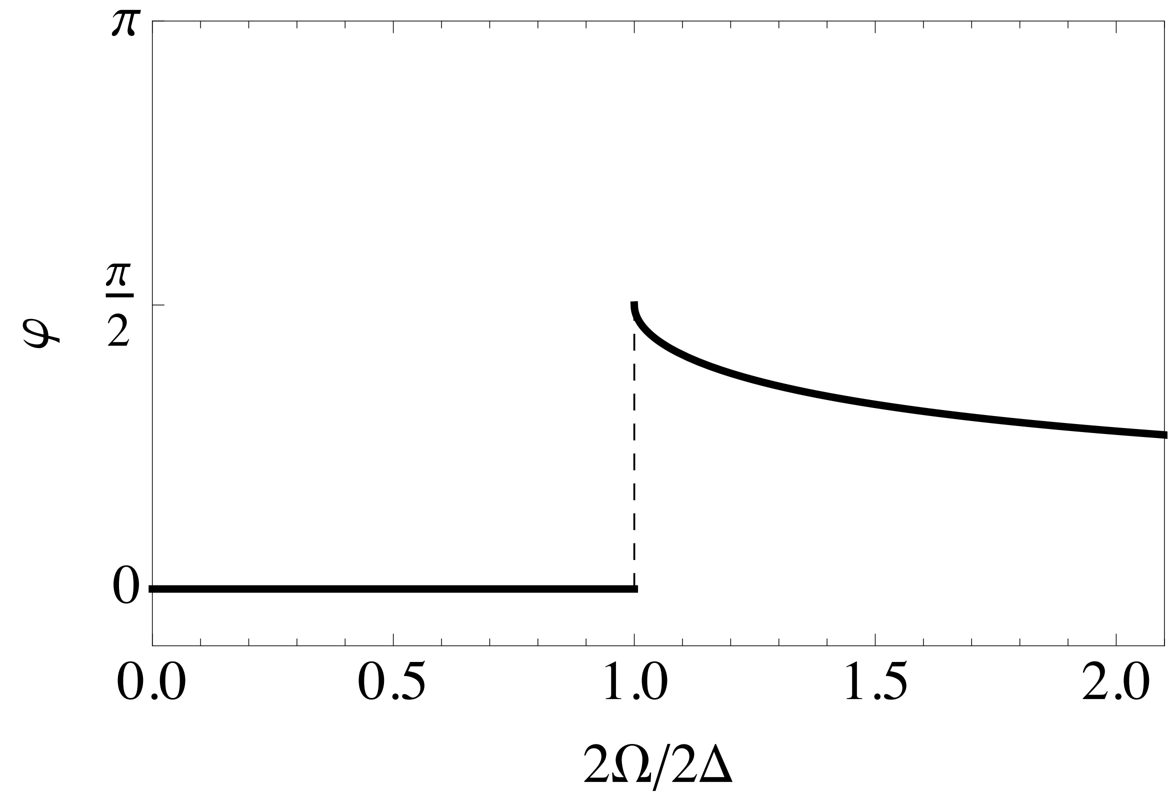
$$
\varphi = \tan^{-1}\left(\frac{\pi/2}{\cosh^{-1}\left(\frac{\Omega}{\Delta}\right)}\right)
$$

→ resonance with Higgs mode at 2Δ=2Ω

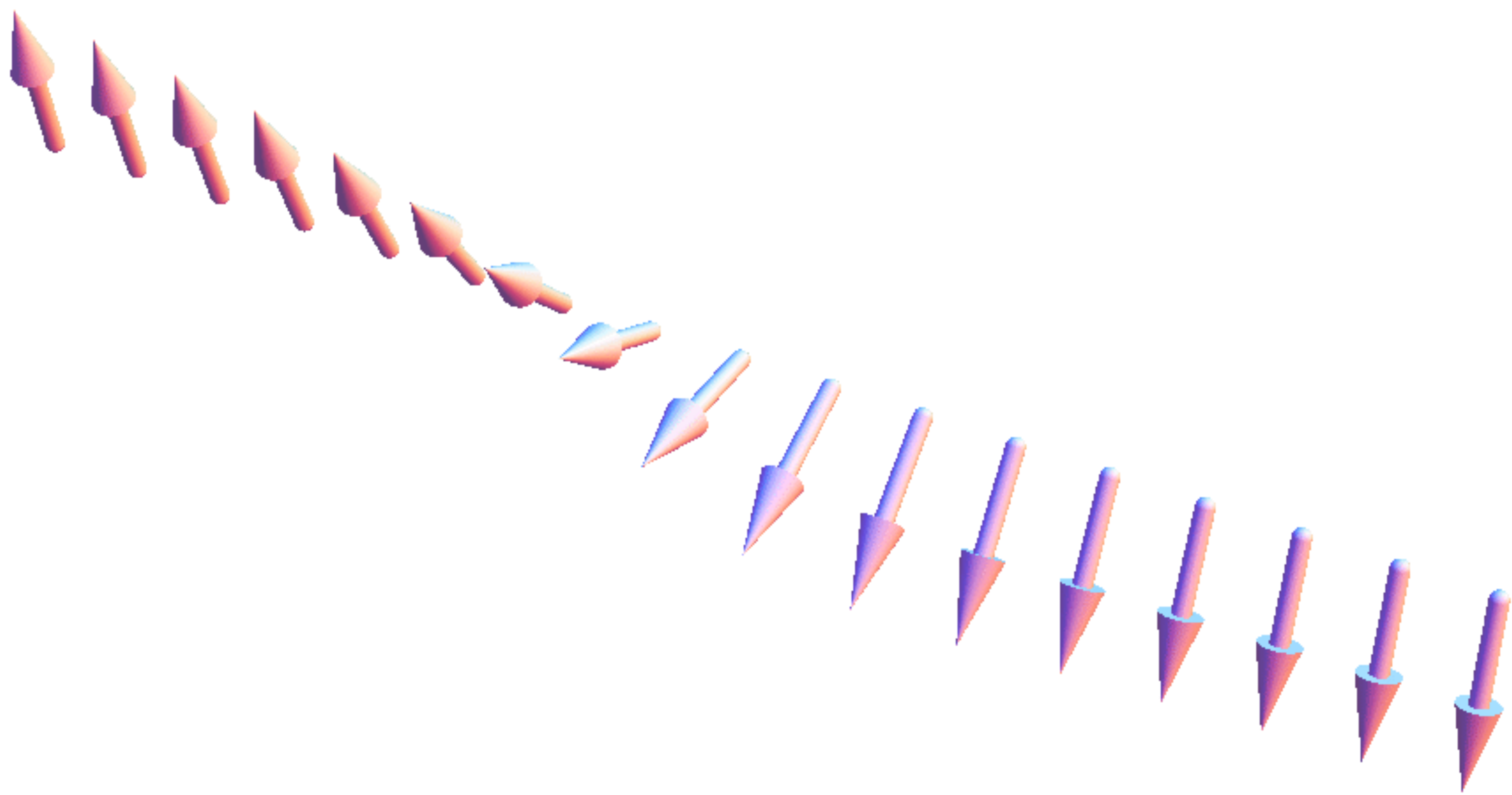
Anderson pseudospin resonance

- 2Ω oscillation of the order parameter: $\delta\Delta(t) \propto \mathscr{A}\cos(2\Omega t - \varphi)$
- $\mathscr{A}$ and φ are the universal function of $2\Omega/2\Delta$.



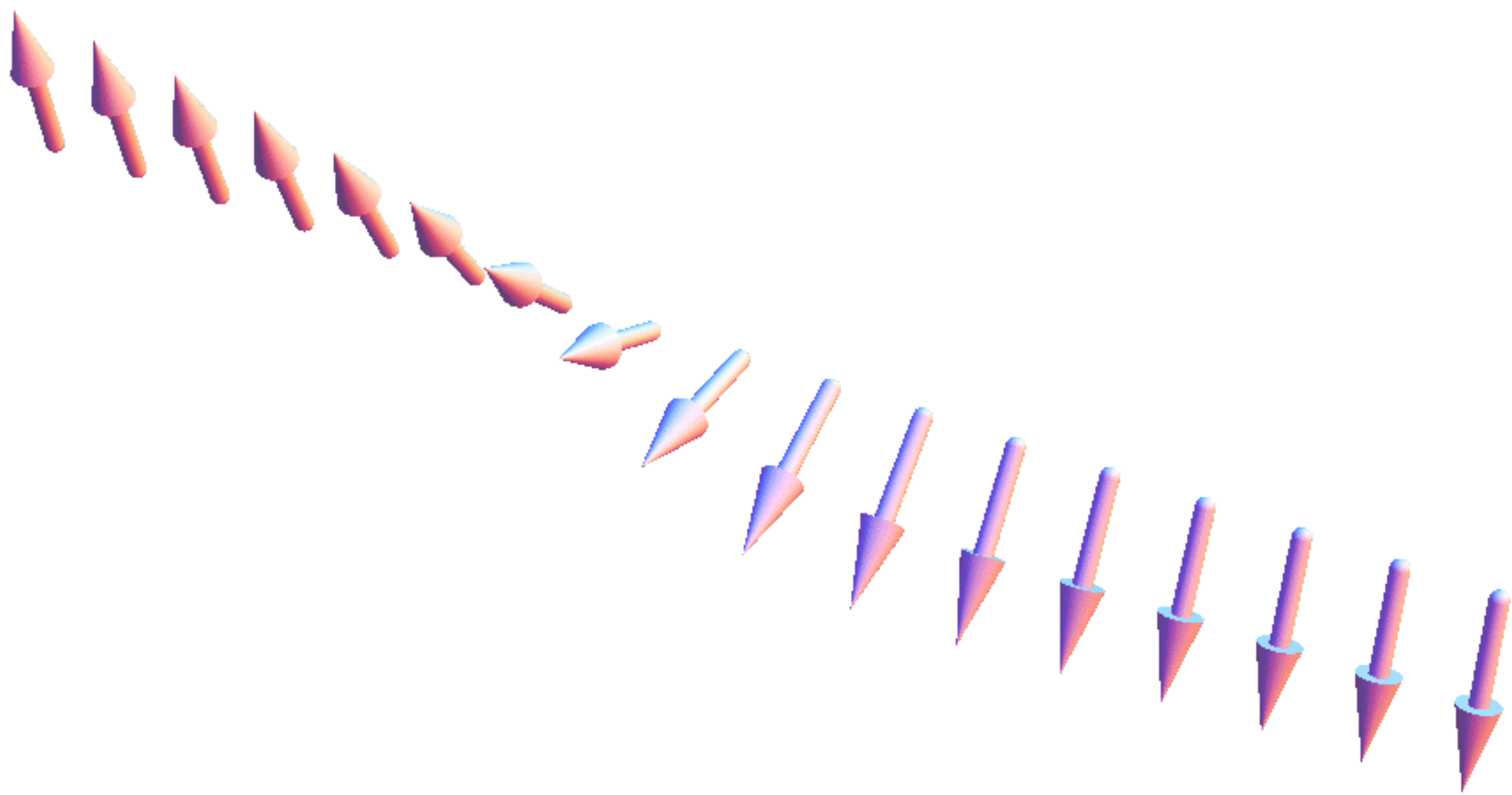


Off resonance



U=2, W=4, flat DOS
T=0, half filling
E=0.15, Ω=0.8Δ

On resonance



U=2, W=4, flat DOS
T=0, half filling
E=0.15, $\Omega=\Delta$

Third harmonic generation (THG)

- Current

$$\boldsymbol{j} = -e \sum_{\boldsymbol{k}\sigma} \boldsymbol{v}_{\boldsymbol{k}-e\boldsymbol{A}(t)} \langle c^{\dagger}_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} \rangle$$

$$= \sum_{\boldsymbol{k}} (-e\, \boldsymbol{v}_{\boldsymbol{k}-e\boldsymbol{A}(t)} + e\, \boldsymbol{v}_{\boldsymbol{k}+e\boldsymbol{A}(t)}) \sigma^z_{\boldsymbol{k}}$$

$$\approx 2e^2 \boldsymbol{A}(t) \sum_{\boldsymbol{k}} \alpha_1 \epsilon_{\boldsymbol{k}} \delta\sigma^z_{\boldsymbol{k}}$$

$$\epsilon_{\boldsymbol{k}} \delta\sigma^z_{\boldsymbol{k}} = \Delta \delta\sigma^x_{\boldsymbol{k}}$$

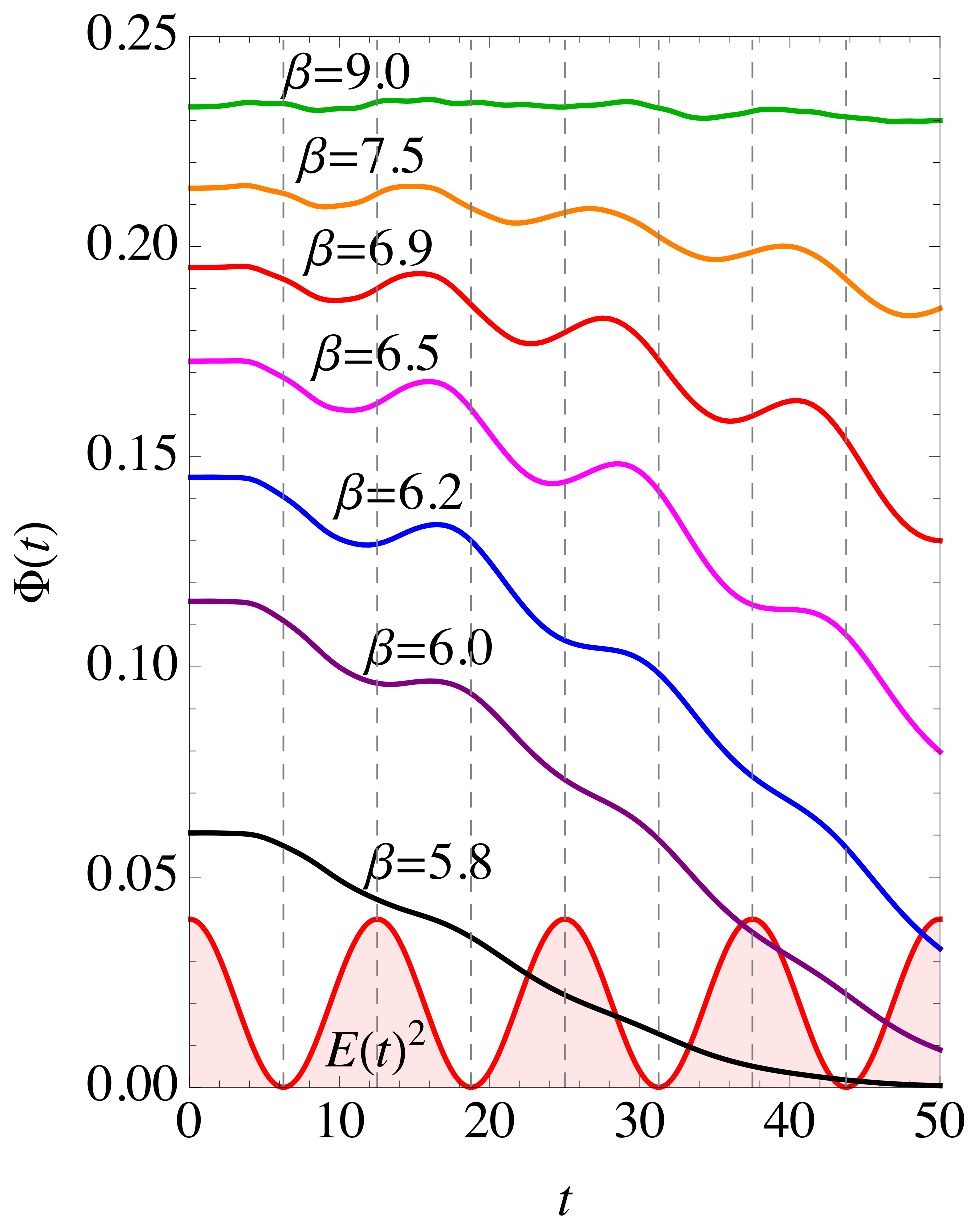
$$= 2e^2 \boldsymbol{A}(t) \sum_{\boldsymbol{k}} \alpha_1 \Delta \delta\sigma^x_{\boldsymbol{k}} = 2\alpha_1 e^2 \boldsymbol{A}(t) \Delta U^{-1} \delta\Delta(t)$$

- Divergent enhancement of THG!

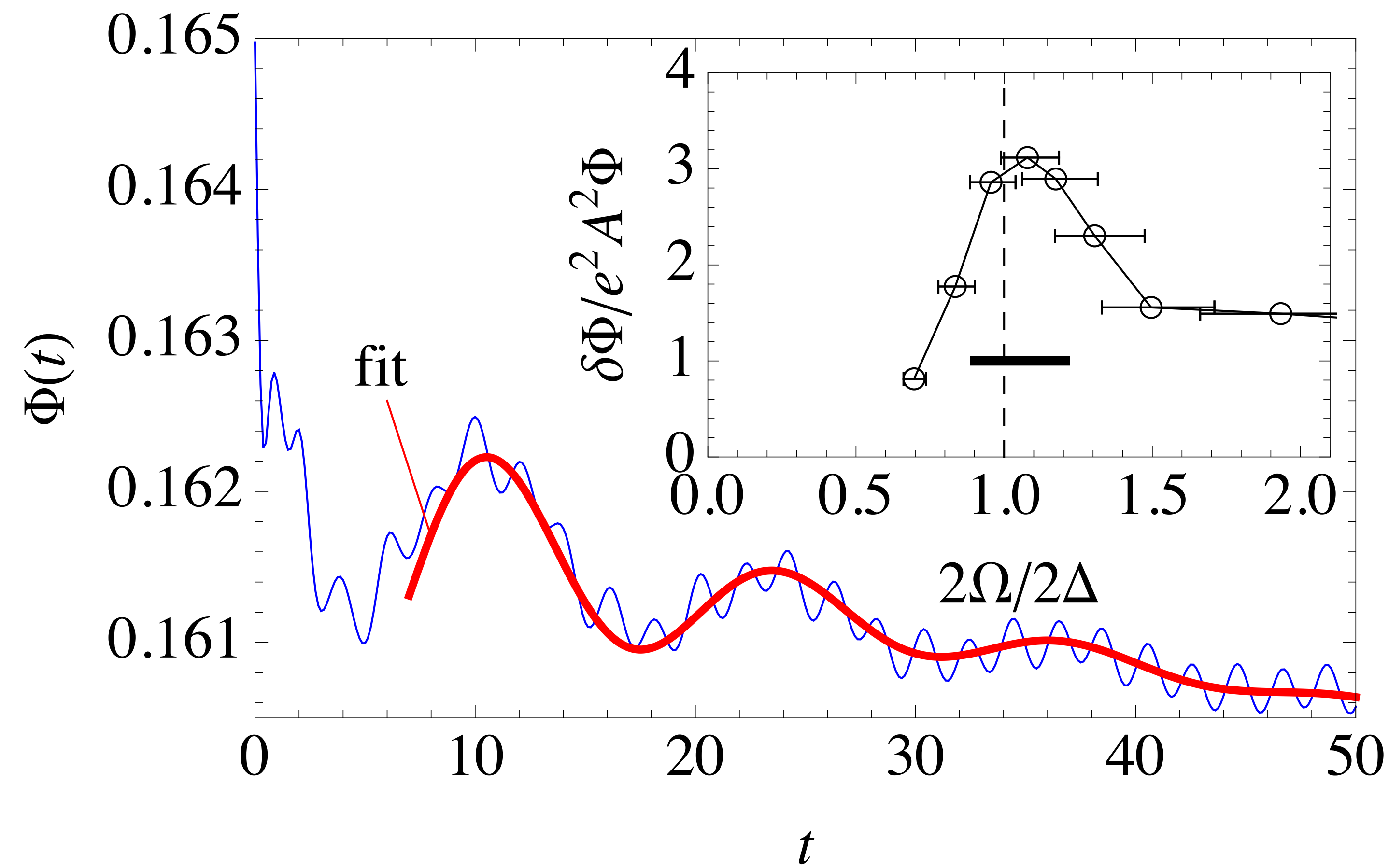
$$\boldsymbol{j}^{(3)}(t) \propto \delta\Delta(t) \boldsymbol{A}(t)$$

Coulomb scattering

- Nonequilibrium DMFT calculation (Aoki, Tsuji et al., Rev. Mod. Phys. in press) of the attractive Hubbard model.



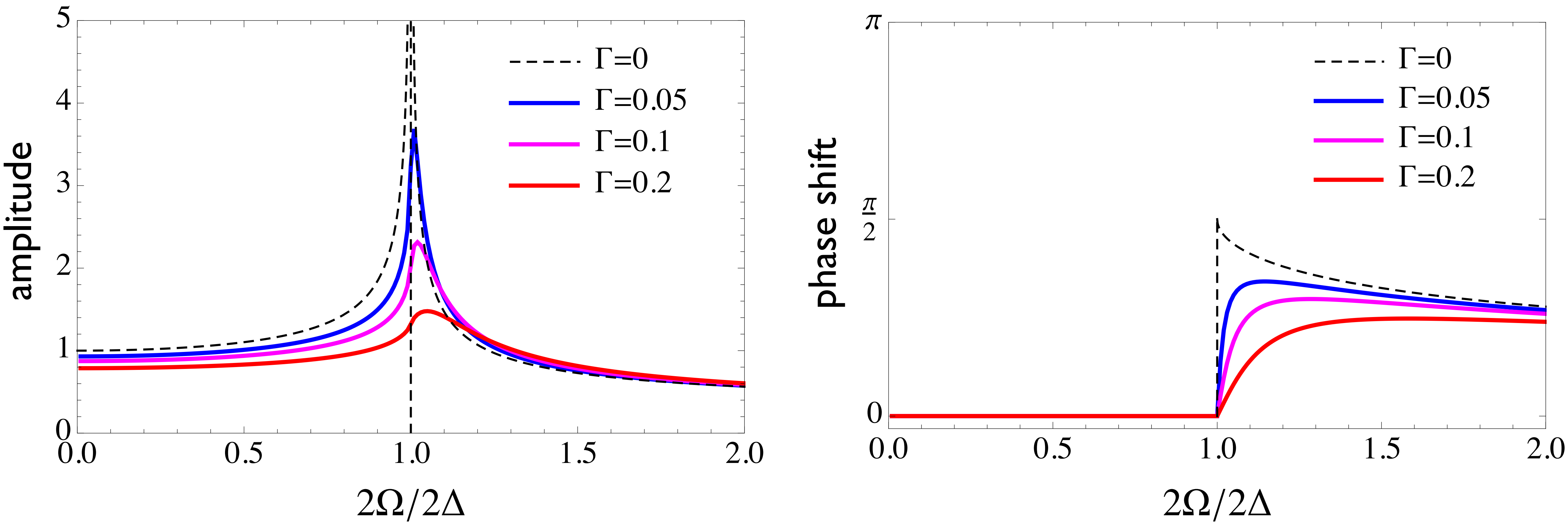
U=3.5, A=0.15, Ω=2π/25



U=3.5, Ω=2π/25

Impurity scattering

- Non-magnetic impurities → 2Δ does not change (Anderson's theorem)
- Calculation based on Abrikosov-Gor'kov theory (λ→0, T=0):



Future directions

- Can one further pursue an analogy between pseudospin and spin?
  — e.g. NMR, ESR, spintronics, ...

- Can one control xy pseudomagnetic fields in addition to z?

- Nonlinear pseudospin dynamics beyond perturbative regime (A^2, A^4, ...)

- What is the dominant pseudospin relaxation process?

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