Far from equilibrium phases of superconducting matter

or

Theory of nonequilibrium superconductivity in the nonadiabatic regime

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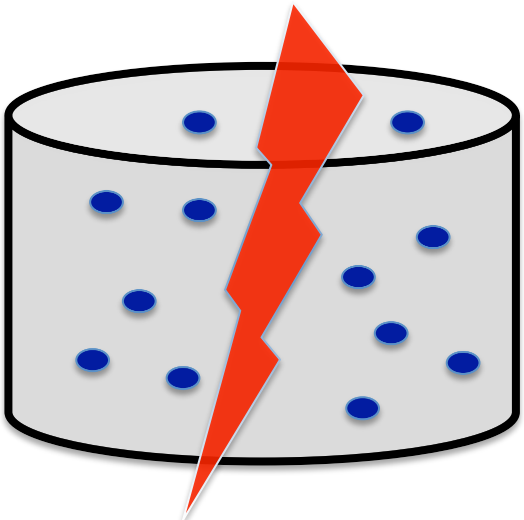
Coherent Many-Body Dynamics

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$



1. Interacting system initially in equilibrium
2. Strong perturbation pulse drives the system far from equilibrium. Easy

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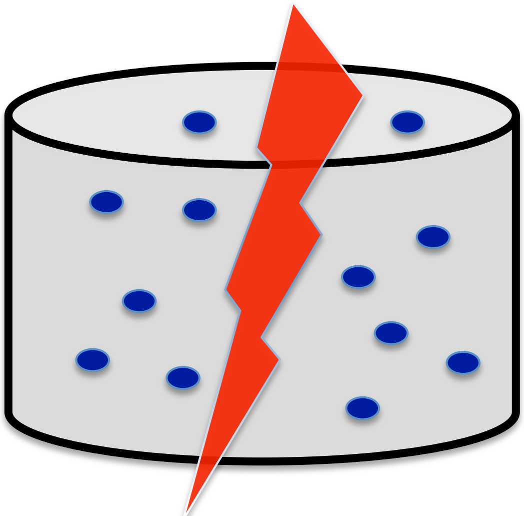


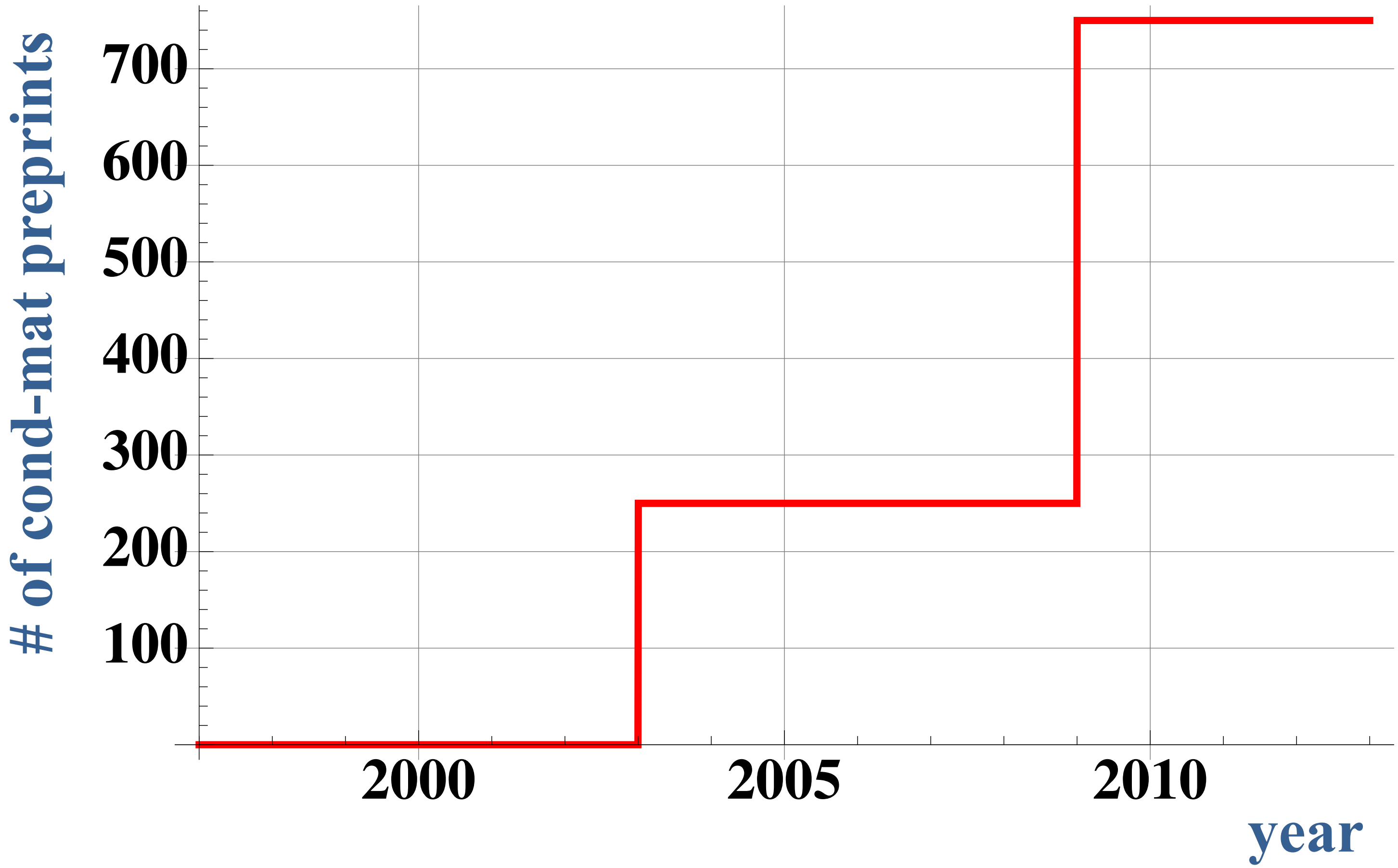
Quantum Quench

1. Interacting system initially in equilibrium
2. Strong perturbation pulse drives the system far from equilibrium. Easy
3. But not too strong. No dissipation, decoherence, controlled interactions. The system evolves coherently with desired Hamiltonian for long time. Very difficult

$$i\frac{\partial|\psi\rangle}{\partial t} = \left(\hat{H}_0 + \hat{H}_{\rm int}\right)|\psi\rangle$$

Coherent *Many-Body Dynamics*

Real experimental access only very recently



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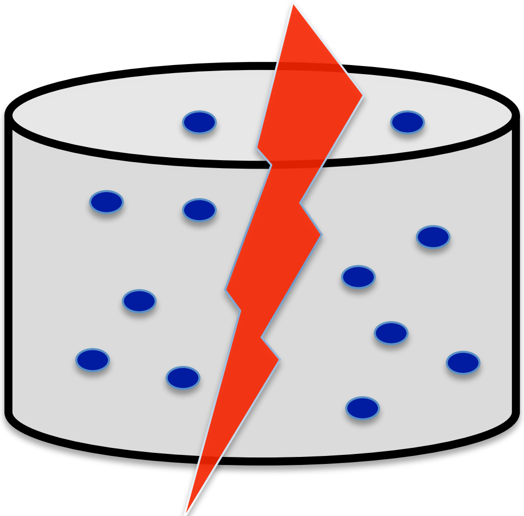

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$$i\frac{\partial|\psi\rangle}{\partial t} = \left(\hat{H}_0 + \hat{H}_{\text{int}}\right)|\psi\rangle$$

Q: What happens to the system in time? Where does it end up as a result of unitary evolution? Does it equilibrate?

$$|\psi(t \to \infty)\rangle = ? \qquad \langle \hat{O}(t \to \infty)\rangle = ?$$

Coherent Many-Body Dynamics: *Quantum Quench*

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$$|\psi(t \to \infty)\rangle = ? \quad \langle \hat{O}(t \to \infty) \rangle = ?$$

A: Depends on the system (on H)

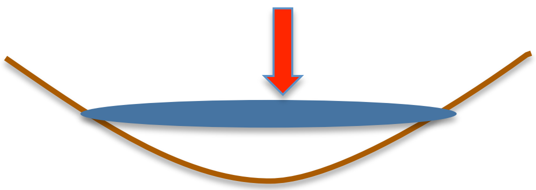
a. Equilibration (thermalization) with some effective T

$$\langle \hat{O}(t \to \infty) \rangle = \mathrm{Tr}\, \hat{O} e^{-\hat{H}_f / T_{\mathrm{eff}}}$$

b. No equilibration – steady state – nonequilibrium "phase" with properties quite distinct from equilibrium phases

$$\langle \hat{O}(t \to \infty) \rangle = ?$$

Free expansion of interacting 1D Bose gas out of a trap

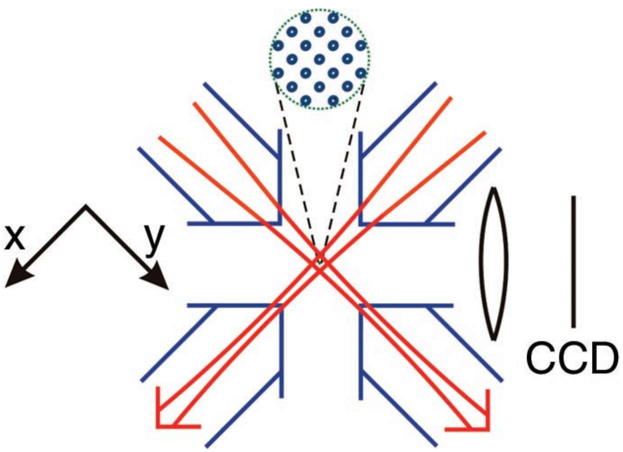


$$H_i = \sum_{\alpha}\left[\frac{p_\alpha^2}{2} + \frac{\omega^2 x_\alpha^2}{2}\right] + g\sum_{\alpha\beta}\delta(x_\alpha - x_\beta)$$

^{87}Rb atoms in a 1D harmonic trap

Kinoshita et. al. Science (2004)

$$|\psi_i\rangle = |\text{ground state of } H_i\rangle$$



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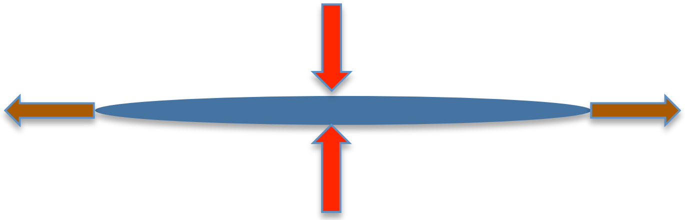

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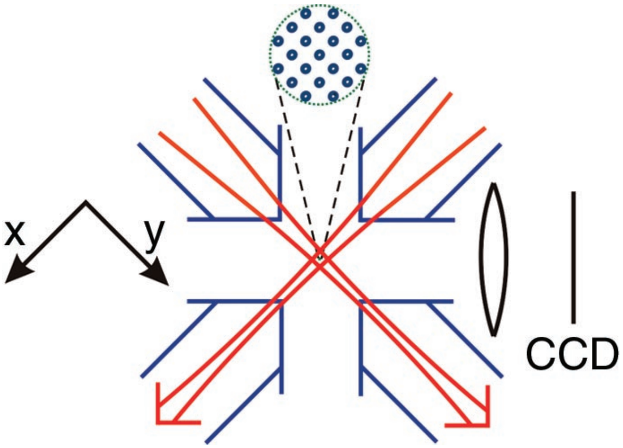

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At $t = 0$ the trap is removed and the gas expands in 1D governed by:

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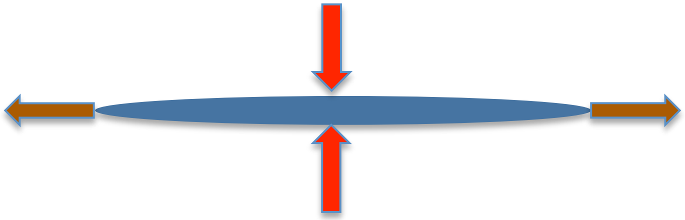

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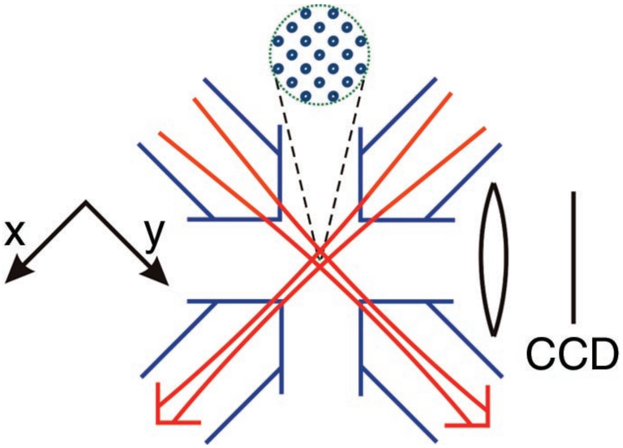

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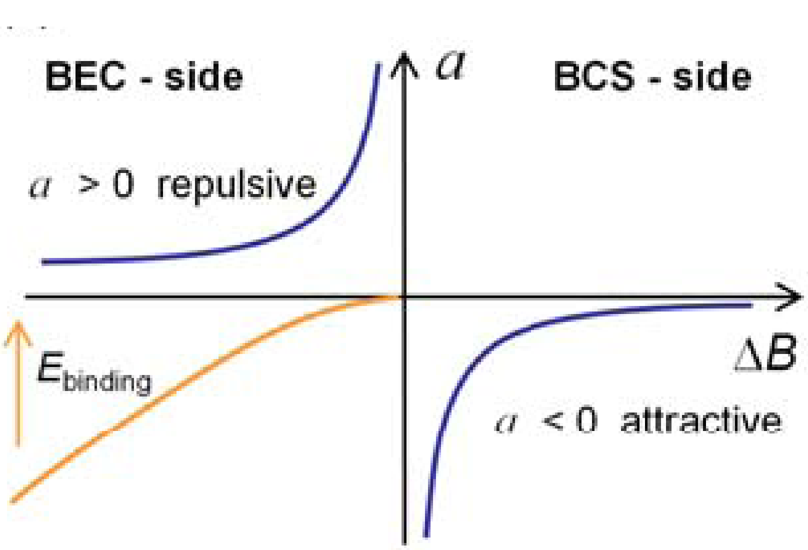
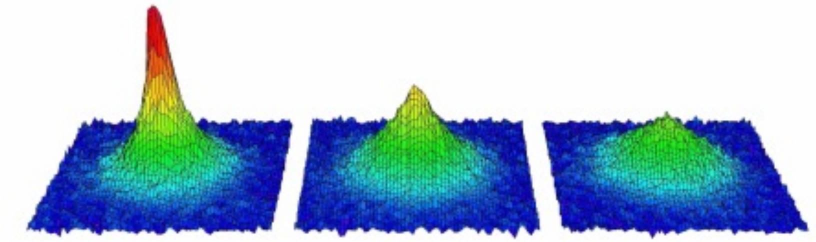
Q: What happens to the system in time? Where does it end up as a result of unitary evolution? Does it equilibrate?

A: Bosons fermionize, momentum distribution approaches Fermi-Dirac, the system does NOT equilibrate (thermalize).

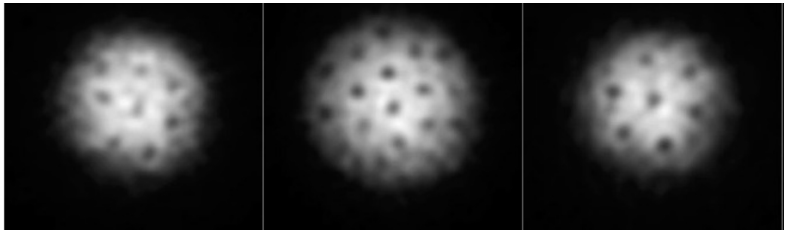
Atomic superconductivity – ultracold fermions (^{40}K, ^{6}Li)



Greiner, Regal &Jin (JILA, ^{40}K)



Optical images of condensate. Regal et. al. ’04



Vortex lattice. Zwierlein et. al. ‘05

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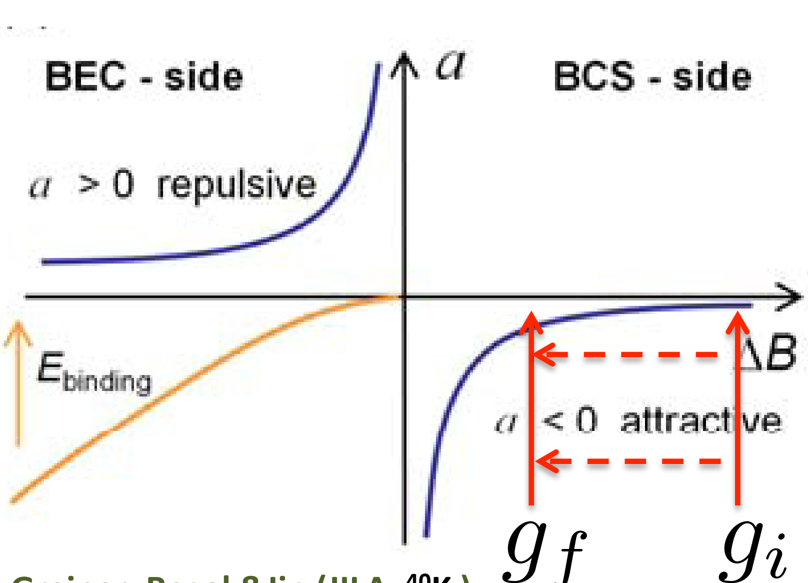

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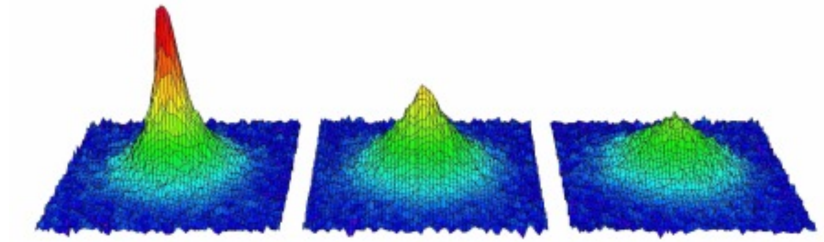

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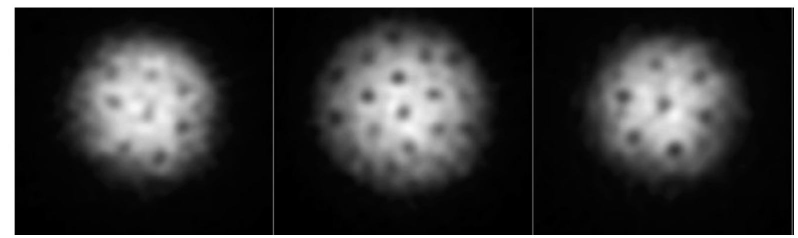


Vortex lattice. Zwierlein et. al. '05

$$\hat{H}_{\mathrm{BCS}} = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} - g \sum_{\mathbf{k},\mathbf{p}} \hat{c}^{\dagger}_{\mathbf{k}\uparrow} \hat{c}^{\dagger}_{-\mathbf{k}\downarrow} \hat{c}_{-\mathbf{p}\downarrow} \hat{c}_{\mathbf{p}\uparrow}$$

Interaction quench: sudden change of the BCS interaction:

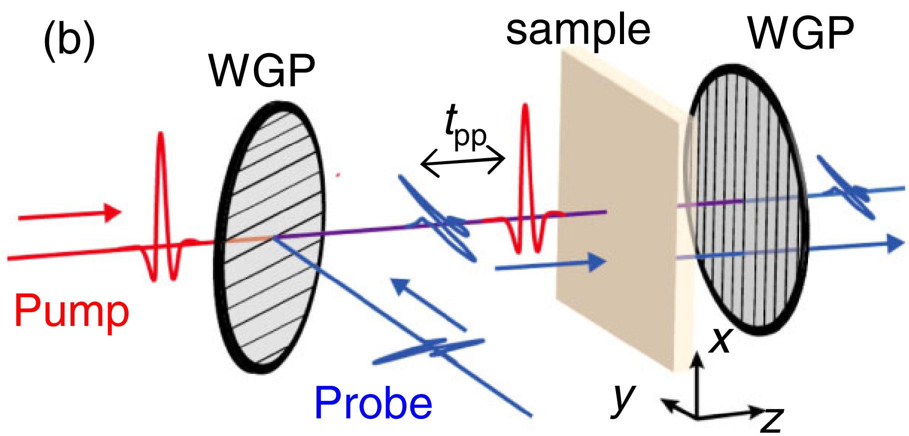
$$g_i \to g_f \ \text{ at } \ t = 0$$

$$|\psi(0)\rangle = |\text{gr. state for } g_i\rangle \qquad |\psi(t)\rangle = ? \quad \Delta(t) = ?$$

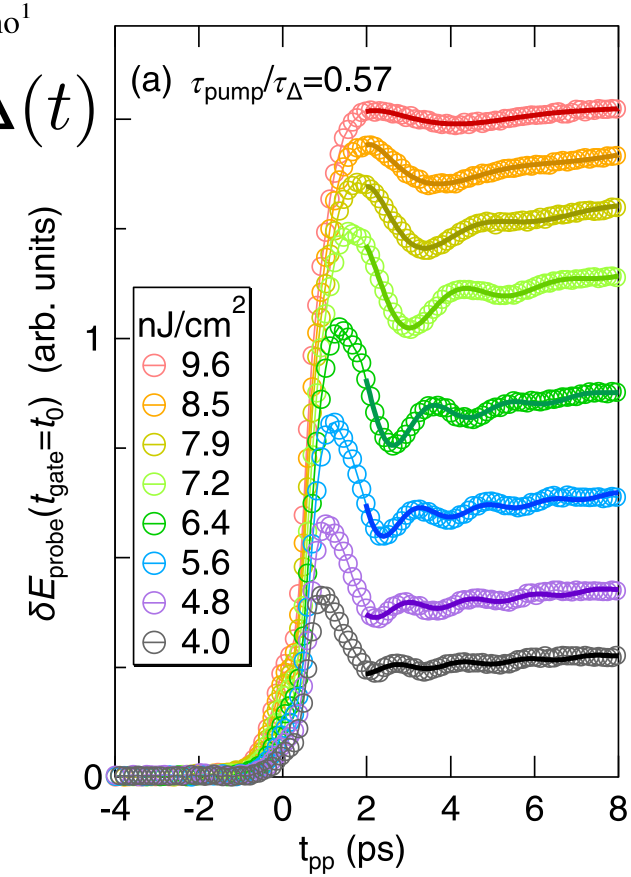
PRL **111,** 057002 (2013) PHYSICAL REVIEW LETTERS week ending 2 AUGUST 2013

Higgs Amplitude Mode in the BCS Superconductors $Nb_{1-x}Ti_xN$ Induced by Terahertz Pulse Excitation

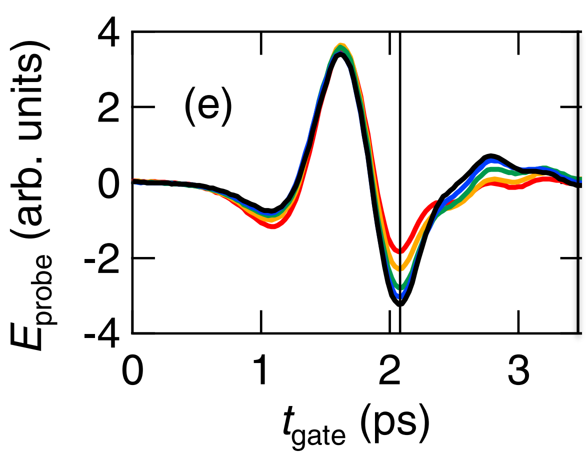
Ryusuke Matsunaga,[1] Yuki I. Hamada,[1] Kazumasa Makise,[2] Yoshinori Uzawa,[3]
Hirotaka Terai,[2] Zhen Wang,[2] and Ryo Shimano[1]



$\Delta(t)$

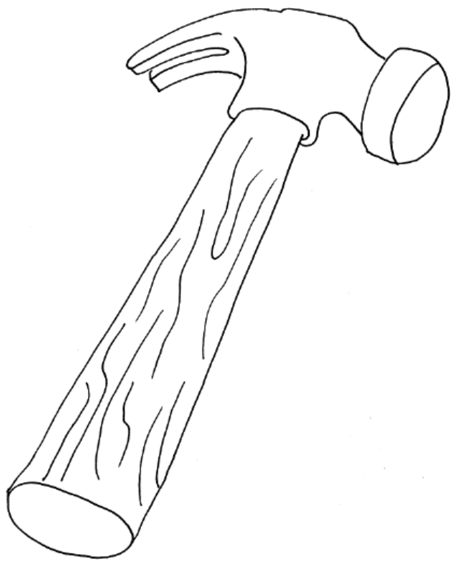


Difficulty: $\tau_\Delta = \hbar/\Delta_0 \approx 3\text{ps}$, Need: $t_{\text{pert}} \sim \tau_\Delta$



"With the recent development of THz technology, such an intense and monocyclelike THz pulse has become available."

Theory of nonequilibrium superconductivity in the nonadiabatic regime



Quantum Quench

Superconductor

Δ_0

?

$$i\frac{d|\psi\rangle}{dt} = \hat{H}_{\mathrm{BCS}}|\psi\rangle$$

t_{pert} – **perturbation time,**

$\tau_\Delta = \hbar/\Delta_0$ – **timescale of Hamiltonian evolution with** $\hat{H}_{\mathrm{BCS}}$**,**

τ_ε– **energy relaxation time, quasiparticle lifetime.**

Nonadiabatic regime: $t_{\mathrm{pert}} \leq \tau_\Delta \ll \tau_\varepsilon$

Long time dynamics of $\hat{H}_{\mathrm{BCS}}$ $(t \to \infty)$ **means** $\tau_\Delta \ll t \ll \tau_\varepsilon$

Traditional Nonequilibrium Superconductivity: Main Approaches

Typically metals are in slow perturbation regime: $t_{\rm pert} \sim \tau_\varepsilon \gg \tau_\Delta$

$$\tau_\varepsilon \approx \frac{\hbar \epsilon_F}{\Delta_0^2} \gg \frac{\hbar}{\Delta_0} = \tau_\Delta$$

Kinetic scheme applies: Boltzmann eqn + selfconsistency eqn for the order parameter

A.I. Larkin & Yu. N. Ovchinnikov, 1968
O. Betbeder-Matibed & P. Nozieres, 1969
A.G. Aronov et al, 1981
⋮

$$\boxed{\frac{\partial f}{\partial t} + \{f, h\} = I_{\rm coll}(f)}$$

Works only for sufficiently slow perturbations. In the nonadiabatic regime quasiparticles and their distribution f are not well formed yet.

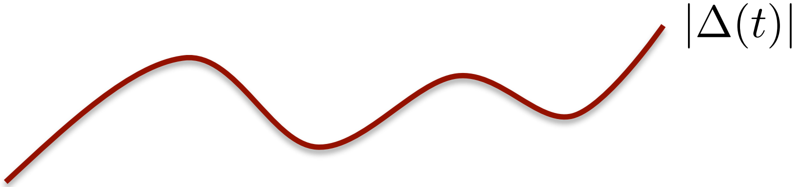
(see N.B. Kopnin, *Theory of Nonequilibrium Superconductivity*, 2001)

Traditional Nonequilibrium Superconductivity: Main Approaches

Time-Dependent Ginzburg-Landau eqn $\tau_\Delta \gg \tau_\varepsilon$

E. Abrahams & T. Tsuneto, 1966
A. Schmid, 1966
L.P. Gorkov & G.M. Eliashberg, 1968

$$\frac{\partial\Delta}{\partial t} = \frac{\delta F}{\delta\Delta^*}, \quad F(\Delta,\Delta^*) = \alpha|\Delta|^2 + \beta|\Delta|^4$$



Local equilibrium assumed.
Works only in very special cases such as gapless superconductivity

Requires: $\tau_\Delta \gg \tau_\varepsilon$ **(adiabatic regime)** – system quickly equilibrates with instantaneous $\Delta(t)$. No other dynamical degrees of freedom.

(see N.B. Kopnin, *Theory of Nonequilibrium Superconductivity*, 2001)

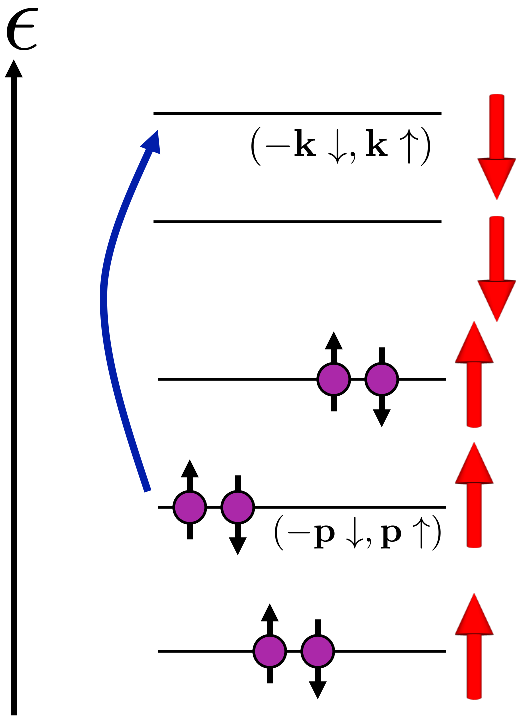
How to address nonadiabatic regime?

P. W. Anderson

Anderson pseudospins

$$\hat{H}_{\mathrm{BCS}} = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} - g \sum_{\mathbf{k},\mathbf{p}} \hat{c}^{\dagger}_{\mathbf{k}\uparrow} \hat{c}^{\dagger}_{-\mathbf{k}\downarrow} \hat{c}_{-\mathbf{p}\downarrow} \hat{c}_{\mathbf{p}\uparrow}$$

$$\hat{s}^z_{\mathbf{k}} = \frac{\hat{n}_{\mathbf{k}} - 1}{2} \qquad \hat{s}^-_{\mathbf{k}} = \hat{c}_{-\mathbf{k}\downarrow}\hat{c}_{\mathbf{k}\uparrow}, \qquad \hat{s}^+_{\mathbf{k}} = \hat{c}^{\dagger}_{\mathbf{k}\uparrow}\hat{c}^{\dagger}_{-\mathbf{k}\downarrow}$$



$$H_{\mathrm{BCS}} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} s^+_{\mathbf{k}} s^-_{\mathbf{p}}$$

P. W. Anderson, Phys. Rev. 112, 1900 (1958)

How to address nonadiabatic regime?

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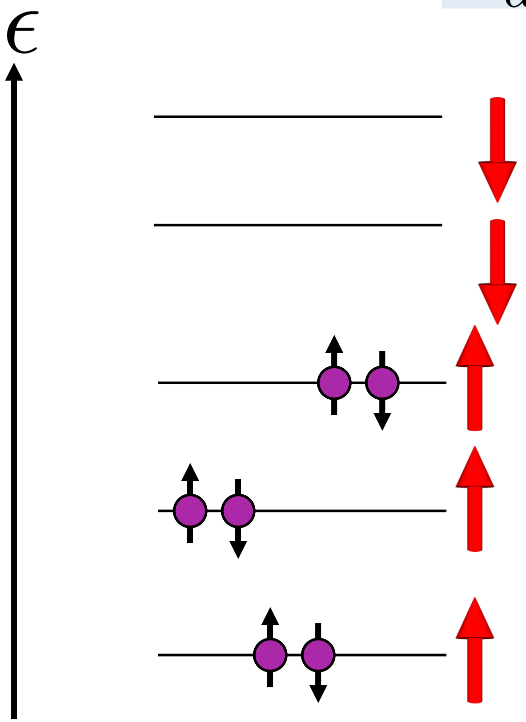
$$H_{\rm BCS} = \sum_{\bf k} 2\epsilon_{\bf k} s^z_{\bf k} - g \sum_{\bf k,p} s^+_{\bf k} s^-_{\bf p}$$

Bloch eqs.

$$i\frac{d|\psi\rangle}{dt} = \hat{H}_{\rm BCS}|\psi\rangle \Rightarrow \dot{\vec{s}}_{\bf k} = (-2\vec{\Delta} + 2\epsilon_{\bf k}\hat{z}) \times \vec{s}_{\bf k}$$



BCS order parameter:

$$\vec{\Delta}(t) = g\sum_{\bf k}(s^x_{\bf k}\hat{x} + s^y_{\bf k}\hat{y})$$

complex representation: $\Delta = \Delta_x - i\Delta_y = |\Delta|e^{-i\phi}$

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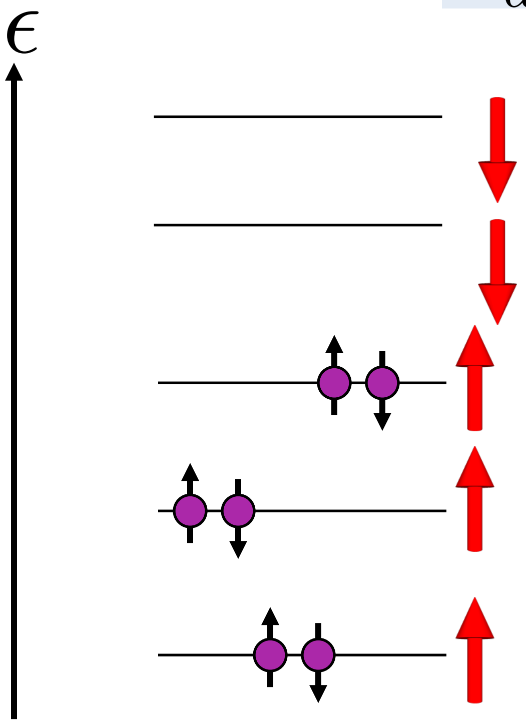

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complex representation: $\Delta = \Delta_x - i\Delta_y = |\Delta|e^{-i\phi}$

Moreover, mean field exact due to the infinite range of interactions. **Can replace quantum spins with classical spins (vectors)!**

P. W. Anderson, Phys. Rev. 112, 1900 (1958)

Gapless phase mode (linear analysis – near equilibrium)

P. W. Anderson

Anderson pseudospins

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Hamiltonian is invariant w.r.t. rotations of all spins around z-axis $\phi \to \phi + \alpha$

⇩

Goldstone-Nambu mode - gapless phase mode

⇩

Acquires mass through coupling to EM gauge field

BCS order parameter: $$\vec{\Delta}(t) = g\sum_{\mathbf{k}} (s^x_{\mathbf{k}}\hat{x} + s^y_{\mathbf{k}}\hat{y})$$

complex representation: $$\Delta = \Delta_x - i\Delta_y = |\Delta|e^{-i\phi}$$

Goldstone + photon = plasmon

Anderson-Higgs mechanism

P. W. Anderson, **Phys. Rev. 130, 439 (1963)**

P. W. Anderson, **Phys. Rev. 112, 1900 (1958)**

Amplitude dynamics in nonadiabatic regime
(linear analysis – near equilibrium)

Collisionless relaxation of the energy gap in superconductors

A. F. Volkov and Sh. M. Kogan

Institute of Radio and Electronics, USSR Academy of Sciences
(Submitted June 15, 1973)
Zh. Eksp. Teor. Fiz. **65**, 2038–2046 (November 1973)

See also:
Galaiko, JETP 34, 203 (1972)
Ivlev, JETP Lett. 15, 313 (1972)
Galperin, Kozub, Spivak, JETP 54, 1126 (1981)
Littlewood, Varma, Phys. Rev. B 26 4883 (1982)

Nonadiabatic regime: $t_{\mathrm{pert}} \leq \tau_\Delta \ll \tau_\varepsilon$

$$\underbrace{\dot{\vec{s}}_{\mathbf{k}} = (-2\vec{\Delta} + 2\epsilon_{\mathbf{k}}\hat{z}) \times \vec{s}_{\mathbf{k}}}_{} + \vec{I}_{\mathrm{coll}}(\mathbf{k})$$

$$H_{\mathrm{BCS}} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} - g\sum_{\mathbf{k},\mathbf{p}} s^+_{\mathbf{k}} s^-_{\mathbf{p}}$$

$$\vec{I}_{\mathrm{coll}}(\mathbf{k}) \sim \delta\vec{s}_{\mathbf{k}}/\tau_\varepsilon, \qquad \dot{\vec{s}}_{\mathbf{k}} \sim \delta\vec{s}_{\mathbf{k}}/\tau_\Delta$$

$\vec{s}_{\mathbf{k}}$ – **classical spins (vectors)**, $|\vec{s}_{\mathbf{k}}| = 1$

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$\vec{s}_{\mathbf{k}}$ – classical spins (vectors), $|\vec{s}_{\mathbf{k}}| = 1$

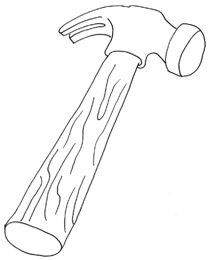
Small deviations from equilibrium – linearize Bloch eqs. around ground state spin configuration and solve ⇨

$$|\Delta(t)| = \Delta_0 + a\frac{\cos(2\Delta_0 t + \alpha)}{\sqrt{\Delta_0 t}}$$

Δ_0 – ground state gap

$\tau_\Delta \ll t \ll \tau_\varepsilon$

What about far from equilibrium superconductivity in the nonadiabatic regime??



Superconductor

Δ_0

?

$$H_{\rm BCS} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} s^+_{\mathbf{k}} s^-_{\mathbf{p}}$$

Quantum Quench

Need to solve full (infinitely) many classical spin evolution :

$$\dot{\vec{s}}_{\mathbf{k}} = (-2\vec{\Delta} + 2\epsilon_{\mathbf{k}}\hat{z}) \times \vec{s}_{\mathbf{k}} \qquad \vec{\Delta}(t) = g \sum_{\mathbf{k}} (s^x_{\mathbf{k}}\hat{x} + s^y_{\mathbf{k}}\hat{y})$$

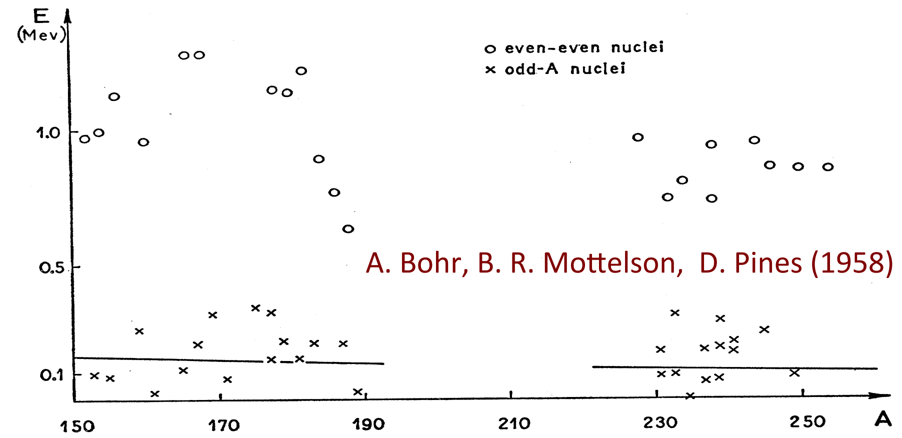
Initial state, $\vec{s}_{\mathbf{k}}(t=0) = \ldots,$ determined by quench (perturbation) details

Nonlinear, many-body, far from equilibrium – normally would be intractable analytically

But it turns out that H_{BCS} is integrable!

Nuclear superconductivity
(s-wave)

Richardson & Sherman (1964)
"Exact eigenstates of the pairing-force Hamiltonian"

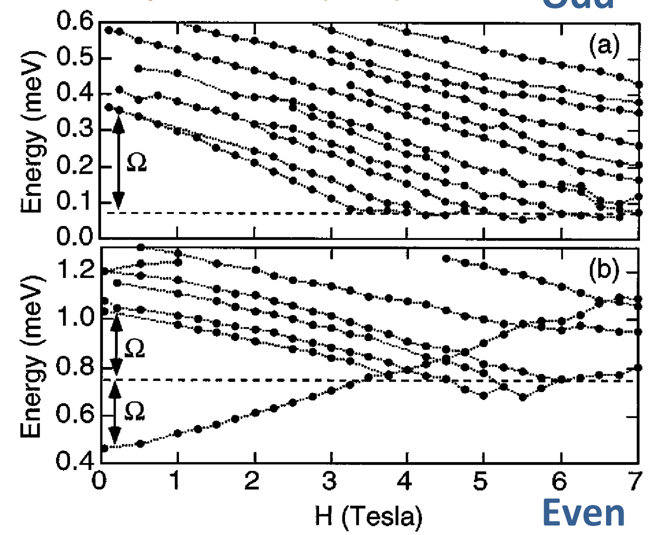


A. Bohr, B. R. Mottelson, D. Pines (1958)

Applications to superconducting qubits
(finite size corrections to the BCS theory):

Von Delft (2001), Dukelsky and Sierra (1999), Schechter et. al. (2001), …

Black, Ralph, Tinkham (1996)



Integrals of motion for H_{BCS} – Gaudin magnets

$$H_{\mathbf{k}} = \sum_{\mathbf{k}} \frac{\vec{s}_{\mathbf{k}} \cdot \vec{s}_{\mathbf{p}}}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{p}}} - \frac{s^z_{\mathbf{k}}}{g}, \quad [H_{\mathbf{k}}, H_{\mathbf{q}}] = 0, \quad H_{\mathrm{BCS}} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} H_{\mathbf{k}}$$

Gaudin (1972 – 1976) *"Diagonalisation d'une classe d'hamiltoniens de spin"*

Advanced approach to BCS integrability
(secret life of BCS integrals):

Sklyanin (1987) *"Separation of variables in the Gaudin model"*
Kuznetsov (1992) *"Quadrics on real Riemannian spaces … connection with Gaudin magnet"*
Takasaki (1998) *"Gaudin Model, KZ Equation, and Isomonodromic Problem on Torus"*
Frenkel (2004) *"Gaudin model and opers"*
⋮

Exact solution for the BCS dynamics:

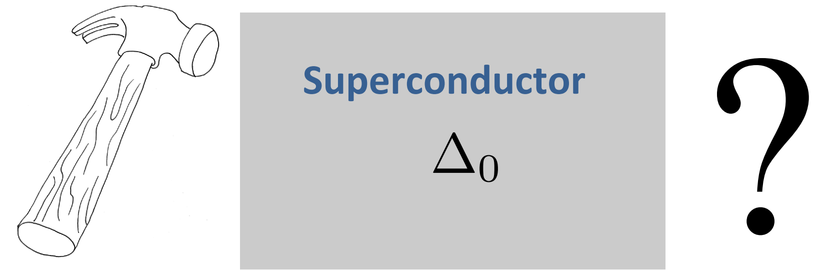
E.Y., Altshuler, Kuznetsov, Enolskii (2005)
E.Y., Altshuler, Tsyplyatyev (2006)
E.Y., Dzero, Gurarie, Foster, unpublished

s-wave

Foster, Dzero, Gurarie, E.Y. (2013)

p-wave

Q: What happens to the system in time? Where does it end up as a result of unitary evolution? Does it equilibrate?



A:

I. No equilibration (thermalization) at all

II. System goes into an <u>exotic steady state with properties unseen in equilibrium</u> (new "phase" of superfluid matter).

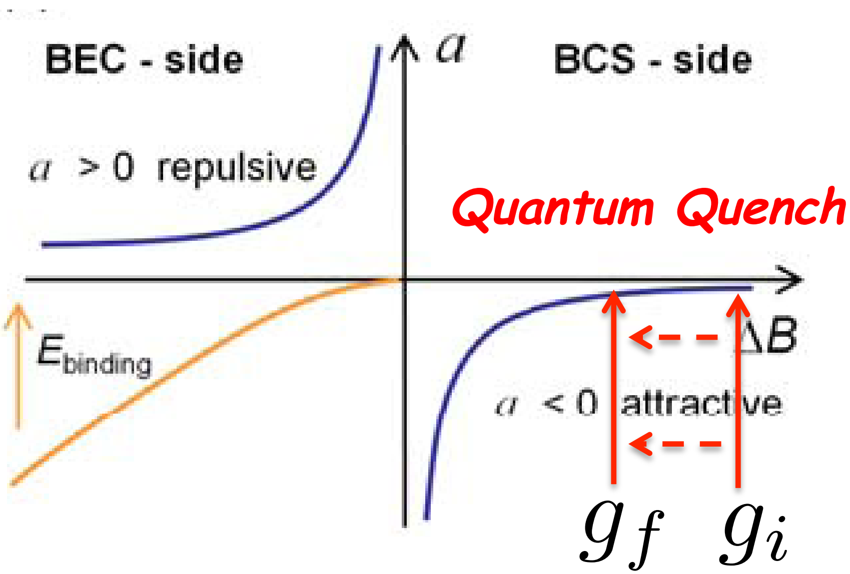
III. <u>Three</u> main possible <u>far from equilibrium "phases"</u> (as opposed to only one in equilibrium at $T=0$)

IV. Which "phase" is realized depends on the strength of the quench

V. Not at all specific to integrable models. More general mechanism at work (spin reduction, synchronization)

$$H=\sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} - \sum_{\mathbf{k},\mathbf{p}} g(\mathbf{k},\mathbf{p}) s^+_{\mathbf{k}} s^-_{\mathbf{p}}, \quad g(\mathbf{k},\mathbf{p}) - \text{any long range interaction}$$

Example: far from equilibrium atomic superconductivity



$$t_{\text{pert}} \sim 10\,\mu\text{s}$$
$$\tau_\Delta \approx 10\,\text{ms}$$
$$\tau_\varepsilon \approx 100\,\text{ms}$$

Naturally in the nonadiabatic regime:

$$t_{\text{pert}} \leq \tau_\Delta \ll \tau_\varepsilon$$

Barankov, Levitov, Spivak, 2004

$$H_{\text{BCS}} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} s^+_{\mathbf{k}} s^-_{\mathbf{p}}$$

Interaction quench away from resonance: sudden change of the BCS interaction: $g_i \to g_f$

$$\dot{\vec{s}}_{\mathbf{k}} = (-2\vec{\Delta} + 2\epsilon_{\mathbf{k}}\hat{z}) \times \vec{s}_{\mathbf{k}}$$

$$\vec{\Delta}(t) = g_f \sum_{\mathbf{k}} (s^x_{\mathbf{k}}\hat{x} + s^y_{\mathbf{k}}\hat{y})$$

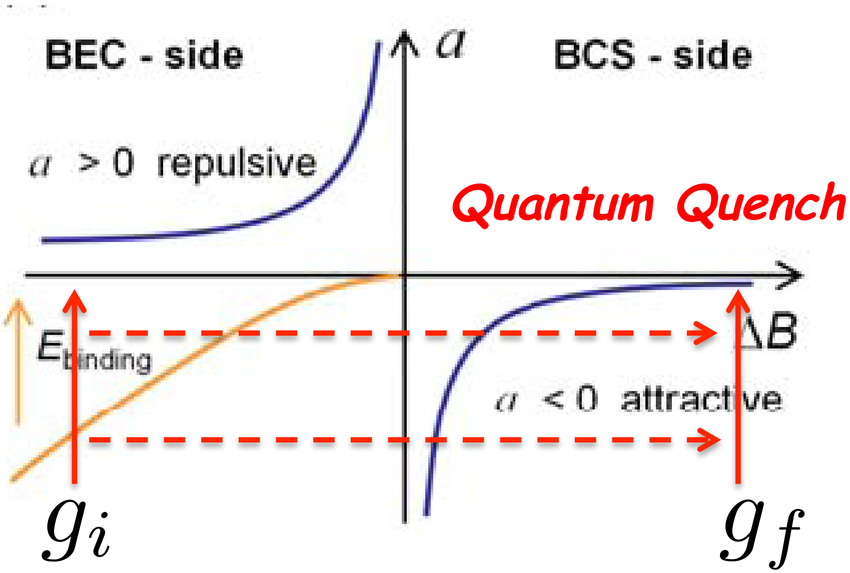
$$\vec{s}_{\mathbf{k}}(t=0) = \text{ground state for } g_i$$

$$\vec{s}_{\mathbf{k}}(t) = ?, \quad \vec{\Delta}(t) = ?$$

Yields full many-body wavefunction, Green's functions etc.

Experiment under way?

Example: far from equilibrium atomic superconductivity



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$$\vec{s}_{\mathbf{k}}(t) = ?, \quad \vec{\Delta}(t) = ? \quad \Rightarrow$$

Yields full many-body wavefunction, Green's functions etc.

Experiment under way?

Exact quench phase diagram: s-wave BCS

Regions II and II':

Order parameter goes to a constant:

$$\Delta(t) \to \Delta_\infty e^{-2\mu_\infty t}$$
$$\Delta_\infty \leq \Delta_{0f}$$

Region I:

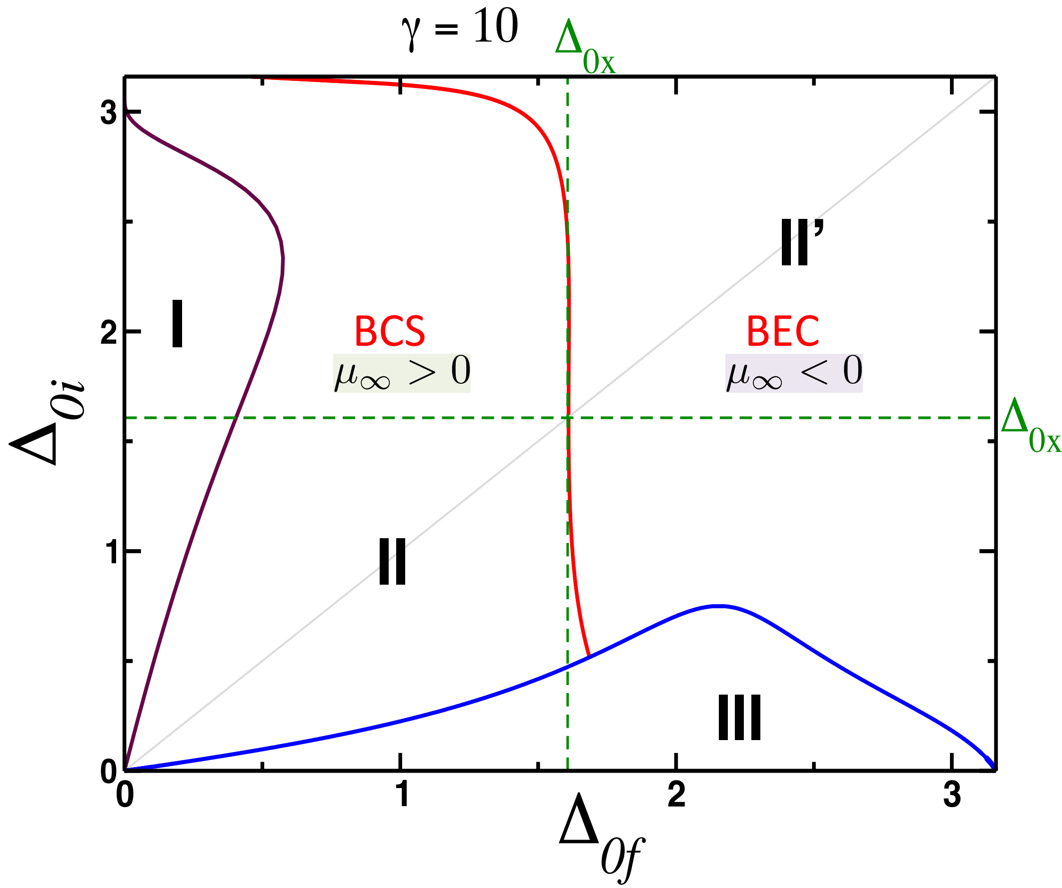
Order parameter vanishes, but nonzero superfluid stiffness (gapless superconductivity):

$$\Delta(t) \to 0,\ n_s = n/2$$

Region III:

Order parameter oscillates periodically:

$$|\Delta(t)| \to \sqrt{a + b^2 \mathrm{dn}^2\left[bt, k'\right]}$$



Steady states of BCS dynamics

Δ_{0i}, Δ_{0f} – *ground state* gaps for g_i, g_f

$\mu_\infty = 0$ line

$$g_i \to g_f \text{ at } t = 0$$

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$$\Delta(t) \to \Delta_\infty e^{-2\mu_\infty t}$$

$$\Delta_\infty \leq \Delta_{0f}$$

$$\Delta_\infty = \Delta_{0i}\cos\eta, \quad \mu_\infty = \epsilon_F,$$

$$\exp\left[-\eta\tan(\eta/2)\right] = \frac{\Delta_{0f}}{\Delta_{0i}}$$

$$\mu_\infty > 0$$

$$|\Delta(t)| = \Delta_\infty + a\frac{\cos(2\Delta_\infty t + \alpha)}{\sqrt{\Delta_\infty t}}$$

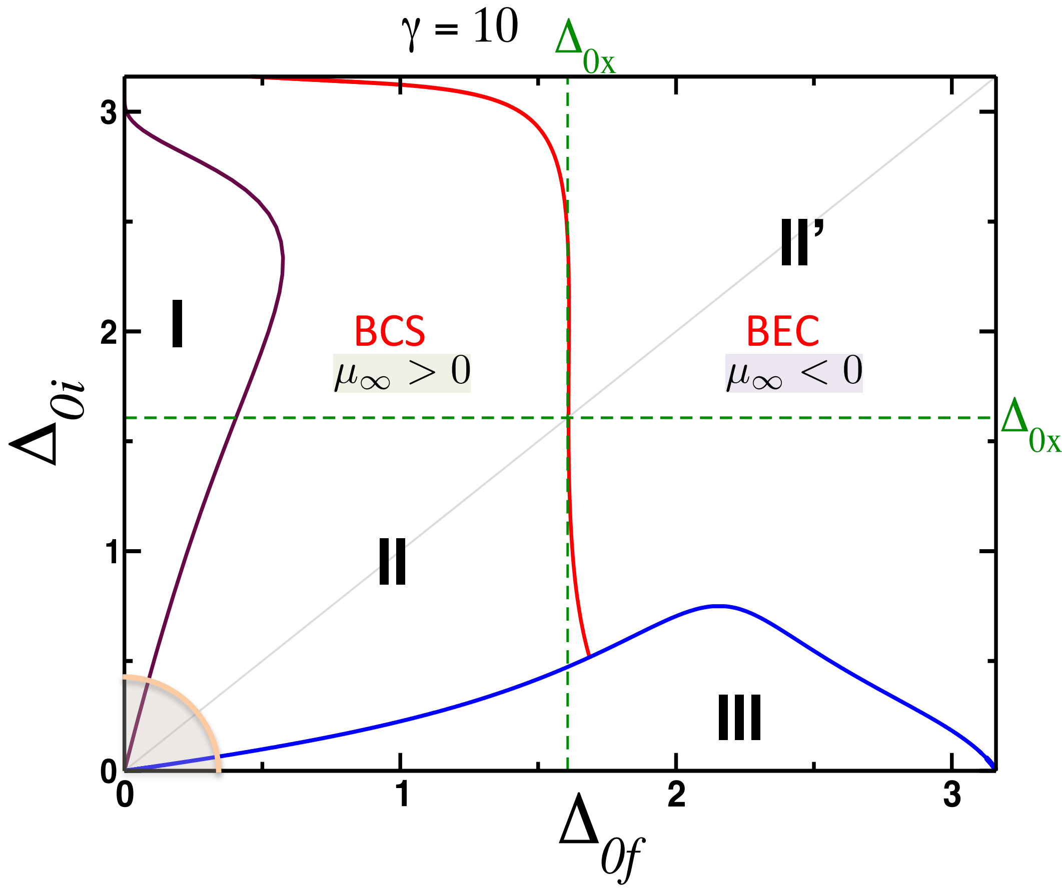
E. Y., Tsyplyatyev, Altshuler, PRL (2006)

$$\mu_\infty < 0$$

$$|\Delta(t)| = \Delta_\infty + b\frac{\cos(2\omega_{\min} t + \alpha)}{(\Delta_\infty t)^{3/2}}$$

$$\omega_{\min} = \sqrt{\mu_\infty^2 + \Delta_\infty^2}$$

E.Y., Dzero, Gurarie, Foster, unpublished



Steady states of BCS dynamics

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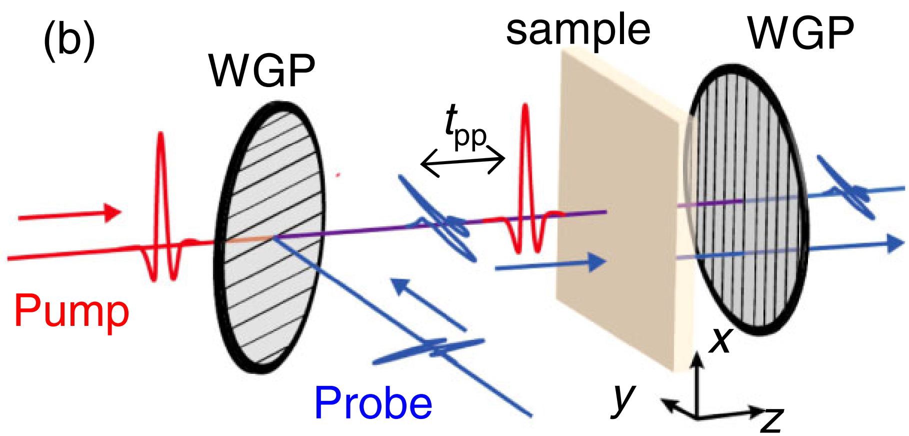
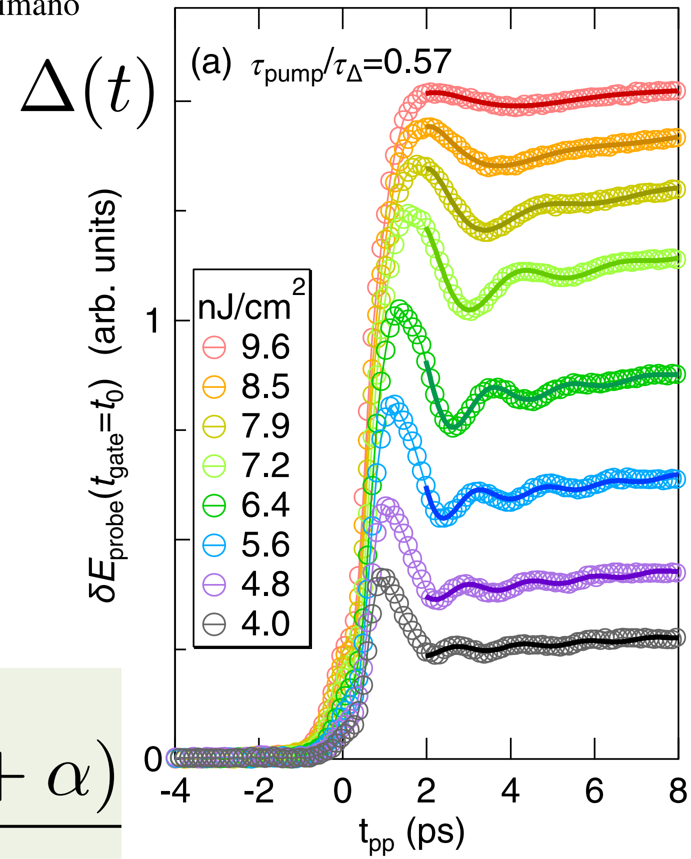
PRL **111**, 057002 (2013) PHYSICAL REVIEW LETTERS week ending 2 AUGUST 2013

Higgs Amplitude Mode in the BCS Superconductors $Nb_{1-x}Ti_xN$ Induced by Terahertz Pulse Excitation

Ryusuke Matsunaga,[1] Yuki I. Hamada,[1] Kazumasa Makise,[2] Yoshinori Uzawa,[3] Hirotaka Terai,[2] Zhen Wang,[2] and Ryo Shimano[1]





$$\mu_\infty > 0$$

$$|\Delta(t)| = \Delta_\infty + a\frac{\cos(2\Delta_\infty t + \alpha)}{\sqrt{\Delta_\infty t}}$$

E. Y., Tsyplyatyev, Altshuler, PRL (2006)

Nature of steady states with constant and zero gap

$$\Delta = \Delta_x - i\Delta_y \to \Delta_\infty e^{-2i\mu_\infty t}$$

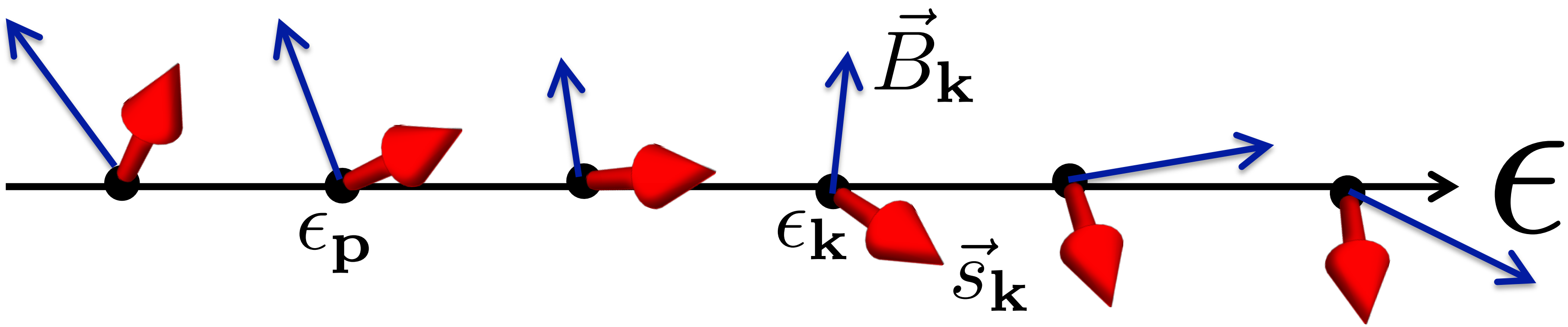
in uniformly rotating frame: $\dot{\vec{s}}_{\mathbf{k}} = (-2\Delta_\infty \hat{x} + 2(\epsilon_{\mathbf{k}} - \mu_\infty)\hat{z}) \times \vec{s}_{\mathbf{k}}$

Each spin (Cooper pair) <u>precesses around</u> its own constant B-field. In equilibrium spins are <u>aligned</u> with the field.

precession frequency: $\omega(\epsilon_{\mathbf{k}}) = 2\sqrt{(\epsilon_{\mathbf{k}} - \mu_\infty)^2 + \Delta_\infty^2}$ nonequillibrium analog of BCS excitation spectrum

This solution is self-consistent at large times. Mechanism: dephasing similar to inhomogeneous line broadening in NMR.

$$\Delta(t) = g\sum_{\mathbf{k}} s_{\mathbf{k}}^- = \int_0^\infty s_{\mathbf{k}\parallel}^- d\epsilon_{\mathbf{k}} + \int_0^\infty A(\epsilon_{\mathbf{k}}) e^{-i\omega(\epsilon_{\mathbf{k}})t} d\epsilon_{\mathbf{k}} \to \Delta_\infty$$

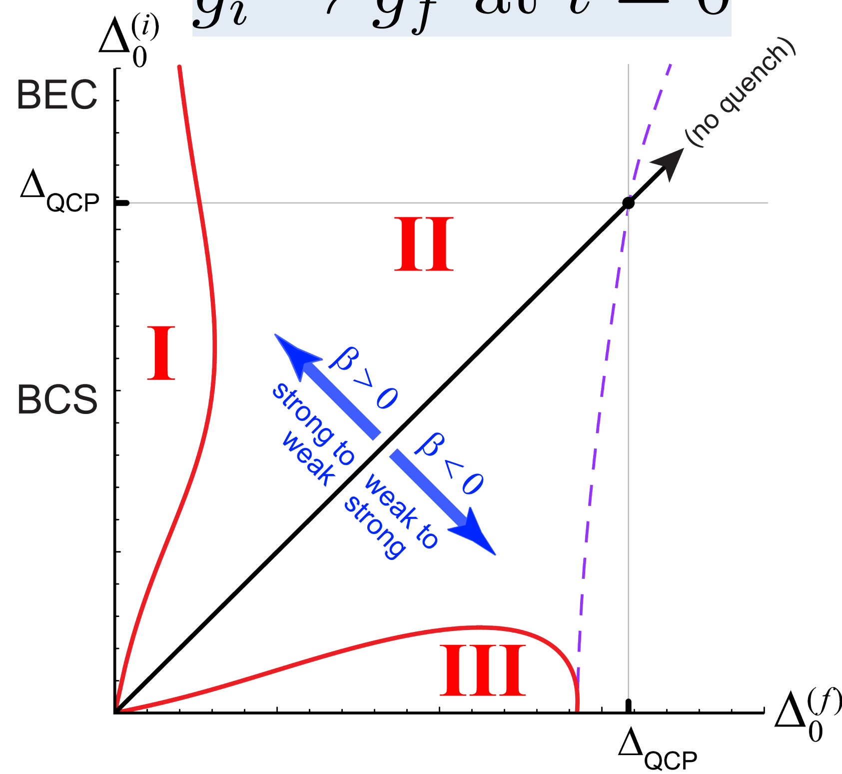


Far from equilibrium topological supercondactivity

2D p-wave BCS Hamiltonian

$$\hat{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{c}^\dagger_{\mathbf{k}} \hat{c}_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} \mathbf{k}\cdot\mathbf{p}\, \hat{c}^\dagger_{\mathbf{k}} \hat{c}^\dagger_{-\mathbf{k}} \hat{c}_{-\mathbf{p}} \hat{c}_{\mathbf{p}}$$

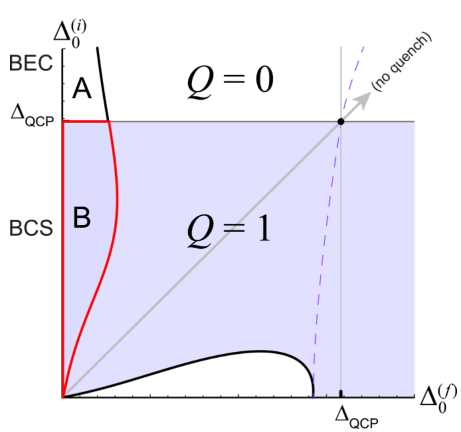
$g_i \rightarrow g_f$ at $t = 0$



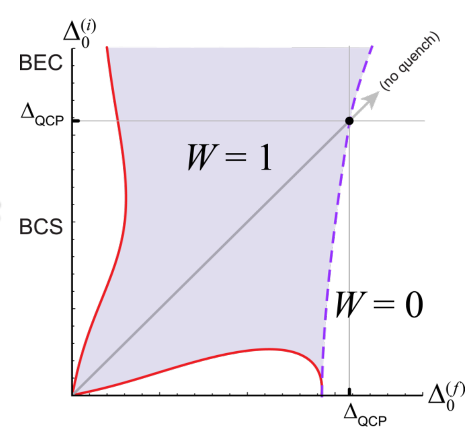
I. Same main three far from equilibrium phases

II. Richer topological features as compared to equilibrium: 2 winding #s instead of 1

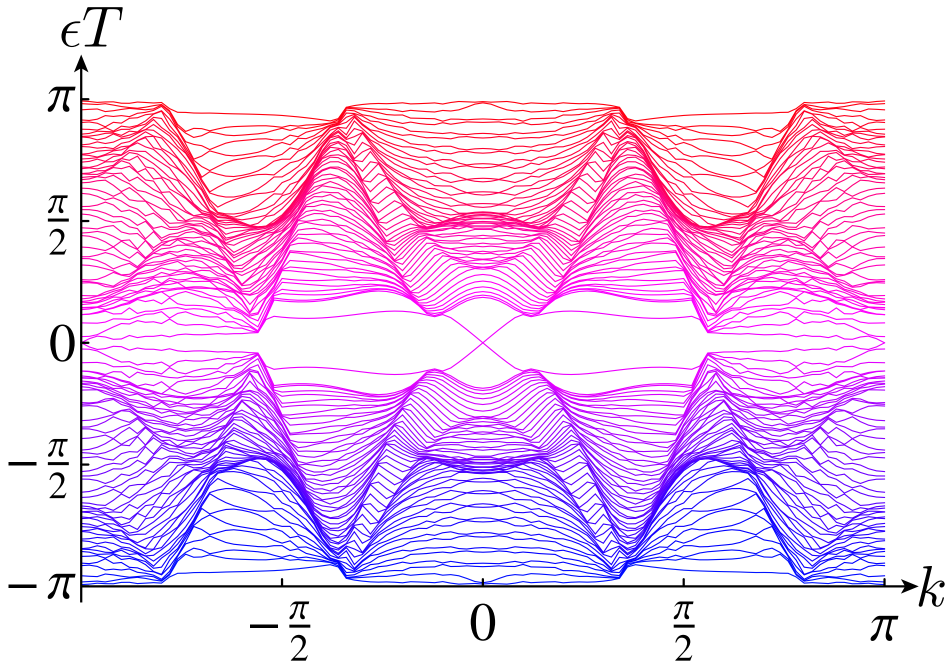
Pseudospin winding Q

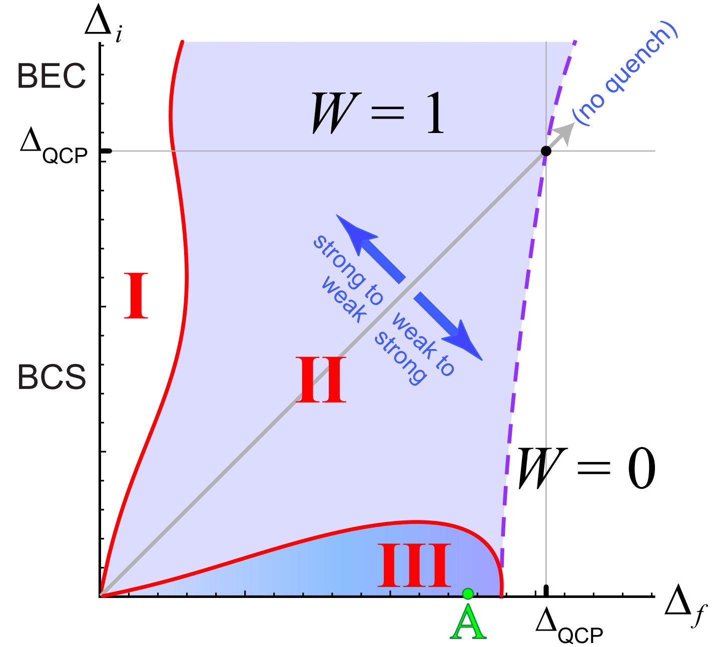


Retarded GF winding W



Quench-induced Floquet topological p-wave superfluids





Floquet spectrum for a quench in Region III, point "A". Majorana edge-modes for a time-dependent state of p-wave superfluidity are xing in the center.

No external drive – quench-induced!

$$g_i \rightarrow g_f \text{ at } t=0$$

All this happens in time. What about space?

$$|\psi(0)\rangle = |\text{gr. state for } g_i\rangle$$ – homogeneous in space

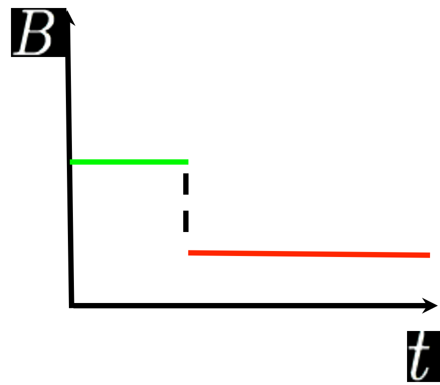
$$g_i \to g_f \text{ at } t = 0$$ – spatially uniform quench

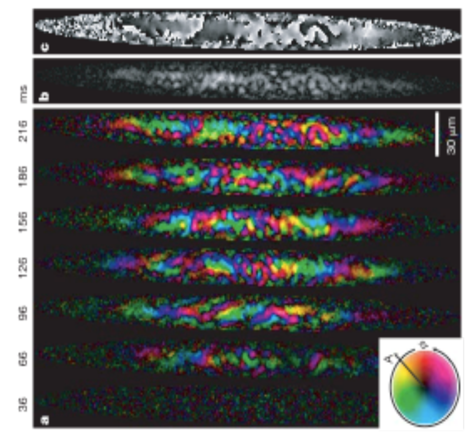
$$\Delta(t)$$ – homogeneous in space

Can spatial inhomogeneities be induced by a uniform quench?

Pattern formation: cosmology in a lab

Parameter (coupling) quench – "Big Bang"





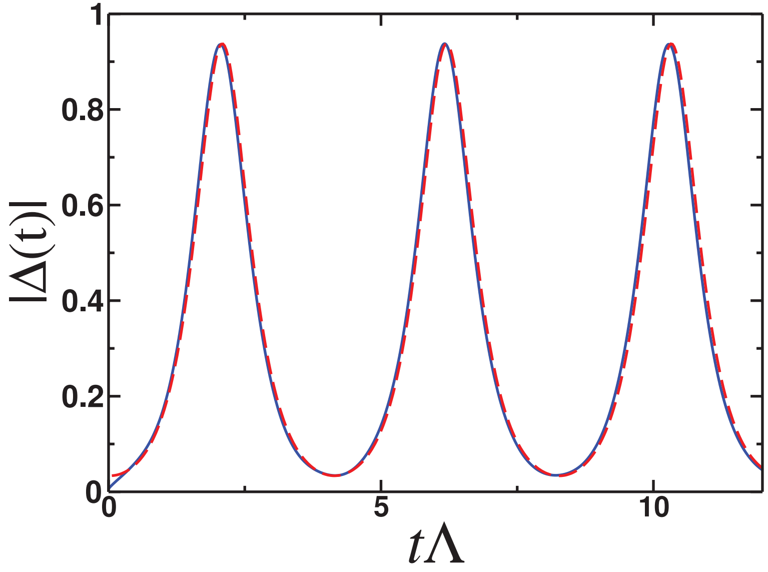
magnetic domain formation in ferromagnetic BEC following a sudden quench of the applied magnetic field, Sadler et al., Nature (London), 2006

Quench-induced parametric resonance??

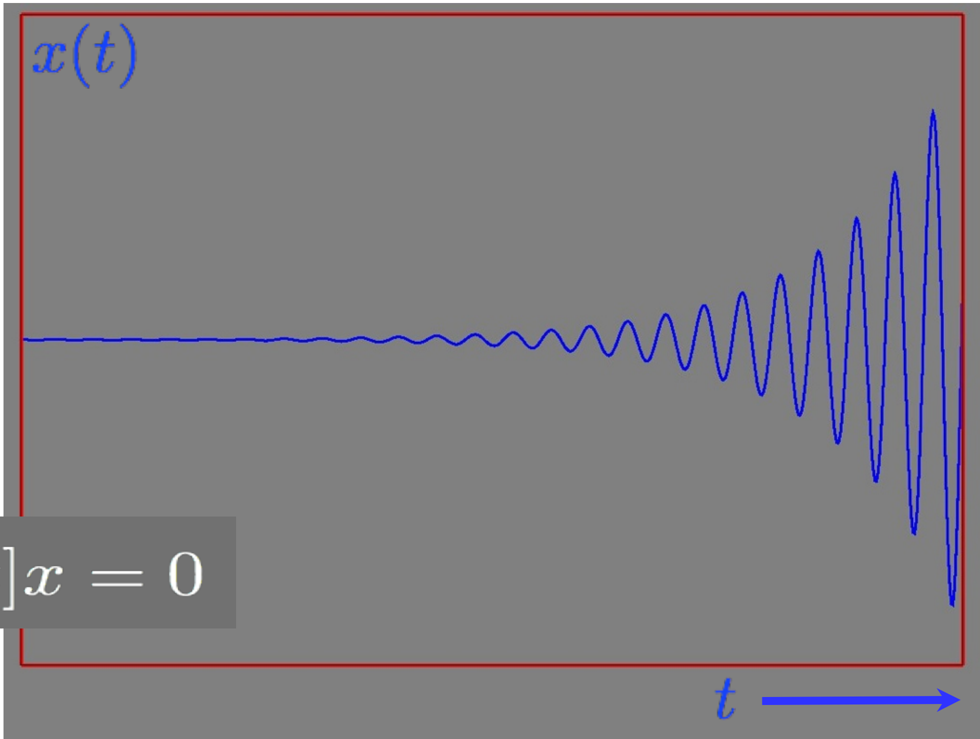
Phase III:
Order parameter oscillates periodically

$$\frac{d\vec{s}_{\mathbf{k}}}{dt} = (-2\vec{\Delta} + 2\epsilon_{\mathbf{k}}\hat{z}) \times \vec{s}_{\mathbf{k}}$$

$$|\vec{b}_{\mathbf{k}}| = \sqrt{\epsilon_{\mathbf{k}}^2 + |\vec{\Delta}|^2}$$

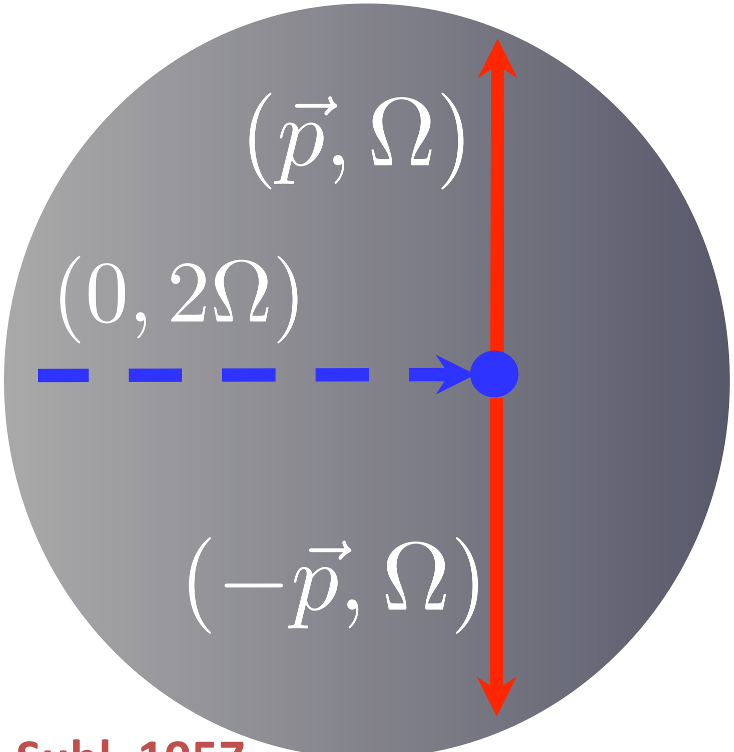


Parametric resonance in continuous media?

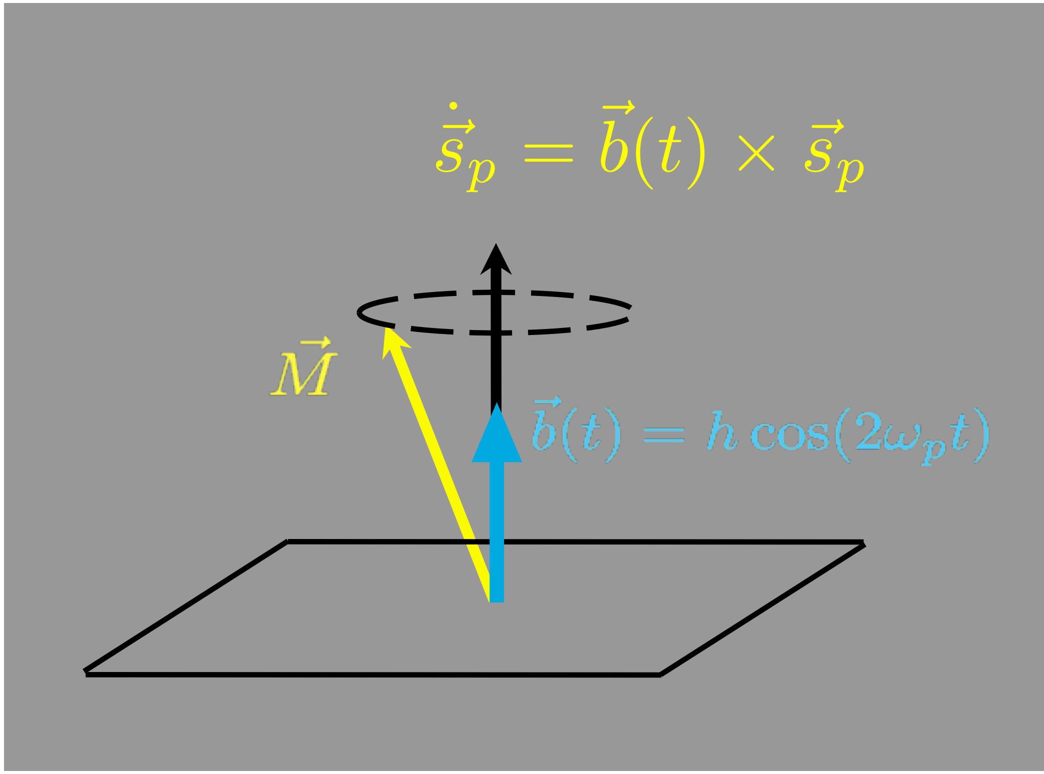


$$\ddot{x} + \omega_0^2[1 + h\cos(2\omega_0 + \varepsilon)t]x = 0$$

Spin wave turbulence



H. Suhl, 1957



dielectric ferromagnet in a uniaxial field (YIG)

microscopic theory of spin wave turbulence

Zakharov, L'vov & Starobinets, 1974

Cooper pair turbulence

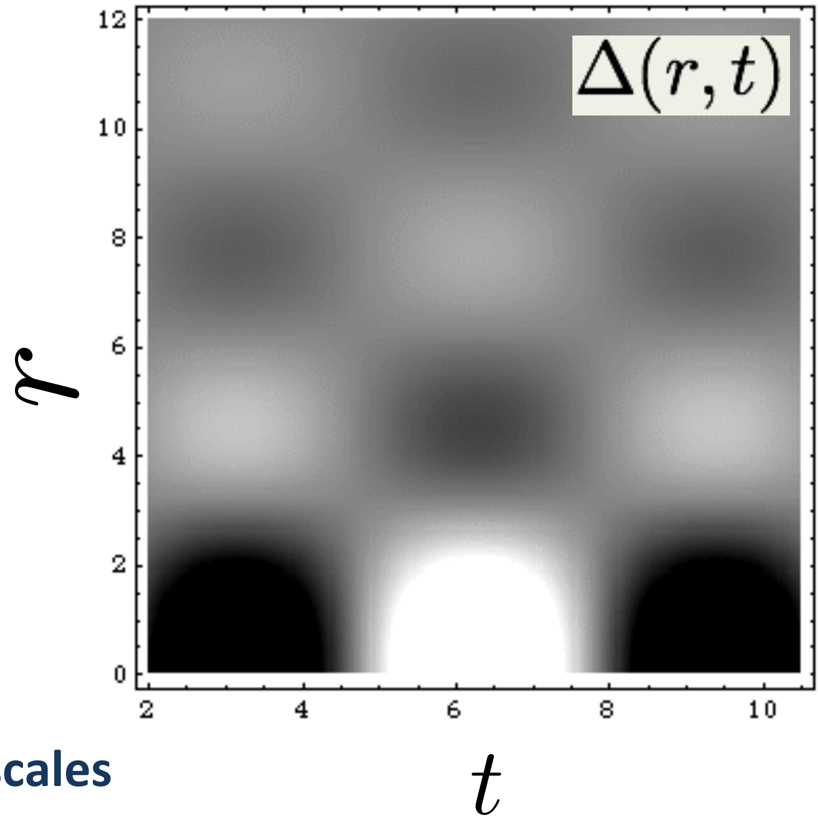
$$\delta\Delta(\vec{r},t) = \frac{\sqrt{q}\Delta_s c_s \sin(k_s|\vec{r}-\vec{r}_0|)}{k_s|\vec{r}-\vec{r}_0|}A(t)$$

$$L \gg \xi$$

"bubble" of a superfluid

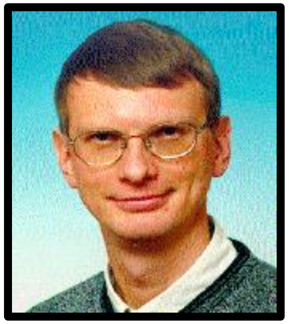
$\vec{r}_0$ – position

$A(t)$ – periodic with random amplitude



Flow of energy to progressively smaller length scales

Typical situation – random superposition of bubbles



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