講義3

強い外場（特に強磁場）中の粒子の性質

荷電粒子

ランダウレベル

性質変化（質量、磁気能率）

synchrotron放射

nonlinear Compton scattering

中性粒子

光子の真空複屈折、崩壊、分裂

π 中間子の崩壊

混合

Charged particles in strong B

磁場中の荷電粒子：古典論

- 運動方程式

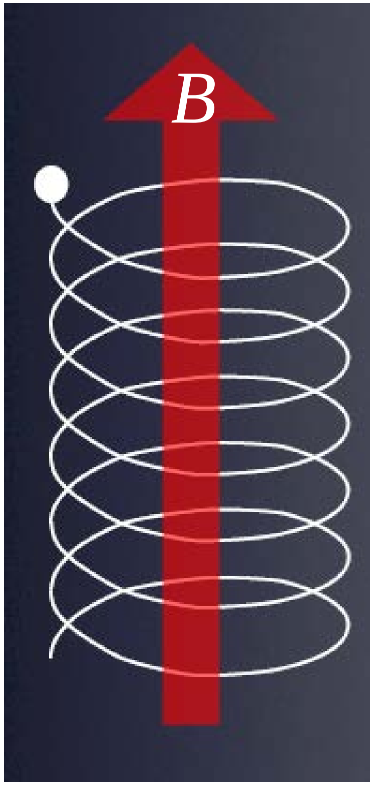
$$m\frac{d\boldsymbol{v}}{dt}=q\,\boldsymbol{v}\times\boldsymbol{B} \qquad \boldsymbol{B}=(0,0,B)$$

- 解　z方向は一様等速運動、x,y平面では円運動

$$\begin{pmatrix}x\\y\end{pmatrix}=r_\perp\begin{pmatrix}\cos\Omega t\\ \sin\Omega t\end{pmatrix},\quad \begin{pmatrix}v_x\\v_y\end{pmatrix}=v_\perp\begin{pmatrix}-\sin\Omega t\\ \cos\Omega t\end{pmatrix}$$

$$\Omega=\frac{qB}{m},\quad r_\perp=\frac{mv_\perp}{qB}$$

Cyclotron (Larmor) frequency, radius



- 電荷の符号が逆だと逆向きに運動。
- 振動数は磁場が強いほど大きくなる。初速度によらない。
- 半径は磁場が強いほど小さく、初速度が大きいほど大きくなる。

磁場中の荷電粒子：量子論（1/2）

- 非相対論的 $$H = \frac{1}{2m}(\boldsymbol{p} - e\boldsymbol{A})^2$$

本質的に調和振動子と同じ構造を持つ

$$\boldsymbol{\Pi} = \boldsymbol{p} - e\boldsymbol{A} \qquad [\Pi_x, \Pi_y] = i\hbar eB$$

Π_x, Π_yが互いに位置と運動量の関係に相当

Cyclotron frequency
は古典論と同じ

$$a = \frac{1}{\sqrt{2}\hbar}\sqrt{\frac{\hbar}{eB}}(\Pi_x + i\Pi_y)$$

$$H = \hbar\omega_c(a^\dagger a + \frac{1}{2})$$

$$\omega_c = \frac{eB}{m}$$

$$E_n(p_z) = \frac{p_z^2}{2m} + \hbar\omega_c\left(n + \frac{1}{2}\right), \qquad n = 0, 1, 2, \cdots$$

Landau 準位

複素座標を導入すると、

$$Z = \frac{1}{\sqrt{2}}(X + iY)\ ,\ \overline{Z} = \frac{1}{\sqrt{2}}(X - iY) \quad \Rightarrow \quad a = \frac{1}{\sqrt{2}}(Z + \frac{\partial}{\partial \overline{Z}})$$ と表現できる

さらに、角運動量 (***L*** = ***r*** x ***p***) に関係した演算子 *b* を導入すると、

$$b = \frac{1}{\sqrt{2}}(\overline{Z} + \frac{\partial}{\partial Z}) \qquad L_z = \hbar(b^\dagger b - a^\dagger a)$$

磁場中の荷電粒子：量子論 (2/2)

一般の固有状態は a^+ , b^+ を最低エネルギー状態に作用させて作る

$$\psi_{n,m}(Z,\overline{Z}) = (b^\dagger)^m (a^\dagger)^n \psi_{0,0}(Z,\overline{Z})$$

ここで、ψ_{00}はa, b で消される状態であるが

$$a\psi_{0,0}(Z,\overline{Z}) = b\psi_{0,0}(Z,\overline{Z}) = 0 \qquad \psi_{0,0}(Z,\overline{Z}) = \exp[-Z\overline{Z}]$$

最低エネルギー状態は a のみについて消される状態なので、
m については足し上げが必要

$$\psi_{0,m}(Z,\overline{Z}) = CZ^m \exp[-Z\overline{Z}]$$

$$= \frac{C}{l^m}(x+iy)^m \exp[-\frac{x^2+y^2}{4l^2}]$$

$$l = \sqrt{\frac{\hbar}{eB}}$$ Larmor radius

古典論との
違いに注意

- 最低ランダウ準位はLarmor半径程度の広がりを持つ
- 高次の準位は磁場と垂直方向の運動量が離散化されることに相当
- ランダウ準位間のエネルギーは $\omega_c=eB/m$ なので、B大で最低ランダウ準位が重要

しつこいようですが

磁場中の荷電粒子：場の量子論

簡単のため、(complex) scalar

Klein-Gordon equation in a magnetic field $\boldsymbol{B}=(0,0,B)$

$$\left(-D_\mu^2 - m^2\right)\phi(x) = 0, \qquad D_\mu = \partial_\mu + ieA_\mu$$

$$\phi(x) = e^{-i\omega t + ip_z z}\varphi(x,y)$$

2次で入る！

$$\left(-D_x^2 - D_y^2\right)\varphi(x,y) = \lambda\varphi(x,y), \quad \lambda = \omega^2 - p_z^2 - m^2$$

量子力学の場合と同様、Dx, Dyが位置と運動量の役割を果たすことに注意する
調和振動子の形となる

$$-D_x^2 - D_y^2 = 2eB\left(a^+ a + \frac{1}{2}\right)$$

mの入り方が今までと違う！

エネルギー固有値

$$E_n(p_z) = \sqrt{m^2 + p_z^2 + 2eB(n+1/2)} \qquad n = 0, 1, 2, \cdots$$

$p_T{}^2$が離散化された

（各準位は角運動量の自由度が縮退している）

スピンの効果

- スピン *s* (z成分s_z), 磁気モーメント(*g*因子) *g*, 電荷 *e*

$$E_n^2(p_z, s_z) = m^2 + p_z^2 + (2n+1)eB - gs_z eB$$

Landau levels　　spin-magnetic effect

- 磁場中の「有効質量」

$$E_{n=0}^2(p_z = 0, s_z) = m^2 + (1 - gs_z)eB \equiv \left[m_{eff}(B)\right]^2$$

- Spin 0 : $m^2 + eB$ (mesons) 重くなる
- Spin 1/2 , *g*=2 : m^2 (electron)
- Spin 1, *g*=2 : $m^2 - eB$ (rho meson) 軽くなる

荷電粒子の磁場応答（まとめ）

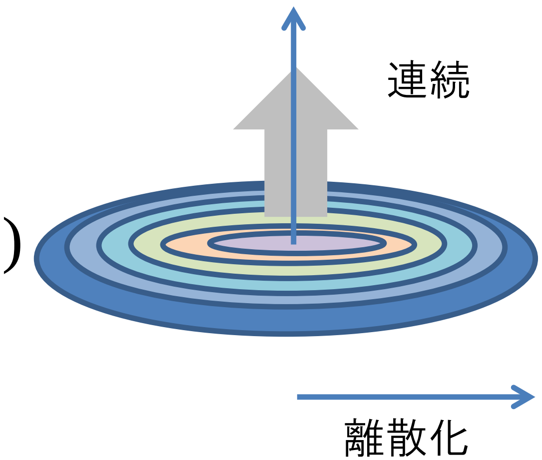
Landau準位が出現し、横運動量が離散化

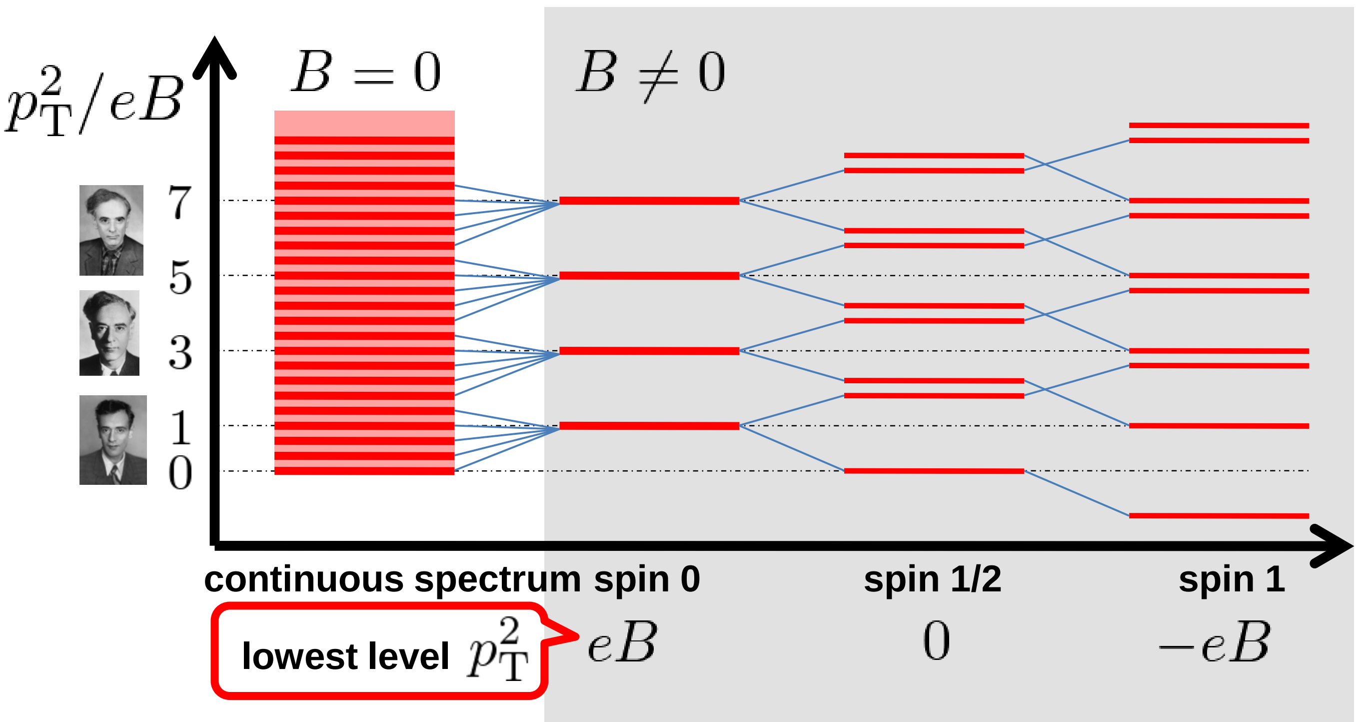
$$p_{\mathrm{T}}^2 = p_x^2 + p_y^2 \to (2n+1)eB \quad (n = 0,1,2,\cdots)$$

スピンと磁場の相互作用でさらにエネルギーがシフト $\pm 2seB \;\; (g = 2)$

各準位は高度に縮退

準位間はB→大でより開く （強磁場では最低Landau準位近似が良い）





Yang-Mills場のNielsen-Olesen不安定性は
スピン1の粒子の不安定性と本質的に同じ

Decay of rho mesons

Chernodub, PRD82 (2010) 085011

- 主要な崩壊モード(>99%) : $\rho^{+/-} \rightarrow \pi^{+/-}\, \pi^{0}$
- 磁場中で質量が変化すると、この崩壊モードが不可能になりうる

$$m_{\rho^\pm}(B_{\rho^\pm}) = m_{\pi^\pm}(B_{\rho^\pm}) + m_{\pi^0}$$

No change

この等式が成り立つ磁場の強さ

$$B_{\rho^\pm} = \frac{1}{2e}\left[m_\rho^2 - m_\pi^2 - m_\pi(m_\pi^2 + 2m_\rho^2)^{\frac{1}{2}}\right]$$

$$\cong 0.36\frac{m_\rho^2}{e} \cong 11\frac{m_\pi^2}{e}$$

この磁場より強い磁場中では、荷電 rho mesons は長寿命になる

Also, neutral rho meson cannot decay into pi+ pi- when masses of charged pions become Large. This happens when $B=(m_\rho^2-4m_\pi^2)/4e \sim 6.5\ m_\pi^2/e$

不安定性??

$$\varepsilon^2_{n,s_z}(p_z) = p_z^2 + (2n - 2s_z + 1)eB_{\rm ext} + m^2$$

- For vector mesons (s=1), the LLL with p_z=0

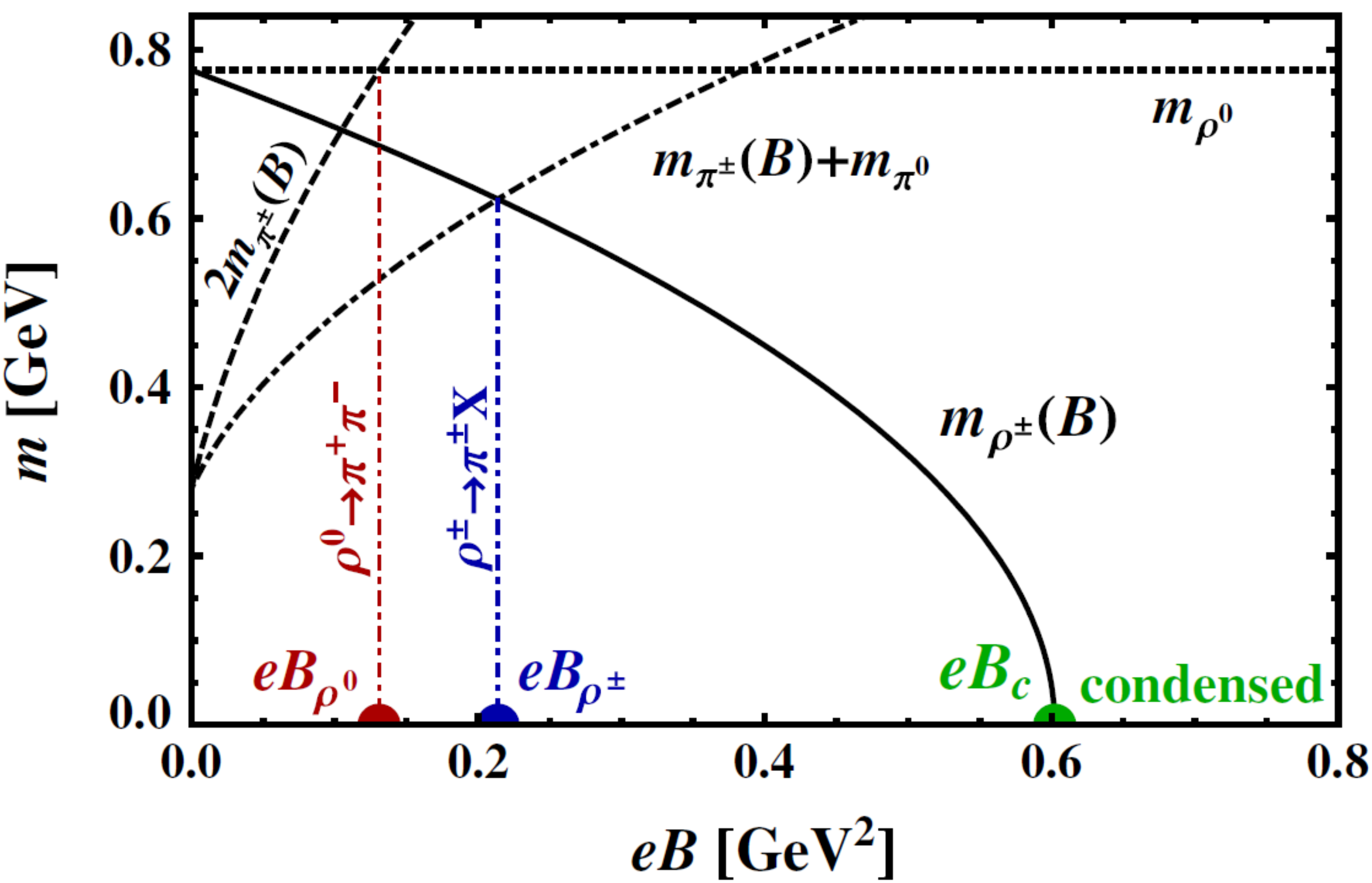
$$\varepsilon^2_{0,1}(p_z = 0) = m_\rho^2 - eB_{ext}$$

can be NEGATIVE when

$$B_{ext} > B_c \equiv \frac{m_\rho^2}{e} \sim 30\frac{m_\pi^2}{e}$$

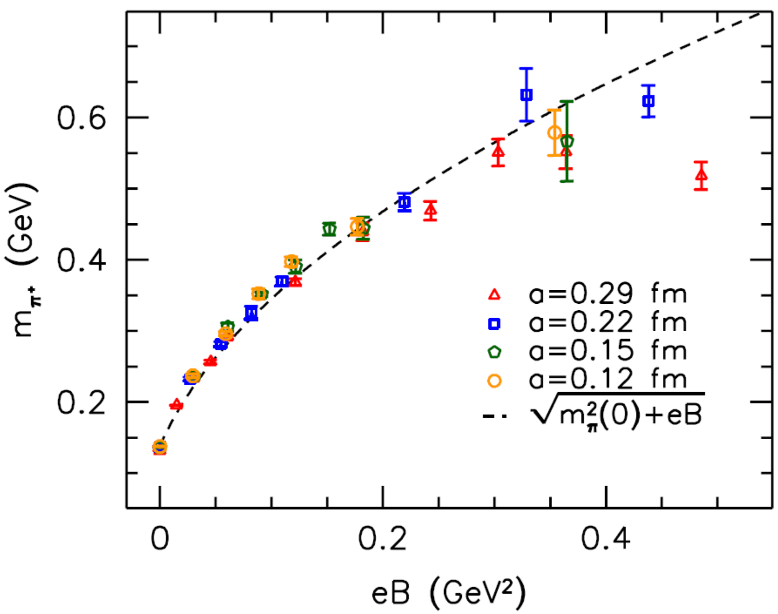
→ Charged rho mesons はこの磁場より強い磁場中では不安定
→ 凝縮の可能性？

Summary of Chernodub



格子計算は不安定性の存在を否定

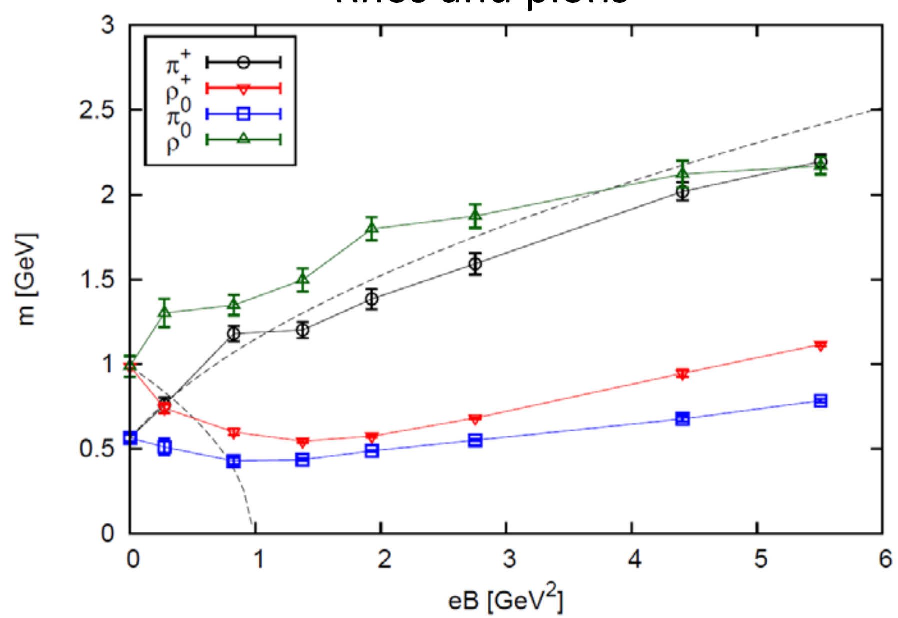
Charged pion



Bali, Bruckmann, Endroedi, Fodor, Katz, Krieg, Schaefera and Szaboeb (2011)

崩壊モードの抑制はありそう

Rhos and pions



Hidaka, Yamamoto (2012)

素朴な計算と合わないのは何故だろうか？
複合粒子性が効いているか？
実は、複合粒子でなくとも非自明な磁場応答をする

電子に対する磁場の高次効果

- 素朴な描像: spin ½, *g*=2

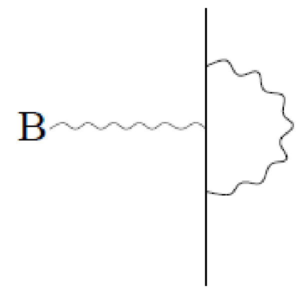
$$\left[m_{eff}(B)\right]^2 = m^2 + (1 - g s_z) eB = m^2$$

→ 磁場中で電子の質量変化はない

<u>しかし、これは2つの点で正しくない</u>

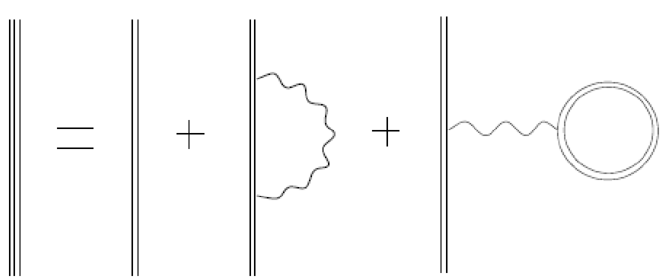
(1) *g*-factor は輻射補正で2からずれる

→ “anomalous” magnetic moment



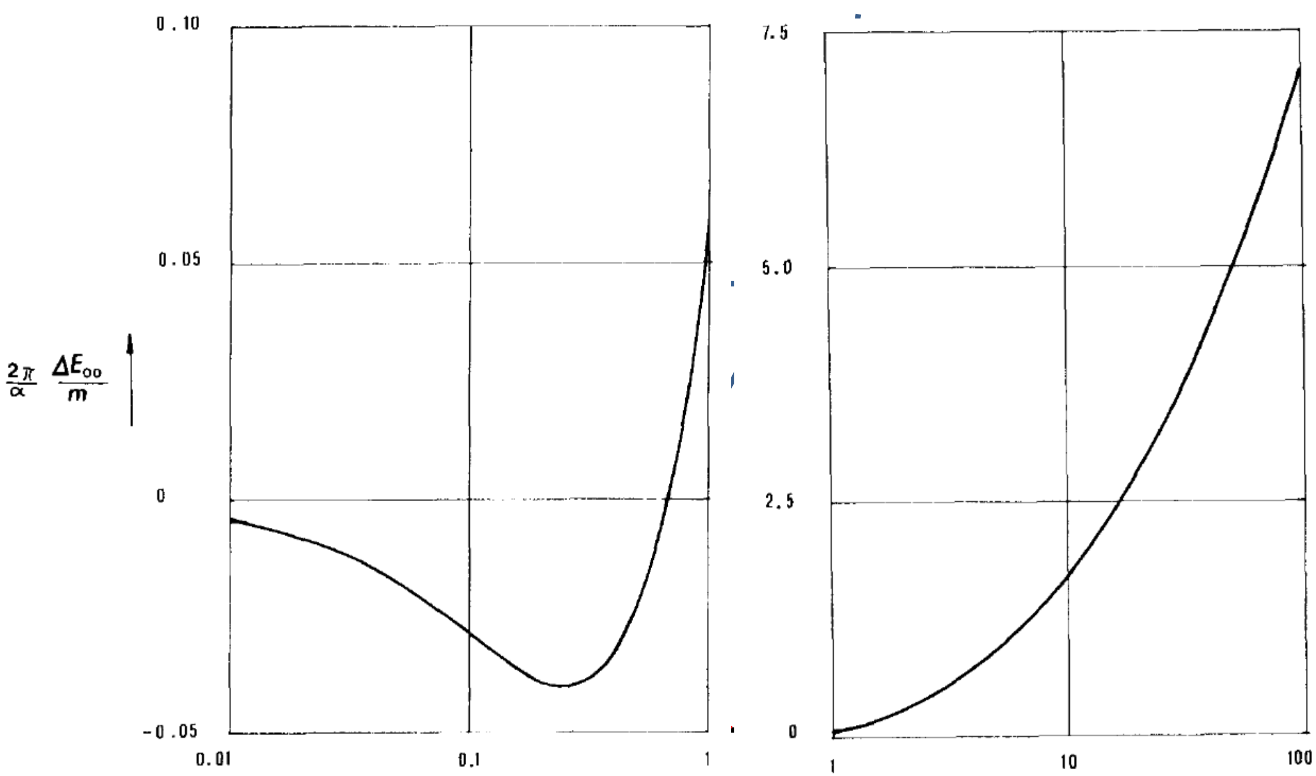
(2) *B* が十分に強いとき、外場との相互作用

について無限次まで足し上げる必要



Double line: dressed electron

磁場中での電子の有効質量



$L=B/B_c$

$$\frac{E_0}{m_e} \sim 1 + \frac{\alpha}{4\pi}\left(\left(\ln\frac{2B}{B_c} - \gamma - \frac{3}{2}\right)^2 + 3.9\right)$$

$E_0 = m_e\left[1 - (\alpha/4\pi)\, B/B_c\right]$

J.Schwinger, PR73(1948)416

Jancovici, 1969

Constantinescu 1972

強磁場中の陽子と中性子

スピン1/2の粒子だからといって$g=2$ではない、また中性だからといって磁場の影響を受けないわけではない　（磁気能率の値は非相対論的クォーク模型で説明されるとされている）

$$H = \boldsymbol{\alpha}\cdot[\boldsymbol{p} - e\boldsymbol{A}(r)] + \beta M_p - \frac{e}{2M_p}\left(\tfrac{1}{2}g_p - 1\right)\beta\boldsymbol{\Sigma}\cdot\boldsymbol{B}.$$

$$g_p = 5.58$$

$$E = \tilde{M}_p = M_p - \frac{e}{2M_p}\left(\tfrac{1}{2}g_p - 1\right)B$$

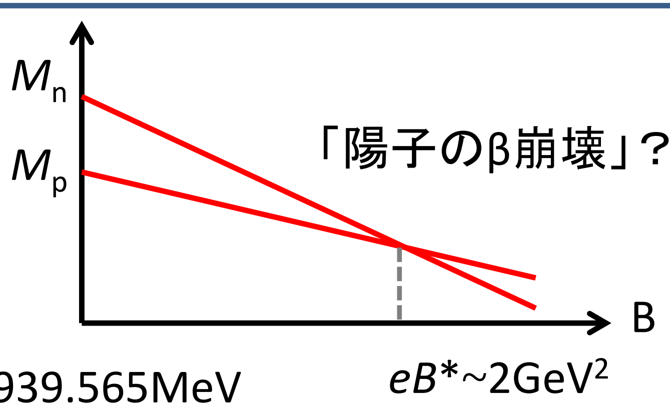
Lowest Landau Level

$$H = \boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta M_n - \frac{e}{2M_n}\frac{g_n}{2}\beta\boldsymbol{\Sigma}\cdot\boldsymbol{B}$$

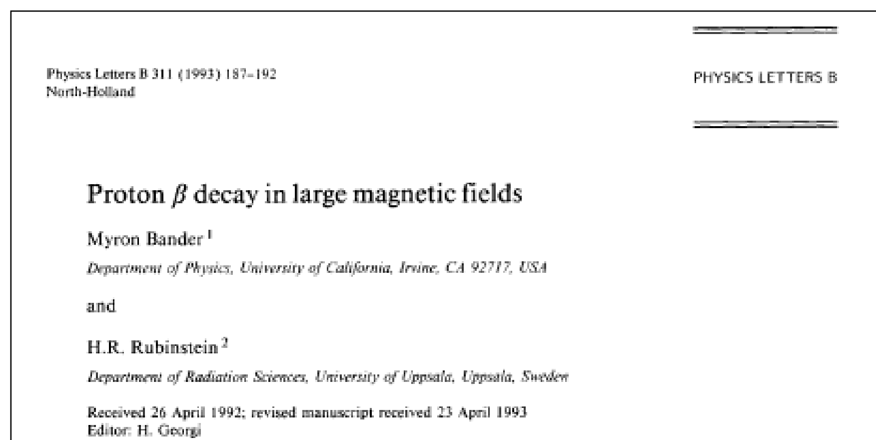
$$g_n = -3.82$$

$$E(\boldsymbol{p}_\perp, p_z = 0) = \frac{e}{2M_n}\frac{g_n}{2}B + \sqrt{\boldsymbol{p}_\perp^2 + M_n^2}$$

No Landau quantization



M_n=939.565MeV
M_p=938.272MeV



Physics Letters B 311 (1993) 187–192
North-Holland

PHYSICS LETTERS B

Proton β decay in large magnetic fields

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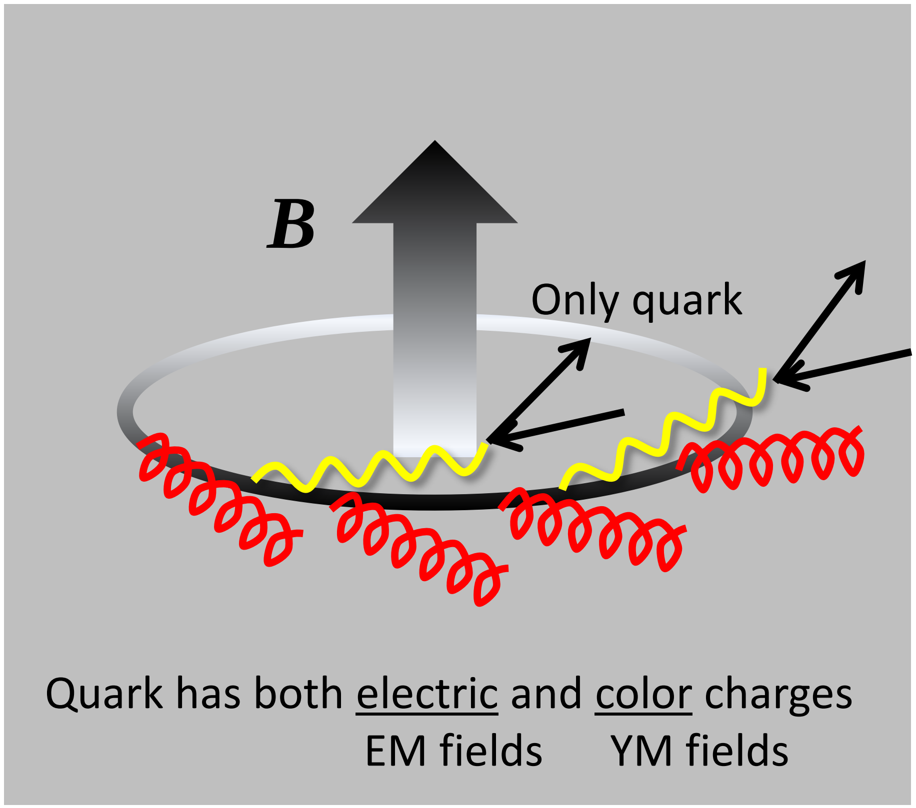
Received 26 April 1992; revised manuscript received 23 April 1993
Editor: H. Georgi

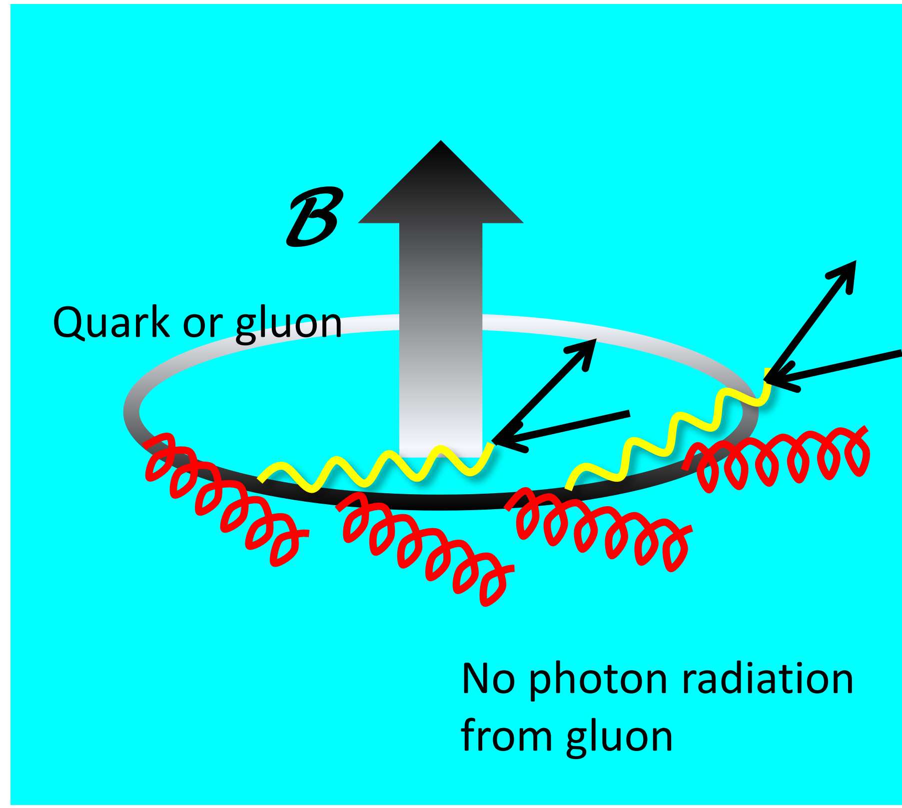
磁場による高次効果、QCD 真空の変化なども考慮する必要がある

Synchrotron 放射

Magnetic field bg



Color magnetic field bg



Also **virtual photons** are emitted → generate dilepton pairs
Gluons will fragment into pions…

Nonlinear Compton scattering

VOLUME 76, NUMBER 17 PHYSICAL REVIEW LETTERS 22 APRIL 1996

Observation of Nonlinear Effects in Compton Scattering

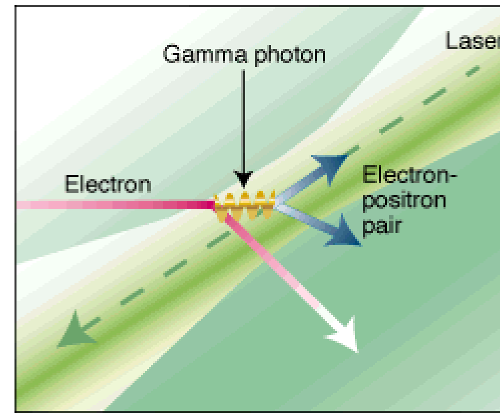
C. Bula, K. T. McDonald, and E. J. Prebys
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544

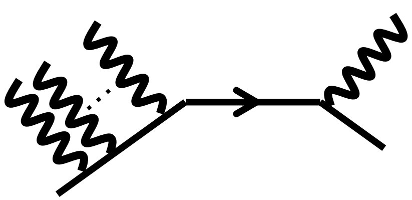
C. Bamber,* S. Boege, T. Kotseroglou, A. C. Melissinos, D. D. Meyerhofer,† and W. Ragg
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627

D. L. Burke, R. C. Field, G. Horton-Smith, A. C. Odian, J. E. Spencer, and D. Walz
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

S. C. Berridge, W. M. Bugg, K. Shmakov, and A. W. Weidemann
Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996
(Received 4 December 1995)

E144 @ SLAC





$$e + n\omega_0 \longrightarrow e' + \omega + m\omega_0$$

e-e+

Phys. Rev. Lett., 79,1626 (1997)

Electron energy 46.6 GeV
Laser Nd:glass 1054 and 527 nm
Peak intensity 10^{18} W/cm2
Measured up to n=4

これらはQCDでは常識的な反応。Initial state radiationとFinal state radiationとして常に考えられている

Extreme limit of nonlinear Compton

例えば最近の例では

PHYSICAL REVIEW A **83**, 032106 (2011)

Nonlinear Compton scattering in ultrashort laser pulses

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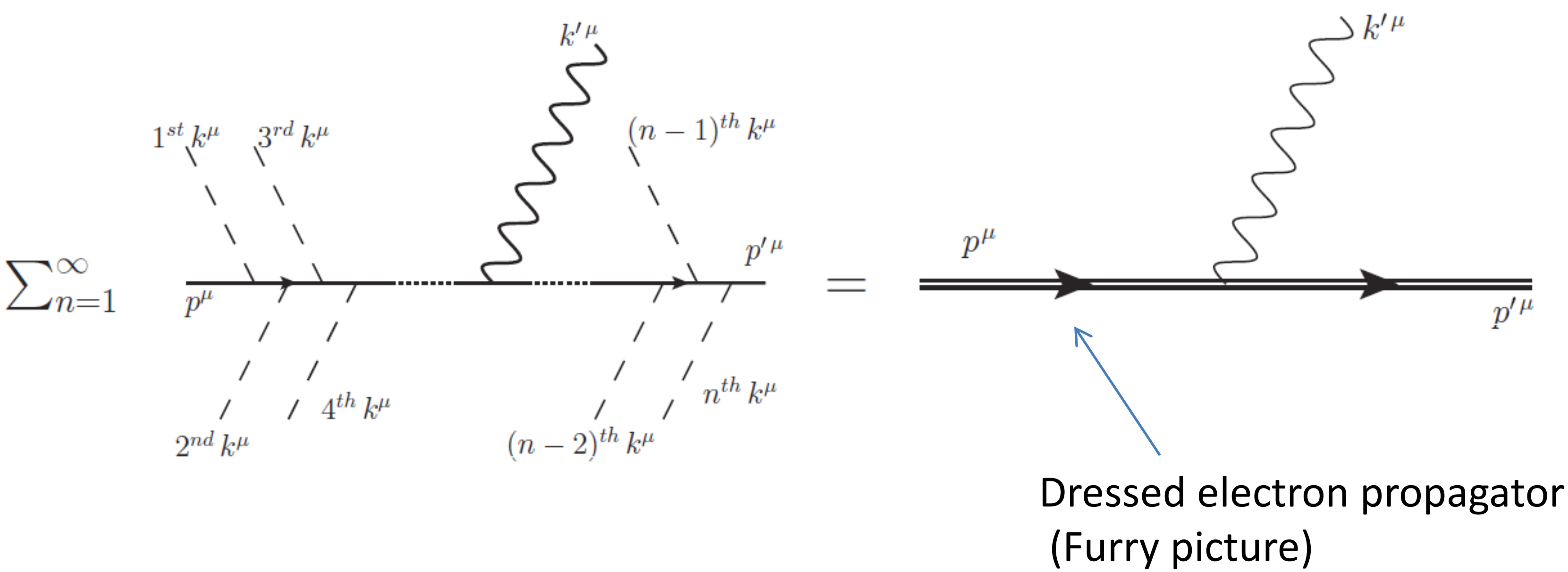
(Received 29 October 2010; published 9 March 2011)

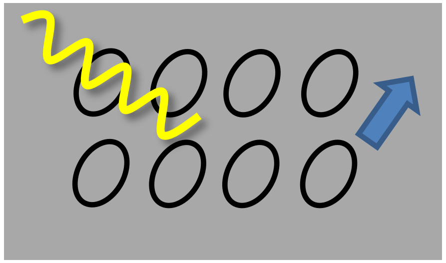


Laser のプロファイルの影響などが詳しく議論されている

Neutral particles in strong B

中性でも、構成粒子、ゆらぎを通じて磁場と相互作用

強磁場中の光子の性質



Propagating photon in strong magnetic field

= probing magnetic vacuum "polarized" by external fields

~ photon couples to virtual excitation of vacuum (cf: exciton-polariton)

***B* dependent anisotropic response of a fermion** (Landau levels)

- discretized transverse vs unchanged longitudinal motion
  - → Two different refractive indices : ***VACUUM BIREFRINGENCE***
- energy conservation gets modified
  - → Pol. Tensor can have imaginary part : ***PHOTON DECAY INTO e+e- PAIR***

(lots of astrophysical applications)

$$\Pi^{\mu\nu}_{\rm ex}(q) = \chi_0(q^2\eta^{\mu\nu} - q^\mu q^\nu) + \underline{\chi_1(q_\parallel^2\eta_\parallel^{\mu\nu} - q_\parallel^\mu q_\parallel^\nu) + \chi_2(q_\perp^2\eta_\perp^{\mu\nu} - q_\perp^\mu q_\perp^\nu)}$$

present only in external fields

II parallel to B

⊥ transverse to B

$$\eta_\parallel^{\mu\nu} = diag(1,0,0,-1)$$

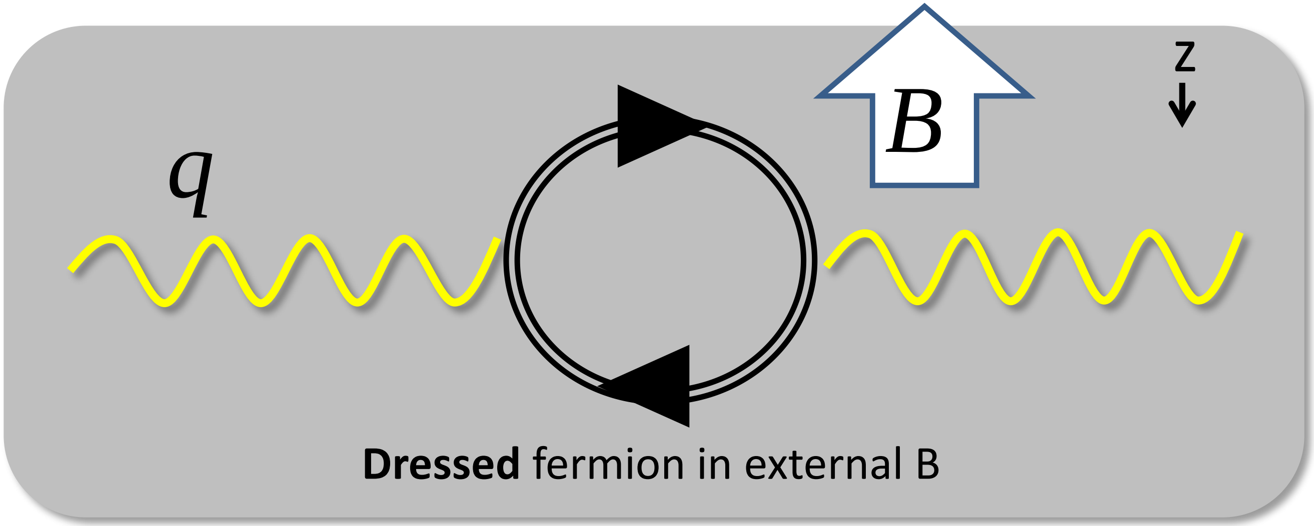
$$\eta_\perp^{\mu\nu} = diag(0,-1,-1,0)$$

$$q^\mu = (q^0, q_\perp, 0, q^3)$$

$$q_\parallel^\mu = (q^0, 0, 0, q^3)$$

$$q_\perp^\mu = (0, q_\perp, 0, 0)$$

真空偏極と屈折率



$$\left(q^2\eta^{\mu\nu} - q^\mu q^\nu + \hat{\Pi}^{\mu\nu}_{\rm ex}\right) A_\nu(q) = 0$$

$$\Pi^{\mu\nu}_{\rm ex}(q) = \chi_0(q^2\eta^{\mu\nu} - q^\mu q^\nu) + \chi_1(q_\parallel^2\eta_\parallel^{\mu\nu} - q_\parallel^\mu q_\parallel^\nu) + \chi_2(q_\perp^2\eta_\perp^{\mu\nu} - q_\perp^\mu q_\perp^\nu)$$

Analytic expression of χ_i now available (Hattori-KI 2013a)

$$n^2 \equiv \frac{|\boldsymbol{q}|^2}{\omega^2} \Rightarrow \begin{cases} n_1^2 = \dfrac{1+\chi_0+\chi_1}{1+\chi_0+\chi_1\cos^2\theta} \\ n_2^2 = \dfrac{1+\chi_0}{1+\chi_0+\chi_2\sin^2\theta} \end{cases}$$

birefringence

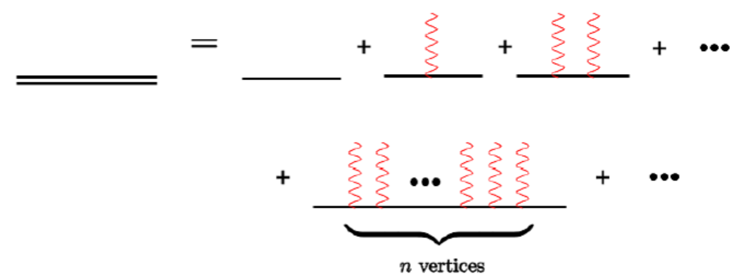
Proper-time method

J. S. Schwinger, Phys. Rev. **82** (1951) 664-679

Electron propagator in external EM field

$$G(p|A_{\rm cl}) = \frac{i}{\not{p} - e\not{A}_{\rm cl} - m}$$

$$= \frac{i}{\not{p} - m}\sum_{n=0}^{\infty}\left[(-ie\not{A}_{\rm cl})\frac{i}{\not{p} - m} \right]^n$$



can be equivalently rewritten as

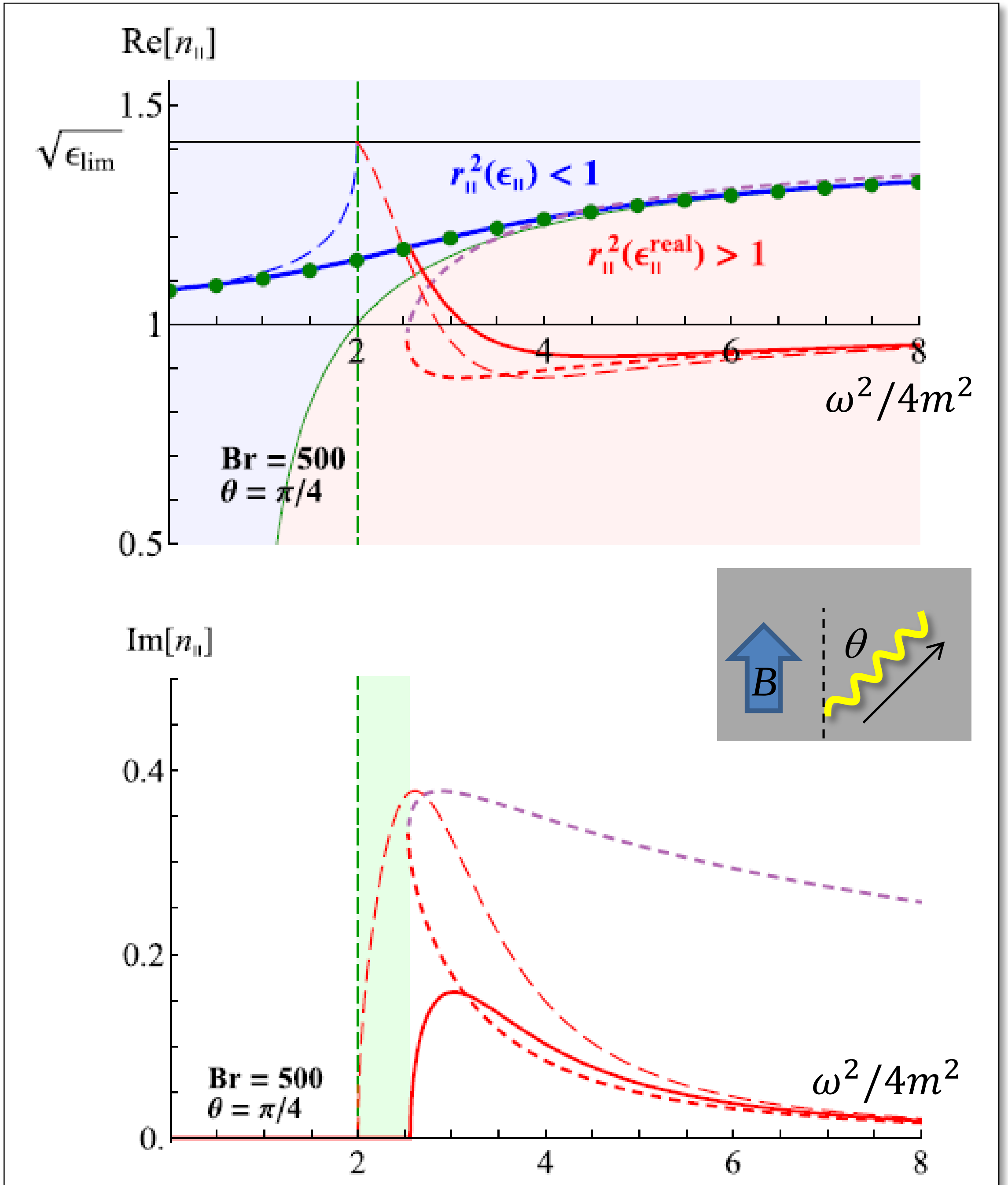
τ: proper time

$$G(p|A_{\rm cl}) = i\left(\not{p} - e\not{A}_{\rm cl} + m\right) \times \frac{1}{i}\int_0^{\infty} d\hat{\tau}\ {\rm e}^{i\hat{\tau}\left\{(\not{p}-e\not{A}_{\rm cl})^2-(m^2-i\varepsilon)\right\}}$$

This form can (relatively easily) incorporate <u>all order</u> contributions w.r.t. the external field.

Hattori-KI 2013b

LLL results



Take the Lowest landau Level approximation
Only n// deviates from 1

B/B_c = 500 (magnetar)

Refractive index $n_{||}$ deviates from 1 and increases with increasing ω

cf: air n = 1.0003, water n = 1.333

Imaginary part appears when photon energy exceeds threshold.
→ Decay into a e+e- pair

Self-consistent treatment of the equation necessary ←backreaction

Consequence?

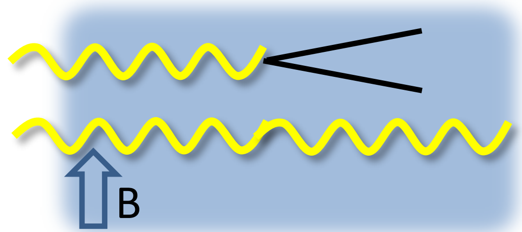
- **Polarization of photons from compact stars**

photon decay strongest when $\theta=\pi/2$ ← only for // modes

→ perp modes survive, generating effectively polarization

Parallel

perpendicular

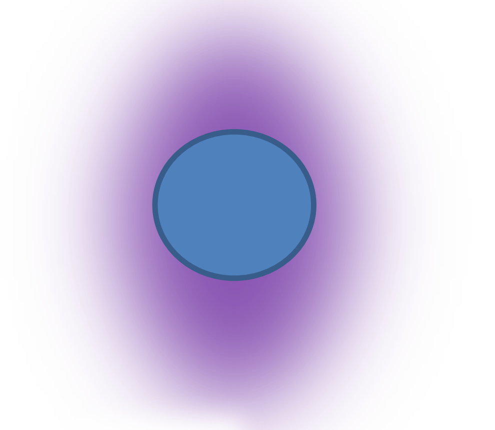


← See perp polarization

- **Magnetic lensing effect**

distorted image of magnetars or stars behind magnetars

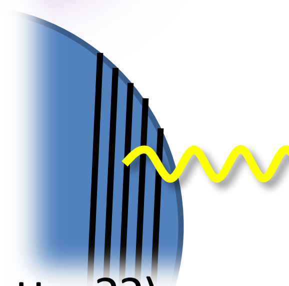
→ present but too small in area



- **Vacuum resonance**

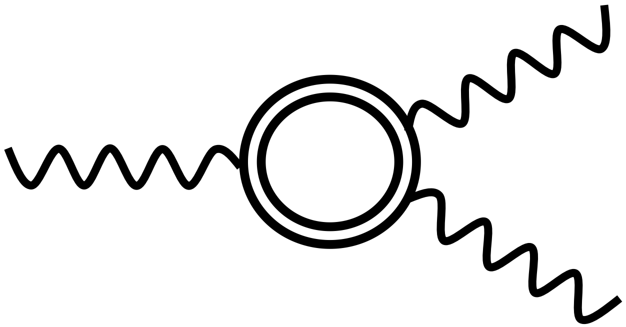
need to include the effects of matter around NS/magnetar

- **Magnetic flux tubes in vortex as a slit**



photons emitted at the surface of NS/magnetar could be affected by the magnetic structure of crust (opacity of nuclear matter??)

光子の分裂 (Photon splitting)



真空中、有限温度では不可能 (Furry's theorem)

外場中、有限密度では可能

まだ、正確な計算がされていない

Recent progress (Hattori and KI, work in progress)

- Proof of vanishing LLL contribution

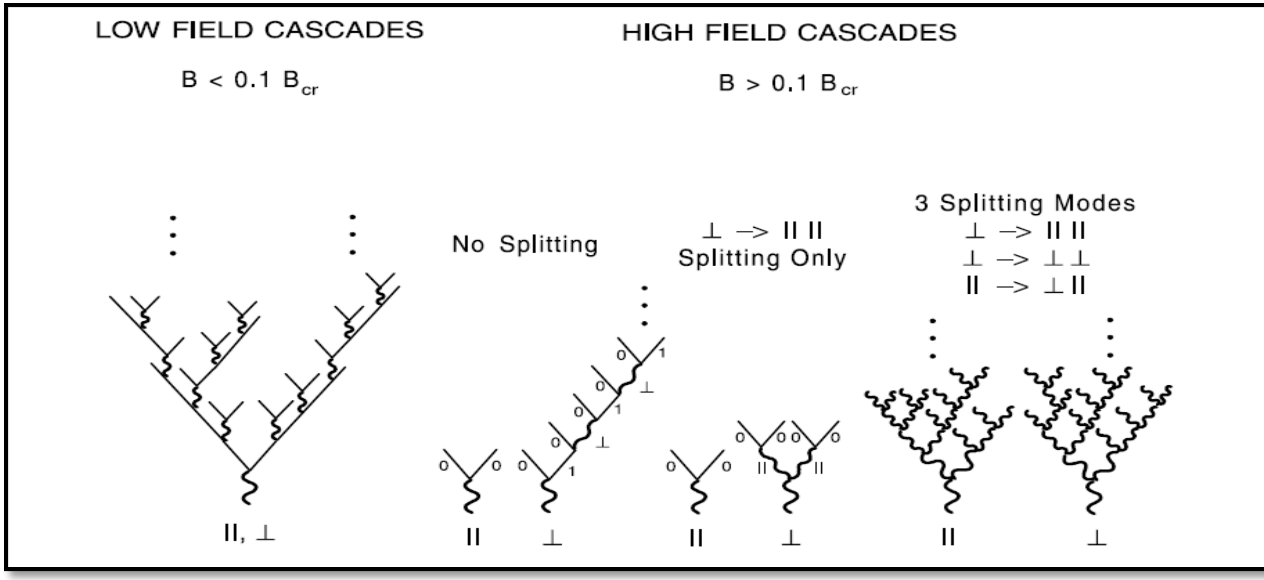
diagram with three LLL propagators is vanishing

at least one propagator must be the second LL

- Proof of vanishing LLL contribution for odd numbers of external photon lines

Photon splittingのもたらす効果

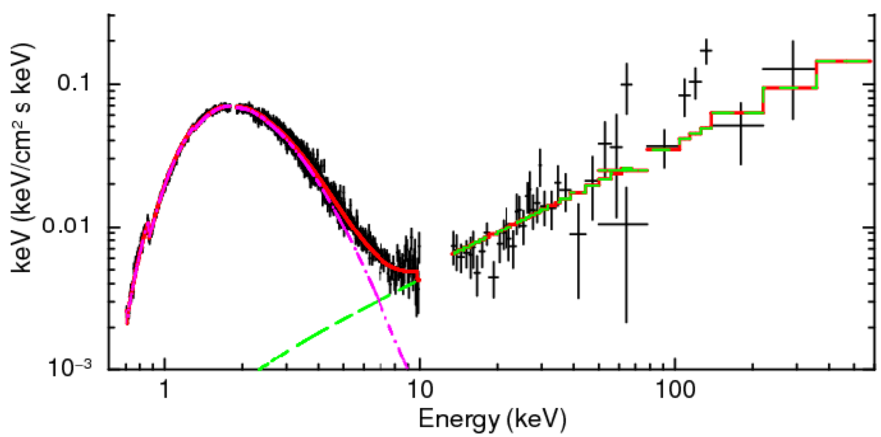
- Photon cascade



Baring, Harding
ApJ 547 (2001)

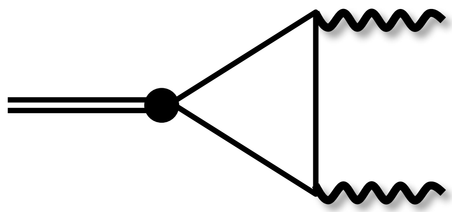
Jet production around magnetic poles

- rising spectrum of photons in magnetars



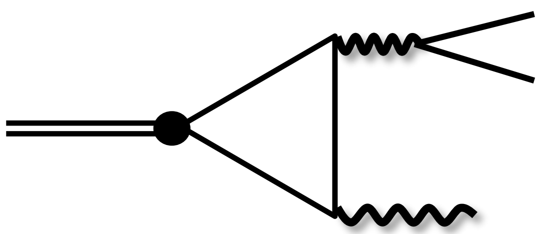
Neutral pion decay

- **Chiral anomaly** induces π^0 decay through triangle diagram



$$\pi^0 \to 2\gamma : \mathcal{O}\left(e^2\right)$$

Dominant (98.798 % in vacuum)



$$\pi^0 \to \gamma + e^+e^- : \mathcal{O}\left(e^3\right)$$

Dalitz decay (1.198 % in vacuum)
NLO contribution

99.996 %

- **Adler-Bardeen's theorem**

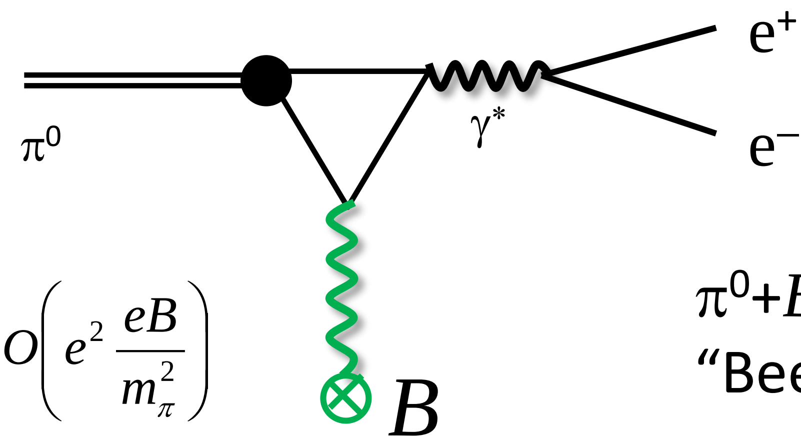
There is no radiative correction to the triangle diagram
Triangle diagram gives the exact result in all-order perturbation theory

→ only two photons can couple to π^0

Neutral pions in strong B

Hattori , KI, Ozaki, arXiv:1305.7224[hep-ph]

- There is only one diagram for a constant external field to be attached



$$O\left(e^2 \frac{eB}{m_\pi^2}\right)$$

$\pi^0 + B \rightarrow e^+e^-$

"Bee" decay

cf: axion
(very light, but
small coupling)

- Also implies

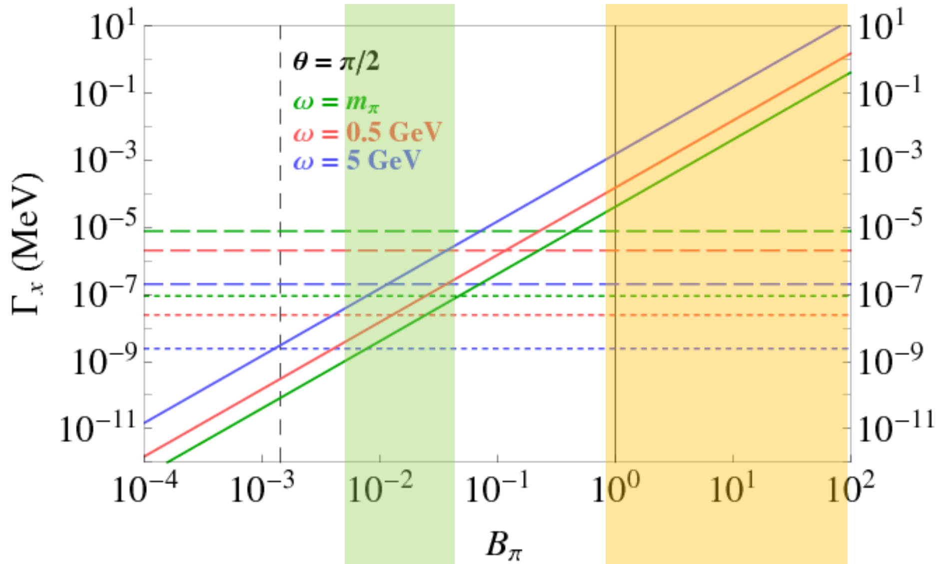
-- conversion into γ with space-time varying B

-- Primakoff process* ($\gamma^* + B \rightarrow \pi^0$): important in HIC

-- mixing of π^0 and γ

* observed in nuclear Coulomb field

Decay rates of three modes

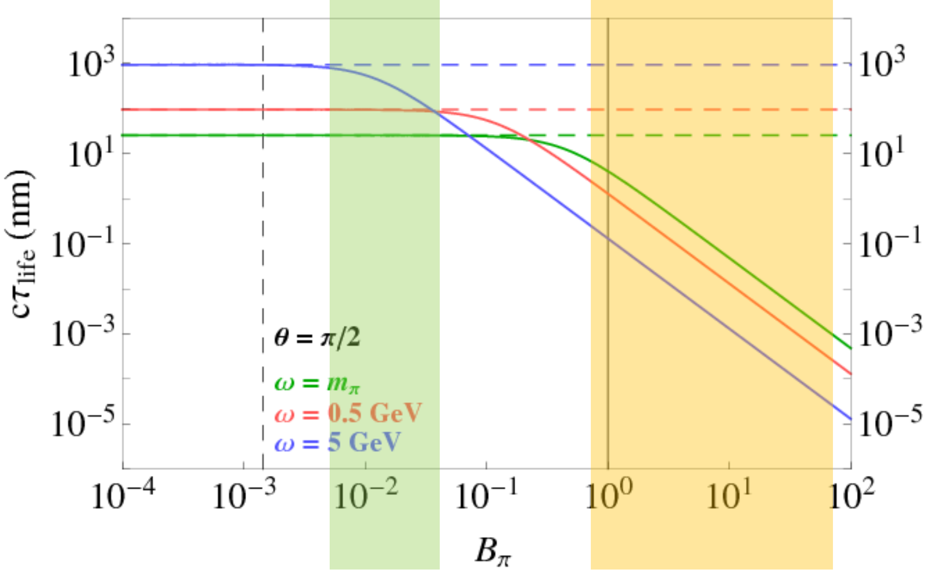


Solid : "Bee" decay
Dashed: 2γ decay
Dotted : Dalitz decay

$$\Gamma_{Be^+e^-} = \frac{q^2 q_\parallel^2}{12\pi\omega_\pi}\left(\lambda\frac{eB}{q^2}\right)^2\left(1+\frac{2m^2}{q^2}\right)\sqrt{1-\frac{4m^2}{q^2}}$$

$$B_\pi = B/m_\pi^{\ 2}$$

Mean lifetime



$$\tau_{life} = \Gamma_{total}^{-1}$$

$$= \frac{1}{\Gamma_{2\gamma} + \Gamma_{Dalitz} + \Gamma_{Bee}}$$

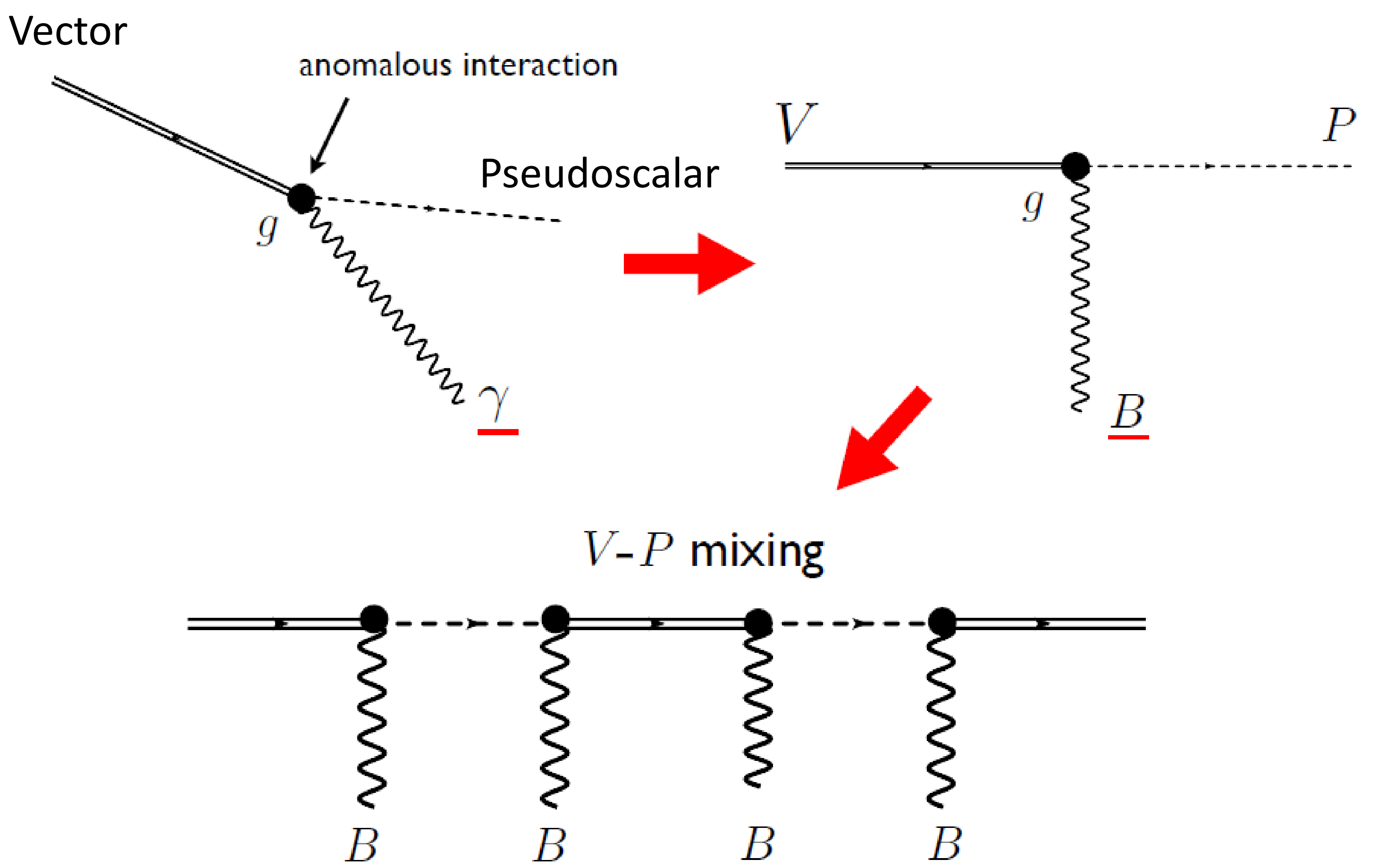
← Picometer

← femtometer

Energetic pions created in cosmic ray reactions will be affected

強磁場中での中性粒子の混合

Radiative decay of hadrons and V-P mixing in magnetic field



Slide by S. Oszaki

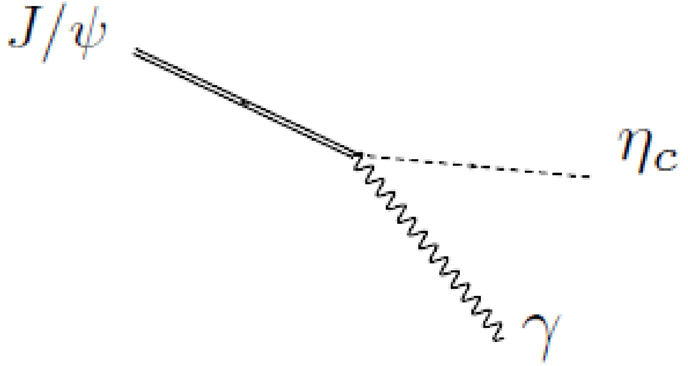
J/psiの電磁的崩壊に応用

$$M_P = M_{\eta_c}$$

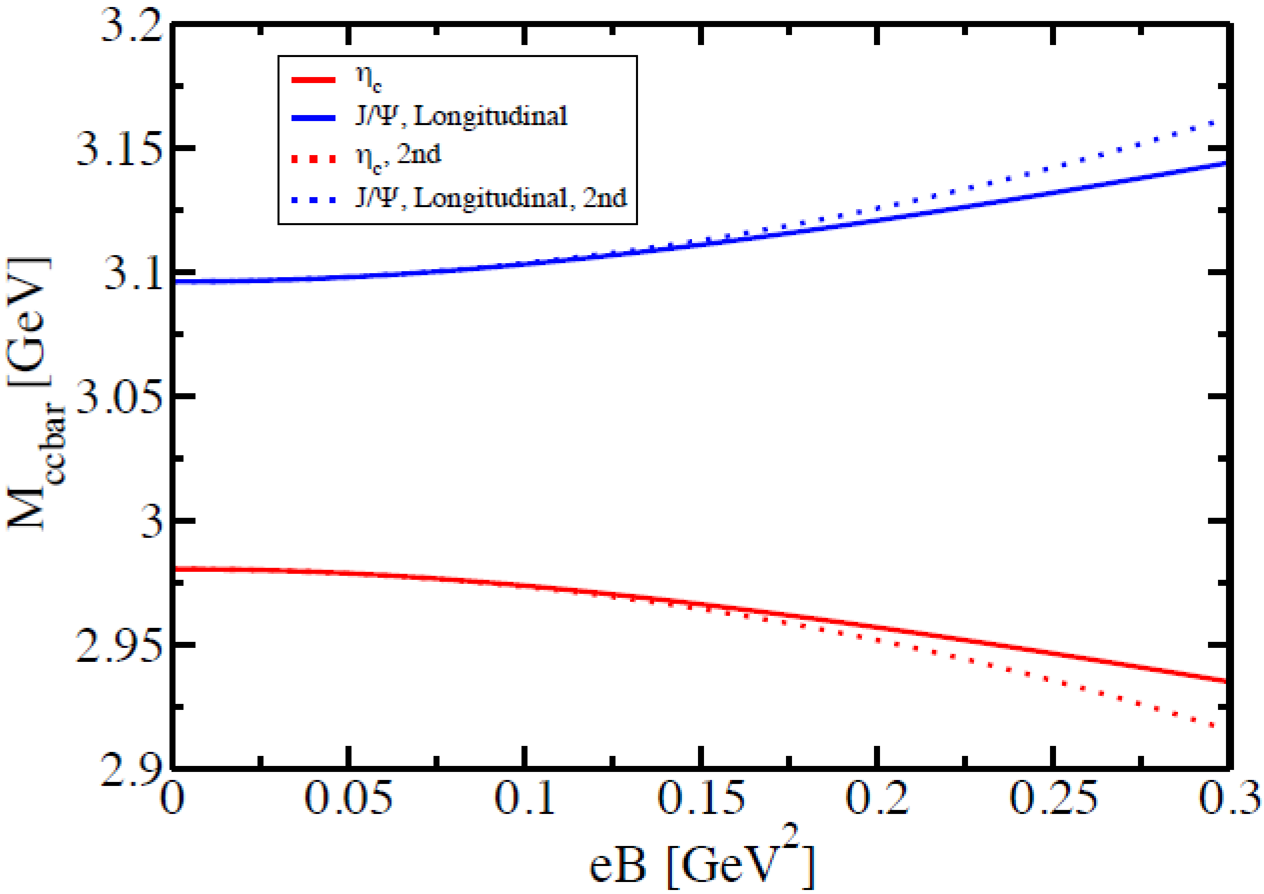
$$M_V = M_{J/\psi}$$

$$g_{\gamma PV} = g_{\gamma \eta_c J/\psi} = \underline{2.055e}$$

This value is determined from J/psi radiative decay.



J/ψ-η_c 混合



QCD sum ruleからの結果と一致する　　　Hattori, Ozaki, Lee, Morita 2014

同じ効果は　V= photon, P= pi-zero でも起こる

講義3で議論したこと

強い外場(特に強磁場)中の粒子の性質

荷電粒子
- ランダウレベル
- 性質変化(質量、磁気能率)
- synchrotron放射
- nonlinear Compton scattering

中性粒子
- 光子の真空複屈折、崩壊、分裂
- π中間子の崩壊
- 混合