Physics with Muon Theoretical Aspects

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1.Introduction

- Muon

Very well observed, established, and defined

Mass :: 105.6583715(35) MeV

Lifetime :: $2.1969811(22) \times 10^{-6}$ s

Charge :: $-e$

Spin :: 1/2

- In the Standard Model , a field theory with gauge $SU(3) \times SU(2) \times U(1)$

Left-handed :: SU(2) doublet with Y=-1/2

Right-handed:: SU(2) singlet with Y=-1

With appropriate input, all quantities, say lifetime, anomalous magnetic moment etc. are all calculable, in principle.

- Well established

Heavier than electron, V-A weak interaction, etc

- Muon as a probe, “light” to watch inside

Muon tomography

Muon spin rotation (spectroscopy) , μSR

- Muon as a material

Muon catalyzed fusion

- Well observed, controlled

Precision measurement of free and bound muon

- To check QED for free and bound state, SM

Muonium hyperfine splitting	Bound QED
Lamb shift of Muonic atom	
Anomalous magnetic moment	Free QED

- To determine physical quantities

Muonium hyperfine splitting	Muon mass

- To test SM

Rare decay, EDM	New physics
Anomalous magnetic moment	SM test & NP

2. Hyperfine Splitting

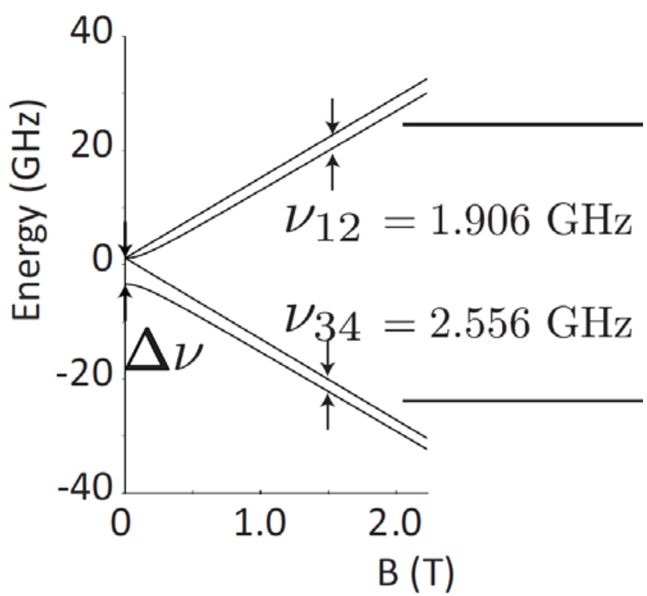
$\mu^+ e^-$ Muonium :: Hydrogen like "atom"

Hamiltonian under static magnetic field

$$\mathcal{H} = h\Delta\nu \mathbf{I}_\mu \cdot \mathbf{J} - \mu_B^\mu g'_\mu \mathbf{I}_\mu \cdot \mathbf{H} + \mu_B^e g_J \mathbf{J} \cdot \mathbf{H}$$

$\mathbf{I}_\mu$ $\mathbf{J}$ Muon and electron spin

μ's are Bohr magneton and g's are those in bound state



JPS Conf Proc 2 010405, 2014

$$\nu_{12} = -\frac{\mu_B^\mu g'_\mu H}{h} + \frac{\Delta\nu}{2}[(1 + x) - \sqrt{1 + x^2}],$$

$$\nu_{34} = +\frac{\mu_B^\mu g'_\mu H}{h} + \frac{\Delta\nu}{2}[(1 - x) + \sqrt{1 + x^2}],$$

$$x = (g_J \mu_B^e + g'_\mu \mu_B^\mu)H/(h\Delta\nu)$$

H is measured by proton precession.
Translation of H to μ_p via $2\mu_p H = h\nu_p$
We can measure

$\Delta\nu$ and μ_μ/μ_p

Test of bound QED

Precise calculation by QED as it is leptonic system.
Exp. And Theor. Values of Hyperfine splitting $\Delta\nu$

Determination of fundamental parameters

Mesurement of μ_μ/μ_p and

Phys.Rev. Lett. 82, 711 (1999).

$$\frac{m_\mu}{m_e} = \left(\frac{g_\mu}{2}\right)\left(\frac{\mu_p}{\mu_\mu}\right)\left(\frac{\mu_B^e}{\mu_p}\right) \qquad 206.768\,277(24)$$

Best Value

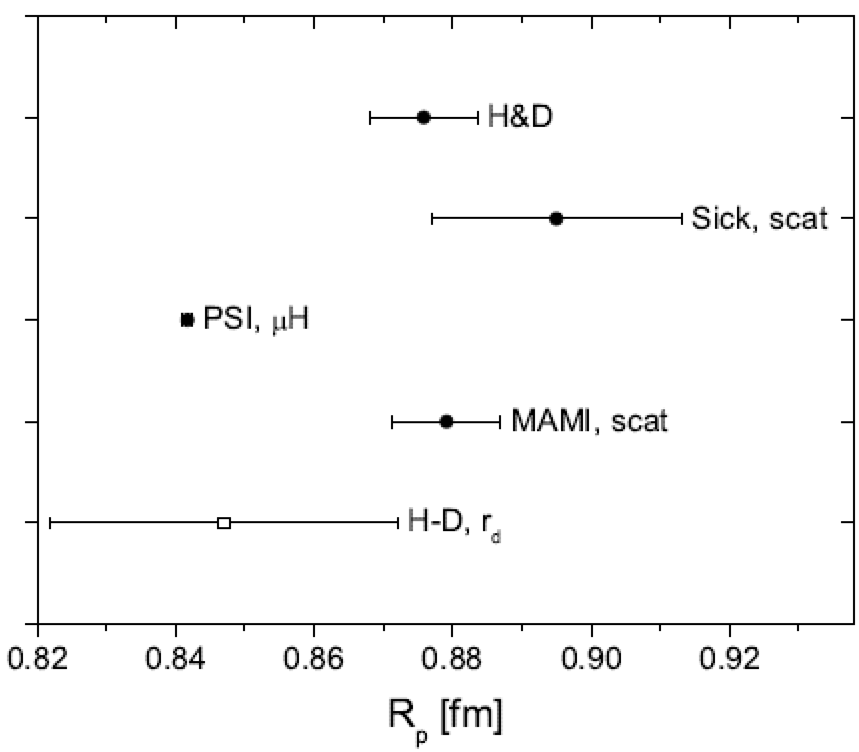
3. Lamb Shift

Lamb Shift in Muonic Hydrogen

root mean square of proton charge radius &

test of bound QED

Karshenboim 1405.6039



RE = 0.840 22(56) fm

Radius from Lamb shift of hydrogen and ep scattering is differ by 7σ

muon-specic dark forces ?

Karshenboim *et al*1401.6154

Nooo!!

Other constraints are too ristrictive !

4. Anomalous Magnetic Moment

Miller et al Rept.Prog.Phys.70:795,2007, & Nyffeler's lecture note

Magnetic moment :: proportional to spin

$$\vec{\mu} = g_\mu \frac{q}{2m_\mu} \vec{s}, \qquad \text{and} \qquad \underbrace{g_\mu}_{\text{Dirac}} = 2(1 + a_\mu)$$

Difference from Dirac theory a_μ

Anomalous magnetic moment :: field theoretical effect

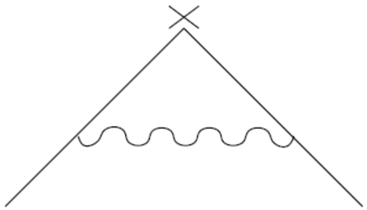
Test of QED and Standard Model

&

New physics

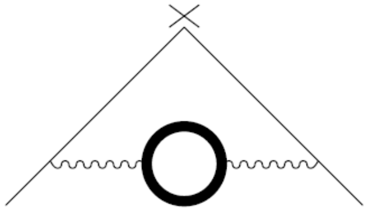
Field theoretical effect :: loop correction

- Anomalous magnetic moments are dimensionless
  To lowest order in perturbation theory in quantum electrodynamics (QED):



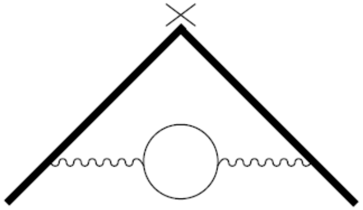
$$= a_e = a_\mu = \frac{\alpha}{2\pi}$$ [Schwinger 1947/48]

- Loops with different masses $\Rightarrow a_e \neq a_\mu$
  - Internal large masses decouple:



$$= \left[\frac{1}{45} \left(\frac{m_e}{m_\mu} \right)^2 + \mathcal{O}\left(\frac{m_e^4}{m_\mu^4} \ln \frac{m_\mu}{m_e} \right) \right] \left(\frac{\alpha}{\pi} \right)^2$$

  - Internal small masses give rise to large log's of mass ratios:



$$= \left[\frac{1}{3} \ln \frac{m_\mu}{m_e} - \frac{25}{36} + \mathcal{O}\left(\frac{m_e}{m_\mu} \right) \right] \left(\frac{\alpha}{\pi} \right)^2$$

Contribution of heavy particle is suppressed by

$$\left(\frac{m_l}{m_h}\right)^2$$ decoupling

Contribution of weak interaction for a_μ is larger by

$$\left(\frac{m_\mu}{m_e}\right)^2$$ than a_e

At 1-loop

$$a_\mu^{\mathrm{W}} = \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}(1-4\sin^2\theta_W)^2\right]$$

Precise value satisfies the relation

$$a_e(\mathrm{weak}) = 0.0297\ (5)\ \times 10^{-12}$$

$$a_\mu(\mathrm{weak}) = 154\ (2) \times 10^{-11}$$ ⟺ QED 4-loop $381.33\ (0.023)$ 10^{-11}

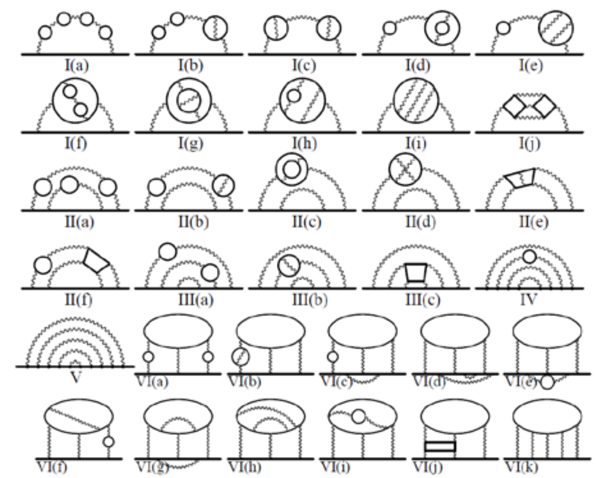
Sensitive to high energy

$$\left(\frac{m_\mu}{m_h}\right)^2 \sim 10^{-6}\left(\frac{100\mathrm{GeV}}{m_h}\right)^2 \sim 10^{-8}\left(\frac{1\,\mathrm{TeV}}{m_h}\right)^2$$

Comparable to QED 3 or 4 loop || 4 or 5 loop

Aoyama et al,Phys.Rev.Lett. 109 (2012) 111808

Current status



Theory

$$
\begin{aligned}
a_\mu^{\text{QED}} &= \frac{1}{2} \times \left(\frac{\alpha}{\pi}\right) \\
&+ 0.765\,857\,425 \underbrace{(17)}_{m_\mu/m_{e,\tau}} \times \left(\frac{\alpha}{\pi}\right)^2 \\
&+ 24.050\,509\,96 \underbrace{(32)}_{m_\mu/m_{e,\tau}} \times \left(\frac{\alpha}{\pi}\right)^3 \\
&+ 130.8796 \underbrace{(63)}_{\text{num. int.}} \times \left(\frac{\alpha}{\pi}\right)^4 \\
&+ 753.29 \underbrace{(1.04)}_{\text{num. int.}} \times \left(\frac{\alpha}{\pi}\right)^5 \\
&= 116\,584\,718.853 \underbrace{(9)}_{m_\mu/m_{e,\tau}} \underbrace{(19)}_{C_4} \underbrace{(7)}_{C_5} \underbrace{(29)}_{\alpha(a_e)} [36] \times 10^{-11}
\end{aligned}
$$

$$a_\mu(\text{had. v.p.}) = 6949.1\ (37.2)_{\text{exp}}(21.0)_{\text{rad}} \times 10^{-11}$$

$$a_\mu(\text{weak}) = 154\ (2) \times 10^{-11}$$

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{hadronic}) + a_\mu(\text{electroweak}) \quad = 116\ 591\ 840\ (59) \times 10^{-11}$$

Comparing with experiment

Muon g-2 Collaboration + Roberts

$$a_\mu(\text{exp}) = 116\ 592\ 089\ (63) \times 10^{-11} \quad [0.5\,\text{ppm}]$$

Discrepancy

$$a_\mu(\text{exp}) - a_\mu(\text{SM}) = 249\ (87) \times 10^{-11}$$

3 σ!?

More aggressive analysis are there.

New Physics ?

Assume that New Physics contribution with $M_{\text{NP}} \gg m_\mu$ decouples:

$$a_\mu^{\text{NP}} = \mathcal{C}\,\frac{m_\mu^2}{M_{\text{NP}}^2}$$

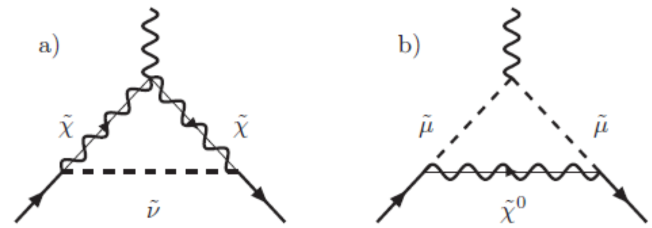
where naturally $\mathcal{C} = \frac{\alpha}{\pi}$, like from a one-loop QED diagram, but with new particles.
Typical New Physics scales required to satisfy $a_\mu^{\text{NP}} = \Delta a_\mu$:

$\mathcal{C}$	1	$\frac{\alpha}{\pi}$	$\left(\frac{\alpha}{\pi}\right)^2$
M_{NP}	$2.0^{+0.4}_{-0.3}$ TeV	100^{+21}_{-13} GeV	5^{+1}_{-1} GeV

a_μ: Supersymmetry

Supersymmetry for large $\tan\beta, \mu > 0$:

$$a_\mu^{\text{SUSY}} \approx 123 \times 10^{-11} \left(\frac{100 \text{ GeV}}{M_{\text{SUSY}}}\right)^2 \tan\beta$$



(Czarnecki, Marciano, 2001)

Explains $\Delta a_\mu = 290 \times 10^{-11}$ if $M_{\text{SUSY}} \approx (93 - 414)$ GeV $\quad (2 < \tan\beta < 40)$.

In some regions of the parameter space, there can also be large two-loop contributions.

5. Electric Dipole Moment

$$-d\mathbf{E}\cdot\frac{\mathbf{S}}{S}$$

P and T violating

0 in Dirac theory

For lepton 4-loop effect in SM

$$d_\mu < 10^{-36} e\ \mathrm{cm}$$

From SM

Experimental bound

Phys. Rev. D 80, 052008

$$|d_\mu| < 1.9\times 10^{-19} e\ \mathrm{cm}$$

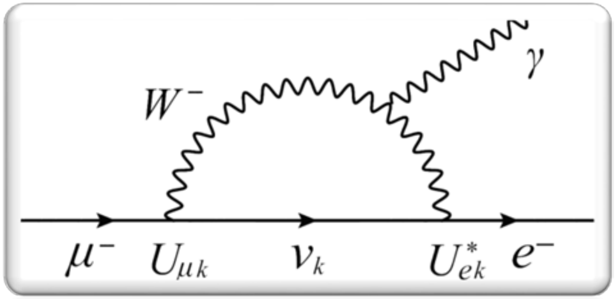
Very good probe for New physics

6.Lepton Flavor Violation

Lepton Flavor is exact symmetry in SM

as long as neutrinos are massless

Charged Lepton Flavor Violation
(cLFV) through Lepton Mixing
in the neutrino oscillation



But …

$$\mathrm{BR}(\mu \to e\gamma) \sim \left(\frac{\delta m_\nu^2}{m_W^2}\right)^2 < 10^{-54}$$

Invisible, eternally

Strong suppression of FCNC by GIM

Detection of the LFV signal

Clear evidence for beyond SM

Indeed, in physics beyond SM,

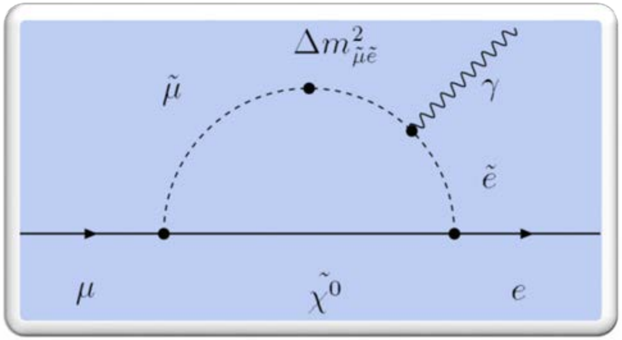
Large FCNC is expected

Particularly Combining with neutrino oscillation
Large FCNC in charged lepton is expected
(must appear ??)

e.g. a supersymmetric model

Enhancement of LFV
through the slepton mixing

⇨ Detectable at future experiments



Two or more flavors are needed to observe cLFV

⇨ ***Search for LFV with muon is inevitable***

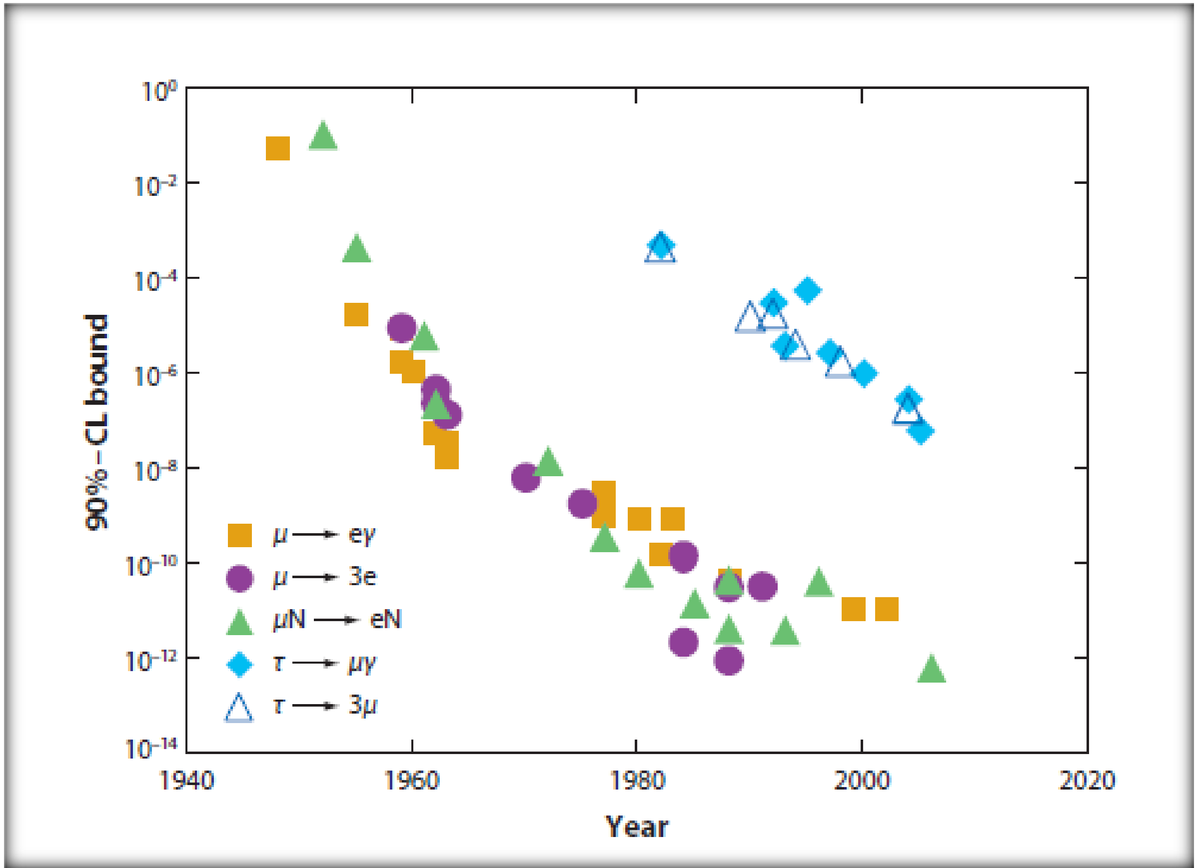
cLFV from muon decay

Upper limit on Br

$\mu^+ \to e^+\gamma$	$<2.4 \times 10^{-12}$
$\mu^+ \to e^+e^+e^-$	$<1.0 \times 10^{-12}$
$\mu^-\mathrm{Ti} \to e^-\mathrm{Ti}$	$<6.1 \times 10^{-13}$
$\mu^-\mathrm{Au} \to e^-\mathrm{Au}$	$<7 \times 10^{-13}$
$\mu^+e^- \to \mu^-e^+$	$<8.3 \times 10^{-11}$

Annu. Ref. Nucl. Part. Sci. 2008. 58:315-41
W. J. Marciano, T.Mori, and J. M. Roney

Long history



Effective operators

A) Loop vs Tree

$\mu^+ \to e^+\gamma$ :: Loop only, dipole

Gauge Symmetry forbids tree contribution

$\mu^+ \to e^+e^-e^+$

$\mu^- N \to e^- N$ :: Loop and Tree

e.g. Loop = dipole + quark bilinear = $\mu^- N \to e^- N$

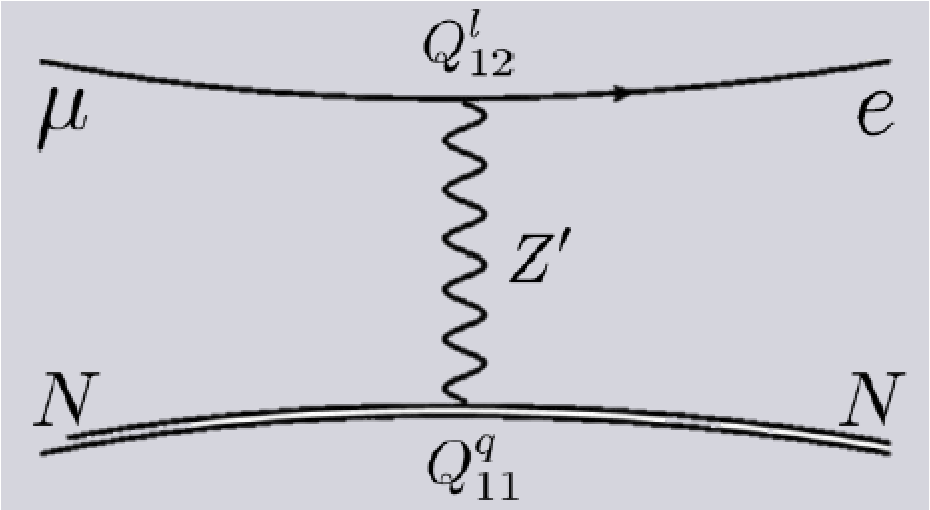
$\sim$ α smaller than μ->eγ

Tree :: singlet particle is necessary for conversion!

Charge 2 is OK for μ -> 3e

Leptoquark is OK for μ -> e

Tree, e.g.



No direct relation with MEG
NO suppression

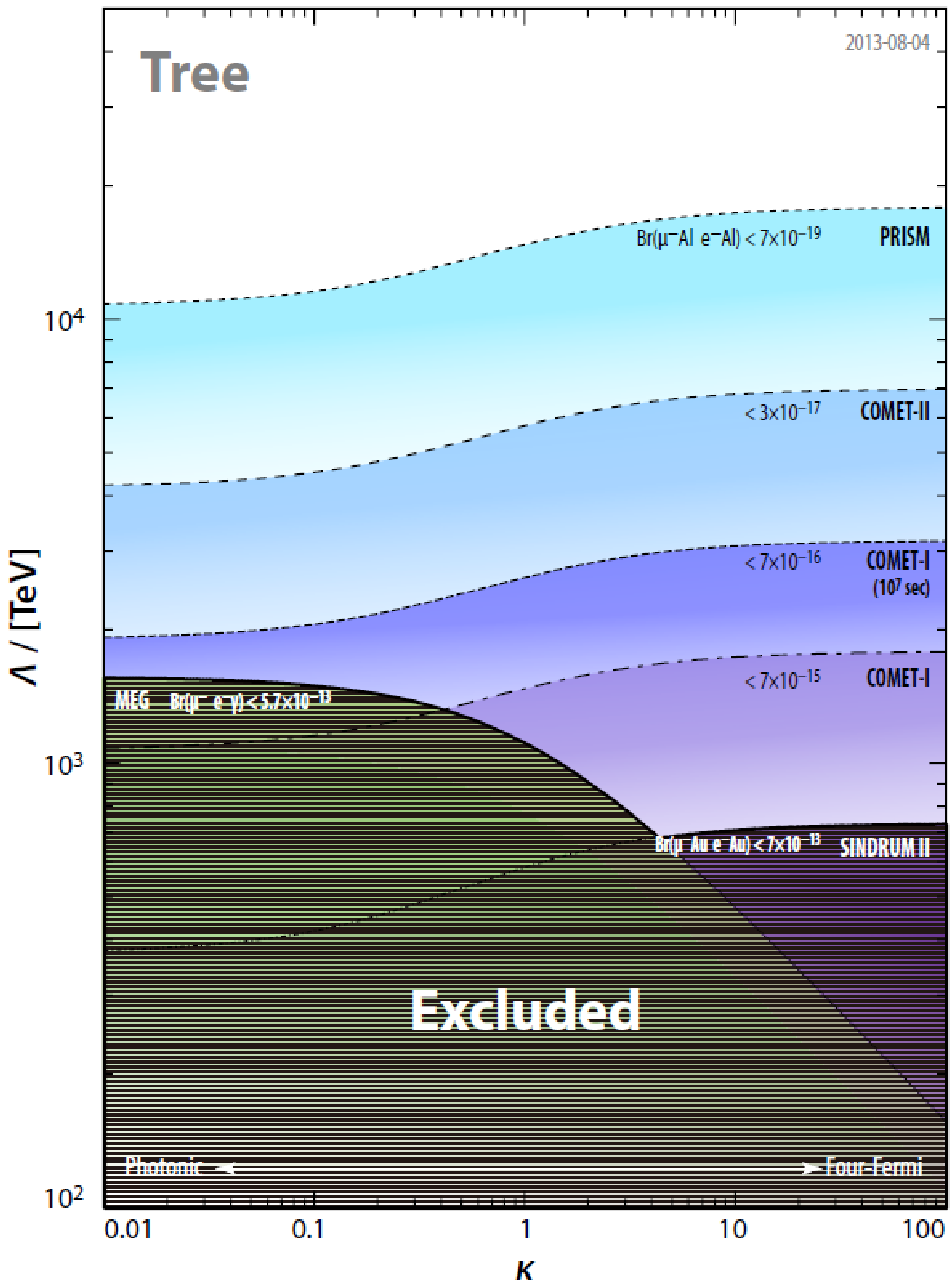
We can parameterize the relative strength

$$\mathcal{L} = \frac{1}{1+\kappa}\frac{m_\mu}{\Lambda^2}\bar{\mu}_{\mathrm{R}}\sigma^{\mu\nu}e_{\mathrm{L}}F_{\mu\nu} + \frac{\kappa}{1+\kappa}\frac{1}{\Lambda^2}(\bar{\mu}_{\mathrm{L}}\gamma^\mu e_{\mathrm{L}})(\bar{q}_{\mathrm{L}}\gamma_\mu q_{\mathrm{L}})$$

$\kappa \sim$ α :: dipole type, say SUSY with R parity

In general, Model Dependent

MEG



PRISM

COMET Phase-II

COMET Phase-I

B) Vector vs Scalar

cLFV is mediated by new particle(s)

Vector Boson ::

Boson with broken gauge

So-called Z' Model, Extra U(1) from SO(10) GUT

Kaluza-Klein mode of gauge

Higher dimensional models have massive modes of gauge bosons

Scalar Boson ::

From symmetry = SUSY

Extension of Higgs :: more 2plet, 3plet for nu mass

Explanation for new physics

Vector type interaction

If Vector boson has no charge

$\mu^+ \to e^+ e^- e^+$ and $\mu^- N \to e^- N$

can occur at tree level

in a wide sense Z' model

$$\mu^+ \to e^+ \gamma \Longleftrightarrow \mu^- N \to e^- N$$

irrelevant

cLFV Interaction

$$\frac{g_{Z'}}{\sin\theta_W}\bar{l}_i Q_{ij}\gamma^\mu l_j Z'_\mu$$

Experiment type	Charges probed
$\mu^- N \to e^- N$	Q_{12}
$\mu^+ \to e^+\gamma$	$Q_{13}Q_{23}$ (and all others)
$\mu^+ \to e^+e^-e^+$	$Q_{11}Q_{12}$
$e^+e^- \to \mu^+\tau^-(\mu^-\tau^+)$	$Q_{11}Q_{23}$ ($Q_{12}Q_{13}$)
$e^+e^- \to \mu^+\mu^-$	$Q_{11}Q_{22}$ (Q_{12})
muon $g-2$	Q_{23} ($Q_{21}Q_{22}$)

Table 1: The CFLV experiments and the corre- sponding Z' charges probed at lowest order pro-

Different Q's !!

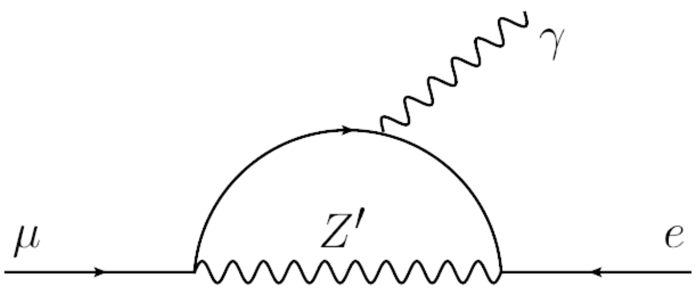


Figure 2: $\mu^+ \to e^+\gamma$ in the Z' models.

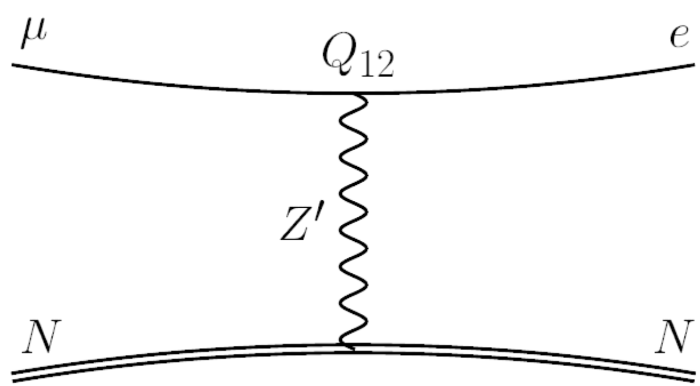
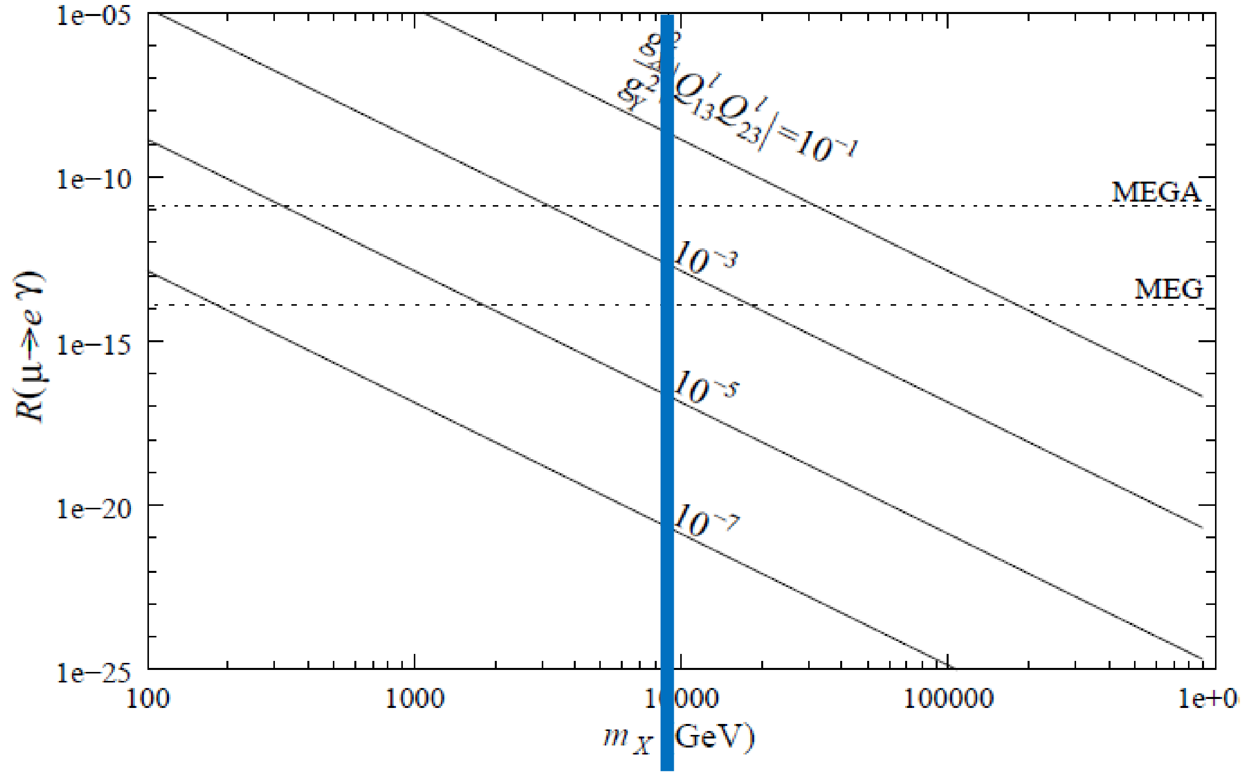


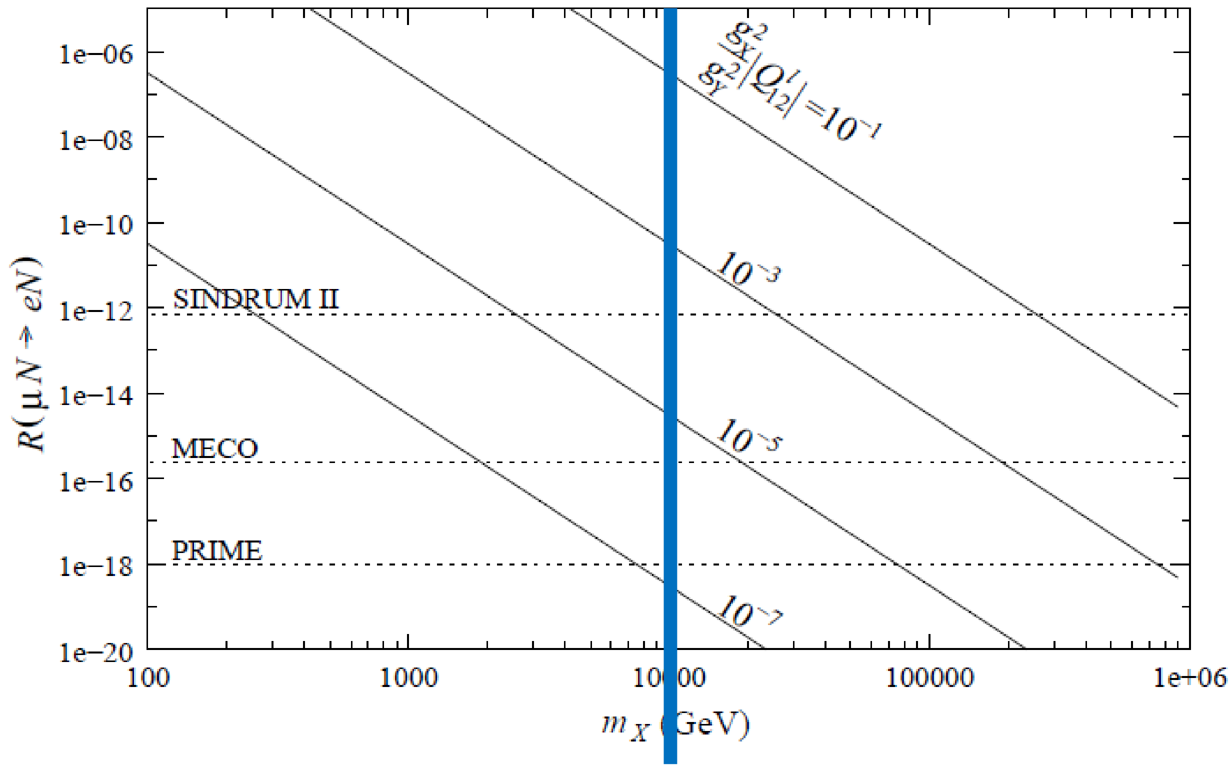
Figure 3: Non-photonic diagram of μ^-–e^- conversion in the Z' models.



10TeV

Figure 4: Constraint of Z' by the current search for $\mu^+ \to e^+\gamma$.

$$B(\mu \to e\gamma) = 1.3 \times 10^{-13} \left(\frac{g_x}{g_Y}\right)^4 \left(\frac{Q_{13}Q_{23}}{10^{-5}}\right)^2 \left(\frac{1\mathrm{TeV}}{m_{Z'}}\right)^4$$



10TeV

Figure 5: Constraint of Z' by the current search for $\mu^- - e^-$ conversion.

$$B(\mu N \to eN) = 3.1 \times 10^{-11} \left(\frac{g_{Z'}}{g_Y}\right)^4 \left(\frac{Q_{12}}{10^{-5}}\right)^2 \left(\frac{1\ \text{TeV}}{m_{Z'}}\right)^4$$

Direct Search at LHC ,excluded < 2.5TeV

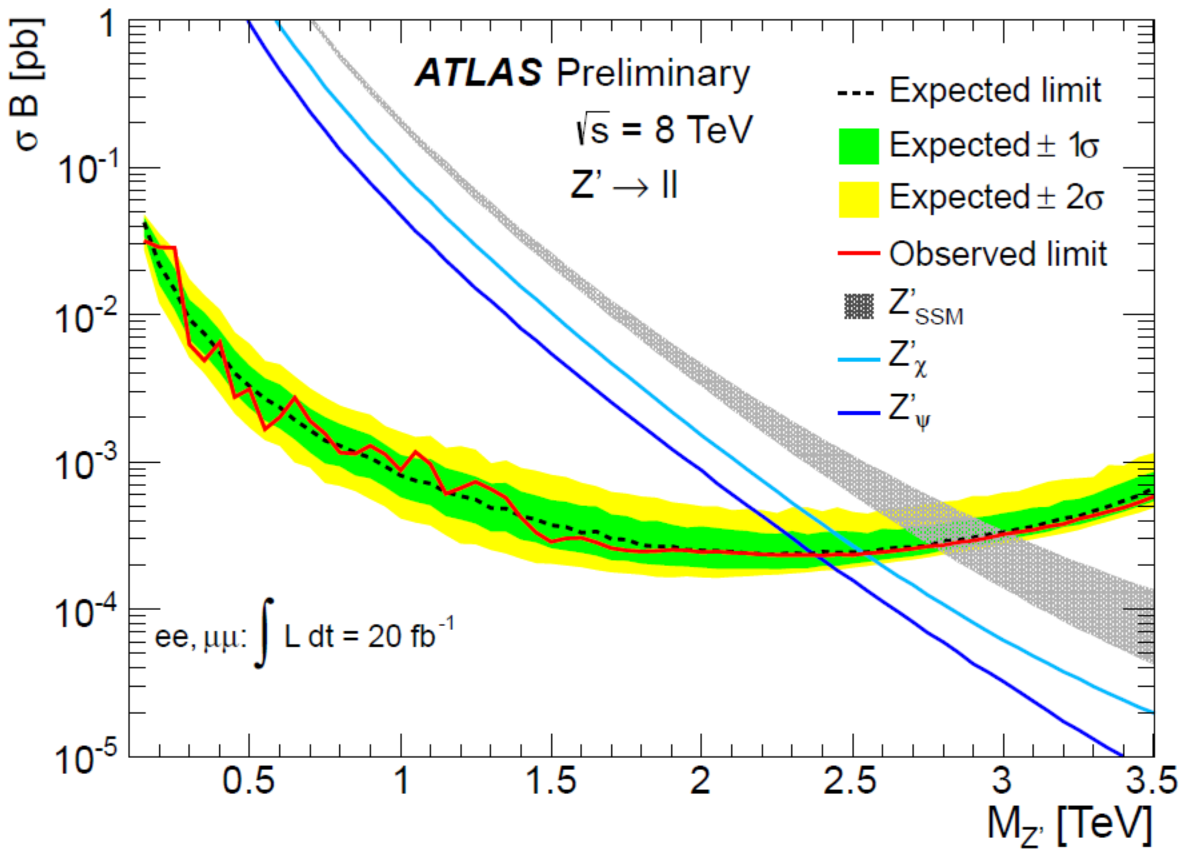


Figure 1: Expected and observed 95% C.L. limits on the combination of the production cross section of Z', σ, and its branching ratio of dilepton decay, B. Z' bosons in the benchmark models lighter than 2.5 TeV has been excluded.

Scalar type

SUSY :: Still main target!?

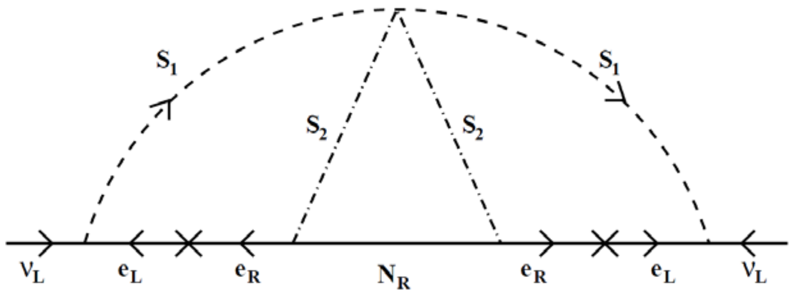
2< doublet higgs :: SUSY is restricted version

{ Higgs triplet :: doubly charged

Radiative generation of neutrino masses

Krauss *etal*

sometimes doubly charged



$$\mu^+ \to e^+ e^- e^+$$

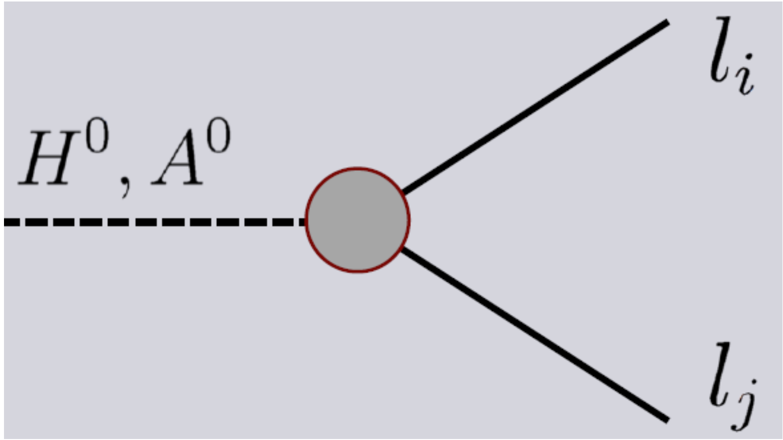
Is more relevant

SUSY

Neutral scalar : Heavy neutral higgs , sneutrino

With R-Parity

Higgs can contribute at tree level



Naïve 2< doublets, this coupling can be large, though…

In SUSY , slepton mixing must be contributed , that is, the couplings has same or less magnitude as dipole

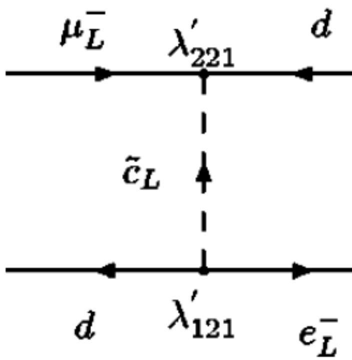
Furthermore, these higgses are probably very heavy

If R parity is broken,

$$W_{RPV}=\frac{\lambda_{ijk}}{2}L_iL_j\overline{E}_k+\lambda'_{ijk}L_iQ_j\overline{D}_k+\lambda''_{ijk}\overline{U}_i\overline{D}_j\overline{D}_k+\mu'_iL_iH_u$$

Tree contribution may dominate for $\mu - e$ conversion

Leptoquark



While $\mu \rightarrow e + \gamma$

Induced by loop

→ distinction of models

Andre´ de Gouveˆa, Smaragda Lola, and Kazuhiro Tobe

	$\frac{Br(\mu\to e\gamma)}{Br(\mu\to 3e)}$	$\frac{R(\mu\to e \text{ in Ti})}{Br(\mu\to 3e)}$	A_P	A_{P_1}	A_{P_2}	A_{P_1}/A_{P_2}
$\lambda'_{121}\lambda'_{221}$	1.1	2×10^5	−100%	−26%	−5%	5.6
MSSM with ν_R	1.6×10^2	0.92	−100%	10%	17%	0.6

Singlet = sneutrino

Optimistic case $\lambda_{132}\lambda'_{311}$ exists only → μe conversion only

Orthodox scenario

Source of LFV
=
Slepton mixing

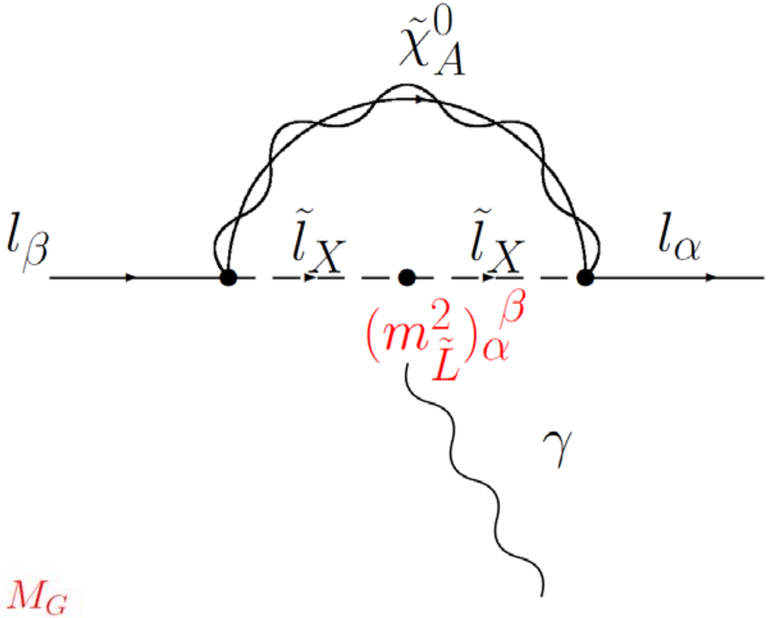
$$m_{\tilde{l}}^2 = \begin{pmatrix} m_{\tilde{e}\tilde{e}}^2 & \Delta m_{\tilde{e}\tilde{\mu}}^2 & \Delta m_{\tilde{e}\tilde{\tau}}^2 \\ \Delta m_{\tilde{\mu}\tilde{e}}^2 & m_{\tilde{\mu}\tilde{\mu}}^2 & \Delta m_{\tilde{\mu}\tilde{\tau}}^2 \\ \Delta m_{\tilde{\tau}\tilde{e}}^2 & \Delta m_{\tilde{\tau}\tilde{\mu}}^2 & m_{\tilde{\tau}\tilde{\tau}}^2 \end{pmatrix}$$

CMSSM + RH neutrino

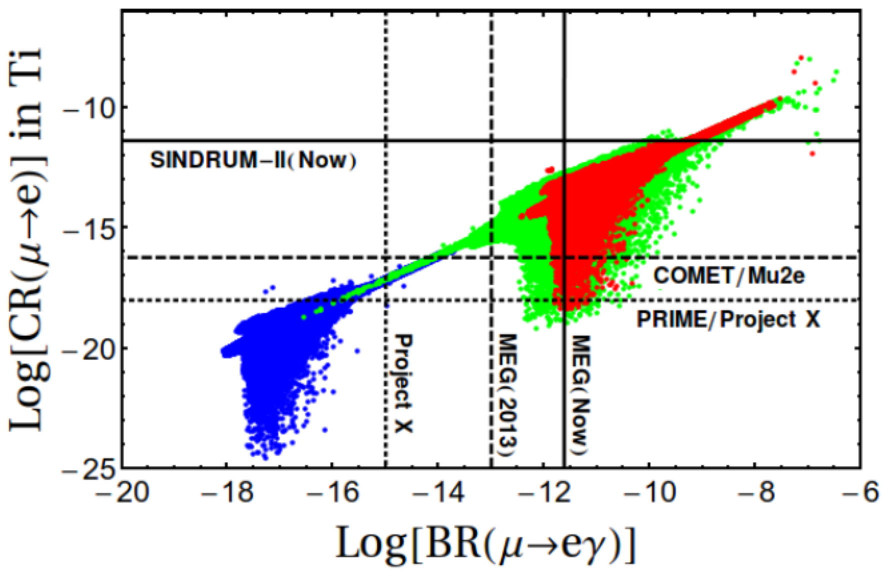
Most exhaustedly studied

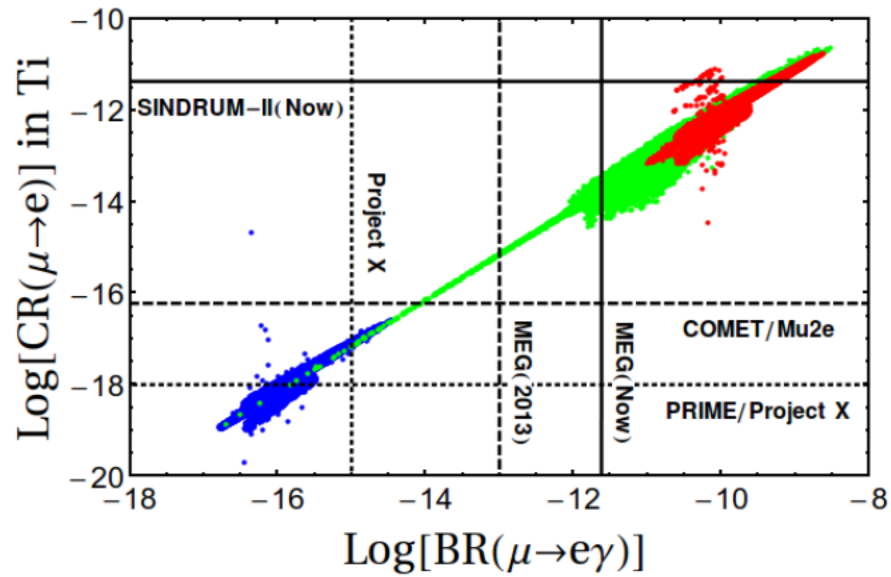
$$W = f_\nu^{i\beta} \bar{N}_i L_\beta H_u$$

$$(m_{\tilde{L}}^2)_\alpha^\beta \simeq -\frac{(6+a_0^2)m_0^2}{16\pi^2}(f_\nu^\dagger f_\nu)_\alpha^\beta \log\frac{M_G}{M_R}$$
$$\simeq -\frac{(6+a_0^2)m_0^2}{16\pi^2} U_{\alpha k}^{Dirac}(U^{Dirac*})^{\beta k}|f_{\nu k}|^2 \log\frac{M_G}{M_R}$$



Dipole dominant


Log[CR(μ→e)] in Ti
Log[BR(μ→eγ)]
SINDRUM−II(Now)
COMET/Mu2e
PRIME/Project X
Project X
MEG(2013)
MEG(Now)
−10
−15
−20
−25
−20
−18
−16
−14
−12
−10
−8
−6

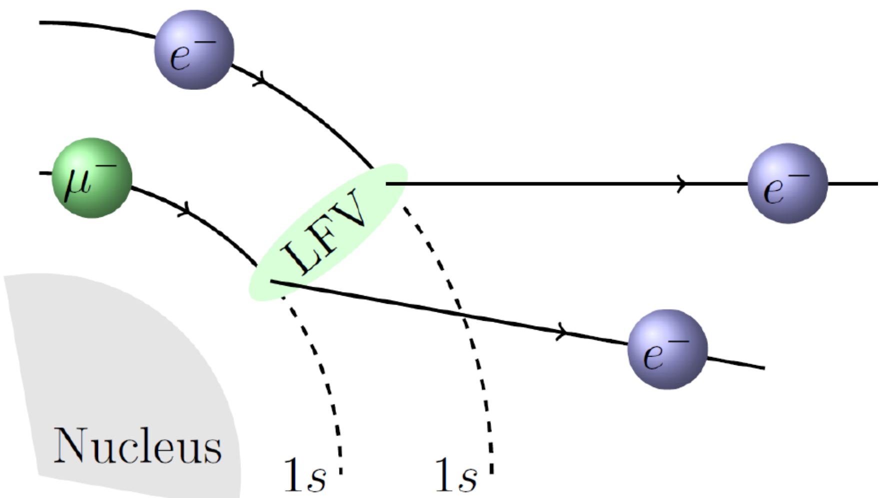

Log[CR(μ→e)] in Ti
Log[BR(μ→eγ)]
SINDRUM−II(Now)
COMET/Mu2e
PRIME/Project X
Project X
MEG(2013)
MEG(Now)
−10
−12
−14
−16
−18
−20
−18
−16
−14
−12
−10
−8

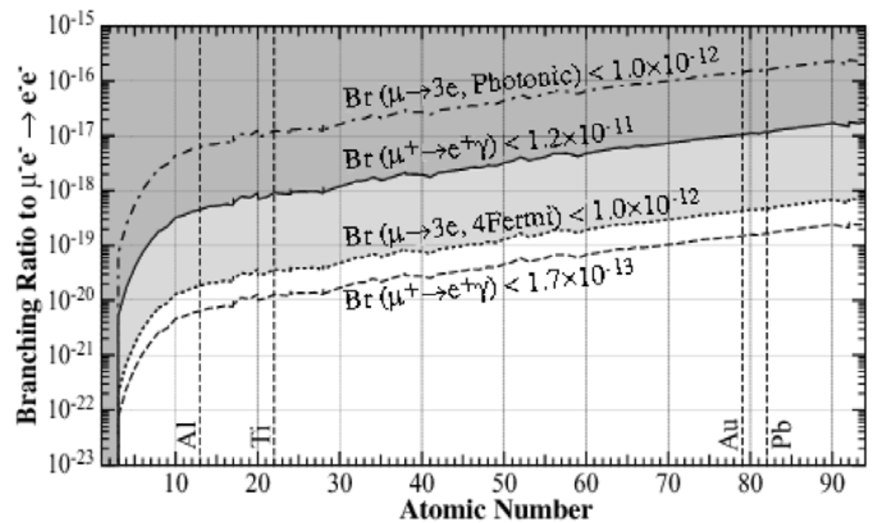
Another idea !

$$\mu^- + e^- \to e^- + e^- \ \textbf{in orbits}$$

Same lagrangian for muon LFV

$$\mathrm{Br}(\mu^- e^- \to e^- e^-) = 3.3 \cdot 10^{-12} \times (Z-1)^3 \left(\frac{\tilde{\tau}_\mu}{\tau_\mu}\right) G$$





7. Muonium ←→ Anti-muonium

$$\mu^+ e^- \leftrightarrow \mu^- e^+$$

Muon # +2 Electron # -2

Latest Result Phys. Rev. Lett. 82, 49 (1999)

Upper limit on conversion probability

$$\mathrm{P_{M\overline{M}}} \leq 8.2 \cdot 10^{-11} / \mathrm{S_B} \ (90\% \ \mathrm{C.L.})$$

$$\mid \lambda_{132} \lambda^*_{231} \mid \leq 3 * 10^{-4}$$

R Parity violation

$$\mathrm{G_{M\overline{M}}} = \mathrm{G_F} \cdot \sqrt{\frac{\mathrm{P_{M\overline{M}}(0T)}}{2.56 \cdot 10^{-5}}}$$

$$\leq 3.0 \cdot 10^{-3} \mathrm{G_F} (90\% C.L.)$$

100GeV sneutriono

2. Hyperfine Splitting

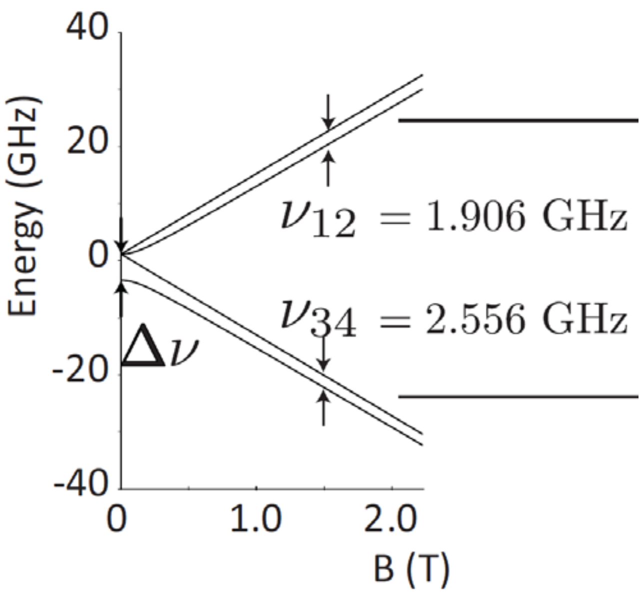
$\mu^+ e^-$ Muonium :: Hydrogen like "atom"

Hamiltonian under static magnetic field

$$\mathcal{H} = h\Delta\nu \mathbf{I}_\mu \cdot \mathbf{J} - \mu_B^\mu g'_\mu \mathbf{I}_\mu \cdot \mathbf{H} + \mu_B^e g_J \mathbf{J} \cdot \mathbf{H}$$

$\mathbf{I}_\mu$ $\mathbf{J}$ Muon and electron spin

μ's are Bohr magneton and g's are those in bound state



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$$\nu_{12} = -\frac{\mu_B^\mu g'_\mu H}{h} + \frac{\Delta\nu}{2}[(1+x) - \sqrt{1+x^2}],$$

$$\nu_{34} = +\frac{\mu_B^\mu g'_\mu H}{h} + \frac{\Delta\nu}{2}[(1-x) + \sqrt{1+x^2}],$$

$$x = (g_J \mu_B^e + g'_\mu \mu_B^\mu)H/(h\Delta\nu)$$

H is measured by proton precession.
Translation of H to μ_p via $2\mu_p H = h\nu_p$
We can measure

$\Delta\nu$ and μ_μ/μ_p

8.Summary

Muon plays many roles

“Completely “ measured :: probe to “see” something

Well controlled & Theoretically well defined

test of theory

QED(Field theory)

HFS, Lamb Shift, magnetic moment

SM

magnetic moment, EDM, rare decay (cLFV)

determination of fundamental parameter(s)

muon mass

HFS