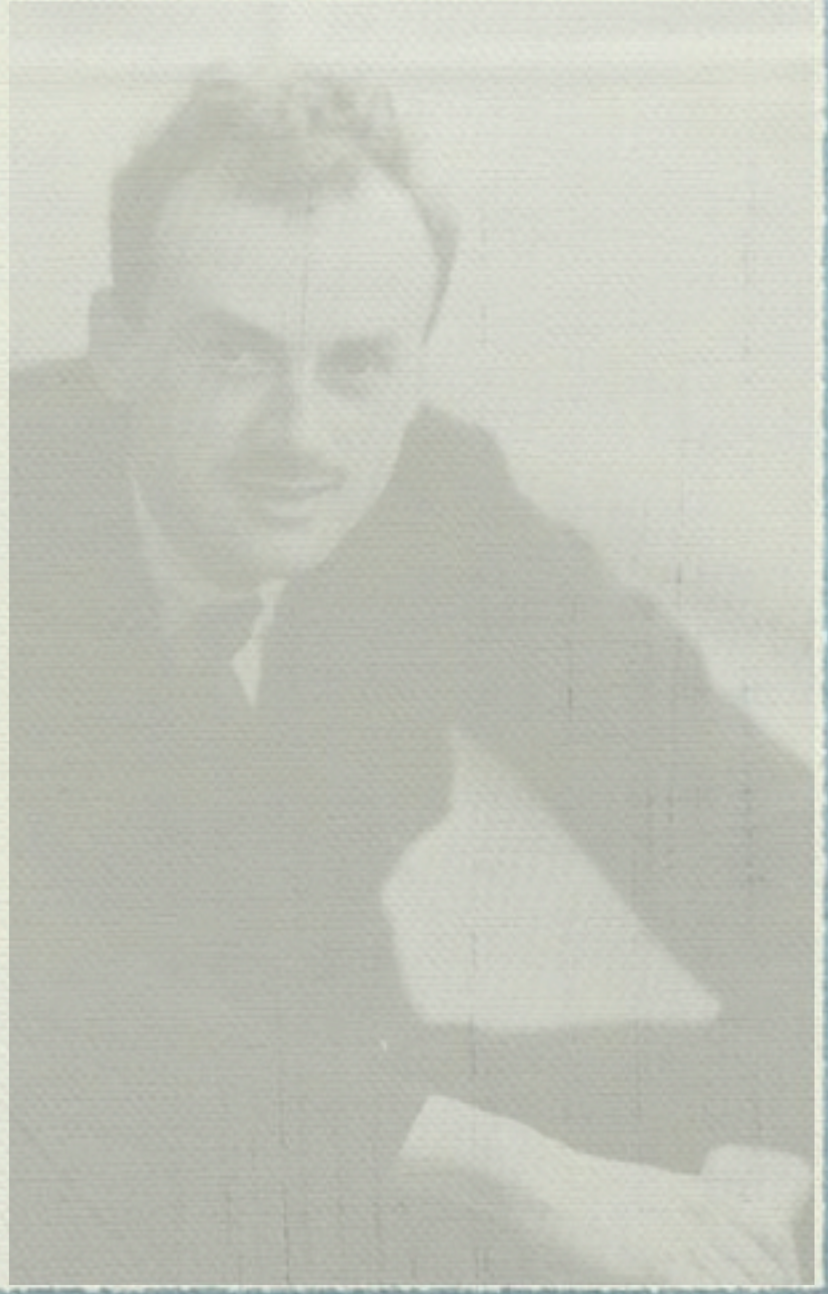
Tamiaki Yoneya
8/5,6/2014

弦理論とは何だろうか

1 Prologue
2 A brief history of string theory
3 The rudiments of string theory: a short course of string quantum mechanics
4 What is (and should be) string theory?



私のこれまでの若手夏の学校講義

- 『量子ゲージ場の相』　1979.8.6

- 『Introduction to conformal symmetry and its applications』　1989.8

「この度、来年度の原子核三者若手夏の学校において弦理論パートの講義をしていただきたく、連絡させていただきました。講義の内容ですが、主に修士の学生向けに弦理論の基礎からお話していただきたいと考えております。その際、現在に至るまでの歴史的なエピソードを交えてお話していただけるとありがたいです。」

２０１３年１０月２４日、重神さんからのメッセージ

1979. 8月6日

夏の学校講義　「量子ゲージ場の相」

米谷民明　北大理

1.序

現在の場の理論の最重要な問題の一つは，ゲージ場のミクロな構造がどのようなマクロ構造に導くかという問題である。これは原子・分子の相互作用から出発して物質の種々の可能な熱力学的相を理解しようとすることと同質の問題であるが，素粒子論においても最近のクオーク・レプトン物理の進展によって現実的な意味を持つようになって来た。この講義では，今までこの問題についてどの程度のことが明らかにされているかを，なるだけ初等的なところから始めて，その考え方・直観的な見通しに重点を置いて述べることにしたい。計算の details や数学的技術，およびここでは触れ得なかった重要事項（例えば Block spin の方法）については，末尾にあげる参考文献を勉強していただきたい。

以下では場の量子論の出発点として，Euclid 空間の path integral をとる。もし Hamiltonian H と，理論をその上で考えるべき Hilbert 空間が与えられていると，Euclidean path integral は

$$\lim_{T\to\infty} \mathrm{Tr}\left(e^{-HT}\right) \equiv Z \qquad (1.1)$$

に他ならない。path integral から出発するということは，最初に Z を書き下すことによって，Hamiltonian と Hilbert 空間を同時に定義したことになる。(1.1) において $T\to\infty$ の極限は，H の最低固有状態つまり真空を取り出すことになり，path

'89. 夏の学校

Introduction to Conformal Symmetry and its applications

— A view as a st

I. Introduction

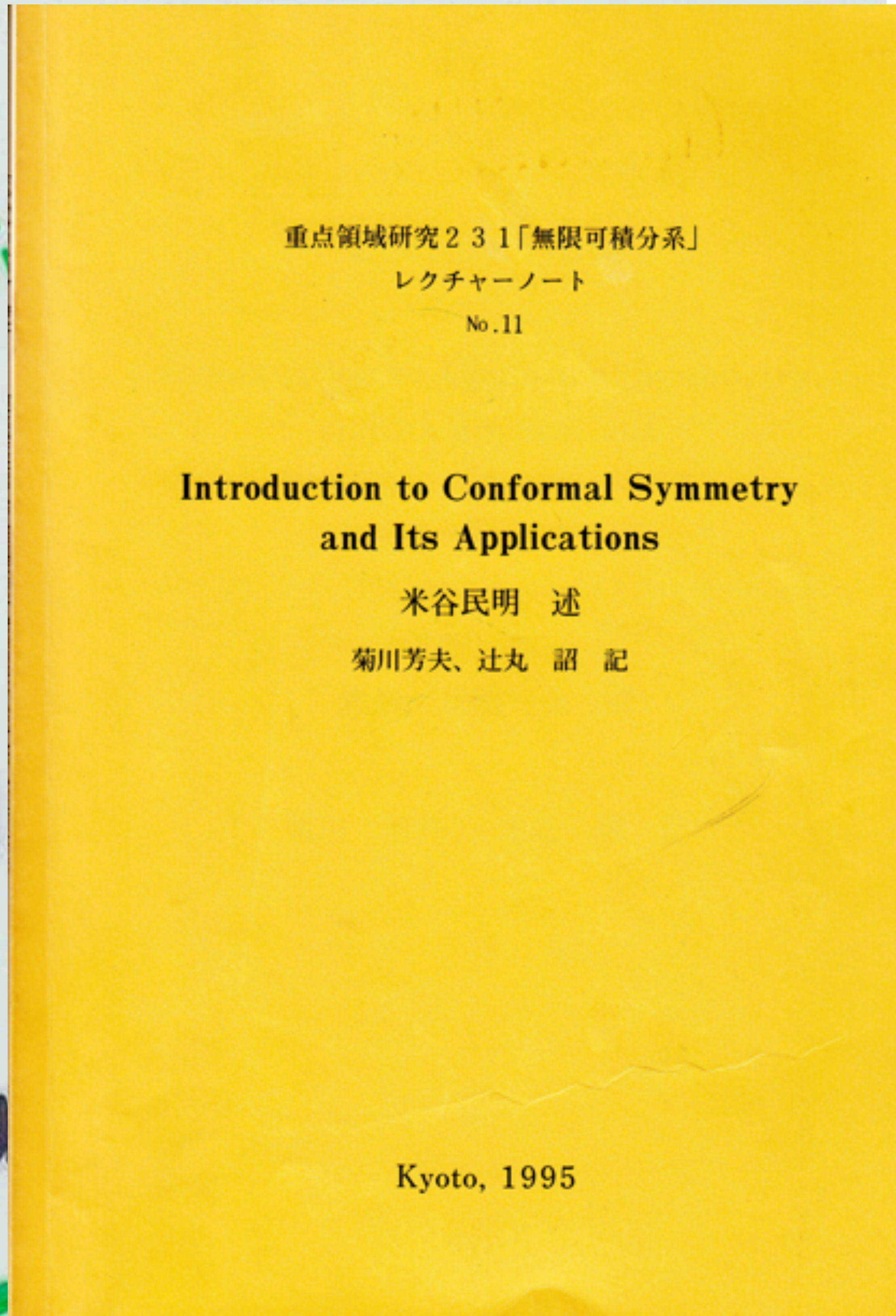
historicals. ...

II. Kinematics of conformal symmetry

Conformal trans. & Weyl transf.

III. Derivation of 2 dim. conformal Ward

connection with conformal anoma



重点領域研究２３１「無限可積分系」
レクチャーノート
No.11

Introduction to Conformal Symmetry and Its Applications

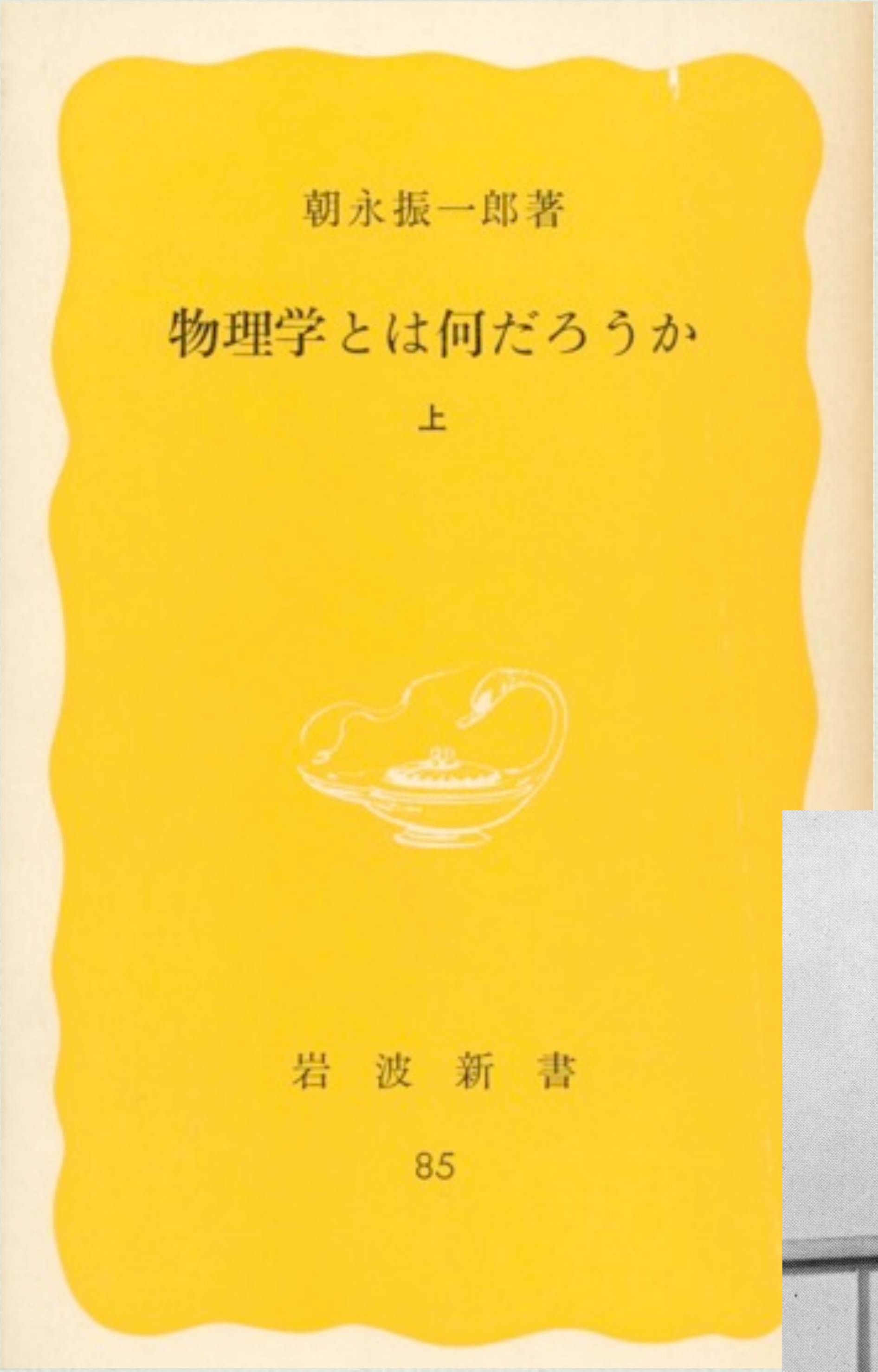
米谷民明　述

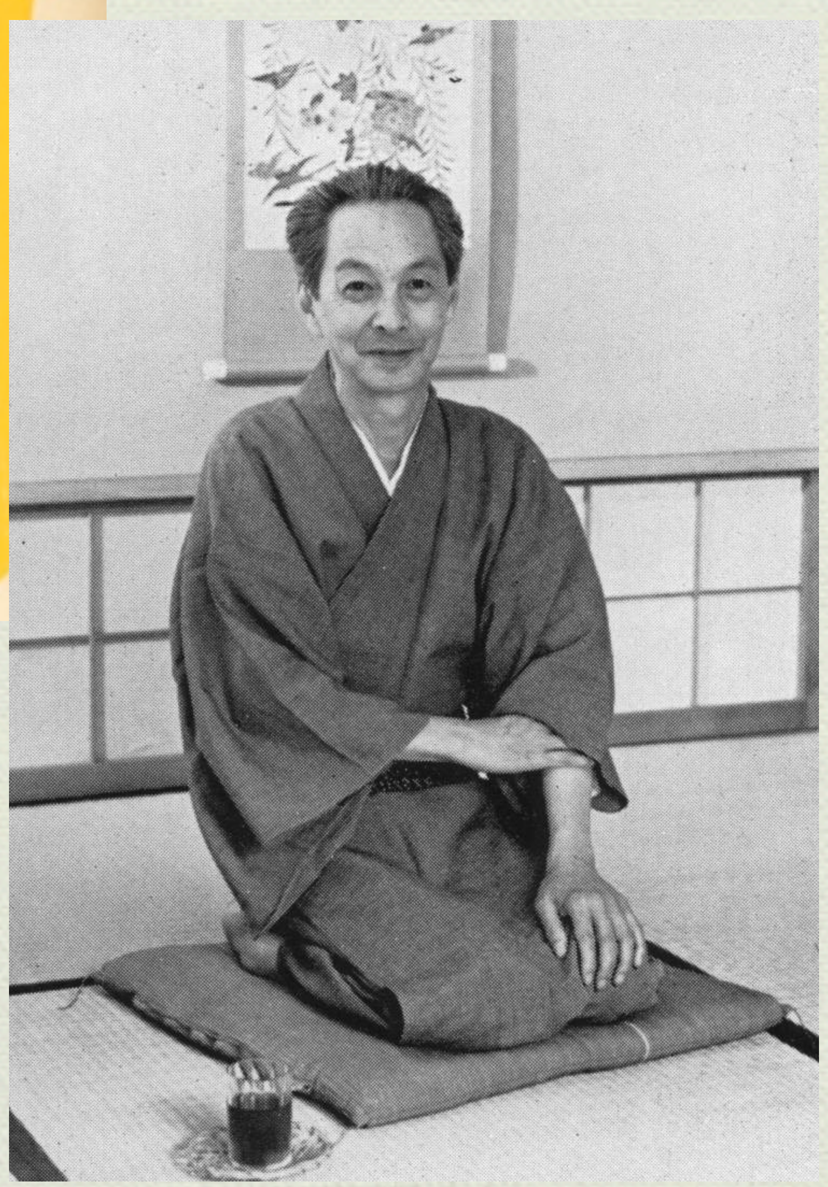
菊川芳夫、辻丸　詔　記

Kyoto, 1995

プロローグ：物理学とは何だろうか

「われわれをとりかこむ自然界に
生起するもろもろの現象
—ただし主として無生物にかんするもの—
の奥に存在する法則を、観察事実に
拠りどころを求めつつ追求すること」





宇宙 10^{28}

銀河 10^{24}

太陽系 10^{14}

地球 10^{10}

人間 10^{2}

ビールス 10^{-4}

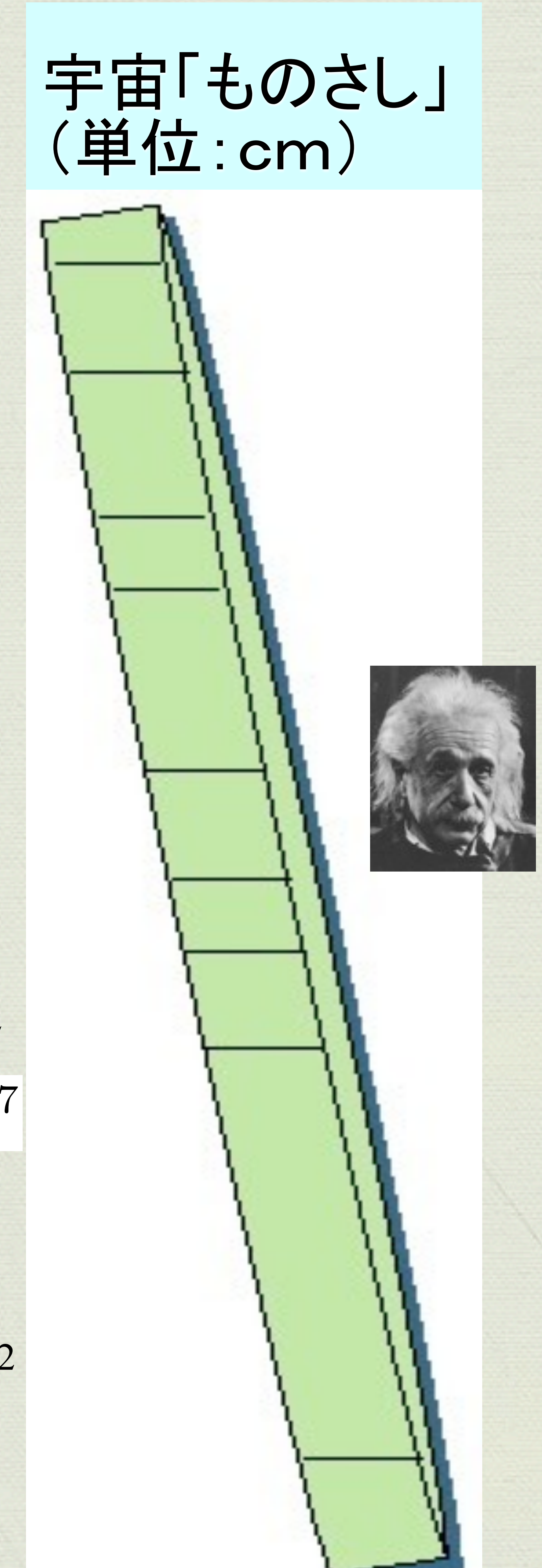
原子 10^{-8}

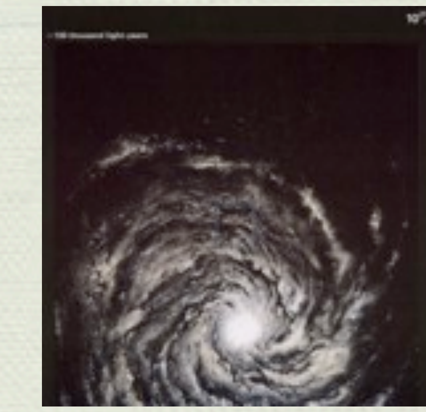
原子核 10^{-12}

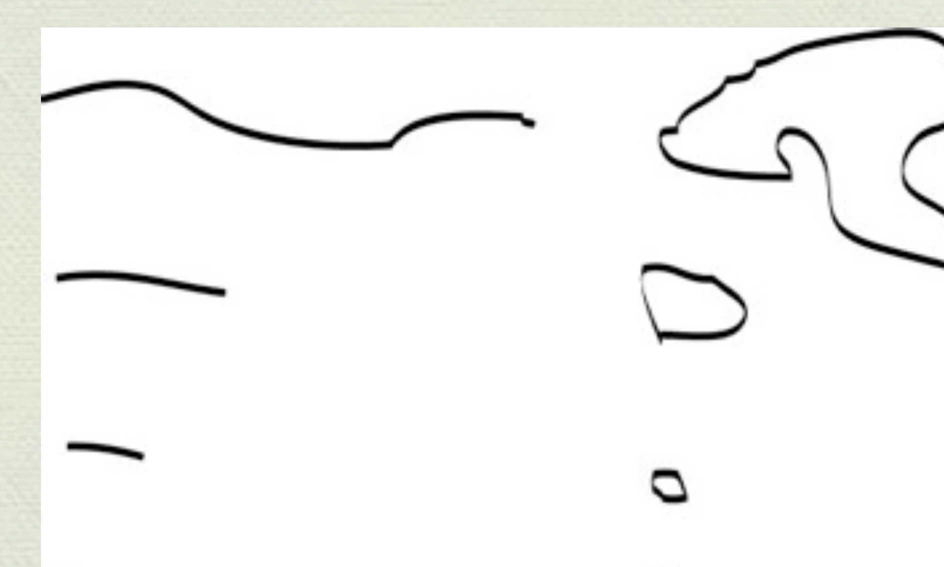
10^{-17}

現存の加速器で到達できる長さのスケール

プランク距離 10^{-32}





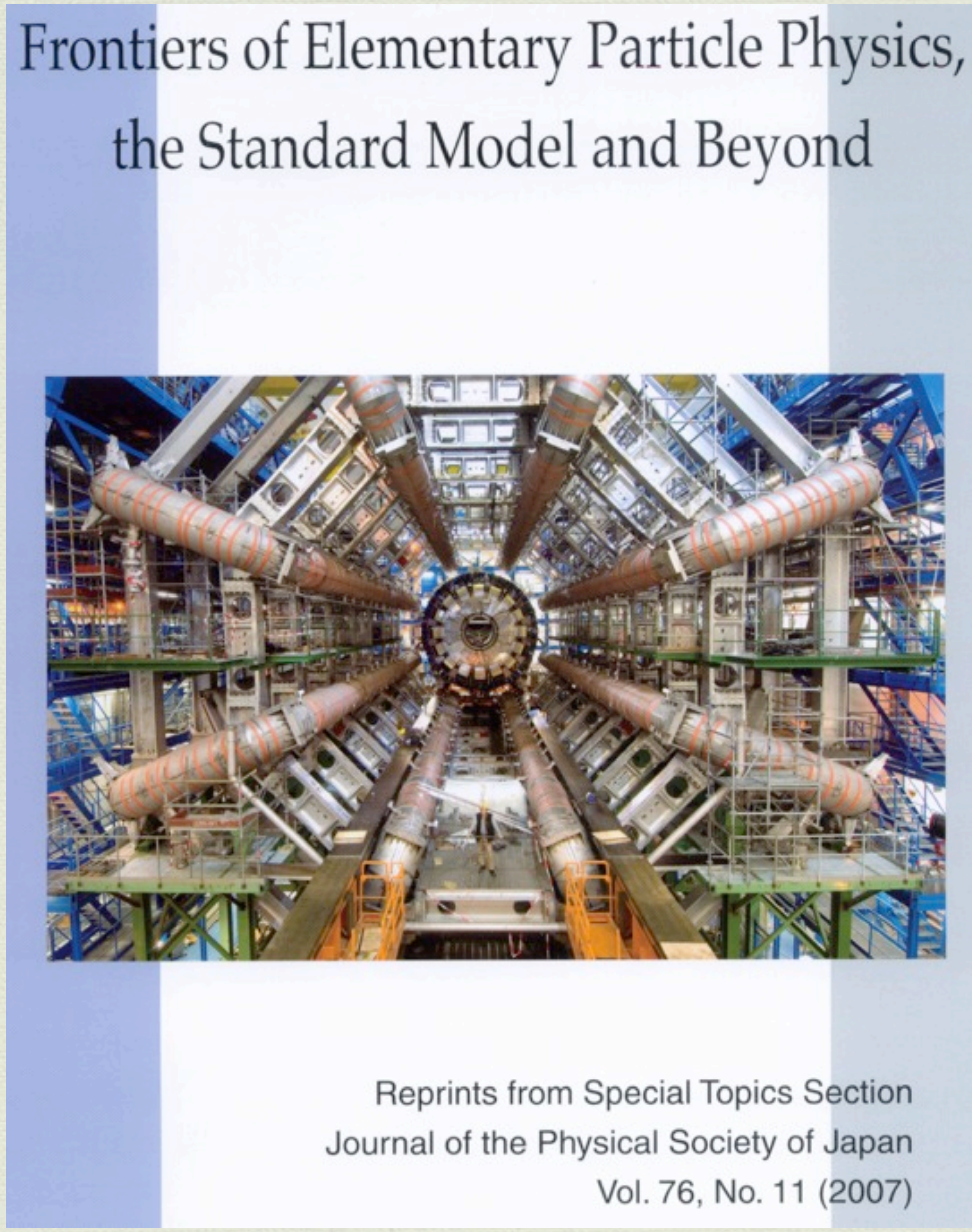


この宇宙の “超ミクロ” から “超マクロ” にわたる多様な現象,
ひいては、宇宙の起源、成り立ち、仕組を
できるだけ**少数の基本的物理法則に基づき探求**

物理学の法則が深化し普遍性が高まるとともに,
適用範囲・応用が広がり,
より多様な現象の理解へと地平が開けてきた

ミクロの方向の理解の深化が, マクロの方向の理解に役立ったり
逆に,
マクロの方向の理解で得られた新しい見方がミクロの方向の探求に役立つ

弦理論はその誕生と発展の経緯と現状から見て、その意義を正しく理解するは、
物理学の 歴史の中で位置づける ことも重要



T.Y. "***String Theory",***
J. Phys. Soc. J. 75(2007)
111020

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Vol. 76, No. 11, November, 2007, 111020

SPECIAL TOPICS

Frontiers of Elementary Particle Physics, the Standard Model and Beyond

String Theory

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This article is devoted to a nontechnical review on the present status of string theory towards an ultimate unification of all fundamental interactions including gravity. In particular, we emphasize the importance of string theory as a new theoretical framework in which the long-standing conflict between quantum theory and general relativity is resolved.

KEYWORDS: string theory, unified theory, quantum gravity, gravity-gauge correspondence
DOI: 10.1143/JPSJ.76.111020

1. Historical Background

The standard model has provided us a good understanding of the basic properties of present-day elementary particles, quarks, leptons and gauge bosons. Given various numerical data, we can in principle compute the probability amplitudes of every possible process involving these particles. However, it seems needless to emphasize that the model is still quite incomplete from a theoretical standpoint. In addition to the fact that the standard model has to assume a large number of input parameters, the model regarded as a fundamental theory of matter and its interactions is yet at a very unsatisfactory level, since it says nothing about quantum gravitational interaction of elementary particles.

As is well known, the mathematical framework of the standard model, gauge-field theory, has been developed in our endeavor towards unification of fundamental interactions: the structure of the standard model is governed by non-Abelian gauge symmetries. Even putting aside universal gravitational force, however, the standard model still has not really achieved desired unification between electroweak and strong nuclear forces. We often expect that the idea of unified gauge theory could be extended to a unification, "grand" unification, of these two fundamental forces. In regard to gravity, however, a majority of us now agree, after intensive efforts of many years, on that the ultimate unification of general relativity with quantum gauge-field theory would require a totally new mathematical framework. Hawking radiation. Of course, in terms of classical physics, the effect of gravity is negligible at present experimental scales when it is compared with other forces. However, the existence of such fundamental difficulties lying beneath the extremely successful framework of modern quantum physics should never be discarded. The situation is analogous to what physicists in the early 20th century were faced with in exploring microscopic laws of physics at atomic scale. The recent development of string theory[1)] strengthens our hope that string theory contains crucial ingredients for achieving a reconciliation between quantum theory and general relativity.

String theory has a quite curious history. It started out from something which was nothing to do with unified theories of interactions. From the 1950s to the 60s, even after a spectacular success of quantum electrodynamics, a large group of high-energy physicists at that time tended to believe that quantum field theory might not be the appropriate framework for describing strong nuclear force. Therefore the so-called "*S*-matrix approach" became a major stream during this period, and string theory actually emerged from this development in the late 60s. However, as our understanding on its nature was becoming deepened, various facets as an ideal theory of all interactions including gravity have gradually been uncovered. Even after almost 40 years since its first discovery, we are still in the midst of this process of exploring true meanings and new outcome of string theory. It is very important to recognize such peculiar

湯川秀樹　(1907-1981)

「素粒子を点と思っていたのでは、'てん'で話にならない」

A brief history of string theory

$$\alpha(s) = \alpha' s + \alpha_0$$

1968 Veneziano model

$$V(s,t) = \int_0^1 dx\, x^{-\alpha' s - \alpha_0 - 1}(1-x)^{-\alpha' t - \alpha_0 - 1} = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$$

$$= \sum_{n=0}^{\infty} \frac{r_n(s)}{t - m_n^2} = \sum_{n=0}^{\infty} \frac{r_n(t)}{s - m_n^2}$$

s-*t* (channel) duality

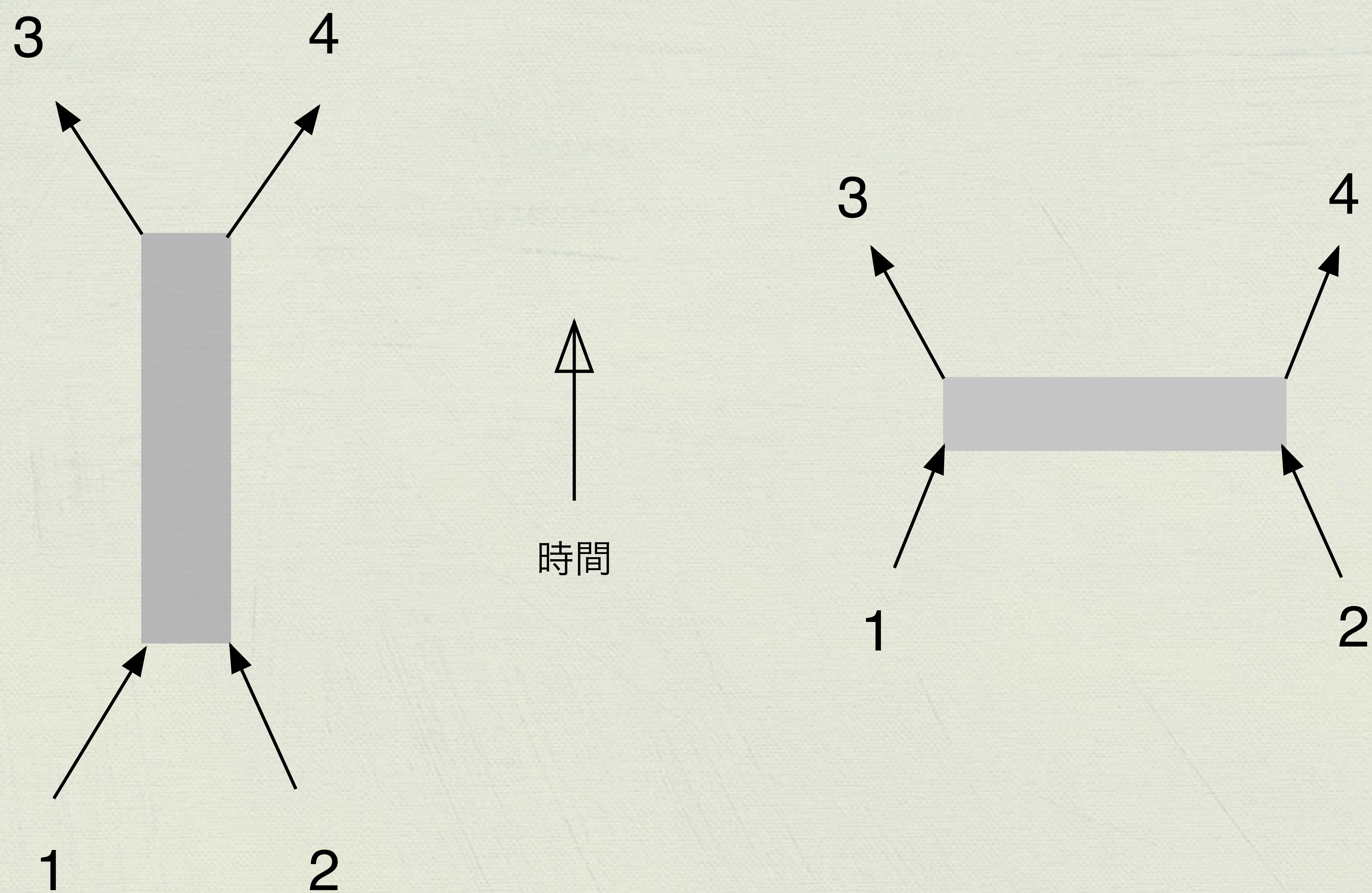
→ an infinite number of poles at $s \text{ or } t = m_n^2 = (n-1)/\alpha'$

spectrum of relativistic open strings

Similar formula (Virasoro-Shapiro), corresponding to closed strings

1970 ~ 1978 Initial developments of string theory (*models for hadronic interactions*)

- Nambu-Goto action
- Light-cone quantization, no-ghost theorem, critical dimensions (26 or 10)
- Ultraviolet finiteness → (modular invariance)
- Neveu-Schwarz-Ramond model (inclusion of fermionic degrees of freedom)
- Space-time supersymmetry

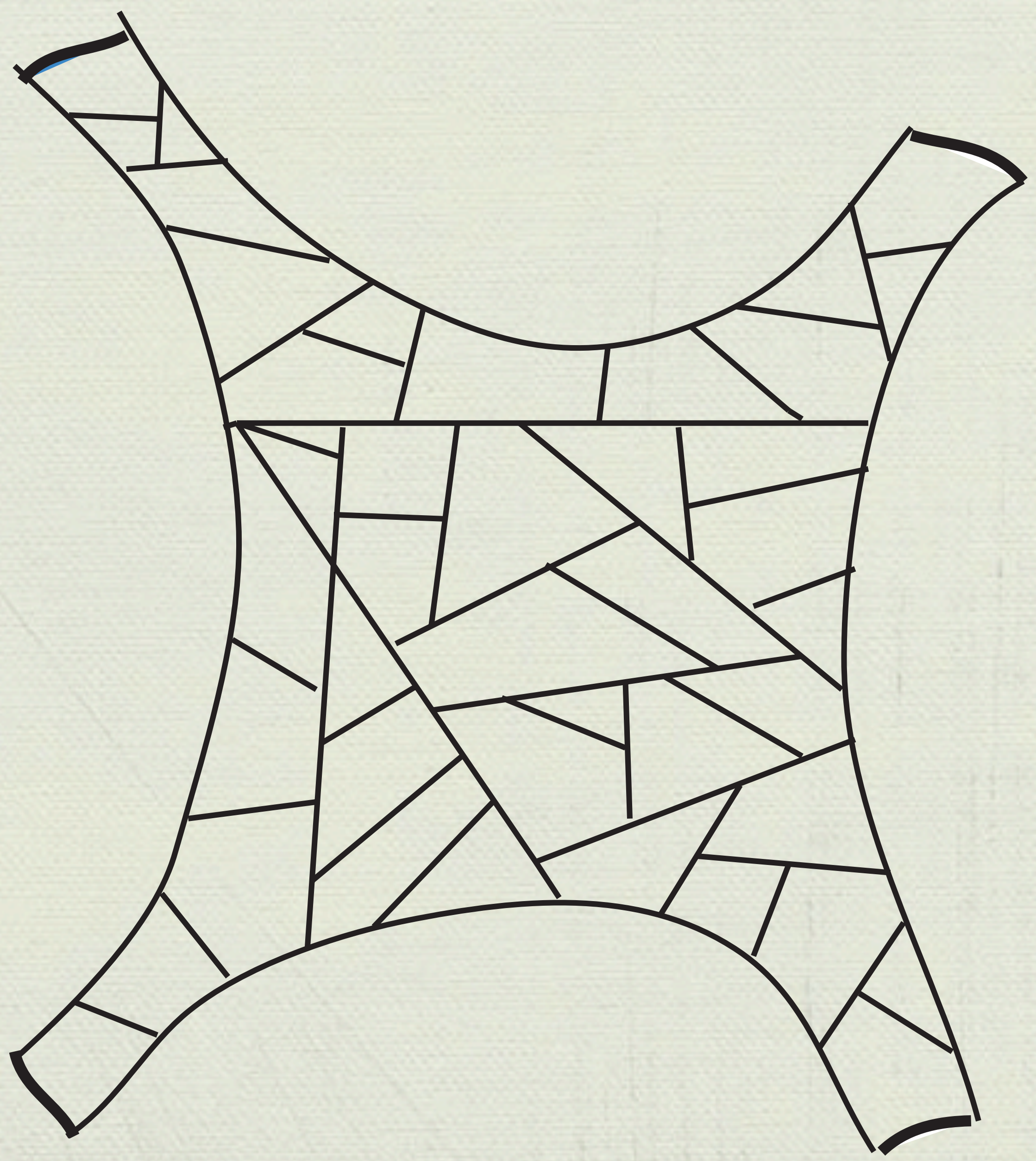


s-channel

t channel

Developments related to **field-theory/string connection**

- `Fishnet' diagram interpretation, Nielsen-Olesen vortex
- Derivation of gauge theory, general relativity and supergravity from strings in the zero-slope limit : *unification including gravity*
- Construction of various supersymmetric gauge and gravity theories
- String picture from strong-coupling lattice gauge theory
- t` Hooft's large N limit

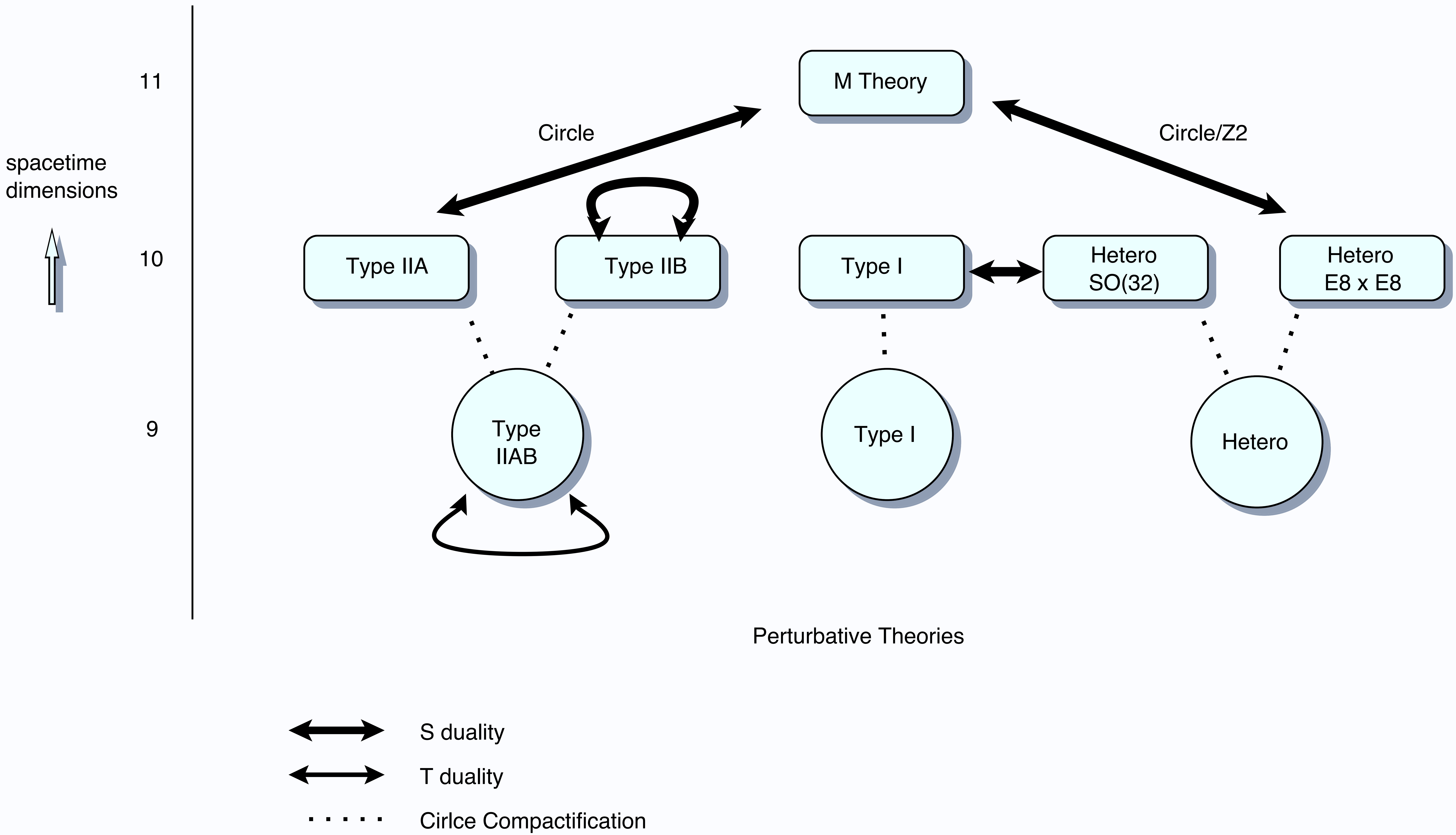


1984~1989 First revolution in string theory

- Green-Schwarz anomaly cancelation
- Five consistent perturbative string vacua (IIA, IIB, I, 2xHetro)
- Compactifications(T-duality, Calabi-Yau, ...), new connections to mathematics
- CFT technique, renormalization group interpretation

1990~1994 Development of "old" matrix models and related models

- Double scaling limit
- c=1 strings, 2D gravity, 'non-critical' strings
- topological field theories and strings


11
10
9
spacetime dimensions
M Theory
Circle
Circle/Z2
Type IIA
Type IIB
Type I
Hetero SO(32)
Hetero E8 x E8
Type IIAB
Type I
Hetero
Perturbative Theories
S duality
T duality
Cirlce Compactification

1995~present

Second revolution in string theory and various steady developments

- discovery of D-branes
- statistical interpretation of black-hole entropy in the BPS or near-BPS limits
- conjecture of M-theory
- New matrix models, membranes, ...
- AdS/CFT correspondence, GKPW relation, ...
- string landscape, ...
- various applications of gauge-gravity correspondence

General idea of gauge-gravity correspondence

unification of two old ideas on strings from the 70s :

- hadronic strings for quark confinement from gauge theory
- string theory for ultimate unification as an extension of general relativity

A.T.

日本物理学会誌1976年１月号４６ページより

What is string theory?

40年来の基本問題

strings (gravity) from gauge theory ?

or

gauge theory from strings (gravity) ?

Dual 模型と場の理論

北 大 理　米 谷 民 明

dual 模型[1,2]はハドロン反応のいくつかの現象論的特徴や理論的要請を総合的に取入れた，散乱振幅に対する数学的模型として出発し，その後素粒子論の種々の分野に刺激を与えつつ発展して来た．しかしこの模型は，今までの素粒子論の基礎である場の理論とは無関係に生まれたものであったため，両者の関係をどう理解するかは，この模型の誕生以来一つの問題であった．これについては二つの考え方が有りうるであろう．一つは，dual 模型は場の理論を越えたものであって，後者は前者のある特定な極限として理解する可能性．もう一つは，この模型は場の理論の従来の標準的な摂動論の枠内には入らないが，摂動論によらない別の観点から理解可能かも知れないという考え方である．この小稿では，後者の立場[3]については他にゆずり，前者の観点に立って最近までなされて来た研究を紹介したい．

§1.　Dual 模型とゲージ場

dual 模型を理論的側面から見た場合，特に注目しなければならないことの一つは，それが無限に多くの粒子をごく少数のパラメターで記述しているということであろう．実際，例えば内部対称群を SU(2) にか

しては含まないという要求(conformal 不変性[4])から，
また外線の数や種類の異なった振幅間の相対的大きさ
は factorization と duality の性質のために一意的
に決まってしまう．こういう事情は，通常の場の理論
とのアナロジーを求めると，ゲージ場[5]の理論とよく
似ている．Regge 軌跡の切片の値が conformal 不変
性から決まるこ
量項を含まない
数を与えること
場の結合定数の
ーをさらに進め
することもでき
互作用バーテッ
しかし dual
ロジーにとどま
統的な理解が可
と dual 模型の
dual 振幅の Re
を提案した．√
あるから，この
できる．彼が
求をゆるめて I
ターにしたもの
$\lambda \equiv g/\sqrt{\alpha'}$ を固定して $\alpha' \to 0$ の極限をとると，dual
振幅は $\lambda\phi^3$ 相互作用のスカラー場の理論のトリー
(tree) 近似の振幅と一致することがわかった．この

Guggenheim museum (New York)
の螺旋回廊

$\alpha' \to 0$ の極限はその後すぐに，Regge 軌跡の切片が 1 に固定された模型へ Neveu と Scherk[10] によって適用された．その結果，外線が 4 本の dual 振幅が g を固定した $\alpha' \to 0$ 極限で通常の Yang-Mills 場のトリー近似振幅に一致することが示された．この場合，内部対称性は Chan と Paton[11] の手続きに従って入れられている．また g はゲージ場の結合定数になる．このようにして冒頭に述べたゲージ場と dual 振幅の関係がより明確にされたわけである．一見場の理論とは全く無関係に提出された模型がこのような性質を持つことは興味深いことであろう．この結果はまた次のようにして予想することもできよう．有質量ベクトル場の理論は，もしそのトリー近似振幅の高エネルギー極限についてある制限（トリーユニタリ性）をおくと，唯一可能なものは Higgs-Kibble 型[12]の理論であることが知られている．[13] dual 振幅はこの制限をこわしてはいないので，もし切片を 1 からずらしベクトル粒子の質量を固定して $\alpha' \to 0$ 極限[14]を考えれば，Higgs-Kibble 型の理論が得られるであろう．Yang-Mills 場のトリー振幅は，Higgs-Kibble 型のものから質量ゼロの極限として得られるから，切片を 1 にした場合には，Yang-Mills 場の振幅が $\alpha' \to 0$ 極限として得られると期待される．（この議論は，いくつか

場のゲージ粒子に一致するわけである．これに対して，閉じた弦の相互作用を記述するいわゆる非平面 (non planar) 型の模型[18]は，同様の要求のもとでは必ず切片が 2 の Regge 軌跡を含み，従って質量ゼロ，スピン 2 の粒子を含む．またこの模型には内部対称性をもたすことができない．これらのことから，非平面 (non planar) 型模型と重力理論との関係が期待される．以下にこの場合の研究を簡単に紹介しよう．

§ 2.　非平面型模型と重力理論

conformal 不変な非平面 (non planar) 型模型 (Virasoro-Shapiro[18] 模型) の $\alpha' \to 0$ 極限は最初次のようにして調べられた (Yoneya[19])．この模型において，n 個の質量ゼロ，スピン 2 粒子（以下，重力子と略する）と 2 個のスカラー粒子の散乱振幅を作り $g^2\alpha'$ を固定して $\alpha' \to 0$ 極限をとる．まず，$n=2$ のときは，振幅は Einstein のラグランジアンを $g_{\mu\nu}=\delta_{\mu\nu}+\kappa h_{\mu\nu}$ とおいて $h_{\mu\nu}$ について展開して得られる場の理論[20]のトリー近似振幅に一致することが示される．このとき，重力定数 $G=\kappa^2/32\pi$ は定数 $g^2\alpha'$ に比例する．なお便宜的に物質場に相当するものとして導入されたスカラー粒子の質量は，空間の次元をふやす工夫をすることによって $\alpha' \to 0$ 極限でも有限にとどまるように

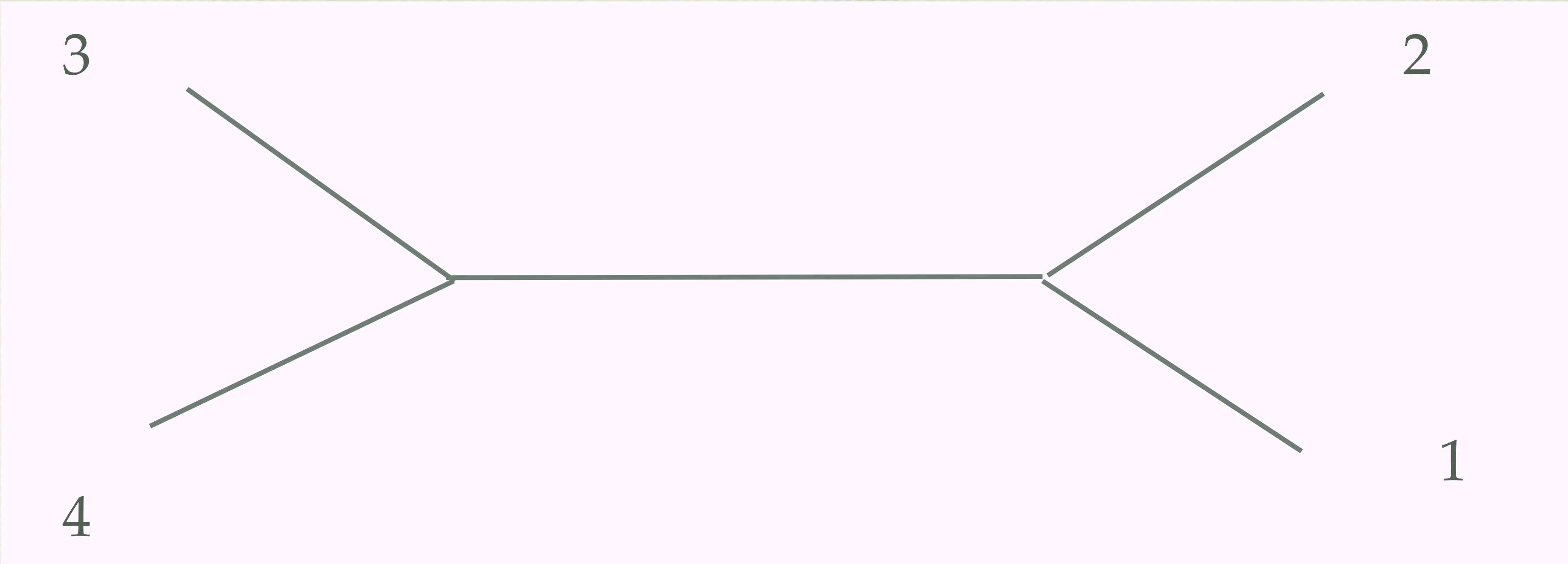
First inception of relativistic strings from the Veneziano amplitude (Nambu, Susskind, ... 1969)

$$V(s,t) = \int_0^1 dx\, x^{-\alpha' s - \alpha_0 - 1}(1-x)^{-\alpha' t - \alpha_0 - 1}$$

$$s = -(p_1+p_2)^2 = -2p_1p_2 - p_1^2 - p_2^2$$

$$t = -(p_2+p_3)^2 = -2p_2p_3 - p_2^2 - p_3^2$$

This can be interpreted in terms of a Feynman-like diagram



$$x \to e^{-\tau}$$

$$\int_0^1 dx x^{-\alpha' s - \alpha_0 + N - 1} \to \int_0^\infty d\tau\, e^{-(\alpha_0 s + \alpha_0 - N)\tau} = \frac{1}{\alpha(s) - N} \qquad \alpha(s) = \alpha' s + \alpha_0$$

Assume

$$\alpha_0 = \alpha(0) = 1$$
$$p_i^2 = \frac{1}{\sqrt{\alpha'}}$$

Then, we can represent the amplitude in terms of **an infinite set of harmonic oscillators**

$$[a_n^\mu, a_m^{\nu\,\dagger}] = \eta^{\mu\nu}\delta_{nm}$$

$$(1-x)^{-\alpha(t)-1} = \exp\left((\alpha(t)+1)\sum_{n=1}^{\infty}\frac{x^n}{n}\right) = \exp\left(-2\alpha' p_2 p_3 \sum_{n=1}^{\infty}\frac{e^{-n\tau}}{n}\right)$$

formula:

$$= \langle 0| \exp\left(ip_3 \sum_{n=1}^{\infty}\frac{\sqrt{\alpha'}a_n}{\sqrt{n}}\right) e^{-\sum_{n=1}^{\infty} n a_n^\dagger a_n \tau} \exp\left(ip_2 \sum_{n=1}^{\infty}\frac{\sqrt{\alpha'}a_n^\dagger}{\sqrt{n}}\right)|0\rangle$$

$\longrightarrow$ $$\int_0^{\infty} dx x^{-\alpha(s)-1}(1-x)^{-\alpha(t)-1} = \langle 0|V(p_3)\frac{1}{\alpha(s)-N}V(p_2)|0\rangle$$

$$N = \sum_{n=1}^{\infty} n a_n^\dagger a_n$$

$$V(p) = \exp\left(ip\sum_{n=1}^{\infty}\frac{\sqrt{\alpha'}a_n^\dagger}{\sqrt{n}}\right)\exp\left(ip\sum_{n=1}^{\infty}\frac{\sqrt{\alpha'}a_n}{\sqrt{n}}\right)$$

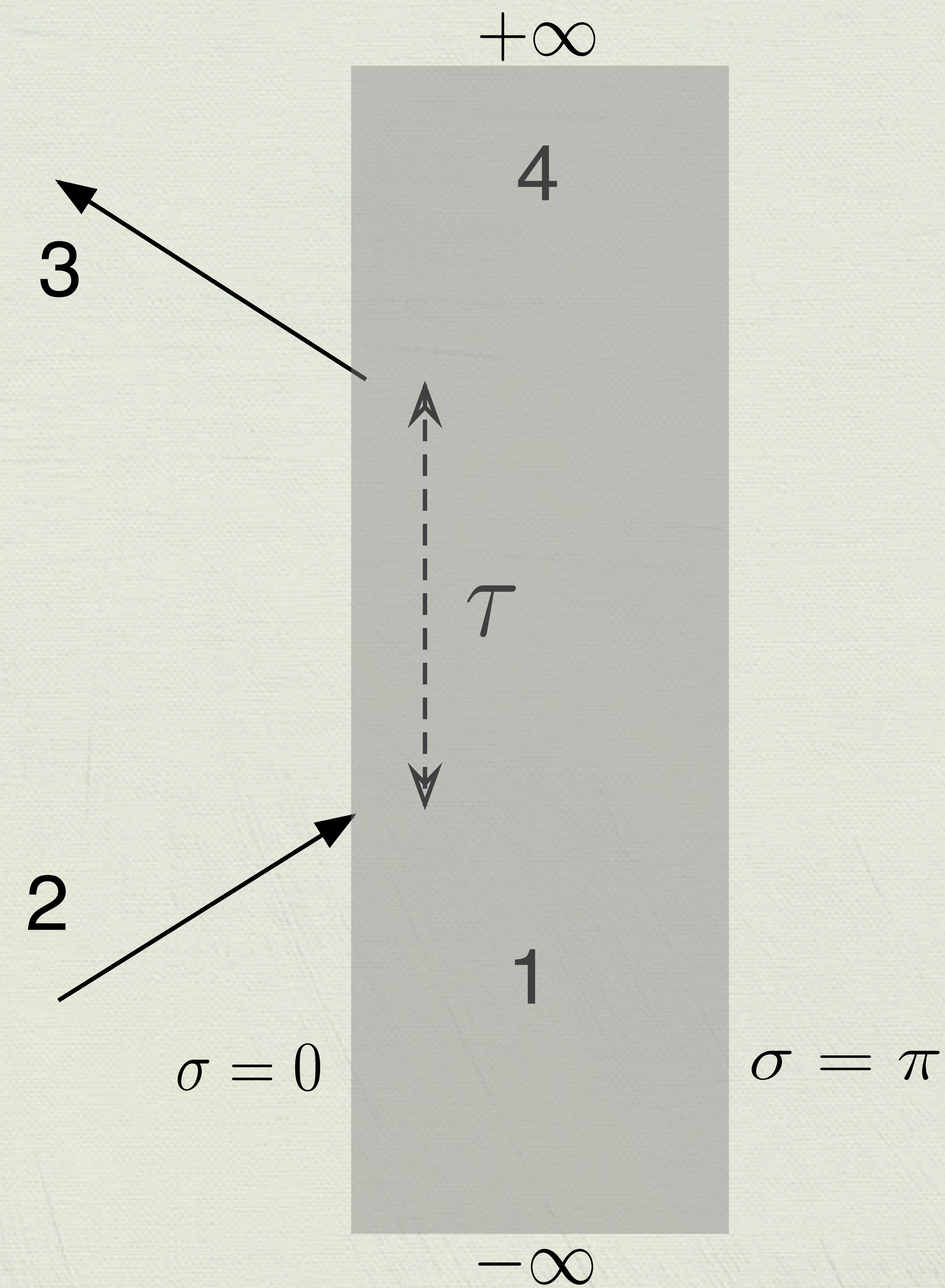
Vertex operator describing emission and absorption of the ground state

$$M^2 = \frac{1}{\alpha'}(N-1)$$

mass (square) operator:

This suggested to define a quantum 'coordinate' operator associated with this infinite-component harmonic oscillators.

$$x^\mu(\tau) = \sum_{n\neq 0}\frac{\sqrt{\alpha'}}{\sqrt{n}}a_n e^{-n\tau}$$



$+\infty$
4
3
τ
2
1
$\sigma = 0$
$\sigma = \pi$
$-\infty$

Problems:

- states of negative metric (ghost)
- ground state = tachyon ⟶ space-time supersymmetry

The first difficulty can be resolved by the existence of WT-like identity

For the first excited state at $s \equiv -k^2 = 0$

$$k a_1 \exp\left(ip_2 \sum_{n=1}^{\infty} \frac{\sqrt{\alpha'} a_n^\dagger}{\sqrt{n}}\right)|0\rangle = ikp_2 \exp\left(ip_2 \sum_{n=1}^{\infty} \frac{\sqrt{\alpha'} a_n^\dagger}{\sqrt{n}}\right)|0\rangle$$

$$kp_2 = \frac{1}{2}[-(k-p_2)^2 + k^2 + p_2^2] = \frac{1}{2}[-p_1^2 + p_2^2 + k^2] = 0$$

Negative metric states are decoupled !

$$1 = \sum_{i=1}^{D-2} \mathbf{a}_1^{i\,\dagger}|0\rangle\langle 0|\mathbf{a}_1^i + (a_1^{D-1\,\dagger} + a_1^{0\,\dagger})|0\rangle\langle 0|(a_1^{D-1} - a_1^0) + (a_1^{D-1\,\dagger} - a_1^{0\,\dagger})|0\rangle\langle 0|(a_1^{D-1} + a_1^0)$$

decouple as in QED $|k^{D-1}| = |k^0|$ frame

The analysis can be extended to arbitrary higher excited states **when D=26, "critical space-time dimensions".**

Important properties and generalizations

- electric circuit analogy ⟶ world-sheet picture
- conformal symmetry ⟶ WT-like identity
- Virasoro-Shapiro model ⟶ closed string
- K(吉川)S(崎田)V(irasoro) program (higher loops)

- Regge behavior with linearly rising trajectory
  (Chew-Frautschi plot)
- massless spinning states ⟶ gauge theory and general relativity

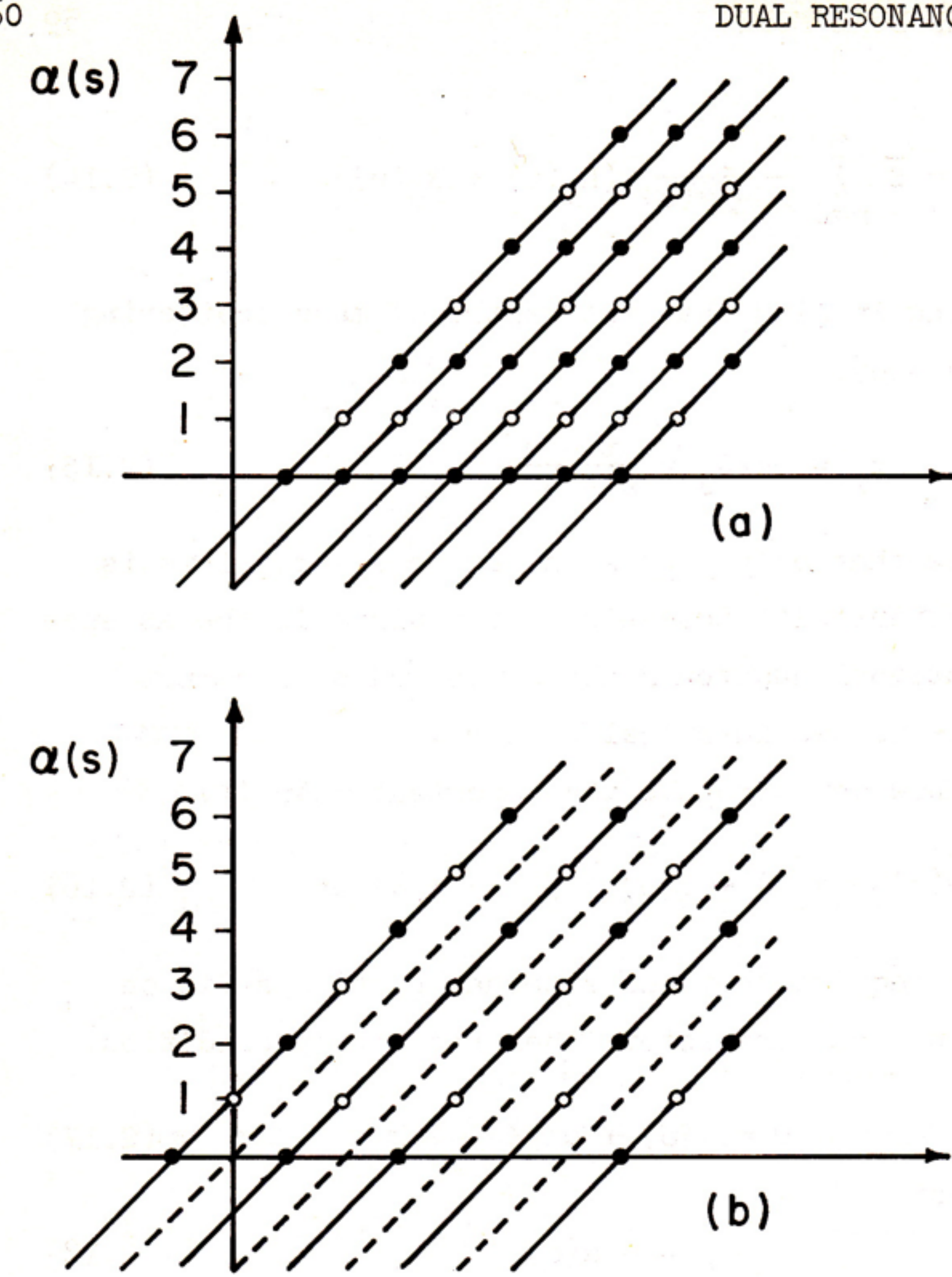


FIGURE 2.1

Chew-Frautschi Plots

$$J = \alpha(s) \quad M^2 = s$$

P. H. Frampton,
Dual Resonance Models,
Academic Express, 1974　より

ゲージ理論、一般相対性理論から(超)弦理論へ　(個人的回想を含めて)

１９７０年代前半の段階では、**弦理論が場の量子論とどういう関係があるのか**さえ、明らかではなかった。関連する問題の追求から、後の発展につながる多くのアイデアがその頃に芽生えた。

Gravity from strings: personal reminiscences of early developments

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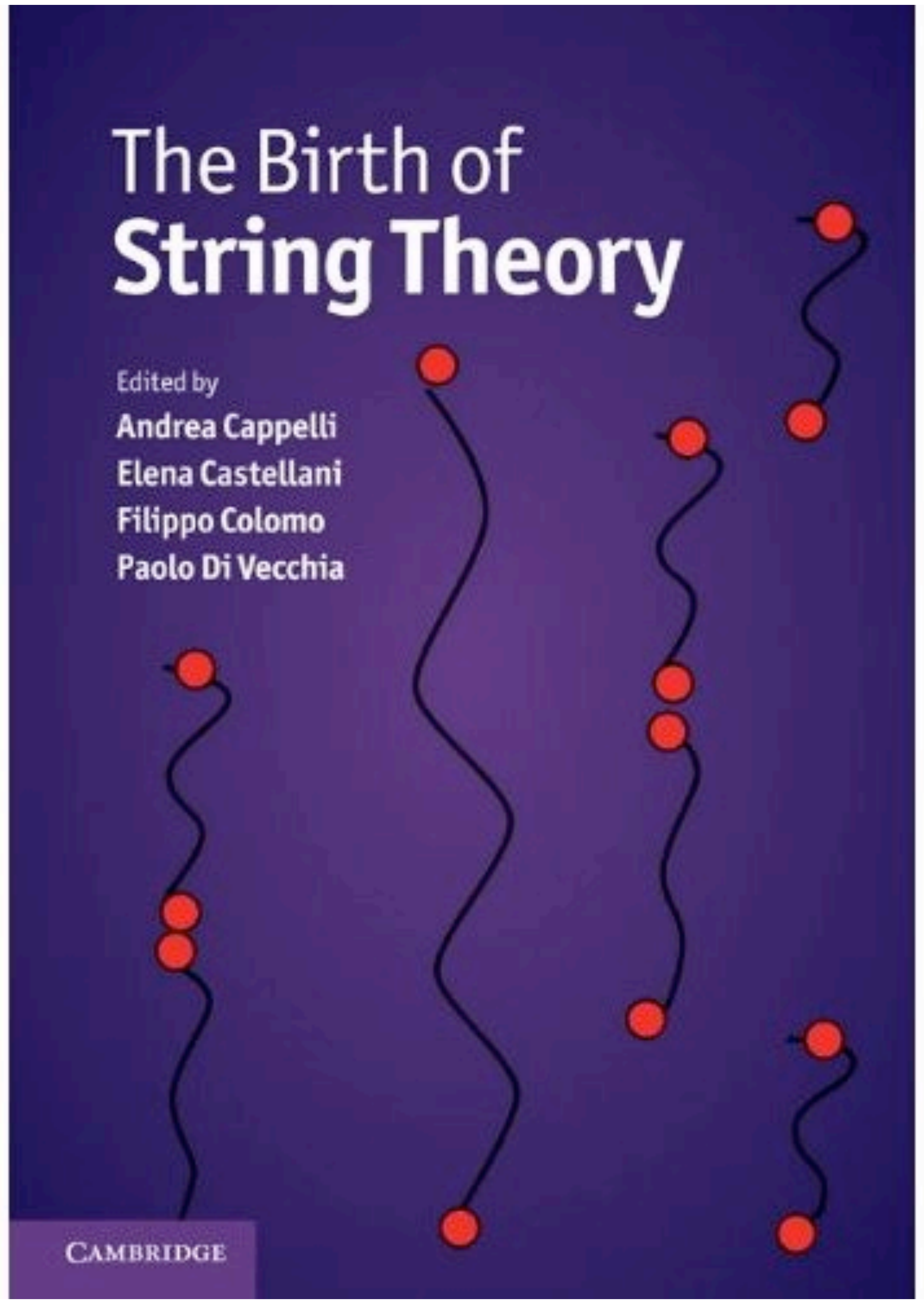
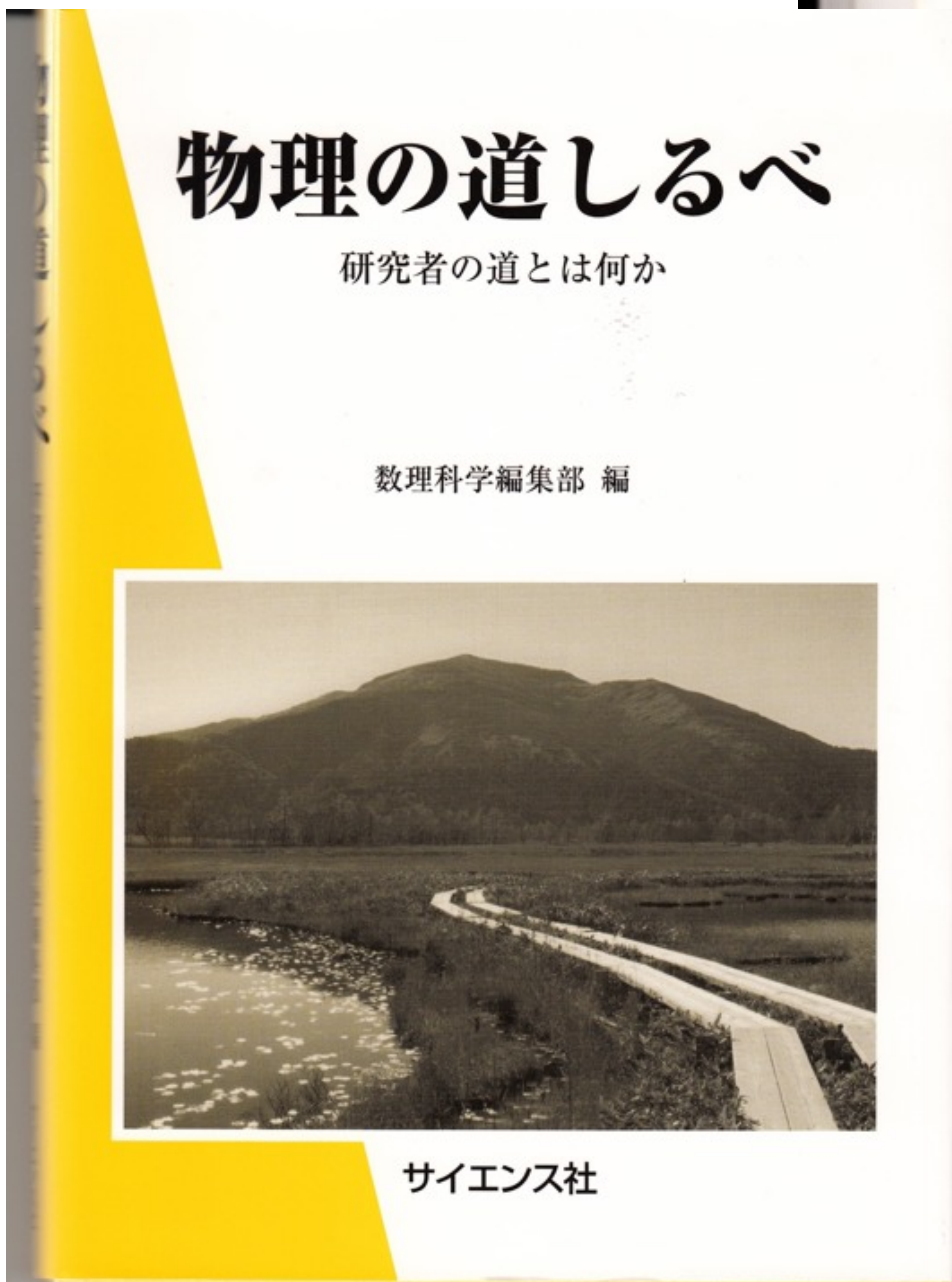
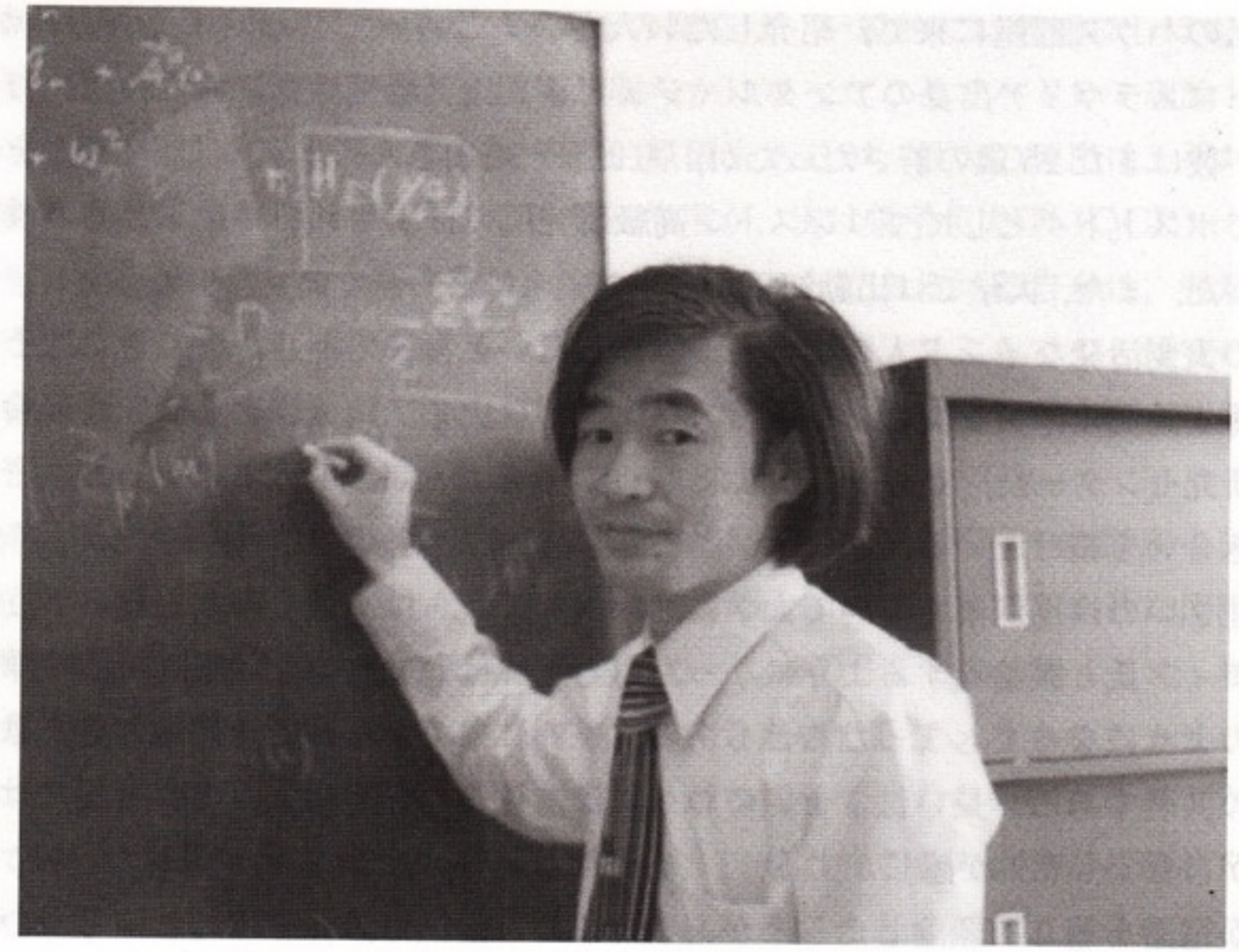
Abstract

I discuss the early developments of string theory with respect to its connection with gauge theory and general relativity from my own perspective. The period covered is mainly from 1969 to 1974, during which I became involved in research on dual string models as a graduate student. My thinking towards the recognition of string theory as an extended quantum theory of gravity is described. Some retrospective remarks on my later works related to this subject are also given.

The Birth of String Theory,
Cambridge Univ. Press, 2012







NY 市立大学のオフィスにて（1978 年）．

音楽，オペラ，絵画などの鑑賞を存分に堪能できた．古い日本映画などについても NY で初めて接し，改めて日本文化の素晴らしさも認識させられた．

物理に関しては，到着後数週間の頃ブルックヘブン研究所で会議があり，トホーフト（G. t' Hooft）氏が講演するというので院生数人と出かけたときのことを衝撃的な出来事として鮮明に思い出す．トホーフト氏の講演は，それまでの乏しい経験の中では，とびきりすばらしいものだった．それは二重の意味で衝撃であった．まず私の目には彼は 24～5 歳の学生にしか見えないほど若かった．こんなに若い人が，これほど堂々たる講演をしたのに圧倒された．実際には，彼は私より 1 歳年上であることが後に分かって，驚きは少し和らいだ．また，話の主題が，それまで私が知らなかった「インスタントン」であったことも大変ショックであった．そのときのセッションでは，確かファン・ニューベンホイゼン（P. van Nieuwenhuizen）氏の「超重力理論」の話もあった．こちらのほうも私には初耳であった．インスタントンについては，その 1～2 カ月後にあった，ウイッテン氏のセミナーも強い印象に残っている．当時まだプリン

ストンの院生だった彼の初々しい
こんなに重要な問題の両方とも，
おらず，私自身も全然認識してい
め，渡米後の数カ月はそれらに関
算をすませていて NY で暇ができ
5 編ほど異なるテーマで論文を書
いたようだ．着眼点は悪くないに
論文がある．

多分，崎田さんもそれをお感じ
をいただいた．もう論文は十分に
年をかけてこれから何をやるべき
これには本当に衝撃を受けたが，
感謝している．実際にそれから 7
たトホーフトの非可換ゲージ理論
理論上での実現に関する少し長い
の後の最初の論文になった．こち
れたようだ．この仕事をきっかけ
明ができないかという考えに取り
び，何をやるべきか悩む日々がそ

この仕事についてのセミナーを
時 NY 市立大学学長で，アメリカ
シャク（R. Marshak）先生が一
は，セミナーに現れることはよほ
分，崎田さんが知らせてくれてい
コメントや質問は実に的確であっ
を記念する国際会議が市立大学の
年であったと思う．この会議につ
知っていた錚々たる顔ぶれ（H. B
A. Pais, L. Glashow, T. Regge,
スピーカーに接することができ

こんなふうに思い出はつきない

1970年代の素粒子論

現在の素粒子理論を支える方法論的・概念的基礎の多くのものは、
７０年代に築かれた。

- クオーク模型がパートン描像と結びつき、ハドロンの複合模型として確立
- 弱い相互作用と電磁相互作用が、Weinberg-Salam理論により統一され、さらに強い相互作用もクオーク間力の color ゲージ理論(QCD)として確立
- クオークの新しい世代が発見され、WS理論と結びつき、３世代標準模型に結実

- **繰り込み理論、格子ゲージ理論、非摂動的古典解に関する発展により、ゲージ場理論の性格についての理解**が飛躍的に高まった
- 一方、50年代から始まった**強い相互作用のS行列理論の発展からは、現在の弦理論につながる新しい展開**が起こった

こうした状況のもとで、「相互作用の統一」への動機づけも高まり、70年代中盤には、「大統一」理論の最初の提案 (Georgi-Glashow,1974)も成された

しかし、素粒子論側からの重力理論(一般相対性理論)との結びつきに関する関心は、全体としては、極めて希薄であった

素粒子物理	(超)弦理論		私	
湯川中間子の発見		1947		生
量子電気力学				
Yang-Mills 理論		1954		
クォーク模型	Regge-pole theory			
電弱統一理論		1967		
パートン模型	Veneziano 模型	1969		北大大学院 助手 (1969-1980)
量子色力学(QCD)	(弦理論の誕生)	1973	弦理論からの	
Charm クオークの発見	.	1974	ゲージ理論と一般相対性理論	
	.			(CUNY 1976-1978)
			ゲージ理論のdynamics	
	.	1980	(クオーク閉じ込め)	東大着任
	.			
W, Zボゾンの生成	.	1983		(CERN 1983-84)
(CERN)	.	1984	低次元量子重力	
	弦理論の復活		弦理論の基礎をめぐる考察	
	“first” revolution		(時空不確定性、背景独立性)	
３世代標準模型の確立	.			
	.		行列模型とブラックホール	(Santa Barbara KITP 1993)
.		1995		
.			時空不確定性関係の展開	
	“second” revolution		ゲージ／重力対応の展開	
.			行列模型とDブレーン	
.	.		Dブレーンの場の理論	
LHCの稼働	.	2010		放送大学着任
HIggs粒子の検出	.			
(CERN)	.			

dual 模型＝弦理論をゲージ理論と一般相対性理論の拡張と看做すべきであることを指摘した初期の仕事

T. Y., "Note on the local gauge principle in conformal dual models",
Prog. Theor. Phys. 48, 616-624 (1972)

T. Y., "Quantum gravity and the zero-slope limit of the generalized Virasoro model",
Lett. Nuovo. Cim. 8, 951-956 (1973)
T. Y., "Connection of dual models to electrodynamics and gravidynamics",
Prog. Theor. Phys. 51, 1907-1920(1974)
T.Y., "Interacting Fermionic and Pomeronic strings : Gravitational interaction of the Ramond fermion",
Nuovo Cim. A27, 440-457 (1975)
T. Y., "Geometry, gravity and dual strings",
Prog. Theor. Phys. 56, 1310-1317 (1976)

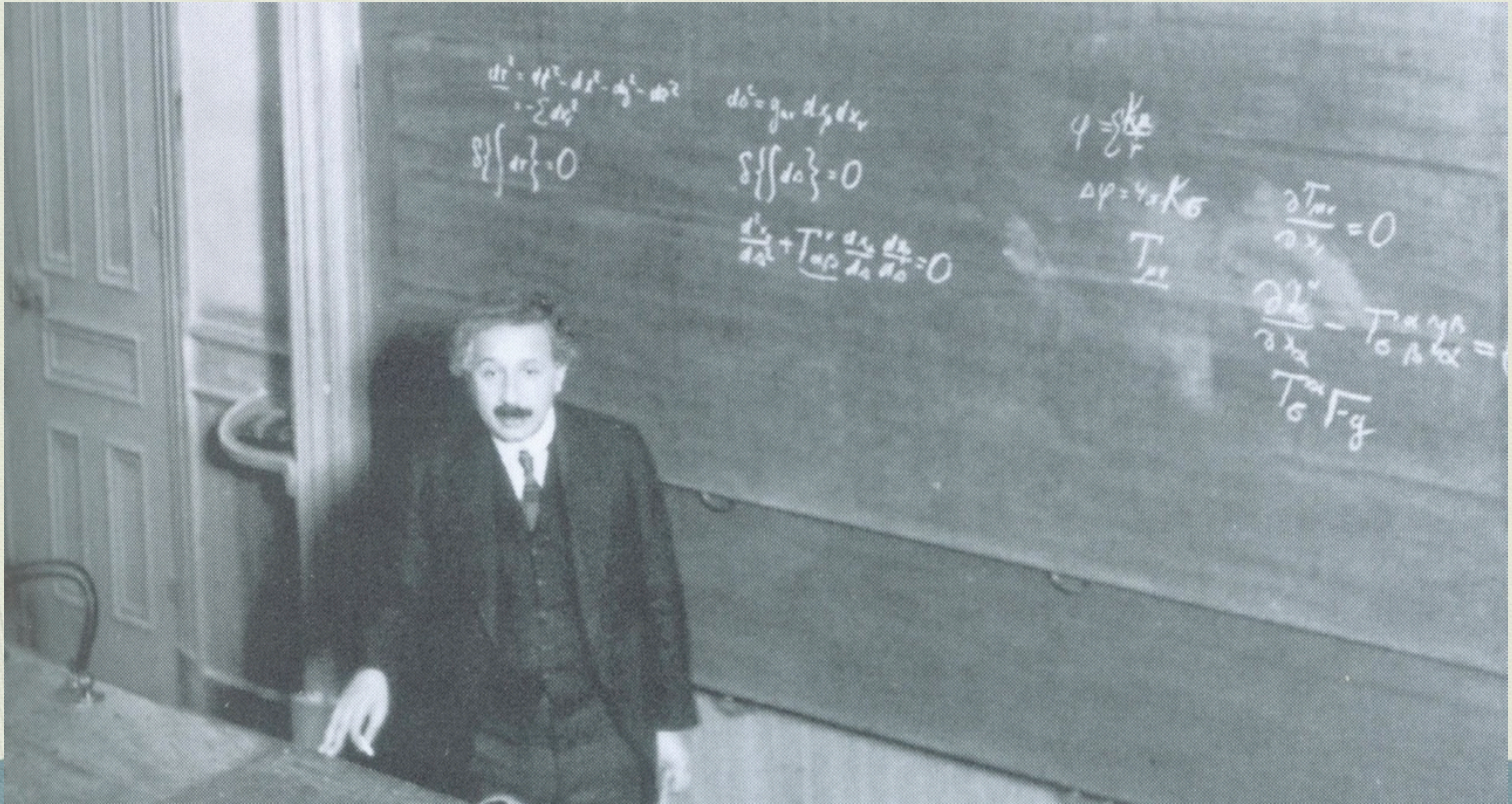
A. Neveu and J. Scherk, "Connection between Yang-Mills fields and dual models", Nucl. Phys. B36, 155-161(1972)

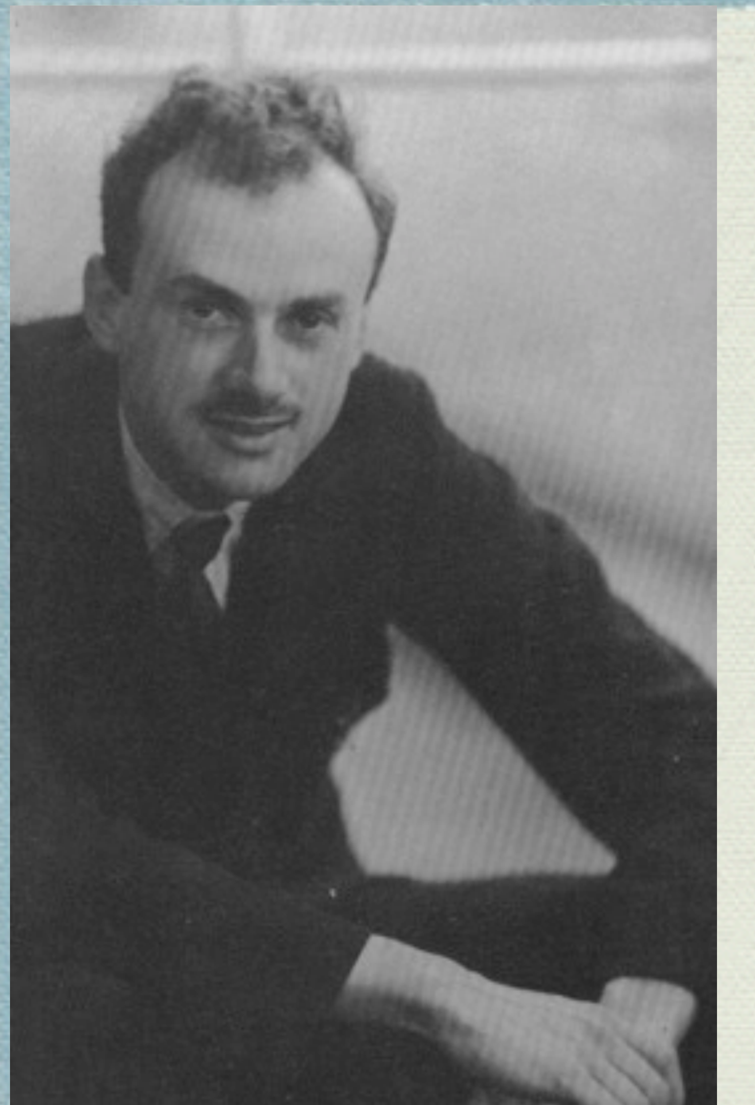
J. Scherk and J. H. Schwarz, "Dual models for non-hadrons",
Nucl. Phys. B81, 118-144 (1974)
J. Scherk and J. H. Schwarz, "Dual models and the geometry of space-time",
Phys. Lett. 52B, 347-350 (1974)
J. Scherk and J. H. Schwarz, "Dual field theory of quarks and gluons",
Phys. Lett. 57B, 463-466 (1975)

アインシュタインとディラックの遺産

The real goal of my research has always been the **simplification and unification of the system of theoretical physics. I attained this goal satisfactorily for macroscopic phenomena**, **but not for the phenomena of quanta and atomic structure**. …

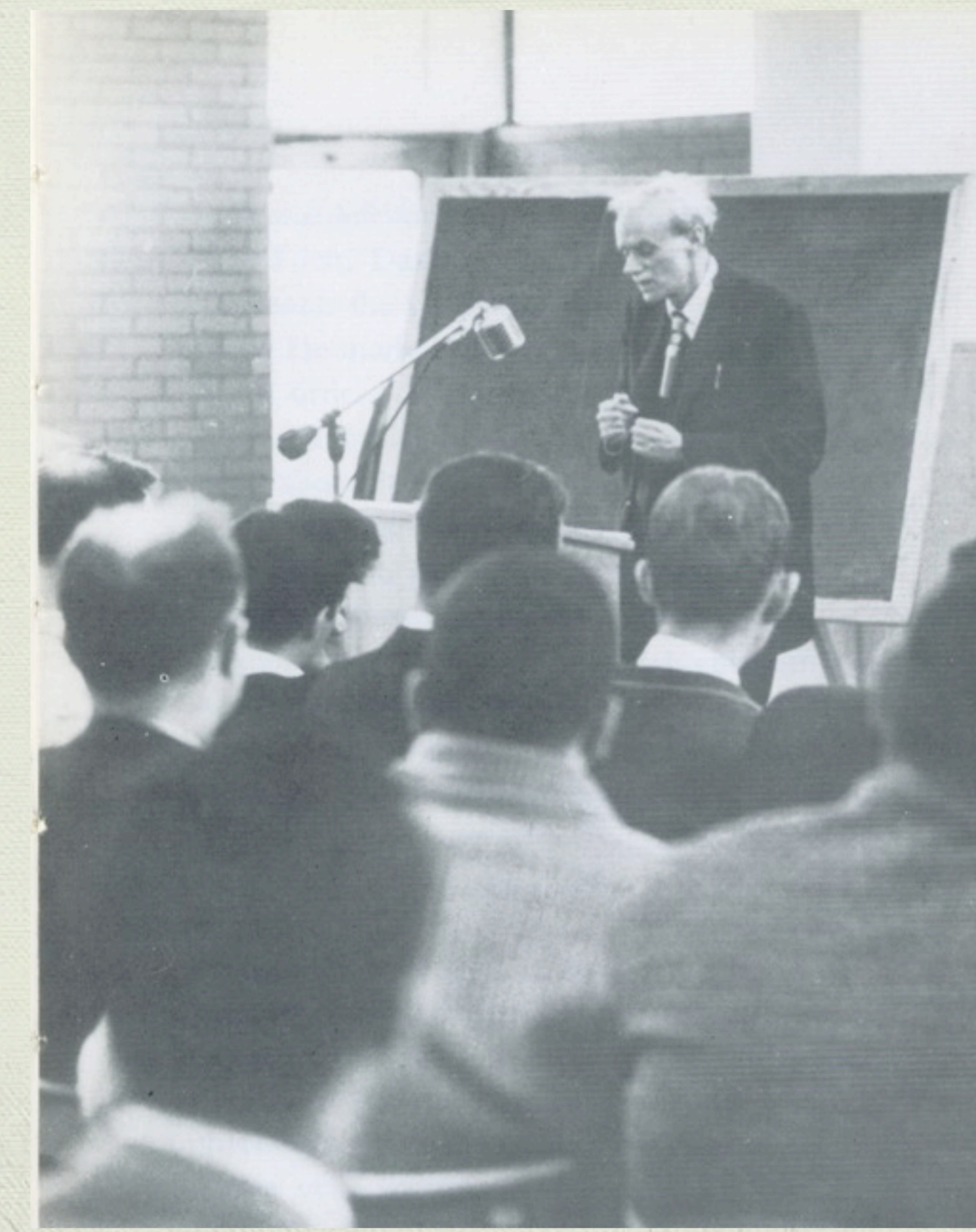
A. Einstein (1879-1955), 1939





The lines would then be the elementary concept in terms of which the whole theory of electrons and the electromagnetic field would have to be built up. **Closed lines would be interpreted as photons and open lines would have their ends interpreted as electrons or positrons**. ...

P. A. M. Dirac (1902-1984), 1955



a precursor to
lattice gauge theory, string theory, ...

Canadian Journal of Physics, 33(1955) 650

GAUGE-INVARIANT FORMULATION OF QUANTUM ELECTRODYNAMICS[1]

By P. A. M. Dirac

ABSTRACT

Electrodynamics is formulated so as to be manifestly invariant under general gauge transformations, through being built up entirely in terms of gauge-invariant dynamical variables. The quantization of the theory can be carried out by the usual rules and meets with the usual difficulties.

It is found that the gauge-invariant operation of creation of an electron involves the simultaneous creation of an electron and of the Coulomb field around it. The requirement of manifest gauge invariance prevents one from using the concept of an electron separated from its Coulomb field.

An extensible model of the electron

By P. A. M. Dirac, F.R.S.
St John's College, University of Cambridge

(*Received* 5 *February* 1962)

It is proposed that the electron should be considered classically as a charged conducting surface, with a surface tension to prevent it from flying apart under the repulsive forces of the charge. Such an electron has a state of stable equilibrium with spherical symmetry, and if disturbed its shape and size oscillate. The equations of motion are deduced from an action principle and a Hamiltonian formalism is obtained. The energy of the first excited state with spherical symmetry is worked out according to the Bohr–Sommerfeld method of quantization, and is found to be about 53 times the rest-energy of the electron. It is suggested that this first excited state may be considered as a muon. The present theory has no electron spin, so it cannot agree accurately with experiment.

"DBI"(Dirac-Born-Infeld) action の起源

Expressed in terms of the new q's with the choice (2) for f, the action is

$$4\pi I_o = -\frac{1}{4}\int_{x^1>0} J g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}\, \mathrm{d}^4x, \tag{5}$$

$$4\pi I_s = -\omega \int_{x^1=0} M\, \mathrm{d}x^0 \mathrm{d}x^2 \mathrm{d}x^3, \tag{6}$$

where $-J^2$ is the determinant of the $g_{\mu\nu}$ and M^2 is the determinant of the g_{ab}, so that

$$M = J(-g^{11})^{\frac{1}{2}}. \tag{7}$$

ω is a positive constant that determines the equilibrium size and mass of the electron.

55年のアイデアと合体させれば
自然に南部後藤作用に行き着くが、
Dirac 自身は、そのことに思い当たら
なかったようだ。

Proc. Roy. Soc. London 144 Ser. A(1934)425

Foundations of the New Field Theory.

By M. Born and L. Infeld,† Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 26, 1934.)

§ 1. *Introduction.*

The relation of matter and the electromagnetic field can be interpreted from two opposite standpoints :—

The first which may be called the *unitarian standpoint*‡ assumes only *one* physical entity, the electromagnetic field. The particles of matter are considered as singularities of the field and mass is a derived notion to be expressed by field energy (electromagnetic mass).

.... Einstein did not find this analogical construction convincing. Infeld and I found it attractive for a long time. We abandoned the theory for completely different reasons, namely, because we did not succeed in reconciling it with the principles of the quantum field theory. In any case this constituted the first attempt to overcome the difficulties of microphysics by means of a non-linear theory. Heisenberg's theory of elementary particles, which is much talked about today, is also non-linear. But I am guessing.

from `The Born-Einstein Letters 1916-1955',
M. Born (1882-1970), published in 1969.

In the papers cited above, the new field theory has been introduced rather dogmatically, by assuming that the Lagrangian underlying Maxwell's theory

$$L = \tfrac{1}{2}(H^2 - E^2) \tag{1.1}$$

(H and E are space-vectors of the electric and magnetic field) has to be replaced by the expression†

$$L = b^2\left(\sqrt{1 + \frac{1}{b^2}(H^2 - E^2)} - 1\right). \tag{1.2}$$

The obvious physical idea of this modification is the following :—

The failure in the present theory may be expressed by the statement that it violates the *principle of finiteness* which postulates that a satisfactory theory should avoid letting physical quantities become infinite. Applying this principle to the velocity one is led to the assumption of an upper limit of velocity c and to replace the Newtonian action function $\frac{1}{2}mv^2$ of a free particle by the relativity expression $mc^2(1 - \sqrt{1 - v^2/c^2})$. Applying the same condition to the space itself one is lead to the idea of closed space as introduced by Einstein's cosmological theory.‡ Applying it to the electromagnetic field one is lead immediately to the assumption of an upper limit of the field strength and to the modification of the action function (1.1) into (1.2).

This argument seems to be quite convincing. But we believe that a deeper foundation of such an important law is necessary, just as in Einstein's mechanics the deeper foundation is provided by the postulate of relativity. Assuming that the expression $mc^2(1 - \sqrt{1 - v^2/c^2})$ has been found by the idea of a velocity limit it is seen that it can be written in the form

$$mc^2(1 - d\tau/dt),$$

where

$$c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2,$$

and therefore it has the property that the time integral of $mc^2 d\tau/dt$ is invariant for all transformations for which $d\tau^2$ is invariant. This four-dimensional group of transformations is larger than the three-dimensional group of transformations for which the time integral of the Newtonian function

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}m(ds/dt)^2; \quad ds^2 = dx^2 + dy^2 + dz^2,$$

is invariant.

† See Born and Infeld, 'Nature,' vol. 132, p. 1004 (1933).
‡ See Eddington, "The Expanding Universe," Cambridge, 1933.

2. The rudiments of string theory:
a short course of string quantum mechanics

弦とは

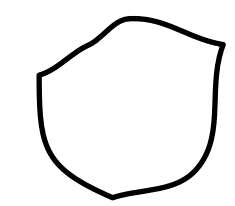
弦 fundamental string (‘ F1’)

・太さ ＝ゼロ

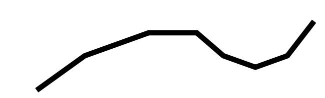
・質量密度＝厳密に一定、弦理論の基本定数＝ $1/2\pi\alpha'$

： 長さの基本単位 $\sqrt{\alpha'} \equiv \ell_s$

閉じた弦 (**closed string**)： 重力子 を含む



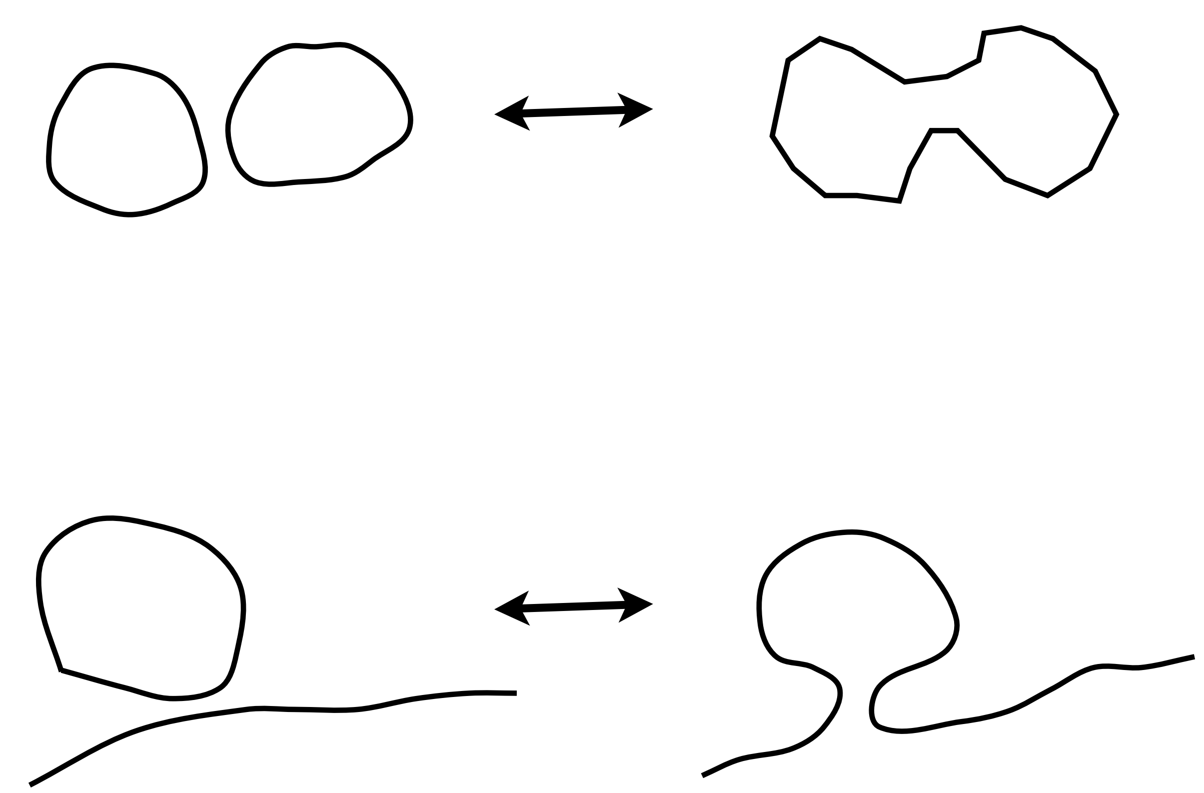
開いた弦 (**open string**) ： ゲージ場 を含む



・**closed string** はすべての弦とある普遍的な仕方で相互作用する

つまり、「万有引力」を説明する

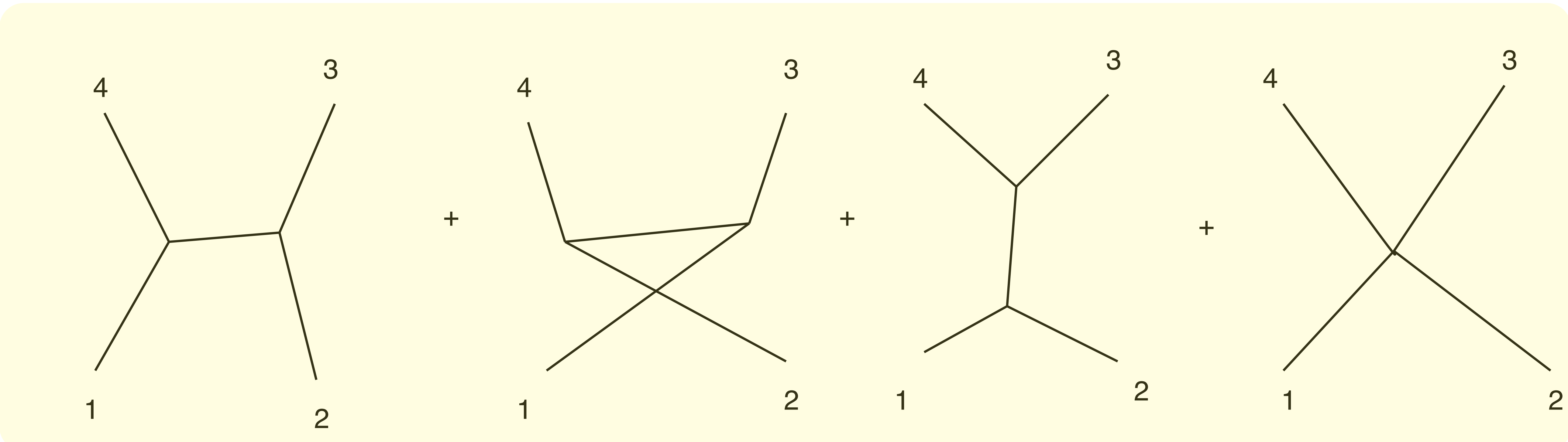
そして，closed string の相互作用の長距離のふるまいは〔拡張された〕一般相対性理論と調和する



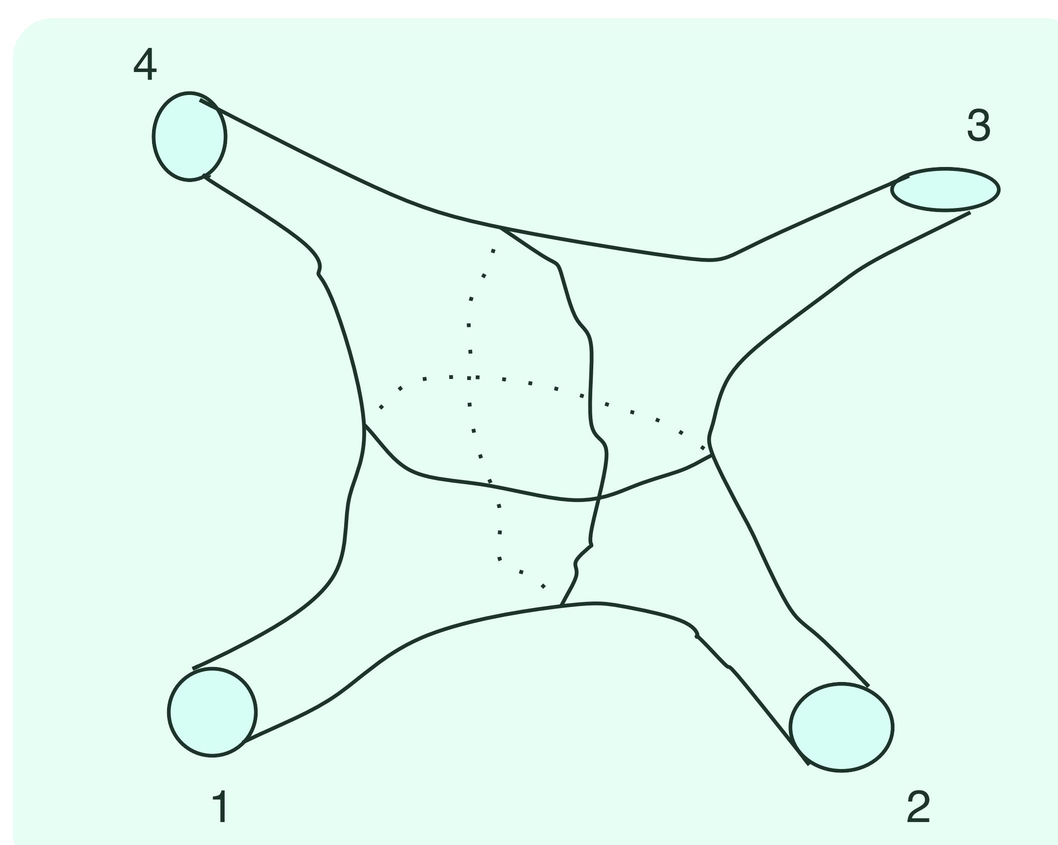
1970年代は、「弦」をよく“Violin string”と呼んでいた.

2.1 Basic ideas of perturbative string theory

particle Feynman rules ⟶ string Feynman rules



Feynman diagrams
for particles



Feynman diagrams
for closed Strings

particle quantum mechanics $x^{\mu}(\tau)$

string quantum mechanics
=2D conformal field theory $x^{\mu}(\tau,\sigma)$

(relativistic free) e. o. m. of particle

$$\frac{dp^\mu(\tau)}{d\tau} = 0 \qquad p_\mu = \frac{dx_\mu}{d\tau}$$

with constraint $p^2 + m^2 = 0$

generator of coordinate transformation

$$\tau \to f(\tau)$$

world-sheet conformal invariance

(free) e. o. m. of string

$$\left(\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial \sigma^2}\right) x^\mu = 0$$

with constraints

$$P_\tau^2 + P_\sigma^2 = 0, \qquad P_\tau \cdot P_\sigma = 0$$

$$P_\tau^\mu = \frac{\partial x^\mu}{\partial \tau}, \qquad P_\sigma^\mu = \frac{\partial x^\mu}{\partial \sigma}$$

generators of conformal transformation:

$$T_{\pm\pm} = \frac{1}{2}(P_\tau \pm P_\sigma)^2 \longleftrightarrow \tau \pm \sigma \to \tau' \pm \sigma' = f_\pm(\tau \pm \sigma)$$

Consistency of constraints

particles : conservation law

$$\frac{d}{d\tau}(p^2 + m^2) = 0$$

$$\{(p^2 + m^2)/2, x^\mu(\tau)\} = -\partial_\tau x^\mu(\tau)$$

relativistic strings :

$$\frac{\partial}{\partial \tau}\left(\frac{\partial x}{\partial \tau}\frac{\partial x}{\partial \sigma}\right) = \frac{1}{2}\frac{\partial}{\partial \sigma}\left(\left(\frac{\partial x}{\partial \tau}\right)^2 + \left(\frac{\partial x}{\partial \sigma}\right)^2\right)$$

$$\frac{\partial}{\partial \sigma}\left(\frac{\partial x}{\partial \tau}\frac{\partial x}{\partial \sigma}\right) = \frac{1}{2}\frac{\partial}{\partial \tau}\left(\left(\frac{\partial x}{\partial \tau}\right)^2 + \left(\frac{\partial x}{\partial \sigma}\right)^2\right)$$

or

$$\partial_\tau T_{10} = \partial_\sigma T_{11} \qquad \partial_\tau T_{00} = \partial_\sigma T_{01}$$

$$\{T_{\pm\pm}(\tau,\sigma), x^\mu(\tau,\sigma')\} = -\delta(\sigma - \sigma')(\partial_\tau \pm \partial_\sigma)x^\mu(\tau,\sigma)$$

Action principles :

relativistic particle mechanics

$$\int_0^1 d\xi \frac{1}{2}\sqrt{\gamma(\xi)}\Big[\gamma(\xi)^{-1}\Big(\frac{dx}{d\xi}\Big)^2 - m^2\Big]$$

e. o. m and constraint

$$\frac{dP}{d\xi} = 0, \quad P^2 = -m^2 \qquad P_\mu = \frac{\partial L_2}{\partial\left(\frac{dx^\mu}{d\xi}\right)} = \frac{1}{\sqrt{\gamma}}\frac{dx_\mu}{d\xi}$$

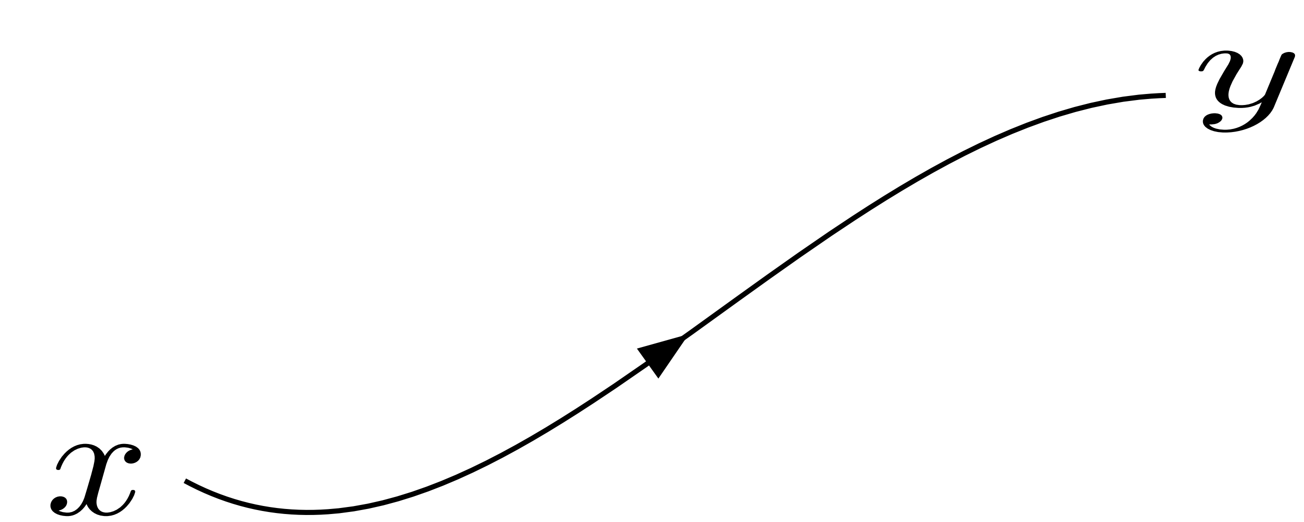
path-integral quantization of this action, using zeta-function regularization, automatically leads to (Feynman) propagator and vacuum loops

$$ds = \sqrt{\gamma}d\xi$$

$$\tau = \int ds$$

(Euclidean form)

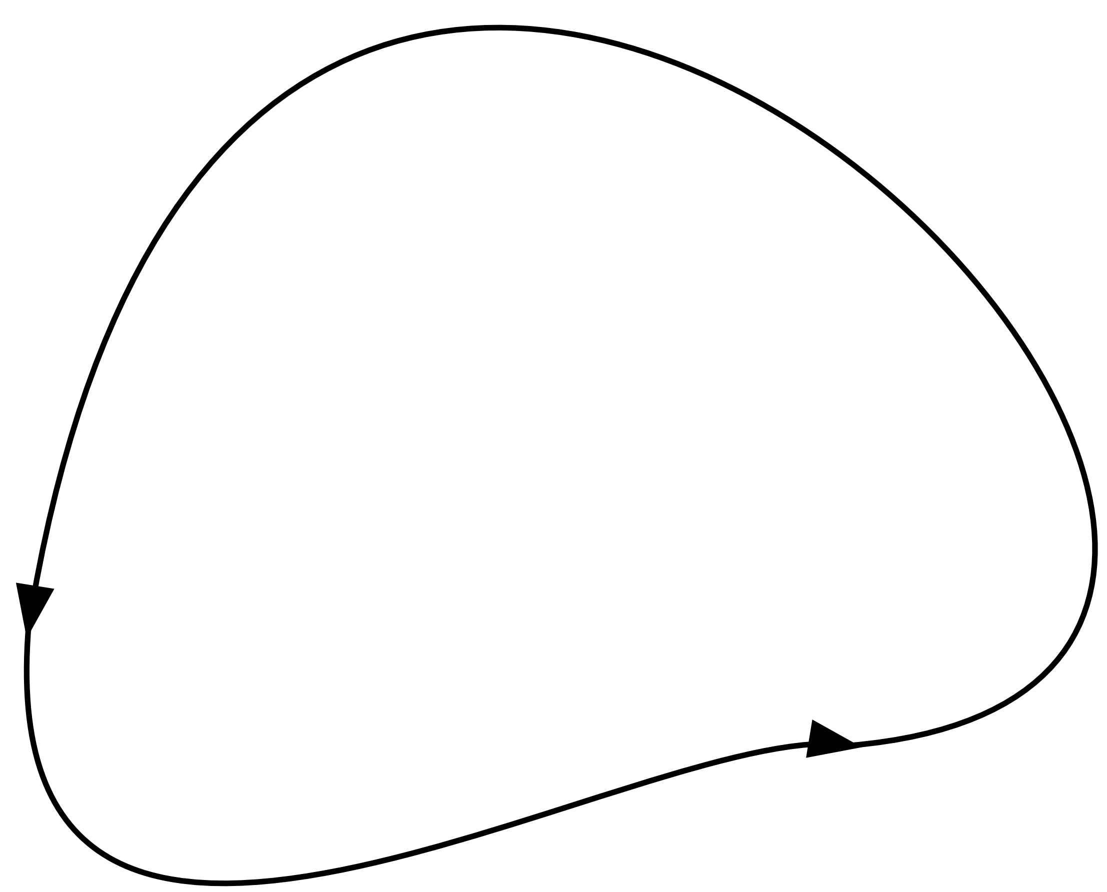
$$\int_0^\infty d\tau \int [\mathcal{D}x(s)] \exp\Big(-\frac{1}{2}\int_0^\tau ds\big[\big(\frac{dx}{ds}\big)^2 + m^2\big]\Big) = \int_0^\infty d\tau \langle x|\mathrm{e}^{-(-\Box+m^2)\tau}|y\rangle$$



$$\frac{1}{2}\int_0^\infty \frac{d\tau}{\tau}\int[\mathcal{D}x(s)]\exp\frac{1}{2}\int_0^\tau ds\Big[-\big(\frac{dx}{ds}\big)^2-m^2\Big] = \int d^Dx\,\frac{1}{2}\int_0^\infty\frac{d\tau}{\tau}\langle x|\mathrm{e}^{(\Box-m^2)\tau/2}|x)$$

$$= VT\frac{1}{2}\int_0^\infty\frac{d\tau}{\tau}\int\frac{d^Dp}{(2\pi)^D}\mathrm{e}^{-(p^2+m^2)\tau}$$

$$= \;T\frac{1}{2}\sum_{\mathbf{p}}\sqrt{\mathbf{p}^2+m^2}$$



zero-point oscillation of local fields

relativistic strings $$S_{\text{string}}[x(\tau,\sigma)] = -\frac{1}{4\pi\alpha'}\iint d^2\xi \sqrt{-\det\gamma(\xi)}\gamma^{ab}(\xi)g_{\mu\nu}(x)\frac{\partial x^\mu}{\partial \xi^a}\frac{\partial x^\nu}{\partial \xi^b}$$

"mass term" $\lambda \iint d^2\xi\sqrt{-\det\gamma_{ab}}$ is not allowed, leading to an inconsistency because of the local symmetry (Weyl invariance) $\gamma_{ab}(\xi) \to \rho(\xi)\gamma_{ab}(\xi)$

relativistic strings are basically massless !

e. o. m and constraint (in the conformal gauge $\gamma_{ab} = \rho(\xi)\eta_{ab}$)

$$\left(\frac{\partial^2}{\partial\tau^2} - \frac{\partial^2}{\partial\sigma^2}\right)x^\mu(\tau,\sigma) = 0 \qquad T_{00}(\sigma) \equiv \frac{1}{2}\left(2\pi\alpha' p(\sigma)^2 + \frac{1}{2\pi\alpha'}\left(\frac{\partial x}{\partial\sigma}\right)^2\right) = 0$$

$$p^\mu(\tau,\sigma) = \frac{1}{2\pi\alpha'}\frac{\partial x^\mu(\tau,\sigma)}{\partial\tau} \qquad T_{01}(\sigma) \equiv p(\sigma)\frac{\partial x(\sigma)}{\partial\sigma} = 0$$

invariant under conformal transformations $z^\pm \equiv \tau \pm \sigma \to (z')^\pm = \tau' \pm \sigma' = f_\pm(\tau \pm \sigma)$

$$(dz')^+(dz')^- = \frac{df_+(z^+)}{dz^+}\frac{df_-(z^-)}{dz^-}dz^+dz^-$$

We can derive various amplitudes in terms of path integral representation, generalizing the same method as in particle quantum mechanics.

classical images of (closed) strings in motion

$$\mathbf{x}(\tau,\sigma) = \mathbf{f}(\tau+\sigma)+\mathbf{g}(\tau-\sigma) \qquad |\mathbf{f}'|^2 = |\mathbf{g}'|^2 = \frac{c^2}{4}$$

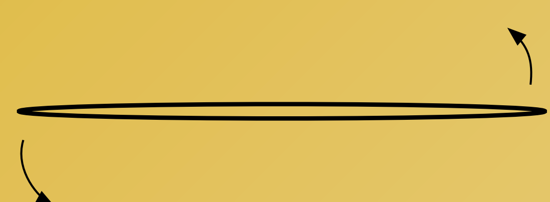
light velocity

$$E = \mathcal{T}c^2\int_0^K d\sigma = c^2K\mathcal{T}$$

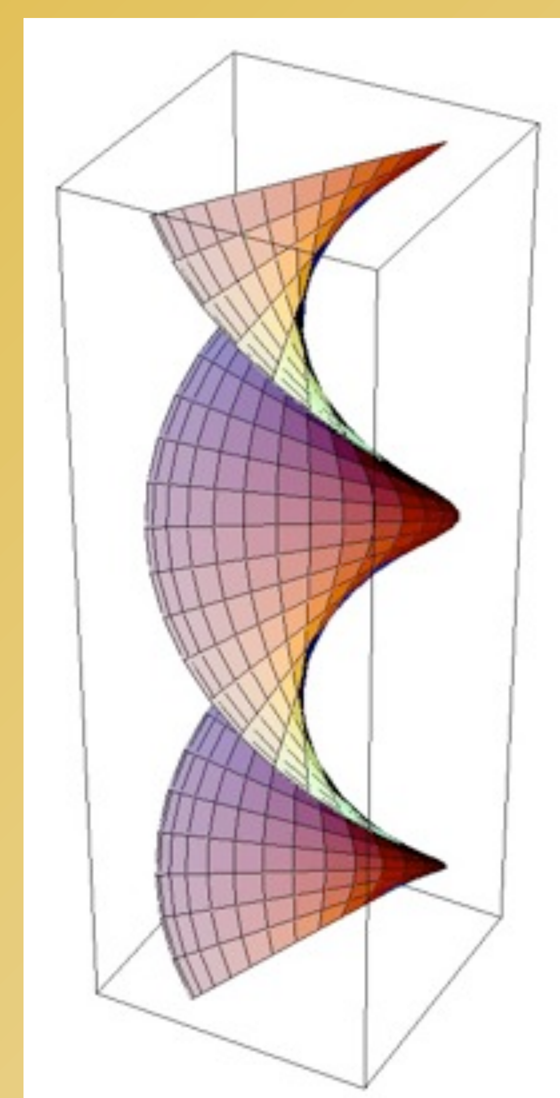
$$\mathbf{f}(x) = \mathbf{g}(x) = \frac{A}{2}\begin{pmatrix}\sin\frac{2\pi}{K}x\\ \cos\frac{2\pi}{K}x\\ 0\end{pmatrix}$$

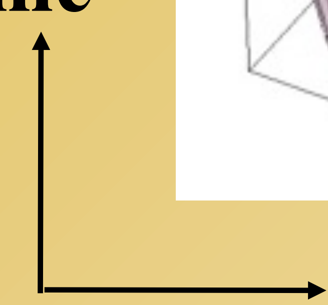
$$\mathbf{f}(x) = \frac{A}{2}\begin{pmatrix}\sin\frac{2\pi}{K}x\\ \cos\frac{2\pi}{K}x\\ 0\end{pmatrix},\quad \mathbf{g}(x) = \frac{A}{2}\begin{pmatrix}\cos\frac{2\pi}{K}x\\ \sin\frac{2\pi}{K}x\\ 0\end{pmatrix}$$

graviton



$$J = \frac{1}{2}\alpha' E^2$$

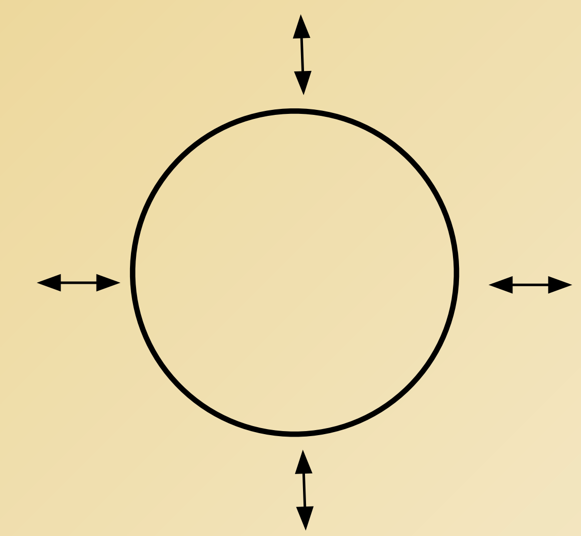




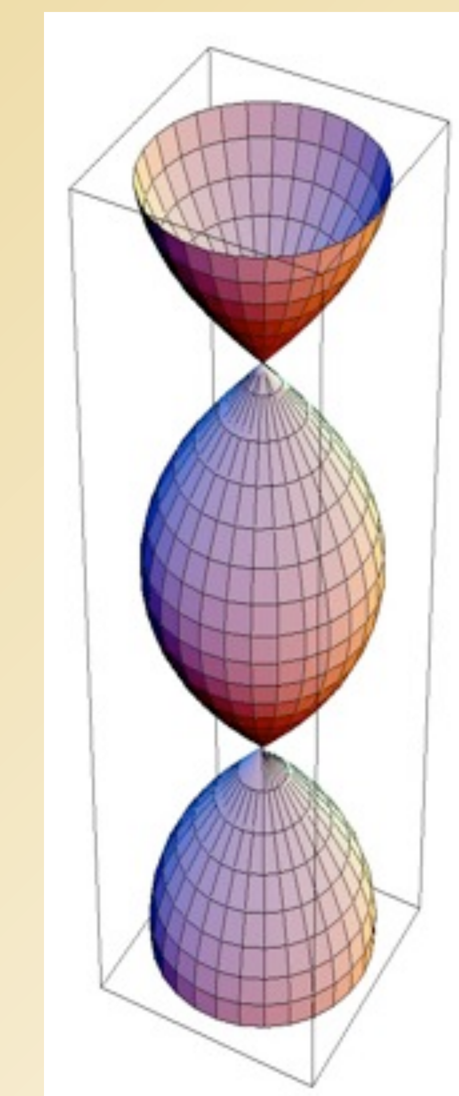
slope parameter

$$\alpha' = \frac{1}{2\pi\mathcal{T}c^2}$$

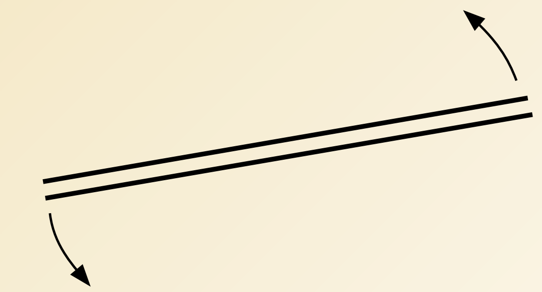
dilaton



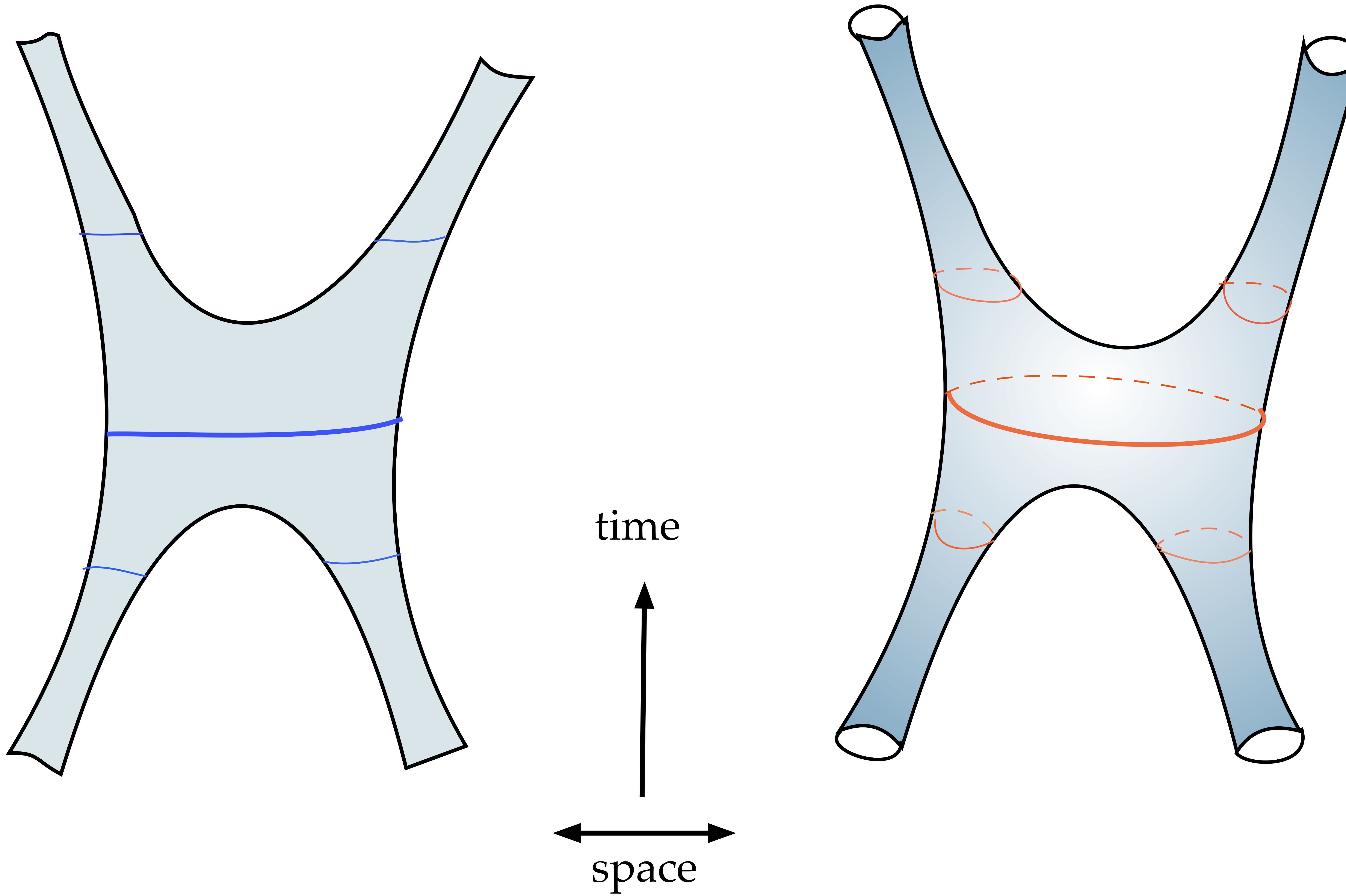
$$J = 0$$



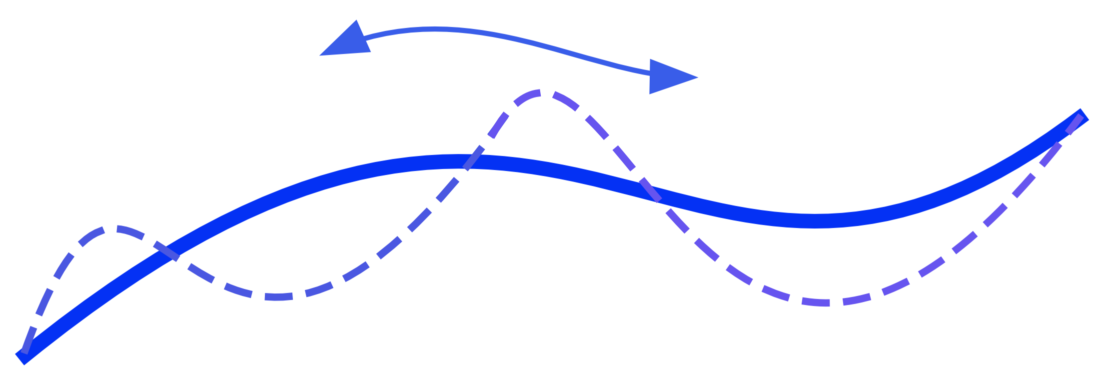
For open strings, $J \to 2J, \quad E \to 2E \quad \Rightarrow \quad J = \alpha' E^2$



world-sheet picture of (open and closed) string scatterings in Minkowsiki spacetime



超対称open stringの基底状態
(boson sector)

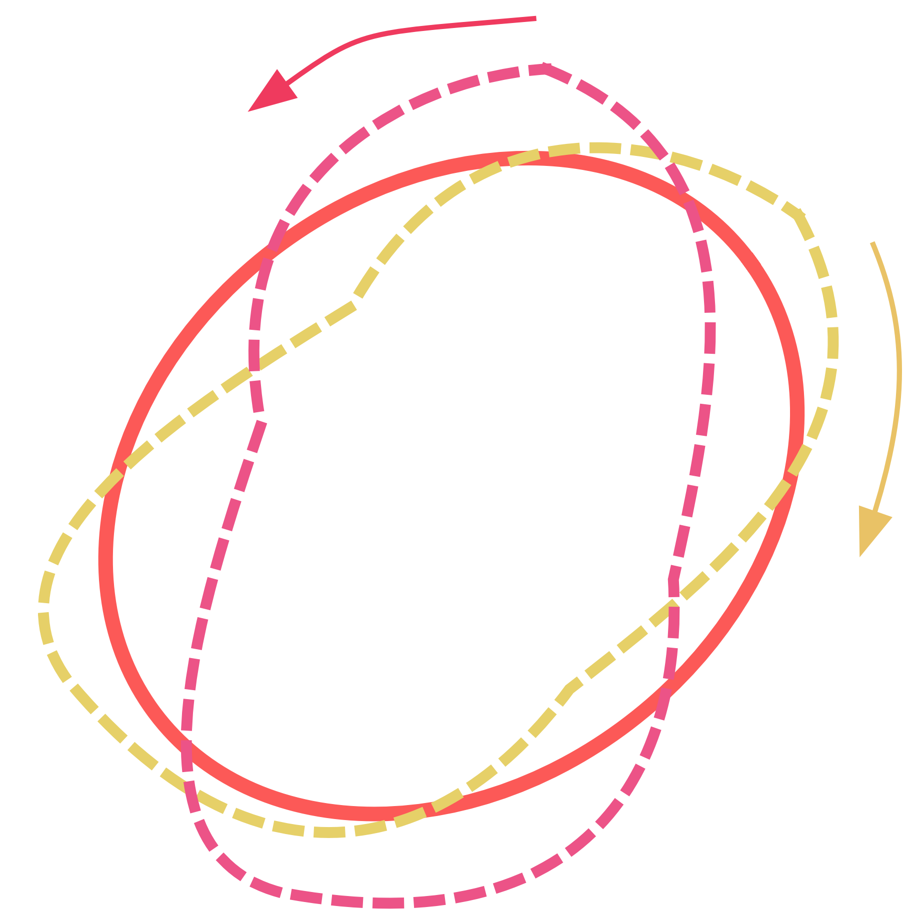


$8 = D - 2 \quad D = 10$ 次元

$(2 = D - 2 \ : \ D = 4$ 次元$)$

ゲージ粒子

超対称closed stringの基底状態
(NS-NS boson sector)



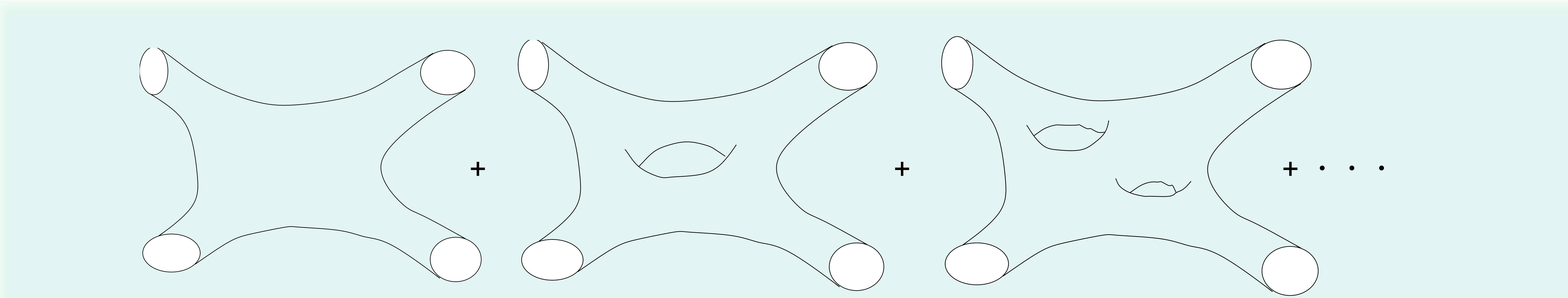
$8 \times 8 = 64 = 35 + 28 + 1$

$35 = D(D-3)/2 \quad D = 10$ 次元

$2 \times 2 = 2 + 1 + 1 \quad (2 = D(D-3)/2 \ : \ D = 4$ 次元$)$

重力子

$p_c = 4$



$h=0$ $h=1$ $h=2$

In terms of path integrals (Euclidean convention),

$$\sum_{\{\Sigma\}} g_s^{-\chi(\Sigma)} \int_{\mathcal{M}} [dX d\psi] \exp\left(-\frac{1}{4\pi\alpha'} S_\Sigma[X,\psi]\right)$$

$$\chi(\Sigma) \equiv 2 - 2h - b - p_c - p_o/2$$: Euler number

- h: # of handles
- b: # of boundaries
- p_c: # of closed-string punctures
- p_o: # of open-string punctures

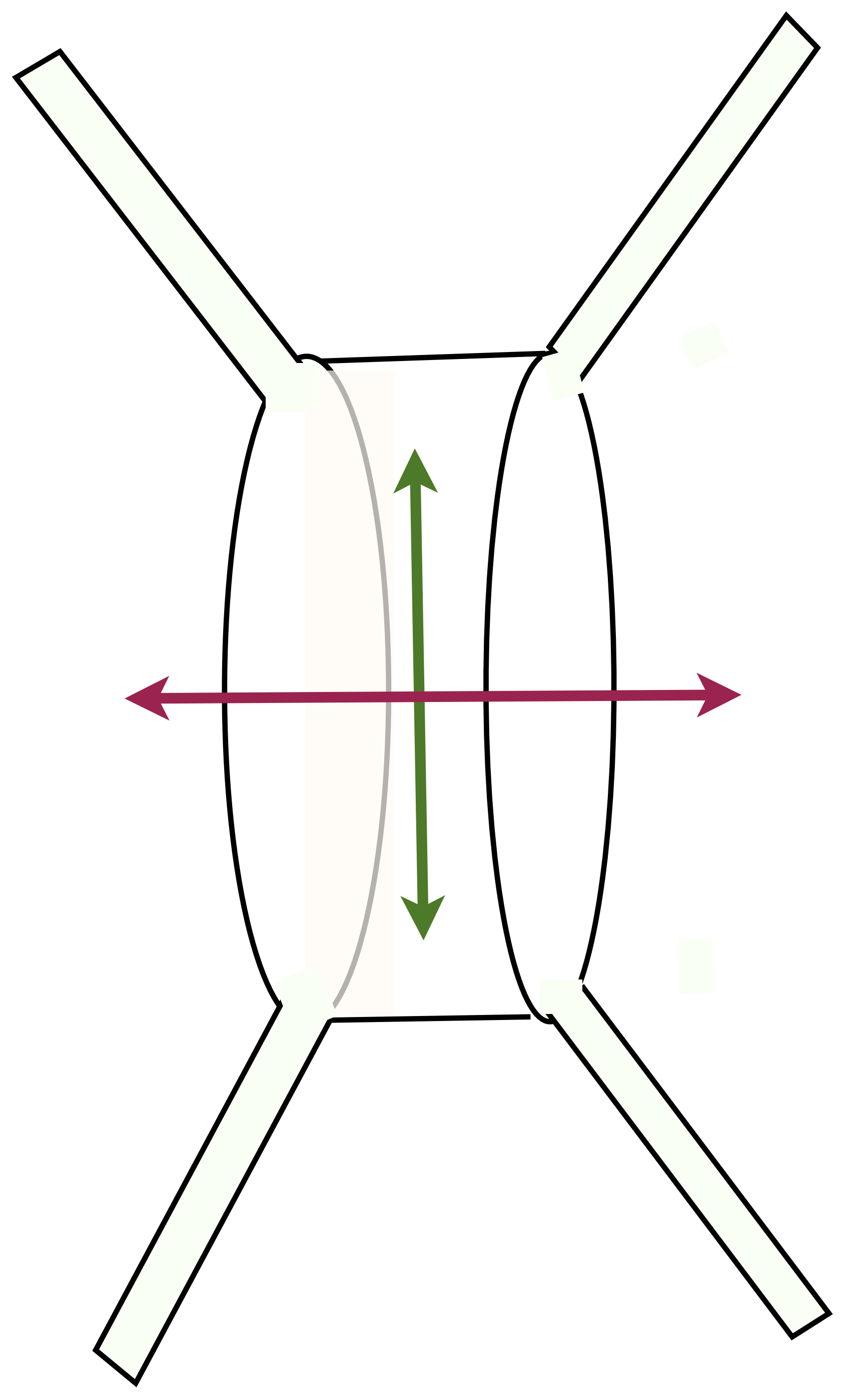
This form is fixed by the requirement of the unitarity of S-matrix

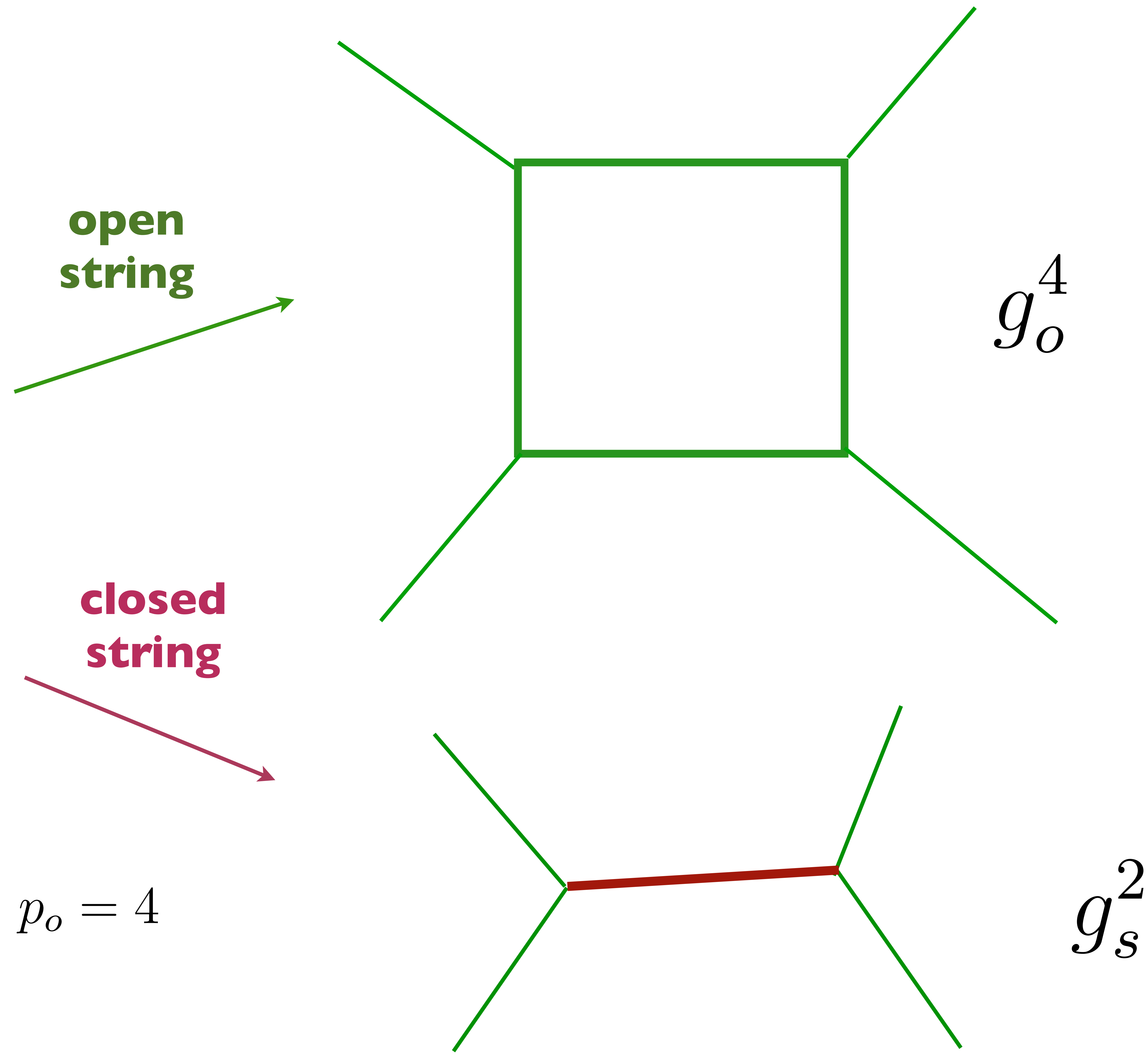
$$g_o^2 \sim g_s$$

g_s : coupling strength of closed string

g_o : coupling strength of open string

open-closed string duality



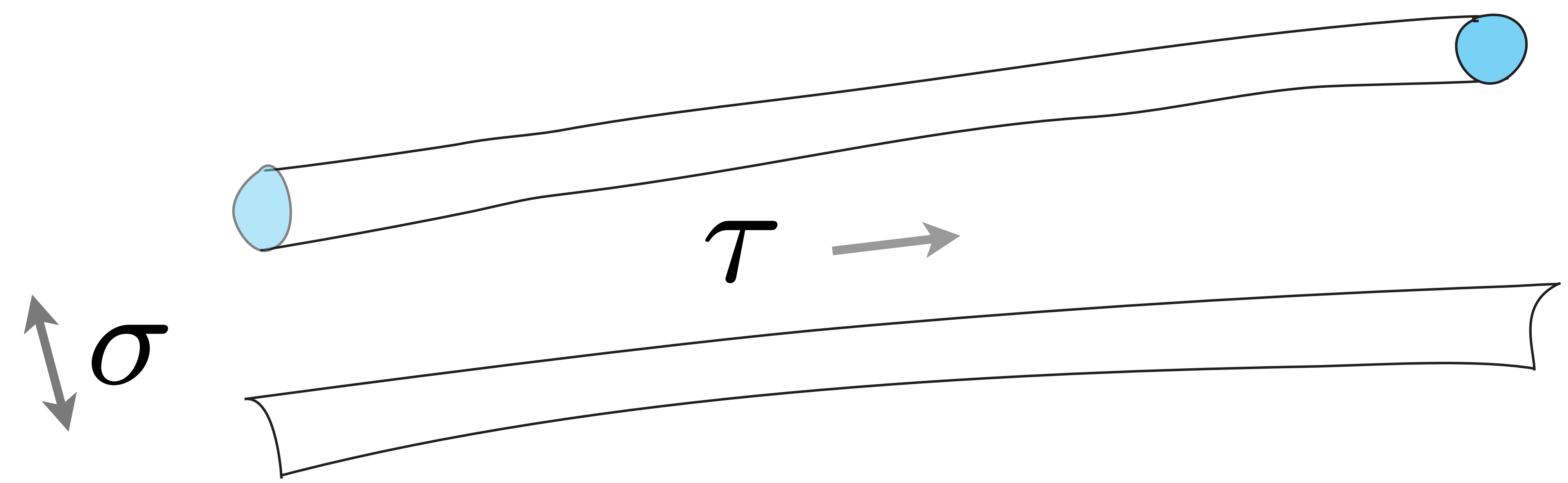


$$h = 0, \quad b = 2, \quad p_c = 0, \quad p_o = 4$$

$$\longrightarrow \quad g_o^4 \quad \sim \quad g_s^2$$

2.2 Spectrum from free propagating strings

intermediate states corresponding to a long cylinder (closed string) or strip (open string)



In the lowest tree approximation, the spectrum is determined by the Hamiltonian corresponding to infinitesimal 'time' translation on these world sheets. $\tau \longrightarrow \tau + \delta\tau$

The simplest way to do all such computations is to start from the (conformal-gauge) action

$$S = \frac{1}{4\pi\alpha'} \int d\tau d\sigma \left[\left(\frac{\partial x}{\partial \tau}\right)^2 - \left(\frac{\partial x}{\partial \sigma}\right)^2 \right]$$

leading to the Hamiltonian and canonical commutation relations

$$H = \frac{1}{4\pi\alpha'} \int d\sigma \left[\left(\frac{\partial x}{\partial \tau}\right)^2 + \left(\frac{\partial x}{\partial \sigma}\right)^2 \right]$$

$$[x^\mu(\sigma), p^\nu(\sigma)] = i\eta^{\mu\nu}\delta(\sigma - \sigma'), \quad p^\mu(\sigma) \equiv \frac{1}{2\pi\alpha'} \frac{\partial x^\mu(\sigma)}{\partial \tau}$$

The constraint operators satisfy the following Poisson algebra corresponding to the Lie algebra of conformal transformations.

$$\{T_{01}(\sigma_1), T_{01}(\sigma_2)\} = T_{01}(\sigma_1)\delta'(\sigma_1 - \sigma_2)$$
$$\{T_{01}(\sigma_1), T_{00}(\sigma_2)\} = T_{00}(\sigma_1)\delta'(\sigma_1 - \sigma_2)$$
$$\{T_{00}(\sigma_1), T_{00}(\sigma_2)\} = T_{01}(\sigma_1)\delta'(\sigma_1 - \sigma_2)$$

$$T_{--}(\tau - \sigma) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} e^{-in(\tau-\sigma)} L_n \qquad L_n = \frac{1}{2}\sum_{m=-\infty}^{\infty} \alpha_{-m}\alpha_{m+n}$$

$$T_{++}(\tau + \sigma) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} e^{-in(\tau+\sigma)} \tilde{L}_n \qquad \tilde{L}_n = \frac{1}{2}\sum_{m=-\infty}^{\infty} \tilde{\alpha}_{-m}\tilde{\alpha}_{m+n}$$

After quantization, the algebra is

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0},$$
$$[\tilde{L}_m, \tilde{L}_n] = (m-n)\tilde{L}_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}$$

correspond to anomaly in the conformal transformation law

$$c\frac{\partial^3}{\partial\sigma_1^3}\delta(\sigma_1 - \sigma_2)$$

c=effective degrees of freedom (central charge)

For the purpose of obtaining the spectrum, it is convenient to solve (the non-zero mode part of) the constraints by choosing light-cone gauge, utilizing the gauge symmetry of conformal transformation.

$$x^+ \equiv x^0 + x^{D-1} \propto P^+ \tau \,, \qquad \frac{\partial}{\partial \sigma} p^+ = 0$$

$$p^+ \equiv \frac{1}{2}(p^0 + p^{D-1}) \Rightarrow P^+$$

center of mass momentum

Then, due to the existence of two constraints, only D-2 directions of the oscillation modes of the transverse space-time coordinates are independent physical degrees of freedom.

$$x^i(\tau, \sigma)\,,\ p^i(\tau, \sigma) \qquad i = 1, 2, \ldots, D-2$$

The zero-mode part of the `Hamiltonian' constraint gives the mass-shell condition

$$\int d\sigma \left[\left(\frac{\partial x^\mu}{\partial \tau} \right)^2 + \left(\frac{\partial x^\mu}{\partial \sigma} \right)^2 \right] = 0 \longrightarrow \qquad -2P^+P^- + \frac{1}{\alpha'} H_\perp = 0$$

$H_\perp$: Hamiltonian with only transverse components

We can also treat all directions, including the time direction, on an equal footing and then impose the constraints afterwards as subsidiary conditions on allowed physical states,
either in the Gupta-Bleuler like covariant formalism or in the BRST formalism.
We always get the same final results in all these different formulations.

Solutions of e. o. m and the commutation relations I : closed strings

$[\tau, \sigma]\quad \tau \in [-\infty, \infty], \sigma \sim \sigma + 2\pi$

general solution of the wave equation (transverse modes)

$$x(\tau,\sigma) = \frac{1}{2}(x(\tau-\sigma) + \tilde{x}(\tau+\sigma))$$

$$x(\tau-\sigma) = x_0 + \alpha' p_0(\tau-\sigma) + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n e^{-in(\tau-\sigma)}$$

$$\tilde{x}(\tau+\sigma) = \tilde{x}_0 + \alpha' \tilde{p}_0(\tau+\sigma) + i\sqrt{2\alpha}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n e^{-in(\tau+\sigma)}$$

$$x(\tau,\sigma) = X + \alpha' P\tau + \sqrt{\alpha'}w\sigma + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}(\alpha_n \mathrm{e}^{-in(\tau-\sigma)} + \tilde{\alpha}_n \mathrm{e}^{-in(\tau+\sigma)})$$

$$[\alpha_n, \alpha_m] = n\delta_{n+m,0}, \quad [\tilde{\alpha}_n, \tilde{\alpha}_m] = n\delta_{n+m,0}$$

$$[X, P] = i, \quad [w, X] = [w, P] = 0$$

If we assume that the (target) space has a finite length R with periodic boundary condition along a particular direction $\sigma \Rightarrow \sigma + 2\pi \longrightarrow x(\sigma) \longrightarrow x(\sigma) + 2\pi R n$

the zero mode parts are quantized as (c. o. m momentum and winding, respectively)

$$P = m/R, \quad \sqrt{\alpha'}w = Rn$$

Zero mode constraints

Hamiltonian constraint : mass-shell condition

$$\frac{\alpha'}{2}M^2 = -\frac{\alpha'}{2}P^2 = \frac{\alpha'}{2}\left(\left(\frac{m}{R}\right)^2 + \left(\frac{Rn}{\alpha'}\right)^2\right) + \sum_{n=1}^{\infty}(\alpha_{-n}\alpha_n + \tilde{\alpha}_{-n}\tilde{\alpha}_n) - \frac{D-2}{12}$$

momentum constraint : level-matching condition

$$mn + \sum_{n=1}^{\infty}(\alpha_{-n}\alpha_n - \tilde{\alpha}_{-n}\tilde{\alpha}_n) = 0 \longleftarrow \int d\sigma \frac{\partial x^\mu}{\partial \tau}\frac{\partial x_\mu}{\partial \sigma} = 0$$

The last term of the Hamiltonian constraint is the contribution of the zero-point oscillations. The divergent sum is regularized by defining it as a particular limiting case of Riemann`s zeta function through an analytic continuation. (zeta-function regularization)

$$\frac{1}{2}\sum_{n=1} n = \frac{1}{2}\zeta(-1) = -\frac{1}{24}$$

$s \rightarrow -1$

$$\zeta(s) \equiv \sum_{n=1} n^{-s} = \frac{1}{\Gamma(s)}\int_0^\infty \frac{x^{s-1}}{e^x - 1} = \prod_{\text{primes}} (1-p^{-s})^{-1}$$

Let us consider the case where the space-time is just flat Minkowski space without any compactified directions. Then the first terms in the mass-shell & level-matching conditions do not appear, and the theory must be Lorentz covariant.

Lorentz covariance requires that $\frac{D-2}{12} = 2 \quad \rightarrow \quad D = 26$ (critical dimension)

since only in that case the first excited states correspond to allowed representations of Lorentz group of massless states.

$$|ij\rangle \equiv \alpha^i_{-1}\tilde{\alpha}^j_{-1}|0\rangle$$

irreducible representations

$$= \left(\alpha^{(i}_{-1}\tilde{\alpha}^{j)}_{-1} - \frac{\delta_{ij}}{D-2}\mathrm{Tr}_i\left[\alpha^i_{-1}\tilde{\alpha}^j_{-1}\right]\right)|0\rangle \rightarrow$$

$$\frac{1}{2}(D-2)(D-1)-1 = \frac{D(D-3)}{2} = \frac{D(D+1)}{2} - 2D$$

$g_{\mu\nu} = g_{\nu\mu}$ graviton

$$+\frac{\delta_{ij}}{D-2}\mathrm{Tr}_i\left[\alpha^i_{-1}\tilde{\alpha}^j_{-1}\right]|0\rangle \rightarrow$$

ϕ dilaton

$$+\alpha^{[i}_{-1}\tilde{\alpha}^{j]}_{-1}|0\rangle \rightarrow$$

$$\frac{1}{2}(D-2)(D-3) = \frac{D(D-1)}{2} - (D-1) - (D-2)$$

B-field

$$B_{\mu\nu} = -B_{\nu\mu}$$

$$\delta B_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu$$

This result in turn causes the problem of tachyonic ground state.

Remedy: space-time supersymmetry

Linearized gravitational fields ("a particle-physicist's derivation of general relativity")

We can arrive at the vector gauge fields starting from $e\int d^4x\, A_\mu j^\mu$

$$\partial_\mu j^\mu = 0 \longrightarrow A_\mu \to A_\mu + \partial_\mu \lambda$$

$\partial^\mu A_\mu = 0 \quad \Box\lambda = 0$

gauge condition and residual gauge degrees

$D \to D-2$

Similarly, we can start from $\kappa\int d^4x\, h_{\mu\nu}T^{\mu\nu}$

$$T^{\mu\nu} = T^{\nu\mu} \qquad \partial_\mu T^{\mu\nu} = 0 \longrightarrow h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$$

However, a big difference! This is valid only at the lowest order in κ

Energy and momentum of $h_{\mu\nu}$ must also be included in the energy-momentum tensor, when we go to higher orders.

In the lowest order on-shell approximation, we can utilize the above gauge invariance.

$$\psi_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_\mu h \qquad h = h^\mu{}_\mu$$

$$\delta\psi_{\mu\nu} = \partial_\mu\xi_\nu + \partial_\nu\xi_\mu - \eta_{\mu\nu}\partial\xi$$

$$\partial^\mu\delta\psi_{\mu\nu} = \Box\xi_\nu$$

$\partial^\mu\psi_{\mu\nu} = 0 \qquad \Box\xi_\mu = 0$ gauge condition and residual gauge degrees

$$\frac{D(D+1)}{2} - 2D = \frac{D(D-3)}{2}$$

Derivation of the action for $h_{\mu\nu}$

Under the requirements that the action is second order with respect to space-time derivatives and is invariant under the gauge transformation, the Lagrangian is unique, up to total derivatives and is equal to the linearized form of the Einstein action.

$$L = \frac{1}{4}\partial_\sigma h^{\mu\nu}\partial^\sigma h^{\mu\nu} - \frac{1}{2}\partial_\sigma h_{\mu\nu}\partial^\nu h^{\mu\sigma} - \frac{1}{4}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu h \partial_\nu h^{\nu\mu}$$

eq. of motion

$$G_{\mu\nu} = 0 \qquad \partial^\mu G_{\mu\nu} = 0$$

$$2G^{\alpha\beta} = \Box h^{\alpha\beta} + \partial^\alpha\partial^\beta h - \partial^\beta\partial_\sigma h^{\alpha\sigma} - \partial_\sigma\partial^\alpha h^{\beta\sigma} + \eta^\alpha(\partial_\mu\partial_\nu h^{\mu\nu} - \Box h)$$

As soon as we add higher order terms to this form, we encounter inconsistency, unless we **extend the gauge transformation recursively** with respect to the coupling constant. The recursive construction is bound to coincide with the expansion of the Einstein action using $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$

A convenient way of convincing this expectation is to start from the first-order form of the linearized Einstein action.

$$A = \int d^D x [\psi^{\mu\nu}(\Gamma^\alpha_{\mu\nu,\alpha} - \Gamma_{\mu,\nu}) + \eta^{\mu\nu}(\Gamma^\alpha_{\mu\nu}\Gamma_\alpha - \Gamma^\alpha_{\beta\mu}\Gamma^\beta_{\alpha\nu})] \qquad \Gamma_\mu = \Gamma^\alpha_{\mu\alpha}$$

treating $\psi_{\mu\nu}$ and $\Gamma^\alpha_{\mu\nu}$ as independent fields.

$$\delta\psi \longrightarrow \quad \partial_\alpha \Gamma^\alpha_{\mu\nu} - \frac{1}{2}\partial_\nu \Gamma_\mu - \frac{1}{2}\partial_\mu \Gamma_\nu = 0$$

$$\delta\Gamma \longrightarrow \quad 2\Gamma^\alpha_{\mu\nu} - \eta^\alpha_\mu \Gamma_\nu - \eta^\alpha_\nu \Gamma_\mu = \partial^\alpha \psi_{\mu\nu} - \partial_\nu \psi^\alpha_\mu - \partial_\mu \psi^\alpha_\nu - \frac{1}{2}\eta_{\mu\nu}\partial_\alpha \partial_\beta \psi^{\alpha\beta}$$

$$\Box \psi^{\mu\nu} - \partial_\alpha \partial^\nu \psi^{\mu\alpha} - \partial_\alpha \partial^\mu \psi^{\nu\alpha} - \frac{1}{2}\Box \psi = 0$$

This is equivalent with the linearized eq. of motion for $h_{\mu\nu}$

The complete non-linear action is obtained from this form simply by the replacement

$$\eta^{\mu\nu} \longrightarrow \eta^{\mu\nu} + \psi^{\mu\nu}$$

which gives the correct first order Einstein action up to total derivatives.

- Vertex operators corresponding to these states can be identified as infinitesimal deformations of the string action from the flat space-time with no background.

$$S[x] = \frac{1}{4\pi\alpha'}\int d^2\xi\sqrt{-h}\left[g_{\mu\nu}(x)h^{ab}\partial_a x^\mu \partial_b x^\nu + B_{\mu\nu}(x)\epsilon^{ab}\partial_a x^\mu \partial_b x^\nu\right] + \frac{1}{2\pi}\int d^2\xi\sqrt{-h}R^{(2)}\phi(x) + \text{surface terms}$$

or in the Euclidean form with the conformal gauge $h_{ab} = e^\rho \delta_{ab}$

$$S_{\rm E}[x] = \frac{1}{4\pi\alpha'}\int |dz|^2\left[g_{\mu\nu}(x)\partial_z x^\mu \partial_{\bar z} x^\nu + B_{\mu\nu}(x)\partial_z x^\mu \partial_{\bar z} x^\nu\right] - \frac{1}{2\pi}\int |dz|^2\,\phi\partial_z\partial_{\bar z}\rho$$

The linearized fields correspond to the vertex operators:

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \qquad \Box h_{\mu\nu} = 0 \quad \partial^\mu h_{\mu\nu} = 0 \quad h^\mu{}_\mu = 0$$

$$B_{\mu\nu} = -B_{\nu\mu} \qquad \Box B_{\mu\nu} = 0 \quad \partial^\mu B_{\mu\nu} = 0$$

$$\Box\phi = 0$$

$$V_h(z,\bar z) \sim h_{\mu\nu}(x)\partial_z x^\mu \partial_{\bar z} x^\nu$$

$$V_B \sim B_{\mu\nu}(x)\partial_z x^\mu \partial_{\bar z} x^\nu$$

$$V_\phi \sim \phi\partial_z\partial_{\bar z}\rho$$

valid in the bulk of world sheet

- These vertex operators satisfy the correct transformation properties under conformal transformation $\delta_\epsilon z = \epsilon(z)$ (related to Weyl invariance).

$$\delta_\epsilon V(z,\bar{z}) = \partial_z\left[\epsilon(z)V(z,\bar{z})\right] + \partial_{\bar{z}}\left[\overline{\epsilon(z)}V(z,\bar{z})\right]$$

$$\updownarrow$$

$$\delta_\epsilon \int |dz|^2 V(z,\bar{z}) = 0$$

- The metric can be redefined by mixing with dilaton by the field redefinition

$$h_{\mu\nu} \rightarrow h_{\mu\nu} - \frac{a}{\alpha'}\phi\eta_{\mu\nu}$$

(`dilaton': some similarity with the NG boson associated with dilation)

- Soft-dilaton theorem: shift of the dilaton field by a constant induces the change of the string coupling

$$\phi(x) \rightarrow \phi(x) + c \qquad \frac{1}{2\pi}\int d^2\xi\sqrt{-h}R^{(2)}\phi(x) \rightarrow \frac{1}{2\pi}\int d^2\xi\sqrt{-h}R^{(2)}\phi(x) + c\chi$$

We can thus make identification $g_s = e^{\langle\phi\rangle}$

(We can also prove this theorem in the framework of string-field theory)

- The string action with nontrivial backgrounds is nothing but a nonlinear sigma model in two dimensions. In the weak-coupling expansion (which is essentially the alpha' expansion), the conditions for Weyl invariance (or fixed-point condition of renormalization group) are known to be

$$R_{\mu\nu} + \frac{1}{4} H_\mu{}^{\alpha\beta} H_{\mu\alpha\beta} - 2D_\mu D_\nu \phi = 0$$

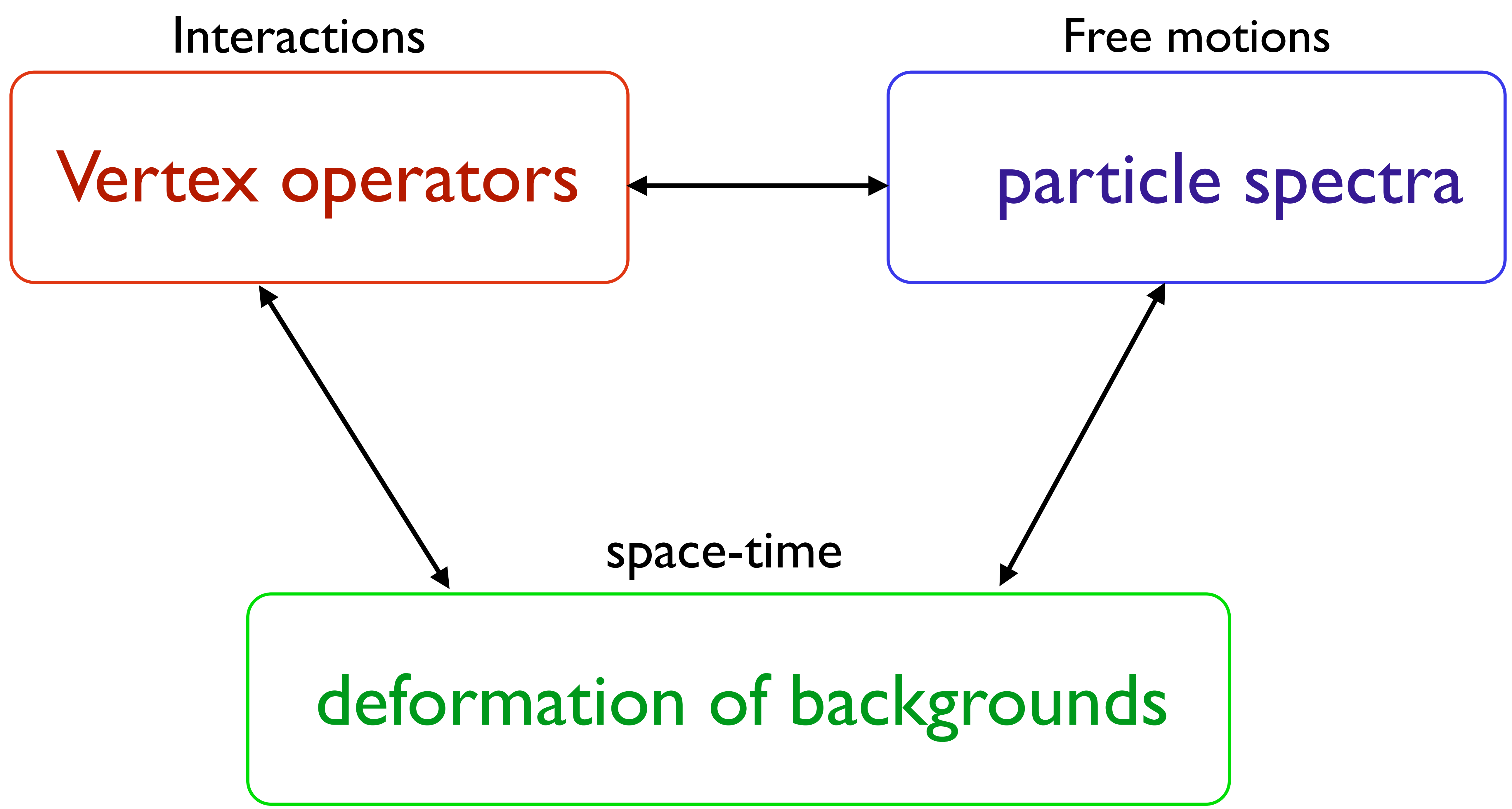
$$D_\alpha H^\alpha_{\ \mu\nu} - 2(D_\alpha \phi) H^\alpha_{\ \mu\nu} = 0$$

$$4(D_\mu \phi)^2 - 4D_\mu D^\mu \phi + R + \frac{1}{12} H_{\alpha\mu\nu} H^{\alpha\mu\nu} = 0$$

In the linearized approximation for backgrounds, these equations reduce to the massless field equations for graviton, B-field and dilaton.

These equations and its action should be
(and have been confirmed by explicit computation in lower orders)
equivalent with the effective action for the S-matrix of string scatterings.

Trinity of string theory



The non-linear structure of interactions automatically emerge from the extendedness of strings which is governed by world-sheet conformal symmetry

This indicates that the structure of string theory can be *basically* background independent.

It is one of the most fundamental (and long-standing) problems to explore manifestly background independent formulations of string theory.

Solutions of e. o. m and the commutation relations II : open strings $0 \leq \sigma \leq \pi$

two typical boundary conditions at the end points, free (Neumann) or fixed (Dirichlet)

N : $\partial_\sigma x = 0$

D : $\partial_\tau x = 0$

consistent with the momentum constraint at the boundary $\partial_\tau x \partial_\sigma x = 0$

Using the same general solution as for closed strings (at $\sigma = 0, \pi$),

$$N \Rightarrow -\alpha'(p_0 - \tilde{p}_0) + i\sqrt{2\alpha'}\sum_{n\neq 0} i(\alpha_n \mathrm{e}^{-in\tau} - \tilde{\alpha}_n \mathrm{e}^{-in\tau}) = 0$$

$$\Downarrow$$

$$p_0 = \tilde{p}_0 \equiv 2P, \quad \alpha_n = \tilde{\alpha}_n$$

P : c.o.m momentum along N direction

$$D \Rightarrow \alpha'(p_0 + \tilde{p}_0) + i\sqrt{2\alpha'}\sum_{n\neq 0}(-i)(\alpha_n \mathrm{e}^{-in\tau} + \tilde{\alpha}_n \mathrm{e}^{-in\tau}) = 0$$

$$\Downarrow$$

$$p_0 = -\tilde{p}_0 \equiv L/\alpha', \quad \alpha_n = -\tilde{\alpha}_n$$

$\ell = \pi L$: string length along D direction

$$N: x(\tau,\sigma) = X + 2\alpha' P\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n \mathrm{e}^{-in\tau}\cos n\sigma$$

$$p(\tau,\sigma) = \frac{1}{2\pi\alpha'}\dot{x} = \frac{1}{2\pi\alpha'}\left(2\alpha' P + \sqrt{2\alpha'}\sum_{n\neq 0}\alpha_n \mathrm{e}^{-in\tau}\cos n\sigma\right)$$

$$x'(\tau,\sigma) = -i\sqrt{2\alpha'}\sum_{n\neq 0}\alpha_n \mathrm{e}^{-in\tau}\sin n\sigma$$

$$D: x(\tau,\sigma) = X - L\sigma - \sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n \mathrm{e}^{-in\tau}\sin n\sigma$$

$$p(\tau,\sigma) = \frac{1}{2\pi\alpha'}\dot{x} = \frac{i}{2\pi\alpha'}\sqrt{2\alpha'}\sum_{n\neq 0}\alpha_n \mathrm{e}^{-in\tau}\sin n\sigma$$

$$x'(\tau,\sigma) = -L - \sqrt{2\alpha'}\sum_{n\neq 0}\alpha_n \mathrm{e}^{-in\tau}\cos n\sigma$$

For simplicity, only NN and DD cases are treated here.

Hamiltonian constraints : $(H-1)|\Psi\rangle = 0$

$$N \Rightarrow H \to \alpha' P^2 + \sum_{n=1}^{\infty}\alpha_{-n}\alpha_n$$

$$D \Rightarrow H \to \frac{1}{4\alpha'}L^2 + \sum_{n=1}^{\infty}\alpha_{-n}\alpha_n$$

$-\dfrac{D-2}{24}$

$D = 26$

Massless vector modes propagating along Neumann directions and massless scalar modes associated with Dirichlet directions

$$\alpha^i_{-1}|0\rangle$$

2.3 T-duality

The Hamiltonian is invariant under

$$\sqrt{1/2\pi\alpha'}\partial_\sigma x \leftrightarrow -\sqrt{1/2\pi\alpha'}\partial_\tau x = -\sqrt{2\pi\alpha'}p$$

In the case of field theories on space-time lattices, this simply corresponds to transformation between the original lattice and its dual lattice (Kramers-Wannier duality)

closed strings :

$$\sqrt{2\pi\alpha'}p(\sigma) = \frac{1}{\sqrt{4\pi}}\sum_n [\alpha_n \mathrm{e}^{in\sigma} + \tilde{\alpha}_n \mathrm{e}^{-in\sigma}] \qquad p_0^\mu = \sqrt{2/\alpha'}\alpha_0^\mu;$$

$$-\sqrt{1/2\pi\alpha'}x'(\sigma) = \frac{1}{\sqrt{4\pi}}\sum_n [\alpha_n \mathrm{e}^{in\sigma} - \tilde{\alpha}_n \mathrm{e}^{-in\sigma}] \qquad \tilde{p}_0^\mu = \sqrt{2/\alpha'}\tilde{\alpha}_0^\mu$$

$$(m,n) \leftrightarrow (n,m), \quad R \leftrightarrow \frac{\alpha'}{R} \qquad \alpha_n \leftrightarrow \alpha_n, \quad \tilde{\alpha}_n \leftrightarrow -\tilde{\alpha}_n$$

open strings :

$$(N,D) \rightarrow (D,N)$$

$$\sqrt{2\alpha'}P \rightarrow L/\sqrt{\alpha'}, \quad \alpha_n \rightarrow \alpha_n$$

There can be no such symmetries in particle mechanics. Historically, T-duality symmetry first explicitly appeared in the calculation of one-loop vacuum amplitude with torus compactification (Kikkawa-Yamasaki, 1984).

By a T-duality transformation along a particular direction, a theory compactified on a circle of radius R is transformed into another theory with radius 1/R (in the string unit), with momentum and winding modes being interchanged for closed strings and, for open strings, with Neumann and Dirichlet boundary conditions being interchanged.

Further analyses of scattering amplitudes indicate that the string coupling is transformed as (being consistent with the lattice analysis)

$$g_s \longrightarrow g_s/R$$

This is consistent with the requirement that the effective gravitational constant in un-compactified part of the space-time must be invariant under T-duality.

$$1/G_{eff} = R/g_s^2 \qquad \text{(string unit} \quad \alpha' = 1 \quad \text{)}$$

$$R \rightarrow 1/R, \quad g_s \rightarrow g_s' \qquad R/g_s^2 = 1/R \times 1/(g_s')^2$$

$$\downarrow$$

$$g_s' = g_s/R$$

T-duality in the lattice-regularized world-sheet picture
(Kramers-Wannier duality)

$$\iint d\tau d\sigma (\partial x)^2 \to \sum_{ij} a^2 \left(\frac{x_i - x_j + 2\pi m_{ij} R}{a}\right)^2 = \sum_{ij} (x_i - x_j + 2\pi m_{ij} R)^2$$

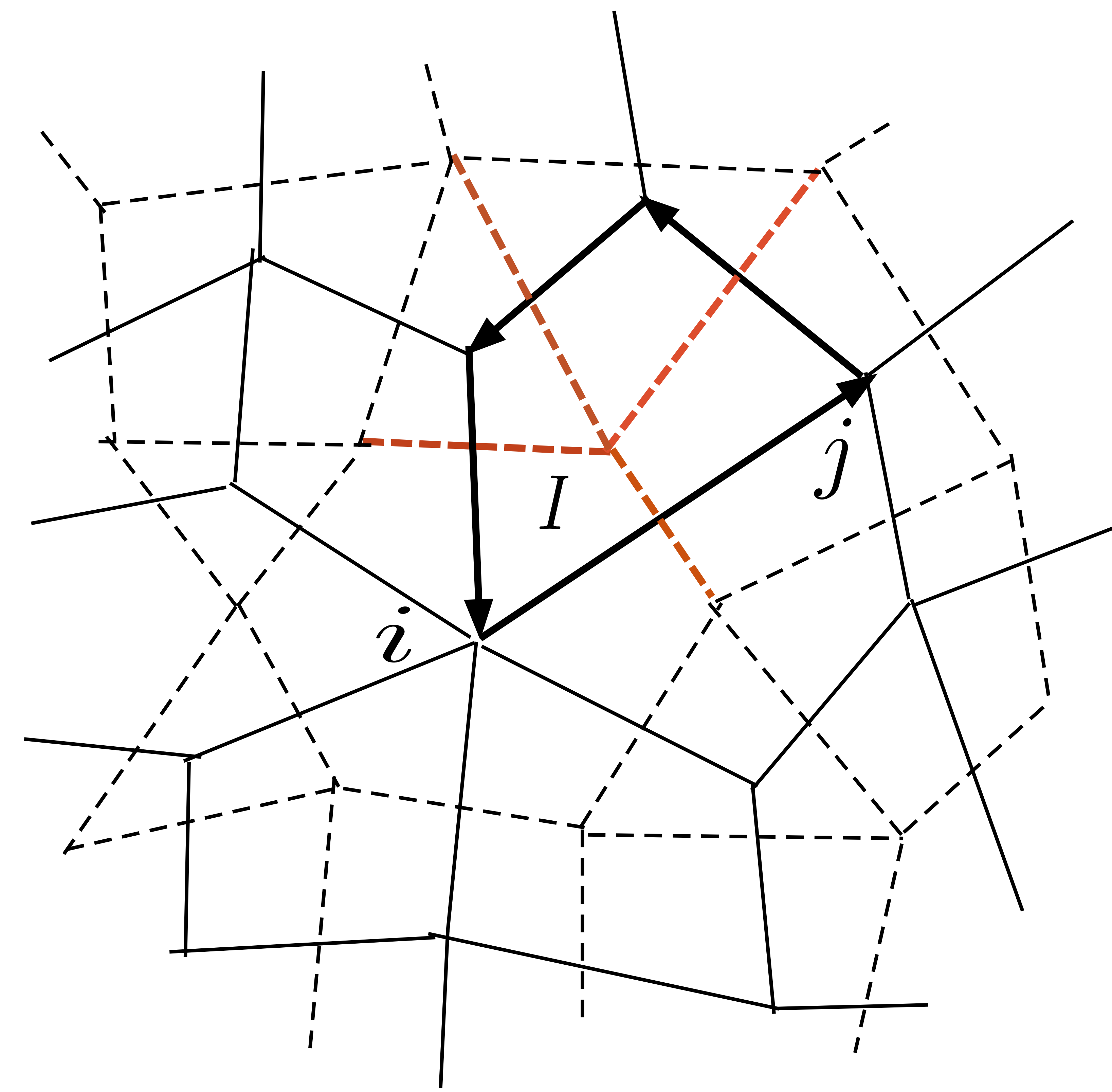
with constraint $\sum_{i,j \in I} m_{ij} = 0$

forbidding the excitation of local vortices.

The periodicity of the compactified circle
is represented by the gauge symmetry

$$x_i \to x_i + 2\pi R n_i$$

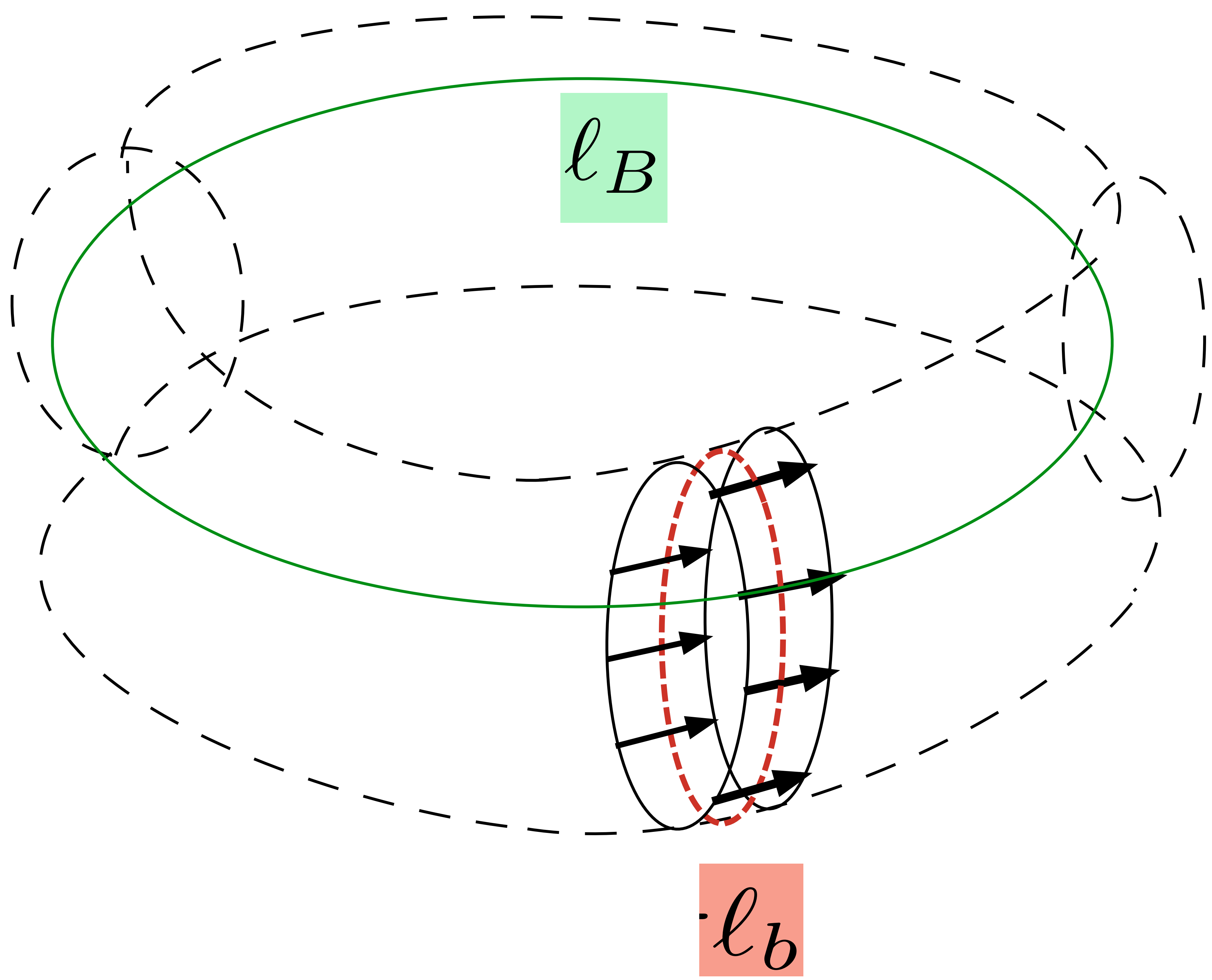
$$m_{ij} \to m_{ij} - 2\pi R(n_i - n_j)$$



lattice and dual lattice

general solution of the constraint :

$$m_{ij} = m_i - m_j + \ell_b \epsilon_{ij}^b, \quad m_i \in Z, \quad \ell_a \in Z, \quad \sum_i m_i = 0$$



representative homology cycles of a torus

$$\epsilon_{ij}^b = \pm 1$$

when the link (ij) crosses the representative homology cycle b on the dual lattice, with signs depending on orientation.

There is *one*-to-*one* correspondence between the homology cycles on the original lattice and the dual lattice.

Partition function $\alpha' = 1$

fixation of the center-of-mass coordinate

$$Z = R \sum_{\{m_{ij}\}} \int_0^{2\pi R} \left[\frac{Dx_i}{2\pi}\right]' \left(\prod_{ij} \exp[-\frac{1}{4\pi}(x_i - x_j + 2\pi m_{ij} R)^2]\right) \times \prod_I \delta_{\sum_{i,j\in I} m_{ij},0}$$

$$= R \sum_{\{\ell_b\}} \int_{-\infty}^{\infty} \left[\frac{Dx_i}{2\pi}\right]' \left(\prod_{ij} \exp[-\frac{1}{4\pi}(x_i - x_j + 2\pi \ell_b \epsilon_{ij}^b R)^2]\right)$$

The Boltzmann factor can be re-expressed as

$$\exp[-\frac{1}{4\pi}(x_i - x_j + 2\pi \ell_b \epsilon_{ij}^b R)^2]$$
$$= \int_{-\infty}^{+\infty} \frac{dp_{ij}}{2\pi} \exp\left[-\frac{1}{4\pi} p_{ij}^2 + \frac{i}{2\pi} p_{ij}(x_i - x_j + 2\pi \ell_b \epsilon_{ij}^b R)\right]$$

Then perform the integration over the coordinates first by fixing the momenta (Fourier transform).

Apart from the overall factor R, the partition function becomes

$$\sum_{\{\ell_b\}} \int_{-\infty}^{\infty} \left[\frac{Dp_{IJ}}{2\pi} \right] \left(\prod_{\{ij\}} \exp\left[-\frac{1}{4\pi} p_{IJ}^2 + i p_{IJ} \ell_b \epsilon_{ij}^b R \right] \right) \prod_i{}' \, 2\pi\delta\Big(\sum_{IJ\in i} p_{IJ} = 0\Big)$$

$$p_{ij} \longleftrightarrow p_{IJ}$$

using the one-to-one correspondence between links and dual links

The delta-function constraint is solved as

$$p_{IJ} = p_I - p_J + P_B \epsilon_{IJ}^B, \quad \sum_I p_I = 0$$

Also the following relation is identically satisfied. $\displaystyle\sum_{ij} (p_I - p_J) \ell_b \epsilon_{ij}^b = 0$

The summation over ℓ_b can be re-expressed using the Poisson resummation formula.

$$\sum_{n=-\infty}^{\infty} e^{in\theta} = 2\pi \sum_{m=-\infty}^{\infty} \delta(\theta - 2\pi m)$$

$$\sum_{\ell_b=-\infty}^{\infty} \exp[i P_B \ell_b \epsilon_{IJ}^B \epsilon_{ij}^b R] = \sum_{\ell_b=-\infty}^{\infty} \exp[i P_B \ell_b R] = \frac{2\pi}{R} \sum_{\ell_B=-\infty}^{\infty} \delta(P^B - 2\pi \ell_B / R)$$

Thus we arrive at the following 'momentum' representation of the partition function.

$$Z = R \sum_{\{\ell_A\}} \int_{-\infty}^{+\infty} [Dp_I]'(2\pi)^{-P+V-1}(\frac{2\pi}{R})^{2h} \times \prod_{\{IJ\}} \exp\Big[-\frac{1}{4\pi}(p_I - p_J + 2\pi\ell_B\epsilon^B_{IJ}/R)^2\Big]$$

Euler theorem $2-2h = S-P+V \rightarrow (2\pi)^{-P+V-1+2h}R^{-2h} = (2\pi)^{-S+1}R^{-2h}$

V : # of vertices

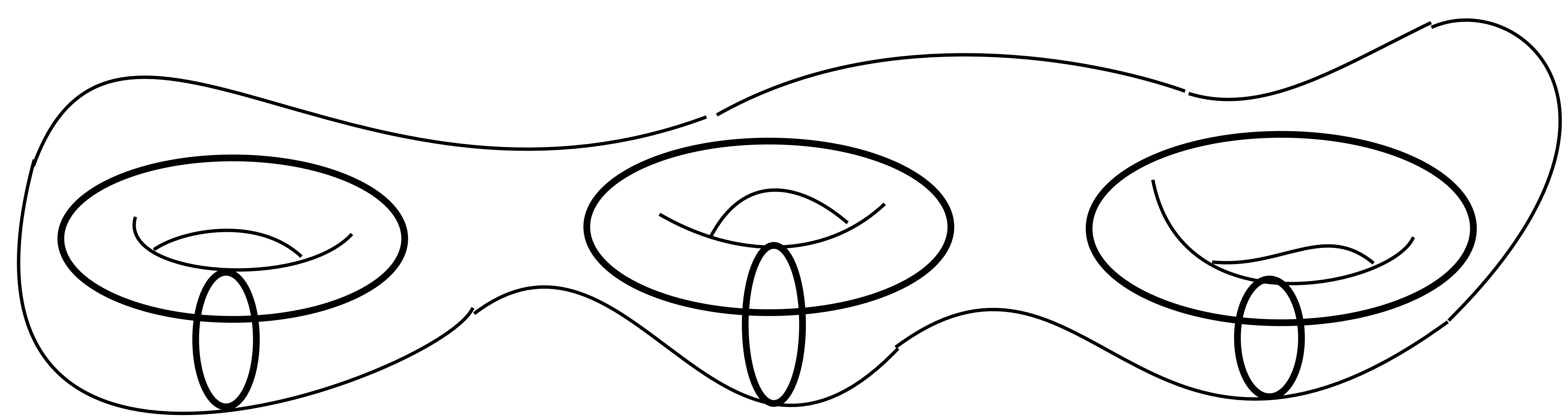
P : # of links

S : # of dual vertices = # of plaquettes

h : # of handles

$h = 3$

$\chi = -4$



We have proved the following identity.

$$
\begin{aligned}
Z_h(R) \equiv & R \sum_{\{m_{ij}\}} \int_{-\infty}^{+\infty} \left[\frac{dx_i}{2\pi}\right]' \prod_{\{ij\}} \exp[-\frac{1}{4\pi}(x_i - x_j + 2\pi\ell_b\epsilon_{ij}^b R)^2] \\
= & R^{2-2h} \frac{1}{R} \sum_{\{\ell_B\}} \int_{-\infty}^{+\infty} \left[\frac{dp_I}{2\pi}\right]' \prod_{\{IJ\}} \exp\left[-\frac{1}{4\pi}(p_I - p_J + 2\pi\ell_B\epsilon_{IJ}^B/R)^2\right] \\
= & R^{2-2h} Z_h(1/R)
\end{aligned}
$$

Or in the grand-canonical form, $Z(g_s, R) \equiv \sum_{h=0}^{\infty} g_s^{2h-2} Z_h(R)$

$$
Z(g_s, R) = Z(g_s/R, 1/R)
$$

T-duality symmetry :

$$
R \to 1/R \qquad\qquad g_s \to g_s/R
$$

The present model is essentially the 2D XY model, which is familiar in the statistical mechanics of spin systems. In that case, however, we do not impose the condition

$$\sum_{i,j\in I} m_{ij} = 0$$

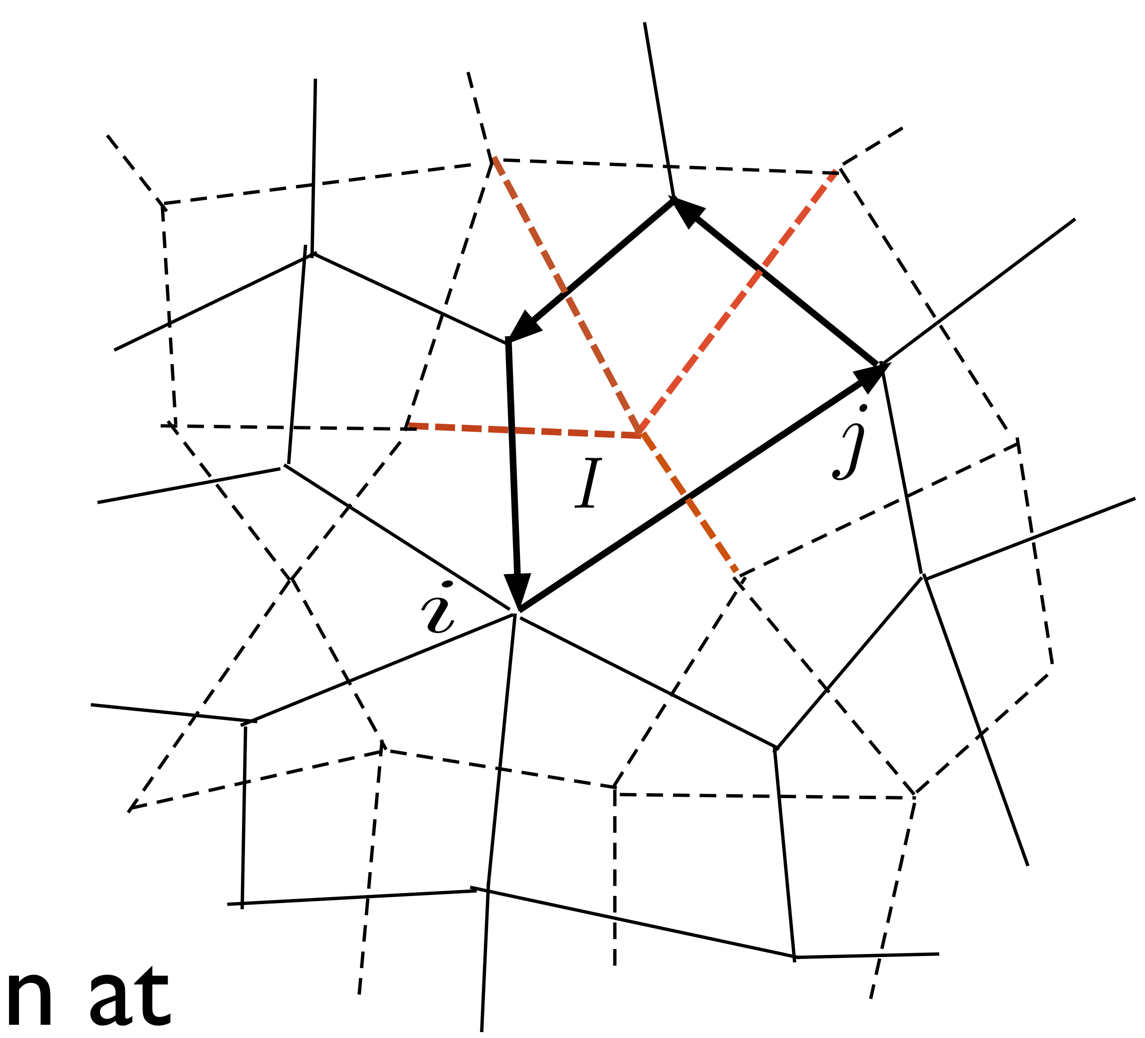
If we allow the excitations of local vortices, without imposing this condition, the dual transformed system becomes the system of Coulomb gas of local vortices,

$$\sum_{\{m_I\}} \int_{-\infty}^{\infty} [dp_I] \exp\left[-\frac{1}{4\pi} \sum_{\{I,J\}} (p_I - p_J)^2 + iR \sum_I m_I p_I \right]$$

where we assumed the base space to be of sphere topology

for simplicity, and $$m_I = \sum_{ij\in I} m_{ij}$$

measures the strength of local vorticity.



Thus self-duality is violated. The system undergoes a phase transition at some finite $R = R_c$ (Kosterlitz-Thouless transition).

$R > R_c$: massless Coulomb phase

$R < R_c$: plasma phase of local vortices

enhanced (**emergent**) symmetries at the self-dual point $R = \sqrt{\alpha'}$

Suppose $(n, m) = (\pm 1, \pm 1)$, then

$$\alpha' M^2 = (n\sqrt{\alpha'}/R)^2 + (mR/\sqrt{\alpha'})^2 + 2\sum(\boldsymbol{\alpha}_{-n}\boldsymbol{\alpha}_n + \tilde{\boldsymbol{\alpha}}_{-n}\tilde{\boldsymbol{\alpha}}_n) - 4$$
$$= 1 + 1 + 2\sum(\boldsymbol{\alpha}_{-n}\boldsymbol{\alpha}_n + \tilde{\boldsymbol{\alpha}}_{-n}\tilde{\boldsymbol{\alpha}}_n) - 4$$

and the level-matching condition becomes

$$\left(1 + \sum(\boldsymbol{\alpha}_{-n}\boldsymbol{\alpha}_n - \tilde{\boldsymbol{\alpha}}_{-n}\tilde{\boldsymbol{\alpha}}_n)\right)|0_{\pm\pm}\rangle = 0$$
$$\left(-1 + \sum(\boldsymbol{\alpha}_{-n}\boldsymbol{\alpha}_n - \tilde{\boldsymbol{\alpha}}_{-n}\tilde{\boldsymbol{\alpha}}_n)\right)|0_{\pm\mp}\rangle = 0$$

⟶ The massless states of the following types are also possible,

$$\tilde{\boldsymbol{\alpha}}_{-1}|0_{\pm\pm}\rangle, \quad \boldsymbol{\alpha}_{-1}|0_{\pm\mp}\rangle$$

in addition to the Kaluza-Klein "vector" gravitons

$$\alpha_{-1}\tilde{\boldsymbol{\alpha}}_{-1}|0\rangle, \quad \boldsymbol{\alpha}_{-1}\tilde{\alpha}_{-1}|0\rangle$$

compactified direction →

← uncompactified directions

These excitations can constitute $\mathrm{SU}(2)$ triplet(s) of massless vector states.

$$(\tilde{\boldsymbol{\alpha}}_{-1}|0_{\pm\pm}\rangle, \alpha_{-1}\tilde{\boldsymbol{\alpha}}_{-1}|0\rangle) \text{ and/or } (\boldsymbol{\alpha}_{-1}|0_{\pm\mp}\rangle, \boldsymbol{\alpha}_{-1}\tilde{\alpha}_{-1}|0\rangle)$$

The corresponding conserved (world-sheet) charges are

$$T^R_{\pm} = \int_0^{2\pi} d\sigma : \exp i(\pm x(\tau-\sigma)/\sqrt{\alpha'}): \qquad T^R_3 = \int_0^{2\pi} d\sigma p(\tau-\sigma)$$

$$T^L_{\pm} = \int_0^{2\pi} d\sigma : \exp i(\pm x(\tau+\sigma)/\sqrt{\alpha'}): \qquad T^L_3 = \int_0^{2\pi} d\sigma p(\tau+\sigma)$$

generating momentum and winding number of unit strength

measuring momentum and winding number

$$L_3 = \frac{\sqrt{\alpha'}}{4\pi} T^R_3 \qquad L_{\pm} = \frac{1}{2\pi} T^R_{\pm}$$

and satisfying the SU(2) algebra

$$[L_3, L_{\pm}] = \pm L_{\pm}, \ [L_+, L_-] = 2L_3$$

- There also appear scalar massless states, which should be interpreted as Nambu-Goldstone modes ('moduli fields') associated with the infinitesimal deformation of the geometry of compactified space.

$$\alpha_{-1}\tilde{\alpha}_{-1}|0\rangle, \quad \tilde{\alpha}_{-1}|0_{\pm\pm}\rangle, \quad \alpha_{-1}|0_{\pm\mp}\rangle$$

Recall old Kaluza-Klein theory

Interpret U(1) gauge field as the 'fifth'-component of the metric tensor of 5 dimensional space-time.

$$g_{\mu 4}(x) = RA_\mu(x),\ g_{44} = 1$$

$$ds^2 = ds_4^2 + \left(dx^4 + RA_\mu(x)dx^\mu\right)^2 \qquad x^4 \sim x^4 + 2\pi R$$

Then the coordinate shift of $x^4 \to x^4 + R\theta(x)$ induces the gauge transformation.

$$A_\mu(x) \to A_\mu(x) + \partial_\mu\theta(x)$$

The corresponding conserved charge is nothing but the momentum operator of the compactified direction. This corresponds to the world-sheet charge

$$T_3^R + T_3^L = \int_0^{2\pi} d\sigma p(\tau-\sigma) + \int_0^{2\pi} d\sigma p(\tau+\sigma)$$

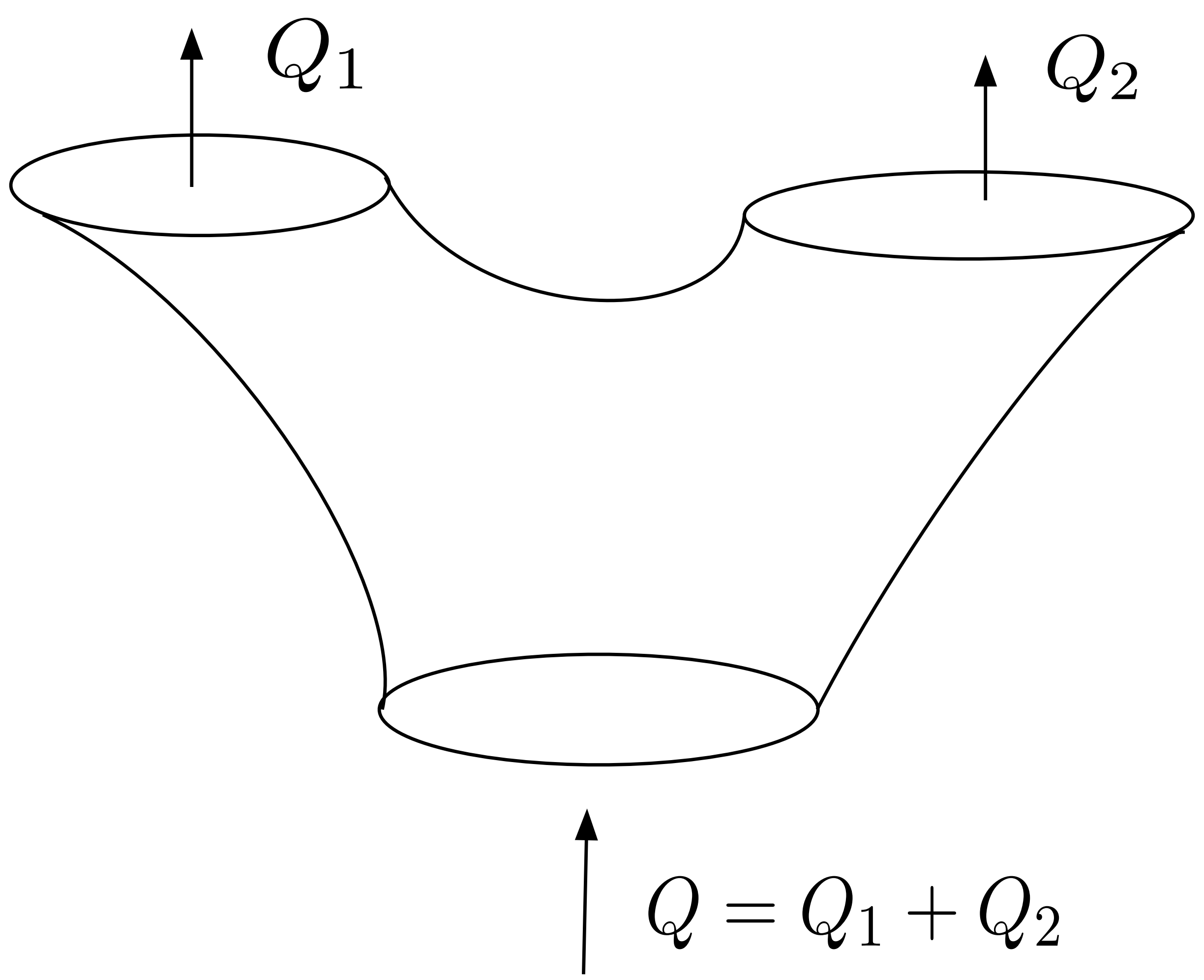
The massive KK states are higher (charged) modes in the Fourier expansion.

$$\phi(x^\mu, x^4) = \sum_{n=-\infty}^{\infty} \phi_n(x^\mu) e^{inx^4/R}$$

$$\phi_1(x^\mu)e^{ix^4/R} \to \phi_1(x^\mu)' e^{ix^4/R}, \quad \phi_1(x^\mu)' = e^{i\theta(x^\mu)}\phi_1(x^\mu)$$

Remark: world-sheet v.s. target space symmetries

- A conservation law (other than the constraints) on the world-sheet leads to the corresponding conservation law in the target space-time, provided that there does not appear any anomaly in the transformation law of current operators under conformal transformation.



$$\partial_\tau j_\tau + \partial_\sigma j_\sigma = 0$$

$$Q = \oint d\sigma\, j_\tau(\tau, \sigma) \qquad \frac{d}{d\tau} Q = 0$$

$$Q_{\rm target} \sim \sum_i Q_i$$

Then, the world-sheet currents can be reinterpreted as target space currents as

$$J^\mu(x) \sim \sum_i \int d\tau \oint d\sigma\, j^a_{(i)}(\tau, \sigma) \partial_a x^\mu_{(i)}(\tau, \sigma) \delta^D\left(x - x_{(i)}(\tau, \sigma)\right)$$

at least in the classical approximation

In the case of open strings, conservation law can be broken at the boundary depending on the boundary condition. In target space-time, this often corresponds to non-linear realization associated with spontaneous symmetry breaking.

Even if not conserved, world-sheet currents may also correspond to target-space symmetries,
provided that the current divergence is proportional to vertex operators of massless physical states, and hence the non-conservation is **compensated by** the shift of background fields.

For instance, general coordinate transformation and the gauge symmetry of the B field are of this type. In the former case, anomalies of conformal transformation law leads to non-linear form in higher orders of string coupling.

$$T_a = v_\mu(x)\partial_a x^\mu$$

$$\partial_a T^a = \frac{1}{2}(\partial_\nu v_\mu + \partial_\mu v_\nu)\partial^a x^\nu \partial_a x^\mu$$

$$\updownarrow$$

$$\delta h_{\mu\nu} = \partial_\nu v_\mu + \partial_\mu v_\nu$$

$$B^a = \epsilon^{ab} u_\mu(x)\partial_b x^\mu$$

$$\partial_a B^a = \frac{1}{2}(\partial_\mu u_\nu - \partial_\nu u_\mu)\epsilon^{ab}\partial_a x^\mu \partial_b x^\nu$$

$$\updownarrow$$

$$\delta B_{\mu\nu} = \partial_\mu u_\nu - \partial_\nu u_\mu$$

2.4 UV finiteness: vacuum amplitudes of closed strings

Consider the generalization of vacuum amplitude of bosonic field theory $A = e^{VTL}$

$$L = \frac{1}{2}\int \frac{d^D p}{(2\pi)^D}\int_0^\infty \frac{d\tau}{\tau} e^{-\tau(p^2+m^2)}$$

$$\longrightarrow \qquad L'_{\rm string} \equiv \frac{1}{2}\int \frac{d^D p}{(2\pi)^D}\int_0^\infty \frac{d\tau}{\tau}\mathrm{Tr}_{\rm phys}\left(e^{-\tau(p^2+M^2)}\right)$$

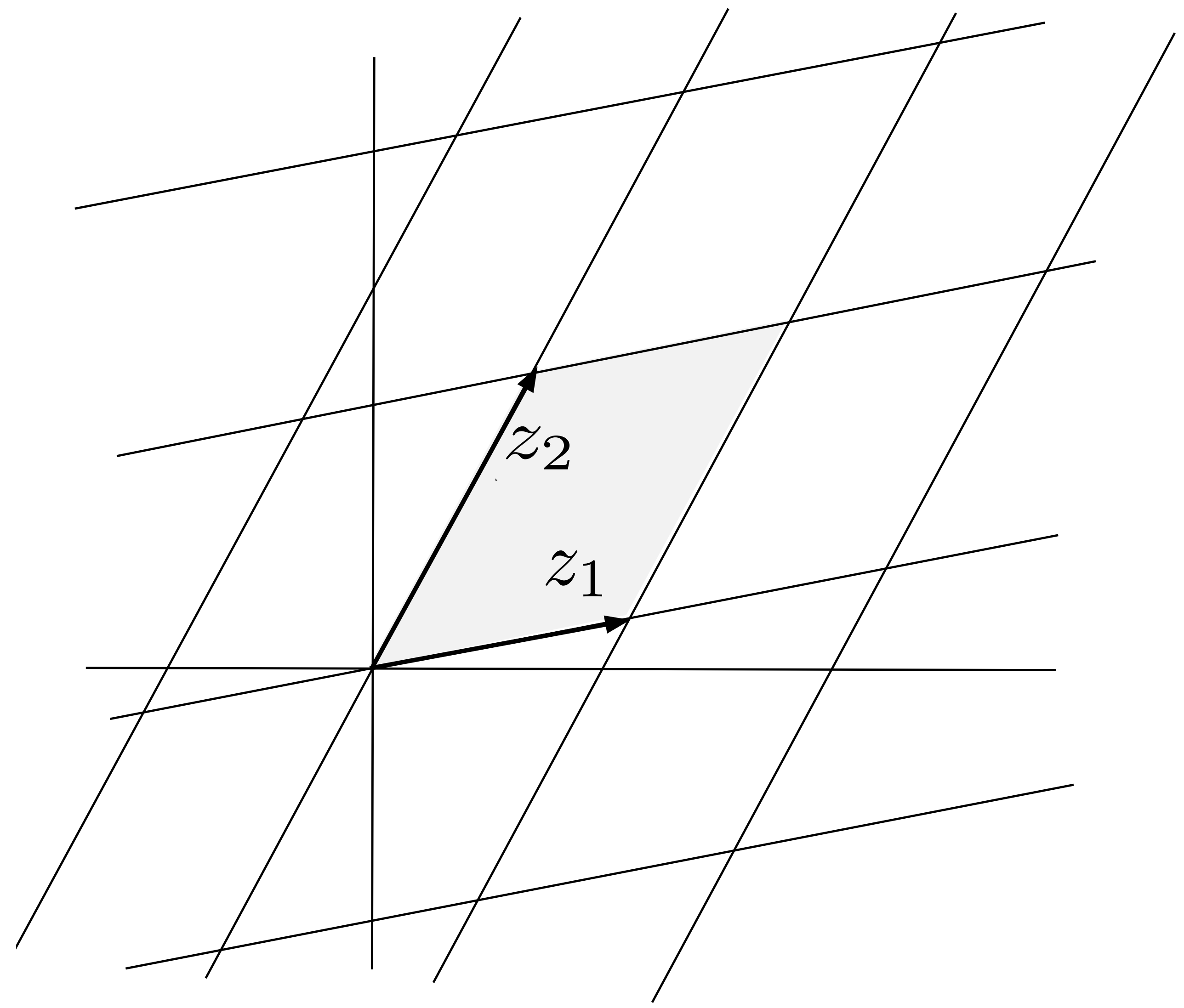
$$\mathrm{Tr}_{\rm phys}\left(e^{-\tau(p^2+M^2)}\right) = e^{-\tau p^2}\int_0^{2\pi}\frac{d\theta}{2\pi}\mathrm{Tr}\left(e^{i\theta\sum_{n=1}^\infty(\alpha_{-n}\alpha_n-\tilde{\alpha}_{-n}\tilde{\alpha}_n)}e^{-\tau M^2}\right)$$

$$M^2 = \frac{2}{\alpha'}\sum_{n=1}^{\infty}(\alpha_{-n}\alpha_n+\tilde{\alpha}_{-n}\tilde{\alpha}_n) - \frac{4}{\alpha'}$$

We can take into account the theta integration by complexifing the proper time variable.

$$\frac{2\tau}{2\pi\alpha'} \to \mathrm{Im}\,\tau, \qquad \frac{\theta}{2\pi} \to \mathrm{Re}\,\tau$$

Moduli parameter of torus



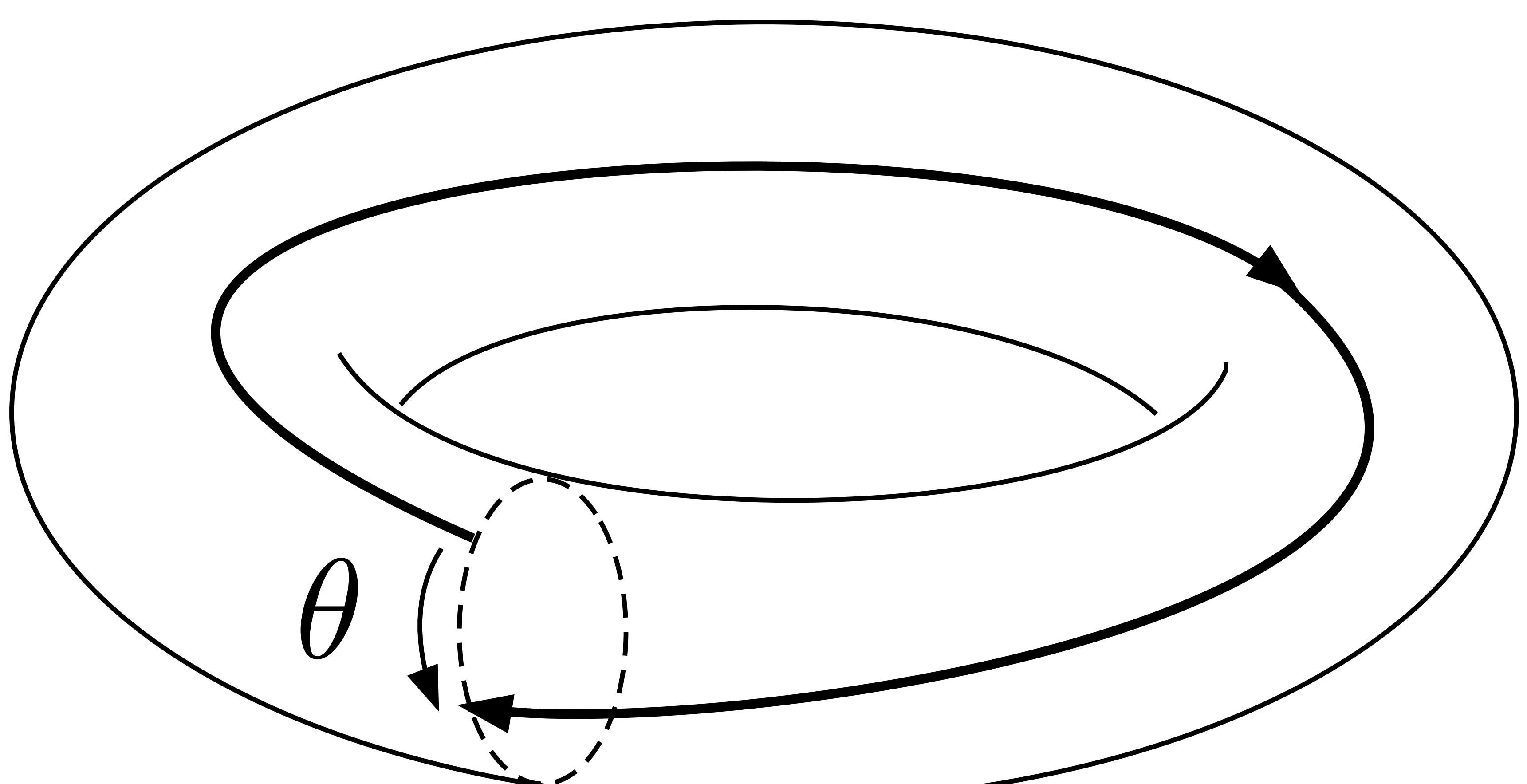
change of basis

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \to \begin{pmatrix} z_1' \\ z_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

$$ad - bc = 1 \quad (a, b, c, d) \in \mathbb{Z}$$

$$\tau = \frac{z_2}{z_1} \qquad \tau \to \tau' = \frac{c + d\tau}{a + b\tau}$$

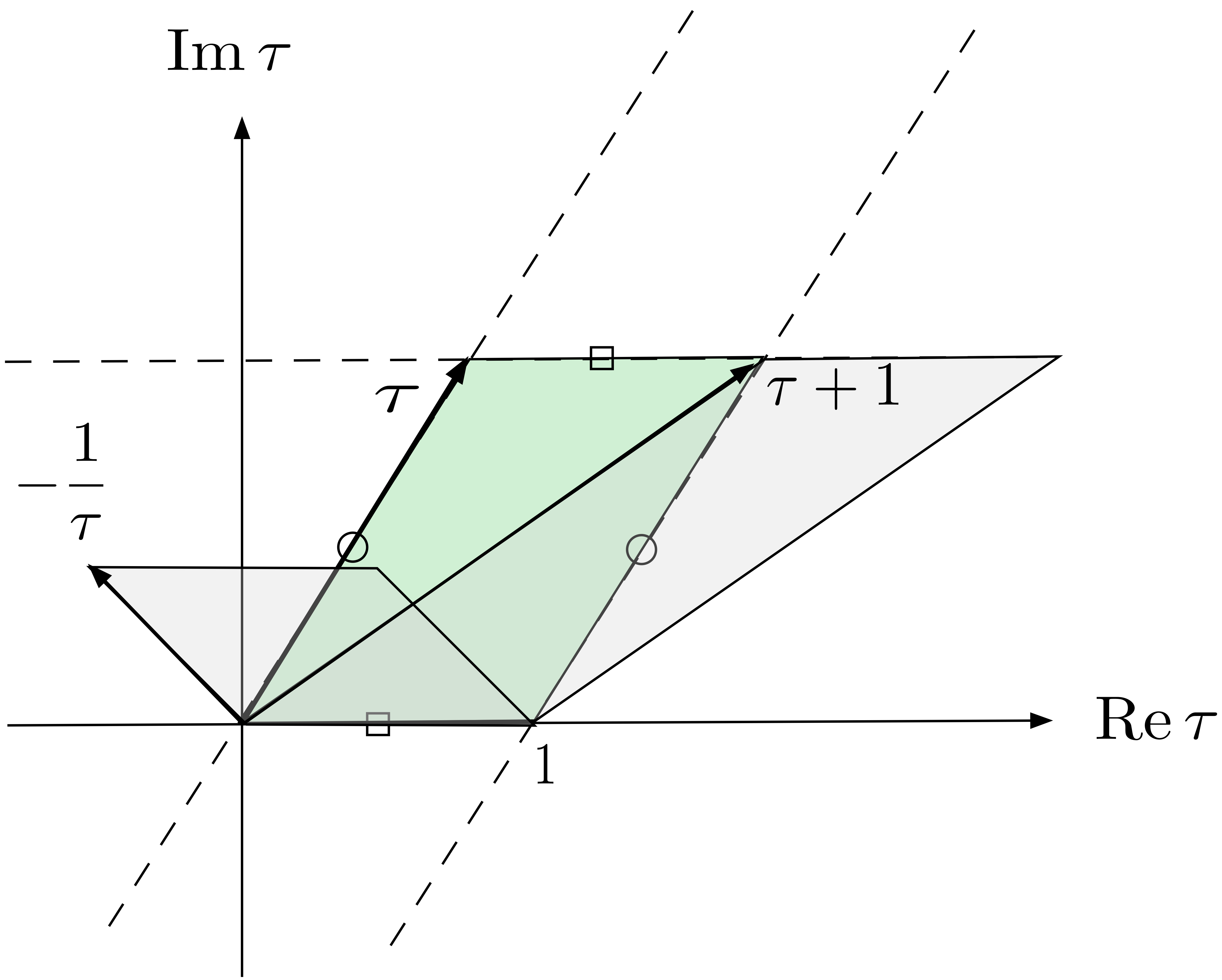
modular transformation $\mathrm{SL}(2, \mathbb{Z})$



The moduli, in general, consist of proper length and twisting angles.

$\mathrm{Im}\,\tau$ $\mathrm{Re}\,\tau$

All modular transformations can be generated by the following two operations.



$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \qquad \tau' = \tau + 1$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \tau' = -\frac{1}{\tau}$$

In terms of the moduli parameter, the vacuum amplitude is

$$
\begin{aligned}
L'_{\text{string}} =& \frac{1}{2} \int \frac{d^2\tau}{\operatorname{Im}\tau} \int \frac{d^D p}{(2\pi)^D} e^{-p^2 \pi \alpha' \operatorname{Im}\tau} \\
& \times \operatorname{Tr}\left(e^{2\pi i \tau (\sum_{n=1} \alpha_{-n}\alpha_n - 1)} e^{-2\pi i \bar{\tau} (\sum_{n=1} \tilde{\alpha}_{-n}\tilde{\alpha}_n - 1)} \right) \\
=& \frac{1}{2} \int \frac{d^2\tau}{\operatorname{Im}\tau} \left(\frac{1}{\alpha' \operatorname{Im}\tau} \right)^{D/2} (q\bar{q})^{-2} \left[\prod_{n=1}^{\infty} (1 - q^{2n})(1 - \bar{q}^{2n}) \right]^{-(D-2)}
\end{aligned}
$$

$$
q \equiv e^{i\pi\tau}
$$

When $D = 26$, we have to redefine the amplitudes in order to circumvent an infinite multiple counting.

$$
L_{\text{string}} = \frac{1}{2(\alpha')^{13}} \int_{\text{F}} \frac{d^2\tau}{(\operatorname{Im}\tau)^2} \left[(\operatorname{Im}\tau)^{1/2} \eta(\tau) \eta(\bar{\tau}) \right]^{-24}
$$

$$\eta(\tau) \equiv q^{1/12} \prod_{n=1}^{\infty} (1 - q^{2n}) \quad \text{satisfies} \quad \begin{aligned} \eta(-\frac{1}{\tau}) &= (-i\tau)^{1/2}\eta(\tau) \\ \eta(\tau+1) &= e^{\pi i/12}\eta(\tau) \end{aligned}$$

Thus $(\mathrm{Im}\,\tau')^{1/2}\eta(\tau')\eta(\bar{\tau}') = (\mathrm{Im}\,\tau)^{1/2}\eta(\tau)\eta(\bar{\tau})$

and $$\frac{d^2\tau}{(\mathrm{Im}\,\tau)^2} = \frac{d^2\tau'}{(\mathrm{Im}\,\tau')^2}$$

The vacuum amplitude is invariant under modulartransformation.
But this implies that the upper half complex plane of tau
covers regions with the one and the same contributions *infinitely many times*.

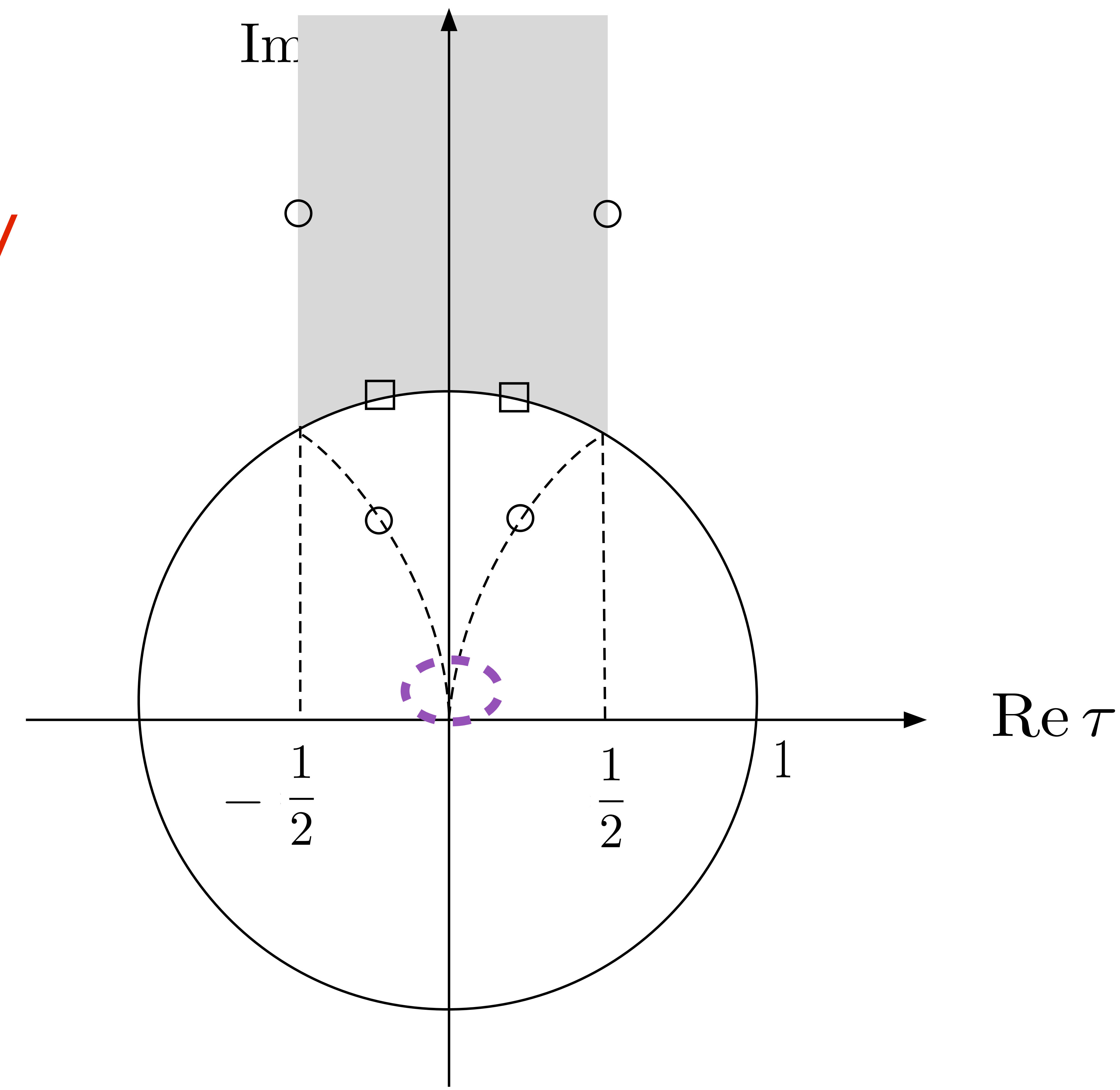
We should restrict the integration region to the **fundamental region F**,
covering only once the region such that it does not have any overlap
with the other regions obtained by modular transformations.

$$L_{\text{string}} = \frac{1}{2(\alpha')^{13}} \int_{\mathrm{F}} \frac{d^2\tau}{(\mathrm{Im}\,\tau)^2} \left[(\mathrm{Im}\,\tau)^{1/2}\eta(\tau)\eta(\bar\tau)\right]^{-24}$$

The fundamental region F does not contain **UV region**, and hence there is no UV divergence!

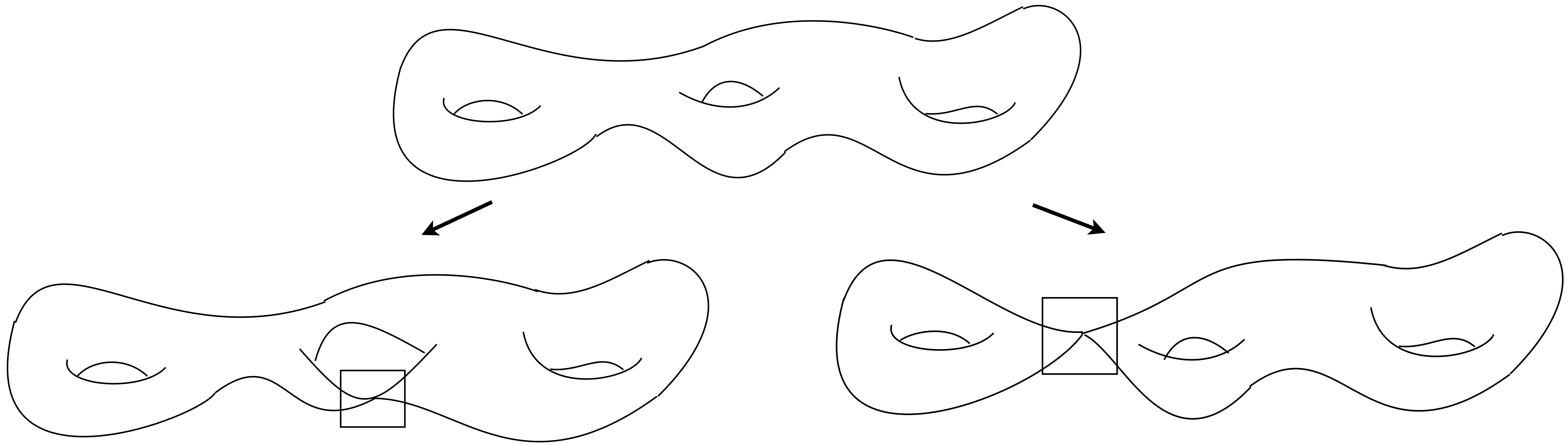
This also suggests that the notion of quantized string fields for closed strings is unnatural for describing full-fledged quantum string theory.

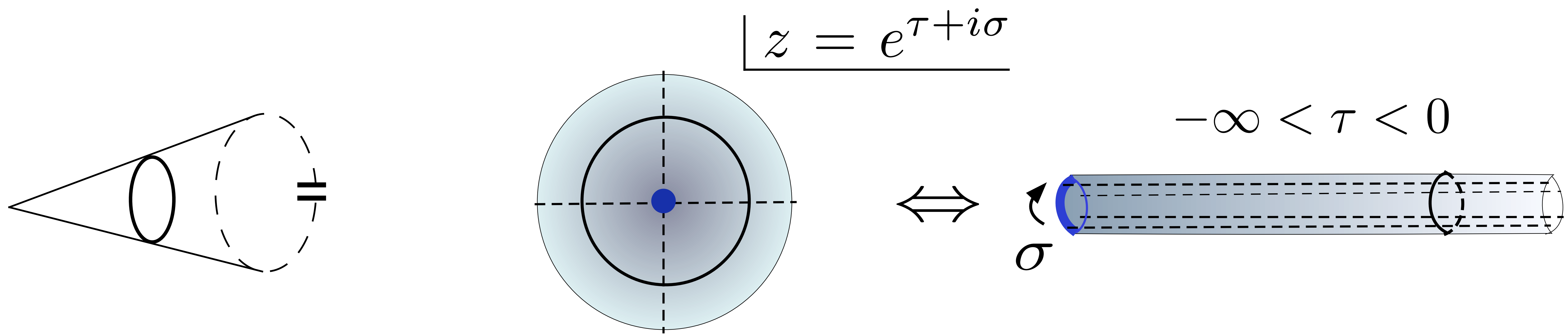
However, the situation is quite different in the case of open strings, as we shall see later.



F $\quad |\tau| \geq 1,\ -1/2 < \mathrm{Re}\,\tau \leq 1/2$

In general, the boundaryies of moduli space of Riemann surfaces consist essentially of only two elements.





Possible singularities correspond only to infinitely long propagations of strings, namely to IR behaviors. All these correspond to unitarity singularities.

dimension of the moduli space (case of closed strings)

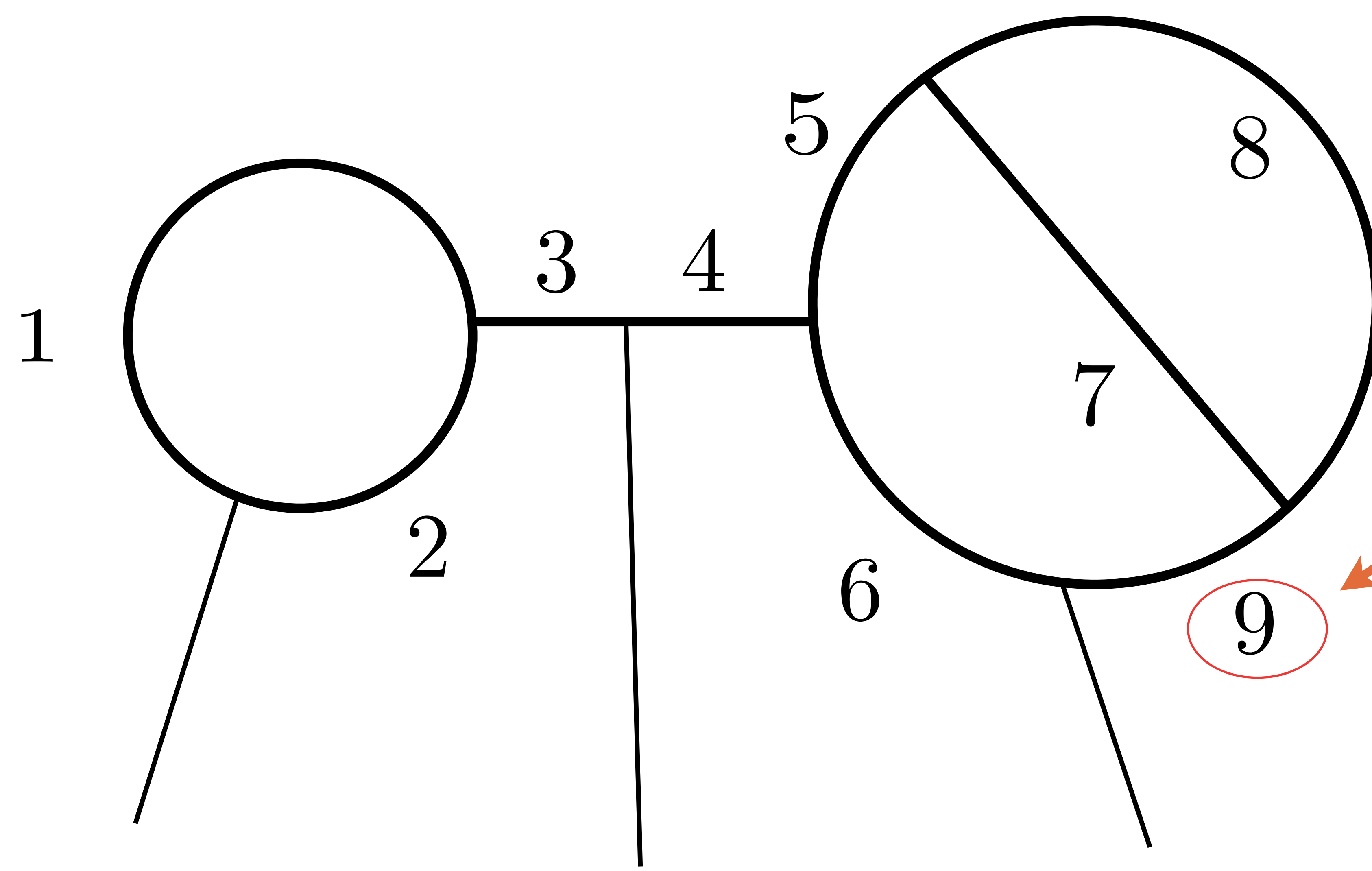
$$d_{\text{moduli}} = 6h - 6 + 2p_c \quad (p_c \geq 3)$$

$h = 3 \quad p_c = 3$

due to twisting angles

$$d_{\text{moduli}} = 18 = 2 \times 9$$

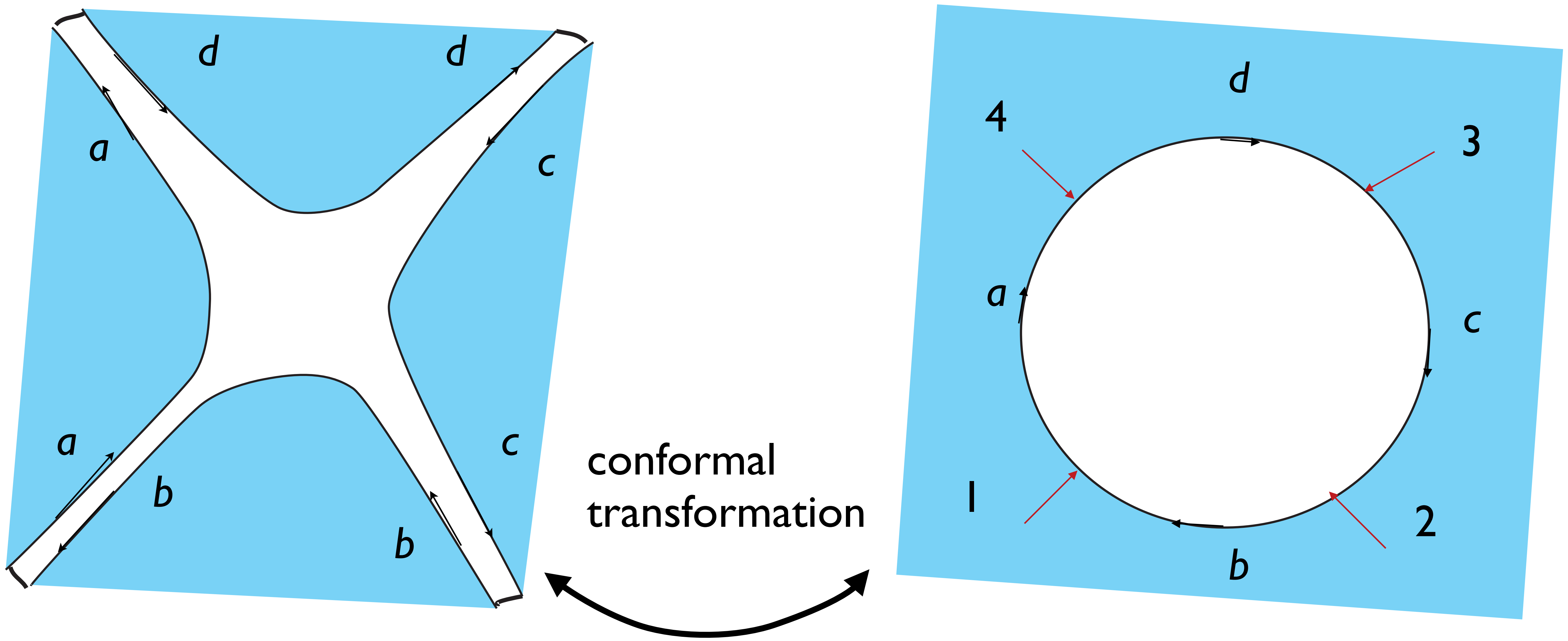
dimension of Schwinger-Feynman parameters



an example of 3-loop 3-point (amputated) Feynman diagrams

2.5 D-branes: Physical interpretation of open strings

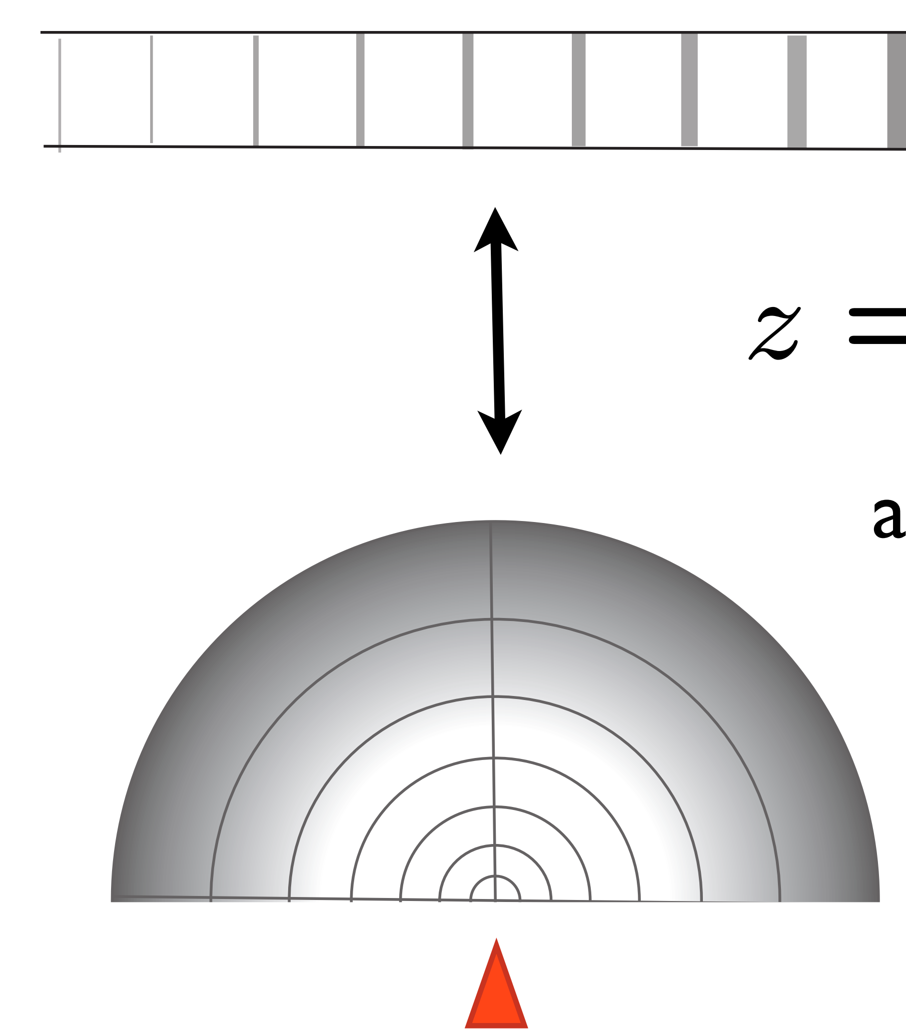
We can introduce internal degrees of freedom associated with the end points of open strings.



Amplitudes are multiplied by the `Chan-Paton factor'

$$\mathrm{Tr}\left(\lambda^{A_1}\lambda^{A_2}\lambda^{A_3}\lambda^{A_4}\right)$$

The external lines of on-shell asymptotic states are represented by the insertion of (**local**) vertex operators on the boundary of world-sheets.



$$z = e^{-i(\sigma + i\tau)}$$

after Wick rotation

$$\tau \to -i\tau$$

The vertex operators can be matrices with respect to the internal indices *a, b, c, d*, ... For orientable strings, only possible internal symmetry which is consistent with the channel duality and factorization of scattering amplitudes is known to be U(N) (or SU(N)).

massless vector modes and their vertex operators with internal symmetry

$$\alpha^i_{-1}|0\rangle \lambda^A_{ab} \quad \xleftarrow[\;e^{-i\tau}\times\;]{\tau\to-\infty} \quad \begin{cases} \lambda^A_{ab}\,\epsilon_i \dot{x}^i(\tau,\sigma=0) & \text{N} \\ \lambda^A_{ab}\,\epsilon_i x'^i(\tau,\sigma=0) & \text{D} \end{cases}$$

T duality (N ↔ D)

apart from the zero-mode momentum part along the Neumann directions

$$e^{ikx} \sim e^{ikx(\tau,\sigma=0)}$$

Physical interpretation of vertex operators (to the first order in the coupling)

N: minimal substitution

ϵ^i : polarization vector

$$p^i(\tau,\sigma)\delta_{ab} \to p^i(\tau,\sigma)\delta_{ab} - \lambda^A_{ab}\epsilon^i e^{ikx(\tau,\sigma)}\delta(\sigma)$$

⇓

charge residing at string endpoints

U(N) gauge fields propagating along the N directions

D: deformation of the Dirichlet boundary condition

$$\frac{1}{4\pi\alpha'}\int d\tau d\sigma\left[(\partial_\tau x)^2 - (\partial_\sigma x)^2\right] + \frac{1}{2\pi\alpha'}\int d\tau \epsilon_i \partial_\sigma x^i$$

Using e.o.m., the last term is equal to the first variation of

$$\frac{1}{4\pi\alpha'}\int d\tau d\sigma\Big[\left(\partial_\tau(x+\delta x)\right)^2 - \left(\partial_\sigma(x+\delta x)\right)^2\Big]$$

$$\delta x^i(\tau,\sigma)|_{\sigma=0} = -\epsilon^i$$ (diagonal a=b)

with a simultaneous insertion of a matrix λ^A_{ab}

$$\Downarrow$$

Massless modes along the Dirichlet directions should be interpreted as collective coordinates corresponding to the change of positions of open string endpoints.

The mass associated with these collective modes are determined by studying the effective low-energy actions.

Remarks on world-sheet currents associated with the massless states of open strings

- gauge symmetry

$$j^a = \epsilon^{ab}\partial_b \lambda \qquad Q = \lambda(x(\sigma))\Big|_0^\pi$$

$$\partial_a j^a = 0$$

$$\frac{dQ}{d\tau} = \frac{d}{d\tau}\int_0^\pi d\sigma j^0 = -j^1\Big|_{\sigma=0}^{\sigma=\pi} \qquad j^1 = -\partial_\mu \lambda \frac{\partial x^\mu}{\partial \tau}$$

$$\delta A_\mu(x) = \partial_\mu \lambda \qquad |\Psi\rangle \to e^{\pm i\lambda(x)}|\Psi\rangle$$

← end points

The massless gauge boson can be interpreted, at least *classically*, as the Goldstone boson associated with a large gauge transformation.

$$\lambda = x^\mu a_\mu \quad \Rightarrow \delta A_\mu = a_\mu$$

translation symmetry

$$k_a^i = \partial_a x^i \qquad K^i = \int_0^\pi d\sigma\, k_0^i = P^i$$

$$\partial^a k_a^i = 0$$

$$\frac{dK^i}{d\tau} = -k_1^\mu \Big|_{\sigma=0}^{\sigma=\pi} = -\partial_1 x^\mu \Big|_{\sigma=0}^{\sigma=\pi}$$

$$\delta X^i = \delta x^i \qquad |\Psi\rangle \to e^{\pm i\delta x_i P^i}|\Psi\rangle$$

This corresponds to a shift of D boundary condition at each end points, and the scalar fields along the D-directions are nothing but the Goldstone bosons associated with spontaneous symmetry breaking of translation symmetry.

The situation is quite analogous to the collective modes associated with translation zero modes of classical soliton-type solutions in ordinary field theories.

The effective actions always contain the kinetic terms of the standard form for the gauge fields and endpoint collective fields (~Higgs fields), in dimensionless unit, as

$$-\frac{1}{g_o^2}\int d^{p+1}x\,\mathrm{Tr}\left(\frac{1}{4}(F_{\mu\nu})^2+\frac{1}{2}(D_\mu X^i)^2+\cdots\right)$$

$$\mu,\nu,\ldots\in(0,1,2,\ldots,p)$$

$$i,j,\ldots,\in(p+1,p+2,\ldots,d-1)$$

where $p+1$ is the dimension (including time direction) of the N directions. The Yang-Mills coupling constant is related to the string coupling by

$$\Downarrow \quad g_o^2\sim g_s$$

The open-string endpoints must be interpreted as being attached to a new physical object whose spatial dimension is equal to p and mass density is proportional to $\frac{1}{g_s}$

These objects are called D-branes.

The internal indices *a, b, c, ...* label different D-branes.

- Each T-duality transformation changes the dimension of D-brane by one.
- The size of the gauge group N corresponds to the number of D-branes.
- Open string theories = "collective" description of D-branes :

diagonal parts of matrix coordinates
~ coordinates of D-branes

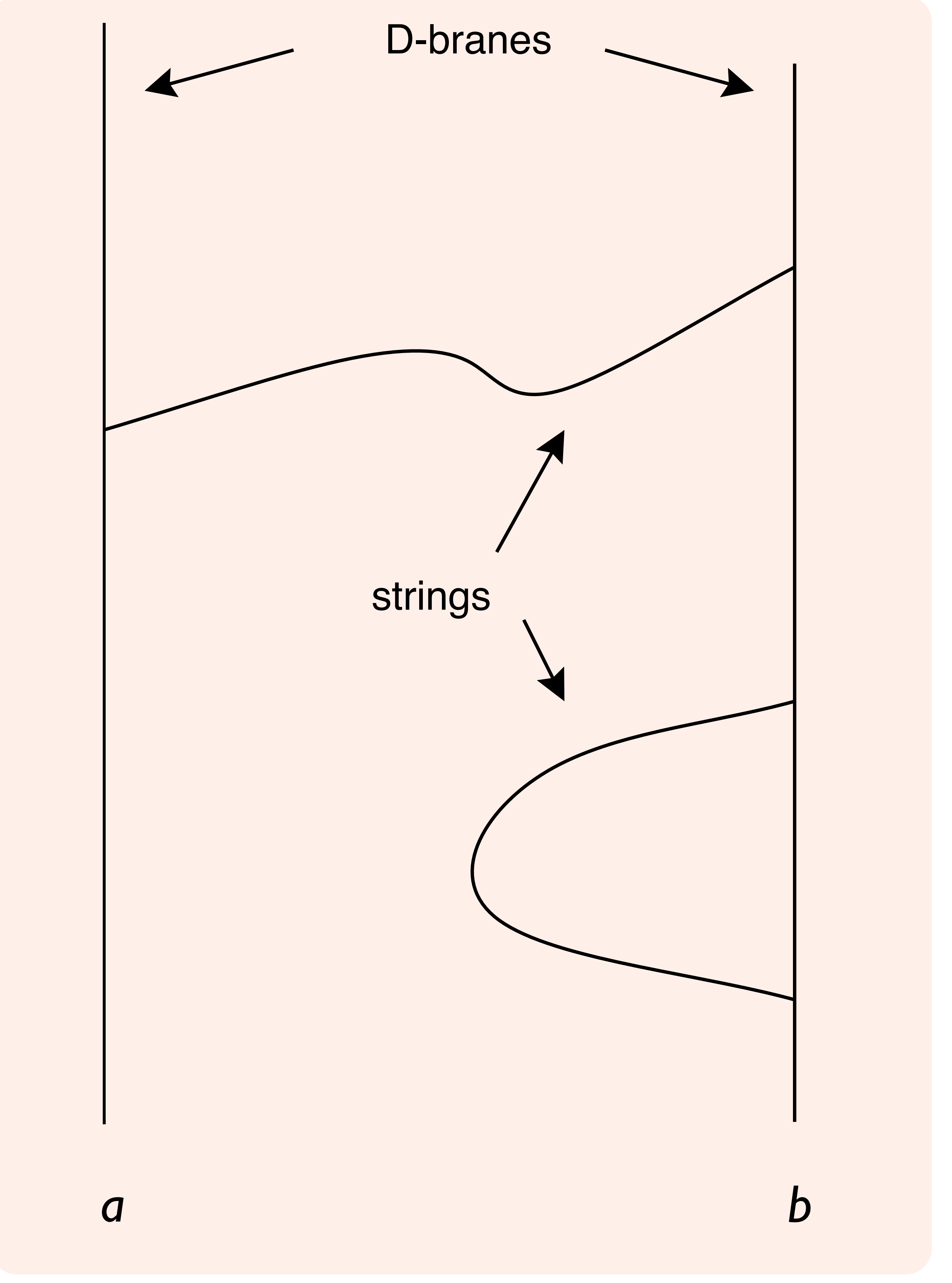
$$X^i_{aa}(x^\mu)$$

off-diagonal parts of matrix coordinates
~ mutual interactions among different D-branes

$$X^i_{ab}(x^\mu)$$

U(N) gauge symmetry
~ extending the role of permutation symmetry for ordinary identical particles

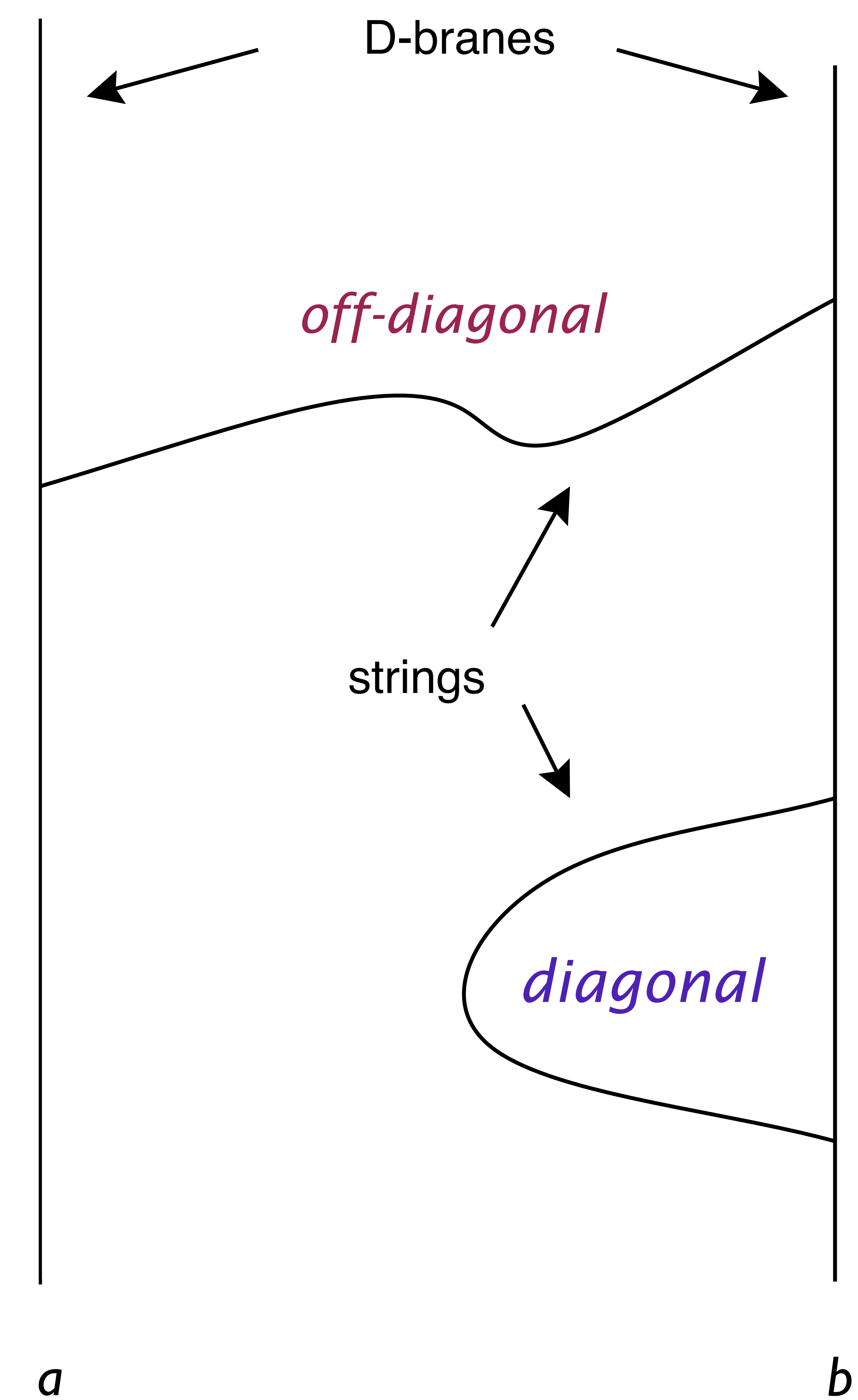
$$X^i(x^\mu) \to U(x)X^i(x^\mu)U(x)^\dagger$$



Dブレーン力学が示唆する統一と新たな量子的構造

Dブレーンとは何か

- open string の端点の自由度
- 単なる境界条件ではなく力学的自由度
- bulkゲージ場（RR 場）の源で安定　（ゲージ電磁荷を持たない不安定ブレーンも考えることができる)
- 一般に広がりを持つ：空間的広がりの次元が p のとき　Dp ブレーンと呼ぶ.　安定ブレーンの可能な次元は摂動的真空による.
- 低速度(非相対論)・低励起エネルギー近似では、supersymmetric Yang-Mills theory により記述できる
- bulk supergravity 近似 (low-energy effective theory) では、古典解（ブラックホール解）として記述できる
 Bekenstein-Hawking エントロピー公式の統計的解釈が可能
(extremal and near extremal blackholes)

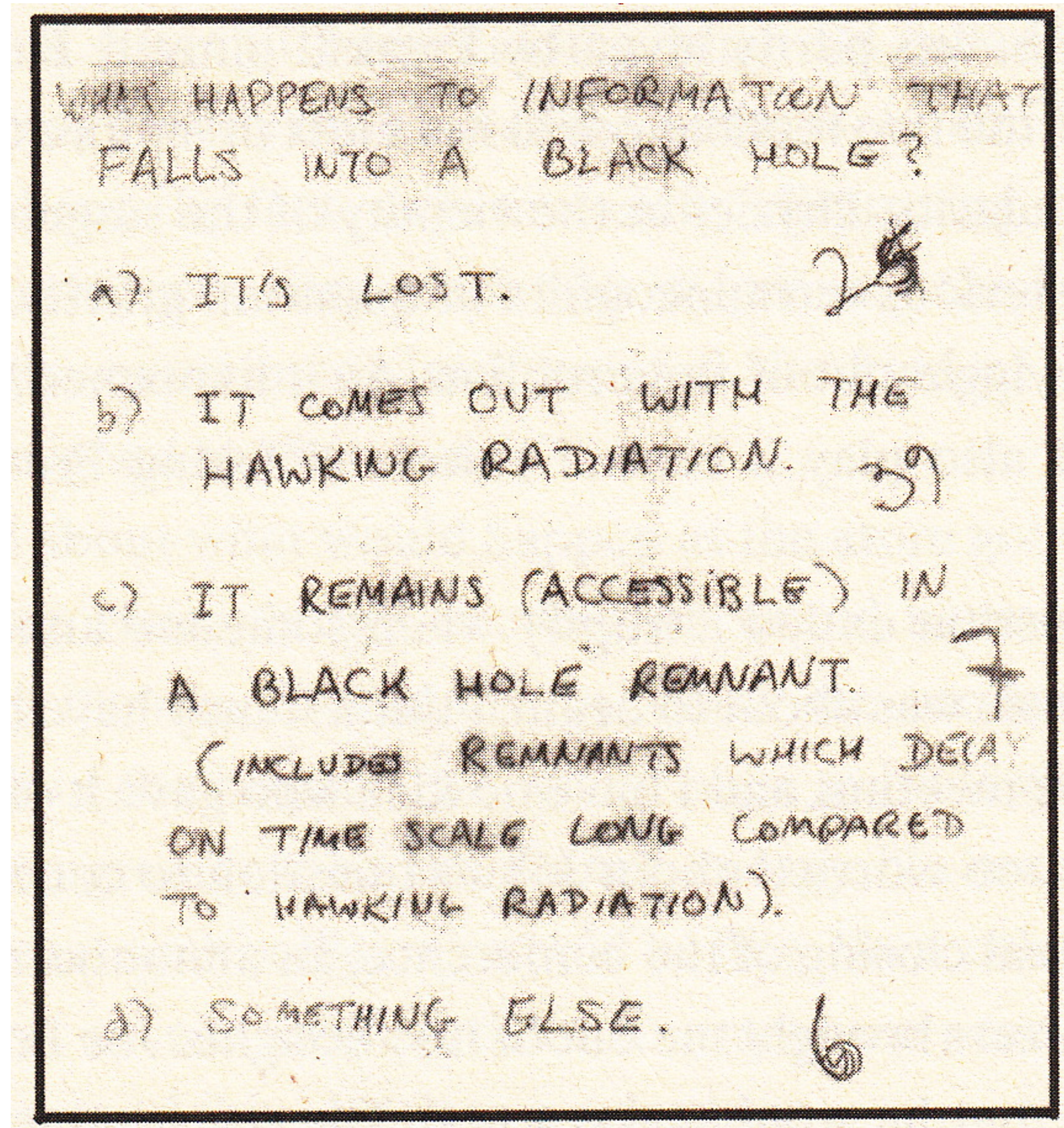


1993年　Santa Babara (K)ITPで開催された重力と弦理論についてのworkshopの最後のconference におけるinformation paradoxに関する参加者の投票 (J. Polchinskiの提案)

THE BLACK HOLE WAR

1. Hawking's option: information that falls into a black hole is irretrievably lost.
2. 'T Hooft and Susskind's option: information dribbles back out among the photons and other particles in the Hawking radiation.
3. Information becomes trapped in tiny Planck-sized remnants.
4. Something else.

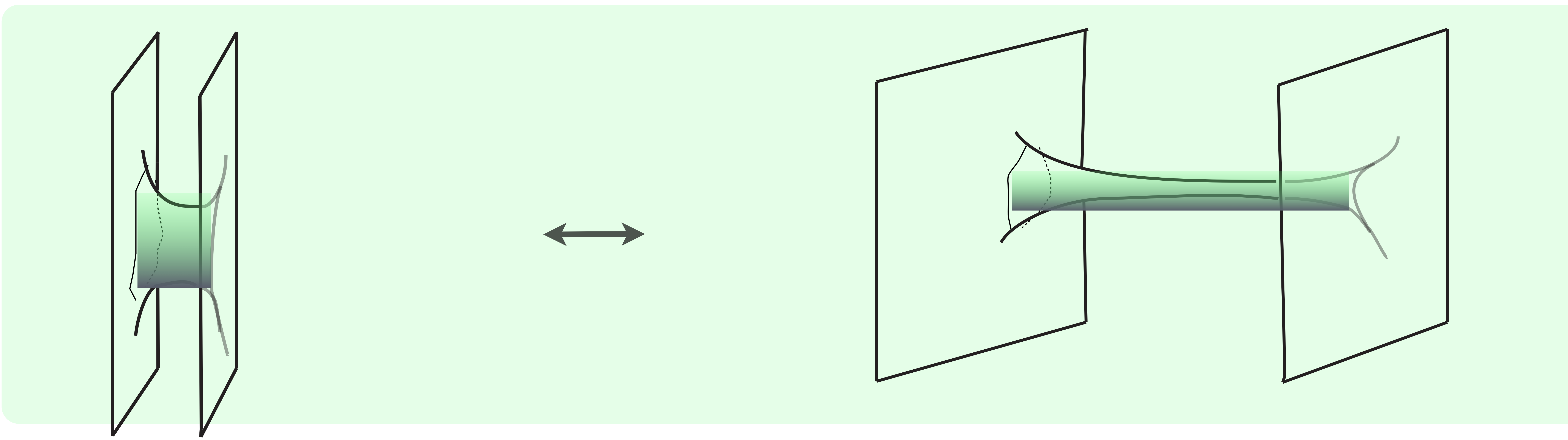
With each show of hands, Joe recorded the result on the whiteboard at the front of the lecture hall. Someone photographed the board for posterity. Here it is, courtesy of Joe.



Dブレーンを通し、**Yang-Mills ゲージ理論と一般相対性理論(重力理論)が互いに「双対的」な関係にある**

最近１２−１３年ほどの弦理論の最大の成果：弦理論を、ミクロな力学の枠組みの「統一」と看做せる

open-closed string duality in string perturbation theory (simplest one-loop case)



creation and annihilation of open strings

exchange of closed strings (includes graviton exchange)

スムーズにつながる

effective theory =gauge theory

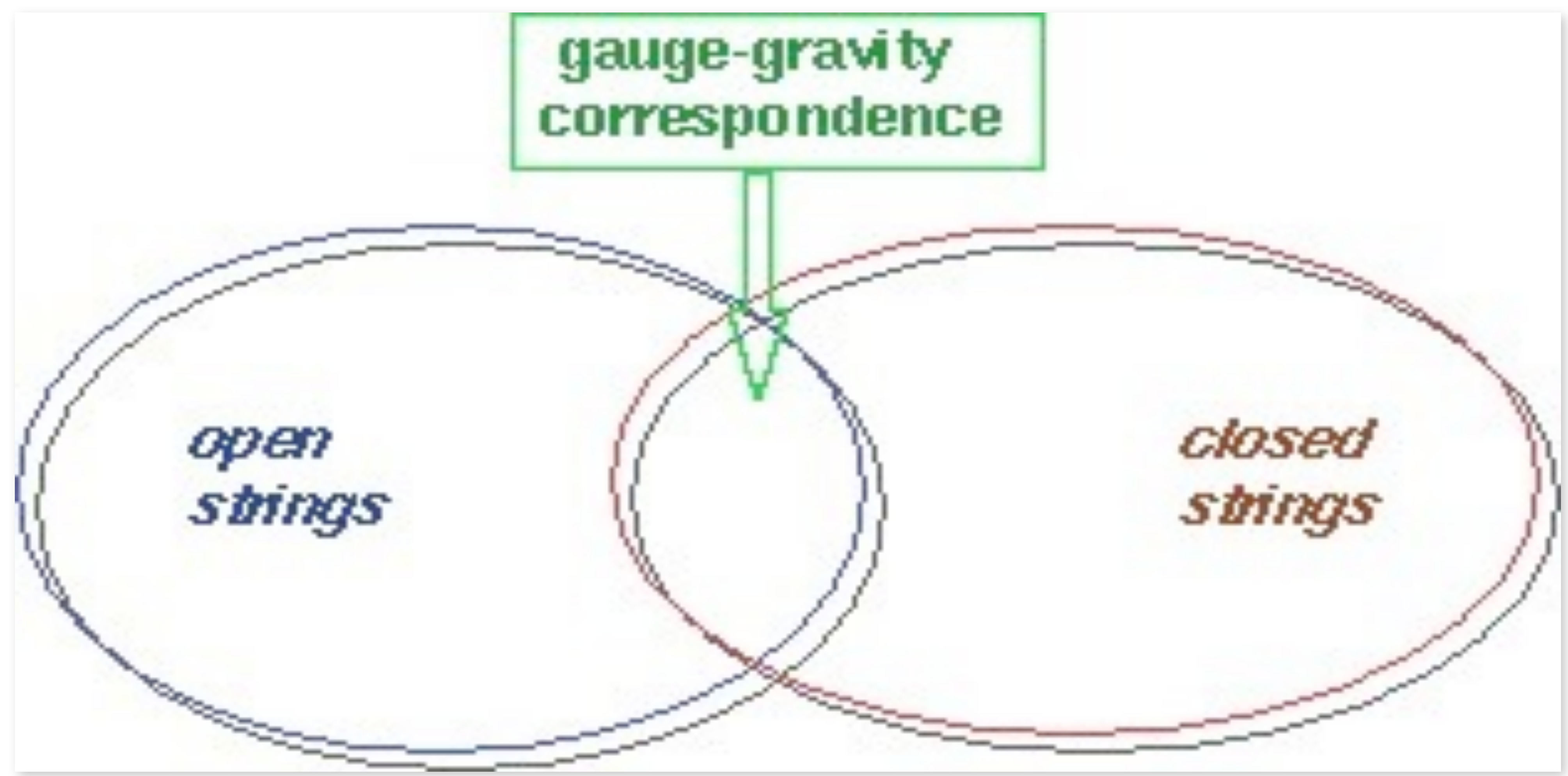
effective theory =gravity

もし、この性質が任意のオーダーまで成り立ち、

さらに non-perturbative　にも有効なら

両者がまったく同等の既述を与える領域の存在も予想される

(CFT/AdS 対応:Maldacena 1997, Witten 1998, Gubser–Polyakov–Klebanov 1998, ...)



gauge/gravity (string) 対応

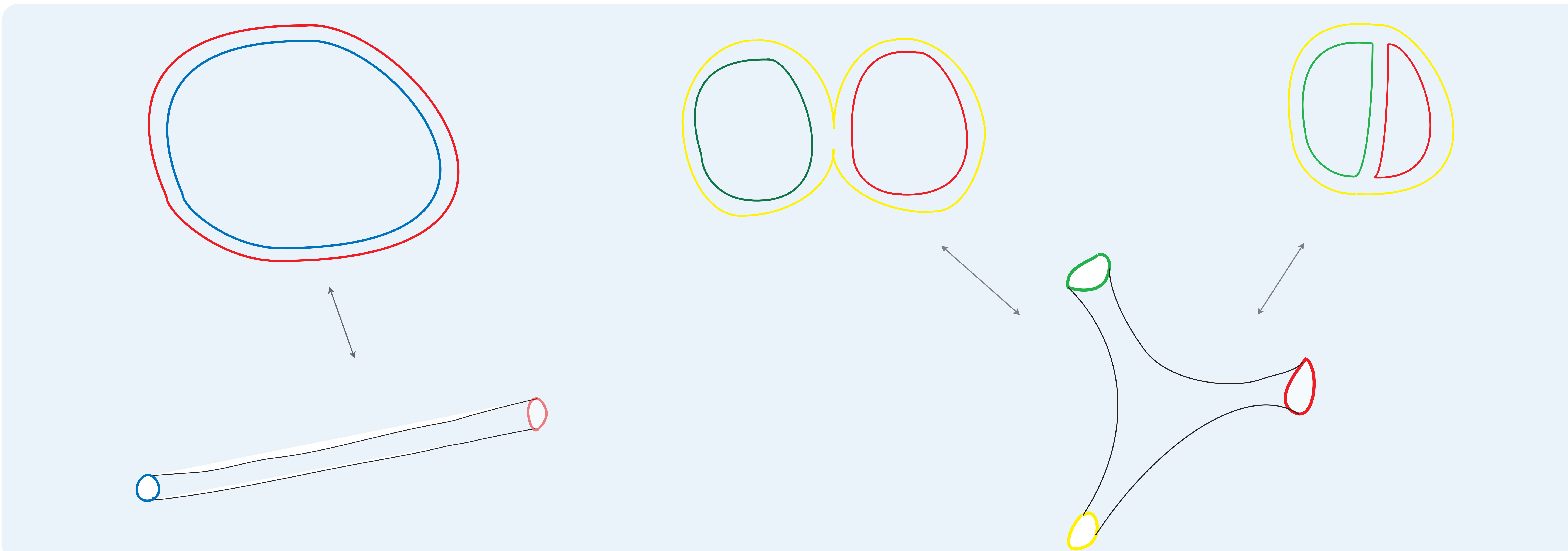
Dブレーンの力学の定式化の観点から、(局所)場の理論と弦理論との関係に新たな認識をもたらし、方法論的にも新たな普遍性を強く示唆しているが、その有効性の範囲については、解明されるべき謎が多く残っている

Dブレーンの運動と相互作用は open string の量子的揺らぎにより支配される.

ゲージ理論からＤ０粒子の一般相対論的３体力と運動方程式を正しく導ける

Y. Okawa-T. Y., NPB538, 67(1999) hep-th/9806108, NPB541, 163(1999) hep-th/9808188

(the resolution of the so-called Dine-Rajaraman problem)



$$L_Y = -\sum_{a,b,c} \frac{(15)^3 N_a N_b N_c}{96(2\pi)^4 R^5 M^{18}} \Big[-v_{bc}^2 v_{ca}^2 (v_{cb}\cdot\nabla_c)(v_{ca}\cdot\nabla_c) + \frac{1}{2} v_{ca}^4 (v_{cb}\cdot\nabla_c)^2 + \frac{1}{2} v_{bc}^4 (v_{ca}\cdot\nabla_c)^2 - \frac{1}{2} v_{ba}^2 v_{ac}^2 (v_{cb}\cdot\nabla_c)(v_{bc}\cdot\nabla_b) + \frac{1}{4} v_{bc}^4 (v_{ba}\cdot\nabla_b)(v_{ca}\cdot\nabla_c) \Big] \Delta(a,b,c)$$

$$\Delta(a,b,c) \equiv \int d^9y \frac{1}{|x_a - y|^7 |x_b - y|^7 |x_c - y|^7}$$

gauge/gravity (or string) 対応の課題

どういう場合に、どこまで有効か、両者を関係づける内在的論理はあるのか、

・大域的対称性(supersymmetry, conformal symmetry, ...)の役割:

perturbative な open-closed string duality は、bosonic string でも成立

・局所的対称性の役割:

bulk: general coordinate invariance ⟷ boundary: local gauge symmetry

・大N極限の役割:

もし、1/N 展開の高次まで成り立つなら、有限の N でも有効か

・「境界側理論=局所場理論」は、どこまで成り立つのか:

ゲージ理論は、弦理論の立場では、lowest mode だけを残した近似にすぎない

・単純には弦理論に埋め込めない(?)ような理論で、どこまで正当化されるか:

3D O(N) vector model の場合 : Vasiliev's higher spin gauge theory (AdS_4)

(tensionless limit of some string theory?)

- ゲージ理論に本質的なスケール依存 (running coupling constant) のダイナミクスを捉えられるか:

  QCD の場合、 asymptotic freedom と confinement を同時に記述できるか

- 様々な非摂動的方法(integrability、resummation, renormalization, ...)や、computer simulation によるゲージ理論の非 摂動的性質に対する重力側予言の検証も重要

PRL **104,** 151601 (2010) PHYSICAL REVIEW LETTERS week ending 16 APRIL 2010

Monte Carlo Studies of Matrix Theory Correlation Functions

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(Received 25 January 2010; published 16 April 2010)

We study correlation functions in (0 + 1)-dimensional maximally supersymmetric U(N) gauge theory, which represents the low-energy effective theory of D0-branes. In the large-N limit, the gauge-gravity duality predicts power-law behaviors in the infrared region for the two-point correlation functions of operators corresponding to supergravity modes. We evaluate such correlation functions on the gauge theory side by the Monte Carlo method. Clear power-law behaviors are observed at $N = 3$, and the predicted exponents are confirmed consistently. Our results suggest that the agreement extends to the M-theory regime, where the supergravity analysis in 10 dimensions may not be justified *a priori*.

Y. Sekino and T.Y. NPB570,174(2000)

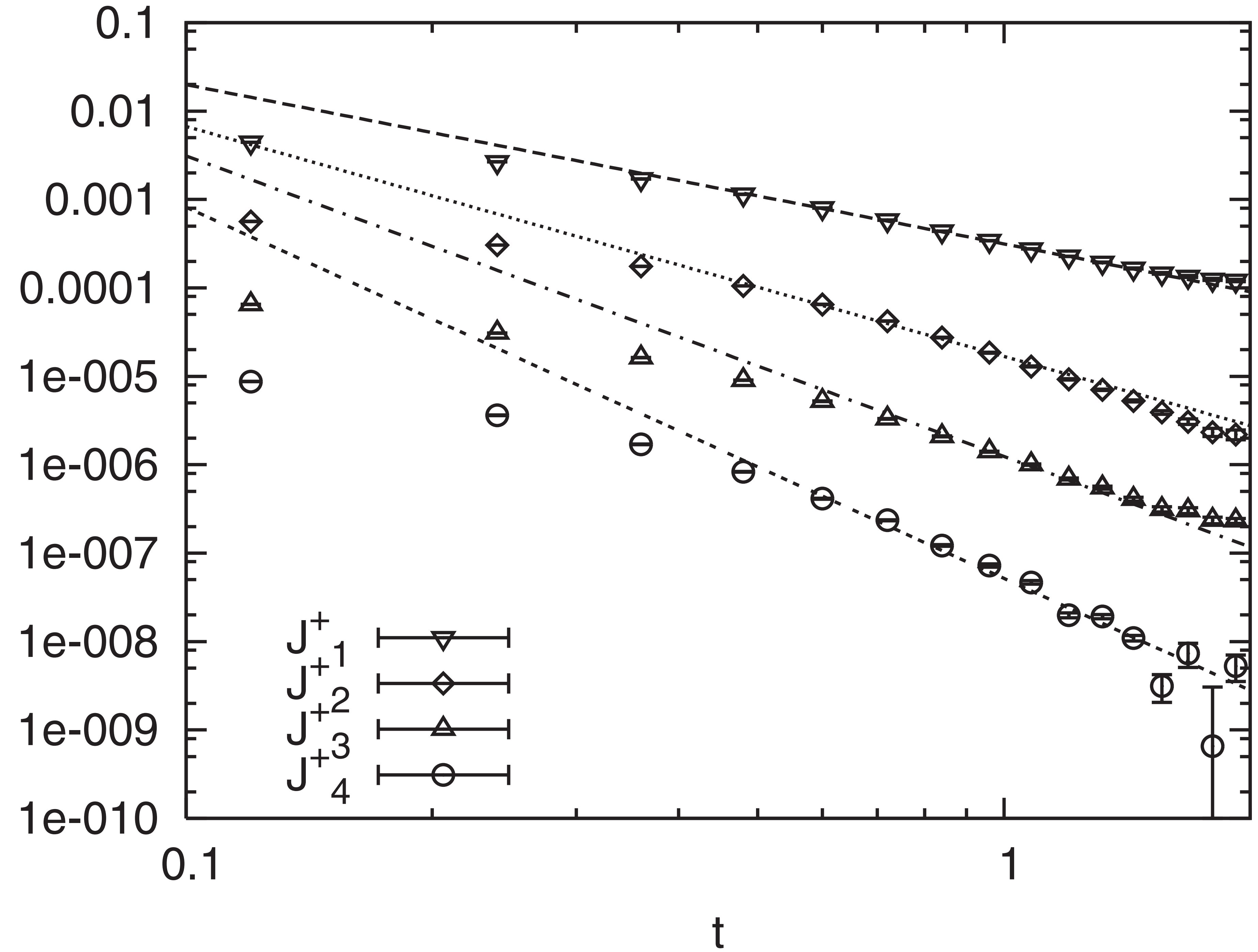


FIG. 1. The real-space two-point correlation functions $\langle J_l^+(t) J_l^+(0)\rangle$ ($l = 1, 2, 3, 4$) are plotted for $N = 3$ and $\beta = 4$. The UV cutoff is $\Lambda = 16$. The straight lines are fits to the predicted power-law behavior.

最近試みられている様々な「現象論的応用」を正当化し基礎づけ、
また、それにより、**本当の意味で新しい知見**を得るためには、
これらの点について理解を深めることが不可欠

可能性を広げることも大事だが、
根拠が薄弱なまま拡大を続けるだけでは、新しい物理を見いだすのは困難

1970年代の疑問
― 弦理論についての二つの見方(or 役割)とその相互関係 ―

は、まだ解明されたとは言えない。

まとめ：弦理論とは何か、何であるべきか

- 重力の古典論＝一般相対性理論(時空構造に関する”熱力学”)は、弦の量子論から導ける
- 弦理論が現実の宇宙を説明できるかどうかは、いまのところは不明だが、重力のミクロレベル理論としての資格をそなえている

  宇宙定数、CP violation の起源、dark matter, ... 等の問題について、具体的な見通しは残念ながら得られてない (潜在的には弦理論を確かめる鍵になる可能性)

- 弦の量子論では、局所性は根本的にくずれる
  （時空そのものに一種の不確定性関係が成立、holographic principle or UV-IR 対応とも密接に関係）
- 時空の幾何学そのものが、「弦-ブレーン」の概念だけに基づき、再構築されなれければならない(string geometry)

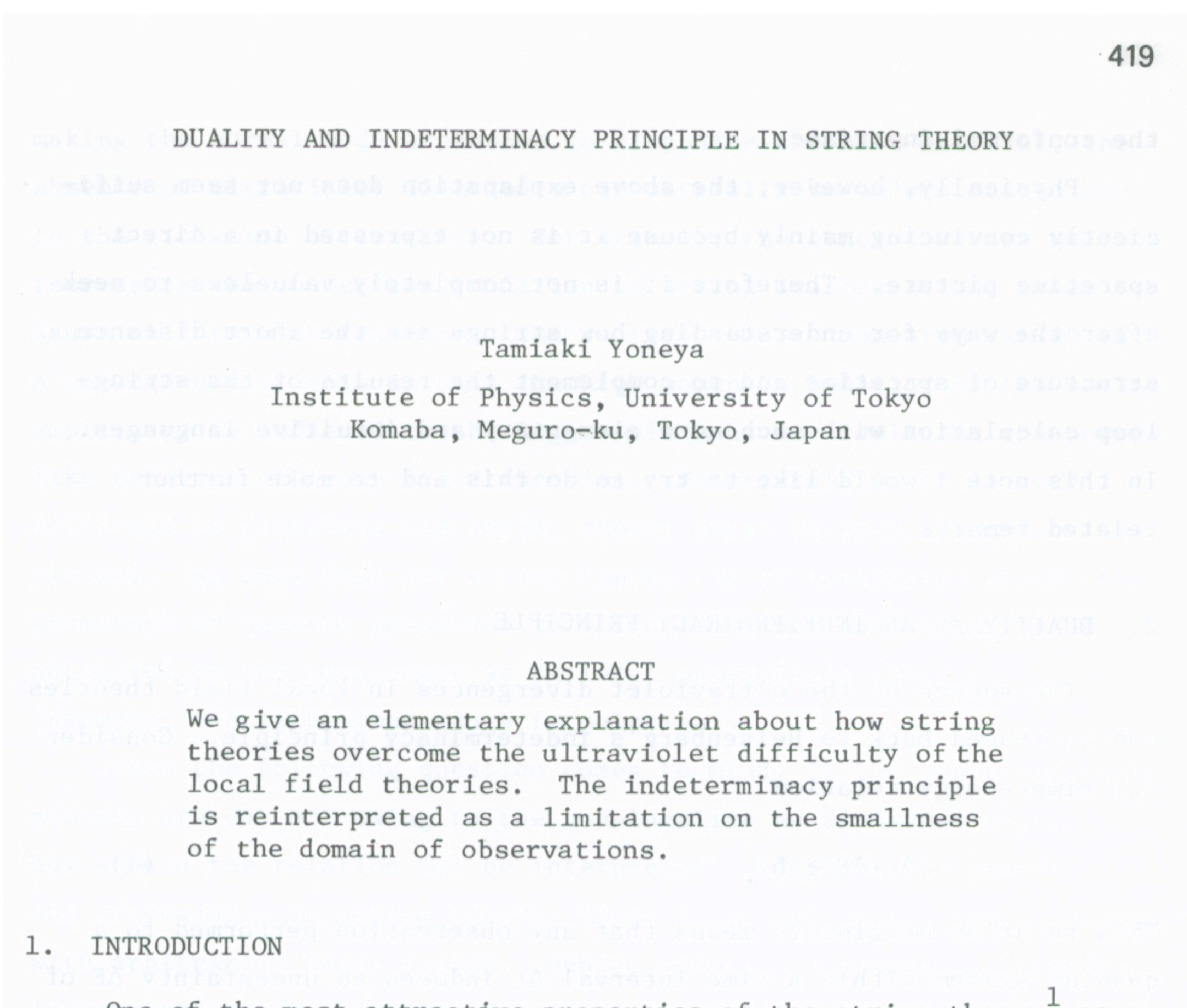
時空不確定性関係：通常の量子力学における時間・エネルギー不確定性関係からの再解釈

$$\Delta E \Delta t \gtrsim h \longrightarrow \Delta X \Delta T \gtrsim \ell_s^2$$

$$\Delta E \sim \Delta X \frac{h}{\ell_s^2} \qquad \Delta t = \Delta T$$

- 共形対称性から導ける
- その時空的表現とみなせる（**時間と空間の間の非可換性**）

In “Wandering in the fields”, Vol. in honor of the 60th birthday of Prof. Nishijima,
World Scientific, 1987



DUALITY AND INDETERMINACY PRINCIPLE IN STRING THEORY

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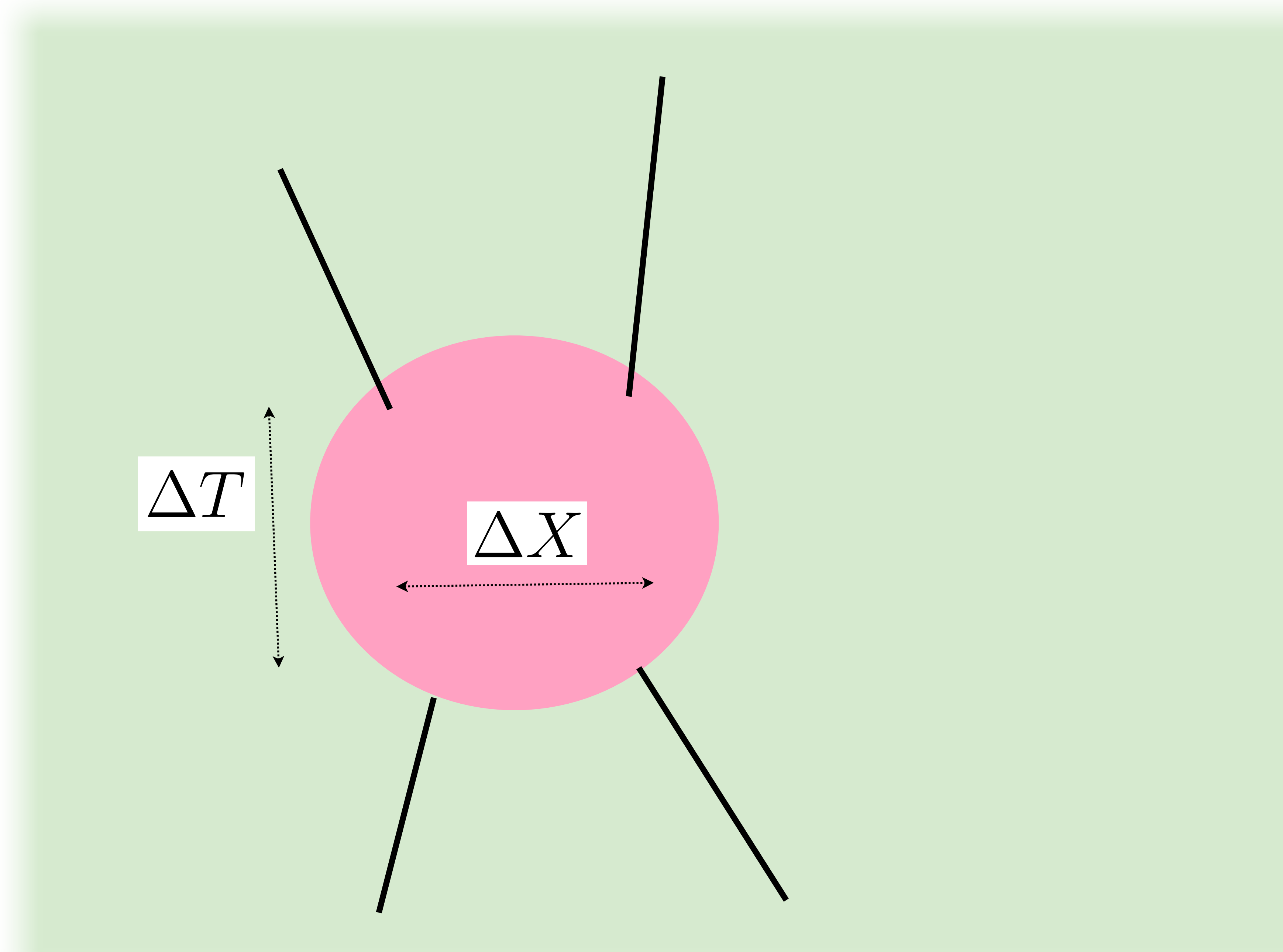
ABSTRACT
We give an elementary explanation about how string theories overcome the ultraviolet difficulty of the local field theories. The indeterminacy principle is reinterpreted as a limitation on the smallness of the domain of observations.

1. INTRODUCTION



このアイデアを提唱した
当初(1987年)はほとんど
受け入れられなかったが、
10年ほどを経て90年代
終盤になりDブレーンの
概念が一般的になってから、
有効性が認識され出した

Polchinski の教科書に
引用なしで触れられている

Pointlike D-brane Dynamics and Space-Time Uncertainty Relation

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(Received 12 November 1996)

We argue that the space-time uncertainty relation of the form $\Delta X \Delta T \gtrsim \alpha'$ for the observability of the distances with respect to time, ΔT, and space, ΔX, is universally valid in string theory including D-branes. This relation has been previously proposed by one (T.Y.) of the present authors as a simple qualitative representation of the perturbative short-distance structure of fundamental string theory. We show that the relation, combined with the usual quantum mechanical uncertainty principle, explains the key qualitative features of D-particle dynamics. [S0031-9007(97)02389-2]

PACS numbers: 11.25.Mj, 04.60.–m

It is often stated that in the fundamental string theory there exists a minimum length of order of $\sqrt{\alpha'} \equiv \ell_s$ beyond which we cannot probe the structure of space-time. This comes about from the properties of string amplitudes in the high-energy limit [1,2] and also in the high-temperature limit [3]. Such a statement is indeed quite natural when we have only the ordinary string states as possible probes for short distances, since string states themselves have an intrinsic extension of the order of length ℓ_s.

Recently, however, we found that string theory, in fact, allows a variety of objects of various dimensions as solitonic excitations and that they are bound to play crucial roles in nonperturbative formulations of string theory. In particular, we have even pointlike objects called D0-branes [4] or D-particles. Recent studies [5–12] of D-particle dynamics revealed the possibility of probing the distance scales of eleven-dimensional (11D) Planck scale of the order $g_s^{1/3}\ell_s$, the natural scale of the M-theory [13], which is indeed much shorter than the string scale ℓ_s for weak string coupling (the importance of shorter length scales in string theory has been suggested earlier in [14]). If

were described by the usual local field theories neglecting quantum gravity, we would be allowed to state that it is determined by the typical wavelength of the objects, namely, $\frac{1}{E}$ for sufficiently high energies. Hence, in principle, we would have no limitation for probing the short-distance scale, provided we neglect quantum gravity. If, on the other hand, the interactions are mediated by fundamental strings, high energies do not necessarily imply that the typical spatial scale is given by the wavelength of the scattering objects, since higher energies dominantly cause larger fluctuations with respect to string excitations during interactions than with respect to the center of mass motion because of the huge degeneracy of string excitation modes. It is easy to see [15,16] that the typical (smeared-out) spatial extension ΔX of strings with energy E is of order $\Delta X \sim \ell_s^2 E$. This implies the simple relation for the indeterminacies of the space and time lengths,

$$\Delta X \Delta T \gtrsim \ell_s^2 , \quad (1)$$

which we call the space-time uncertainty relation. In Ref. [15], this relation was proposed as a natural space-time representation of the *st*-duality properties of string

重点領域研究２３１「無限可積分系」
レクチャーノート
No.11

Introduction to Conformal Symmetry and Its Applications

米谷民明　述

菊川芳夫、辻丸　詔　記

Kyoto, 1995

これは、主に
を念頭において
BPZ のオリジナ
の理論の立場か
とと、後半部で
議論してみたこ
等、不十分なと
頂ければ幸いで

終わりに、こ
夫、辻丸詔の両
に深く感謝いた

僕自身の期待というか予想を少し言っておくことにします。conformal field theory というのは integrable であれば string theory になる。その場合、target space が時空だと言いました。しかしその時空の細かいことをいってもしかたない。large scale で見れば、Riemann 幾何学になっている。かつ、Planck scale というのはまさに量子化して始めて出てくる定数です。Planck がこれを発見して一番喜んだことは、Planck constant があれば長さの単位を作れる、ということだったのです。つまり長さの単位は量子化して初めて出てくる。だから、僕は Riemann 幾何学を自然に量子化したら、今はまだわからない量子化の手続きがあって、本来の量子化をしたら、実は自然に integrable な conformal field theory がでてくると期待しています。こういうことを結び付ける何か、量子化を特徴付ける関係がいろいろあるはずです。一つとしては、時空の短い構造が見えなくなるある種の不確定性原理、そういうものがあるはずです。そういうものをどう定式化するかが問題であって、前から色々考えているわけです。上で言ったことをひとまとめに見るような見方が将来できあがるのではないかというのが、ぼく自身の考えです。

この立場でいうと、universality というのは、時空はどういうものであってもいいとい

うのですから、ある意味でいえば、general covariance とか equivalence principle の一般化になってる。general covariance というのは 19 世紀の物理学からすれば、革命的だったわけです。つまり 20 世紀の初めでも慣性系が重要な役割を果していた。慣性系でないものを考えるのはあまり意味がないと思われていた。だけど Einstein が言ったことは慣性系でない座標系を使っても物理が同じように記述できるということで、これが general covariance でした。equivalence principle というのも加速度をもたせるということと重力があることは同じであるという、一見違ったものを同一視することでした。そういう意味での拡張だと思われます。universality というのもいろいろな model が全部同等であるといっているのですから。これを general covariance というべきか、equivalence principle というべきかは知りませんが、全部、理論を同じように記述できるという非常に大きな対称性があるということです。

こういうことが出来れば、たぶん将来、最終的には現在の量子論と幾何学というものが完全に融合したような理論ができあがる、というのが一つの夢です。ですから、conformal field theory は非常に technical な側面が強いのですが、実は物理的にも非常に重要な意味を持っているに違いないと思われます。ですから、その内容を数学的な面からも物理的な面からもいろいろ追求するということは今後も一番重要な課題の一つと考えています。

- 摂動的な定式化(ユニーク！）しか知られていないが、**非局所的な相互作用の理論を相対論と量子力学の unitary 性とに矛盾せずに両立させ、非摂動的定式化を得る方向を強く示唆している**

  もともと、局所的な量子場には概念的困難がある　(Landau-Peierls, ..., 湯川, ...)重力以外の相互作用については、繰り込み理論によって実用的な定式化が存在する．しかし、重力相互作用を考慮すると繰り込み理論は破綻する

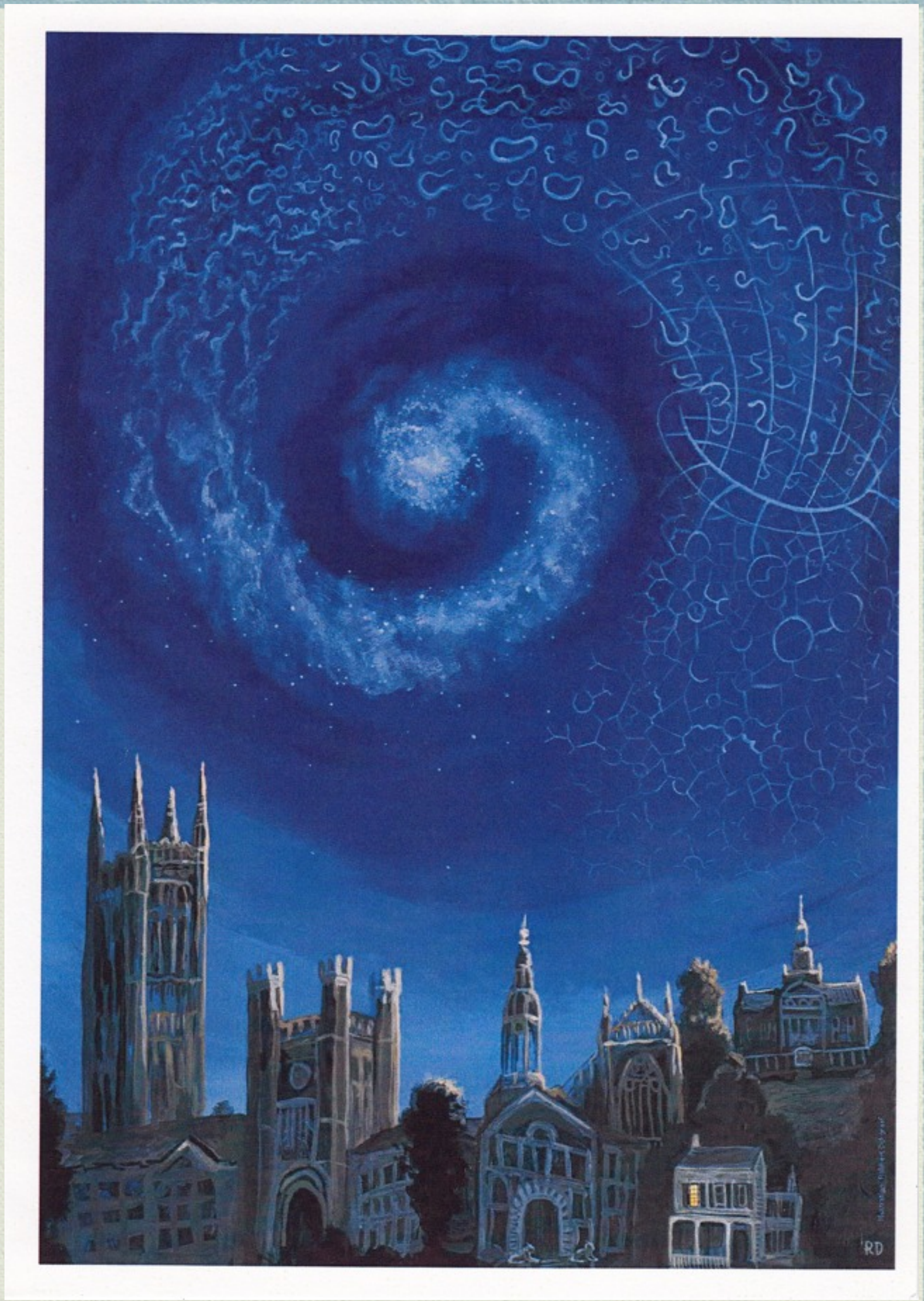
  （ultraviolet catastrophe & unitarity violation)

  note: N=8 4D sugra の finiteness conjecture？(もし正しいとしても、susy だけでは説明できないし、その正則化は弦理論に埋め込む以外にないと思われる）

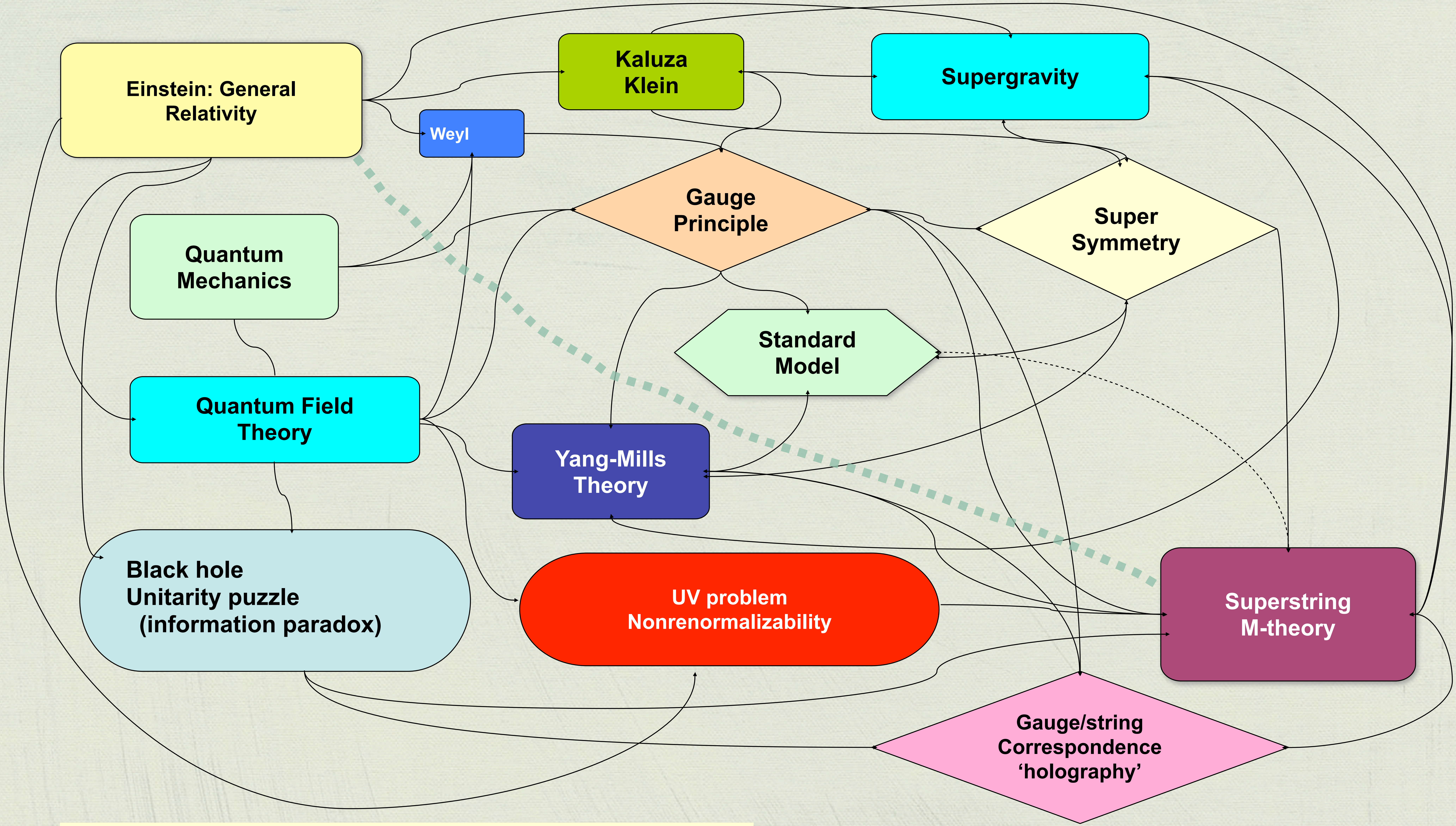
- **gauge/gravity (string) 対応**は、(局所)場の理論と弦理論との関係に新たな認識をもたらし、方法論的にも新たな普遍性を強く示唆しているが、その有効性の範囲については、解明されるべき謎が多く残っている

- 統一理論、および、より根源的理論へ向けた過去の様々な試みを、あたかもジグゾーパズルのように自然に含む　— **アイデアの統一**

一般相対性理論、Weyl の理論、Kaluza-Klein理論、Born-Infeld の非線形電磁場理論、Diracの石鹸膜模型、Yang-Mills 理論、湯川の非局所場（素領域）理論, 非可換時空（Snyder）, ...



Strings2014 (Princeton) ポスター*(Dijkgraaf* 画)



Web of Unification, and unification of ideas

長年の課題

非摂動的定式化のための原理を探ること

そのような定式化ができれば、
量子論の基礎およびその解釈についても大きな
影響を及ぼす可能性が高い

- 背景独立な定式化:　背景を微小変形するための自由度が弦の状態として存在する意味では、弦理論はもともと背景独立な理論であるが、明白に背景独立な形式を探ること
- Dブレーン・反Dブレーン多体系を取り扱うための方法論の開発
- 開弦–閉弦双対性の非摂動的な定式化

いずれにしても、最終的な定式化には何らかの概念的飛躍が必要だろう

歴史の教訓

独創的研究には “Center” にいることは、必ずしも重要ではない

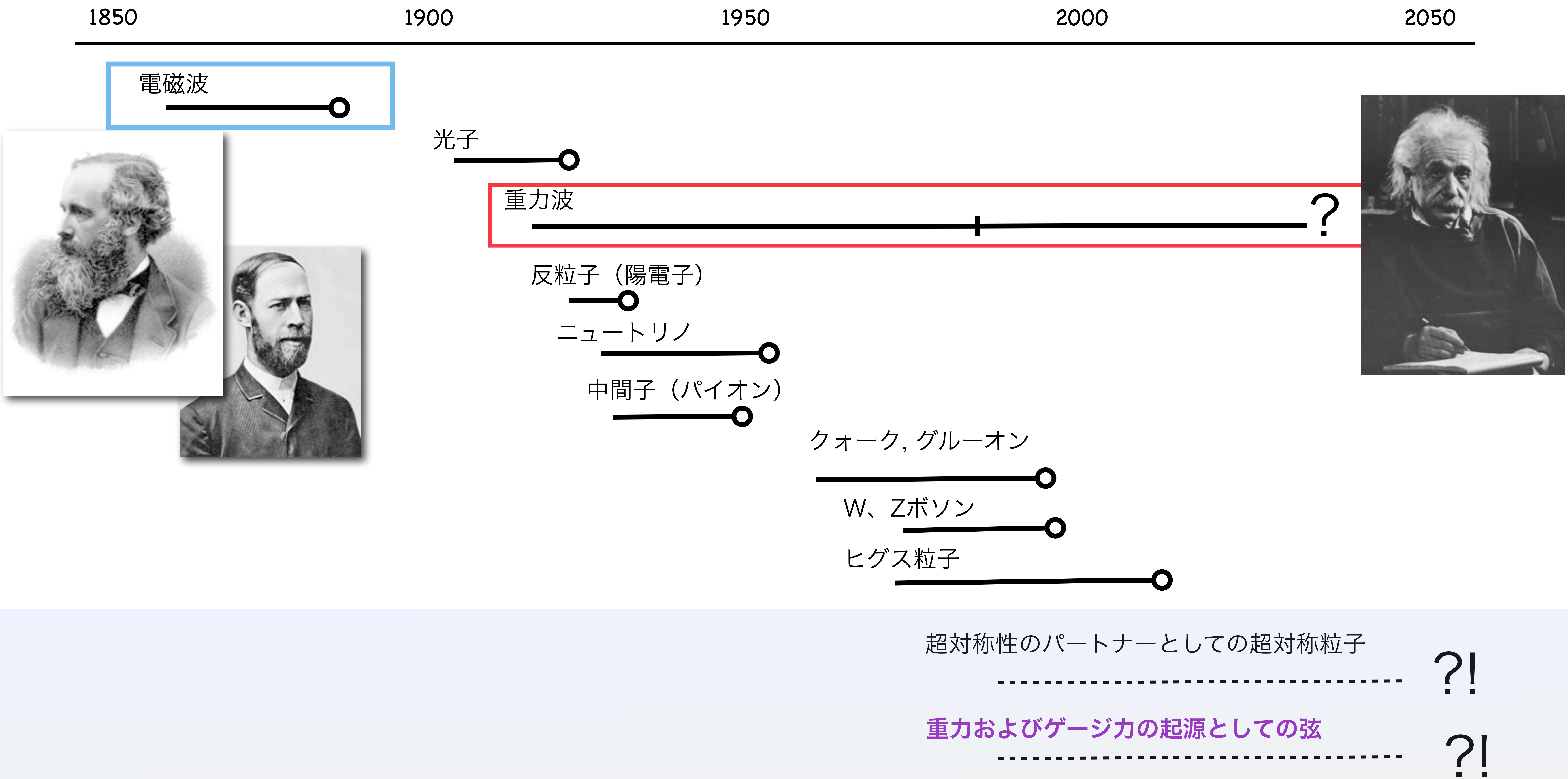
“Outsider” (or 周辺的) 的立場にいることが有利であることもある

同時に自分の外部、異なる考え方、アプローチとの交流は研究を進める上での生命線

- 1903年の 長岡半太郎：原子模型
- 1911-12年の 石原純：相対論、量子論
- 1905年の Einstein：特殊相対性理論、光量子論
- 1923-24年の de Broglie：物質の波動性
- 1924年の Bose：ボース・アインシュタイン統計
- 1928年の 菊池正士：物質波検証実験
- 1934年の 湯川秀樹：中間子論
- 1940年代の 坂田昌一、朝永振一郎：２中間子論、量子電気力学、複合模型、...

etc..........

物理学における新たな自由度の予言と実験的検証（過去、未来）



結論：

超弦理論は，弦の**量子論から一般相対論を導き，さらに素粒子相互作用を記述するためのゲージ相互作用も自然に含む**．その意味では，すでに「統合」の材料を用意している．
しかし,

・その背後にある原理的基礎が不明
・非摂動的な定義がなされていない
・現実の宇宙を説明できるか否か示されていない

未だ不完全な発展途上の理論であるが、その潜在的意義は計り知れない

最大の課題

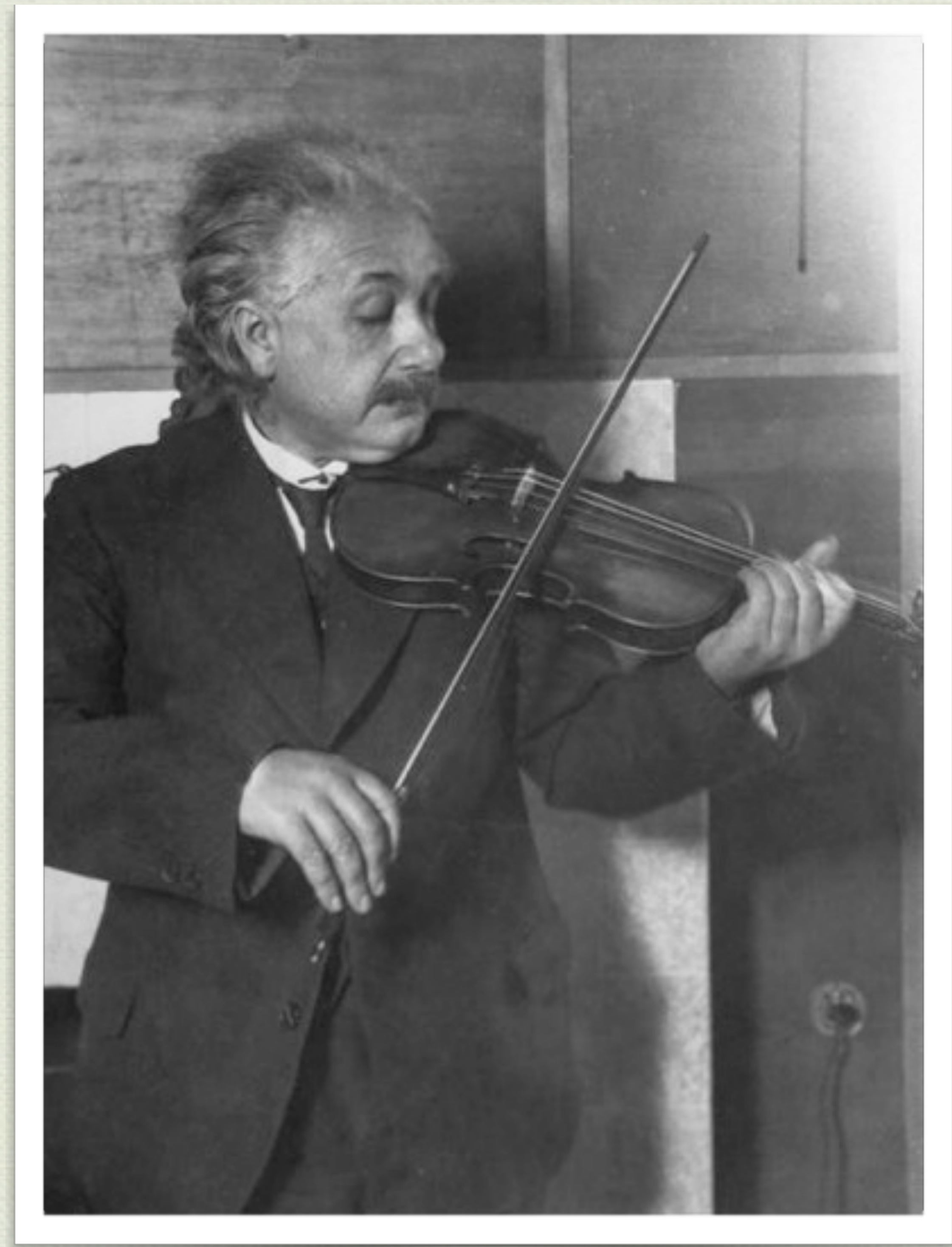
・理論の全貌と背後にある原理を解明し、実験・観測で実際に検証可能な予言を与えること

今後ますます精密化すると期待される宇宙論的データ（背景輻射ゆらぎ、暗黒物質、暗黒エネルギー、重力波、等々）が検証の鍵になるだろう

超弦理論により，重力を含む自己矛盾のない量子論による力の統一の可能性が，初めて明らかにされたことの重要性は何人にも否定しがたい意味がある.

アインシュタインの死後およそ６０年、そして、
一般相対性理論の誕生後100年、を経て，
私たちは，今，ようやく，彼が夢みて，
悩み，果たせなかった統一への
「夜明け前」に近づいているのではなかろうか！

多分，アインシュタインにとっても



“Raffiniert ist der Herr Gott, aber Boshaft ist er nicht”

（神様というのは巧妙だが、意地悪いわけではない）