A spin-polarized state of quark matter with color superconductivity

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1) One-gluon-exchange model (3 color, 2 flavor)

\[ I_{\text{int}} = -g^2 \frac{1}{2} \int d^4x \int d^4y \left[ \bar{\psi}(x) \gamma^\mu \frac{\lambda^a}{2} \psi(x) \right] D_{\mu\nu}(x, y) \left[ \bar{\psi}(y) \gamma^\nu \frac{\lambda^a}{2} \psi(y) \right] \]

Fierz trans. \[ \langle \bar{\psi} \psi \rangle, \quad \langle \bar{\psi} \gamma_5 \psi \rangle, \quad \langle \bar{\psi} \gamma_\mu \psi \rangle, \quad \langle \bar{\psi} \gamma_5 \gamma_\mu \psi \rangle \quad \rightarrow \quad U(p) \]
1-2 Nambu formalism

\[ I_{MF} = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \left( \begin{pmatrix} \bar{\psi} \\ \psi_c \end{pmatrix} \right)^T G^{-1}(p) \left( \begin{pmatrix} \psi \\ \psi_c \end{pmatrix} \right) \]

Charge conjugation.

\[ G^{-1}(p) = \begin{pmatrix} \Delta(p) & \gamma_0 \Delta^+(p) \gamma_0 \\ \gamma_0 \Delta^+(p) \gamma_0 & p - m - \gamma_0 \mu + \overline{U}(p) \end{pmatrix} \]

\[ \psi_c(p) = C \bar{\psi}^T(-p) \]
\[ \overline{U}(p) = CU^T(p)C^{-1} \]

Self-consistent condition to stabilize the MF ground state is that self-energy from residual interaction vanishes:

\[ \Sigma_{Res}(p) = 0. \]

Self-consistent equation from the first-order perturbation:

\[ \Sigma_{MF}(k) = g^2 \int \frac{d^4 p}{i(2\pi)^4} \Gamma^a D_{ab}(k-p) G(p) \Gamma^b \]

\[ \Sigma_{MF}(k) = G_0^{-1}(k) - G^{-1}(k) = \begin{pmatrix} 0 & \gamma_0 \Delta^+(p) \gamma_0 \\ \Delta(p) & 0 \end{pmatrix} \]

Quark propagator under the external mean field \( U \):

\[ G_0^{-1}(k) = p - m + \gamma_0 \mu + U(p) \]
1-3 Gap equation

\[ G(p) = \begin{pmatrix} G_{11}(p) & G_{12}(p) \\ G_{21}(p) & G_{22}(p) \end{pmatrix} \text{ Nambu propagator} \]

\[ U(k) = -\frac{g^2}{4} \int \frac{d^4p}{i(2\pi)^4} D_{\mu\nu}(k - p) \text{Tr}\{\lambda_a\gamma_\mu G_{11}(k)\lambda_a\gamma_\nu\} \]

\[ \Delta(k) = -\frac{g^2}{4} \int \frac{d^4p}{i(2\pi)^4} D_{\mu\nu}(k - p) \lambda_a^T\gamma_\mu G_{21}(k)\lambda_a\gamma_\nu \]

2)

2-1 Axial-vector mean field

\[ U(p) = \gamma_5\gamma_\mu U_A^\mu(p), \quad U^0_A(p) = 0 \]

\[ G_0^{-1}(k) = p - m + \gamma_0\mu + \gamma_5U_A(p) \]

Single particle energy

\[ \varepsilon_\pm(p) = \sqrt{p^2 + U_A^2 + m^2 + 2\sqrt{(p \cdot U_A)^2 + m^2U_A^2}} \]

(exchange splitting) \[ \langle \text{Spin} \rangle \neq 0 \]
2-2 Gap structure

(1) first step

We define the spinor $u_n(p)$ that satisfies the equation $G_0^{-1}(p_0 = \varepsilon_n) u_n(p) = 0$ and introduce its projector $\Lambda_n(p) = u_n(p)u_n^+(p)$ with properties:

$$\Lambda_n(p)\Lambda_m(p) = \delta_{nm}, \quad \sum_n \Lambda_n(p) = 1$$

Spectral representation

$$G_0(p) = \sum_n \frac{\Lambda_n(p)}{(p_0 - \varepsilon_n)} \gamma_0$$

(2) second step

We assume that the gap function $\Delta(k)$ has a form in Dirac space:

$$\Delta(k) = \sum_n \Delta_n(k) B_n(k)$$

$$B_n(k) = \gamma_0 u_n(k)u_{-n}(k)$$
The anti-symmetric nature of the fermion field puts a constraint on the gap function:
\[ C \Delta(k) C^{-1} = \Delta^T(-k) C \quad \text{(on the whole internal space)} \]

In the present case
\[ CB_n(k) C^{-1} = B_n^T(-k) \]
so that \( \Delta_n(k) \) must be anti-symmetric in the color and flavor space:
\[ \{\Delta_n(k)\}_{ij}^{\alpha\beta} = \varepsilon^{\alpha\beta} \varepsilon_{ij} \phi_n(k) \quad \alpha\beta : \text{color}, \quad ij : \text{flavor} \]

2-3 Explicit form of self-consistent equation

Equation for \( U_A \)
\[
G_{11}(p) = \sum_n \left\{ \frac{1-v_n^2(p)}{p_0 - E_n + i\eta} + \frac{v_n^2(p)}{p_0 + E_n - i\eta} \right\} \Lambda_n(p) \gamma_0 \otimes 1_F
\]
\[
U_A(k) = \frac{4}{9} g^2 \int \frac{d^3 p}{(2\pi)^3} D(k-p) \sum_n \left\{ 2v_n^2(p) + \theta(\mu - \varepsilon_n) \right\} \text{Sp}_n(p)
\]

\[ \text{Sp}_n(p) \equiv u_n^+(p) \sigma_z u_n(p) = \frac{U_A(p) \pm \sqrt{p_z^2 + m^2}}{\varepsilon_n(p)}, \quad \sigma_z = \gamma_0 \gamma_5 \gamma_{30} \quad \text{(Spin op.)} \]

where
\[
\begin{align*}
v_n^2(p) &= \frac{1}{2} \left\{ 1 - \frac{\varepsilon_n(p) - \mu}{E_n(p)} \right\}, \\
E_n(p) &= \sqrt{\left(\varepsilon_n - \mu\right)^2 + \phi_n^2}
\end{align*}
\]

(occupation prob.) (Quasi-particle energy.)
Equation for $\phi_n$, we assume that the gap is formed only near the Fermi surface.

\[
G_{21}(p) = \sum_n \frac{\gamma_0 B_n(p)\gamma_0}{p_0^2 - (\varepsilon_n - \mu) - \phi_n^2 + i\eta} \phi_n(p) \otimes \tau_2 \otimes \lambda_2
\]

\[
\phi_n(k) = \frac{2}{3} g^2 \int \frac{d^3 p}{(2\pi)^3} D(k-p) \sum_m T_{nm}(k, p) \frac{\phi_m(p)}{2E_m(p)} \theta(\delta - |\varepsilon_n - \mu|)
\]

The individual gap function $\phi_n(k)$ couples each other via a function $T_{nm}(k, p)$ near the Fermi surface

\[
\begin{align*}
T_{nm}(k_F, p_F) &\sim 2 & \text{for } n = m \\
T_{nm}(k_F, p_F) &\sim 0 & \text{for } n \neq m
\end{align*}
\]

which shows that the gap functions become almost decoupled for large momentum.

Since the gap function becomes uniform for the horizontal angle, $\phi_n(k)$ behaves as a function of azimuthal angle $\theta_k$ near the Fermi surface.

\[
\phi_n(\theta_k, k_F) \approx \frac{2}{3} g^2 \int \frac{d\theta_p dp}{(2\pi)^2} p^2 \sin(\theta_p) D(\theta_p, \theta_k, k_F, p) \frac{\phi_n(\theta_p, p)}{E_n(p)}
\]
3) Numerical calculations and Discussions

Storner approximation for boson propagator: \( g^2 D(k-p) \Rightarrow \tilde{g}^2 \)

constant mean fields

\[
1 = \frac{2}{3} \tilde{g}^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_n(p)} \\
\geq \frac{2}{3} \tilde{g}^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{(\epsilon_0 - \mu)^2 + \phi_\pm ^2 + \epsilon_\pm U_A}}
\]

Coupled eqs.

\[
U_A = \frac{4}{9} \tilde{g}^2 \int \frac{d^3p}{(2\pi)^3} \sum_n \{2v_n^2(p) + \theta(\mu - \epsilon_n)\}Sp_n(p)
\]

\[
Sp_\pm (p) = u_\pm (p) \sigma_z u_\pm (p) = \frac{U_A + \sqrt{p_z^2 + m^2}}{\epsilon_\pm (p)}
\]

\( U_A \) (Spin exp.) \( \Rightarrow \phi_- > \phi_+ \)

deformation in p-space

Gap enhances Spin exp.
Magnetic Moment as a tensor field: \( \left( \overline{\psi} \sigma_{\mu \nu} \psi \right) F^{\mu \nu} \)

\[
M_x = M_y = 0 \\
M_z = \langle \overline{\psi} \sigma_{12} \psi \rangle \neq 0 \quad \rightarrow \quad \text{Ferromagnetism}
\]

\[
\overline{u}_\pm(p) \sigma_{12} u_\pm(p) = \pm \frac{m}{\sqrt{p_z^2 + m^2}}
\]
\[ \rho_B (\text{fm}^{-3}) \]

\[ \Delta (\text{MeV}) \]

\[ \Delta_+ \]

\[ \Delta_- \]

\[ m_q = 55\text{MeV} \]

\[ m_q = 50\text{MeV} \]
\[ \rho_B (\text{fm}^{-3}) \]

> Spin-Polarization

\[ \langle \sigma/N^2 \rangle \]

- Super
- Normal

- \( m_q = 55 \text{MeV} \)
- \( m_q = 50 \text{MeV} \)
- \( g = 0.03 \text{MeV} \)
\[ \langle \sigma_z/N_q \rangle \]

Spin-Polarization

Super

Normal

\( m_q \) (MeV): $\mu = 500$ MeV, $\mu = 550$ MeV, $\mu = 600$ MeV, $\mu = 650$ MeV, $\mu = 700$ MeV