Spectral function of the charmonium near the deconfining transition on the lattice

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Introduction

In this work, we focus on charmonium correlators at finite temperature.

— Mass shift near $T_c$
  T. Hashimoto et al., Phys. Rev. Lett. 57 (86) 2123

— $J/\psi$ suppression above $T_c$

$\rightarrow$ Signal of QCD phase transition.

Our goal is to investigate nature of charmonium correlators at $T > 0$ using lattice QCD simulation.

— Mass shift near $T_c$ ?
— No collective excitation mode above $T_c$ ?

Analysis of correlators in Euclidean temporal direction.
$\rightarrow$ Spectral function of charmonium at $T > 0$. 
Our Approach (1)

Detailed analysis of correlators in t-direction at $T > 0$.
- Need sufficient number of d.o.f.
  $\Leftarrow$ restricted by short temporal extent $\sim 1/Ta_\tau$.
$\Rightarrow$ Anisotropic lattice
  $a_\sigma > a_\tau \ (\xi = a_\sigma/a_\tau : \text{anisotropy})$
  QCD-TARO, Phys. Rev. D63 (01) 054501

We use $O(a)$ improved Wilson quark action.
  T. Umeda et al., Int. J. Mod. Phys. A16 (01) 2215
  J. Harada et al., Phys. Rev D64 (01) 074501

We focus on low energy structure of spectral function.
  -$\rightarrow$ Smearing technique.
    (with wave function at $T = 0$)
    $\rightarrow$ enhances low energy part
Our Approach (2)

Correlators measured in lattice simulation
⇒ Spectral function

\[ C(t) = \sum_{\vec{x}} \langle O(\vec{x}, t), O^\dagger(0) \rangle \]

\[ \Downarrow \]

\[ C(t) = \int d\omega K(t, \omega) A(\omega) \]

\[ K(t, \omega) = \frac{e^{-\omega t} + e^{-\omega(N_t-t)}}{1 - e^{-N_t\omega}} \]

We apply two analysis procedures.

1. Maximum Entropy Method (MEM)
   Y. Nakahara, et al., Phys. Rev. D60 (99) 091503
   - direct determination of \( A(\omega) \)
     - successful at \( T = 0 \)
     - application to \( T > 0 \) may be not straightforward

2. Fit with ansatz for spectral function
   - need information on the form of \( A(\omega) \)
   - with given form \( A(\omega) \), more quantitative

These procedures are complementary.

With estimate of spectral function from MEM, quantitative evaluation is given by fit.
Maximum Entropy Method (MEM)

Reconstruction of a spectral function with Maximum Entropy Method


\[ C(\tau) = \int d\omega K(\tau, \omega)A(\omega) \]

kernel :

\[ K(t, \omega) = \frac{e^{-\omega t} + e^{-\omega(Nt-t)}}{1 - e^{-Nt\omega}} \]

Standard \( \chi^2 \)-fit \( \rightarrow \) ill-posed problem

\[ \rightarrow \text{MEM ( based on Bayes’ theorem )} \]

Maximization of \( Q = \alpha S - L \)

\[ S = \int d\omega \left[ A(\omega) - m(\omega) - A(\omega) \ln \frac{A(\omega)}{m(\omega)} \right] \]

\[ m(\omega) = m_0\omega^2 : \text{default model} \]

\( \alpha : \text{constant} \rightarrow \text{to be integrated out} \)

\( L : \text{Likelihood function (} \chi^2 \text{-term)} \)
**Singular Value Decomposition**

\[ K(\tau, \omega) = V(\tau, \tau')w(\tau', \tau'')U(\omega, \tau'')^t \]

\( w(\tau, \tau') \): diagonal matrix, \( VV^t = V^tV = U^tU = 1 \)

\[ A(\omega) = m(\omega) \prod_{i=1}^{N} \exp \{ b_i u_i(\omega) \} \]

\( u_i(\omega) \): basis in singular space

\( b_i \): parameters

- need for large \( N \) (fine temporal resolution)
- difficult to reconstruct

narrow peak at \( \omega \gg 1 \)
wide peak at \( \omega \ll 1 \)
Fit with ansatz

We suppose the shape of spectral function. (MEM gives rough estimate of shape of spectral function.)

- Pole type:
  \[ \rho(\omega) = C\delta(m - \omega) \]

- Breit-Wigner type:
  \[ \rho(\omega) = \frac{C\Gamma m}{(\omega^2 - m^2)^2 - \Gamma^2 m^2} \]
  \[ A(\omega) = \omega^2 \rho(\omega) \]

  \[ C : \text{overlap, } m : \text{mass, } \Gamma : \text{width} \]

<table>
<thead>
<tr>
<th>Fit type</th>
<th>lowest peak</th>
<th>second peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-pole fit</td>
<td>pole</td>
<td>pole</td>
</tr>
<tr>
<td>1-BW fit</td>
<td>BW</td>
<td></td>
</tr>
<tr>
<td>BW+pole fit</td>
<td>BW</td>
<td>pole</td>
</tr>
</tbody>
</table>

*2-pole : expected to suffice below \( T_c \)
*1-BW : workable for large \( L_{\text{min}} \)
*BW+pole : for subtraction of contribution from large \( \omega \) part

Fit range: \( L_{\text{min}} - L_{\text{max}} \)

We study \( L_{\text{min}} \) dependencies of the fitting parameters. The region in which there value are constant \( \implies \) ansatz is successfully applicable.
Anisotropic quark action

Quark action:

\[ S_F = \sum_{x,y} \bar{\psi}(x)K(x,y)\psi(y) \]

\[ K(x,y) = \delta_{x,y} - \kappa_\tau \left[ (1 - \gamma_4)U_4(x)\delta_{x,\hat{4},y} + (1 + \gamma_4)U_4^\dagger(x - \hat{4})\delta_{x,\hat{-4},y} \right] \]

\[ -\kappa_\sigma \sum_i \left[ (r - \gamma_i)U_i(x)\delta_{x,\hat{i},y} + (r + \gamma_i)U_i^\dagger(x - \hat{i})\delta_{x,\hat{-i},y} \right] \]

\[ -\kappa_\sigma c_E \sum_i \sigma_4i F_4i(x)\delta_{x,y} + r\kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x)\delta_{x,y} \]

T. Umeda et al., Int. J. Mod. Phys. A 16 (2001) 2215

- Constructed following the Fermilab approach.

- \( r = 1/\xi \) (action retains explicit Lorentz invariant form)
  (cf. another choice \( r = 1 \) was adopted in several works.)

- Tadpole improvement: \( c_E = 1/u_\sigma^2 u_\tau \), \( c_B = 1/u_\sigma^3 \)
  \( u_\sigma, u_\tau \): mean-field values of spatial and temporal link variables

- Parameters varied in simulations: \( (\kappa, \gamma_F) \)

\[ \gamma_F \equiv \frac{\kappa_\tau u_\tau}{\kappa_\sigma u_\sigma}, \quad \frac{1}{\kappa} = \frac{1}{\kappa_\sigma u_\sigma} - 2(\gamma_F + 3r - 4) \quad (= 2(m_0 \gamma_F + 4)) \]

\( \gamma_F \): bare anisotropy parameter
\( m_0 \): bare quark mass (in temporal lattice units)
Simulation parameters

Gauge parameters:
— Anisotropic plaquette action in quenched approximation

\( (\beta, \gamma_G) = (6.10, 3.2108) \)
- \( \xi = a_\sigma/a_\tau = 4 \)

- \( a_\sigma^{-1} = 2.030(13) \text{ GeV} \)
  Scale is set by hadronic radius \( r_0 \)

\( \Box \) Size : \( 20^3 \times N_t \)

<table>
<thead>
<tr>
<th>( N_t )</th>
<th>( T/T_c )</th>
<th>( N_{\text{conf}} \times N_{\text{source}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>160</td>
<td>( \sim 0 )</td>
<td>( 500 \times 16 )</td>
</tr>
<tr>
<td>32</td>
<td>( \sim 0.9 )</td>
<td>( 1000 \times 16 )</td>
</tr>
<tr>
<td>26</td>
<td>( \sim 1.1 )</td>
<td>( 1000 \times 16 )</td>
</tr>
</tbody>
</table>

Quark parameters :
— \( O(a) \) improved Wilson quark action

\( (\kappa, \gamma_F) = (0.1120, 4.000) \)
- \( \rightarrow \) roughly correspond to charm quark mass

\[ \text{H. Matsufuru et al., Phys. Rev. D 64 (2001) 114503} \]

\( \Box \) Smeared-smeared correlators in PS and V channels

\( \Box \) Smearing function : wave function V meson measured at \( T = 0 \) in Coulomb gauge

\( \Box \) Source points: \( N_{\text{source}} \) points on each configuration
- \( \rightarrow \) reduce statistical fluctuation
**Numerical Results**

**Point-Point correlators**

<table>
<thead>
<tr>
<th></th>
<th>$t_{\text{min}}$</th>
<th>$t_{\text{max}}$</th>
<th>$t_{\text{sep}}$</th>
<th>$N_{\text{DF}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type-I</td>
<td>1</td>
<td>48</td>
<td>1</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>24</td>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>16</td>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>12</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>Type-II</td>
<td>1</td>
<td>48</td>
<td>1</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>48</td>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>48</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>48</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

- $t_{\text{max}} \simeq 16$ is not acceptable
- at least $O(10)$ degree of freedom is necessary
- $t_{\text{max}} a_T$ of the order of 1 fm is necessary

⇒ This requirement cannot be fulfilled at $T > 0$.  

![Graphs showing numerical results](image)
**Smeared-Smeared Correlators**

In the following, we focus on the low frequency part (lowest peak) of $\rho(\omega)$.

□ Effective mass plot

$m_{eff}$ defined through

\[
\frac{C(t)}{C(t + 1)} = \frac{\cosh [m_{eff}(t)(N_t/2 - t)]}{\cosh [m_{eff}(t)(N_t/2 - t - 1)]}
\]

□ Spectrum at $T = 0$

— determined by two-pole fit

<table>
<thead>
<tr>
<th>state</th>
<th>$m_{PS}$</th>
<th>$m_{V}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ground</td>
<td>0.36856(9)</td>
<td>0.37769(12)</td>
</tr>
<tr>
<td>first exc.</td>
<td>0.500(22)</td>
<td>0.479(23)</td>
</tr>
<tr>
<td>fit range</td>
<td>17–80</td>
<td>15–80</td>
</tr>
</tbody>
</table>
Results at $T = 0$

- **Result of MEM**

  Pseudoscalar

  - $t_{\text{max}} = 48$
  - $t_{\text{max}} = 24$
  - $t_{\text{max}} = 16$
  - $t_{\text{max}} = 12$

  ![Graph showing MEM results for pseudoscalar](image)

  Vector

  - $t_{\text{max}} = 48$
  - $t_{\text{max}} = 24$
  - $t_{\text{max}} = 16$
  - $t_{\text{max}} = 12$

  ![Graph showing MEM results for vector](image)

- **Result of Fits**

  - 2-pole, 1-BW, BW+pole forms

  Ground state: mass

  ![Graph showing ground state mass](image)

  width

  ![Graph showing ground state width](image)

  - In MEM, ground state peak is stable with change of $t_{\text{max}}$
  - Consistency of methods
  - No indication of finite width for ground state, as expected
Results at $T = 0.9T_c$

- **Result of MEM**

  ![Graph showing the results of MEM for Pseudoscalar and Vector fields.](image)

  - **Ground state peaks** locate at almost the same as $T = 0$
  - **Width from fit** is consistent with zero.
    - No indication of finite width for ground state

- **Result of Fits**

<table>
<thead>
<tr>
<th>Ground state: mass</th>
<th>width</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Graph showing lowest peak mass and width for different fits." /></td>
<td><img src="image" alt="Graph showing lowest peak width for different fits." /></td>
</tr>
</tbody>
</table>

- 2-pole, 1-BW, BW+pole forms

- ![Graph showing the results of different fits for the lowest peak width at Lmax=13.](image)
**Results at $T = 1.1$**

### Result of MEM

#### Pseudoscalar

<table>
<thead>
<tr>
<th>$m_0$</th>
<th>$A(\omega)/\omega^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.10</td>
<td></td>
</tr>
<tr>
<td>41.0</td>
<td></td>
</tr>
<tr>
<td>0.41</td>
<td></td>
</tr>
</tbody>
</table>

$smeared$ $N_t=26$, $1000-conf.$, $t_{max}=13$, $Ps$

#### Vector

<table>
<thead>
<tr>
<th>$m_0$</th>
<th>$A(\omega)/\omega^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.40</td>
<td></td>
</tr>
<tr>
<td>24.0</td>
<td></td>
</tr>
<tr>
<td>0.24</td>
<td></td>
</tr>
</tbody>
</table>

$smeared$ $N_t=26$, $1000-conf.$, $t_{max}=13$, $V$

### Result of Fits

- 2-pole, 1-BW, BW+pole forms

#### Ground state: mass

- lowest peak mass $N_t=26$, smear-smear $L_{max}=13$

#### width

- lowest peak width $N_t=26$, smear-smear $L_{max}=13$

- There observed hadron-like peak.
- Finite width is observed ($\Gamma \sim 200$ MeV).
- Peak position is almost same as $T < T_c$. 
Spectral function determined by fit

( BW + pole fit, $L_{\text{min}} = 4$, $L_{\text{max}} = 13$ )

Only lowest peak is shown.
Summary

We analyzed smeared charmonium correlators measured in quenched lattice QCD with two methods. —results of fit and MEM are qualitatively consistent

○ Below $T_c$ ($T \sim 0.9T_c$ )
  - No indication of finite width
  - Mass is almost the same as $T = 0$

○ Above $T_c$ ($T \sim 1.1T_c$ )
  - There observed hadron-like peak.
  - Finite width is observed ($\Gamma \sim 200$ MeV ).
  - Peak position is almost the same as $T = 0$.

Complementary use of these procedures is preferable for reliable analysis.

Outlook

- More quantitative study in wide range of $T$
- Effect of dynamical quark