Viscous Hydrodynamics of YM systems from AdS/CFT

Shin Nakamura

(Center for Quantum Spacetime and Hanyang Univ.)

Based on collaboration with Sang-Jin Sin

(hep-th/0607123, to be published in JHEP)

RHIC Experiment

- The created particles are still **strongly** coupled.
- Quark-gluon **liquid** is created rather than Quark-gluon plasma (QGP).

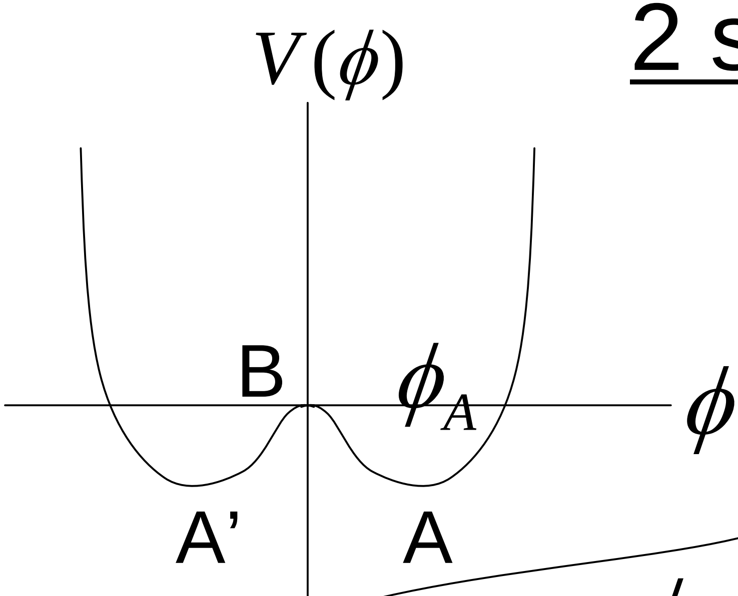
Press release at APS annual meeting 2005

- Relativistic hydrodynamics is useful.
- Perturbative QCD is not applicable.

Why don't we try to apply **AdS/CFT**?

What is AdS/CFT?

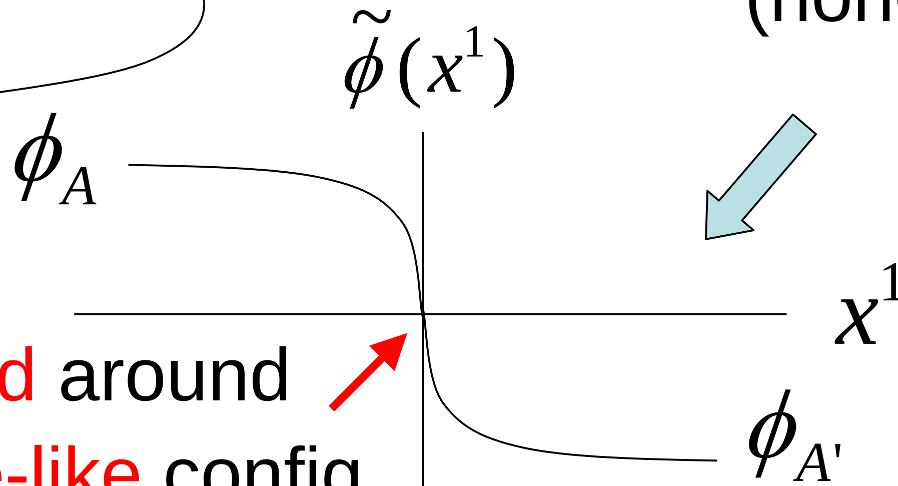
Example: ϕ^4 theory $\qquad V(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4$



2 solutions:

$\phi = \phi_A$ constant solution

$\phi = \tilde{\phi}(x^1)$ kink solution (non-constant)



Energy is localized around $x^1 = 0$: brane-like config.

Physics around the **kink** solution

2 equivalent methods:

$$S(\tilde{\phi} + \varphi)\Big|_{\text{around kink}} = \hat{S}(\phi_A + \hat{\varphi})\Big|_{\text{around }\phi_A} + J(\hat{\varphi})$$

kink, dynamical, const. vacuum, **source term**

Perturbation theory around the kink

||

Perturbation theory around the const. vacuum + **source**

Generalization to string theory?

Type IIB Superstring Theory

Defined in 10d spacetime

Theory of closed strings (perturbatively)

Low energy: 10d type IIB supergravity

Many different vacua. Two of them:

1. Non-trivial vacuum: **black 3-brane** solution

Asymptotically flat

Charged black hole

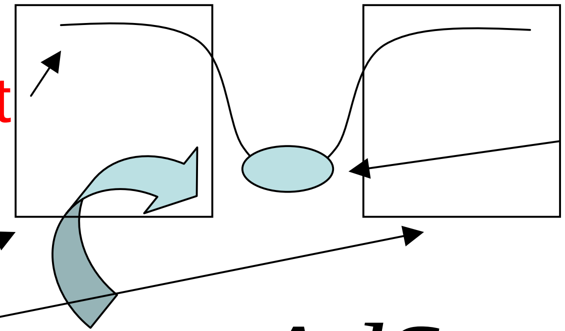
2. **Flat** vacuum

"**Source** of closed strings": D3-brane

3+1 dim. hypersurface, **gauge theory** on it

Superstring theory around black 3-brane geometry

asymptotically flat



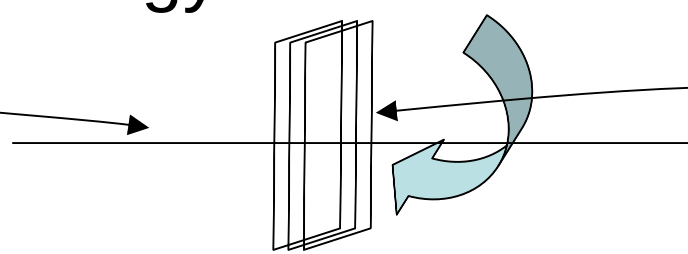
charged black hole
(3+1 dim. object)

The near horizon limit : $AdS_5 \times S^5$

We do not want here.

= ?

U(Nc) 3+1 dim N=4 Super YM theory
at low energy on the D3-branes



Superstring theory around flat geometry

+ source term (Nc D3-brane)

AdS/CFT

(Weak version)

Classical Supergravity on $AdS_5 \times S^5$

|| conjecture

Maldacena '97

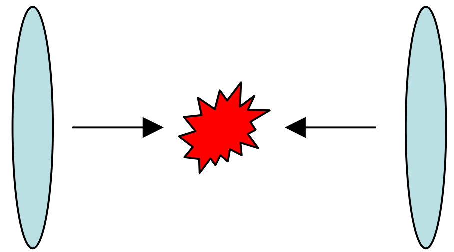
4dim. Large-Nc SU(Nc)

N=4 Super Yang-Mills

at the large 't Hooft coupling

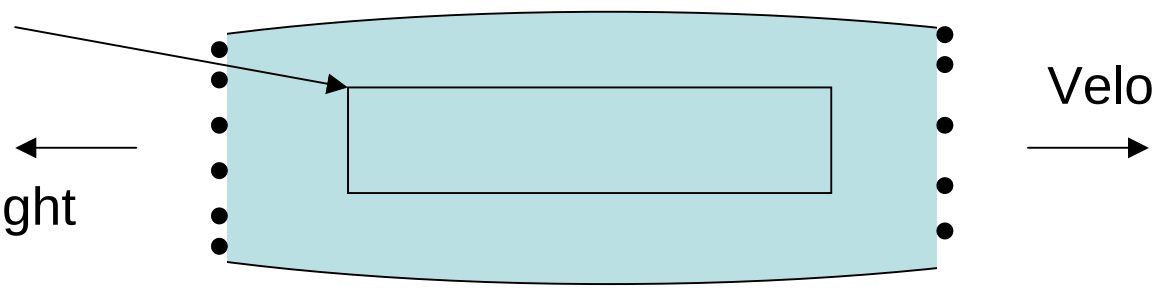
Strongly interacting quantum YM !!

The system we consider



Relativistically accelerated heavy nuclei

Central Rapidity Region (CRR)



Velocity of light

Velocity of light

After collision

- (Almost) **one-dimensional expansion**.
- We have **boost symmetry** in the CRR.

⇨ Time dependence of the physical quantities are written by the **proper time.**

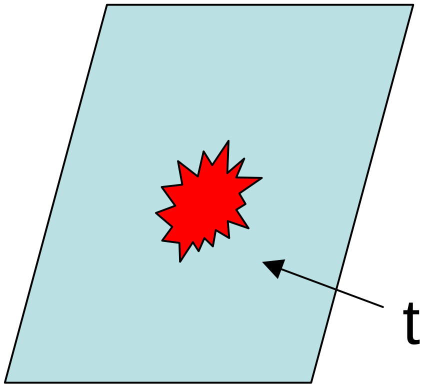
However,

- The system is time-dependent.
- The system is not exactly at the thermal equilibrium.

It is important to construct a **non-static** generalization of AdS/CFT.

If such a generalization exists, it should contain the hydrodynamics of gauge theory plasma.

What we did



time-dependent plasma

non-extremal D3-branes

Boundary condition comes from 4d Hydro.

local rest frame
and
$T^{\mu\nu}(\tau)$

Solving the SUGRA equation
(5d Einstein's equation)

Time dependent
5d Bulk Geometry

Horizon, Hawking temp.

⇨ Something more than Hydro.

The late time geometry

$$ds^2 = \frac{1}{z^2}\left[-\frac{(1-\frac{\rho z^4}{3})^2}{1+\frac{\rho z^4}{3}} d\tau^2 \right.$$

$$\left. + (1+\frac{\rho z^4}{3})\left(\frac{1+\frac{\rho z^4}{3}}{1-\frac{\rho z^4}{3}}\right)^{-2\gamma} \tau^2 dy^2 + (1+\frac{\rho z^4}{3})\left(\frac{1+\frac{\rho z^4}{3}}{1-\frac{\rho z^4}{3}}\right)^{\gamma} dx_\perp^2 \right] + \frac{dz^2}{z^2}$$

$$\gamma = \frac{\eta_0}{\rho_0 \tau^{2/3}}, \qquad \rho(\tau) = \frac{\rho_0}{\tau^{4/3}} - \frac{2\eta_0}{\tau^2}$$

This is correct up to the order of $O(\gamma^2)$.

This looks to be a **black hole** with **time-dependent horizon**.

Various quantities from the geometry

Stefan-Boltzmann: $\rho = \frac{3}{8}\pi^2 N_c^2 T_H^4(\tau)$

Entropy creation: Numerical coefficient is given.

$$S = \frac{A}{4G} = \left(\frac{N_c}{2\pi}\right)^{1/4}\left(\frac{\pi}{3}\right)^{3/4} 2\sqrt{2}\,\rho_0^{\,3/4}\left(1 - \frac{3}{2}\frac{\eta_0}{\rho_0 \tau^{2/3}} + O(\tau^{-4/3})\right)$$

Integration constant is given.

From hydrodynamics:

$$S = S_\infty - 2\frac{\eta_0}{T_0}\tau^{-2/3} + O(\tau^{-4/3})$$

Not only consistent with hydro but also more information in the holographic dual.

Why is AdS/CFT greater than hydrodynamics?

Hydrodynamics

Time evolution of macroscopic quantities

temperature, pressure, entropy, energy, transport coefficients (viscosity,…)

↑

microscopic quantities

correlation functions, quark-antiquark potential, counting of entropy, equation of state,…

AdS/CFT

Conclusions

Details: see my poster or paper.

Caution!

Analogy with ϕ^4-theory is just a quick sketch!

Basics of superstring theory are necessary to understand AdS/CFT properly.

Conclusions

- We considered a holographic dual of hydro-dynamics that is relevant for the analyses of RHIC experiment.
- We obtained the bulk geometry in the late time regime.
- The holographic dual is not only consistent with the hydrodynamics but also contains more information.

Future directions

- We provided a tool: we should compute physical quantities based on the holographic dual.
- More realistic models.
- Inclusion of heat conductivity, bulk viscosity, chemical potential…..
- 3d expansion (work in progress).
- Hadronization
- How far can AdS/CFT be extended into non-equilibrium? Can we go beyond the hydrodynamic region where local thermal equilibrium is not achieved?

Details of our works

Relativistic Hydrodynamics

- We take local rest frame (LRF).

⇨ Our case: (τ, y, x^2, x^3)

Proper-time (→ τ), Rapidity (→ y)

$$ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2$$

- The energy-momentum tensor on this frame:

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & \tau^2\left(p - \frac{4}{3}\frac{\eta}{\tau}\right) & 0 & 0 \\ 0 & 0 & p + \frac{2}{3}\frac{\eta}{\tau} & 0 \\ 0 & 0 & 0 & p + \frac{2}{3}\frac{\eta}{\tau} \end{pmatrix}$$

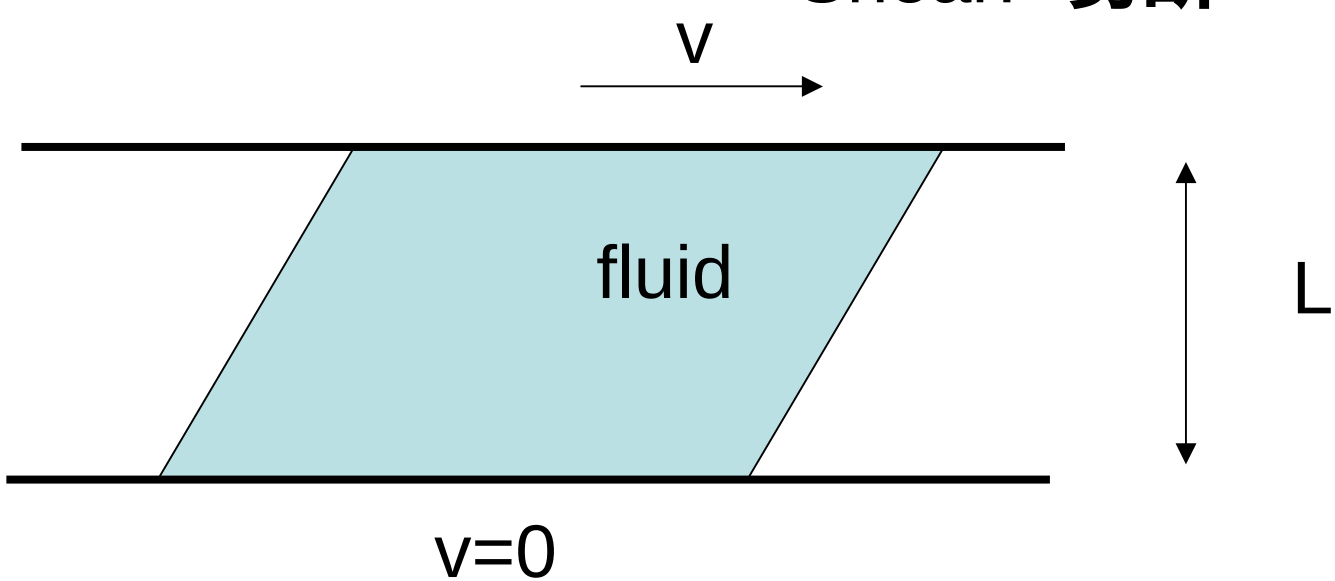
energy density (→ ρ), pressure (→ p), shear viscosity (→ η)

The bulk viscosity is set to be zero.

Shear viscosity

Viscosity: **粘性度**

Shear: **剪断**



$$\frac{F}{A} = \eta \frac{v}{L}$$

3 independent components: $T_{\tau\tau}, T_{yy}, T_{22}$

or ρ, p, η

2 independent constraints:

$$T_{\mu}^{\mu} = 0$$ Conformal invariance (or equation of state)

$$\nabla_{\mu} T^{\mu\nu} = 0$$ Energy-momentum conservation

Only 1 independent quantity: $T_{\tau\tau} = \rho(\tau)$

$$p = \rho / 3$$

$$\frac{d\rho}{d\tau} = -\frac{4}{3}\left(\frac{\rho}{\tau} - \frac{\eta}{\tau^2}\right)$$

Solution:

$$\eta(\tau) = \frac{\eta_0}{\tau^{\beta}}, \qquad \rho(\tau) = \frac{\rho_0}{\tau^{4/3}} + \frac{4\eta_0}{1-3\beta}\frac{1}{\tau^{1+\beta}}$$

$$(\beta \neq 1/3)$$

In the **static** N=4 SYM system:

$\rho \propto T^4$ Stefan-Boltzmann's low

$\eta \propto T^3$ from AdS/CFT

Polocastro-Son-Starinets
hep-th/0104066

$$T(\tau) = T_0\left(\frac{1}{\tau^{1/3}} - \frac{\eta_0}{\rho_0(1-3\beta)}\frac{1}{\tau^{\beta}} + \ldots\ldots\ldots\right)$$

$$\eta(\tau) = \frac{\eta_0}{\tau}, \qquad (\beta = 1)$$

in the slowly varying region

Entropy creation

$$S(\tau) = \int d\tau \frac{4\eta}{3\tau T} = S_{\infty} - 2\frac{\eta_0}{T_0}\tau^{-2/3} + O(\tau^{-4/3})$$

The dissipation creates the entropy.
(散逸)

The entropy (per unit volume on the LRF) at the infinitely far future is not determined in this framework. (Integration constant)

AdS/CFT gives the final entropy in terms of the initial conditions.

Gravity dual

- 5d metric: asymptotically AdS

Boost symmetry

⇨ Solution of 5d Einstein's eq. with $\Lambda < 0$

- 4d part of the metric depends only on τ, z

$$ds^2 = \frac{g_{\mu\nu}(\tau, z)dx^\mu dx^\nu + dz^2}{z^2}$$

$T_{\mu\nu}$

0

Boundary cond.

$$g_{\mu\nu}(\tau, z) = g_{\mu\nu}^{(0)}(\tau) + g_{\mu\nu}^{(2)}(\tau)z^2 + g_{\mu\nu}^{(4)}(\tau)z^4$$

Boundary cond.

$$+ g_{\mu\nu}^{(6)}(\tau)z^6 + \ldots\ldots\ldots\ldots$$

Haro-Skenderis-Solodukhin
hep-th/0002330

The Minkowski metric on the LRF

The higher-order terms are determined **iteratively**.

Now, the energy-momentum tensor is:

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & \tau^2(-\rho - \tau \frac{d}{d\tau}\rho) & 0 & 0 \\ 0 & 0 & \rho + \frac{1}{2}\frac{d}{d\tau}\rho & 0 \\ 0 & 0 & 0 & \rho + \frac{1}{2}\frac{d}{d\tau}\rho \end{pmatrix}$$

with $\rho(\tau) = \dfrac{\rho_0}{\tau^{4/3}} - \dfrac{2\eta_0}{\tau^2}$

The hydrodynamics is input, but we obtain more from its holographic dual.

Caution

The input (hydro.) is valid only at the late time (slowly varying region).

We employ the late time approximation:

$$\tau \to \infty, \text{ with } \frac{z}{\tau^{1/3}} \equiv v \text{ fixed}$$

Janik-Peschanski
hep-th/0512162

$g_{\tau\tau}, g_{yy}\tau^{-2}, g_{xx}$ have the structure of

$$f^{(1)}(v) + \eta_0 f^{(2)}(v)/\tau^{2/3} + f^{(3)}(v)/\tau^{4/3} + \ldots\ldots$$

We discard the higher-order terms.

Solving the Einstein's equation

$$g_{\tau\tau} = -1 + 3a$$

$$-\frac{2\eta_0 v^4}{\tau^{2/3}}\left(1\right.$$

$$\frac{g_{yy}}{\tau^2} = 1 + a$$

$$-\frac{2\eta_0 v^4}{\tau^{2/3}}\left(1\right.$$

$$g_{xx} = 1 + a$$

$$a \equiv \frac{\rho_0 v^4}{3}$$

Solving the Einstein's equation

$$g_{\tau\tau} = -1 + 3a - 4a^2 + 4a^3 - 4a^4 + 4a^5 - 4a^6 + 4a^7 + ...$$

$$- \frac{2\eta_0 v^4}{\tau^{2/3}} \left(1 + \frac{4a}{3} \left(-2 + 3a - 4a^2 + 5a^3 - 6a^4 + 7a^5 + ... \right) \right)$$

$$\frac{g_{yy}}{\tau^2} = 1 + a$$

$$- \frac{2\eta_0 v^4}{\tau^{2/3}} \left(1 + \frac{2a}{3} \left(1 + \frac{a}{3} + \frac{a^2}{3} + \frac{a^3}{5} + \frac{a^4}{5} + \frac{a^5}{7} + ... \right) \right)$$

$$g_{xx} = 1 + a$$

$$+ \frac{2\eta_0 v^4}{\tau^{2/3}} \frac{a}{3} \left(1 + \frac{a}{3} + \frac{a^2}{3} + \frac{a^3}{5} + \frac{a^4}{5} + \frac{a^5}{7} + ... \right)$$

$$a \equiv \frac{\rho_0 v^4}{3}$$

The metric can be re-summed to be:

$$g_{\tau\tau} = -\frac{(1-\frac{\rho_0}{3}v^4)^2}{1+\frac{\rho_0}{3}v^4} + \frac{\eta_0 v^4}{3\tau^{2/3}} \frac{(1-\frac{\rho_0}{3}v^4)(3+\frac{\rho_0}{3}v^4)}{(1+\frac{\rho_0}{3}v^4)^2}$$

$$g_{xx} = 1+\frac{\rho_0}{3}v^4 + \frac{\eta_0 v^4}{3\tau^{2/3}}\left(1-\frac{1}{2}\frac{1+\frac{\rho_0}{3}v^4}{\frac{\rho_0}{3}v^4}\log\left(\frac{1+\frac{\rho_0}{3}v^4}{1-\frac{\rho_0}{3}v^4}\right)\right)$$

$$\frac{g_{yy}}{\tau^2} = 1+\frac{\rho_0}{3}v^4 + \frac{\eta_0 v^4}{3\tau^{2/3}}\left(1+\frac{1+\frac{\rho_0}{3}v^4}{\frac{\rho_0}{3}v^4}\log\left(\frac{1+\frac{\rho_0}{3}v^4}{1-\frac{\rho_0}{3}v^4}\right)\right)$$

The late time geometry

$$ds^2 = \frac{1}{z^2}\left[-\frac{(1-\frac{\rho z^4}{3})^2}{1+\frac{\rho z^4}{3}} d\tau^2 \right.$$

$$\left. +(1+\frac{\rho z^4}{3})\left(\frac{1+\frac{\rho z^4}{3}}{1-\frac{\rho z^4}{3}}\right)^{-2\gamma} \tau^2 dy^2 + (1+\frac{\rho z^4}{3})\left(\frac{1+\frac{\rho z^4}{3}}{1-\frac{\rho z^4}{3}}\right)^{\gamma} dx_\perp^2 \right] + \frac{dz^2}{z^2}$$

$$\gamma = \frac{\eta_0}{\rho_0 \tau^{2/3}}, \qquad \rho(\tau) = \frac{\rho_0}{\tau^{4/3}} - \frac{2\eta_0}{\tau^2}$$

This is correct up to the order of $O(\gamma^2)$.

This looks to be a **black hole** with **time-dependent horizon**.

More about the consistency

From gravity:

$$S = \frac{A}{4G} = \left(\frac{N_c}{2\pi}\right)^{1/4}\left(\frac{\pi}{3}\right)^{3/4} 2\sqrt{2}\,\rho_0^{\;3/4}\left(1 - \frac{3}{2}\frac{\eta_0}{\rho_0 \tau^{2/3}} + O(\tau^{-4/3})\right)$$

From hydrodynamics:

$$S = S_\infty\left(1 - 2\frac{\eta_0}{S_\infty T_0}\tau^{-2/3} + O(\tau^{-4/3})\right)$$

$$T(\tau) = T_0 / \tau^{1/3} + \ldots\ldots\ldots$$

$$\rho(\tau) = \rho_0 / \tau^{4/3} + \ldots\ldots\ldots$$

$$S_\infty = \frac{4}{3}\frac{\rho_0}{T_0} = \frac{4}{3}\frac{\rho\tau}{T}\bigg|_{\tau=\infty}$$

Thermal equiv.

Correct relationship from thermodynamics

More about the late time limit

$$\tau \to \infty, \text{ with } \frac{z}{\tau^{1/3}} \equiv v \text{ fixed}$$

The position of the horizon:

$$z_H = (3/\rho)^{1/4} \approx \tau^{1/3}$$

On the (τ, v) coordinate, the position of the horizon is constant. We are keeping our eyes close to the horizon along the time evolution.

The above limit is valid around the horizon.

If we take the limit with fixing z, we cannot see the horizon.

Supplement on the bulk singularity

Solution:

If $\eta = 0,$

$$\rho(\tau) = \frac{\rho_0}{\tau^{4/3}}$$

Bjorken scaling

Bjorken, PRD27(1983)140

In fact, AdS/CFT says $s = 4/3$ for

$$\rho(\tau) = \frac{\rho_0}{\tau^{s}}$$ if no viscosity.

Janik-Peschanski
hep-th/0512162

Obtained as a condition for **absence of singularity** in the gravity-dual geometry.

Janik-Peschanski analyzed the **late-time geometry** under the limit of

Janik-Peschanski
hep-th/0512162

$$\tau \to \infty, \text{ with } \frac{z}{\tau^{s/4}} \equiv v \text{ fixed}$$

Definition of s:

$$T_{\tau\tau} = \rho(\tau) = \rho_0 / \tau^s$$

R^2_{5d} is non-singular iff $s = 4/3.$

$s = 4/3$ case, the late-time 5d geometry is a AdS-BH with time-dependent horizon:

$$ds^2 = \left[-\frac{\left(1 - \frac{A}{3}\frac{z^4}{\tau^{4/3}}\right)^2}{1 + \frac{A}{3}\frac{z^4}{\tau^{4/3}}} d\tau^2 + \left(1 + \frac{A}{3}\frac{z^4}{\tau^{4/3}}\right)(\tau^2 dy^2 + dx_\perp^2) \right] + \frac{dz^2}{z^2}$$

Hawking temperature $\propto \tau^{-\frac{1}{3}} \propto \rho^4$

Consistent with Bjorken hydrodynamics

$$\rho(\tau) = \frac{\rho_0}{\tau^{4/3}} + \frac{4\eta_0}{1-3\beta}\frac{1}{\tau^{1+\beta}} \xrightarrow{\tau\to\infty} \frac{4\eta_0}{1-3\beta}\frac{1}{\tau^{1+\beta}} \quad \text{if } \beta < \frac{1}{3}.$$

This asymptotic behavior is **forbidden** since $1+\beta \neq \frac{4}{3}.$

AdS/CFT says: $\beta > \frac{1}{3}.$

Key point:

The arguments in the hydrodynamics does not strongly rely on the details of the interaction of the fluid.

⇩

The results can be applicable even for **non-susy** fluid, as far as it has the same equation of state.

Actually, this constraint is **consistent**:

In the **static** N=4 SYM system

$\eta \propto T^3$ from AdS/CFT

Polocastro-Son-Starinets
hep-th/0104066

Together with $T \propto (\tau^{-1/3} + ...)$

$$\eta(\tau) \propto 1/\tau \qquad \left(\beta = 1 > \frac{1}{3}\right)$$

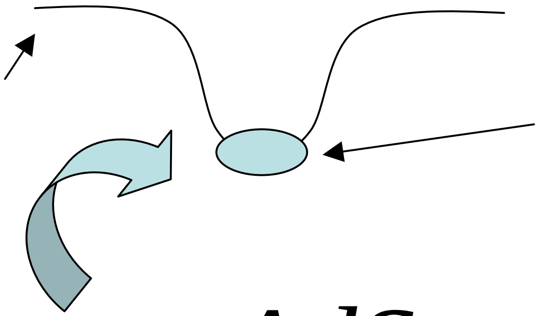
will be the most natural behavior of the shear viscosity in the **non-static** system at the late time.

Questions on the regularity

- How much the regularity gives constraint?
- How much can we justify the regularity argument?

Superstring theory around black 3-brane geometry

asymptotically flat

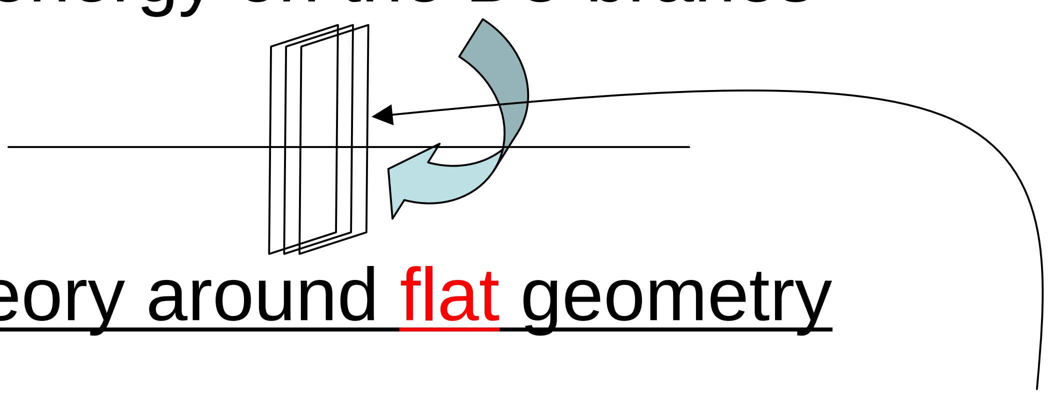


charged black hole
(3+1 dim. object)

The near horizon limit : $AdS_5 \times S^5$

II ?

U(Nc) 3+1 dim N=4 Super YM theory
at low energy on the D3-branes



Superstring theory around flat geometry

+ source term (Nc D3-brane)