Constant mode
in charmonium correlation functions

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This is based on the Phys. Rev. D75 094502 (2007)
[hep-lat/0701005]

Thermal Field Theory and their applications
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Introduction

J/ψ suppression is one of the most promising probe to find the QGP formation in HIC experiment.

Lattice QCD studies of charmonium spectral function suggest the survival of J/ψ state above T_c (1.5T_c?)

- Indirect (sequential) J/ψ suppression

\[
\text{total yield of J/ψ} = \\
\text{direct production of J/ψ (60%)} + \text{decay from higher states, } ψ' & χ_c (40%) \\
\]


⇒ A part of the J/ψ suppression may be observed at \( T_{\text{dis.}}(ψ' \text{ or } χ_c) \) when \( T_{\text{dis.}}(ψ' \text{ or } χ_c) < T_{\text{dis.}}(J/ψ) \)
**Lattice QCD results**

**Lattice setup**

- Quenched approximation (no dynamical quark effect)
- Anisotropic lattices (tadpole imp. Clover quark + plaq. gauge)
  - lattice size: \(20^3 \times N_t\)
  - lattice spacing: \(1/a_s = 2.03(1)\) GeV,
  - anisotropy: \(a_s/a_t = 4\)
- Quark mass
  - charm quark (tuned with J/ψ mass)
  - \(r_s=1\) to reduce cutoff effects in higher energy states

\[ F. \text{Karsch et al., PRD68, 014504 (2003).} \]

<table>
<thead>
<tr>
<th>(N_T)</th>
<th>160</th>
<th>32</th>
<th>26</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T/T_C)</td>
<td>≈ 0</td>
<td>0.88</td>
<td>1.08</td>
<td>1.4</td>
</tr>
<tr>
<td># of conf.</td>
<td>60</td>
<td>300</td>
<td>300</td>
<td>300</td>
</tr>
</tbody>
</table>

- equilib. is 20K sweeps
- each config. is separated by 500 sweeps
Quenched QCD at $T>0$

- Small change in S-wave states
  - Survival of $J/\psi$ & $\eta_c$ at $T>T_c$
- Drastic change in P-wave states
  - Dissociation of $\chi_c$ just above $T_c$ (?)

S. Datta et al., PRD69, 094507 (2004). etc...
A constant mode

Now we consider the meson correlator with $p=0$ & $m_{q1}=m_{q2}$

\[ \exp(-m_q t) \times \exp(-m_q t) = \exp(-2m_q t) \]

$m_q$ is quark mass or single quark energy

\[ \exp(-m_q t) \times \exp(-m_q(L_t-t)) = \exp(-m_q L_t) \]

$L_t$ = temporal extent

- in imaginary time formalism
  
  $L_t = 1/\text{Temp.}$

  gauge field : periodic b.c.
  quark field : anti-periodic b.c.

- in confined phase: $m_q$ is infinite
  
  the effect appears only in deconfined phase

Pentaquark (KN state):
  two pion state:
    $\rightarrow$ Dirichlet b.c.

c.f. T.T.Takahashi et al.,
Physical interpretation

Spectral function at high temp. limit

\[
\rho_{\Gamma}(\omega) = \Theta(\omega^2 - 4m_q^2) \frac{N_c}{8\pi\omega} \sqrt{\omega^2 - 4m_q^2} [1 - 2n_{\Gamma}(\omega/2)] \\
\times [\omega^2(a_H^{(1)} - a_H^{(2)}) + 4m^2(a_H^{(2)} - a_H^{(3)})] \\
+ 2\pi\omega\delta(\omega) N_c [(a_H^{(1)} + a_H^{(2)}) l_1 + (a_H^{(2)} - a_H^{(3)}) l_2]
\]

<table>
<thead>
<tr>
<th>$\Gamma$</th>
<th>$a_H^{(1)} + a_H^{(2)}$</th>
<th>$a_H^{(2)} - a_H^{(3)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS \gamma_5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>V \gamma_i</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>S \gamma_i 1</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>Av \gamma_5 \gamma_5</td>
<td>2</td>
<td>-4</td>
</tr>
</tbody>
</table>

constant mode remains in the continuum & infinite volume

The constant term is related to some transport coefficients. From Kubo–formula, for example, a derivative of the SPF in the V channel is related to the electrical conductivity $\sigma$.

\[
\sigma = \frac{1}{6} \frac{\partial}{\partial \omega} \rho_V(\omega) \bigg|_{\omega=0}
\]
Without constant mode

We would like to investigate thermal effects of SPFs at the region $\omega \gg T$

bound state peaks exist or not? in QGP phase

from "An Introduction to Quantum Field Theory"
Removing the constant mode

An analysis to avoid the constant mode

Midpoint subtracted correlator

\[ \tilde{C}(t) = C(t) - C(N_t/2) \]

\[ \frac{\tilde{C}(t)}{\tilde{C}(t+1)} = \frac{\sinh^2 \left[ \frac{1}{2} m_{\text{sub}}^2(t)(N_t/2 - t) \right]}{\sinh^2 \left[ \frac{1}{2} m_{\text{eff}}^2(t)(N_t/2 - t - 1) \right]} \]

Free quarks

Thermal 2007  
T. Umeda (Tsukuba)
Midpoint subtraction analysis

\[
\bar{C}(t) = C(t) - C(N_t/2) \quad \bar{C}(t+1) = \sinh^2 \left[ \frac{1}{2} m_{\text{eff}}^{\text{sub}}(t)(N_t/2 - t) \right] \
\]

The drastic change in P-wave states disappears in \( m_{\text{eff}}^{\text{sub}}(t) \) \( \Rightarrow \) the change is due to the constant mode

usual effective masses at \( T > 0 \)

subtracted effective mass
Results with extended op.

- extended op. enhances overlap with const. mode
- small constant effect is visible in V channel
- no large change above $T_c$ in $m_{\text{eff}}^{\text{sub}}(t)$
**Conclusion**

- There is the constant mode in charmonium correlators above $T_c$
- The drastic change in $\chi_c$ states is due to the constant mode
  - the survival of $\chi_c$ states above $T_c$, at least $T=1.4T_c$.

The result may affect the scenario of $J/\psi$ suppression.
Conclusion

- There is the constant mode in charmonium correlators above $T_c$.
- The drastic change in $\chi_c$ states is due to the constant mode.
  - the survival of $\chi_c$ states above $T_c$, at least $T=1.4T_c$.

The result may affect the scenario of $J/\psi$ suppression.

In the MEM analysis, one has to check consistency of the results using, e.g., midpoint subtracted correlators.

\[
\tilde{C}(t) = C(t) - C(N_t/2)
\]

\[
\tilde{C}(t) = \int_0^\infty d\omega \rho_T(\omega) K^{\text{sub}}(\omega, t),
\]

\[
K^{\text{sub}}(\omega, t) = \frac{\sinh^2(\frac{\omega}{2}(N_t/2 - t))}{\sinh(\omega N_t/2)}
\]
MEM analysis fails?

MEM test using T=0 data

A.Jakovac et al., PRD75, 014506 (2007).
(also S. Datta et al., PRD69, 094507 (2004).)

FIG. 19: The scalar spectral function for $\beta = 6.1$ at $T = 1.16 T_c$ and at zero temperature reconstructed using $N_{\text{data}} = 12$. At finite temperature two default models $m(\omega) = 0.01$ and $m(\omega) = 0.038\omega^2$ have been used.

MEM sometimes fails when (# or quality) of data point is not sufficient.