Quark spectrum in the ladder gauge theory

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arXiv:0708.3351
Introduction

RHIC suggests strong coupling!

It is important to include non-perturbative effects.

We study the quark spectrum above $T_c$.  

Collective excitation

Kapusta and Gale "Finite-temperature Field Theory"
Le Bellac "Thermal Field Theory"

Thermal mass ; chirally invariant mass

$$-iS_R(p_0, \vec{p}) = \frac{1}{2}(\gamma_0 - \bar{\gamma} \cdot \vec{p}) + \frac{1}{2}(\gamma_0 + \bar{\gamma} \cdot \vec{p})$$

Dispersion: $D_{\pm}(p_0, p) = 0$

Hard Thermal Loop (HTL) approximation ($g << 1$)

$$M^2 = \frac{g^2 T^2}{6}$$

E: plasmino
Introduction

Typical studies of the thermal mass or quark spectrum near $T_c$

- **Lattice QCD (quench) + MEM** Petreczky et al. Nucl.phys.proc.suppl106(2002)513
  \[ \frac{M}{T} = 3.9 \pm 0.2 \quad (T = 1.5T_c) \]
  plasmino was not seen

- **Lattice QCD (quench) + 2 pole fit** Karsch and Kitazawa arXiv:0708.0299
  \[ \frac{M}{T} = 0.800(15) \quad (T = 1.5T_c) \]

  fluctuation effects, (respect $\chi_{\text{sym}}$) 3 peak

- **Yukawa model** Kitazawa,Kunihiro,Nemoto Prog.Theor.Phys.117(2007)103
  (massive boson) 3 peak
  (massive fermion)
  Mitsutani, Kitazawa, Kunihiro, Nemoto arXiv:0704.1710

- **Brueckner-type many-body scheme** Mannarelli, Rapp Phys.Rev.C 72(2005)064905
  heavy quark potential (3D) (non-relativistic treatment)
  plasmino was not studied

- **Our approach**
  Schwinger-Dyson equation (gauge theory, respect $\chi_{\text{sym}}$)
  We study the quark spectrum over a wide range of $g$. 

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Schwinger-Dyson Equation

Ladder Schwinger-Dyson equation at finite $T$ (Imaginary time formalism)

\[
S^{-1}(i\omega_n, \vec{p}) - S_{\text{free}}^{-1}(i\omega_n, \vec{p}) = -C_F g^2 T \sum_{m=-\infty}^{\infty} \int \frac{d^3 k}{(2\pi)^3} \gamma_\mu S(i\omega_m, \vec{k}) \gamma_\nu D^{\mu\nu}_{\text{free}}(i\omega_n - i\omega_m, \vec{p} - \vec{k})
\]

fermion propagator at finite $T$ (chiral limit)

\[
S(i\omega_n, \vec{p}) = \frac{1}{B(i\omega_n, p) + A(i\omega_n, p) \vec{p} \cdot \vec{\gamma} - \frac{C(i\omega_n, p) i\omega_n \gamma_0}{2}}
\]

free gauge boson propagator (Feynman gauge)

\[
D^{\mu\nu}_{\text{free}}(i\omega_n - i\omega_m, \vec{p} - \vec{k}) = \frac{g^{\mu\nu}}{(i\omega_n - i\omega_m)^2 - \vec{p} \cdot \vec{k}}
\]

- We determine the $T_c$ by solving the Schwinger-Dyson Equation in the imaginary time formalism.
- We derive spectral function through the analytic continuation to Minkowski space at $T > T_c$
Schwinger-Dyson Equation

Analytic continuation

Marsiglio et al. Phys.Rev.B 37, 4965

SDE in the imaginary time formalism

\[ S^{-1}(i\omega_n, \vec{p}) - S^{-1}_{\text{free}}(i\omega_n, \vec{p}) = -C_F g^2 T \sum_{m=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \gamma_\mu S(i\omega_m, \vec{p}) \gamma_\nu D^\mu_\nu \text{free}(i\omega_n - i\omega_m, \vec{p} - \vec{k}) \]

Spectral representation of the fermion propagator and gauge boson propagator

\[
\begin{align*}
S(i\omega_m, \vec{k}) &= -\int_{-\infty}^{\infty} dz \frac{\rho_F(z, \vec{k})}{i\omega_m - z} = -\frac{1}{\pi} \int_{-\infty}^{\infty} dz \frac{\text{Im} S_R(z, \vec{k})}{i\omega_m - z} \\
D_{\mu\nu}(i\omega_n - i\omega_m, \vec{p} - \vec{k}) &= -\int_{-\infty}^{\infty} dz \frac{\rho_B^{\mu\nu}(z, \vec{p} - \vec{k})}{(i\omega_n - i\omega_m) - z}
\end{align*}
\]

We perform Matsubara summation

\[ i\omega_n \rightarrow p_0 + i\epsilon \]

Self-consistent equation for retarded propagator

\[
\begin{align*}
iS_R^{-1}(p_0, \vec{p}) - iS_R^{-1}_{\text{free}}(p_0, \vec{p}) &= -C_F g^2 T \sum_{m=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} dz \gamma_\mu \left[ \frac{S(i\omega_m, \vec{k})}{p_0 - z - i\omega_m} \right] \gamma_\nu \rho_B^{\mu\nu}(z, \vec{p} - \vec{k}) \\
&\quad + C_F g^2 \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} dz \gamma_\mu iS_R(p_0 - z, \vec{k}) \gamma_\nu \rho_B^{\mu\nu}(z, \vec{p} - \vec{k}) \frac{1}{2} \left[ \tanh \frac{p_0 - z}{2T} + \coth \frac{z}{2T} \right]
\end{align*}
\]
Numerical results at the weak coupling

\[ g = 0.1 \quad \frac{T}{\Lambda} = 0.3 \]

\[ -iS_R(p_0, p) = \frac{1}{2D_+(p_0, p)} (\gamma_0 - \vec{\gamma} \cdot \vec{p}) + \frac{1}{2D_-(p_0, p)} (\gamma_0 + \vec{\gamma} \cdot \vec{p}) \]

\[ \rho_{\pm}(p_0, p) = -\frac{1}{\pi} \text{Im} \frac{1}{D_{\pm}(p_0, p)} \]

These results almost coincide with the ones obtained by HTL approximation! These agreement implies the validity of our analysis!
As the coupling increases, the peak position moves up toward the high energy region and peaks become broad.
In the strong coupling region, the spectral function is already broad at one-loop. Owing to non-perturbative effects, the peak is even broader and thermal mass is reduced.
Coupling dependence of the thermal mass

The thermal mass is reduced owing to non-perturbative effects.

for small $g$: $M$ is proportional to $g$

for large $g$: $M$ does not depend on $g$

$M/T \sim 1$
The ratio $M/T$ is almost constant, $M \sim T$, for $T/\Lambda \lesssim 0.4$. 

**Temperature dependence of the thermal mass**

for fixed coupling $g = 6$
Structure of the spectral function

Spectral function at \( p=0 \) (\( g=3 \))

Spectral function has 3 peak structure in some parameter region!

SDE

\[
\begin{align*}
\left[ - \quad \right]^{-1} - \left[ - \quad \right]^{-1} &= \\
\left( \quad \right) &= \\
\end{align*}
\]

free gauge boson (in this work)

fermion with thermal mass

Yukawa model


Mitsutani, Kitazawa, Kunihiro, Nemoto arXiv:0704.1710

massless fermion

massive boson

massive fermion (Dirac mass)
3 peak structure

\[
\Sigma(i\omega_n, \vec{p}) = -C_F g^2 T \sum \int \frac{d^3 k}{(2\pi)^3} \gamma_\mu S_{\text{HTL}}(i\omega_m, \vec{k}) \gamma_\nu D_{\text{free}}^{\mu\nu}(i\omega_n - i\omega_m, \vec{p} - \vec{k})
\]

\[
\rho^{\text{HTL}}(p_0, p) = Z_{\pm}(p) \delta(p_0 - \omega_{\pm}(p)) + Z_{\mp}(p) \delta(p_0 + \omega_{\pm}(p)) + \rho_0^{\pm}(p_0, p)
\]

\[
\rho_{\pm}(p_0, 0) = -\frac{1}{\pi} \text{Im} \frac{1}{p_0 - \Sigma_0(p_0, 0)}
\]

\[
\Sigma_0(i\omega_n, 0) = \frac{1}{4} \text{tr}[\gamma_0 \Sigma(i\omega_n, 0)]
\]

**Imaginary part of \(\Sigma_0\)**

**Spectral function at rest \(p = 0\)**

\(g = 3, \ T/\Lambda = 0.4\)
• We derived fermion spectral function by performing analytic continuation for the solution of the Schwinger-Dyson equation to Minkowski space.

• weak coupling: 2 sharp peaks
  Dispersion: SDE ≈ HTL
  \( M / T \approx g \) for small \( g \)

• strong coupling: 2 broad peaks
  (quasi-particle picture in the low momentum region is unclear)
  \[ M / T \sim 1 \]
  \( M \sim T \) is consistent with other model analyses.

• We can see 3-peak structure even in our SDE analysis (massless gauge boson) owing to non-perturbative effects.
Summary

- We studied the quark spectrum above the critical temperature of the chiral phase transition from the Schwinger-Dyson equation, in which we performed an analytic continuation of the solution for the imaginary time axis to the real time axis with a method of an integral equation.

- In the weak coupling region \((g \ll 1)\), the quark spectrum is quite similar to that in the hard thermal loop (HTL) approximation:
  - 2 sharp peaks, thermal mass is almost same

- In the strong coupling region \((g > 1)\), we find that there exist the normal quasi quark and the plasmino as in the weak coupling region. However, their spectra deviate from those in the HTL approximation:
  - 2 broad peaks, \( M \) saturates at large gauge coupling \((M \sim T)\)

- We can see that the appearance of a three-peak structure in the quark spectrum as a non-perturbative effect in some parameter region.