On the statistical properties of the energy transfer between two stochastic systems coupled to different thermal baths

- Two coupled electric circuits
- Two Brownian particles

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Outline

• Motivation for experiments on energy transfer
• Two circuits coupled by thermal fluctuations
• The experimental set up
• Statistical properties of the energy transfer
• The mechanical equivalence
• Two coupled Brownian particles
• The difference and analogies between the electric system
• The transient FT
• Conclusions
On the heat flux and entropy produced by thermal fluctuations in electric circuits

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On the heat flux between two reservoirs at different temperatures

A) In the stationary case for the heat flux between two reservoirs at different temperatures

\[ \ln \frac{P(Q_\tau)}{P(-Q_\tau)} = \left( \frac{1}{T_C} - \frac{1}{T_H} \right) \frac{Q_\tau}{k_B} \]

Theory: no experiments

The Nyquist problem

In 1928 well before Fluctuation Dissipation Theorem (FDT), this was the second example, after the Einstein relation for Brownian motion, relating the dissipation of a system to the amplitude of the thermal noise.

\[ |\tilde{\eta}|^2 = 4k_BRT \]
What are the consequences of removing the Nyquist equilibrium conditions?

What are the statistical properties of the energy exchanged between the two conductors kept at different temperature?

We analyse these questions in an electric circuit within the framework of FT.
What are the consequences of removing the Nyquist equilibrium conditions?

What are the statistical properties of the energy exchanged between the two conductors kept at different temperature?

We analyse these questions in an electric circuit within the framework of FT.

How the variance of $V_1$ and $V_2$ are modified because of the heat flux?

What is the role of correlation between $V_1$ and $V_2$?
Electric Circuit

$T_1$ is changed with a nitrogen vapor circulation

$T_2 = 296K$ is kept fixed

$C$ is the coupling capacitance = 100pF and 1000pF

$C_1$ and $C_2$ are the cable and amplifier capacitances $\approx 500pF$

$R_1 = R_2 = 10M\Omega$

$\tau_o \approx 0.01s$
Electric Circuit and the mechanical equivalent

\( T_1 \) is changed with a nitrogen vapor circulation

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$R_1 = R_2 = 10M\Omega$

$\tau_o \approx 0.01s$
Joint probability of $V_1$ and $V_2$

\[
\log_{10} P(V_1, V_2) \quad \text{at } T_1 = T_2 = 296k
\]

\[
\log_{10} P(V_1, V_2) \quad \text{at } T_1 = 88K \text{ and } T_2 = 296K
\]
Electric Circuit and the dissipated energy

\[ \dot{Q}_m = V_m i_m \]

Power dissipated in the resistance \( m=1,2 \)

\[ i_m = i_C - i_{C_m} \]

\[ i_{C_m} = C_m \frac{dV_m}{dt} \]

\[ i_C = C \frac{d(V_2 - V_1)}{dt} \]

\[ \dot{Q}_m = V_m i_m = \frac{V_m}{R_m} (V_m - \eta_m) = V_m \left[ (C_m + C) \dot{V}_m - C \dot{V}_{m'} \right] \]

\( m' = 2 \) if \( m = 1 \), and \( m' = 1 \) if \( m = 2 \)
Electric Circuit and the dissipated energy

Power dissipated in the resistance $m=1,2$

$$\dot{Q}_m = V_m i_m = V_m [(C_m + C)\dot{V}_m - CV_m']$$

$m' = 2$ if $m = 1$, and $m' = 1$ if $m = 2$

Integrating on a time $\tau$

$$Q_{m,\tau} = W_{m,\tau} - \Delta U_{m,\tau}$$

$$Q_{m,\tau} = \int_{t}^{t+\tau} i_m V_m \, dt$$ heat flowed in the time $\tau$

from reservoir $m'$ to reservoir $m$

$$W_{m,\tau} = \int_{t}^{t+\tau} CV_m \frac{dV_{m'}}{dt} \, dt$$ work performed by the circuit $m$

on $m'$ in the time $\tau$

$$\Delta U_{m,\tau} = \frac{(C_m + C)}{2} (V_m(t + \tau)^2 - V_m(t)^2)$$ Potential energy change of

the circuit $m$ in the time $\tau$. 
Statistic of the work and heat

$T_1 = T_2 = 296K$
$
\tau = 0.5s$

$T_1 = 88K$
$\tau = 0.1s$

$S(X_m, \tau) = \log \frac{P(X_m, \tau)}{P(-X_m, \tau)} = \Delta \beta \frac{X_m, \tau}{k_B T_2}$

with $\Delta \beta = (T_2 / T_1 - 1)$
On the heat flux and entropy produced by thermal fluctuations

\[
S(X_{m,\tau}) = \log \frac{P(X_{m,\tau})}{P(-X_{m,\tau})} = \Delta \beta \frac{X_{m,\tau}}{k_BT_2}
\]

with \( \Delta \beta = \frac{T_2}{T_1} - 1 \)

\[
T_{fit} = \frac{T_2}{\Delta \beta + 1}
\]
The heat flux as a function of $T_2 - T_1$

\[
\langle \dot{Q}_1 \rangle = A (T_2 - T_1) = \frac{C^2 \Delta T}{XY},
\]

\[
X = C_2 C_1 + C (C_1 + C_2)
\]

\[
Y = [(C_1 + C)R_1 + (C_2 + C)R_2] \quad \text{and} \quad A = C^2/(XY) \]
How the equilibrium variance of $V_1$ and $V_2$ is modified

\[ \sigma_m^2 \text{ is the variance of } V_m \]

\[ \sigma_m^2(T_m, T_{m'}) = \sigma_{m,eq}^2(T_m) + < \dot{Q}_m > R_m \]

\[ \sigma_{m,eq}^2(T_m) = k_B T_m (C + C'_m) / X \]

which is an extension to two temperatures of the Harada-Sasa relation.
On the entropy produced by thermal fluctuations

\[ \Delta S_{r,\tau} = Q_{1,\tau}/T_1 + Q_{2,\tau}/T_2 \]
related to the heat exchanged with the reservoirs

Following Seifert, (PRL 95, 040602, 2005) who developed this concept for a single heat bath, we introduce a trajectory entropy for the evolving system

\[ S_s(t) = -k_B \log P(V_1(t), V_2(t)) \]

and the entropy production on the time \( \tau \)

\[ \Delta S_{s,\tau} = -k_B \log \left[ \frac{P(V_1(t + \tau), V_2(t + \tau))}{P(V_1(t), V_2(t))} \right]. \]

The total entropy is:

\[ \Delta S_{tot,\tau} = \Delta S_{r,\tau} + \Delta S_{s,\tau} \]
Statistical properties of the total entropy

\[ \Delta S_{tot, \tau} = \Delta S_{r, \tau} + \Delta S_{s, \tau} \]

independently of \( \Delta T \) and of \( \tau \), the following equality always holds

\[ \langle \exp(-\Delta S_{tot}/k_B) \rangle = 1 \]
Statistical properties of the total entropy

\[ \langle \exp(-\Delta S_{tot}/k_B) \rangle = 1 \]

implies that \( P(\Delta S_{tot}) \) satisfies a FT

\[ \log\left[ \frac{P(\Delta S_{tot})}{P(-\Delta S_{tot})} \right] = \frac{\Delta S_{tot}}{k_B} \]

\( \forall \tau, \Delta T \)
On the heat flux and entropy produced by thermal fluctuations
Summary of the experimental and theoretical results
“On the heat flux and entropy produced by thermal fluctuations”

- The mean heat flux \( \langle \dot{Q} \rangle \propto (T_2 - T_1) \)
- The pdf of \( W_m / \langle W_m \rangle \) satisfies an asymptotic FT whose prefactor is the entropy production rate \( \langle W_m \rangle (1/T_m - 1/T_{m'}) \).
- The out of equilibrium variance:
  \[
  \sigma_m^2(T_m, T_{m'}) = \sigma_{m, eq}(T_m) + \langle \dot{Q}_m \rangle R_m
  \]
  (Extension of Harada-Sasa relation)
- The total entropy \( \Delta S_{tot} \) satisfies a conservation law which implies the second law and imposes the existence of a FT which is not asymptotic in time.
- \( \Delta S_{tot} \) is rigorously zero in equilibrium, both in average and fluctuations
- The electrical-mechanical analogy makes these results very general and useful
On the heat flux and entropy produced by thermal fluctuations

Theory

$q_m$ is the charge flowed in the resistance $R_m$

\begin{align*}
q_1 &= (V_1 - V_2)C + V_1 C_1 \\
q_2 &= (V_1 - V_2)C - V_2 C_2
\end{align*}

\begin{align*}
R_1 \dot{q}_1 &= -q_1 \frac{C_2}{X} + (q_2 - q_1) \frac{C}{X} + \eta_1 \\
R_2 \dot{q}_2 &= -q_2 \frac{C_1}{X} + (q_1 - q_2) \frac{C}{X} + \eta_2
\end{align*}

\[\langle \eta_i(t) \eta_j(t') \rangle = 2\delta_{ij} k_B T_i R_j \delta(t - t')\]
Electric Circuit and the mechanical equivalent

\[ R_1 \dot{q}_1 = -q_1 \frac{C_2}{X} + (q_2 - q_1) \frac{C}{X} + \eta_1 \]
\[ R_2 \dot{q}_2 = -q_2 \frac{C_1}{X} + (q_1 - q_2) \frac{C}{X} + \eta_2 \]

\[ \langle \eta_i(t) \eta_j(t') \rangle = 2 \delta_{ij} k_B T_i R_j \delta(t - t') \]

\[ X = C_2 C_1 + C (C_1 + C_2) \]

\( q_m \) the displacement of the particle \( m \)

\( i_m \) its velocity

\( K_m = 1/C_m \) the stiffness of the spring \( m \)

\( K = 1/C \) the stiffness of the coupling spring

\( R_m \) the viscosity.
On the heat flux between two particles at two different temperatures

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Energy flow between two hydrodynamically coupled particles kept at different effective temperatures
A. Bérut, A. Petrosyan and S. Ciliberto, EPL, 107 (2014) 60004

Stationary and transient Fluctuation Theorem for effective heat flux between hydrodynamically coupled particles
A. Bérut, A. Imparato, A. Petrosyan and S. Ciliberto, in preparation
Two Brownian particles trapped by two laser beams.
Two Brownian particles trapped by two laser beams.

Difficulty of having an harmonic coupling between the particles. The main source of coupling is hydodynamic (viscous)
Two Brownian particles trapped by two laser beams.

Difficulty of having an harmonic coupling between the particles. The main source of coupling is hydodynamic (viscous)

Difficulty of having two close Brownian particles at two different temperatures

The temperature gradient is done by forcing the motion of one particle with an external random force
Experimental results

Spectra of excited particle

![Graph showing power spectra vs frequency for different temperatures: T_{room} (300 K), 470 K, 790 K, 1330 K, 2100 K, and Lorentzian fit.](image)
\[ \sigma_{11}^2 = \langle x_1 x_1 \rangle \]
\[ \sigma_{12}^2 = \langle x_1 x_2 \rangle \]
\[ \sigma_{22}^2 = \langle x_2 x_2 \rangle \]

Variances and cross variances as a function of the random driving voltage (force) at

\[ d = 3.2 \, \mu m \]
\[ \sigma_{11}^2 = \langle x_1 x_1 \rangle \]
\[ \sigma_{12}^2 = \langle x_1 x_2 \rangle \]
\[ \sigma_{22}^2 = \langle x_2 x_2 \rangle \]

Variances and cross variances as a function of the distance between the beads for a fixed driving of 1.5V
From a suitable hydrodynamic model one can compute the variances

\[
\sigma_{11}^2 = \langle x_1 x_1 \rangle = \frac{k_B(T+\Delta T)}{k_1} - \frac{k_2}{k_1} \frac{\epsilon^2 k_B \Delta T}{k_1 + k_2}
\]
\[
\sigma_{12}^2 = \langle x_1 x_2 \rangle = \frac{\epsilon k_B \Delta T}{k_1 + k_2}
\]
\[
\sigma_{22}^2 = \langle x_2 x_2 \rangle = \frac{k_B T}{k_2} + \frac{\epsilon^2 k_B \Delta T}{k_1 + k_2}
\]

where:

\(\epsilon\) is the coupling coefficient of the particle. It has to depend on the distance but not on the random driving amplitude.

\(\Delta T\) is the temperature difference induced by the random driving.

\(k_1\) and \(k_2\) are the stiffness of the optical traps.
From a suitable hydrodynamic model one can compute the variances

\[
\sigma_{11}^2 = \langle x_1 x_1 \rangle = \frac{k_B(T + \Delta T)}{k_1} - \frac{k_2 \epsilon^2 k_B \Delta T}{k_1 + k_2}
\]

\[
\sigma_{12}^2 = \langle x_1 x_2 \rangle = \frac{\epsilon k_B \Delta T}{k_1 + k_2}
\]

\[
\sigma_{22}^2 = \langle x_2 x_2 \rangle = \frac{k_B T}{k_2} + \frac{\epsilon^2 k_B \Delta T}{k_1 + k_2}
\]

where:

\[\epsilon, T \text{ and } \Delta T \text{ are the unknowns.}\]

\[\epsilon \text{ is the coupling coefficient of the particle.}\]
\[\text{It has to depend on the distance but not on the random driving amplitude.}\]

\[\Delta T \text{ is the temperature difference induced by the random driving.}\]

\[k_1 \text{ and } k_2 \text{ are the stiffness of the optical traps.}\]
Values of the parameters from the experiment

\[ \sigma_{11}^2 = \langle x_1 x_1 \rangle = \frac{k_B (T + \Delta T)}{k_1} - \frac{k_2 \epsilon^2 k_B \Delta T}{k_1 + k_2} \]
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\( \epsilon, T \) and \( \Delta T \) are the unknown
Values of the parameters from the experiment

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\sigma_{22}^2 = \langle x_2 x_2 \rangle = \frac{k_B T}{k_2} + \frac{\epsilon^2 k_B \Delta T}{k_1 + k_2}
\]

Can we interpret the term proportional to \(\Delta T\) as the heat flux between the two particles?
The standard hydrodynamic model

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{pmatrix} = \mathcal{H} \times \begin{pmatrix}
F_1 \\
F_2
\end{pmatrix}
\]

two coupled Langevin equations
coupling Rotne-Prager diffusion tensor

\[
\mathcal{H} = \begin{pmatrix}
1/\gamma & \epsilon/\gamma \\
\epsilon/\gamma & 1/\gamma
\end{pmatrix}
\]

\[
\epsilon = \frac{3R}{2d} - \left(\frac{R}{d}\right)^3
\]

and forces

\[
F_i = -k_i \times x_i + f_i
\]

in equilibrium

\[
\langle f_i(t) \rangle = 0
\]

\[
\langle f_i(t)f_j(t') \rangle = 2k_B T (\mathcal{H}^{-1})_{ij} \delta(t - t')
\]
The standard hydrodynamic model

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\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{pmatrix} = \mathcal{H} \times \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}
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two coupled Langevin equations

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\langle f_i(t)f_j(t') \rangle = 2k_B T (\mathcal{H}^{-1})_{ij} \delta(t-t')
\]

Out of Equilibrium: forcing on bead 1 \( f^* = k_1 x_0(t) \)

\( f^* \) is a delta correlated noise

Bead 1 has an effective temperature \( T^* = T + \Delta T \)
The standard hydrodynamic model

It follows that the system of equations is:

\[
\begin{align*}
    \gamma \dot{x}_1 &= -k_1 x_1 + \epsilon (-k_2 x_2 + f_2) + f_1 + f^* \\
    \gamma \dot{x}_2 &= -k_2 x_2 + \epsilon (-k_1 x_1 + f_1 + f^*) + f_2
\end{align*}
\]
The correlation functions

\[ \langle x_1(t)x_2(0) \rangle \]  

\[ \langle x_1(0)x_2(t) \rangle \]
The standard hydrodynamic model

It follows that the system of equations is:

\[
\begin{align*}
\gamma \dot{x}_1 &= -k_1 x_1 + \epsilon(-k_2 x_2 + f_2) + f_1 + f^* \\
\gamma \dot{x}_2 &= -k_2 x_2 + \epsilon(-k_1 x_1 + f_1 + f^*) + f_2
\end{align*}
\]

comparison with the electric case

\[
\begin{align*}
R_1 \dot{q}_1 &= -q_1 \frac{C_2}{X} + (q_2 - q_1) \frac{C}{X} + \eta_1 \\
R_2 \dot{q}_2 &= -q_2 \frac{C_1}{X} + (q_1 - q_2) \frac{C}{X} + \eta_2
\end{align*}
\]
The standard hydrodynamic model

It follows that the system of equations is:

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\begin{align*}
\gamma \dot{x}_1 &= -k_1 x_1 + \epsilon(-k_2 x_2 + f_2) + f_1 + f^* \\
\gamma \dot{x}_2 &= -k_2 x_2 + \epsilon(-k_1 x_1 + f_1 + f^*) + f_2 
\end{align*}
\]

heat exchanged by the bead \( i \) in the time \( \tau \)

\[
Q_i(\tau) = \int_0^\tau (\gamma \dot{x}_i - \gamma \xi_i) \dot{x}_i \, dt
\]

\[
\xi_1 = \frac{1}{\gamma} (f_1 + \epsilon f_2 + f^*) \\
\xi_2 = \frac{1}{\gamma} (f_2 + \epsilon f_1 + \epsilon f^*)
\]

\[
Q_i(\tau) = k_i q_{ii} + \epsilon k_j q_{ij}
\]

\[
q_{ii} = -\int_0^\tau x_i \dot{x}_i \, dt \\
q_{ij} = -\int_0^\tau x_j \dot{x}_i \, dt
\]
The heat flux

\[ Q_i(\tau) = k_i q_{ii} + \epsilon k_j q_{ij} \]

\[ q_{ii} = - \int_0^\tau x_i \dot{x}_i \, dt \]

\[ q_{ij} = - \int_0^\tau x_j \dot{x}_i \, dt \]

potential energy

\[ \langle q_{ii} \rangle = 0 \]

\[ \langle Q_{i,j} \rangle = \epsilon k_j \langle q_{i,j} \rangle \]
The heat flux

\[ \langle Q_{i,j} \rangle = \epsilon k_{j} \langle q_{i,j} \rangle \]

As for the electric case, one obtains that

\[ \sigma_{i}^{2} - \sigma_{i,\text{equilibrium}}^{2} \propto \langle Q_{i} \rangle \]

but \[ \langle Q_{2,1} \rangle = -\frac{k_{1}}{k_{2}} \langle Q_{1,2} \rangle \]

and

\[ \langle Q_{2,1} \rangle + \langle Q_{1,2} \rangle \neq 0 \]
The Fluctuation Theorem and the effective Temperature

\[ S(Q_{2,1}) = \log \frac{P(Q_{2,1})}{P(-Q_{2,1})} = \Delta \beta_{2,1} \frac{Q_{2,1}}{k_B T_2} \]

with \( \Delta \beta_{2,1} = \frac{k_2}{k_1} (1 - T_2/T_1) \)

\[ S(Q_{1,2}) = \log \frac{P(Q_{1,2})}{P(-Q_{1,2})} = \Delta \beta_{1,2} \frac{Q_{1,2}}{k_B T_2} \]

with \( \Delta \beta_{1,2} = (1 - T_2/T_1) \)
Dependence of $\Delta \beta$ on $\Delta T$

FT is satisfied both for $Q_{2,1}$ and $Q_{1,2}$ but with different $\Delta \beta$
What does it occur when the high temperature is switched on?

This question has been theoretically analyzed in a system with conservative coupling.


The main results of this study is that during the transient the energy flux from the hot reservoir satisfied an FT for any time whereas the FT is satisfied only asymptotically for the heat going into the cold reservoir.

We checked this idea in our system which presents a dissipative coupling.
The transient behavior

Experimental procedure in
the two beads system with $k_1 = k_2$

The two beads are kept at the same temperature $T_1 = T_2$

At $t = 0$ the temperature $T_1$ is suddenly increased by $\Delta T = 330 K$ and it is kept constant for about 1s.

$Q_1$ and $Q_2$ are measured during the transient.

$$Q_i(\tau) = k_i q_{ii} + \epsilon k_j q_{ij}$$

$$q_{ii} = -\int_0^\tau x_i \dot{x}_i \, dt$$

$$q_{ij} = -\int_0^\tau x_j \dot{x}_i \, dt$$

The integrals are computed in the interval $0 < t < \tau$
The transient behavior

Experimental procedure in the two beads system with $k_1 = k_2$

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$Q_1$ and $Q_2$ are measured during the transient.

$$Q_i(\tau) = k_i q_{ii} + \epsilon k_j q_{ij}$$

$$q_{ii} = -\int_0^\tau x_i \dot{x}_i \, dt$$

$$q_{ij} = -\int_0^\tau x_j \dot{x}_i \, dt$$

At $t = 1s$ we set $\Delta T = 0$ and we let the system to relax.

This quenching procedure is repeated 4500 times to construct the statistics of $Q_1$ and $Q_2$ during the transient.
The transient behavior

\[ Q_i(\tau) = k_i q_{ii} + \epsilon k_j q_{ij} \]

\[ q_{ii} = -\int_0^\tau x_i \dot{x}_i \, dt \]

\[ q_{ij} = -\int_0^\tau x_j \dot{x}_i \, dt \]

Theoretical Prediction in the case of conservative coupling


\[ \Sigma(Q_i) = \log \frac{P(Q_i)}{P(-Q_i)} = \Delta \beta_i \frac{Q_i}{k_B T_2} \]

where

\[ \Delta \beta_1 = (1 - T_2/T_1) \] for any time \( \tau \)

\[ \Delta \beta_2 = (1 - T_2/T_1) \] only for \( \tau \to \infty \)

\( Q_1 \) and \( Q_2 \) have a different statistical behavior
The transient behavior

\[ \Sigma(Q_i) = \log \frac{P(Q_i)}{P(-Q_i)} = \Delta \beta_i \frac{Q_i}{k_B T_2} \]

where

\[ \Delta \beta_1 = (1 - T_2/T_1) \text{ for any time } \tau \]
\[ \Delta \beta_2 = (1 - T_2/T_1) \text{ only for } \tau \to \infty \]

\( Q_1 \) and \( Q_2 \) have a different statistical behavior in the case of viscous coupling.
Conclusions on particle interactions

- The difference between out-equilibrium and equilibrium variance is proportional to the heat flux.

- A hydrodynamic model precisely describes the experimental data.

- The FT correctly estimates the effective temperature within experimental errors.

- The definition of heat is doubtful!

- During the transient the FT for the heat has a different statistical behaviors for the cold and the hot sources.