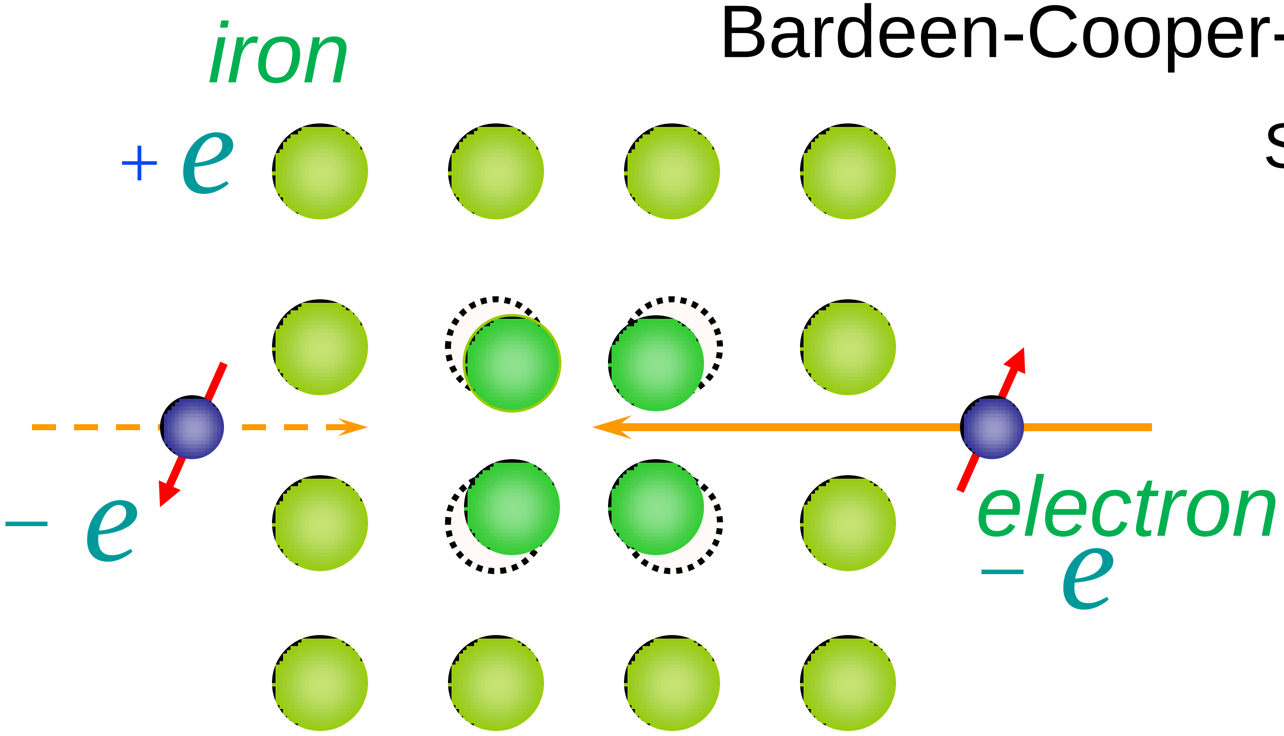
d-wave superconductivity in cuprates

Y. Matsuda

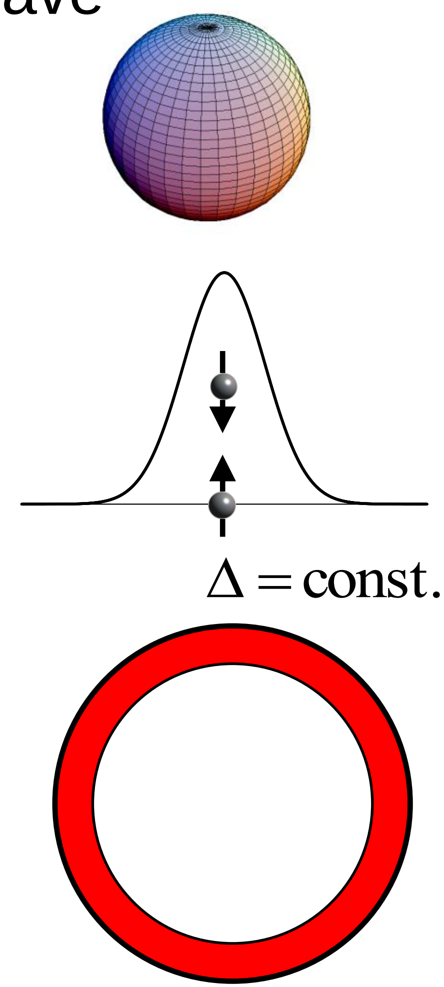
Department of Physics
Kyoto University
Kyoto, Japan

Conventional Superconductor

Bardeen-Cooper-Schrieffer theory



S-wave

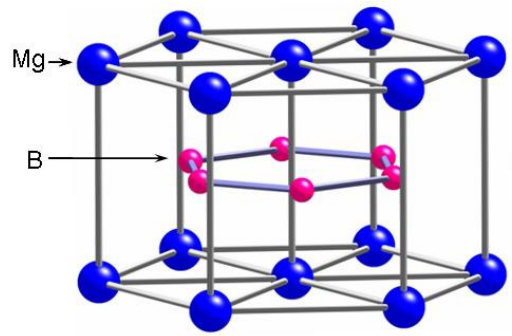


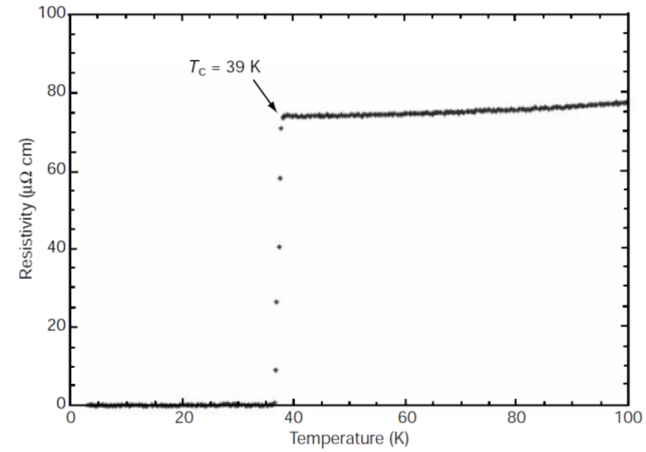
Cooper pair

Attractive electron-phonon interactions

MgB_2 (T_c = 39 K)

J. Nagamatsu *et al*., Nature (2001)





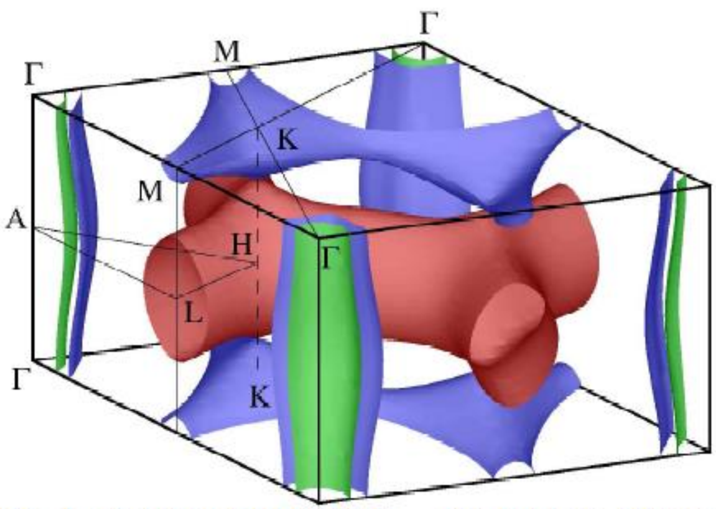
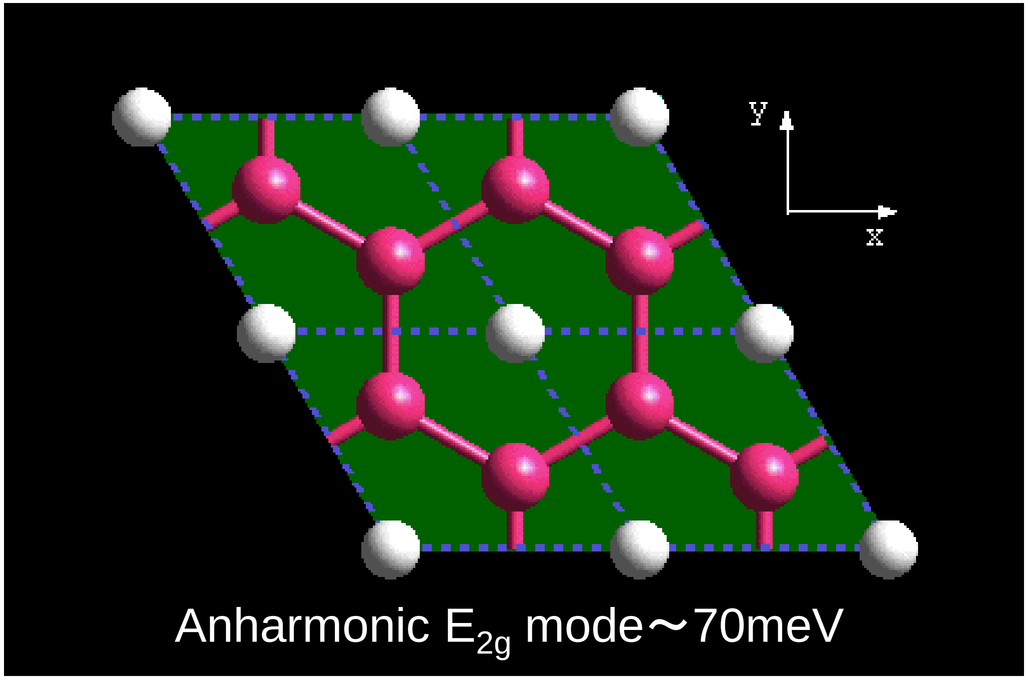


FIG. 1. Fermi surface of MgB_2. The figure is taken from Ref. [5]. Holes in the σ-band form cylinders around the ΓA-line. The π-band has electron and hole pockets located near the H- and K-points, respectively.



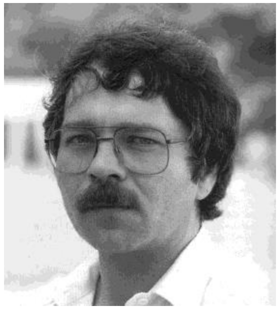
High-T_c cuprates

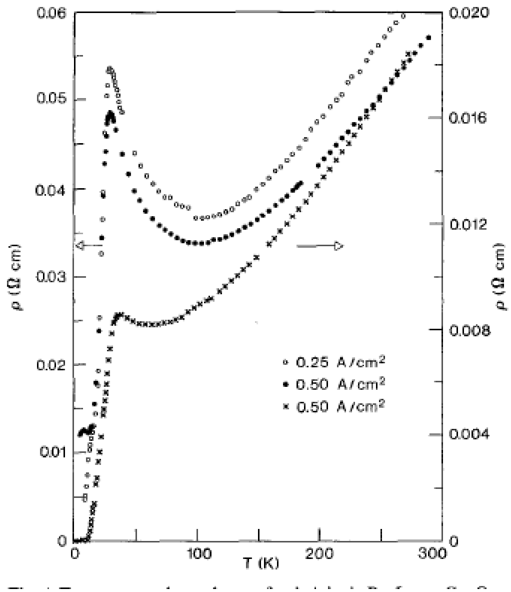
Possible High T_c Superconductivity in the Ba – La – Cu – O System

J.G. Bednorz and K.A. Müller
IBM Zürich Research Laboratory, Rüschlikon, Switzerland

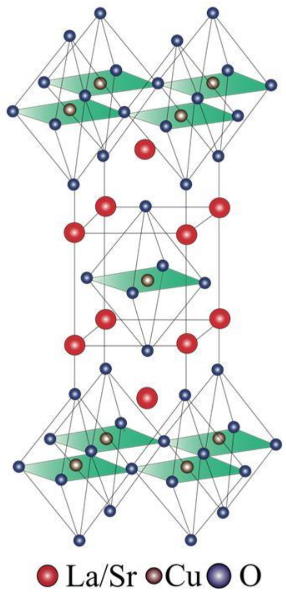
Received April 17, 1986

Metallic, oxygen-deficient compounds in the Ba – La – Cu – O system, with the composition $Ba_xLa_{5-x}Cu_5O_{5(3-y)}$ have been prepared in polycrystalline form. Samples with $x=1$ and 0.75, $y>0$, annealed below 900 °C under reducing conditions, consist of three phases, one of them a perovskite-like mixed-valent copper compound. Upon cooling, the samples show a linear decrease in resistivity, then an approximately logarithmic increase, interpreted as a beginning of localization. Finally an abrupt decrease by up to three orders of magnitude occurs, reminiscent of the onset of percolative superconductivity. The highest onset temperature is observed in the 30 K range. It is markedly reduced by high current densities. Thus, it results partially from the percolative nature, bute possibly also from $2D$ superconducting fluctuations of double perovskite layers of one of the phases present.

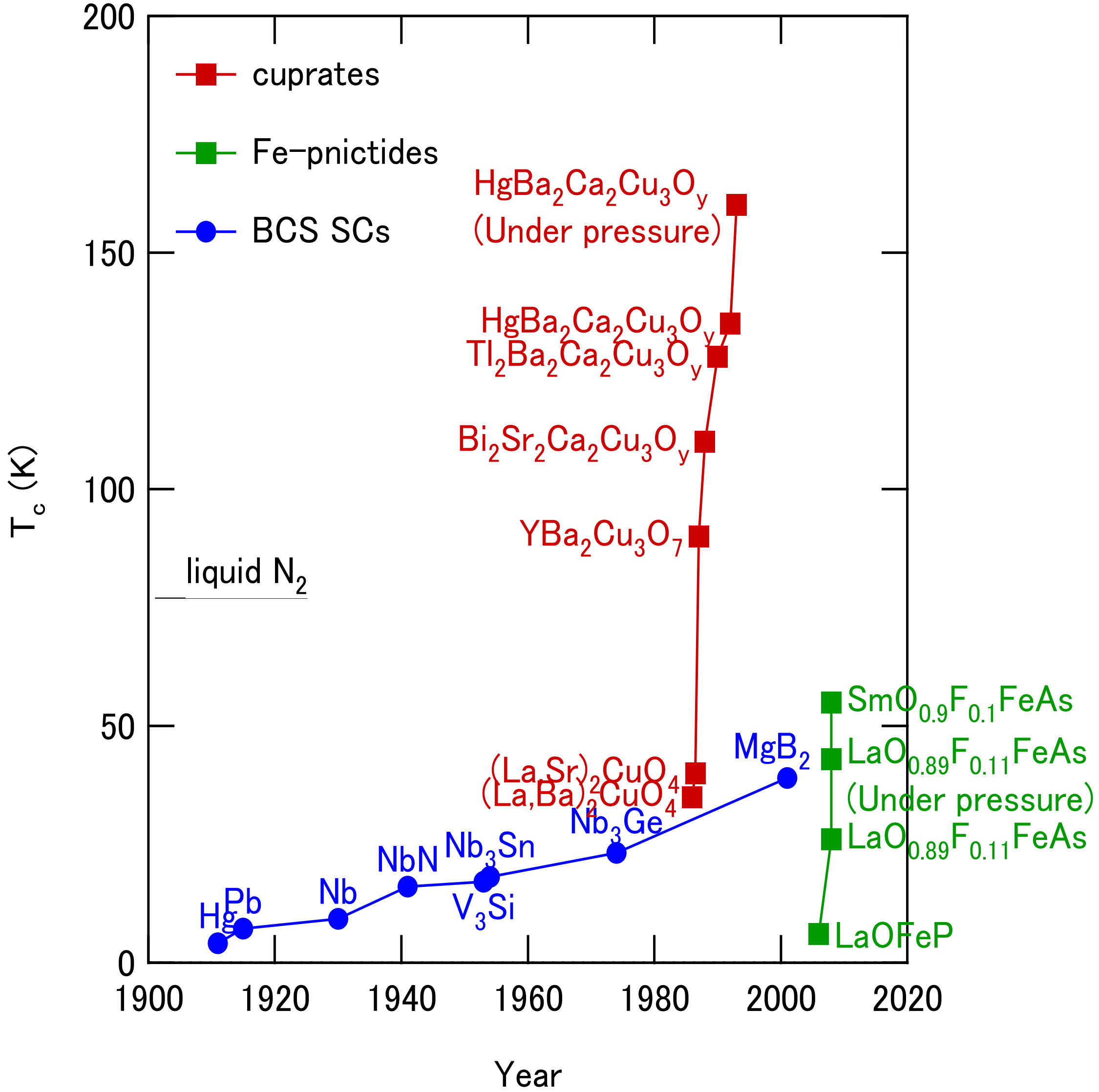




$La_{2-x}Sr_xCuO_4$

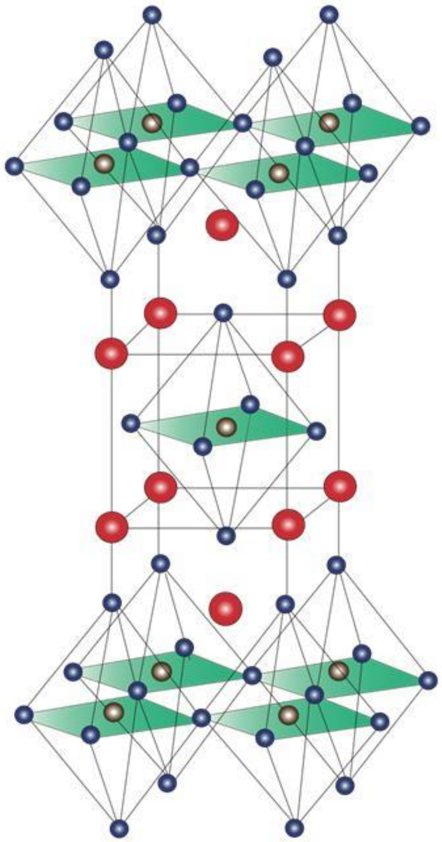


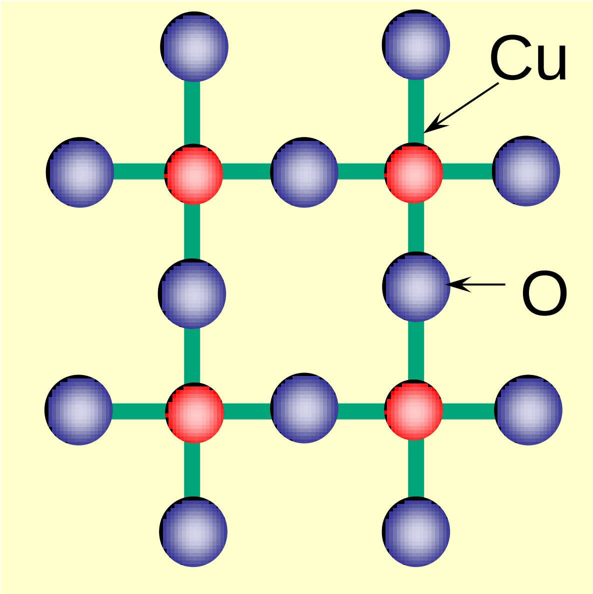
J. G. Bednorz and K.A. Müller, Zeitschrift für Physik B **64**, 189 (1986).



cuprates
Fe-pnictides
BCS SCs
HgBa2Ca2Cu3Oy
(Under pressure)
HgBa2Ca2Cu3Oy
Tl2Ba2Ca2Cu3Oy
Bi2Sr2Ca2Cu3Oy
YBa2Cu3O7
liquid N2
(La,Sr)2CuO4
(La,Ba)2CuO4
MgB2
Nb3Ge
Nb3Sn
NbN
V3Si
Nb
Pb
Hg
SmO0.9F0.1FeAs
LaO0.89F0.11FeAs
(Under pressure)
LaO0.89F0.11FeAs
LaOFeP
Tc (K)
Year
0
50
100
150
200
1900
1920
1940
1960
1980
2000
2020

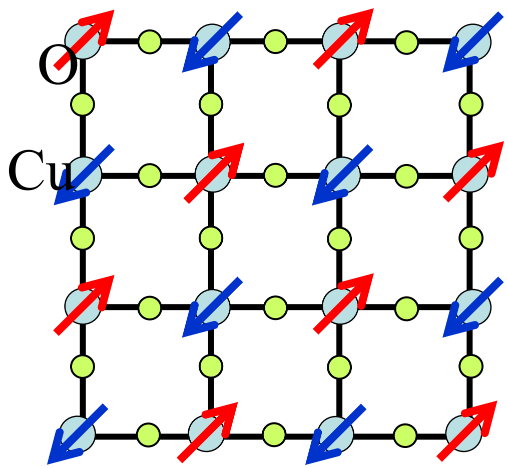
Superconductivity occurs in CuO_2 2D planes





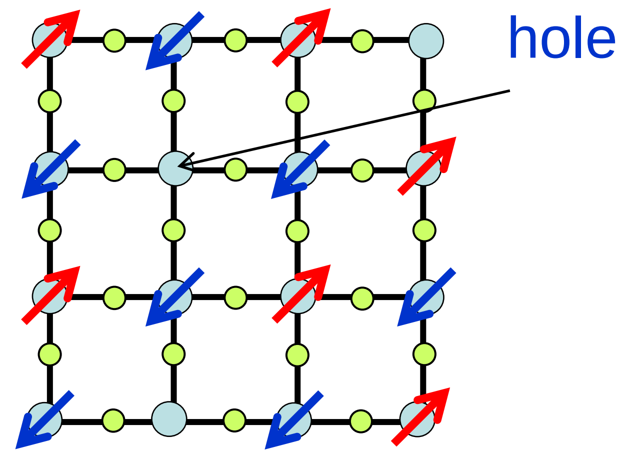
Enhanced fluctuations → suppression of magnetic order

La_2CuO_4

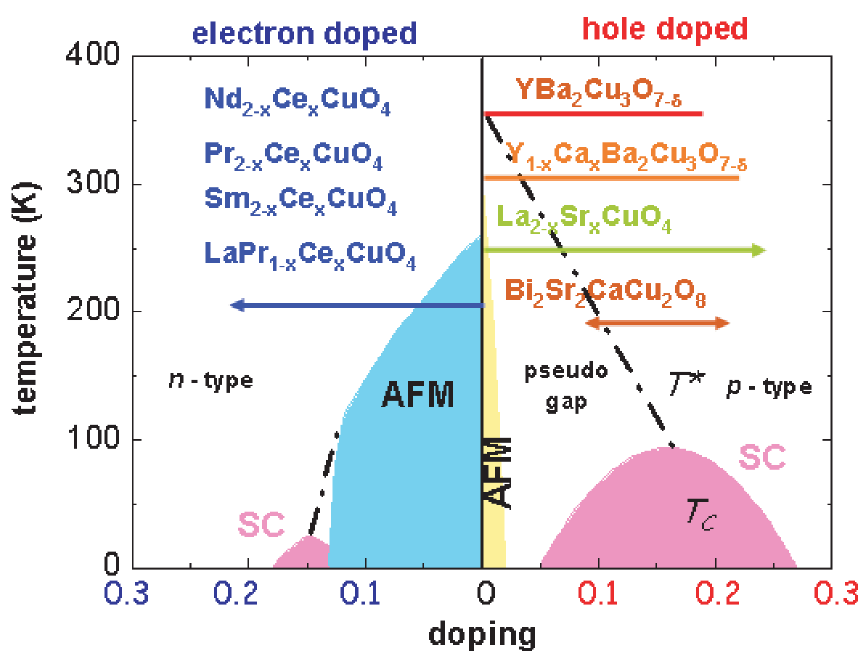


Mott insulator

$(La_{1-x}Sr_x)_2CuO_4$ $La^{3+} \rightarrow Ba^{2+}$



High-T_c superconductor

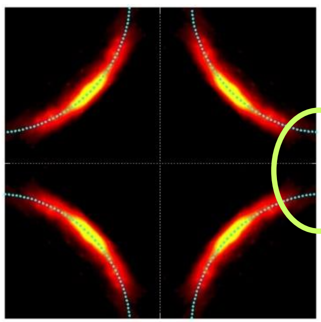


Superconductivity appears by doping holes or electrons

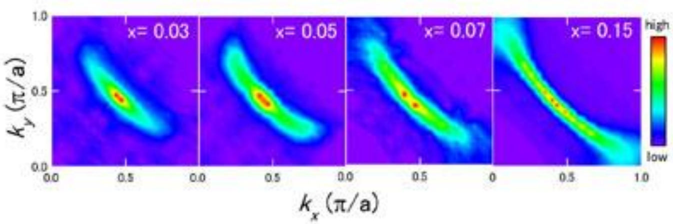
銅酸化物高温超伝導体最大の謎

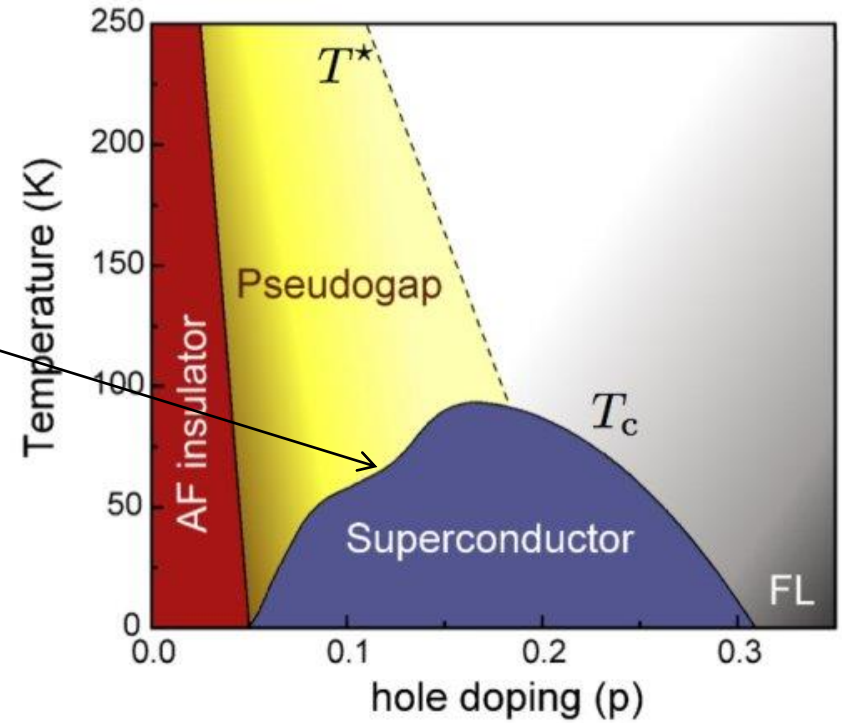
擬ギャップ

フェルミアーク



エネルギーギャップ





擬ギャップの起源

クロスオーバー　　超伝導ゆらぎ

相転移　　超伝導と競合する何らかの秩序

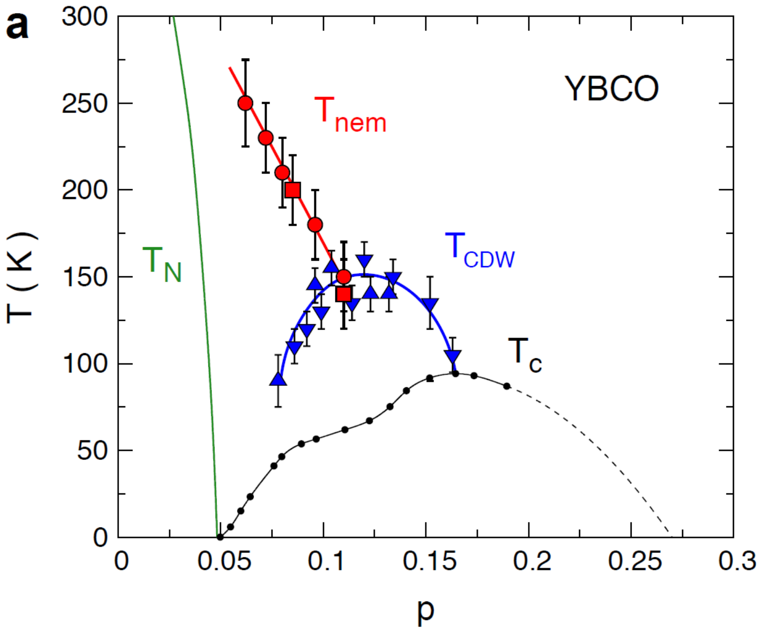
フェルミ面の再構成

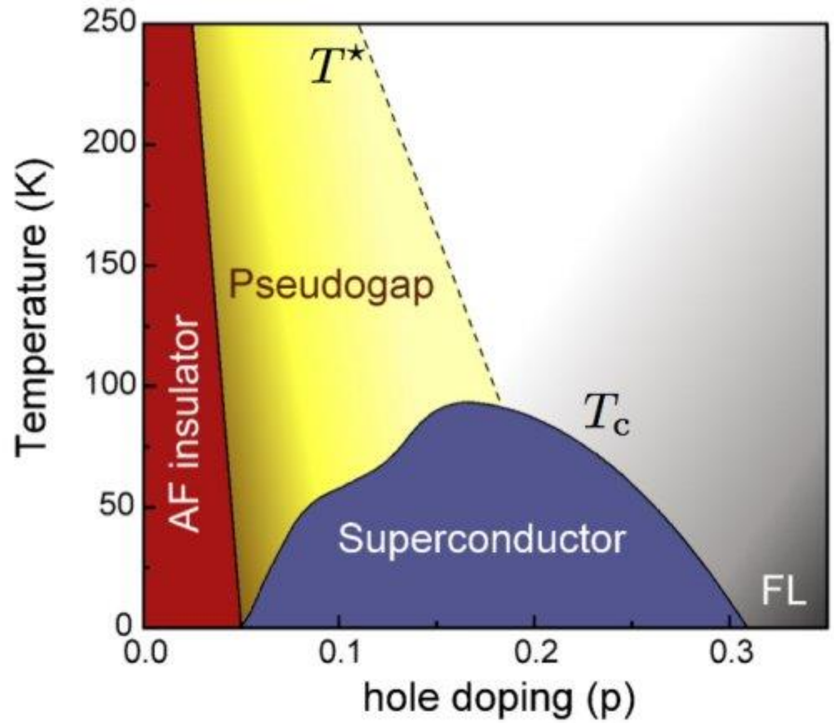
電荷ストライプ相(並進対称性の破れ)

軌道電流反磁性(時間反転対称性の破れ)

銅酸化物高温超伝導体最大の謎

擬ギャップ





超伝導ゆらぎ

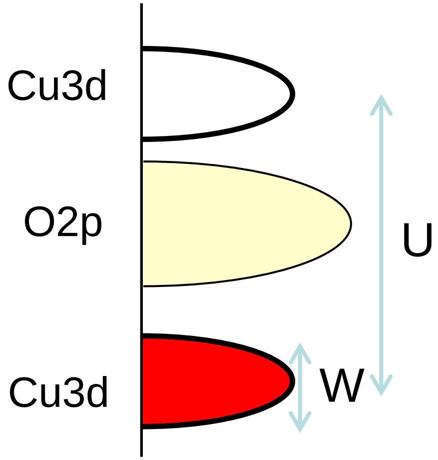
超伝導と競合する何らかの秩序

フェルミ面の再構成

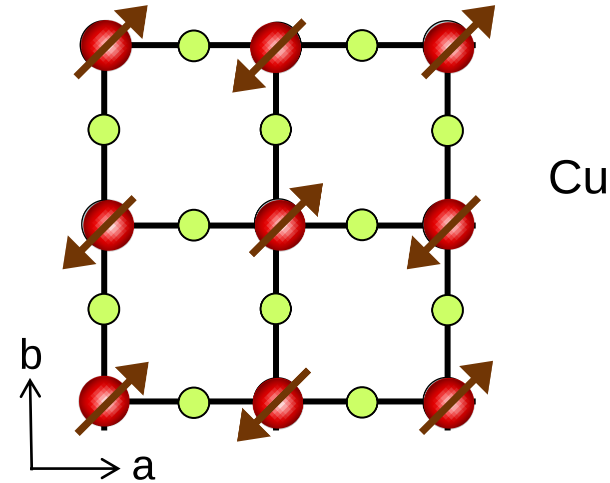
電荷ストライプ相(並進対称性の破れ)

軌道電流反磁性(時間反転対称性の破れ)

Parent compound



AFM insulator



U : Coulomb ~ 8eV

W : Band width ~ 3eV

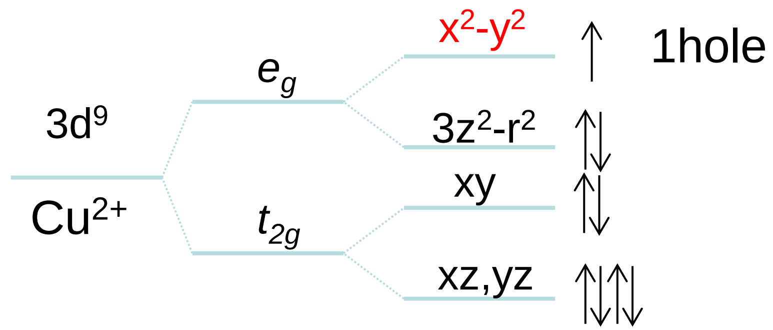
Strong electron-electron correlation

Mott insulator

S=1/2 1 μ_B

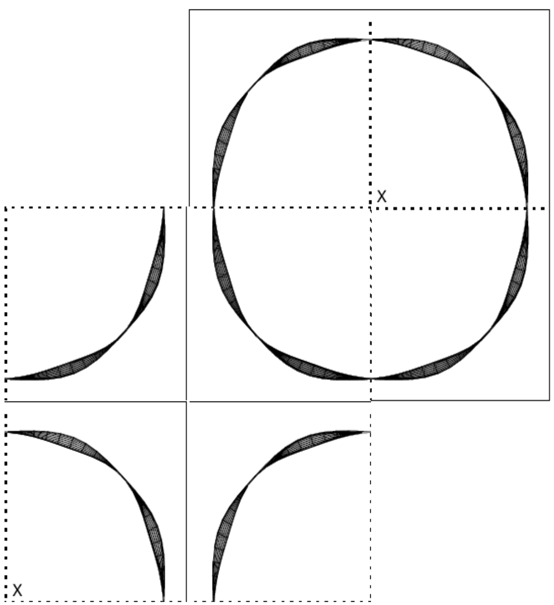
~0.6 μ_B

2D quantum fluctuation



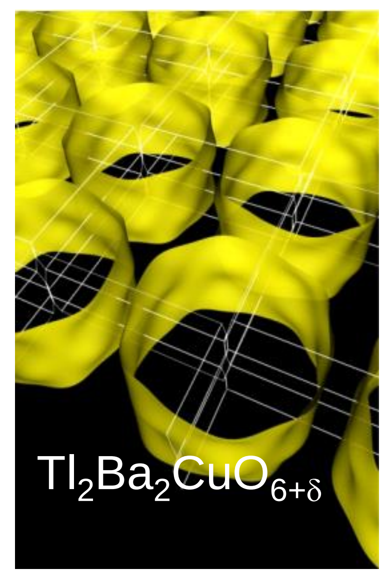
Large crystal field ~2-3 eV

hole



One orbital

x^2-y^2

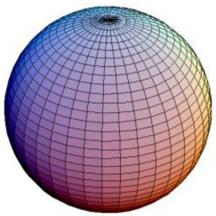


single band

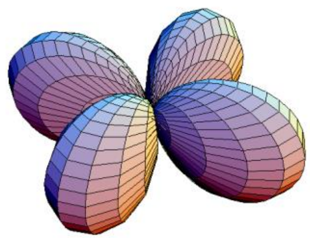
d-wave superconductivity in cuprates

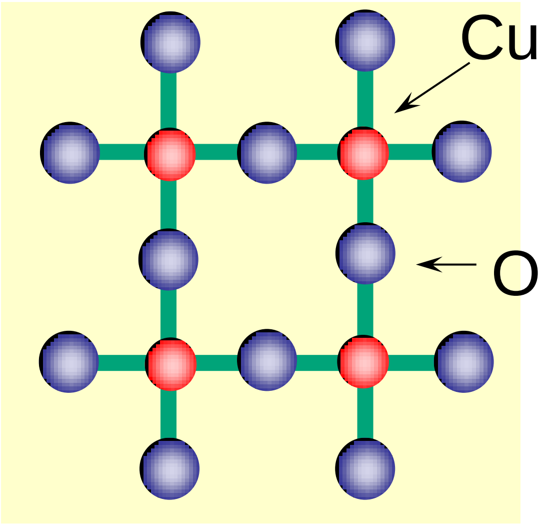
Copper pair with finite angular momentum

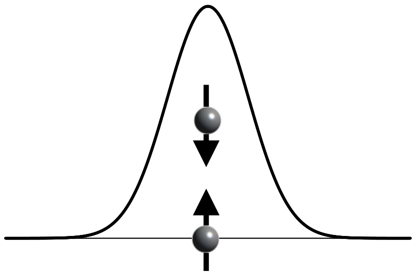
s-wave

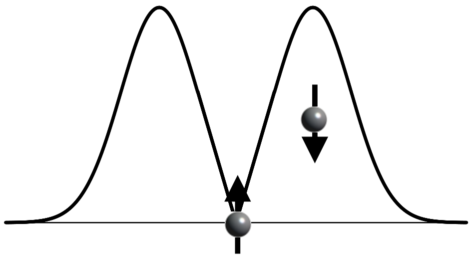


***d*-wave**



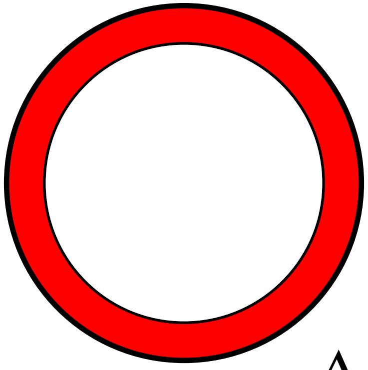




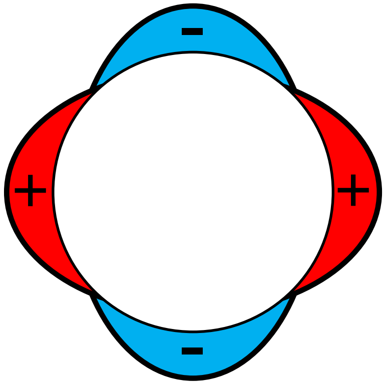


Attractive

Onsite repulsive



$\Delta = \text{const.}$

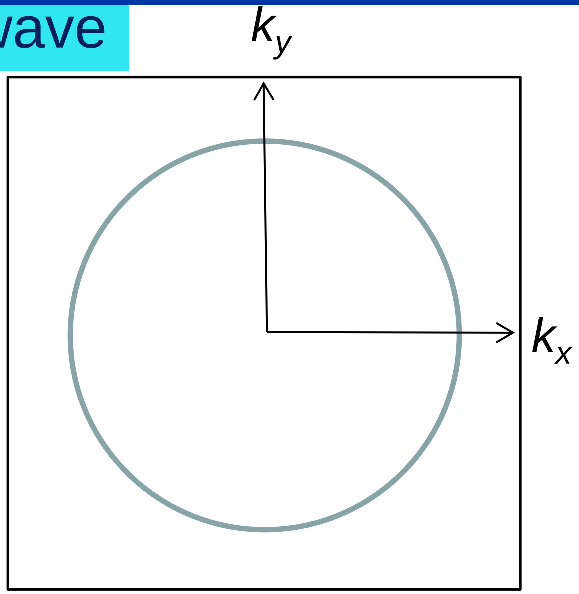


$d_{x^2-y^2}$ $(k_x^2-k_y^2)$

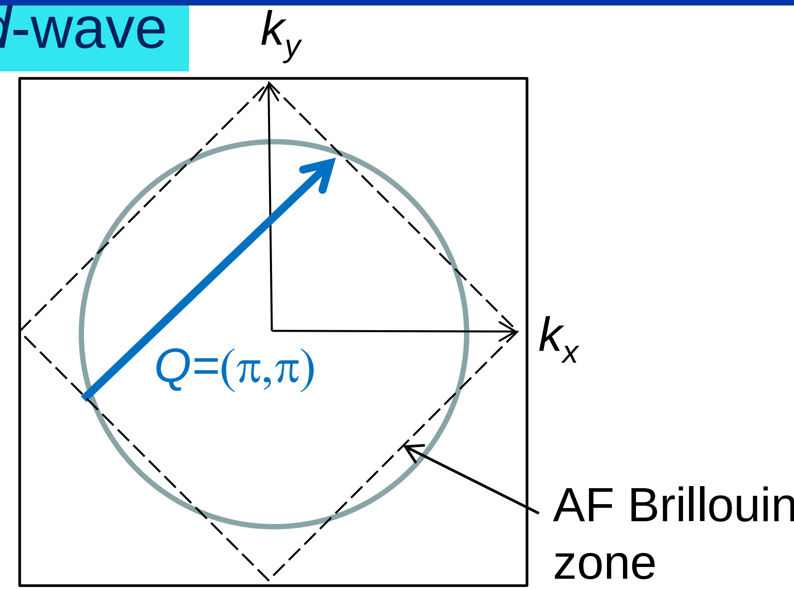
zeros at $k_x=+k_y, -k_y$

d-wave superconductivity in cuprates

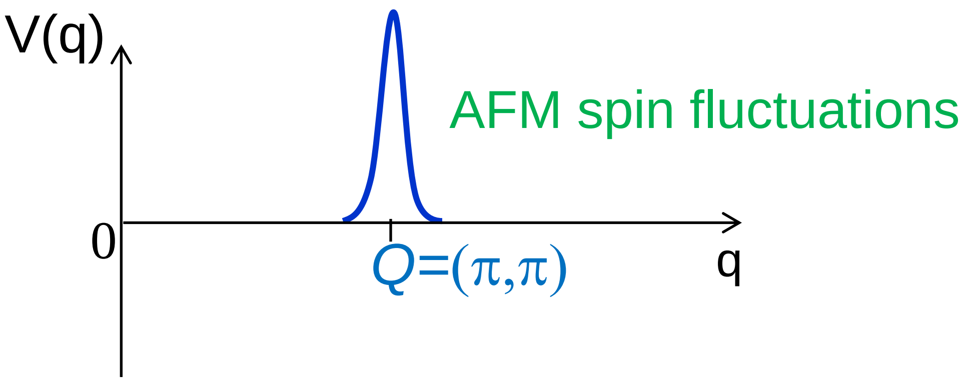
s-wave



d-wave



V(q): pairing interaction



V(q) is negative and constant
(attractive)

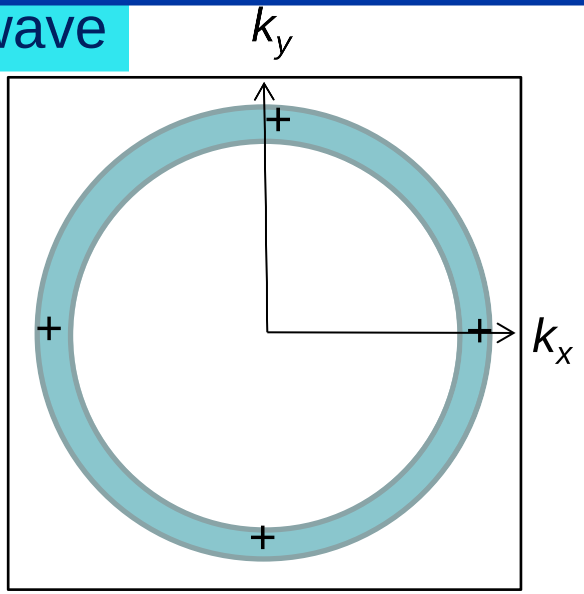
V(q) is positive and peaks at q=*Q*
(repulsive)

$$V_{kp} \simeq \underbrace{\tfrac{3}{2}U^2}_{\text{Coulomb}}\underbrace{\chi(k-p)}_{\text{Magnetic fluctuation}} \qquad \chi(q) \sim \delta(q-Q)$$

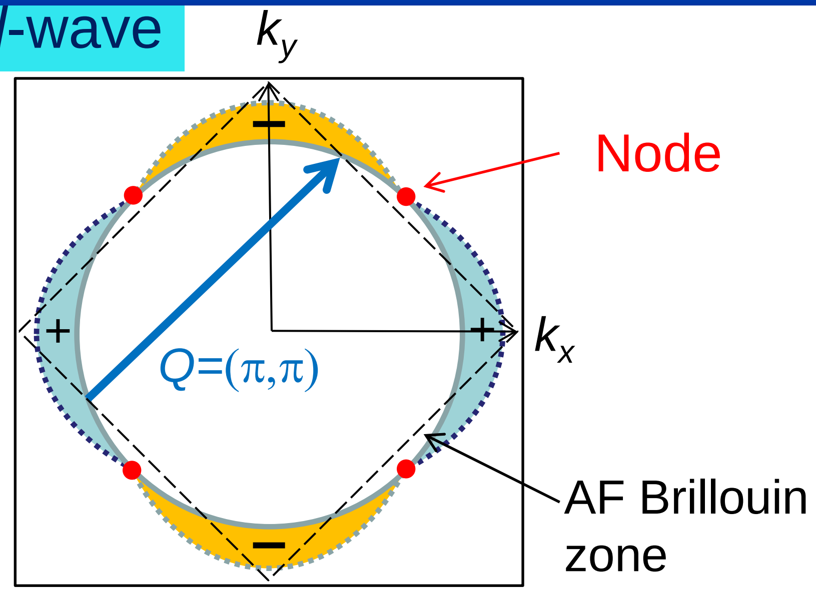
Coulomb Magnetic fluctuation

d-wave superconductivity in cuprates

s-wave



d-wave



Gap equation

$$\Delta(k) = -\sum_p V_{kp} \frac{\tanh(\varepsilon_p/2T)}{2\varepsilon_p} \Delta(p) \quad \varepsilon_p = \sqrt{\Delta_p^2 + \xi_p^2}$$

$$\Delta(k) = \Delta$$

$$\Delta = -\sum_p V_{kp} \underbrace{\frac{\tanh(\varepsilon_p/2T)}{\varepsilon_p}}_{>0} \Delta$$

$$V_{kp} = V < 0$$

$$V(r) \sim -\delta(r)$$

$$V_{kp} \simeq \underbrace{\frac{3}{2}U^2}_{\text{Coulomb}} \underbrace{\chi(k-p)}_{\text{Magnetic fluctuation}} \quad \chi(q) \sim \delta(q-Q)$$

$$Q = (\pi, \pi)$$

$$\Delta(k) \sim -\sum_p U^2 \delta(k-p+Q) \frac{\tanh(\varepsilon_p/2T)}{2\varepsilon_p} \Delta(p)$$

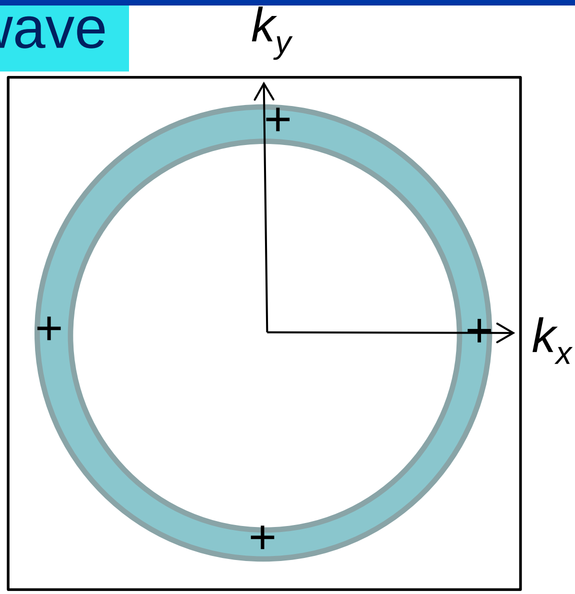
$$= -U^2 \frac{\tanh(\varepsilon_{k+Q}/2T)}{2\varepsilon_{k+Q}} \Delta(k+Q)$$

$\Delta(k+Q)\Delta(k) < 0$ sign change

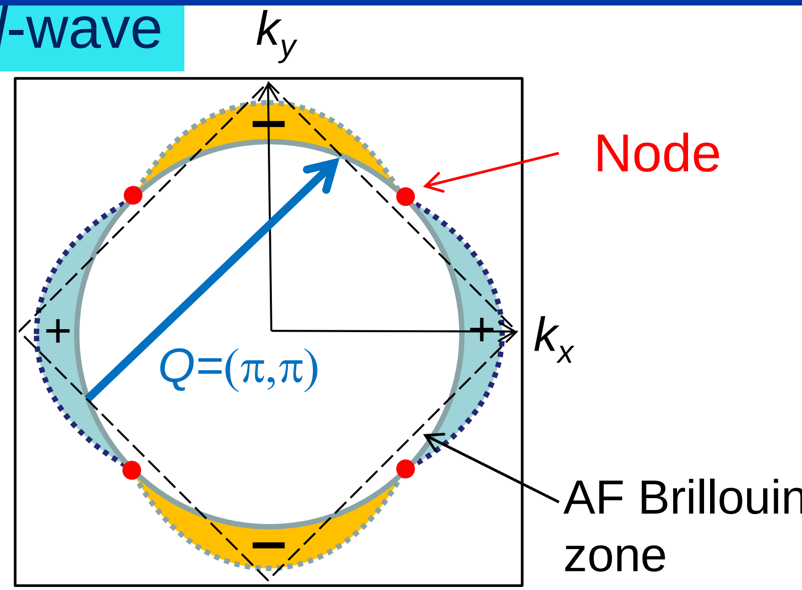
$$V(x,y) \sim \cos\pi(x+y) + \cos\pi(x-y)$$

d-wave superconductivity in cuprates

s-wave



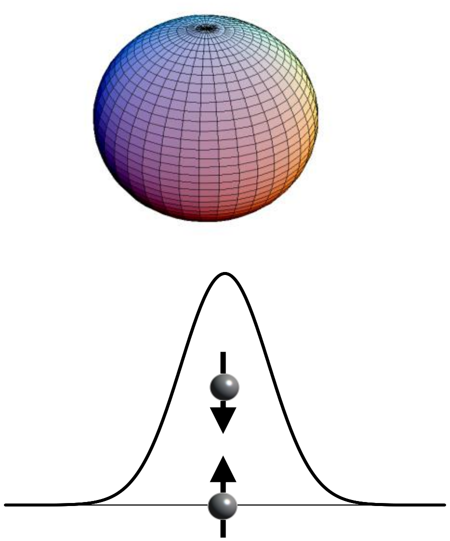
d-wave

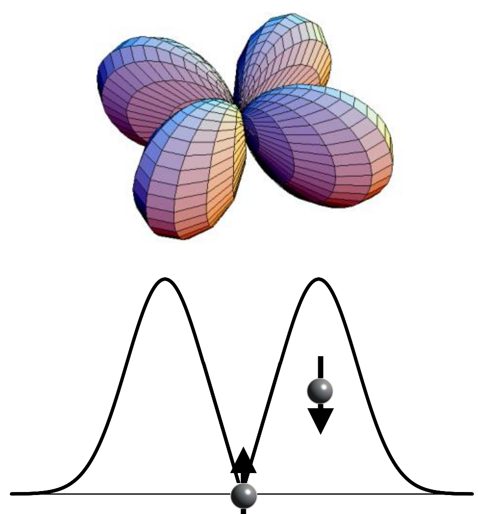


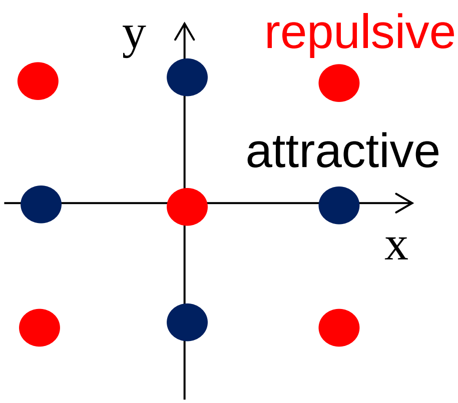
$\Delta(\boldsymbol{k}+\boldsymbol{Q})\Delta(\boldsymbol{k}) < 0$ sign change

$V(r) \sim -\delta(r)$

$V(x,y) \sim \cos\pi(x+y) + \cos\pi(x-y)$

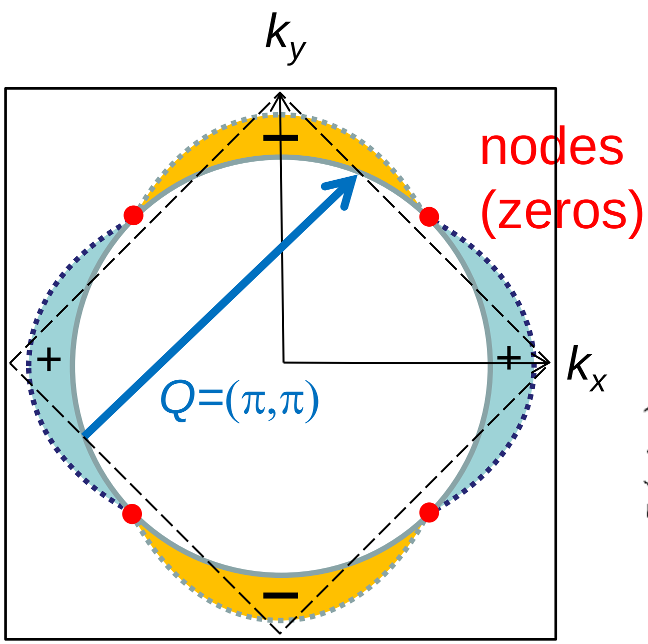






Real space

d-wave superconductivity in cuprates

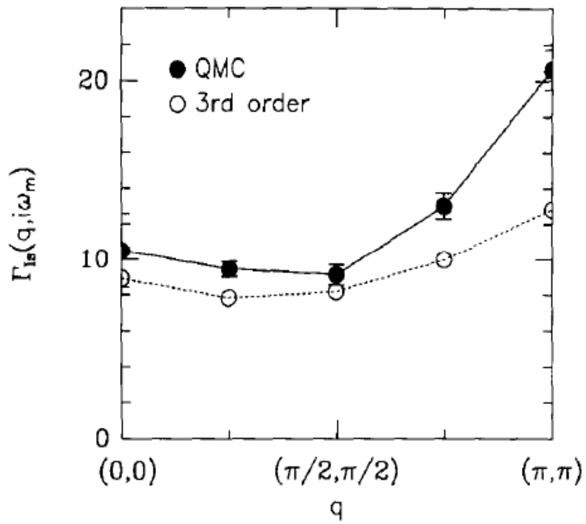


$d_{x^2-y^2}$ $(k_x^2-k_y^2)$

zeros at $k_x=+k_y, -k_y$

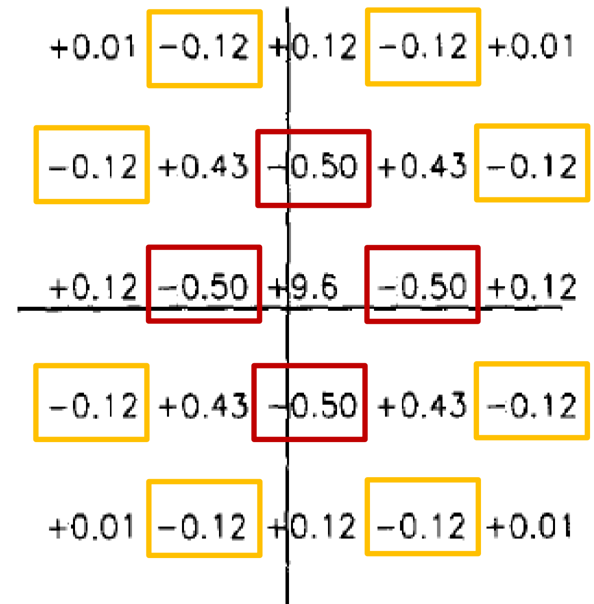
V(q) broadly peaks at (π, π)

Repulsive *V*(q)>0



q-space

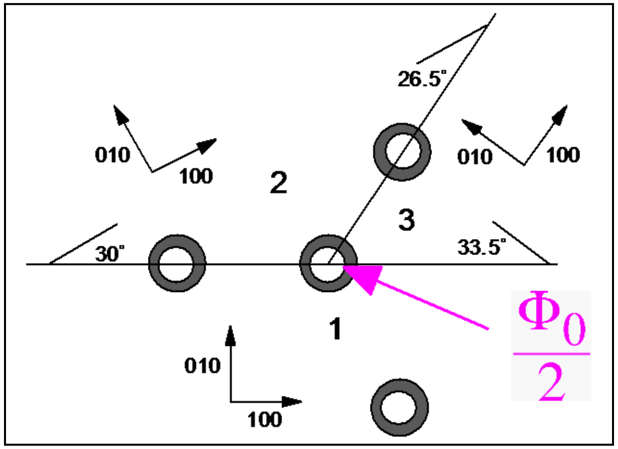
Repulsive on-site and
attractive off-site interaction

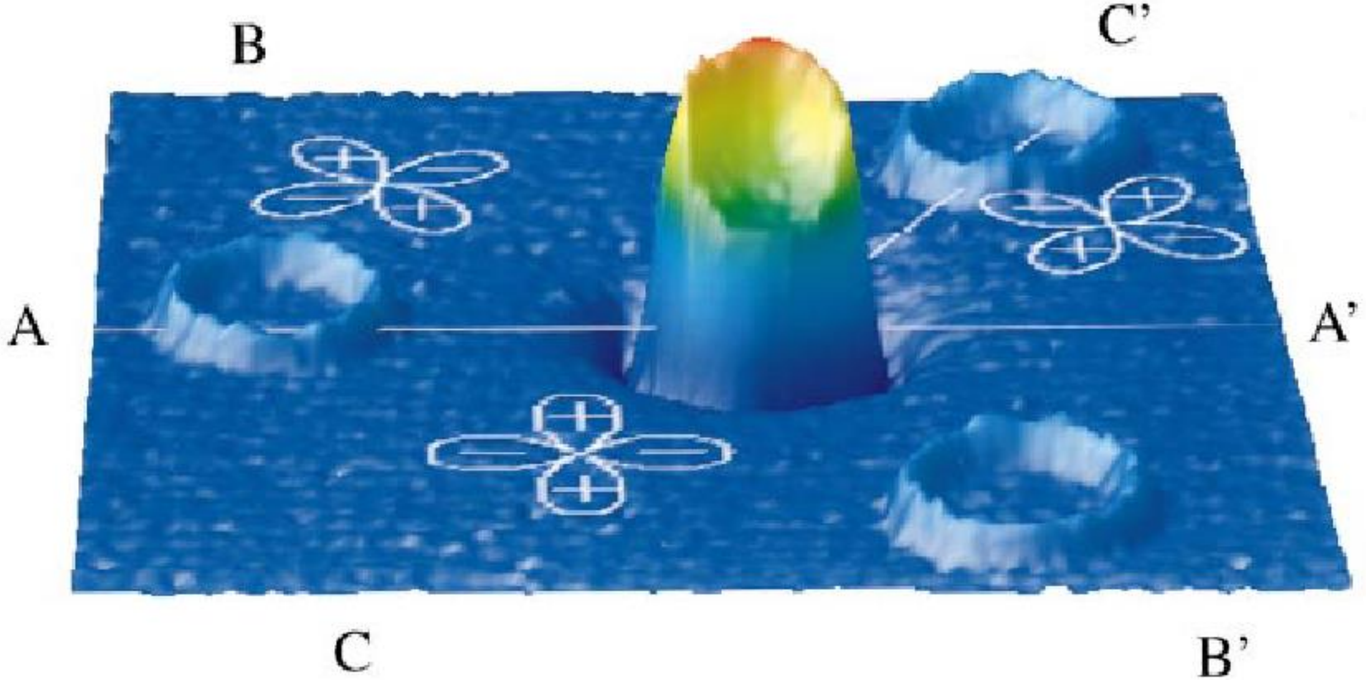


r-space

D. J. Scalapino, Phys. Rep. (1995).

YBCO tricrystal superconducting ring (1994)



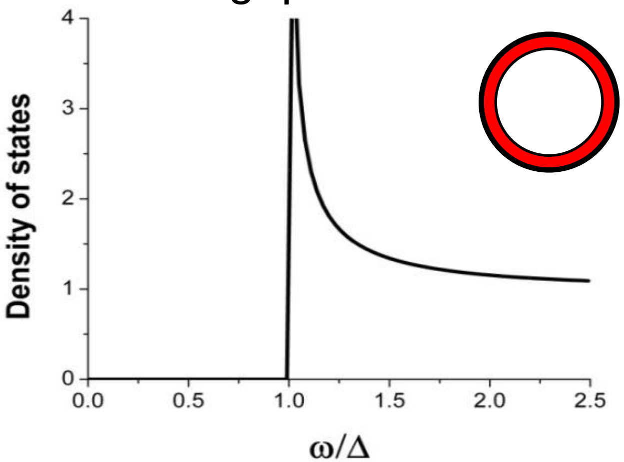


C.C. Tsuei *et al*. PRL (94)

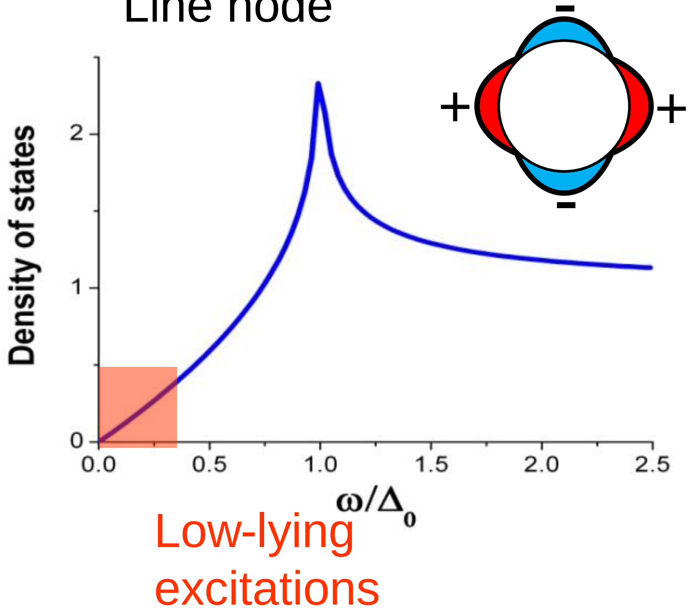
超伝導ギャップ構造の決定方法

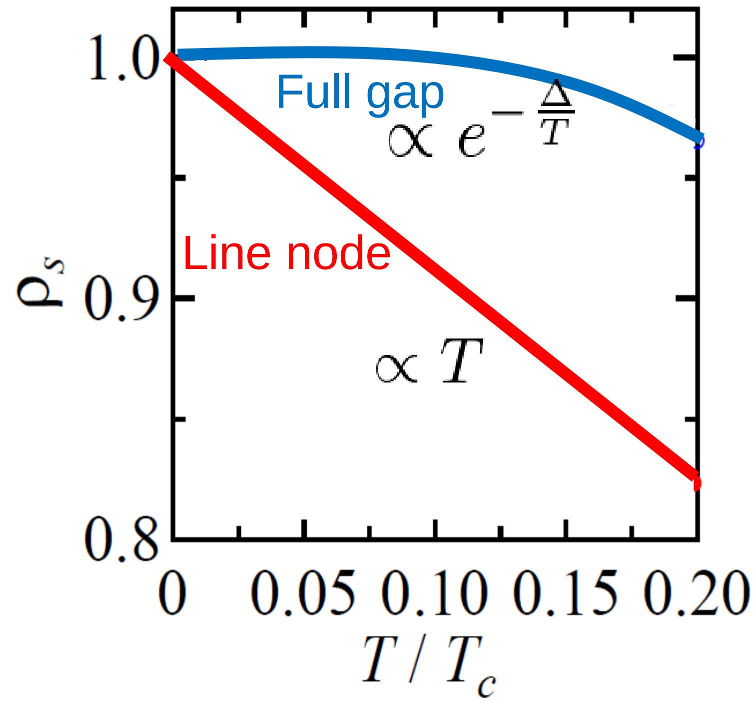
How to determine the gap structure

Full gap



Line node

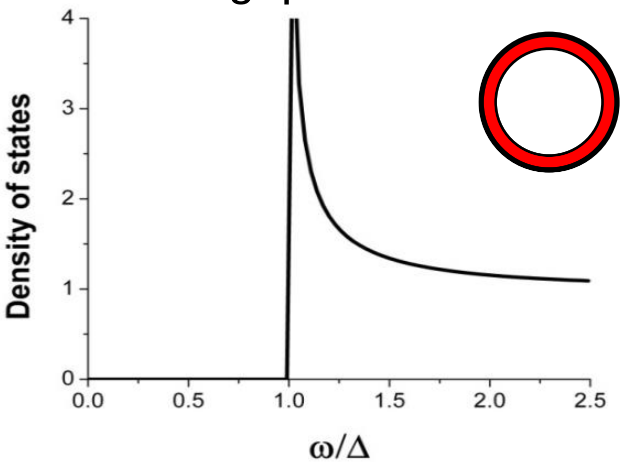




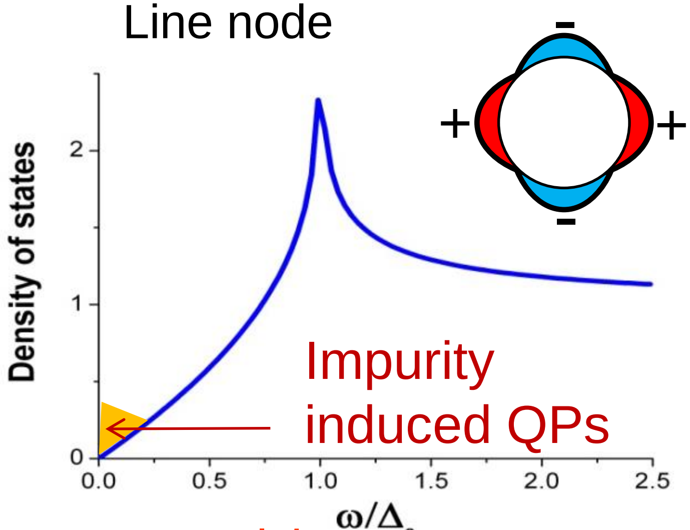
Line node	Full gap
$\lambda_L^{-2} \propto T$	$\lambda_L^{-2} \propto e^{-\frac{\Delta}{T}}$
$C \propto T^2$	$C \propto e^{-\frac{\Delta}{T}}$
$1/T_1 \propto T^3$	$1/T_1 \propto e^{-\frac{\Delta}{T}}$

How to determine the gap structure

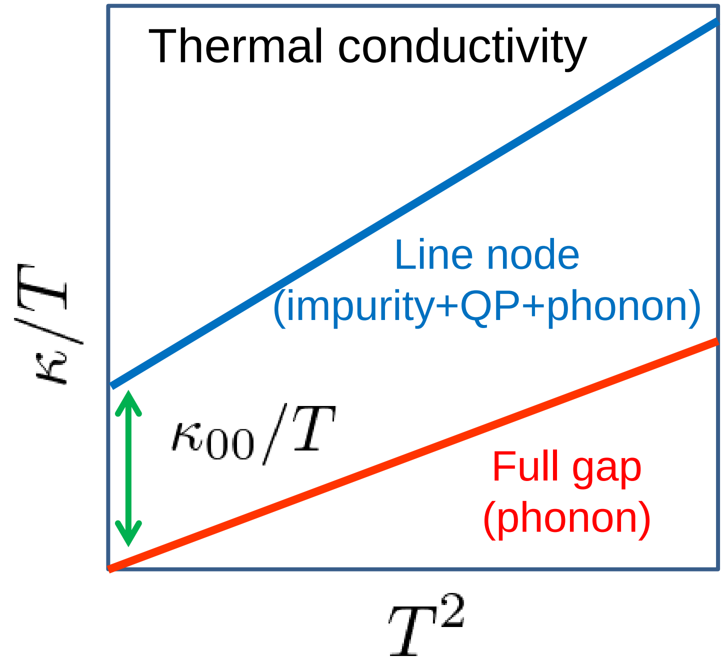
Full gap



Line node



Low-lying
excitations



Superfluid does not carry the heat

$$\kappa_e = C_e v_F^2 \tau \qquad \kappa/T = \alpha + \beta T^2$$

$$\alpha = \kappa_{00}/T$$

$$\kappa_{00}/T = N_{imp}(0) v_F^2 \tau_{imp}$$

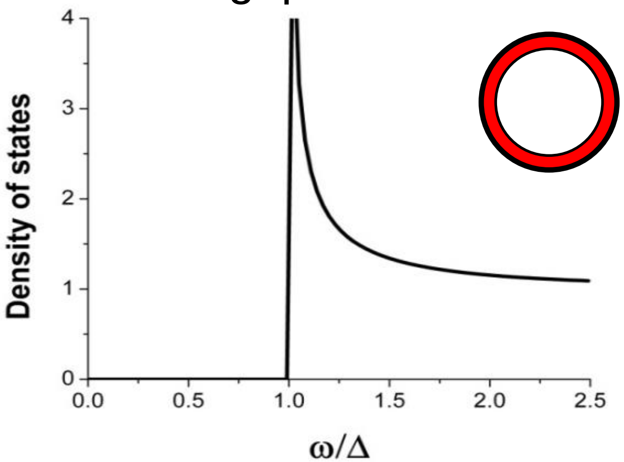
$$\tau_{imp} \propto 1/N_{imp}(0)$$

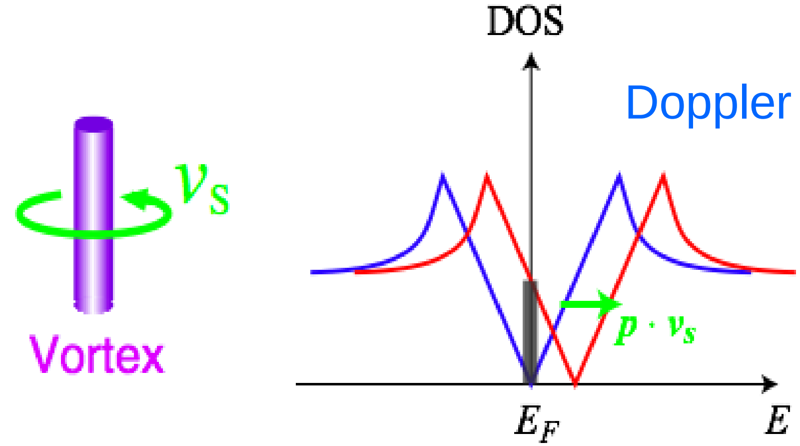
Independent of impurity

Universal thermal conductivity

How to determine the gap structure

Full gap



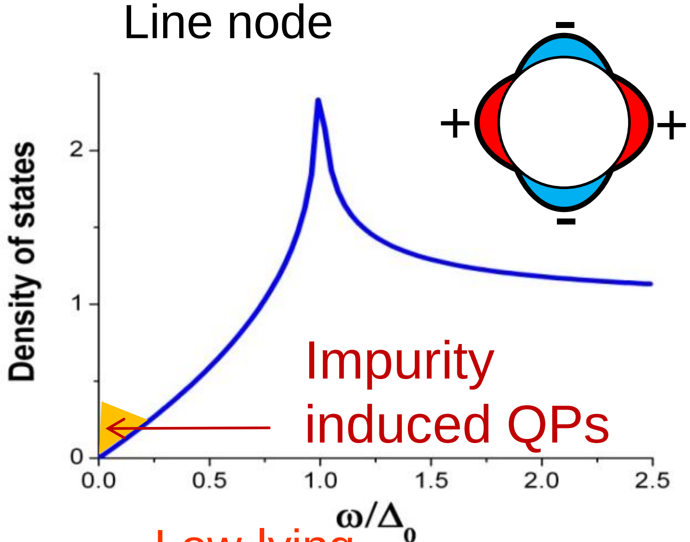


$$E'(\boldsymbol{p}) \to E(\boldsymbol{p}) + \boldsymbol{v_s} \cdot \boldsymbol{p}$$

v_s: supercurrent

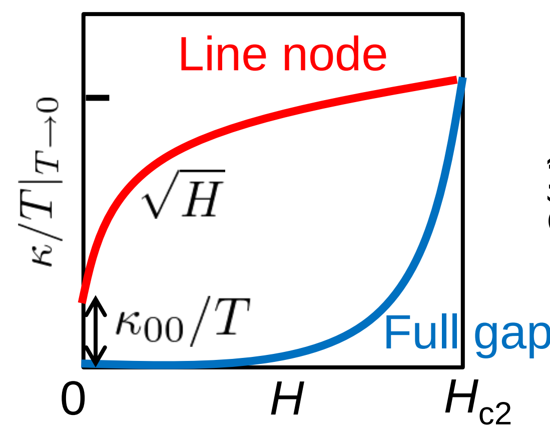
G.E. Volovik, JETP Lett. **58**, 469 (1993)

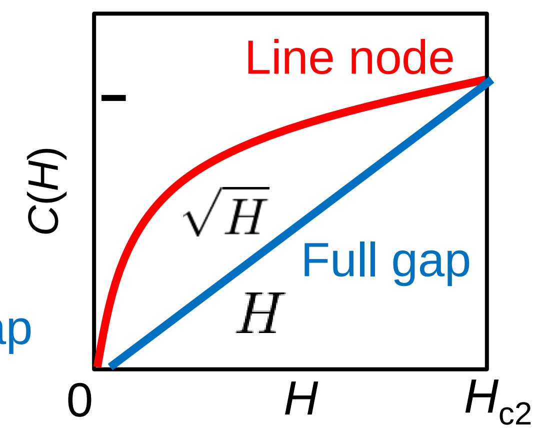
Line node



Low-lying
excitations

$$N(0) \sim \sqrt{H} \longrightarrow C(H),\ \kappa(H) \sim \sqrt{H}$$

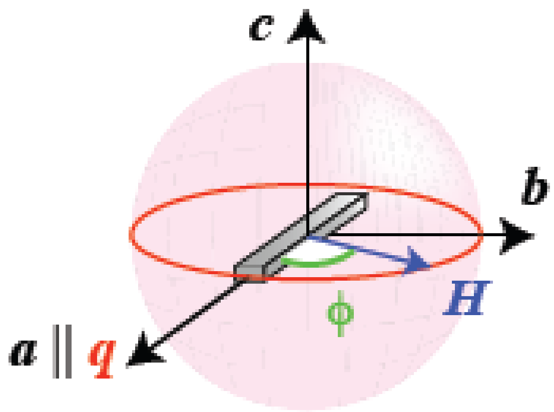




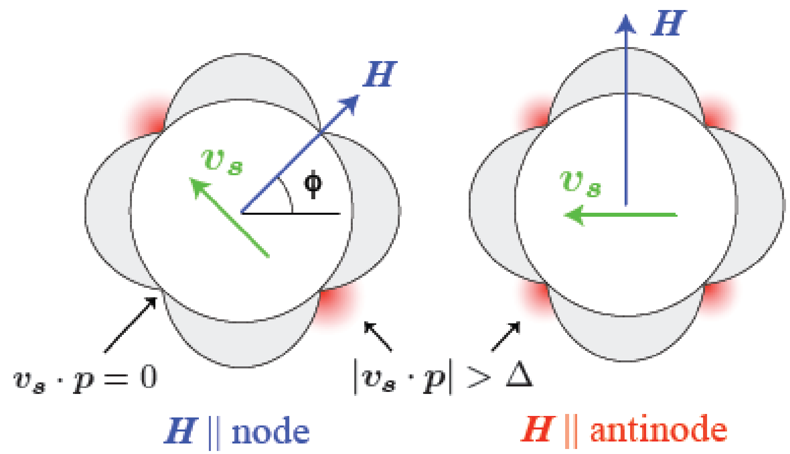
Thermal conductivity is governed by QPs **outside** of vortex core.

How to determine the gap structure

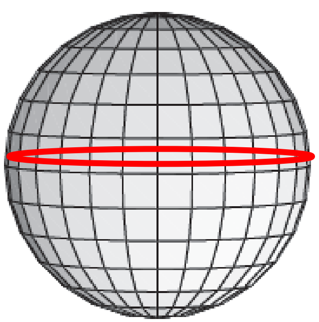
Doppler shift



Angular dependent DOS

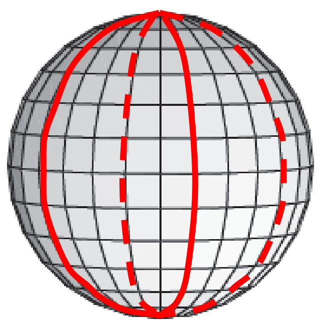


$$E'(\boldsymbol{p}) \rightarrow E(\boldsymbol{p}) + \boldsymbol{v_s} \cdot \boldsymbol{p}$$

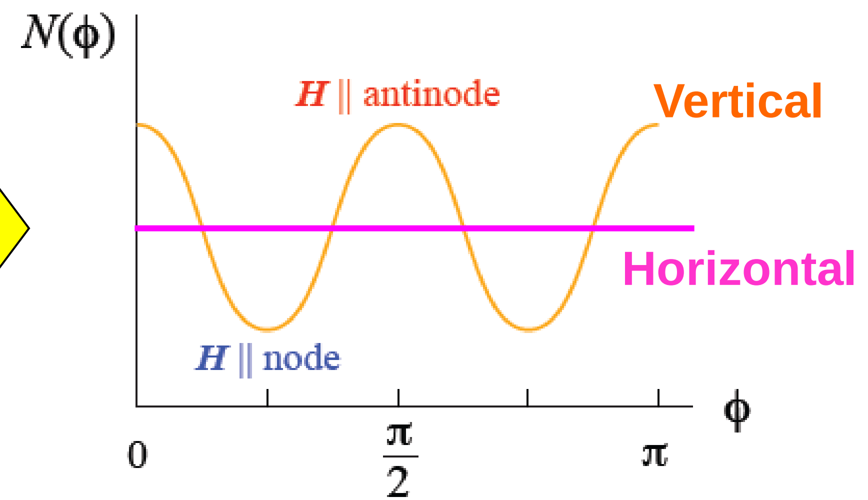


Horizontal

or

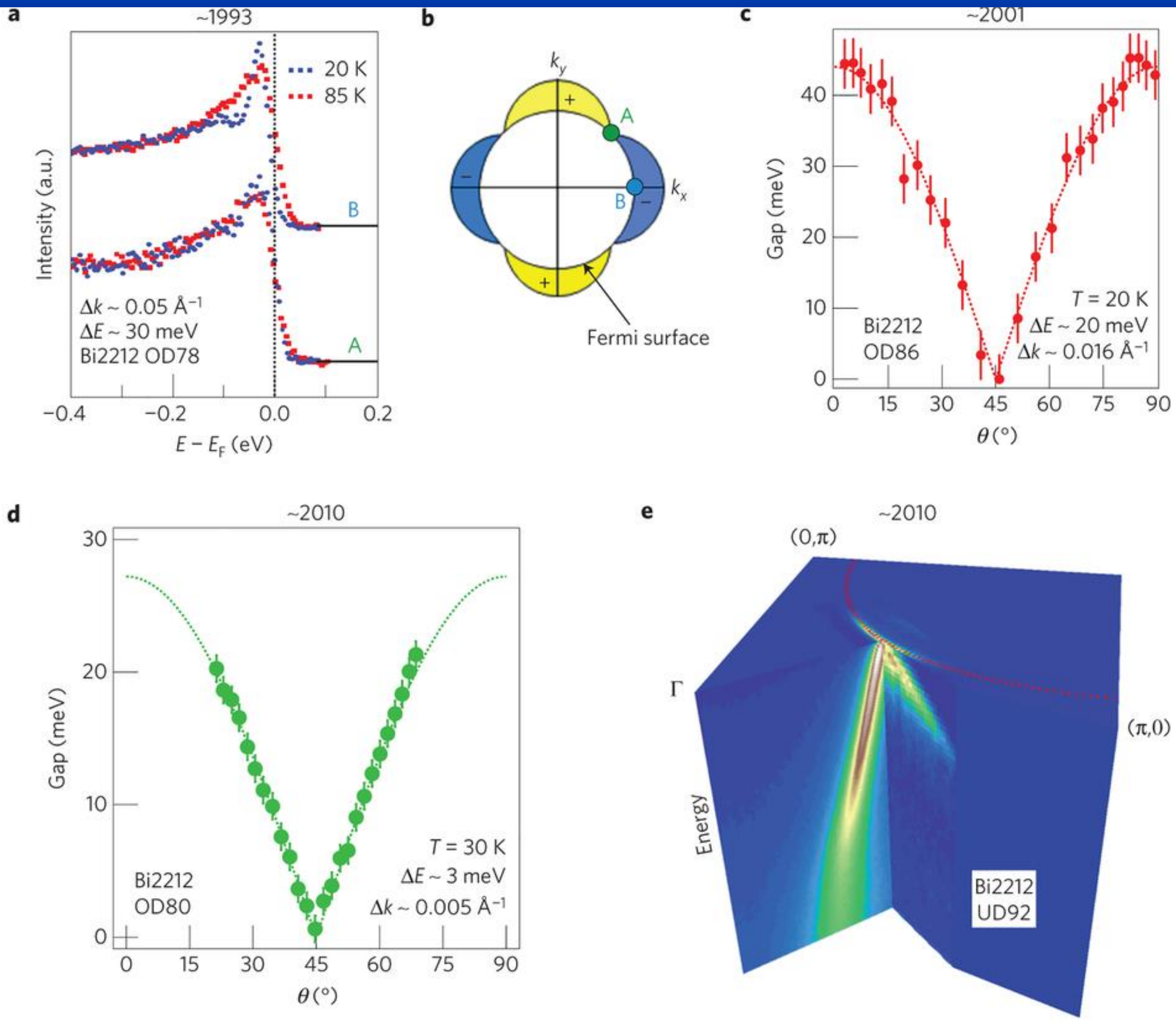


Vertical



Y.Matsuda, K.Izawa and I.Vekhter, J.Phys. C (2006)

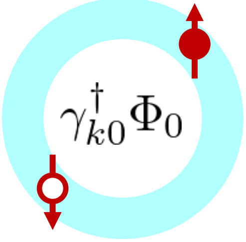
How to determine the gap structure

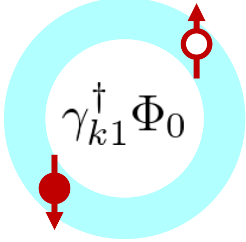


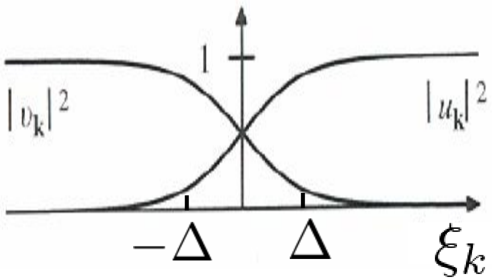
Sign change or no sign change?

Quasiparticle excitations from the SC ground state

$$\gamma_{k0}^{\dagger} = u_k c_{k\uparrow}^{\dagger} - v_k c_{-k\downarrow}$$
$$\gamma_{k1}^{\dagger} = u_k c_{-k\downarrow}^{\dagger} + v_k c_{k\uparrow}$$







$$|u_k|^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{\sqrt{\Delta_k^2 + \xi_k^2}}\right) \qquad |v_k|^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{\sqrt{\Delta_k^2 + \xi_k^2}}\right) \qquad \xi_k \equiv \frac{\hbar^2 k^2}{2m} - \varepsilon_F$$

B-quasiparticle: a superposition of an electron and a hole

$$\mathbf{k}\sigma \rightarrow \mathbf{k}'\sigma'$$

$$\mathcal{H}_1 = \sum_{k\sigma,k'\sigma'} B_{k\sigma,k'\sigma'} c_{k\sigma}^{\dagger} c_{k'\sigma'} - \begin{cases} B_{k\sigma,k'\sigma'} c_{k\sigma}^{\dagger} c_{k'\sigma'} \\ B_{-k'-\sigma',-k-\sigma} c_{-k'-\sigma'}^{\dagger} c_{-k-\sigma} \end{cases}$$

connected by time-reversal symmetry

Coherence factor

Scattering of QPs

$$(u_k u_{k'} \pm v_k v_{k'})^2 = \frac{1}{2}\left(1 \pm \frac{\Delta^2}{E_k E_{k'}}\right)$$

Creation and annihilation of two QPs

$$(v_k u_{k'} \pm u_k v_{k'})^2 = \frac{1}{2}\left(1 \pm \frac{\Delta^2}{E_k E_{k'}}\right)$$

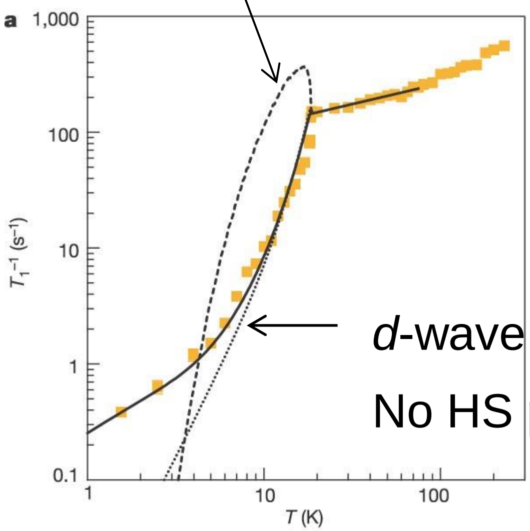
Sign change or no sign change?: NMR

$$\frac{1}{T_1T} \propto \sum_{kk'} \left(1+\frac{\Delta_k\Delta_{k'}}{E_kE_{k'}}\right)\left[-\frac{\partial f(E_k)}{\partial E_k}\right]\delta(E_k - E_{k'})$$

s-wave

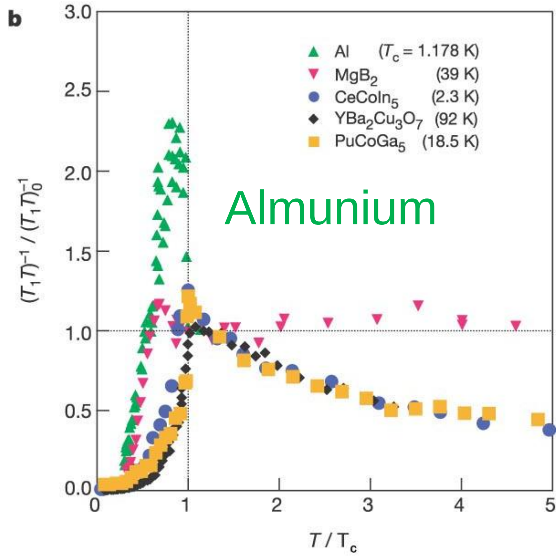
$$\frac{1}{T_1} \propto \int_{\Delta(T)}^{\infty} dE\, \frac{E^2+\Delta^2}{\underline{E^2-\Delta^2}}\, \mathrm{sech}^2\left(\frac{E}{2T}\right)$$

Hebel-Slichter peak

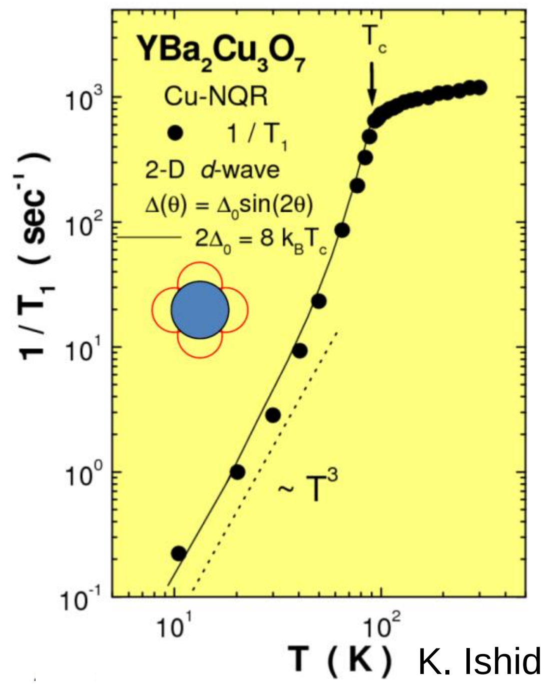


d-wave

No HS peak



N. Curro *et al*. Nature (12)



K. Ishida *et al*. JPSJ (93)

Sign change? Neutron resonance peak

In the superconducting state

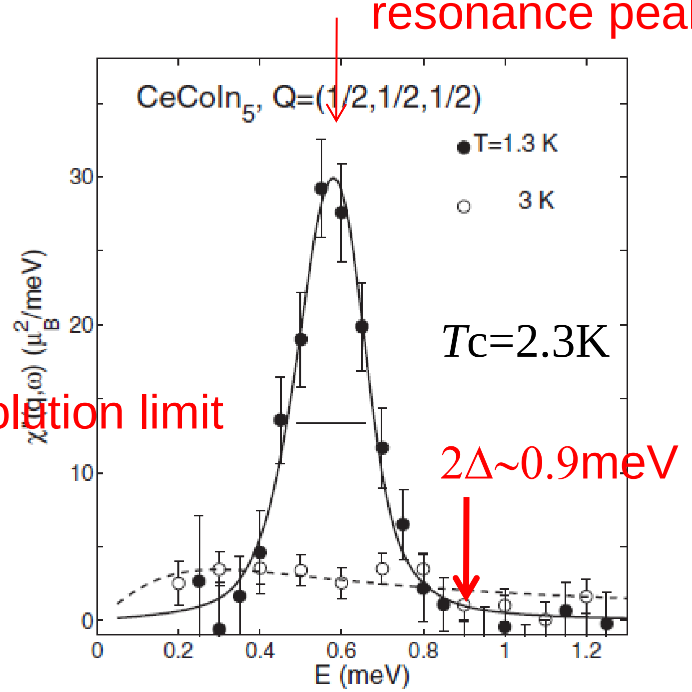
$$\mathrm{Im}\chi_0(\mathbf{q},\omega) = \frac{1}{4}\frac{1}{(2\pi)^3}\int d^3k \left(1 - \frac{\Delta_k \Delta_{k+q}}{E_{k+q}E_k}\right)\delta(\omega - E_{k+q} - E_k) \qquad E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$$

The coherence factor becomes 2 for $\Delta_{\mathbf{k+Q}} = -\Delta_{\mathbf{k}}$

Sharp resonance peak at $\omega_{res} < 2\Delta$

$CeCoIn_5$ *d*-wave

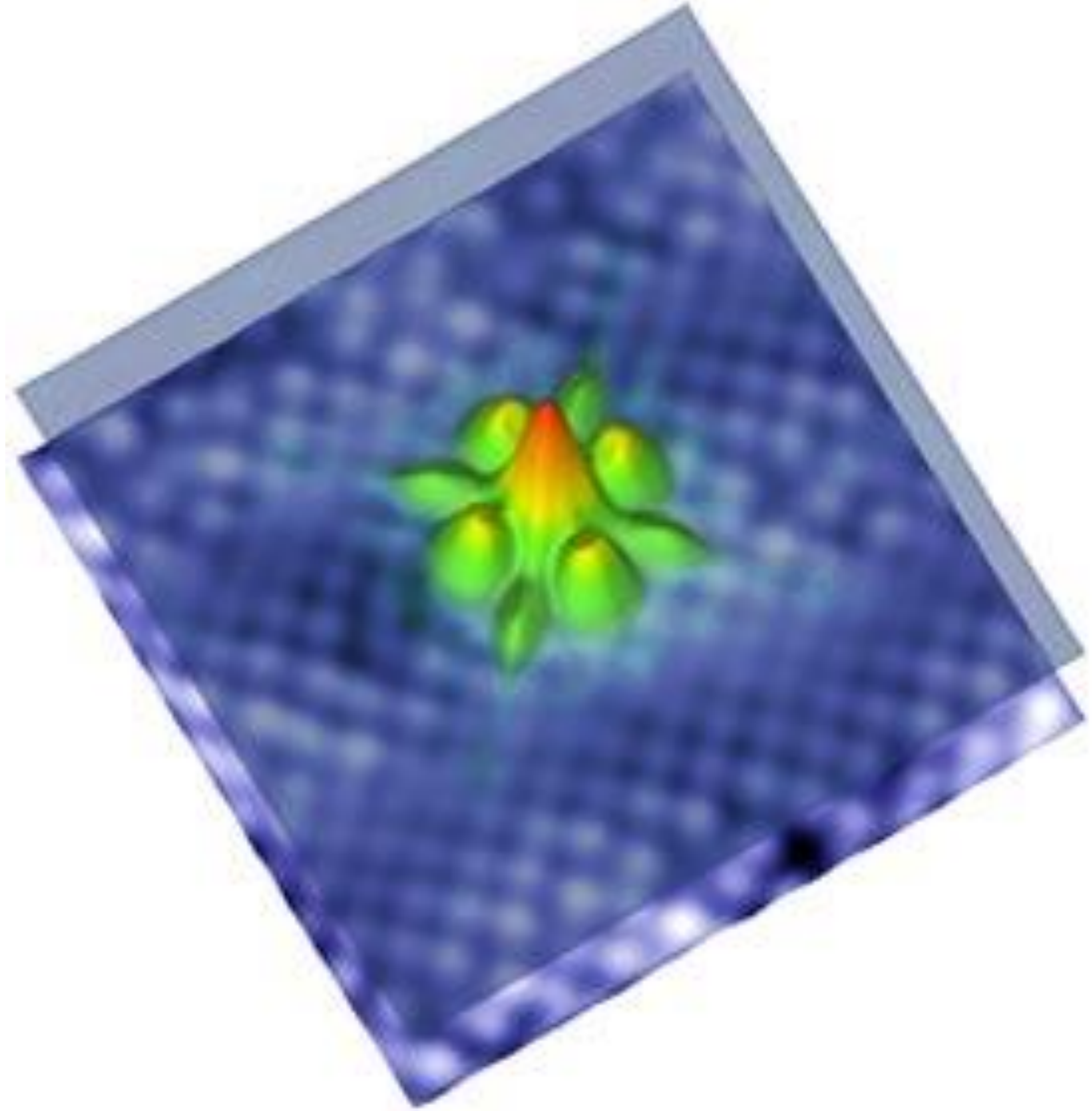
resonance peak



Stock *et al.*, PRL **100**, 087001 (2008)

Sign change or no sign change?: STM

Bi:2212 Zn不純物周りの電子状態



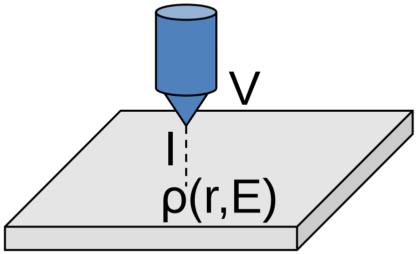
Quasiparticle interference (QPI)

Quasi-Particle Interference

Tunnel conductance

$$Z(\mathbf{r},E) \equiv \frac{dI/dV(\mathbf{r},+E)}{dI/dV(\mathbf{r},-E)} = \frac{\rho(\mathbf{r},+E)}{\rho(\mathbf{r},-E)} \Rightarrow Z(\mathbf{q},E)$$

FT



No impurity (no scattering) $Z(\mathbf{q},E)=0$ for $\mathbf{q}\neq 0$

Nonmagnetic impurity

QP scattering probability (SC state)

$$w(\mathbf{k}\sigma \to \mathbf{k}'\sigma) \propto |V(\mathbf{k},\mathbf{k}')|^2 (u_k u_{k'} - v_k v_{k'})^2$$

Nonmagnetic (no spin flip)

matrix element

coherence factor

$$(u_k u_{k'} - v_k v_{k'})^2 = \frac{1}{2}\left(1 - \frac{\Delta_k \Delta_{k'}}{E_k E_{k'}}\right)$$

sign-preserving scattering

$\Delta_k \Delta_{k'} > 0$ $\quad (u_k u_{k'} - v_k v_{k'})^2$ small

sign-reversing scattering

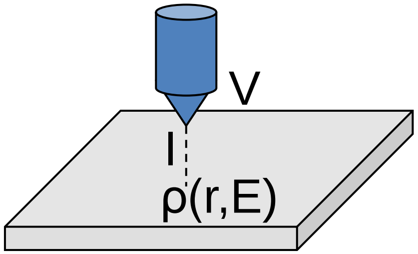
$\Delta_k \Delta_{k'} < 0$ $\quad (u_k u_{k'} - v_k v_{k'})^2$ large

Quasiparticle interference (QPI)

Quasi-Particle Interference

Tunnel conductance

$$Z(\mathbf{r},E) \equiv \frac{dI/dV(\mathbf{r},+E)}{dI/dV(\mathbf{r},-E)} = \frac{\rho(\mathbf{r},+E)}{\rho(\mathbf{r},-E)} \xrightarrow{\text{FT}} Z(\mathbf{q},E)$$



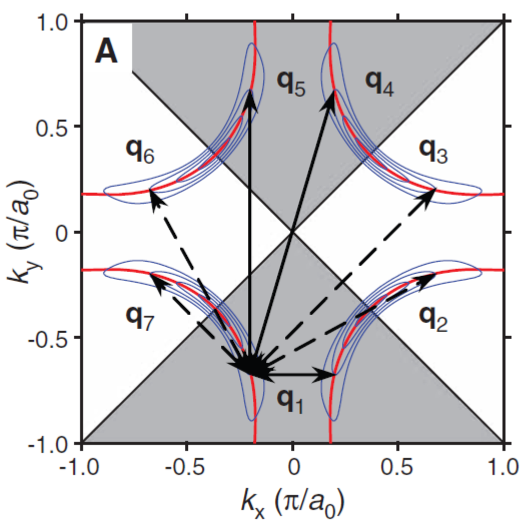
No impurity (no scattering) $Z(\mathbf{q},E)=0$ for $\mathbf{q}\neq 0$

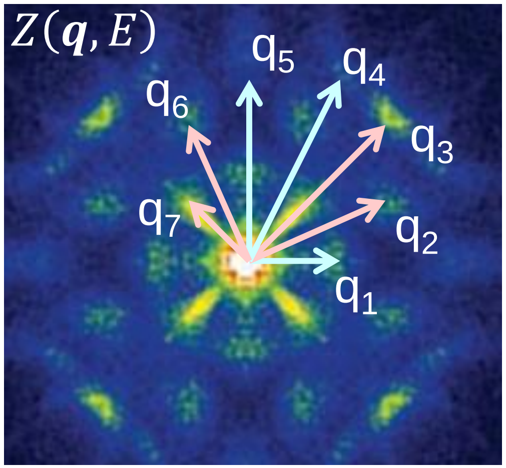
Nonmagnetic impurity

Cuprate : Octet Model

J. Hoffman *et al.*, Science (2002), K. McElroy, *et al.*, Nature (2003).

Δ_k and Δ_{k+q} { sign-preserving scattering => suppression
sign-reversing scattering => enhancement





q_y

q_x

sign-preserving ($\mathbf{q}_1$, $\mathbf{q}_4$, $\mathbf{q}_5$)

sign-reversing ($\mathbf{q}_2$, $\mathbf{q}_3$, $\mathbf{q}_6$, $\mathbf{q}_7$)

T. Hanaguri *et al*.