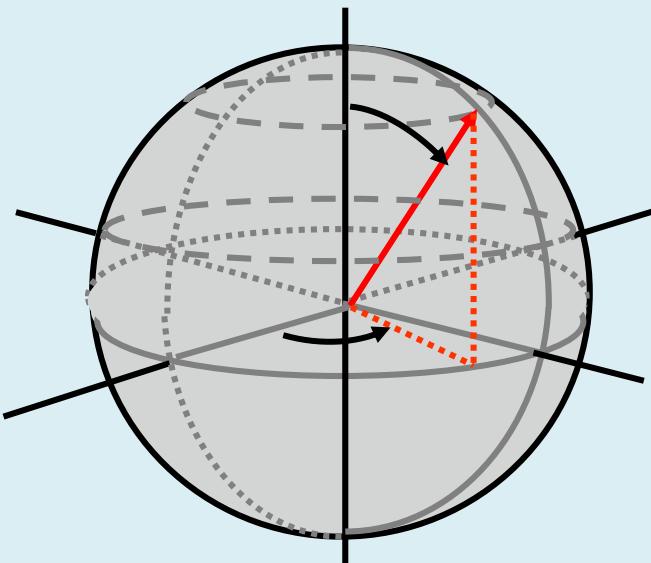


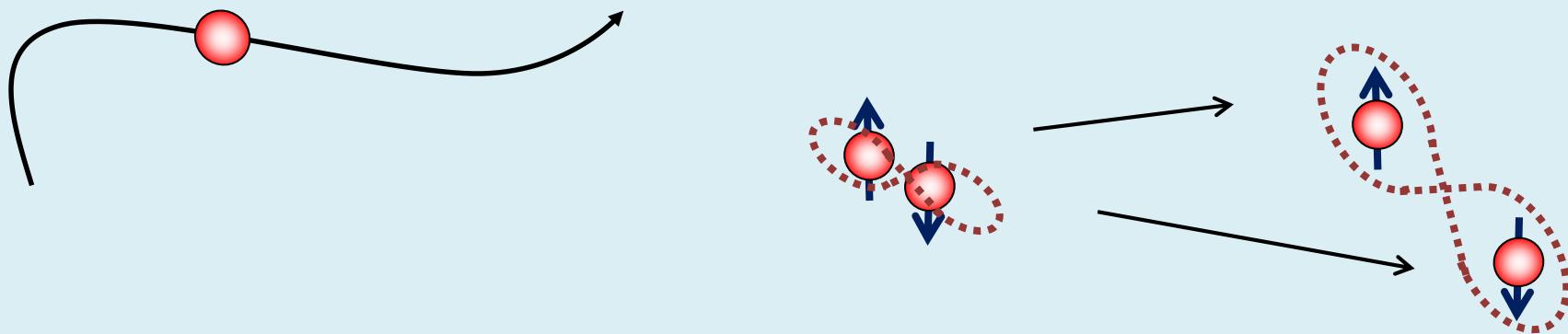
第60回物性若手夏の学校  
平成27年7月30日  
岐阜長良川温泉



## 固体中の電子の量子操作と計測

樽茶 清悟

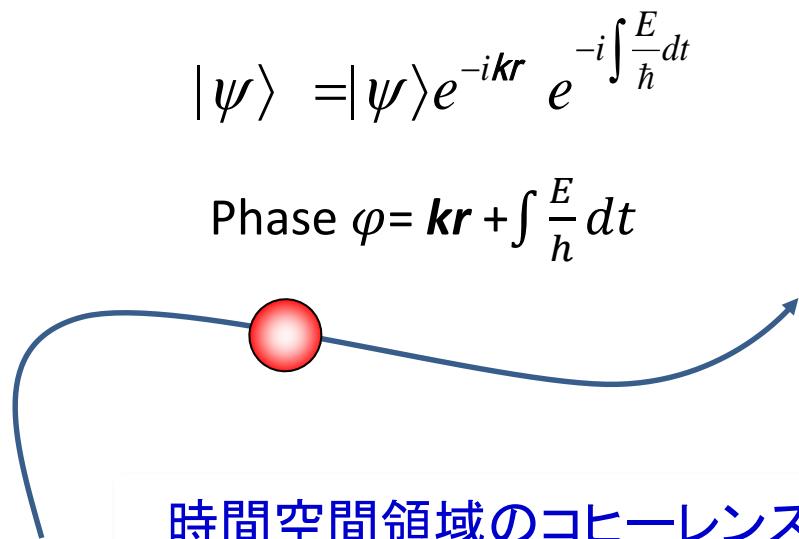
東京大学大学院工学系研究科物理工学専攻  
理化学研究所創発物質科学研究中心



# 電子の位相: 単一粒子

$$|\Psi\rangle = |\psi(r)\rangle |\chi(\sigma)\rangle$$

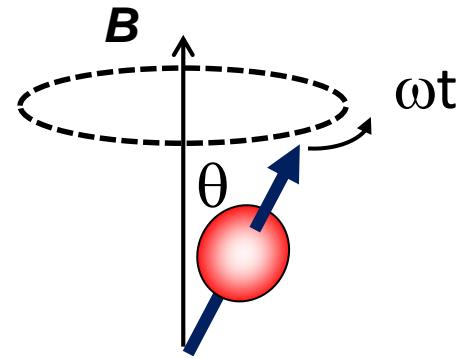
軌道



時間空間領域のコヒーレンス

スピン

$$|\chi\rangle = \cos(\theta/2) | \uparrow \rangle + e^{i\omega t} \sin(\theta/2) | \downarrow \rangle$$



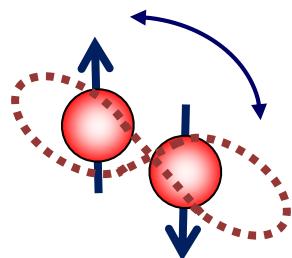
ベクトルポテンシャル: AB位相  
ハミルトニアンの断熱変化: Berry位相

○ ○ ○

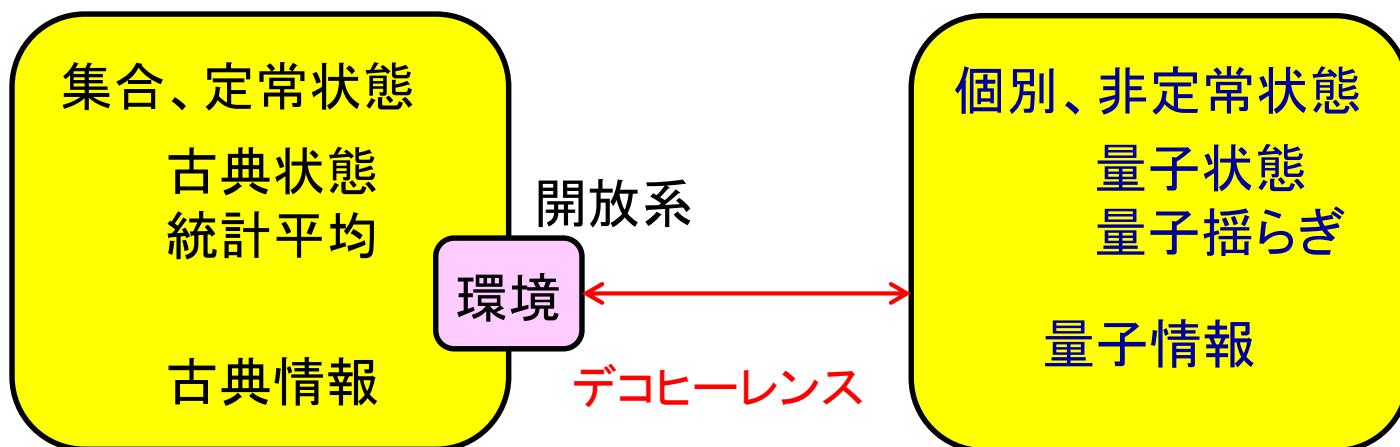
# 電子の位相:2粒子相關

スピン1重項、3重項

$$|\Psi(r, \sigma)\rangle = (|\psi_1(1)\rangle |\psi_2(2)\rangle \pm |\psi_2(1)\rangle |\psi_1(2)\rangle) \otimes (| \uparrow \rangle | \downarrow \rangle \mp | \downarrow \rangle | \uparrow \rangle)$$



$$JS_1 \cdot S_2 : J=4t^2/U \quad (t \neq 0)$$



# 内容

## 1. 量子コヒーレンスと量子情報

单一・結合量子ドットの電子状態と量子ゲートへの応用

単一電荷、单一生スピンの検出

量子ビットと量子もつれの操作と射影測定

多重ドットへの拡張

## 2. 環境とデコヒーレンス

電気的雑音と磁気的雑音

スピン緩和(スピン軌道相互作用と超微細相互作用)

デコヒーレンスのフィードバック制御

非エルゴート領域での不均一デコヒーレンス時間の拡張

核スピン環境の除去

## 3. 位相制御と計測

位相の時空間発展

2経路干渉計による位相制御と検出

量子ドットのフリーデル総和則

近藤位相の実証実験

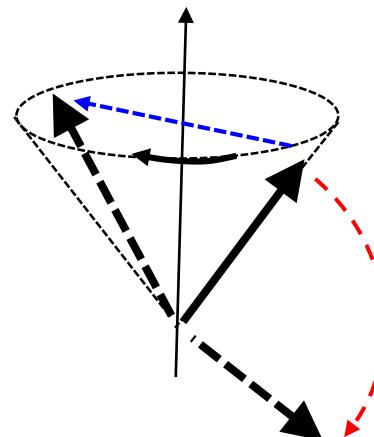
非局所量子もつれの生成と検証

# 量子制御に必要な状態操作=量子情報のゲート

状態ベクトルのユニタリー変換  
(反転と相対位相)

量子もつれの生成と解消

状態緩和(環境との相互作用)



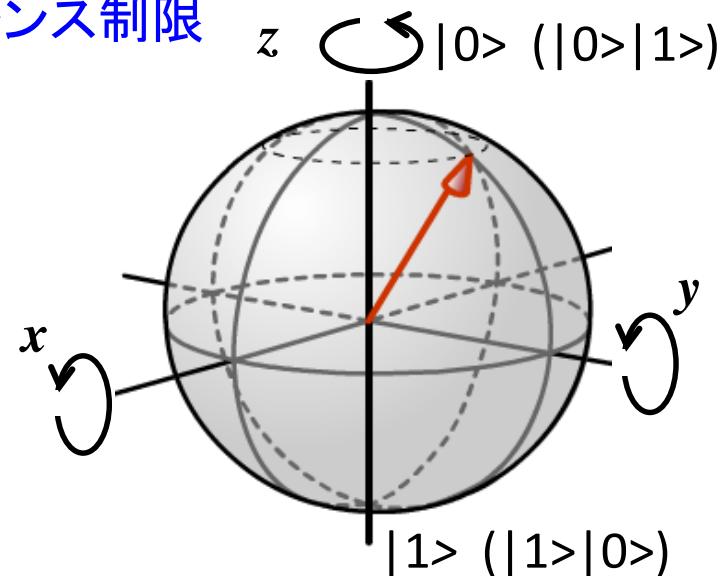
基底状態の選択

観測

$|0\rangle - |1\rangle$  ブロッホ球上の回転  
(量子ビットのXゲートと位相ゲート)

$|01\rangle - |10\rangle$  ブロッホ球上の回転

デコヒーレンス制限



初期化  $|0\rangle|0\rangle, |0\rangle|1\rangle, \dots$

読み出し: 射影測定 ( $|0\rangle, |1\rangle$ )  
ベル測定 ( $|01\rangle \pm |10\rangle, |00\rangle \pm |11\rangle$ )

# 状態操作のハミルトニアン: ベクトル回転

Well-defined two states  
 $|0\rangle$  and  $|1\rangle$

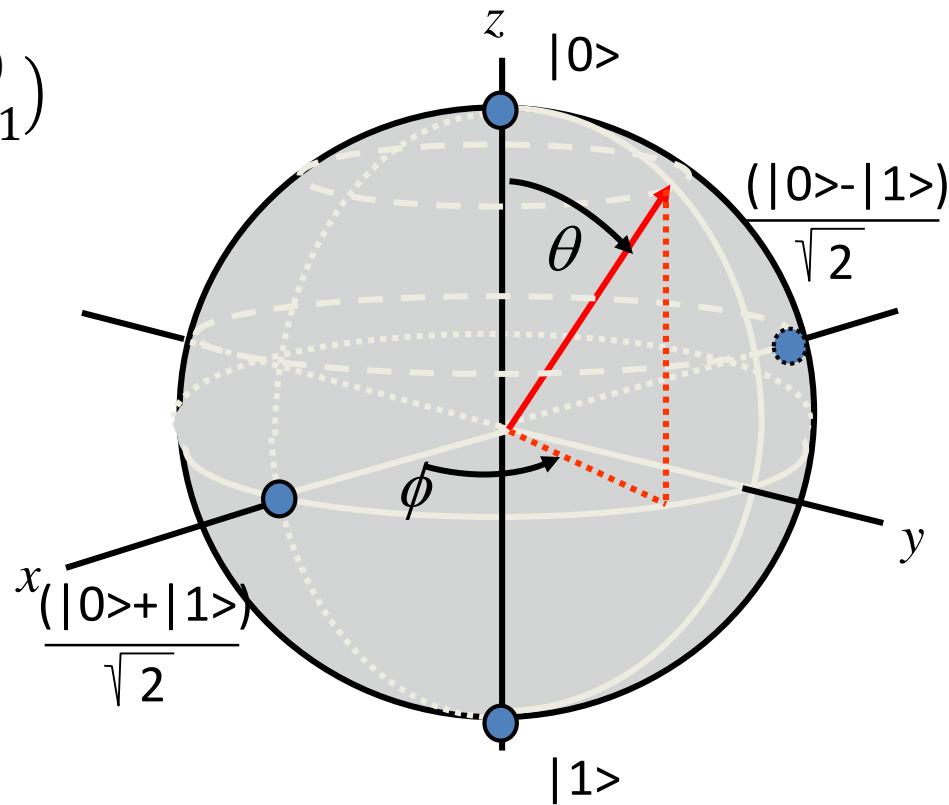
Hybridization of two states

$$H_{\text{qubit}} = \varepsilon_0 |0\rangle\langle 0| - \varepsilon_1 |1\rangle\langle 1| + (\varepsilon_x/2)(|0\rangle\langle 1| + |1\rangle\langle 0|)$$

$$= (\varepsilon_z/2) \sigma_z + (\varepsilon_x/2) \sigma_x$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

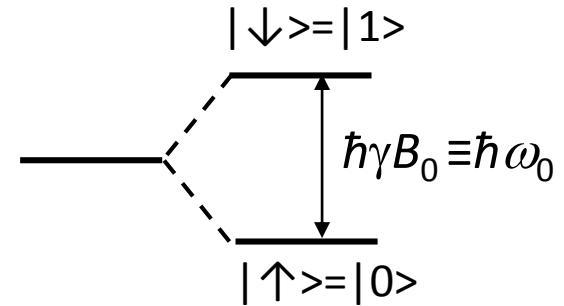
$$\sigma_z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



# スピン回転のハミルトニアン

Steady state Hamiltonian for  $B_z = B_0$  along z-axis

$$H_0 = -\hbar\gamma B_0 S_z = \begin{bmatrix} -\hbar\omega_0/2 & 0 \\ 0 & \hbar\omega_0/2 \end{bmatrix} = -\hbar\omega_0 \sigma_z / 2$$

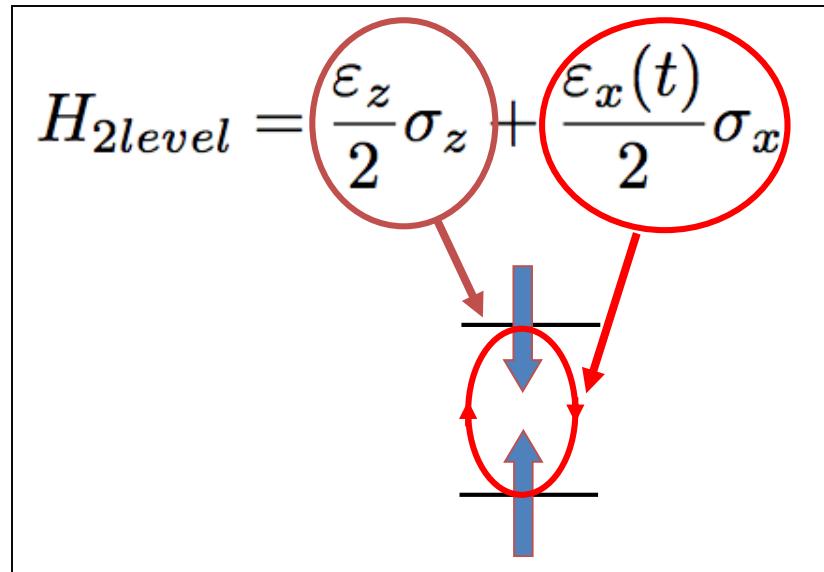


Rotation on Bloch sphere for  $B_x = B_1 \cos(\omega t)$  along x-axis ( $B_1 \ll B_0$ )

↓

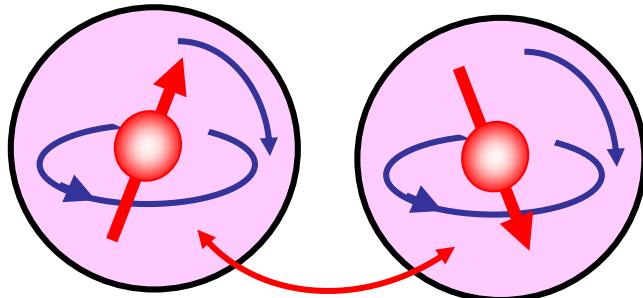
$$H_1 = -\hbar\gamma B_1 \cos(\omega t) S_x = \begin{bmatrix} 0 & -\hbar\omega_0/2 \\ -\hbar\omega_0/2 & 0 \end{bmatrix} \cos(\omega t) = -\hbar\gamma B_1 \cos(\omega t) \sigma_x / 2$$

$$H_{\text{total}} = (-\hbar\gamma/2)[B_0 \sigma_z + B_1 \cos(\omega t) \sigma_x]$$



# 量子もつれ操作のハミルトニアン

スピンの間に働く交換相互作用



$$\text{Spin exchange} = (-J/4)\sigma_1 \cdot \sigma_2$$

Pulsed voltage to on/off exchange coupling  $J$  for  $|\uparrow\rangle|\downarrow\rangle$  and  $|\downarrow\rangle|\uparrow\rangle$

Generate entanglement

$$U_{\sqrt{\text{SWAP}}} |\uparrow\rangle|\downarrow\rangle = (|\uparrow\rangle|\downarrow\rangle + i|\downarrow\rangle|\uparrow\rangle)/(1+i)$$

Swap two qubits

$$U_{\text{SWAP}} |\uparrow\rangle|\downarrow\rangle = |\downarrow\rangle|\uparrow\rangle$$

$$\begin{aligned} H_{\text{exc}} &= (-J/4)\sigma_1 \cdot \sigma_2 = -J(1/4)(I + \sigma_1 \cdot \sigma_2) + (J/4)I \\ &= (-J/2)U_{\text{SWAP}} + (J/4)I \end{aligned}$$

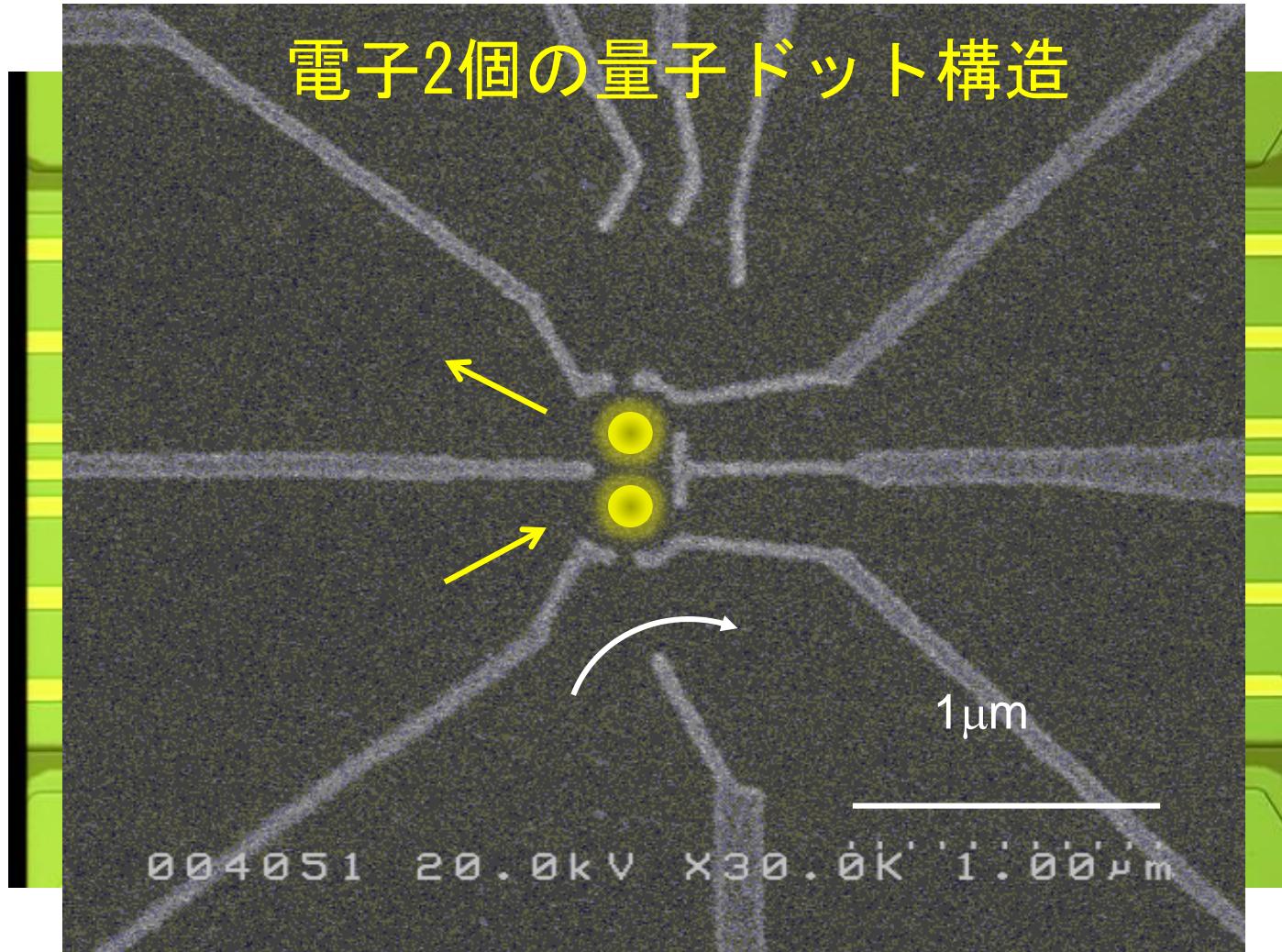
$$\exp[-iH_{\text{exc}}t/\hbar] = \exp[iJU_{\text{SWAP}}t/2\hbar]\exp[-iJt/4\hbar]$$

$$= I\cos(Jt/2\hbar) + iU_{\text{SWAP}}\sin(Jt/2\hbar)$$

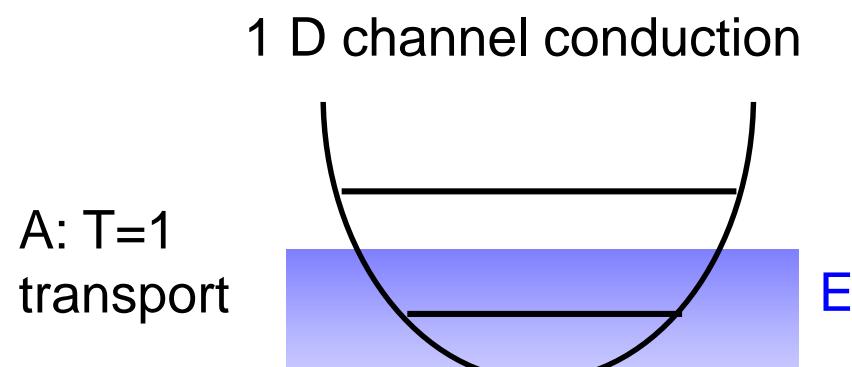
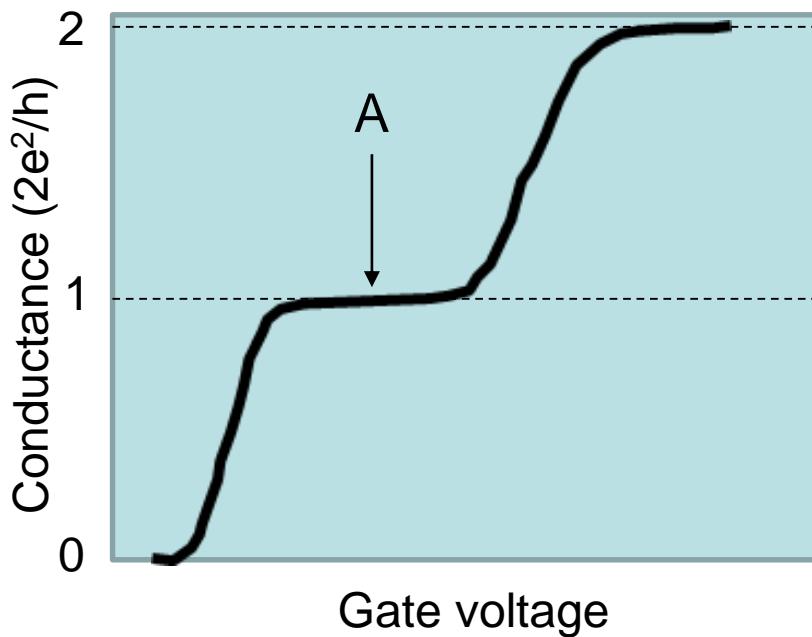
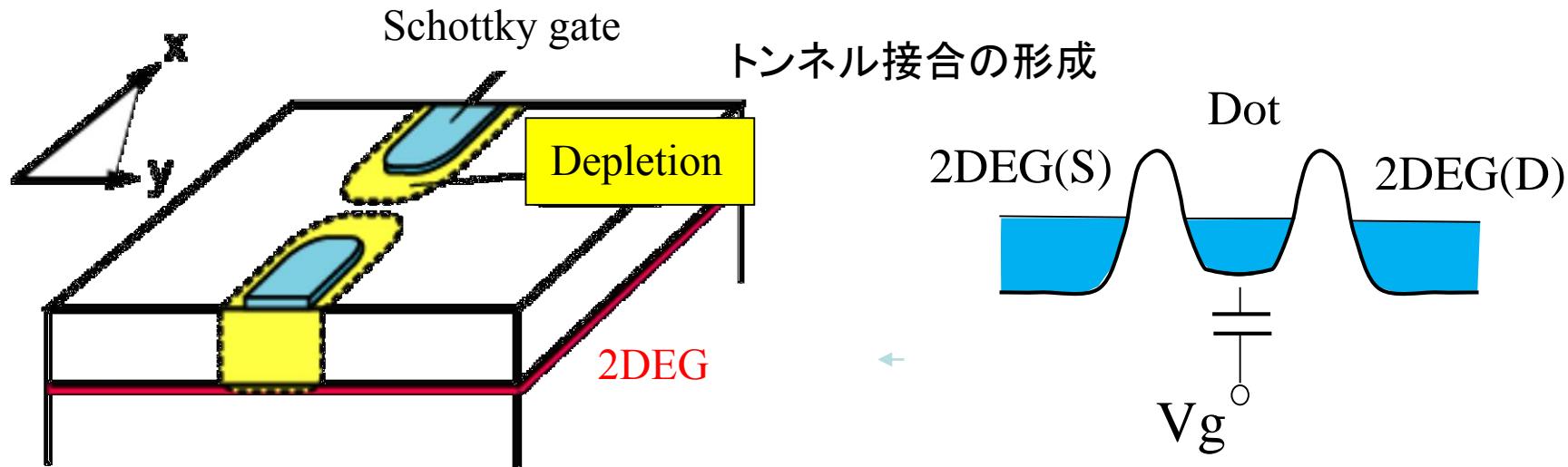
$$= iU_{\text{SWAP}} \text{ for } Jt=\hbar/2$$

$$= \frac{1+i}{\sqrt{2}}U_{\sqrt{\text{SWAP}}} \text{ for } Jt=\hbar/4$$

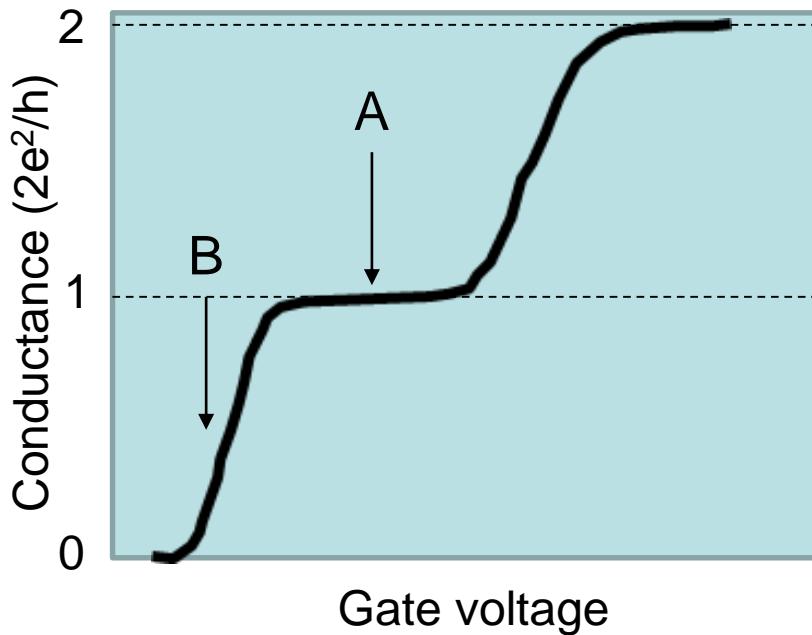
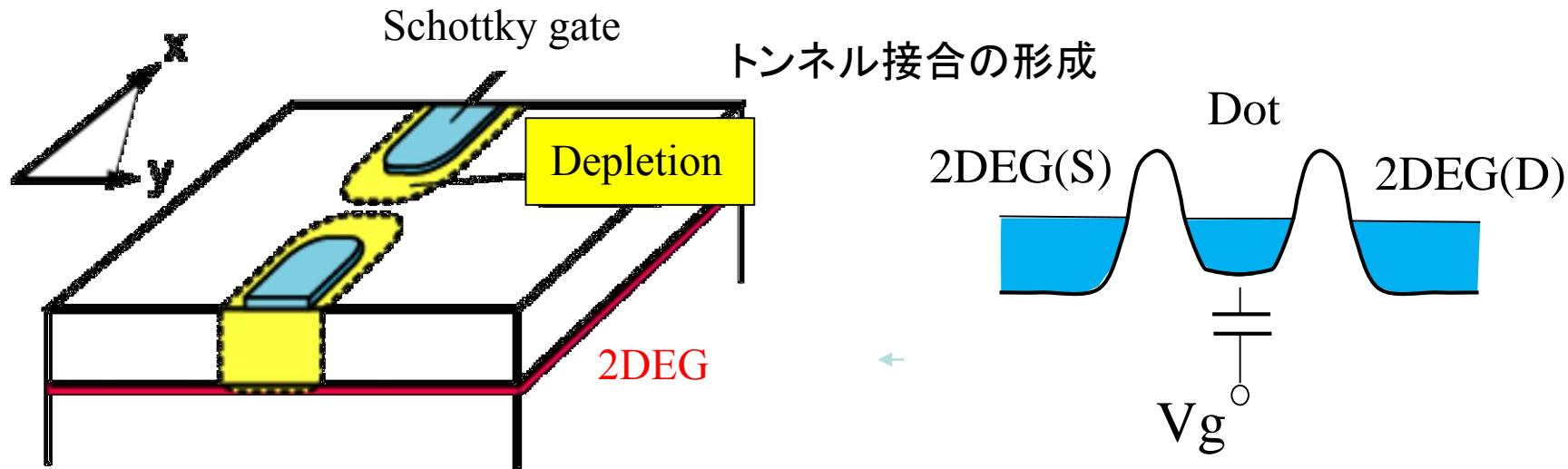
# 量子制御を行うための量子構造の例



# 2次元電子ガスの面内を閉じ込めた構造 (表面ショットキーゲートと量子ポイントコンタクト)

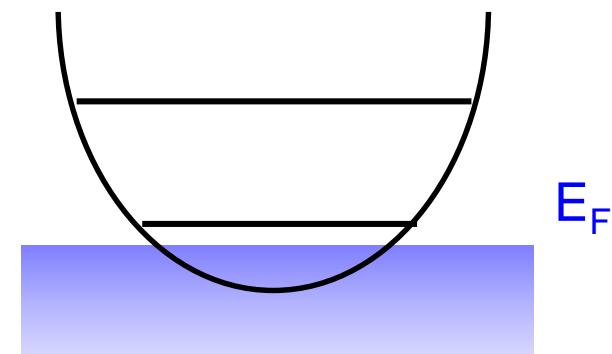


# 2次元電子ガスの面内を閉じ込めた構造 (表面ショットキーゲートと量子ポイントコンタクト)



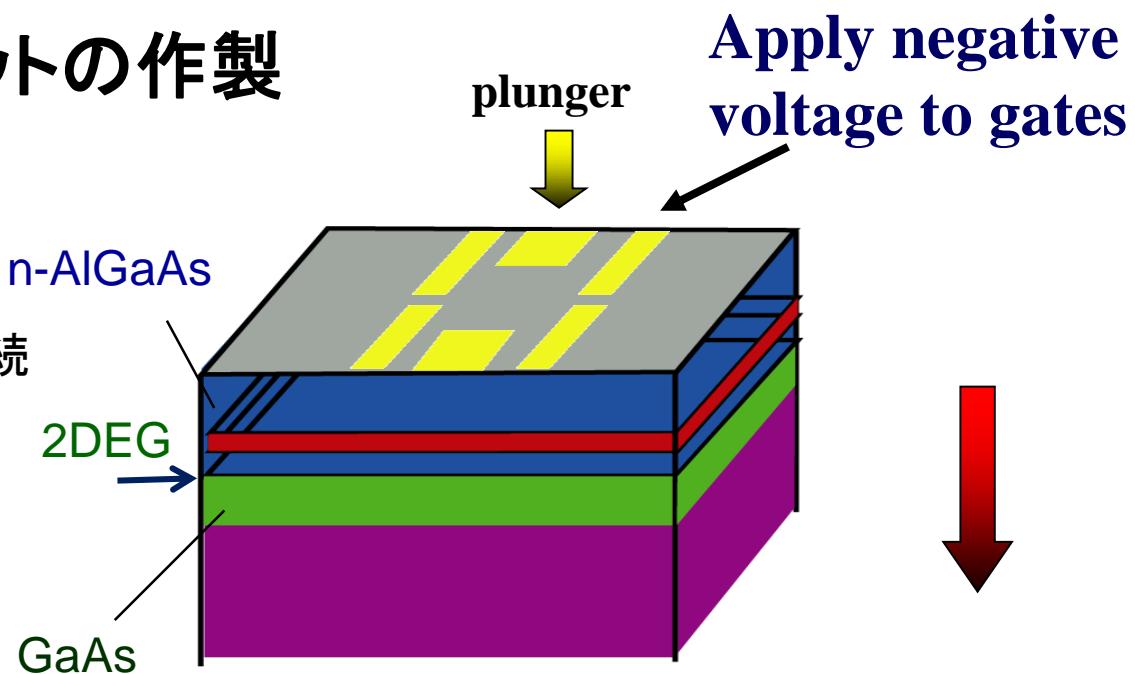
B: Tunneling  
with  $T < 1$

1 D channel conduction

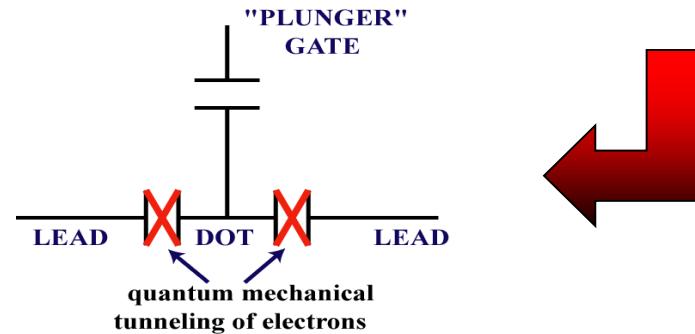
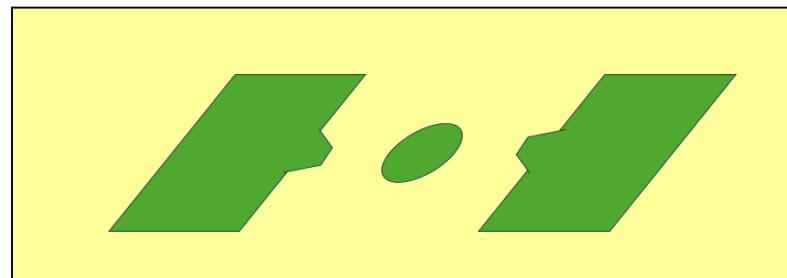
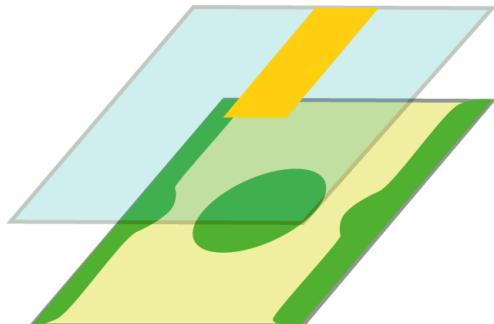


# 量子ドットの作製

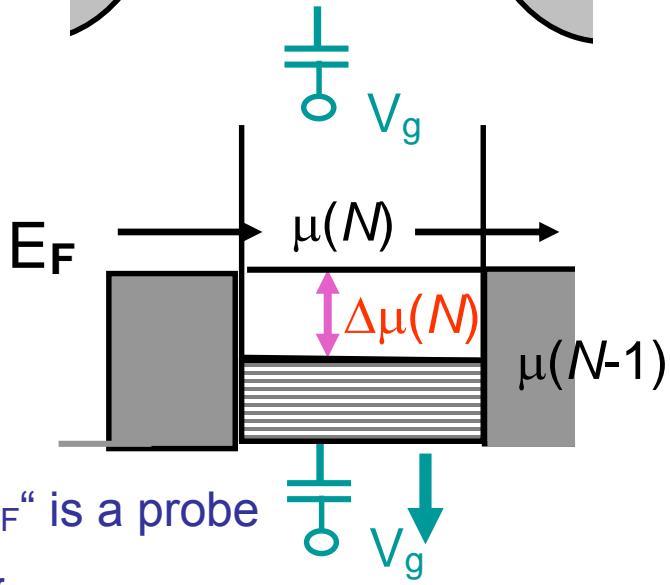
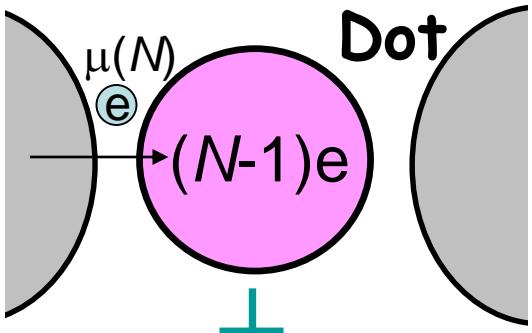
トンネル接合でオーム電極に接続  
容量結合でゲートに接続



プランジャーゲート  
(電子数を変える)



# 量子ドットの電子状態



Total energy for N electrons

$$U(N) = \sum_1^N E_n + E_{\text{int}}(N)$$

Chemical potential

$$\mu(N) = U(N) - U(N-1)$$

$$= E_N + E_{\text{int}}(N) - E_{\text{int}}(N-1)$$

… Energy necessary for putting the  $N$ th electron

$$\Delta\mu(N) = \mu(N) - \mu(N-1) : \text{Increment}$$

$$= \Delta E_N + E_{\text{int}}(N) - 2E_{\text{int}}(N-1) \\ + E_{\text{int}}(N-2)$$

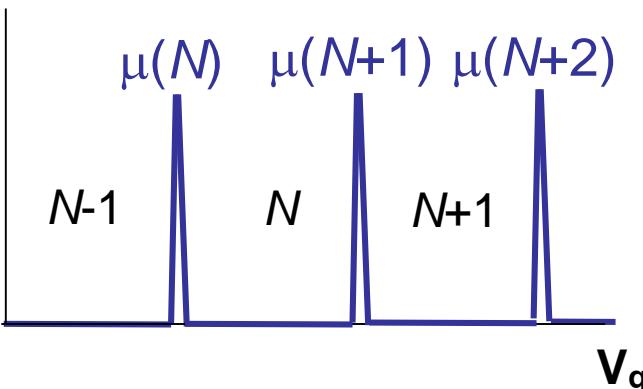
Approximation : large  $N$ , spinless, capacitive

$$\Delta E_N = E_F/N, \quad E_{\text{int}} = (Ne)^2/2C$$

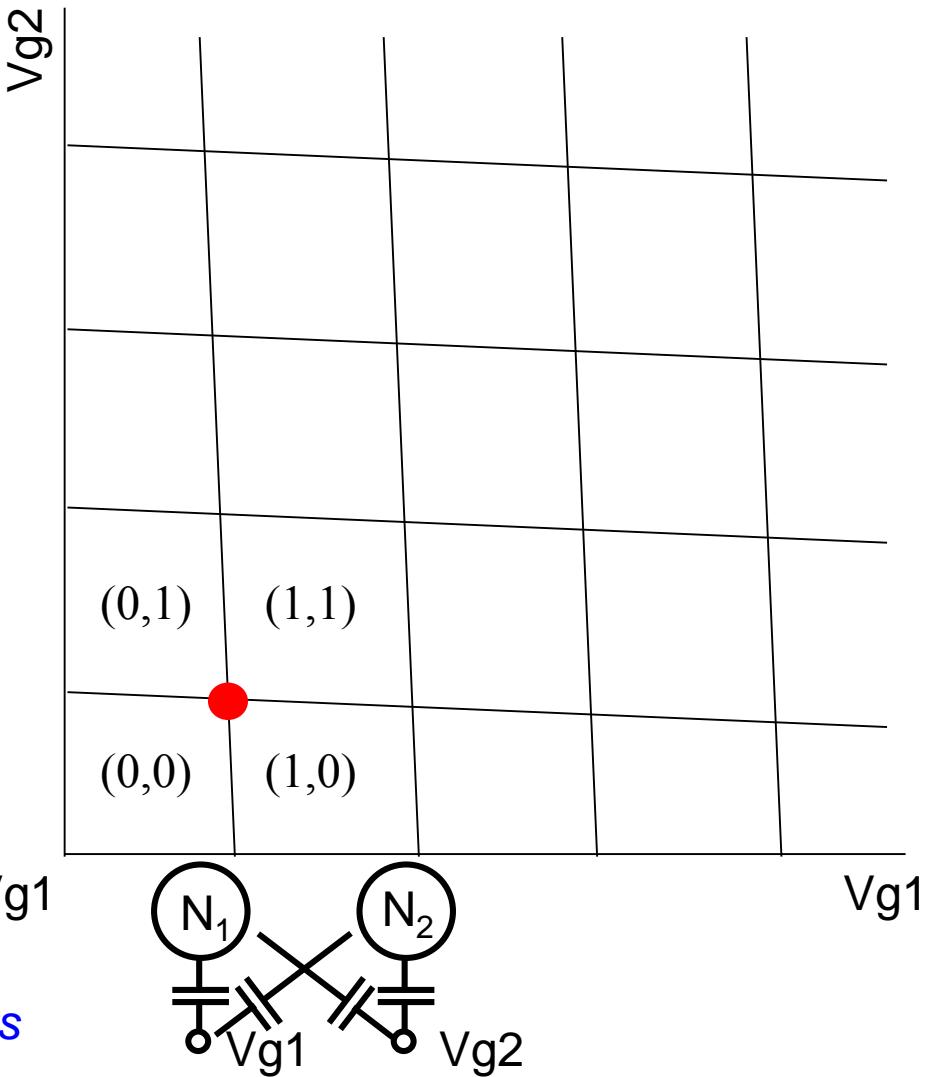
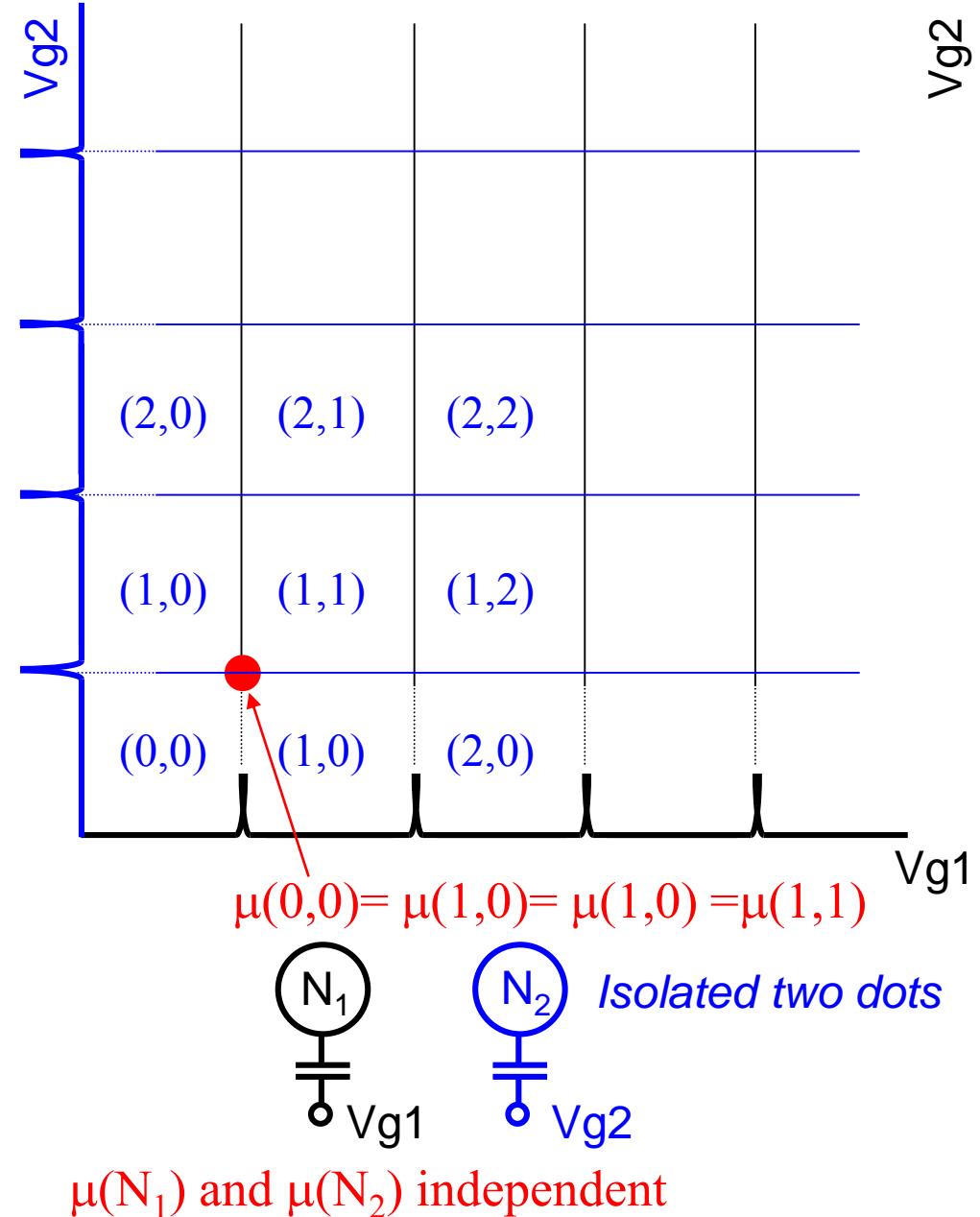
$$E_{\text{int}}(N) - 2E_{\text{int}}(N-1) + E_{\text{int}}(N-2) = \frac{e^2}{C}$$

$$\Delta\mu(N) = E_F/N + e^2/C \approx e^2/C$$

单一電子帯電エネルギー (クーロンギャップ)



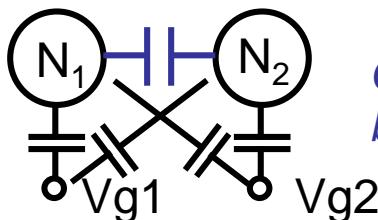
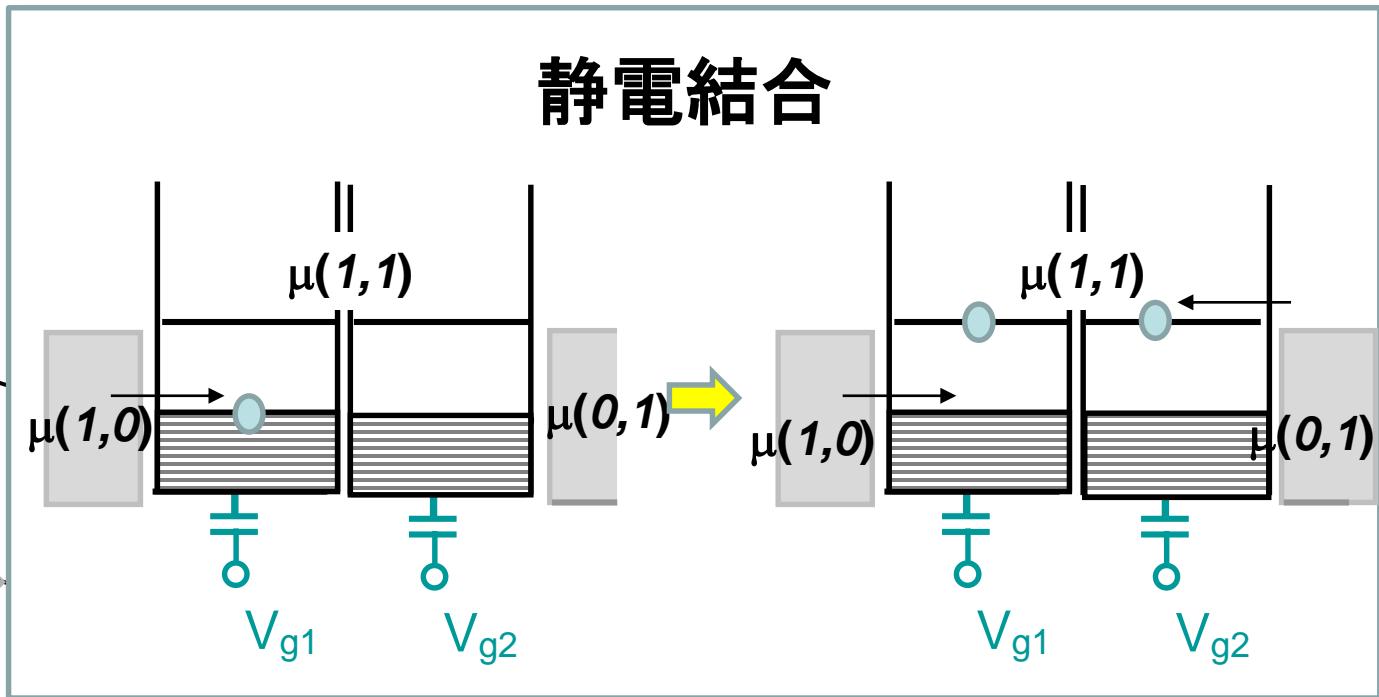
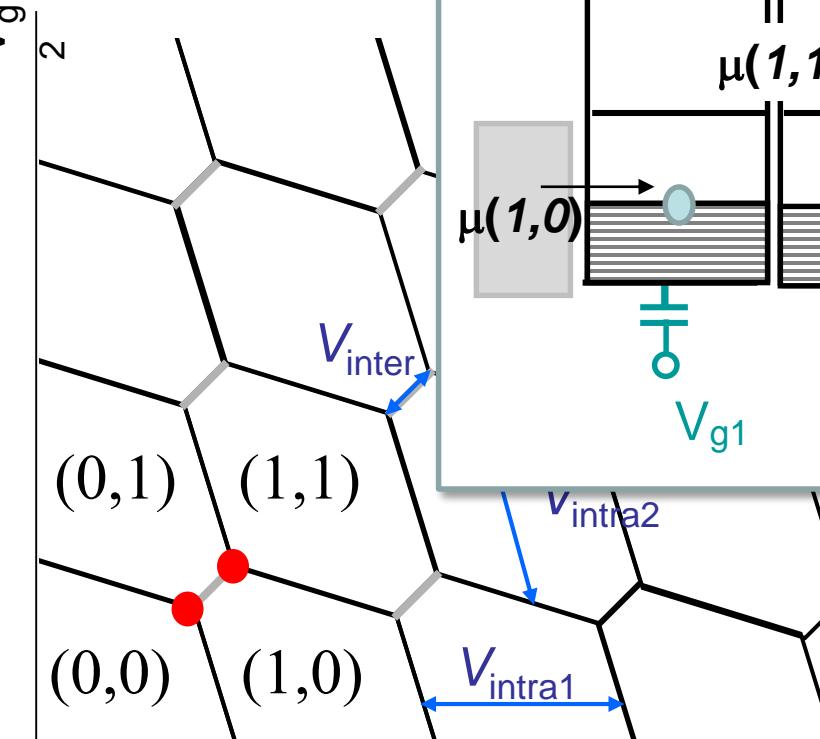
# 結合量子ドットの電子状態



Two dots are only coupled through the gates.

# 静电結合

*Electrostatic*

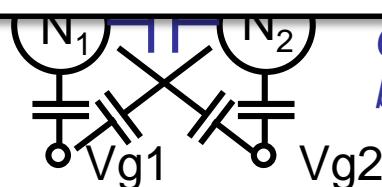
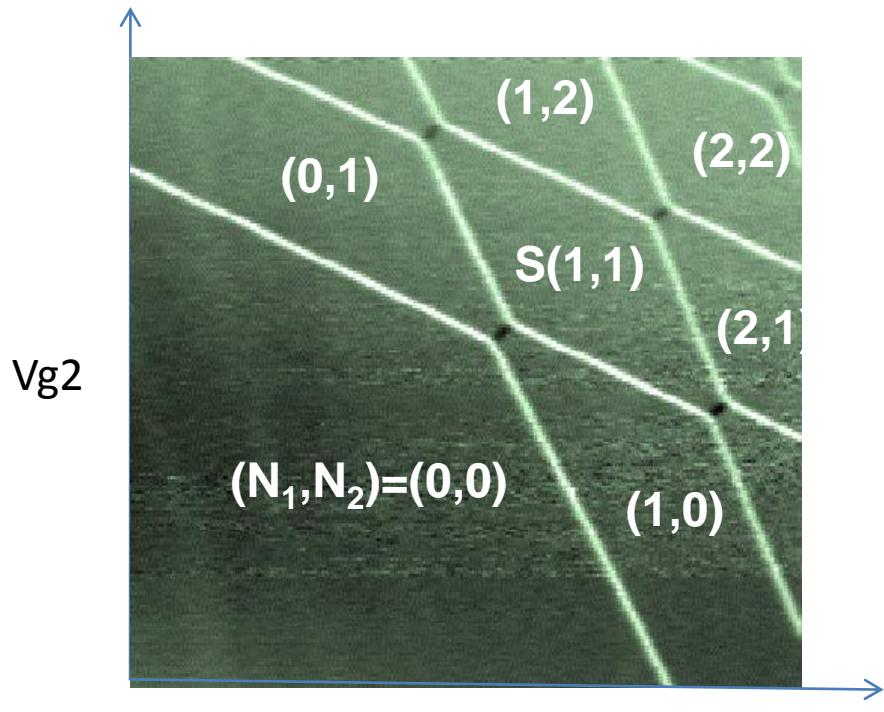


*Capacitive coupling  
between two dots*

One-electron charging in one dot raises the electrostatic potential of the other dot by  $E_c = e^2/C_{\text{inter}}$ , which is equivalent to  $\mu(1,1) - \mu(0,0)$ .

# 結合量子ドットの電子状態

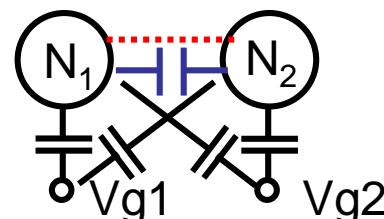
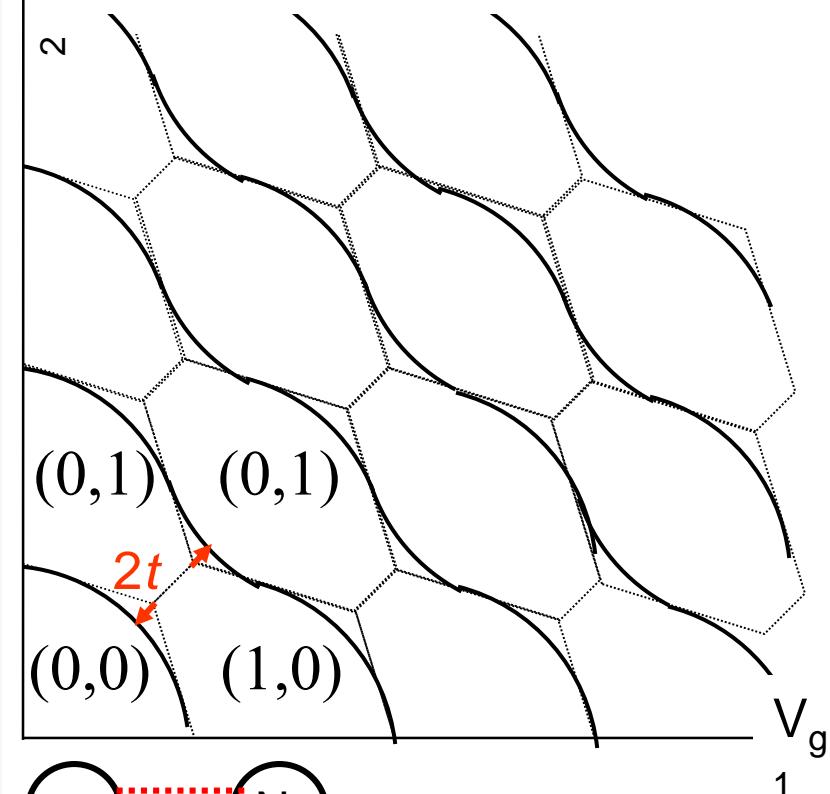
## Electrostatic



*Capacitive coupling  
between two dots*

One-electron charging in one dot raises the electrostatic potential of the other dot by  $E_c = e^2/C_{\text{inter}}$ , which is equivalent to  $\mu(1,1) - \mu(0,0)$ .

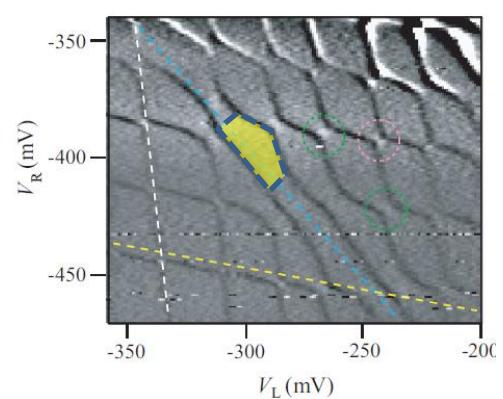
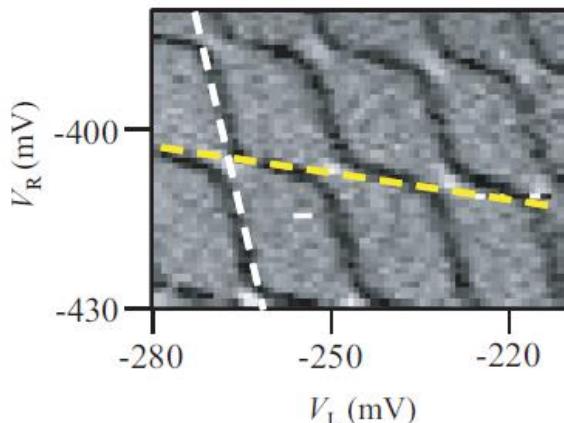
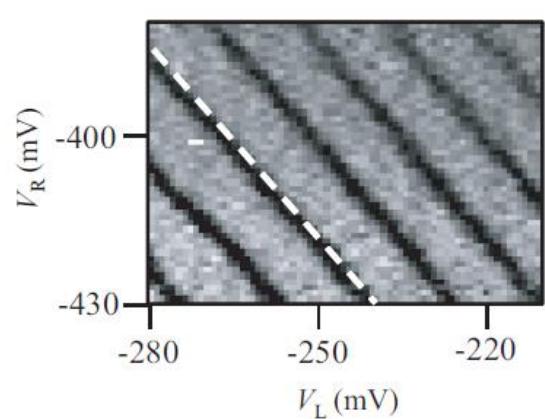
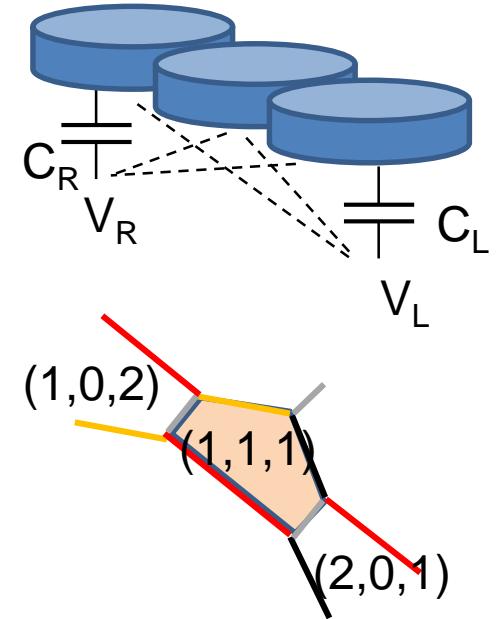
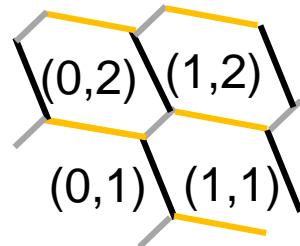
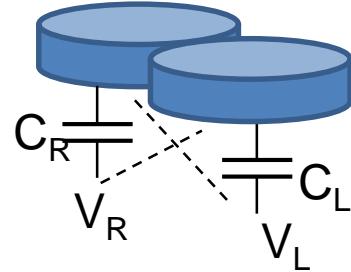
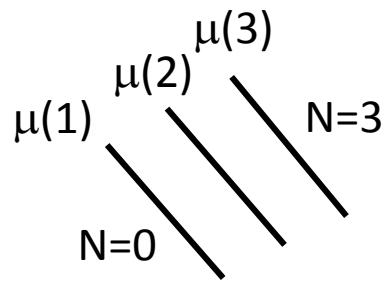
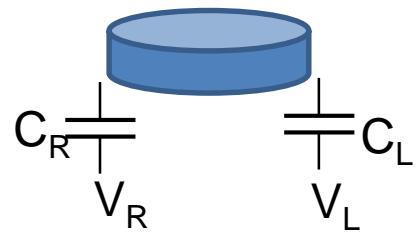
## Electrostatic+Quantum mechanical



*Tunnel coupling  
between two dots*

Degeneracies between different charge states are lifted by the tunnel coupling. The total electron number is only well defined.

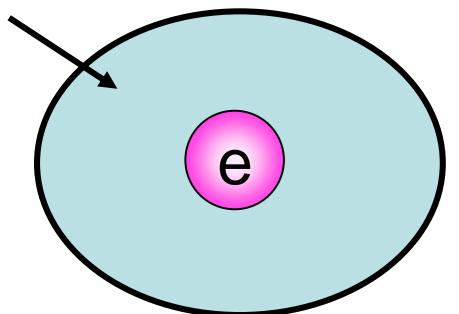
# 多重量子ドットの電荷状態



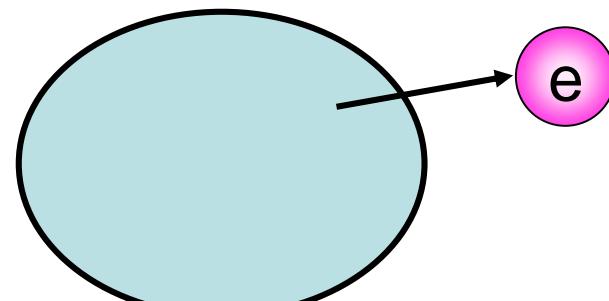
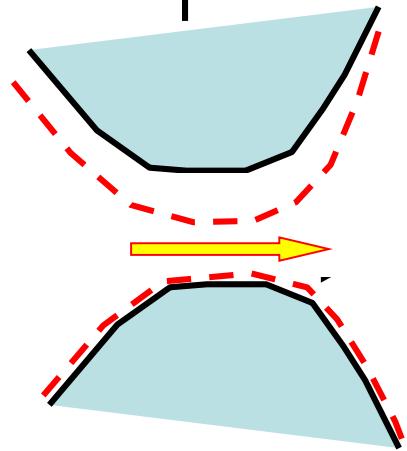
# 単一電荷の実時間測定

静電ポテンシャルの増加分

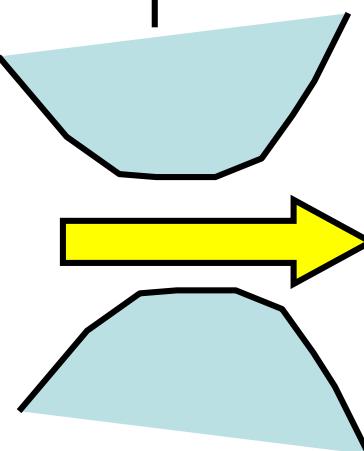
$$\Delta V = e/C \text{ ("small")}= 0.1V$$



$$\begin{array}{c} + \\ \parallel \\ - \end{array} C_g$$

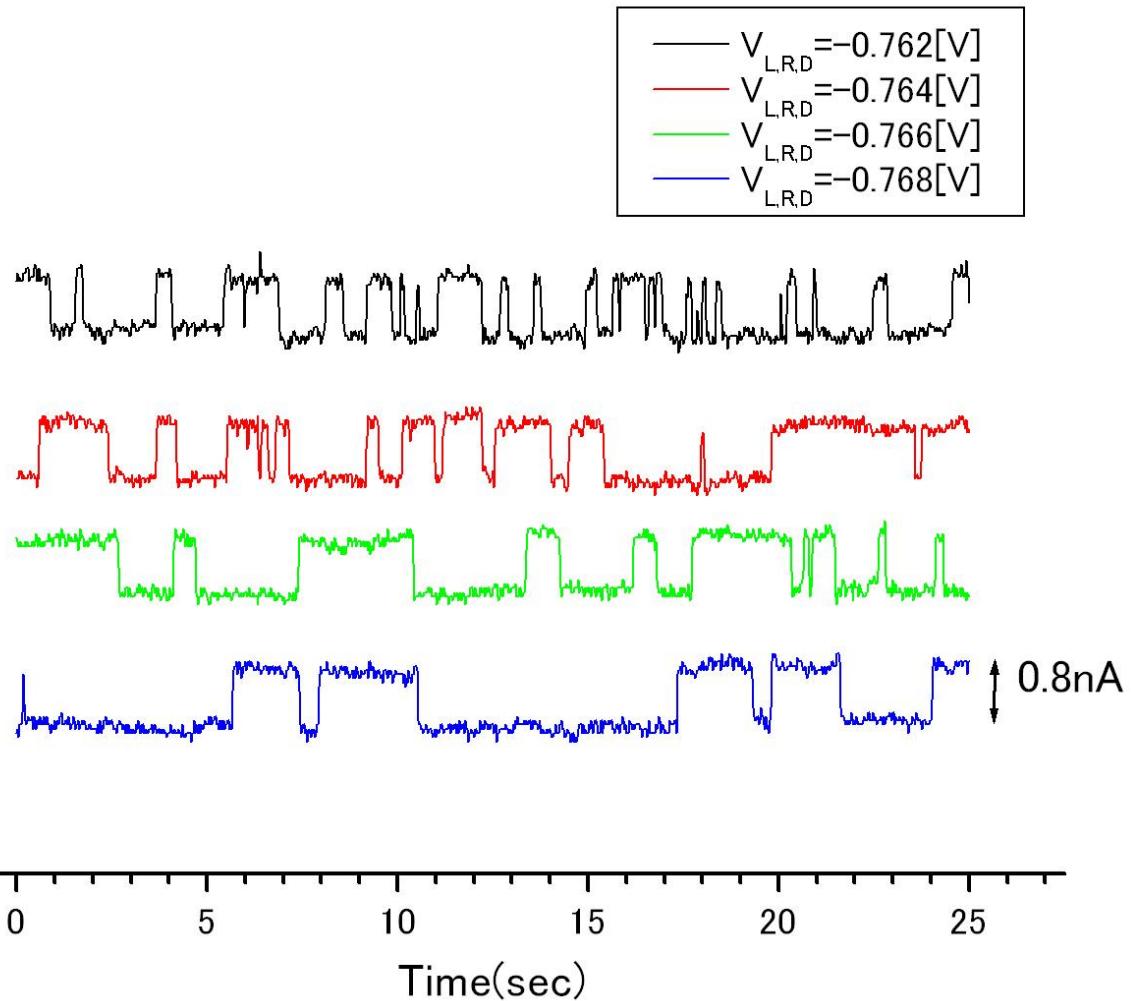
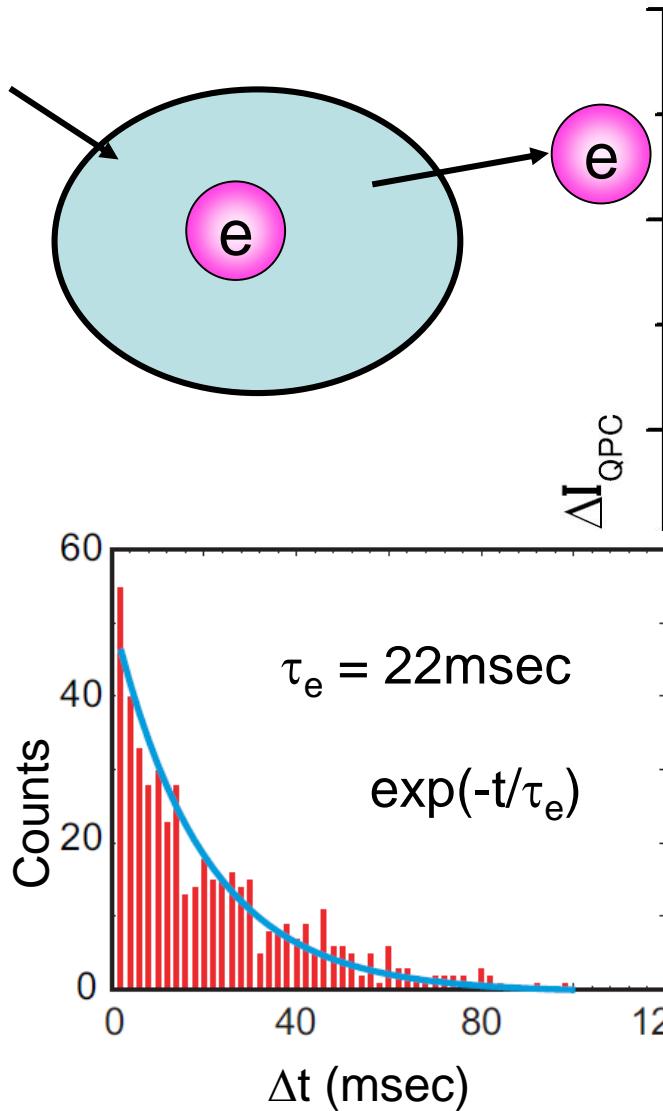


$$\begin{array}{c} + \\ \parallel \\ - \end{array} C_g$$



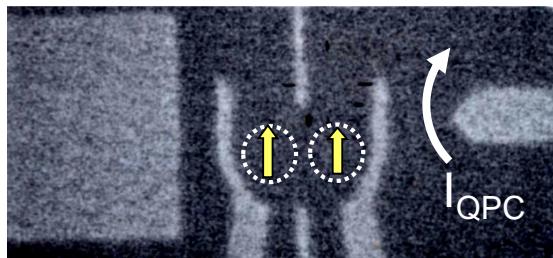
# 電子1個の実時間測定

1個単位の電子トンネルの実時間観測



# 初期化と読み出し

Double dots

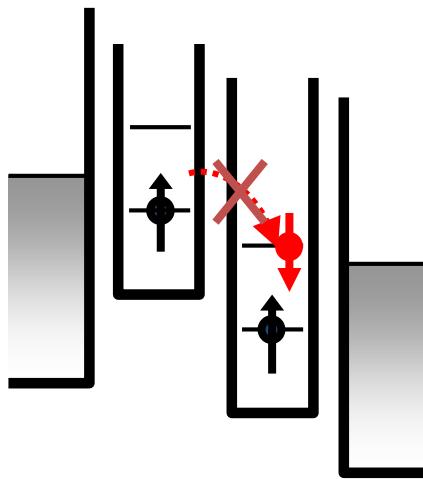


Ono and ST, *Science* 02; *PRL* 04

Petta et al. *Science* 05

Koppens et al. *Nature* 07

Nowack et al. *Science* 07



# 初期化と読み出し

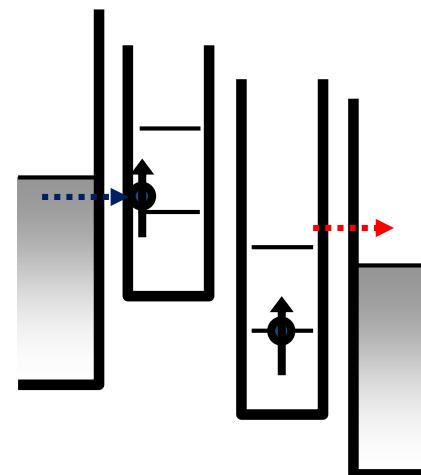
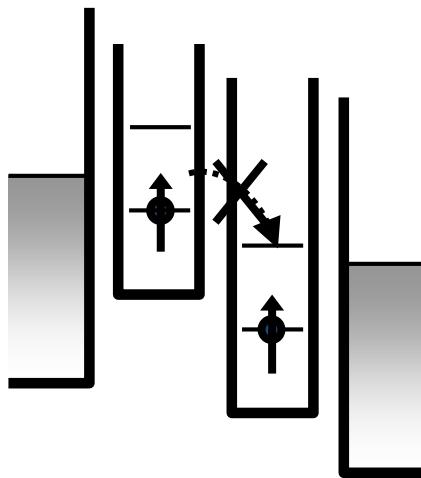
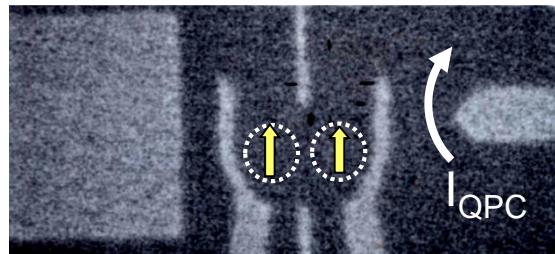
Double dots

Ono and ST, *Science* 02; *PRL* 04

Petta et al. *Science* 05

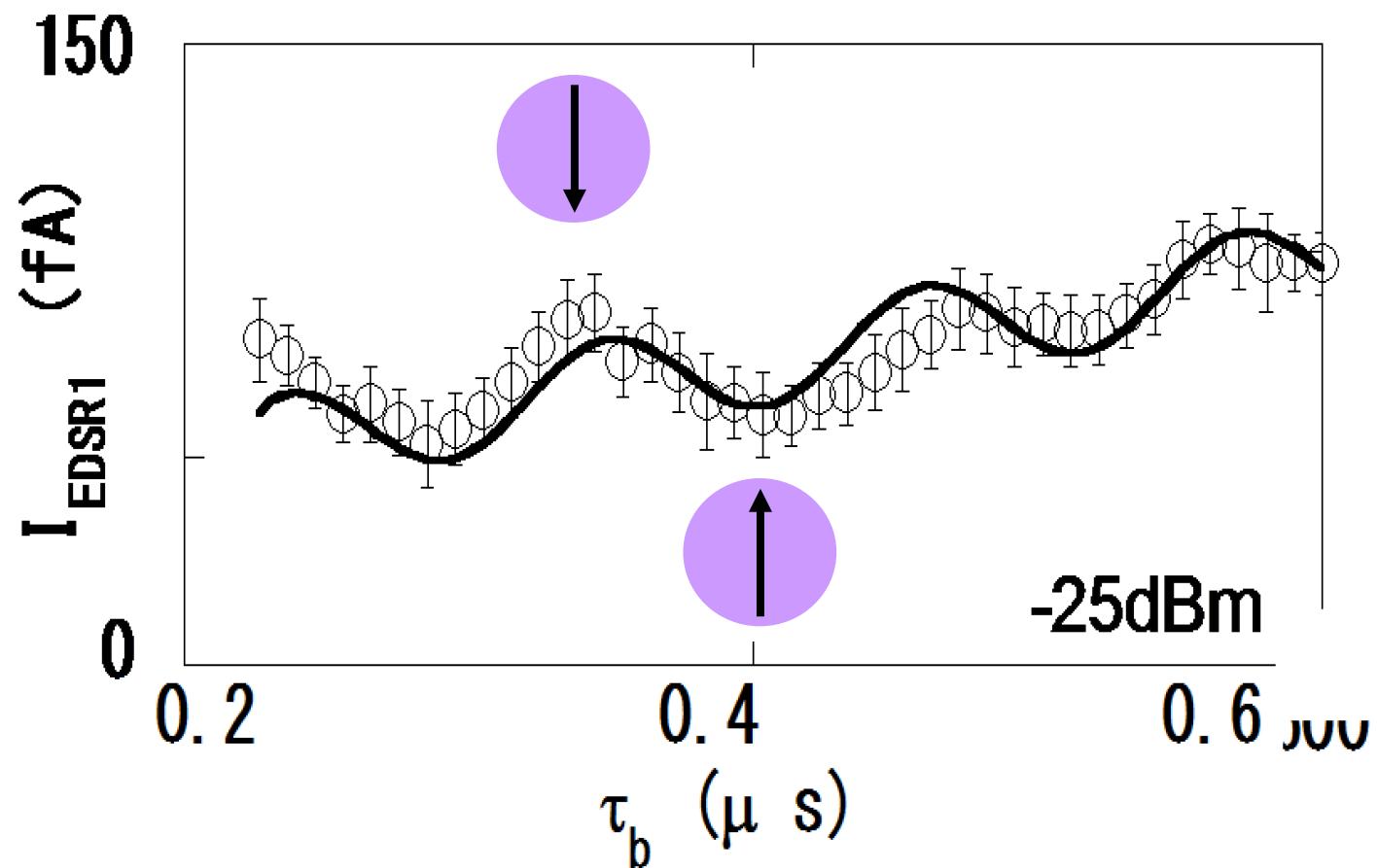
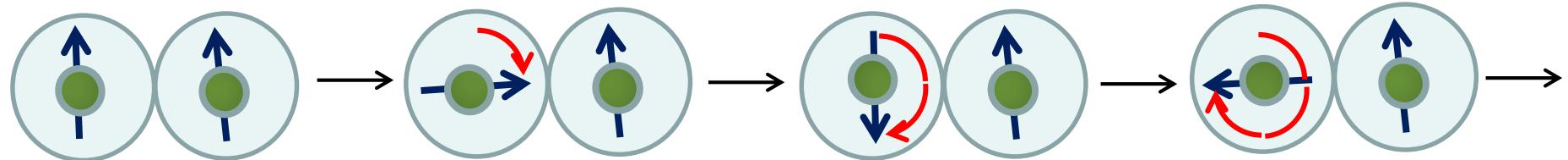
Koppens et al. *Nature* 07

Nowack et al. *Science* 07



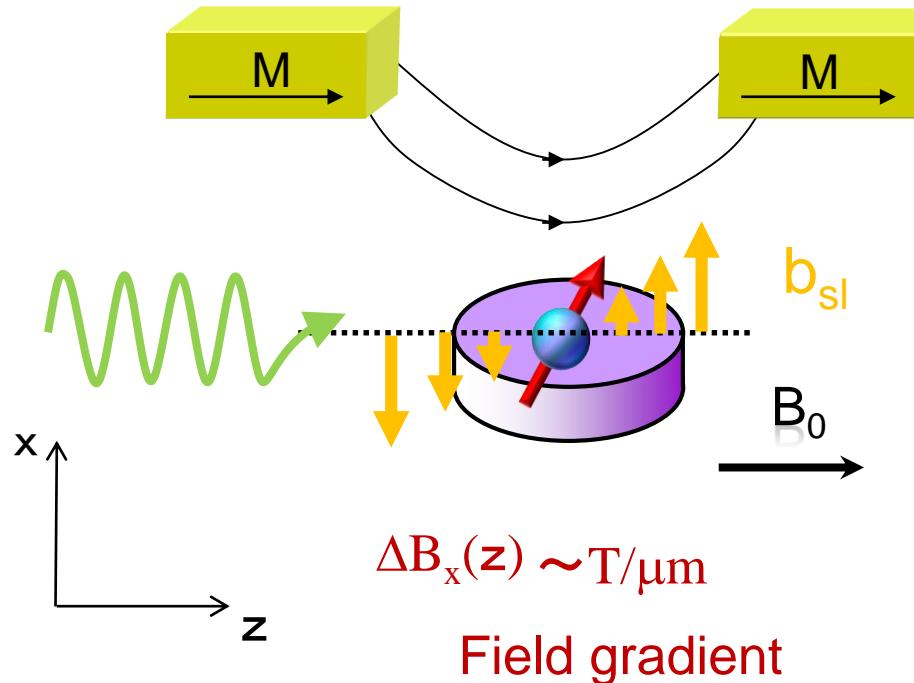
Transport is terminated by  
forming a triplet state  
= Initialization

# スピン回転の検出



# 微小磁石を用いた量子ビット(Xゲート)

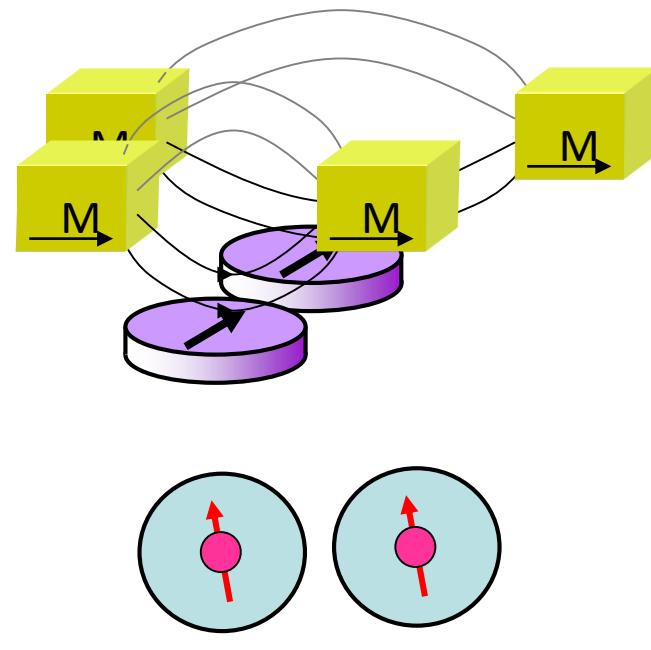
Nat. Phys. 2010; PRL2010, 2011



$$\Delta B_z \longrightarrow$$

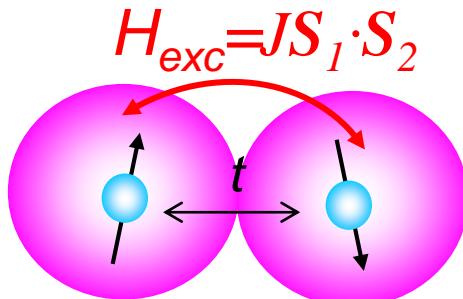
10 to 50 mT/0.1 μm

Inhomogeneous Zeeman field

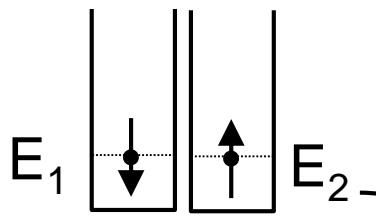


$\Delta B_z$  is the in-plane stray magnetic field// $B_{ext}$

# 2重量子ドットの2スピン量子操作



$$\delta = 0$$



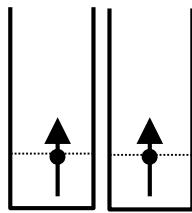
Singlet

$$(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)/\sqrt{2}$$

$S(1,1)$

Triplet

$$(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle)/\sqrt{2}$$



$$|S(1,1)\rangle \pm |S(0,2)\rangle$$

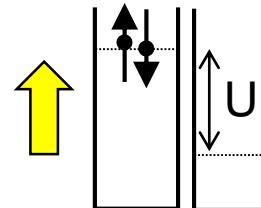
$$\frac{\sqrt{2}}{J \sim t}$$

$$|\downarrow\rangle|\downarrow\rangle \quad T_{-}(1,1)$$

$$J = E_{T0} - E_S > 0$$

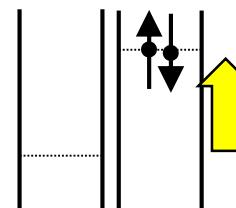
Singlet is ground state

$$U \sim \Delta E \gg J, t$$



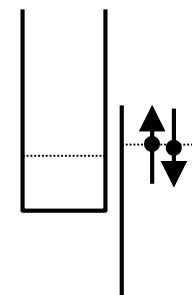
Doubly occupied Singlet

$$|\uparrow\downarrow\rangle|V\rangle S(2,0)$$



$$|V\rangle|\uparrow\downarrow\rangle S(0,2)$$

Energy detuning  $\delta = E_1 - E_2$



When  $\delta \sim U$ ,  $S(0,2)$  and  $S(1,1)$  are hybridized.

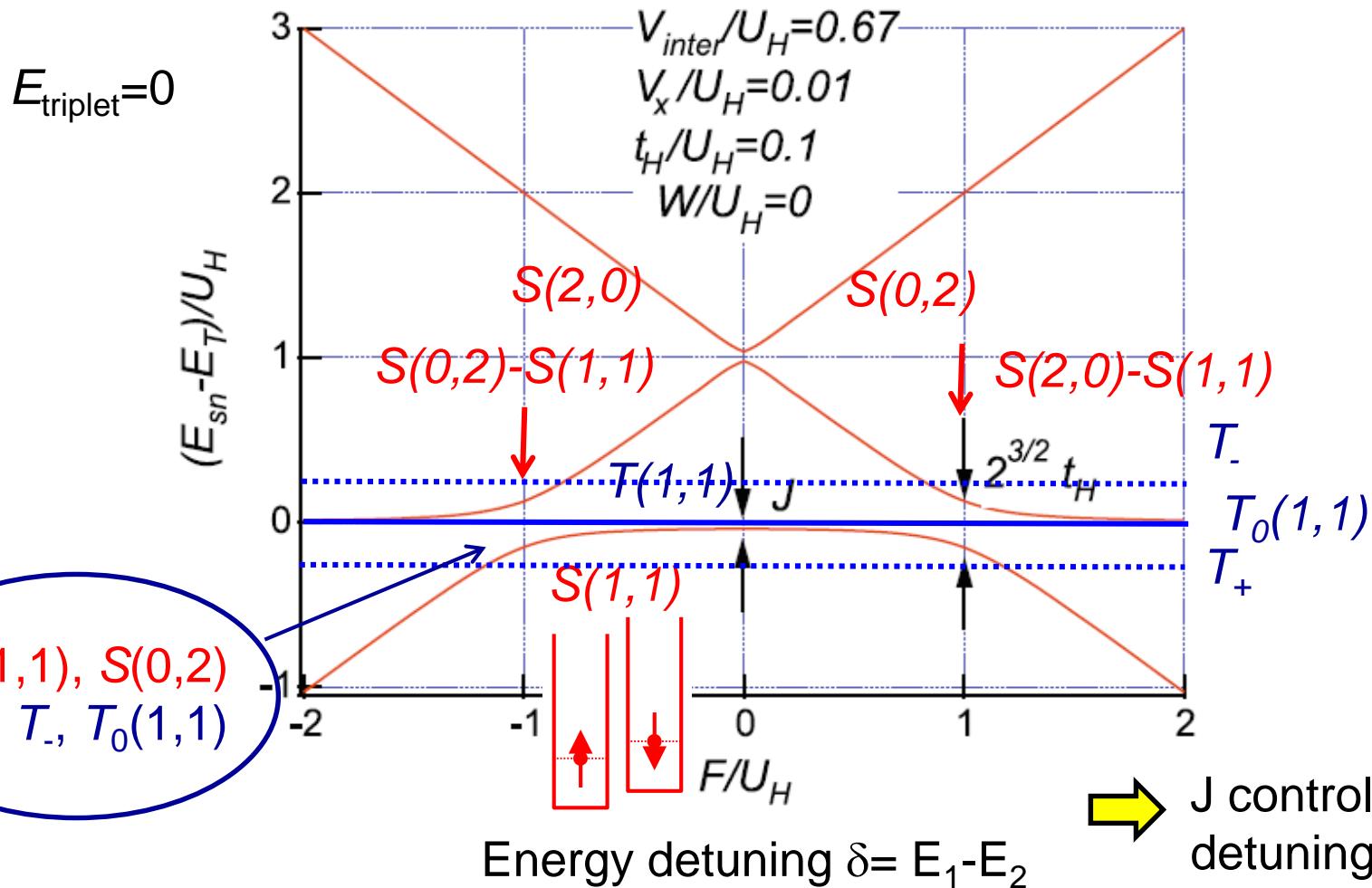
*Used for quantum gate operation*

# 2電子状態エネルギーの離調依存性

Small detuning :  $J=4t^2/U_H$

Tokura et al. Springer 2009

Large detuning:  $J = \frac{1}{2}(\sqrt{U_H^2 + (4t)^2} - U_H) - 2V_x - b\delta W^2$

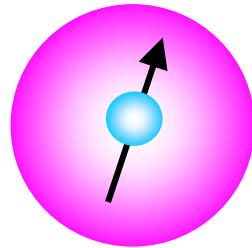
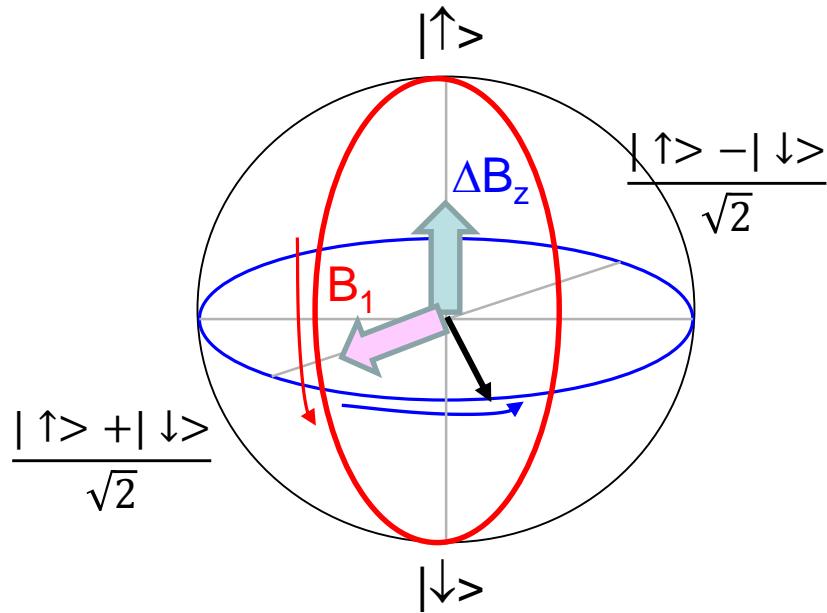


# ブロッホ球上の状態ベクトル操作

回転座標系:  $E_z = \hbar\omega$

$$H_{\text{total}} = (-\hbar\gamma/2)[B_0\sigma_z + B_1\cos(\omega t)\sigma_x]$$

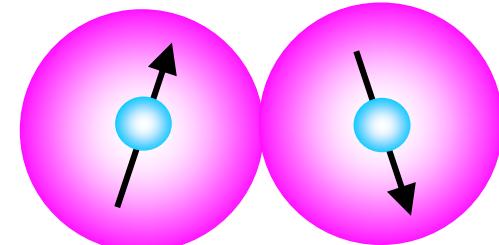
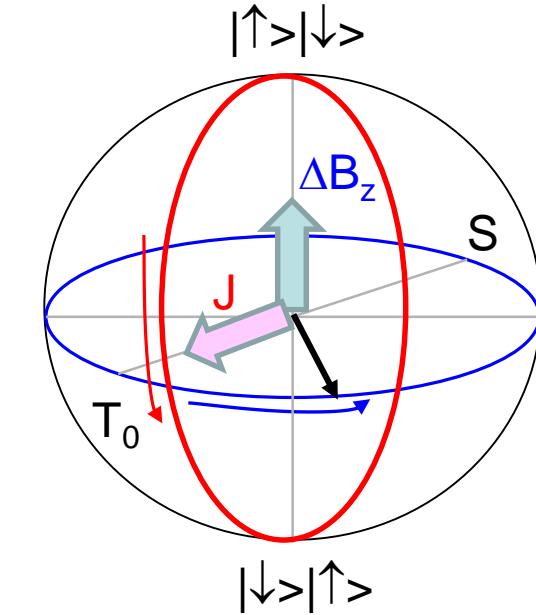
$\Delta B_z$ : 余剰ゼーマン磁場



実験室座標系

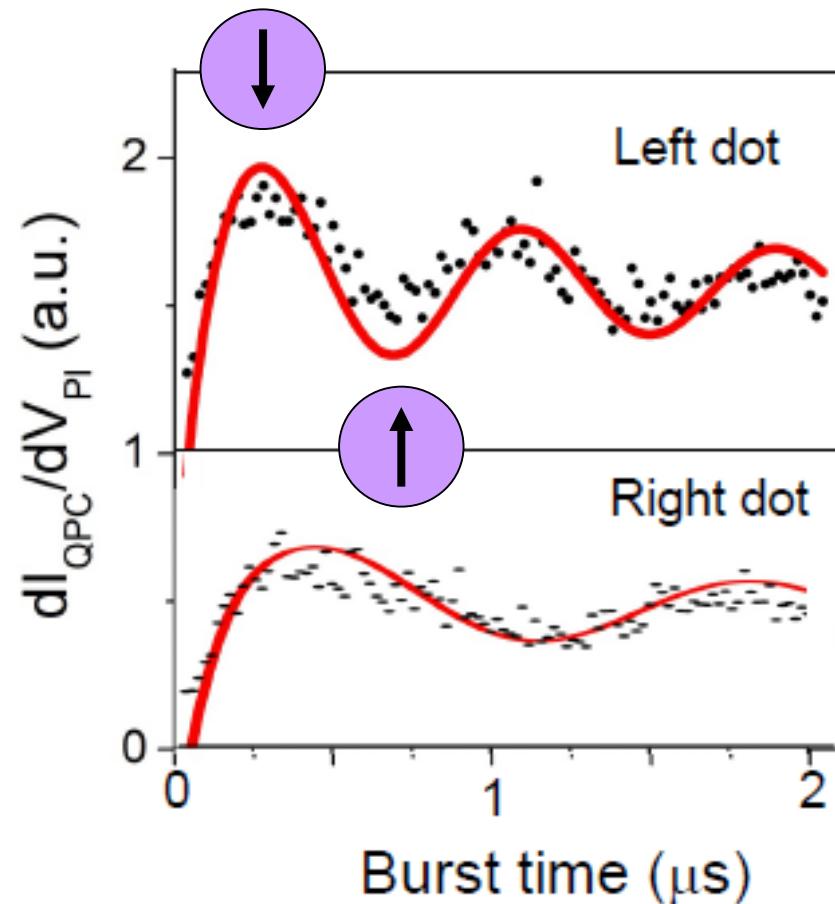
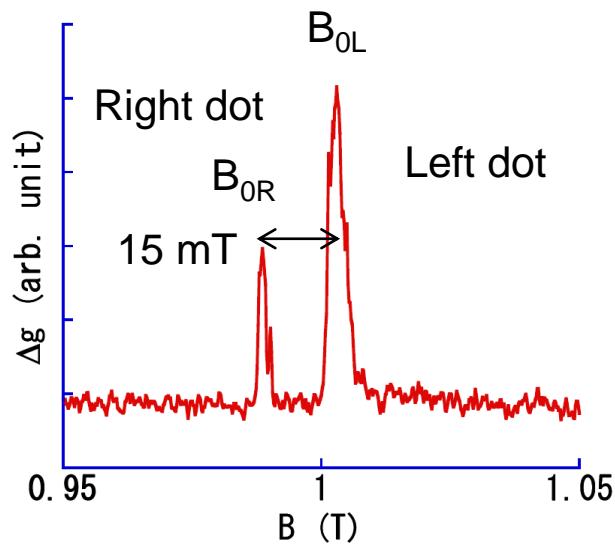
$J$ : ドット間交換結合

$\Delta B_z$ : ドット間のゼーマン磁場差



# 2重量子ドットによる2ビット

QPC charge sensing to detect to charge change in the double dot

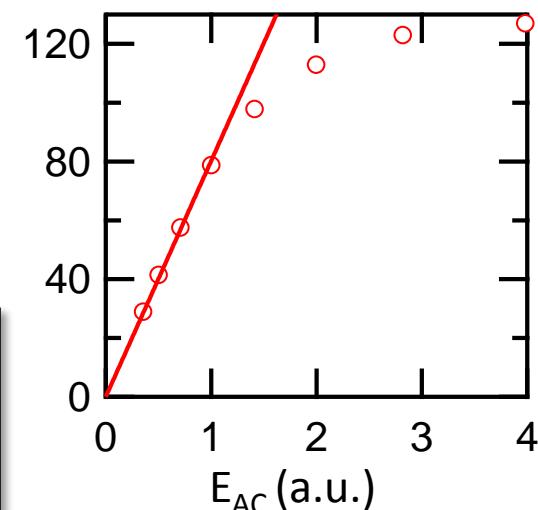
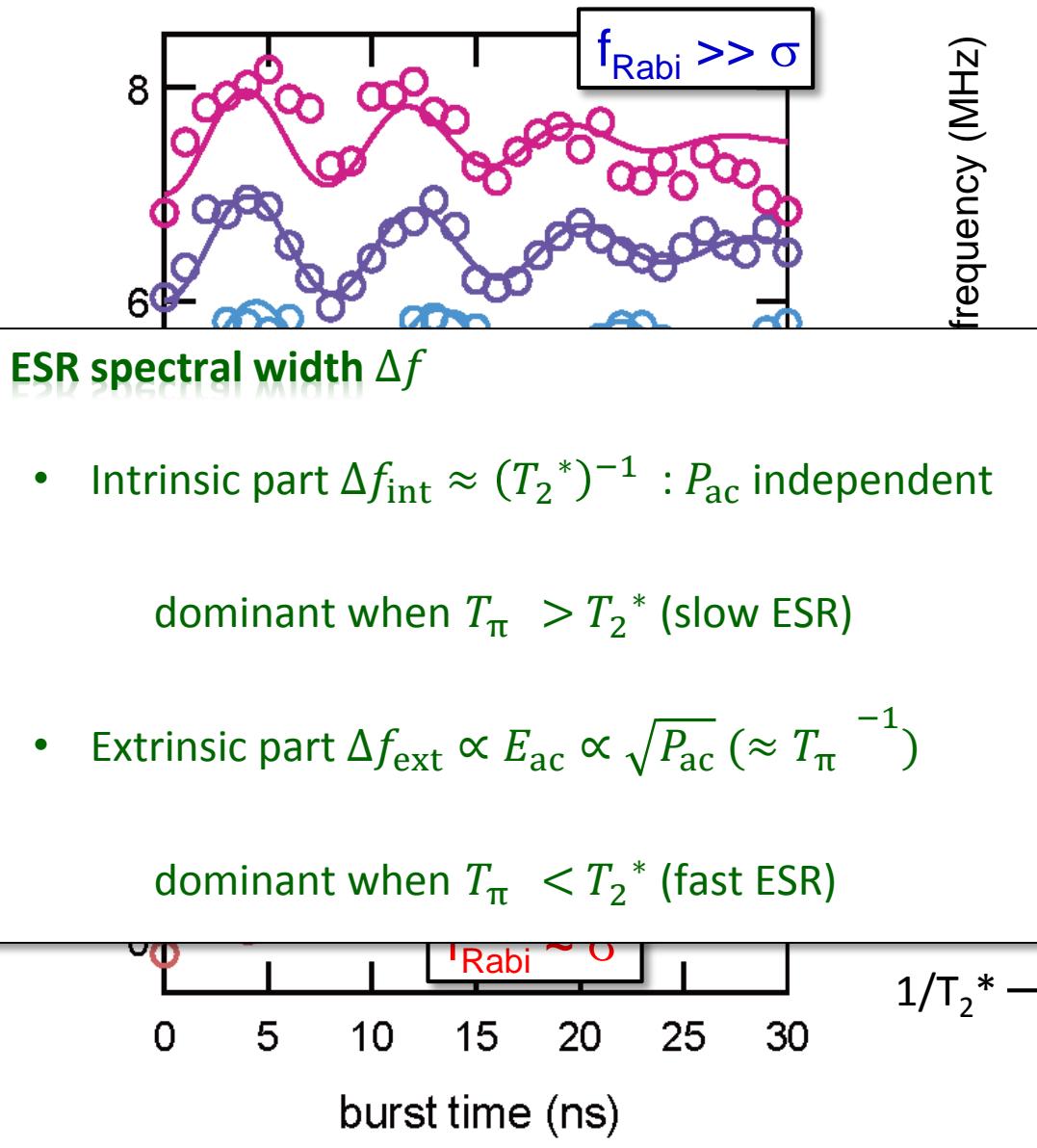


T. Obata et al. *PRB* 2010

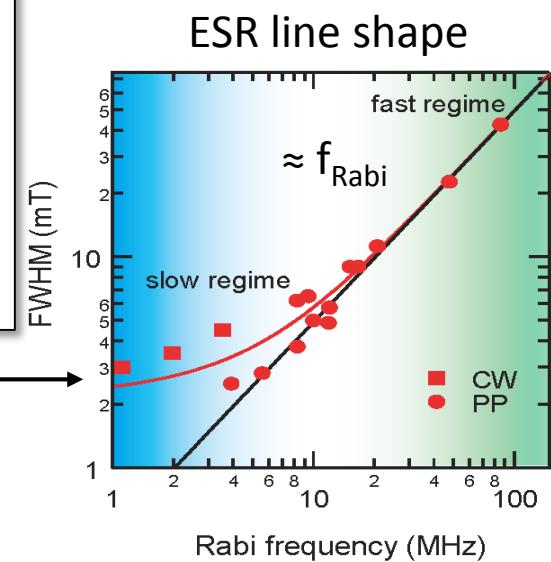
R. Brunner et al. *PRL* 2011

# 最高速度ラビ振動

J. Yoneda et al. PRL 2015



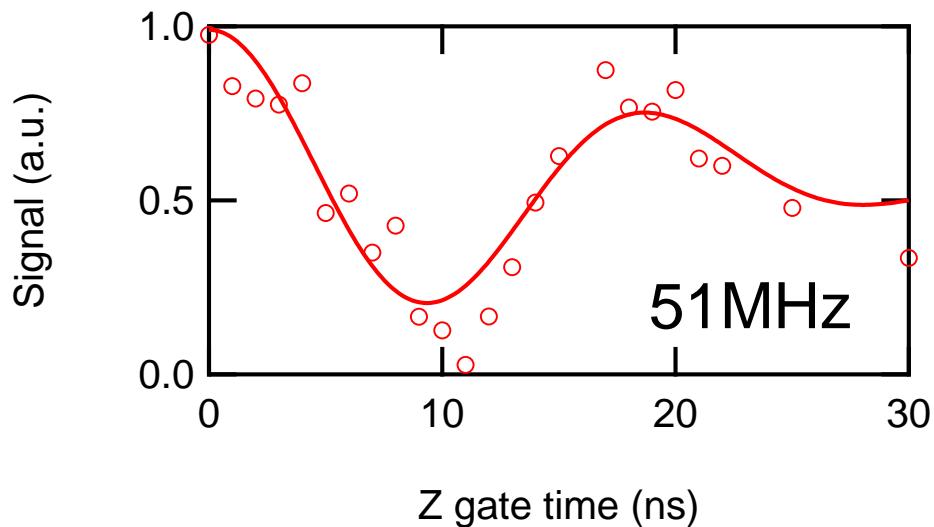
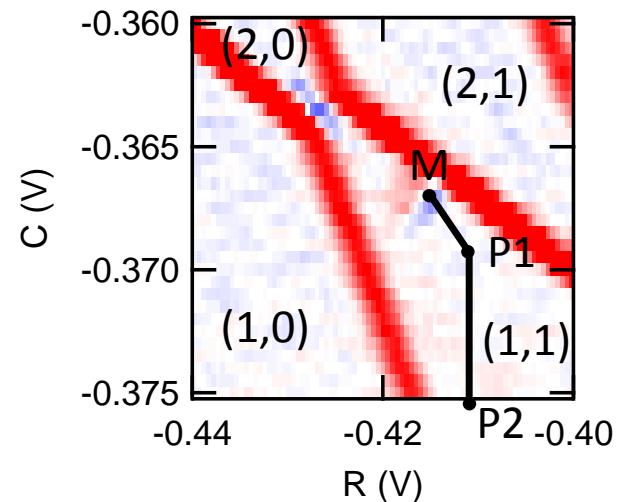
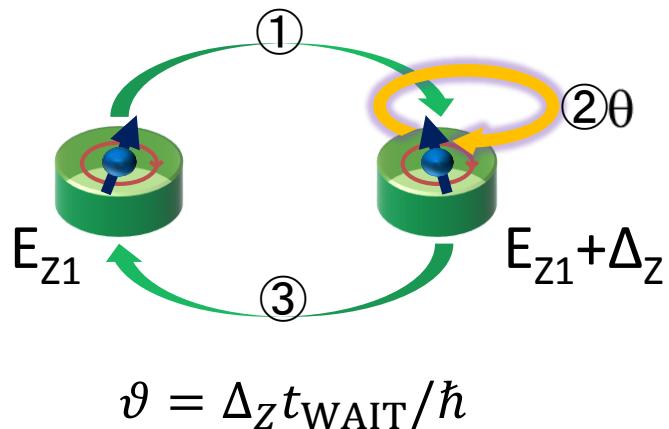
shortest X-gate = 2 nsec  $\ll T_2^*$



# 位相ゲート

Phase shift of Larmor precession = Rotation about z

J. Yoneda et al. PRL 2015

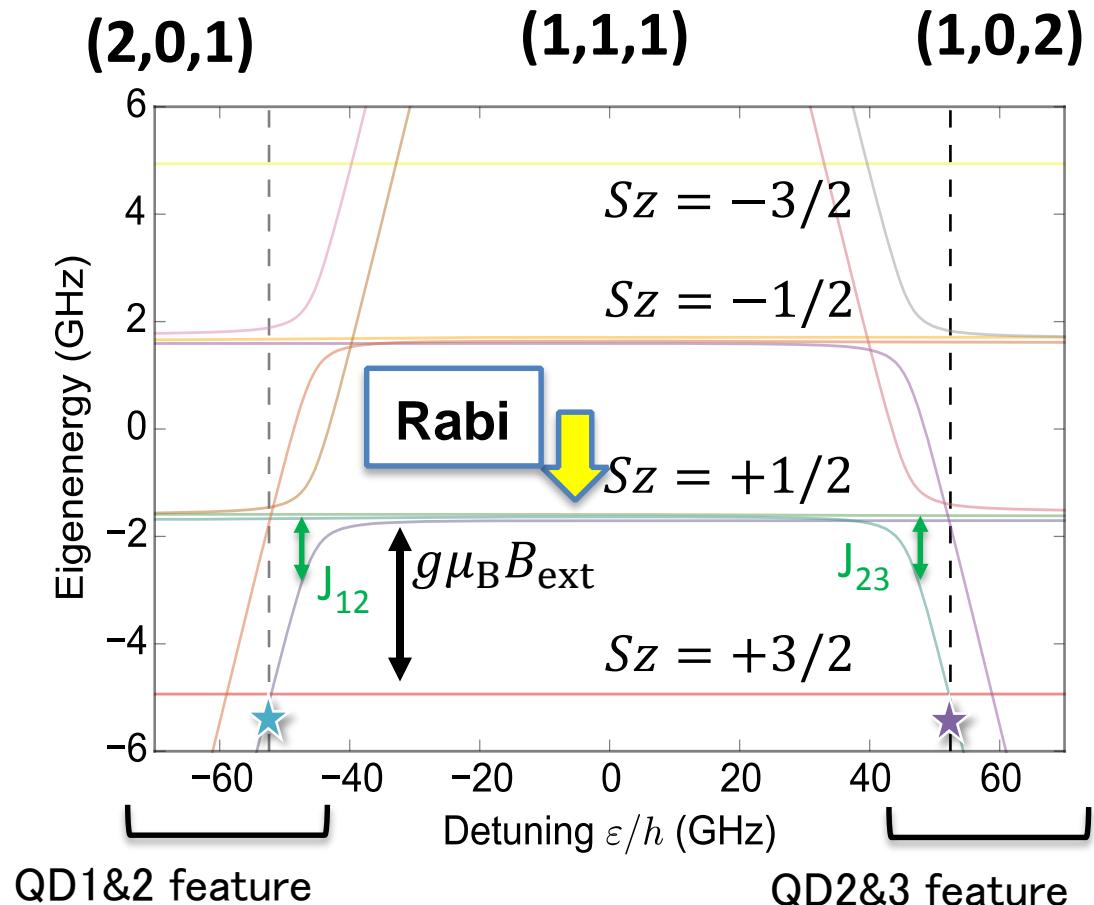
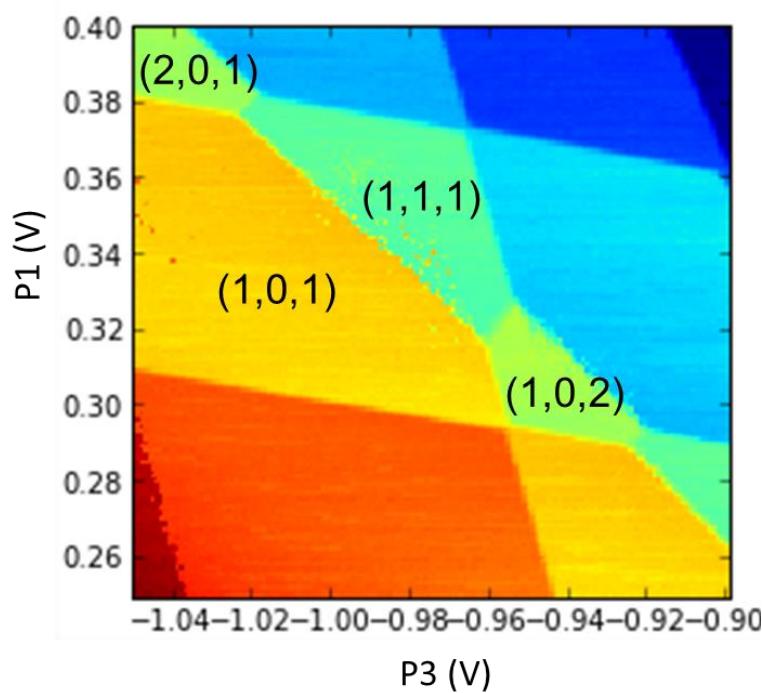
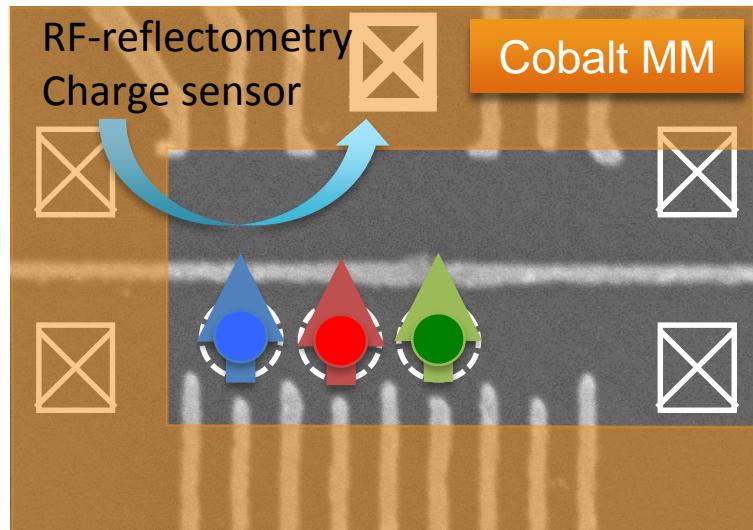


Min.  $Z(\pi/2)$  time = 5 nsec  $\ll T_2^*$

$$R_z(\theta) = R_x(\pi/2)R_y(\theta)R_x(-\pi/2)$$

...Temporal detuning of  $\Delta E_{\text{Zeeman}}$   
 $2\pi\Delta t\Delta f_L$  ( $\Delta E_{\text{Zeeman}} = h\Delta f_L$ )

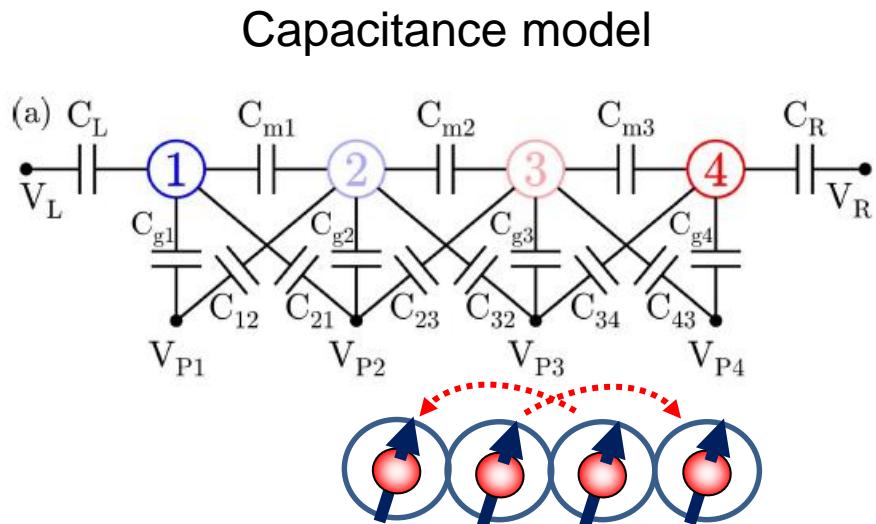
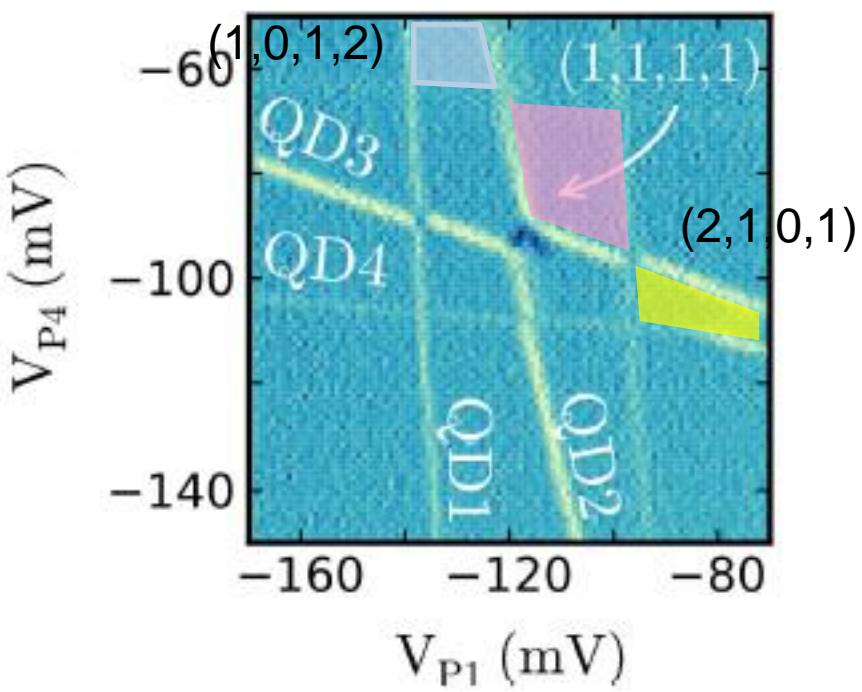
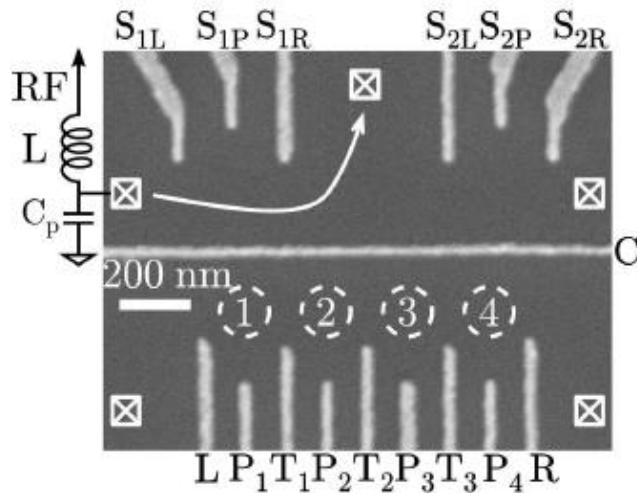
# Single-spin Based Qubits in TQD



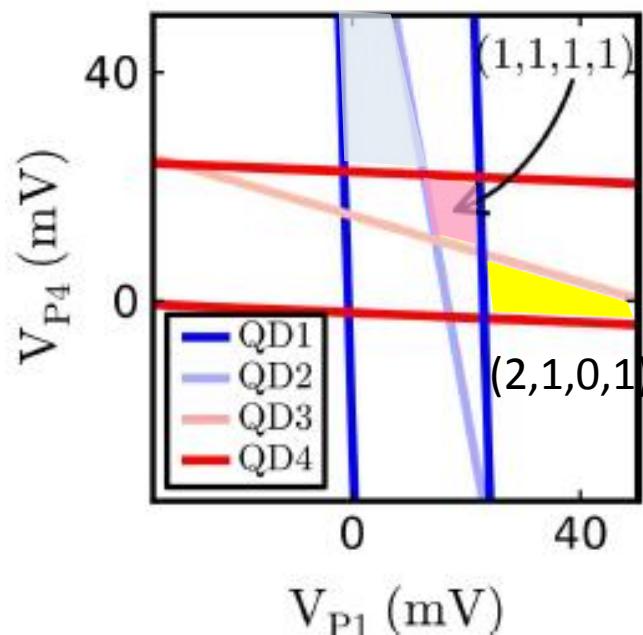
E. A. Laird et al., PRB (2010)  
L. Gaudreau et al., Nat. Phys. (2011)  
J. Medford et al., PRL (2014)

# 4重量子ドット

APL M. Delbeqc et al. 2014



Calculation (1,0,1,2)

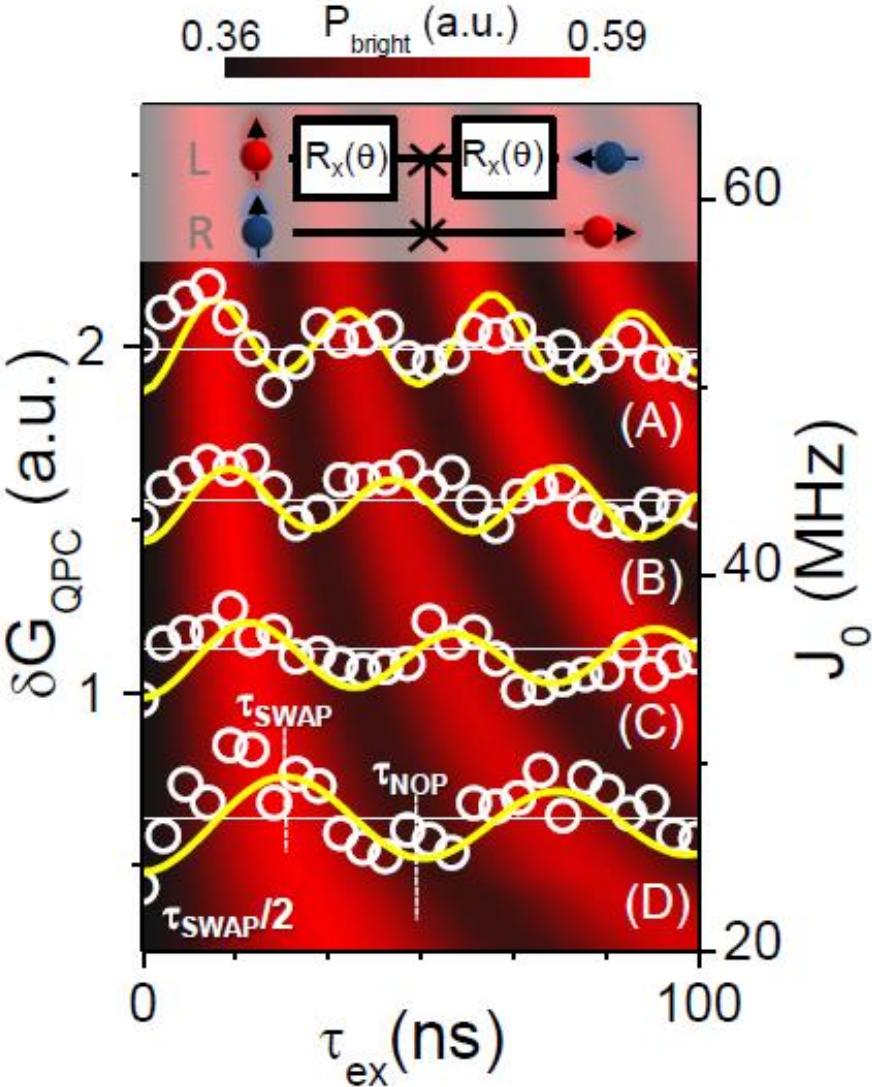


# 量子もつれゲート: $|\uparrow\rangle|\downarrow\rangle \leftrightarrow |\downarrow\rangle|\uparrow\rangle$

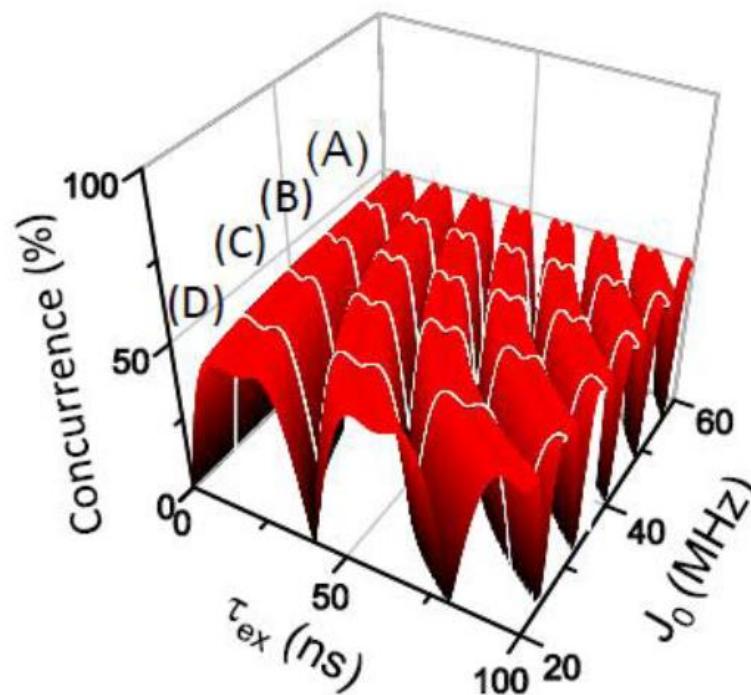
R. Brunner et al. *PRL* 2011

Probability of finding the singlet in the output  $|\psi_2\rangle$

$$P_{bright} \equiv |\langle|\uparrow\rangle|\downarrow|\psi_2\rangle|^2 + |\langle|\downarrow\rangle|\uparrow|\psi_2\rangle|^2$$



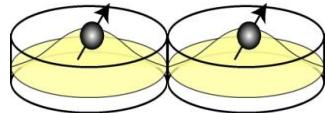
Calculation of concurrence using parameters derived from experiments



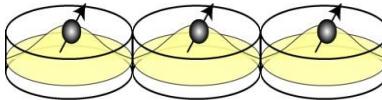
SWAP fidelity  $\sim 98\%$   
Rabi fidelity  $\sim 50\%$

# 多重量子ドットの電荷状態の制御と検出

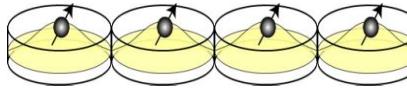
Double QD



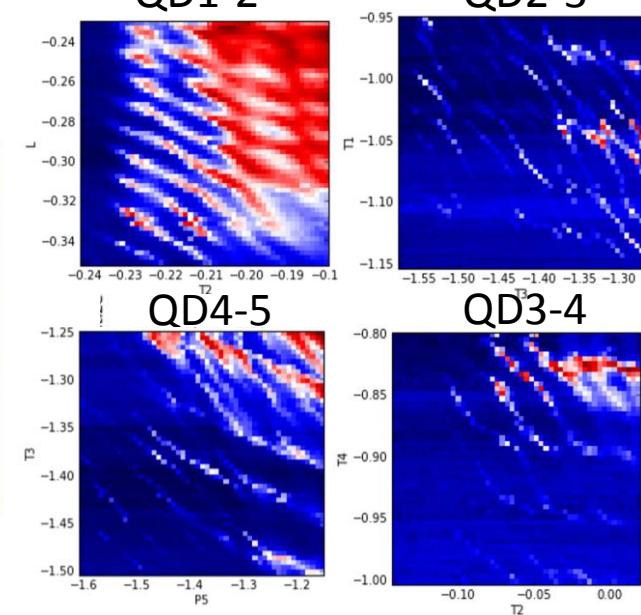
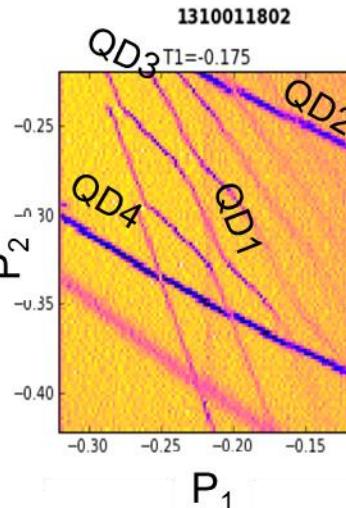
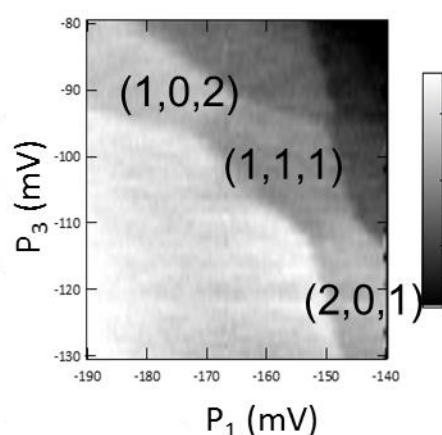
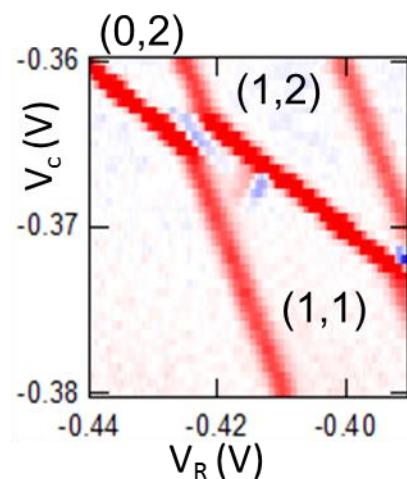
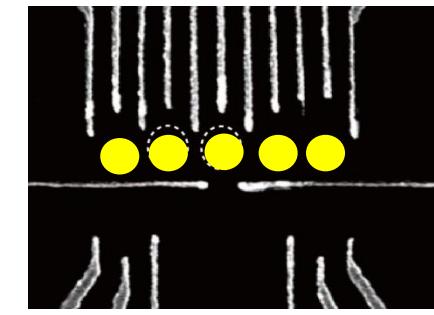
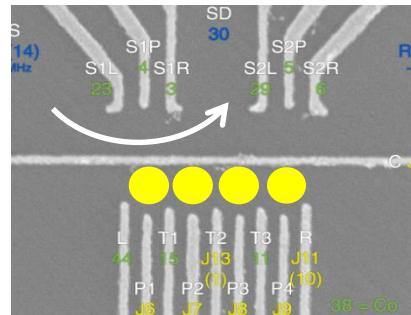
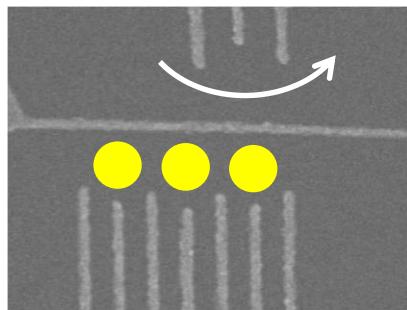
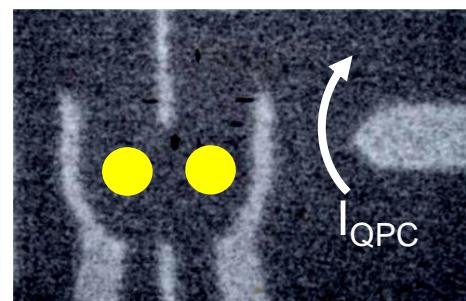
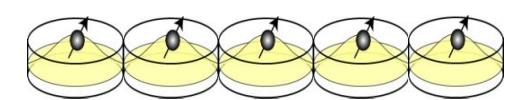
Triple QD



Quadruple QD



Quintuple QD ...



# 内容

## 1. 量子コヒーレンスと量子情報

量子ドットの電子状態と量子ゲートへの応用

単一電荷、单一生スピンの検出

量子ビットと量子もつれの操作と射影測定

多重ドットへの拡張

## 2. 環境とデコヒーレンス

電気的雑音と磁気的雑音

スピン緩和(スピン軌道相互作用と超微細相互作用)

デコヒーレンスのフィードバック制御

非エルゴート領域での不均一デコヒーレンス時間の拡張

核スピン環境の除去

## 3. 位相制御と計測

位相の時空間発展

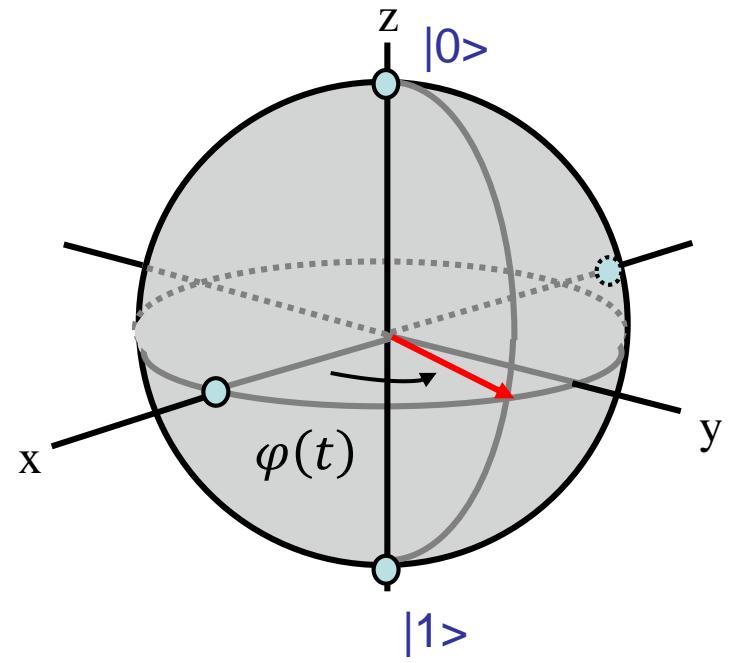
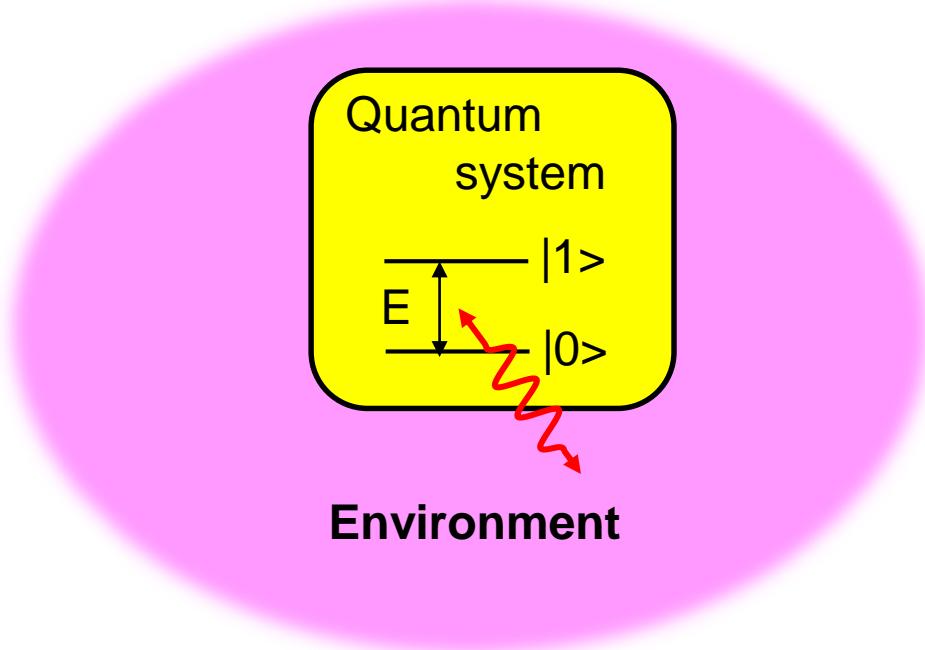
2経路干渉計による位相制御と検出

量子ドットのフリーデル総和則

近藤位相の実証実験

非局所量子もつれの生成と検証

# 量子コヒーレンス



Dephasing

$$\varphi(t) = \frac{E}{\hbar}t + \Delta\varphi(t)$$

# 環境の影響

電荷(軌道)

電気的環境

フォノン: エネルギーに依存  
ピエゾエレクトリック  
変形ポテンシャル  
高周波

抵抗揺らぎ:  $1/f$  低周波

電荷揺らぎ:  $1/f^2$  低周波

ショット雑音: 白色 高周波

$T_2 = 2T_1$

電荷状態のデコヒーレンス

スピン

磁気的環境

スピン軌道相互作用  
: 電場、波数、方位  
に依存  
角運動量保存  
高周波

核スピン: 超微細相互作用  
中～高周波

磁性不純物、近藤効果、。。。



$T_1 > T_2 \gg T_2^*$

スピン状態のデコヒーレンス

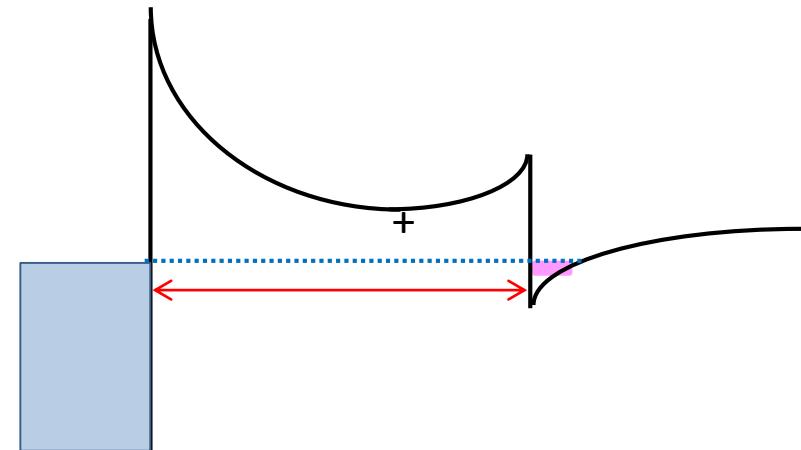
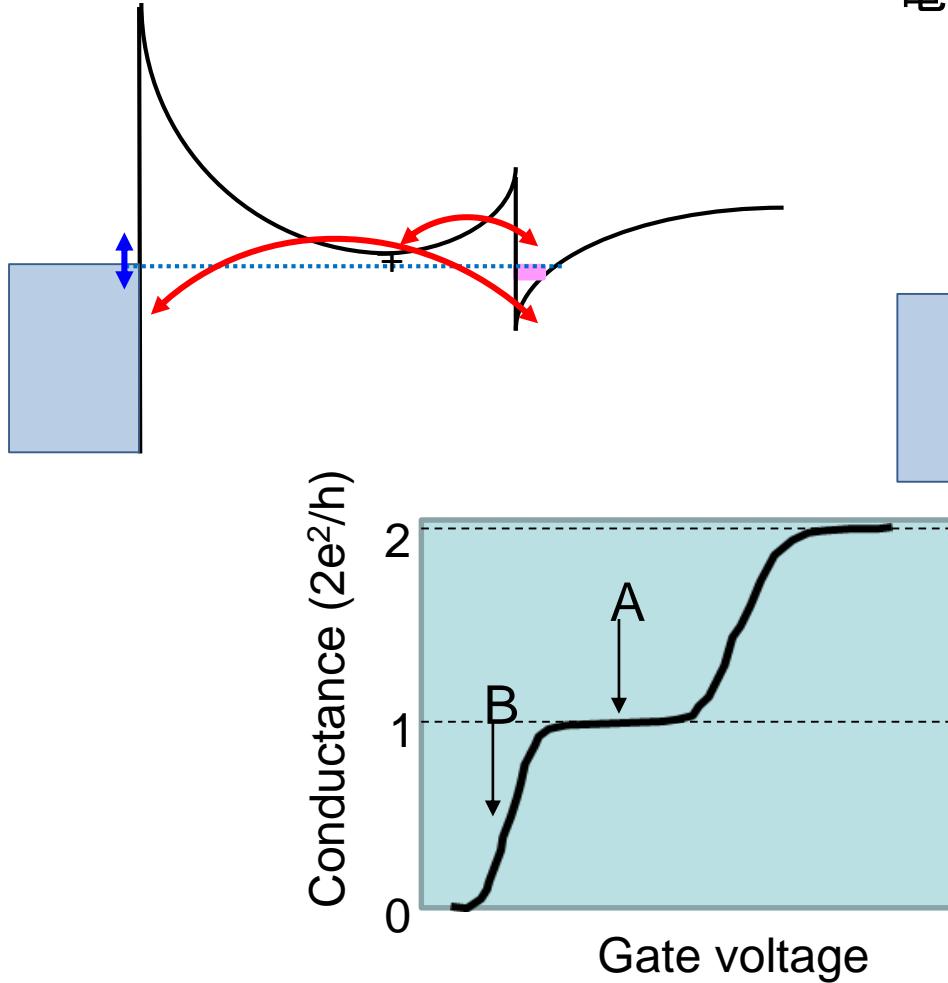
# $1/f$ and $1/f^2$ 雜音: 量子ポイントコンタクト

静電的揺らぎ: 低周波数

抵抗揺らぎ(ゲート電圧の揺らぎ)

電荷捕獲と放出

電荷密度減少による電荷雑音の抑制

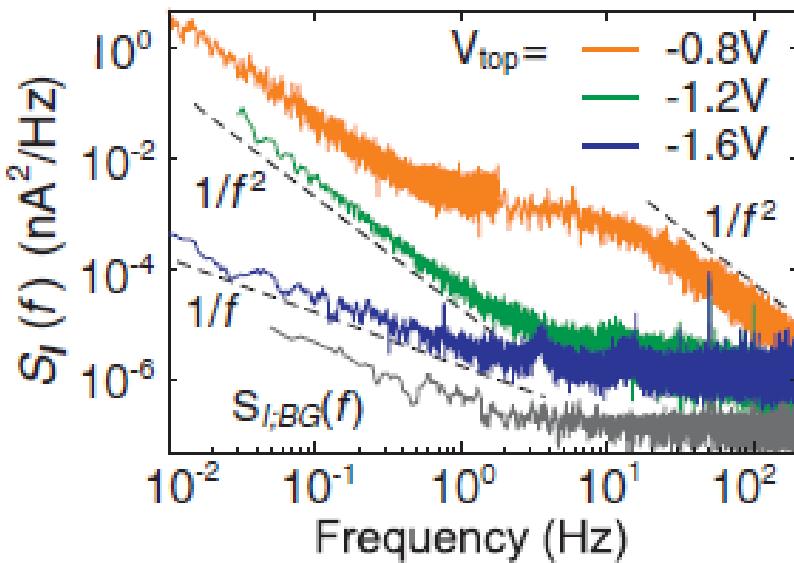
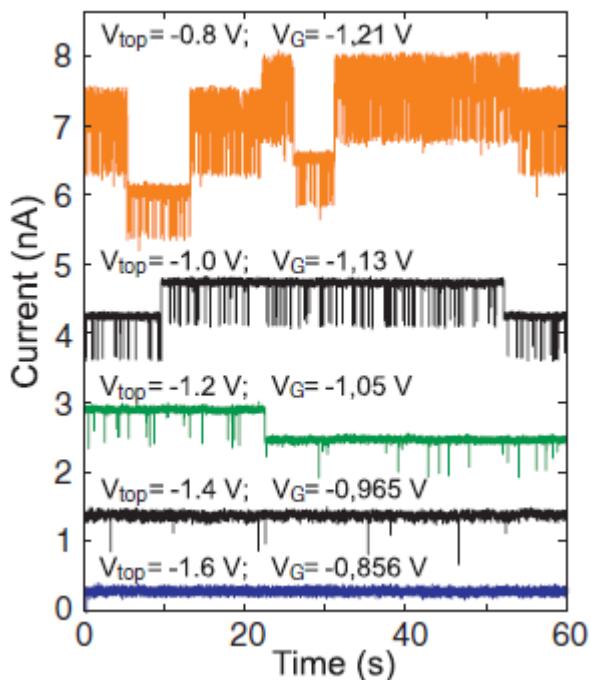
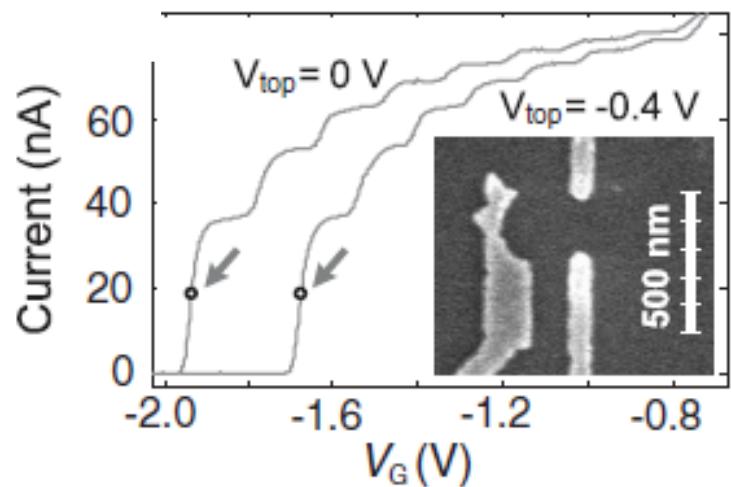


A:  $T=1$   
transport

B: Tunneling  
with  $T < 1$

# $1/f$ and $1/f^2$ 雜音の抑制: 量子ポイントコンタクト

Buisert et al. PRL 2008

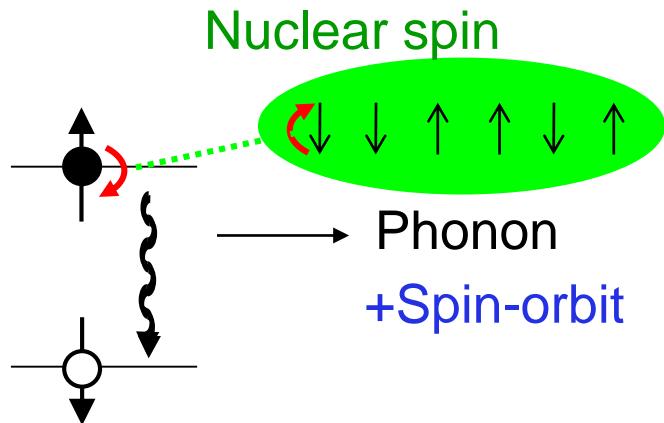


$\cdots 1/f^2 \Rightarrow$  2-level fluctuation  
(switching noise)

$\cdots 1/f \Rightarrow$  homogeneous background  
charge noise

# 軌道とスピンの自由度

*Energy relaxation ( $T_1$ )*



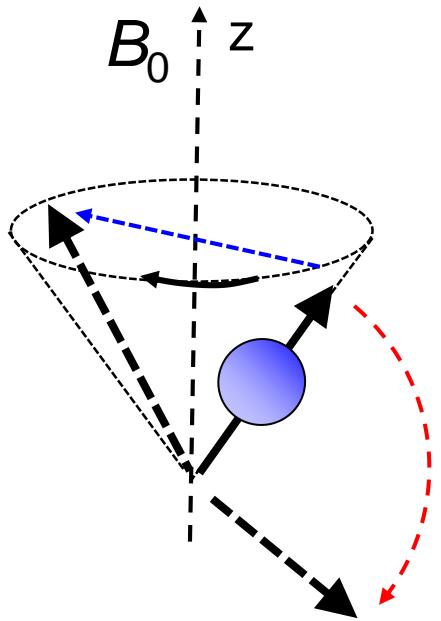
“Slow” due to weak  
coupling to environment  
 $> 0.1 \text{ msec}$

# デコヒーレンス: スピン緩和

Phase flip

:  $T_2, T_2^*$

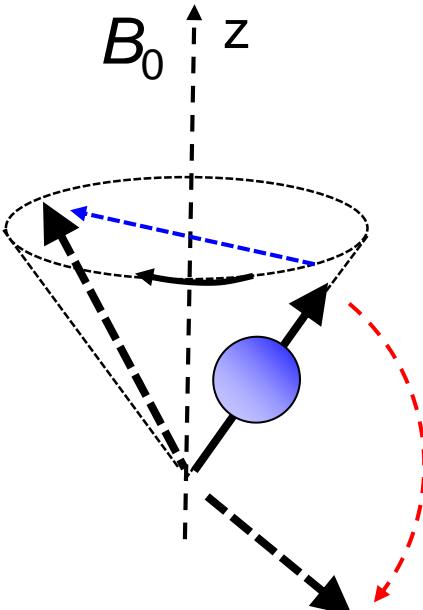
....caused by  $\sigma_z$



Spin flip:  $T_1$

....caused by  $\sigma_{\pm} = \sigma_x \pm i\sigma_y$

# 位相反転( $T_2$ )とスピン反転 ( $T_1$ )



Phase flip

: $T_2, T_2^*$

....caused by  $\sigma_z$

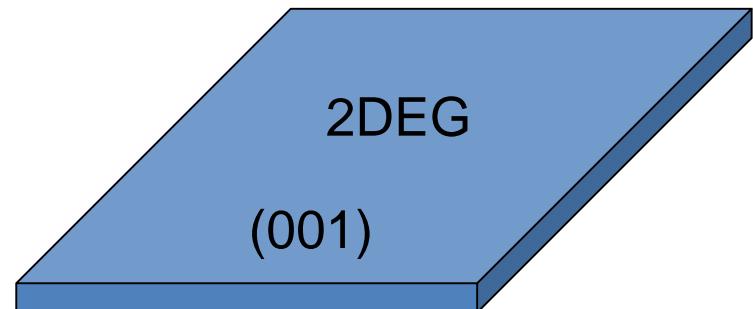
Spin-orbit

$$H_{SO} = \alpha(-p_y\sigma_x + p_x\sigma_y) + \beta(-p_x\sigma_x + p_y\sigma_y)$$

For QD,  $\langle p_x, p_y \rangle = 0$  and only  $\sigma_{\pm}$  included

Spin flip:  $T_1$

....caused by  $\sigma_{\pm} = \sigma_x \pm i\sigma_y$



Only weakly influence “Spin flip”

## Coupling to lattice nuclei

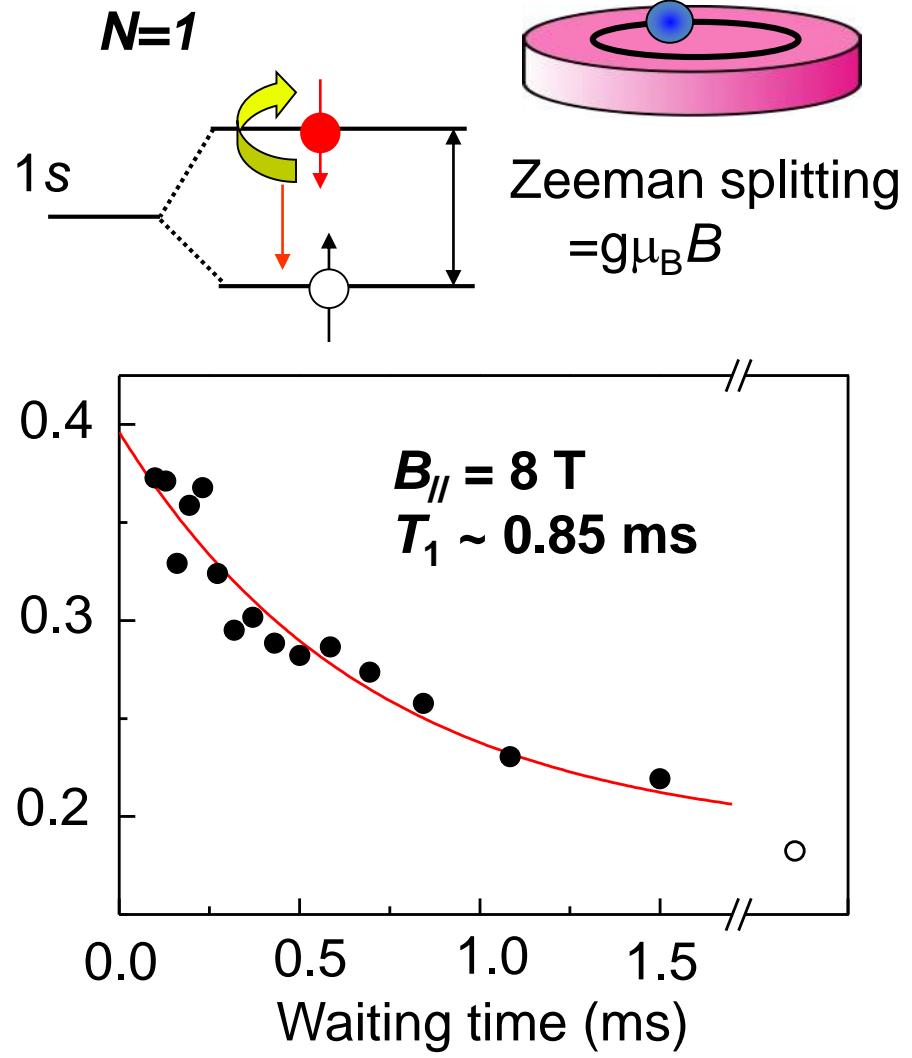
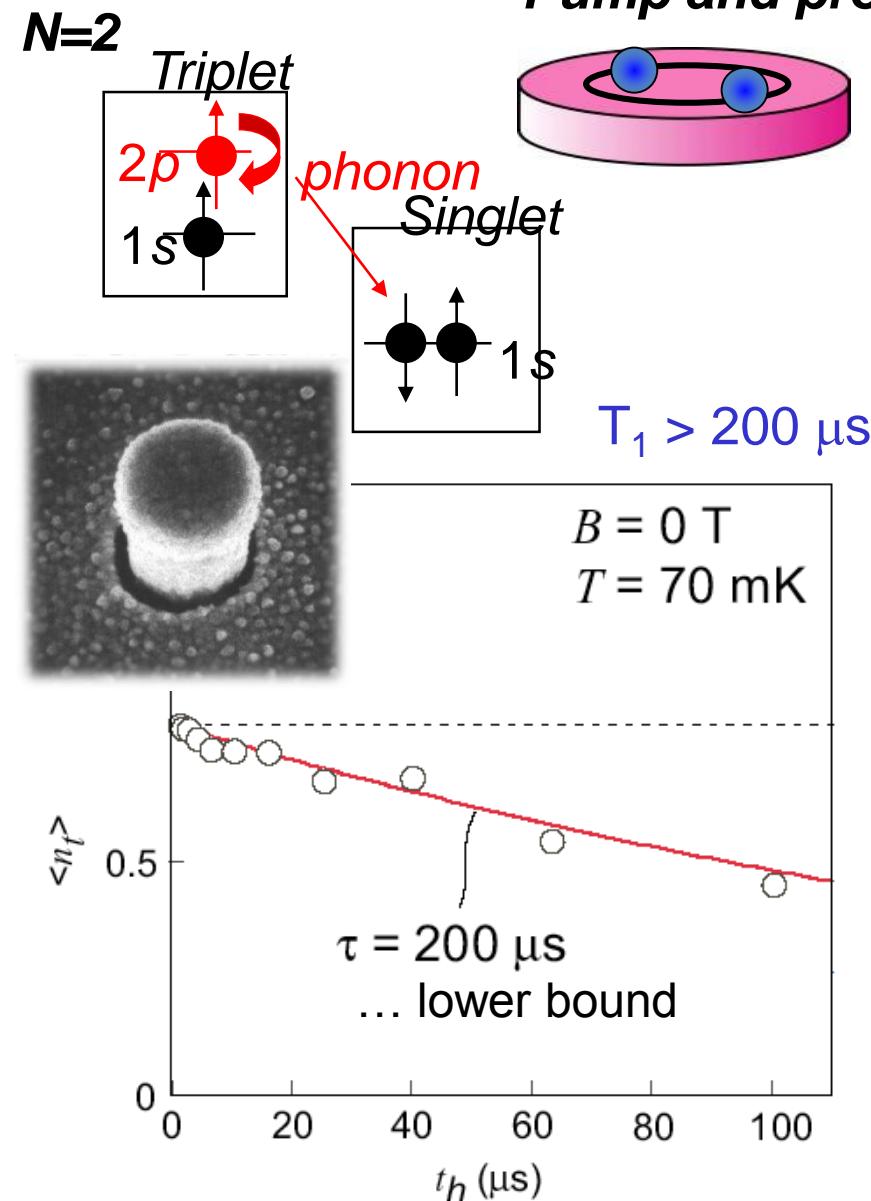
$$H_{HF} = A|\psi(x)|^2 \left( \frac{I_+S_- + I_-S_+}{2} + I_z S_z \right)$$

“Spin flip”

“Phase flip”

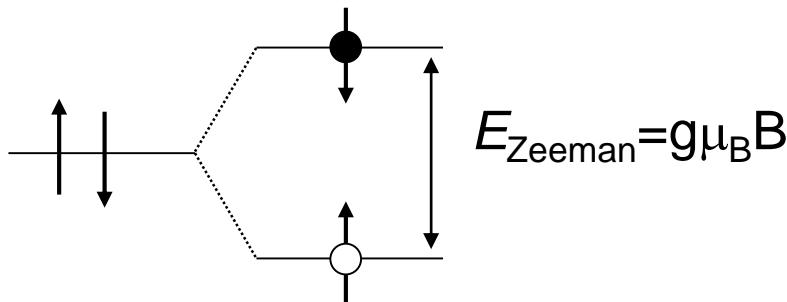
# スピン軌道相互作用によるスピン緩和の $T_1$

*Pump and probe measurement*, Fujisawa, ST PRL 02



$T_1 \sim \text{sec}$  Amasha et al. *PRL*08

# スピン軌道相互作用によるゼーマン分離状態間の緩和

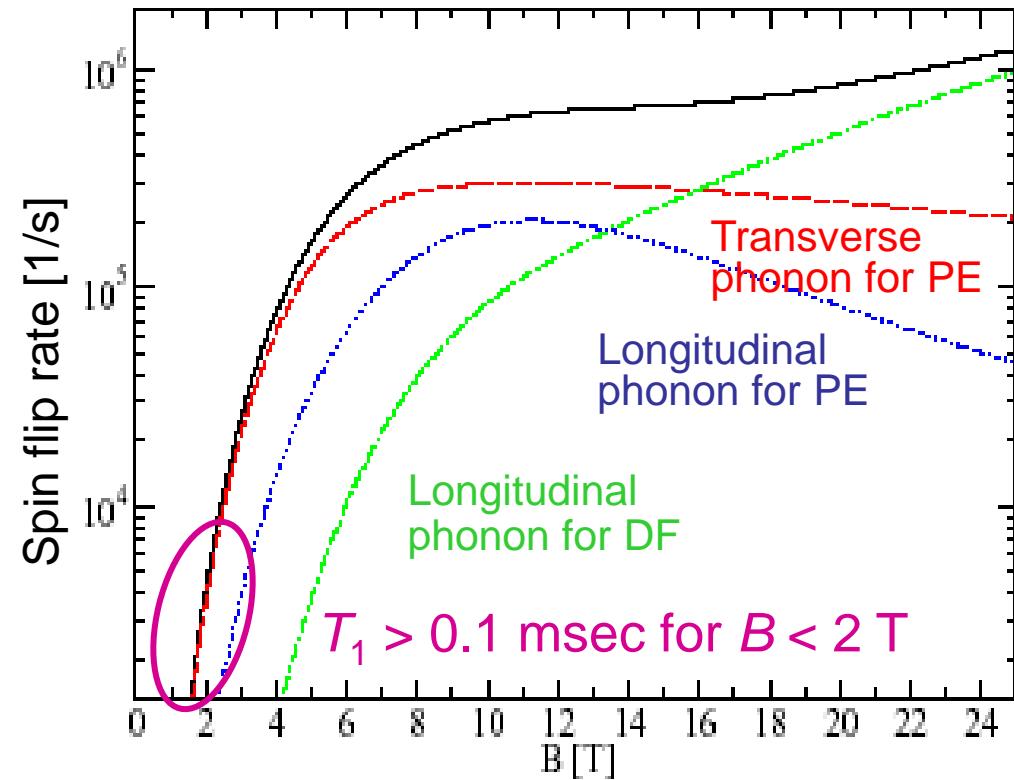


Small energy for piezo phonon  
Large energy for deformation phonon  
Spin flip rate for small Zeeman energy  
 $\Gamma_{\uparrow\downarrow} \propto B^5 \beta^2/\omega_0^4$   
with  $B^3$  from phonon DOS

$\Lambda_p$ : spin–piezo-phonon coupling  
 $\beta$ : piezotensor

$\Gamma_{\uparrow\downarrow} \approx (1 \text{ ms})^{-1}$  for  $\omega_0=1 \text{ meV}$ ,  $B=1 \text{ T}$   
and  $\beta=10^{-2}\sim10^{-1}$  (GaAs)

Golovach et al. PRL 04



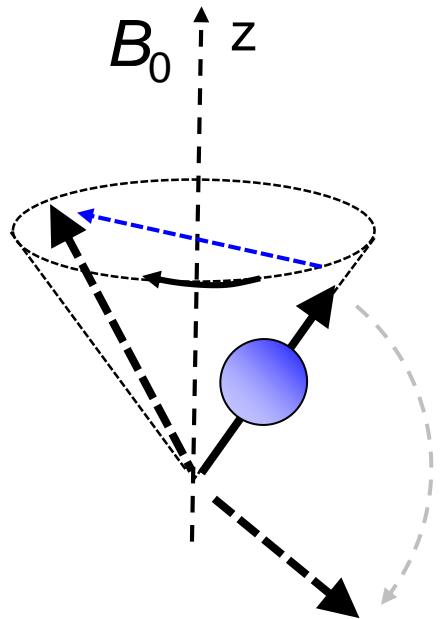
$T_2=2T_1\ldots\text{very long } (> \text{ms})$

# スピニ緩和

Phase flip

: $T_2, T_2^*$

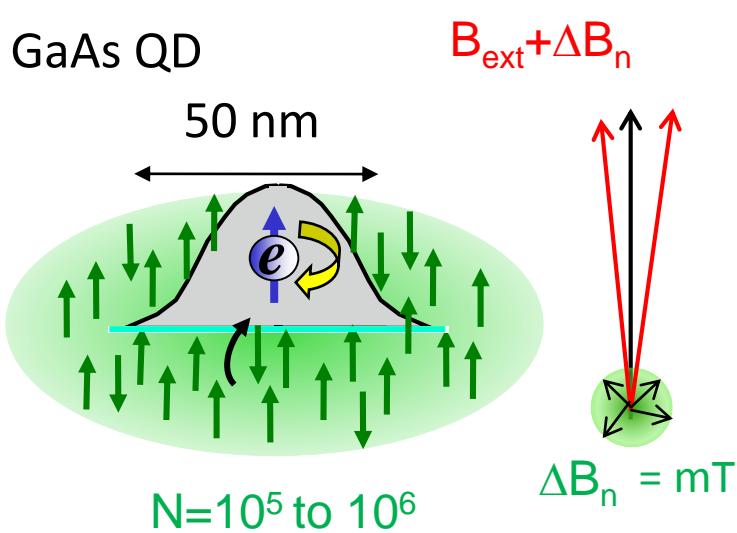
....caused by  $\sigma_z$



Spin flip:  $T_1$

....caused by  $\sigma_{\pm} = \sigma_x \pm i\sigma_y$

# 核スピン環境の影響



$$H_{\text{HF}} = A|\psi(x)|^2 \left( \frac{I_+S_- + I_-S_+}{2} + I_z S_z \right)$$

Dynamical  
Nuclear polarization

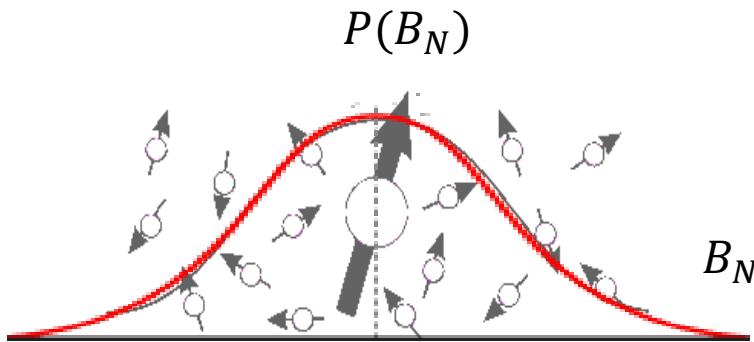
Slow shift of  
resonance ESR field

Statistical fluctuation of  
nuclear field =  $\frac{A}{\sqrt{N}} = mT$

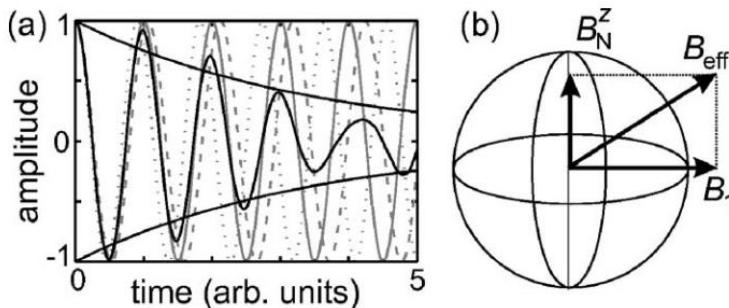
Standar deviation of ESR  
line :  $\sigma = \text{a few MHz}$

# 半古典近似を超える取扱い

## Classical dynamics



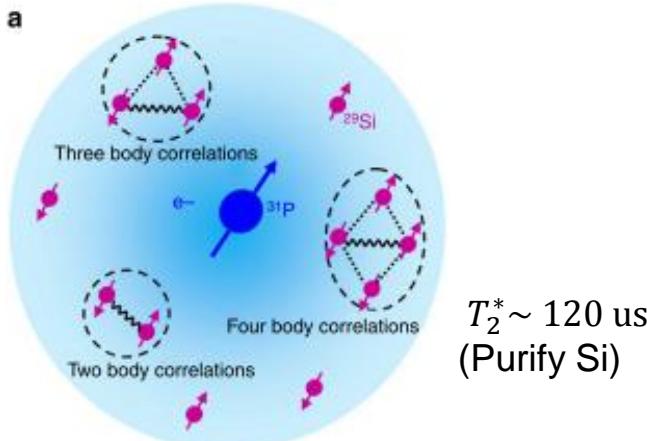
Nuclear field is approximated  
**Gaussian distribution** of nuclear fields  
with deviation  $\sqrt{\langle (B_N)^2 \rangle}$



The decay is fitted as Gaussian  
of the form  $\exp[-t^2/(T_2^*)^2]$ .

$$T_2^* = \frac{\hbar\sqrt{2}}{g\mu_B\sqrt{\langle (B_N)^2 \rangle}} \sim 10 \text{ ns (GaAs)}$$

## Central spin problem



Central-spin problem is to investigate  
**many-body correlations** in nanoscale  
nuclear spin bath.

Ex) phosphorous donors in silicon, silicon quantum dot, GaAs quantum dot

# 核スピン相互作用のラビ振動への影響: 不均一幅と位相シフト

$$P(t) = \int_{-\infty}^{\infty} d\omega \frac{e^{-\frac{\omega^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} \times \frac{b_1^2}{2\Omega_1^2} [1 - \cos(\Omega_1 t)]$$

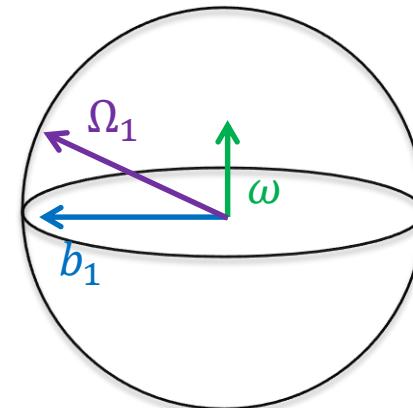
where  $\Omega_1 = \sqrt{b_1^2 + \omega^2}$   
(generalized Rabi freq.)

Average over      Spin flip prob.  
Overhauser field      for detuning  $\omega$   
distribution

- $b_1 > \sigma$  (**strong driving case**)

For  $|\omega| \lesssim \sigma$ , the integrand is roughly constant.

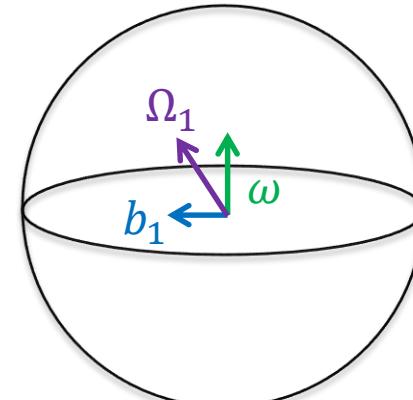
$$P(t) \sim \frac{1}{2} [1 - \cos(b_1 t)] \quad \text{for } t \sim \frac{1}{b_1}$$



- $b_1 < \sigma$  (**weak driving case**)

$$P(t) \sim C + \frac{1}{2\sigma} \sqrt{\frac{b_1}{t}} \cos\left(b_1 t + \frac{\pi}{4}\right) \quad \text{for } t > \frac{2}{b_1}$$

FHL Koppens et al., PRL (2007)

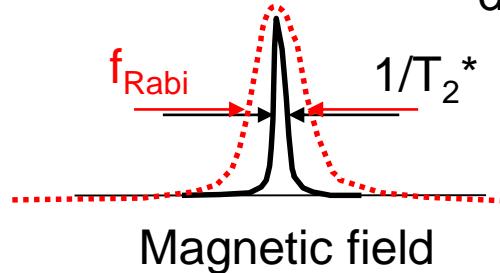


# 核スピン相互作用の影響

Interaction time < Rabi (spin flip) time  $\rightarrow$  Interaction time >> Rabi time

Electron spin resonance (ESR)

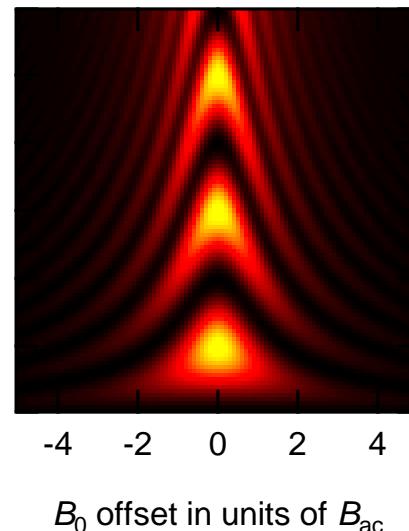
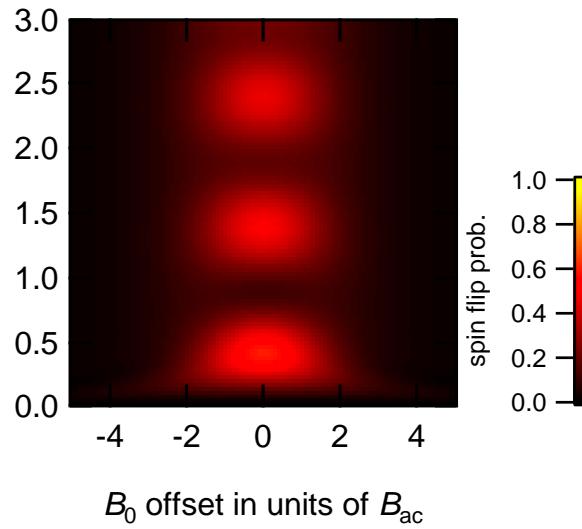
Line shape broadening due to  
dephasing rate



Rabi oscillations

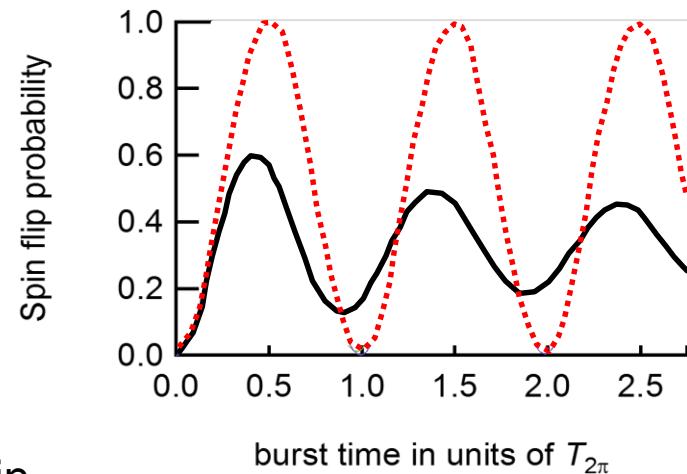
Disrupted chevron pattern in the B-t domain

burst time in units of  $T_{2\pi}$



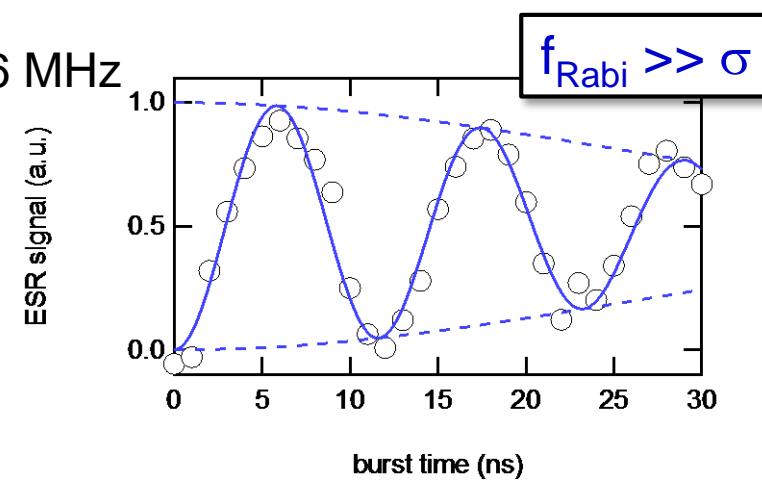
X - gate fidelity

Low fidelity



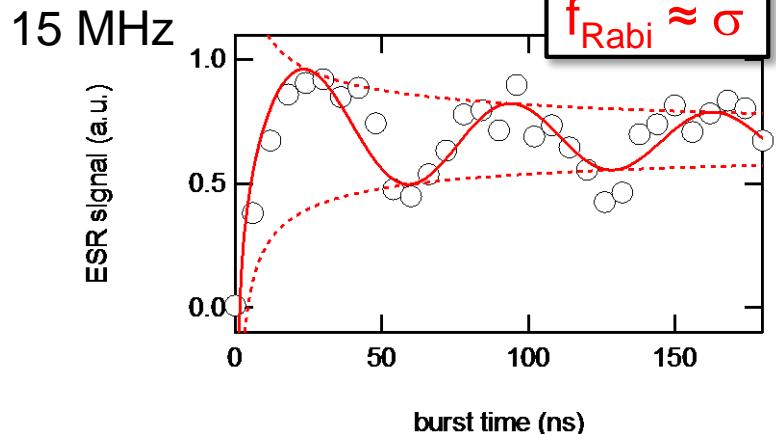
# ラビ高速化による核スピンの影響の解消

$$\frac{1}{2} e^{-(t/2\tau)^2} [1 - \cos(b_1 t)] \text{ (Strong drive Rabi)}$$

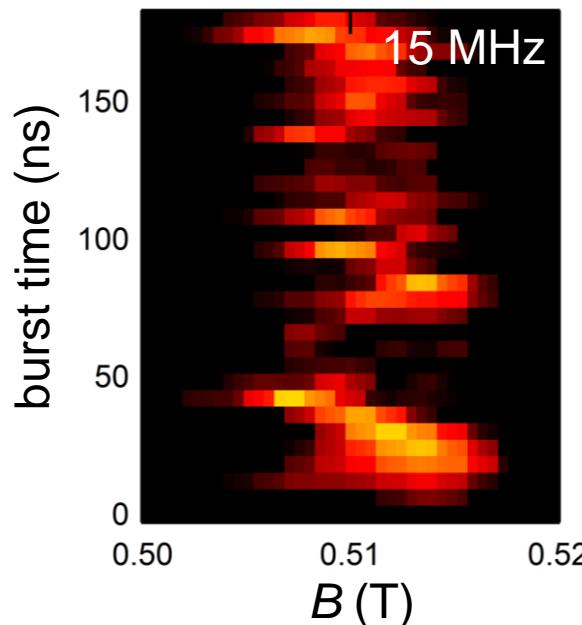
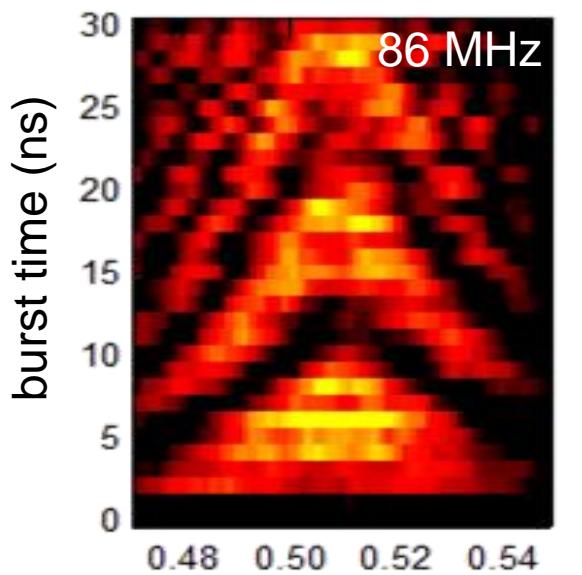


Average X gate fidelity > 94%

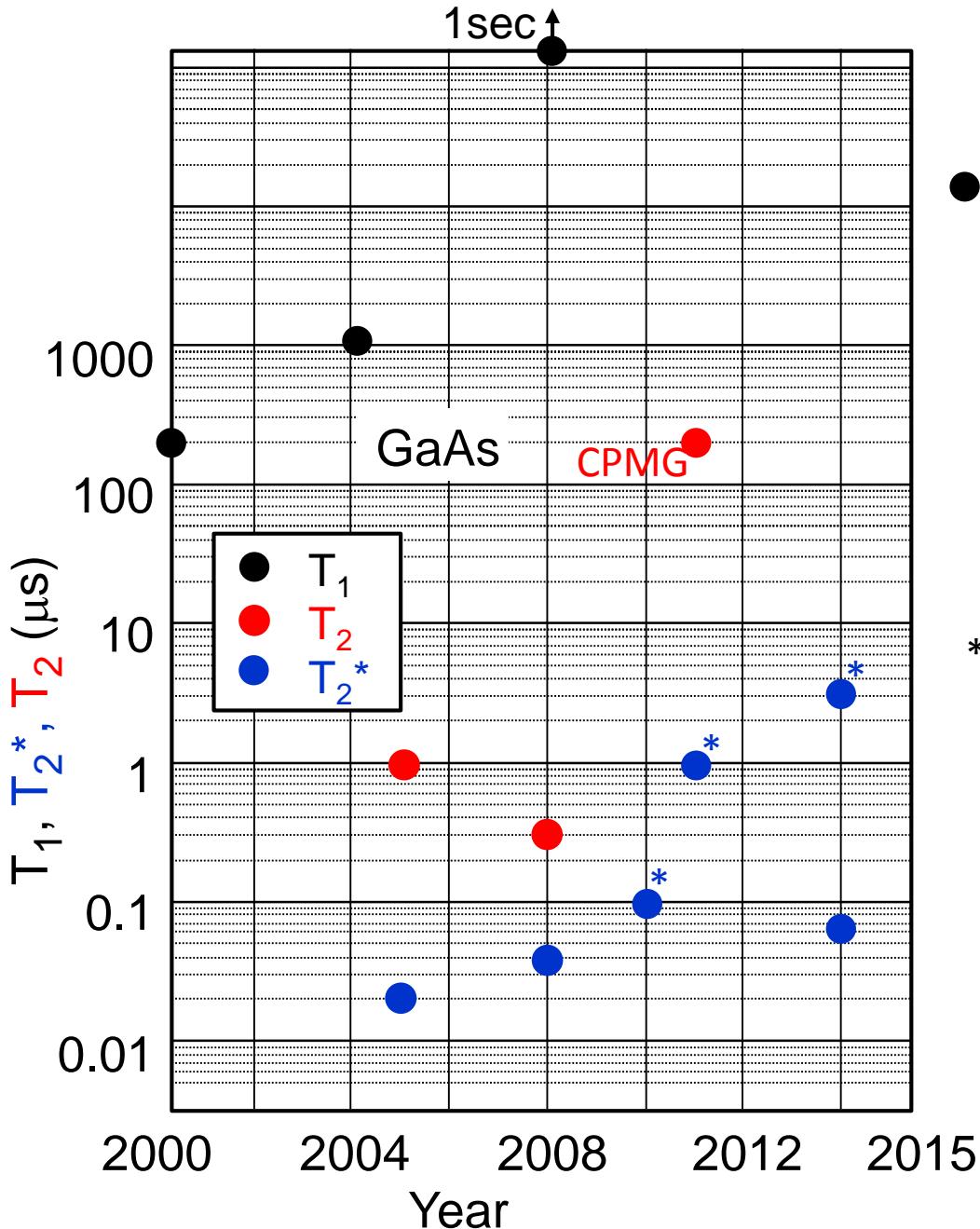
$$\frac{1}{2} \sqrt{\frac{\tau}{t}} \cos\left(b_1 t + \frac{\pi}{4}\right) \text{ (Weak drive Rabi)}$$



J. Yoneda et al. PRL 2015



# 緩和とデフェージング: 量子ドットのスピン



$$T_1 \gg T_2$$

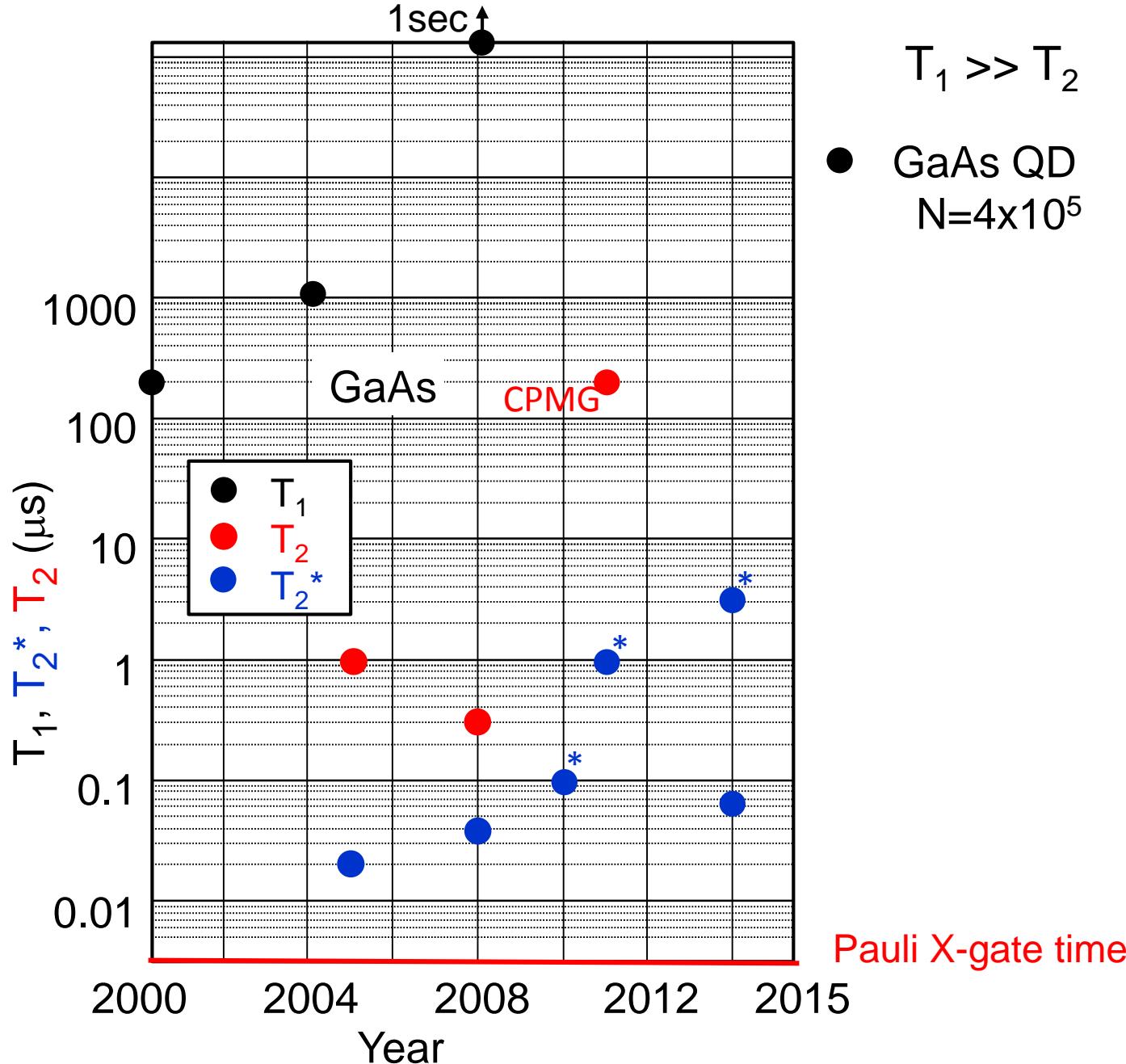
- $T_1 >> T_2$   
 $N=4 \times 10^5$

\* Feedback

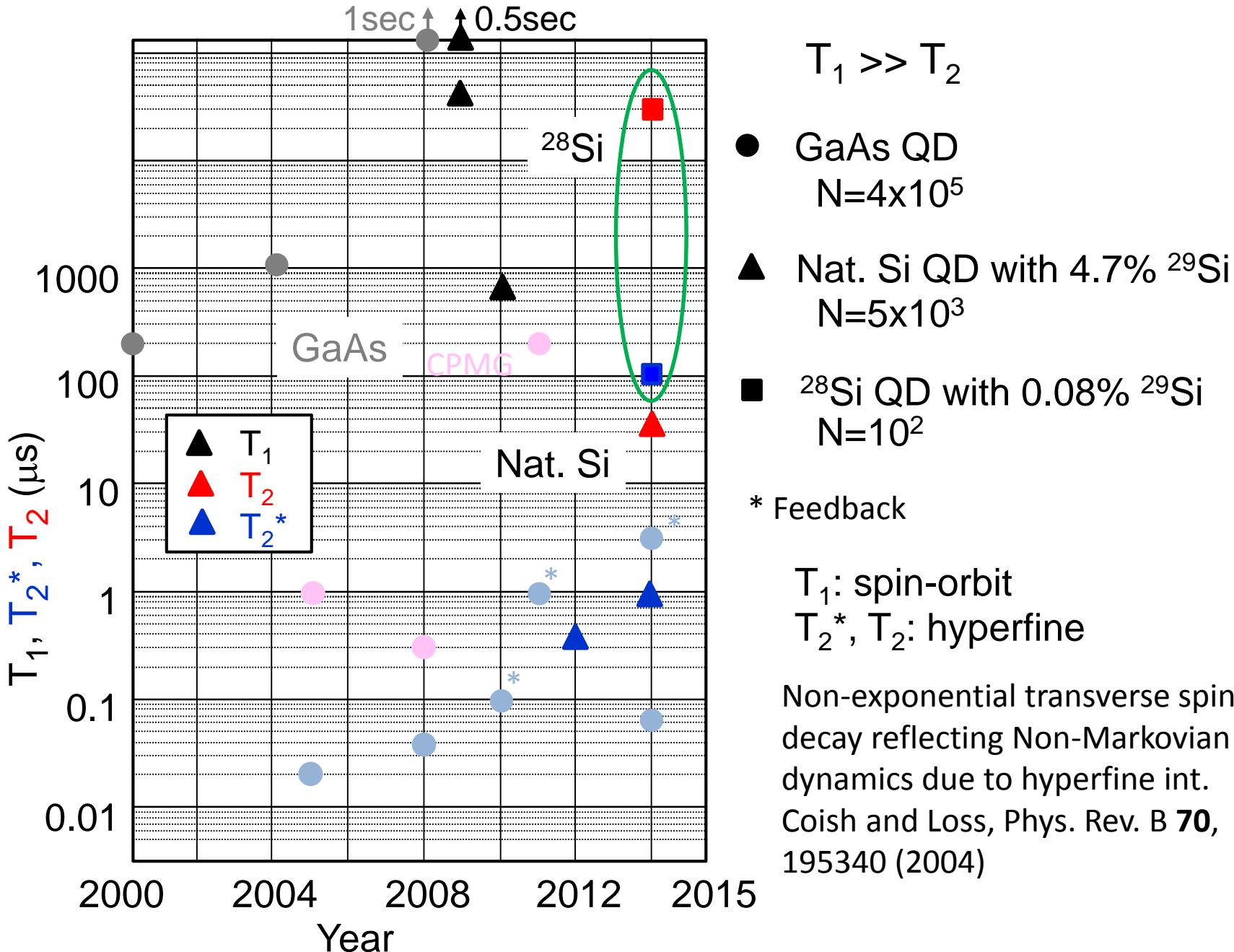
$T_1$ : spin-orbit  
 $T_{2^*}, T_2$ : hyperfine

Non-exponential transverse spin decay reflecting Non-Markovian dynamics due to hyperfine int.  
Coish and Loss, Phys. Rev. B **70**, 195340 (2004)

# 緩和とデフェージング: 量子ドットのスピン



# 緩和とデフェージング: 量子ドットのスピン



# フィードバックによる環境揺らぎ補正

Feedback loop:

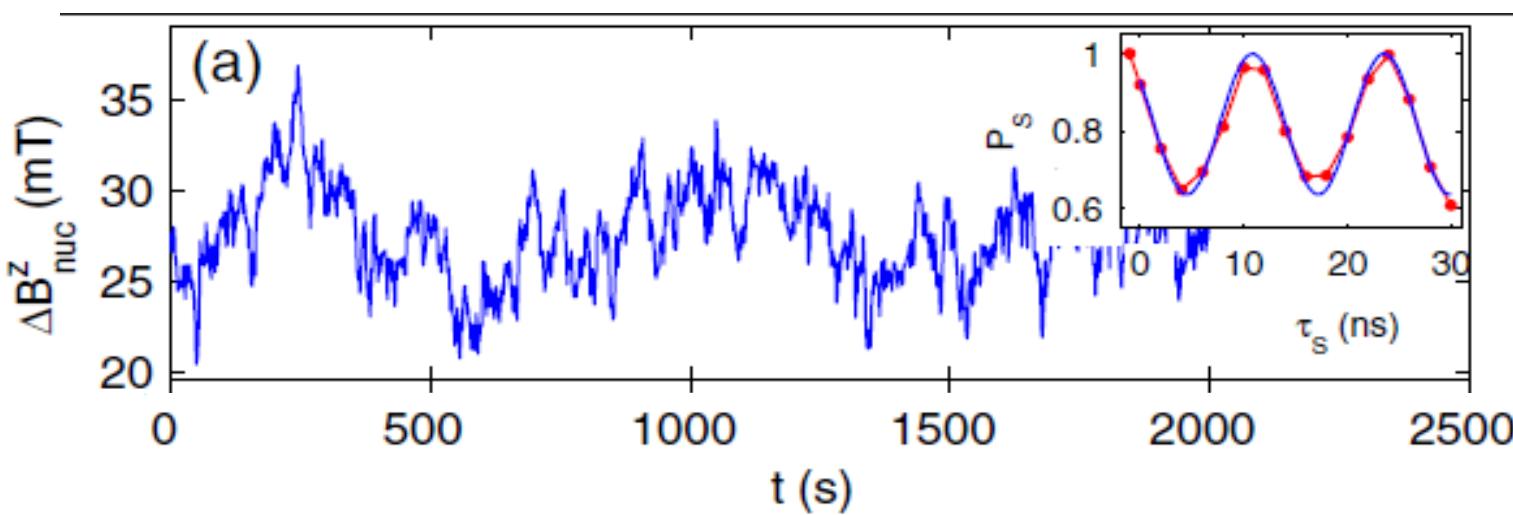
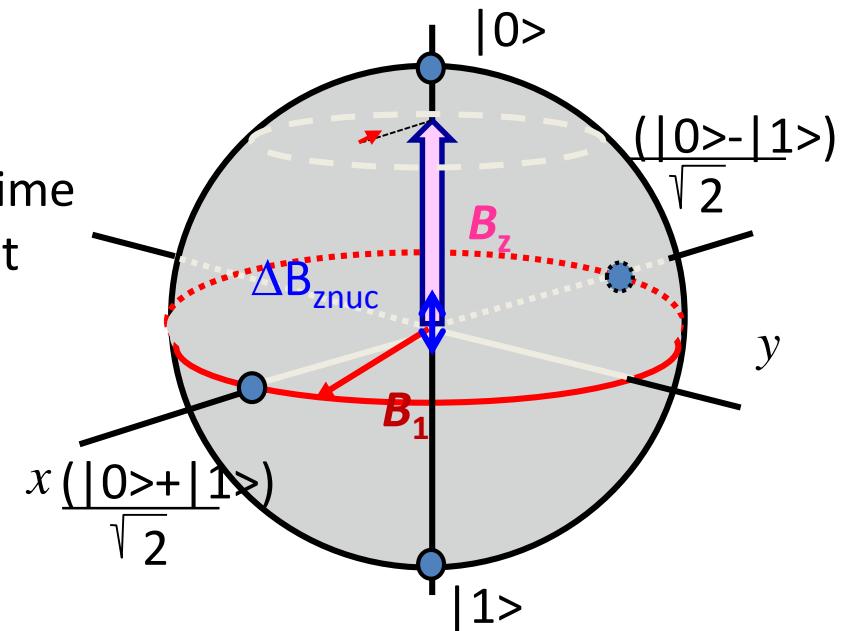
H. Bluhm et al. PRL 105, 216803 (2010)

Measure standard oscillations

Measure a few data points for a short time  
and fit the standard oscillation to extract  
deviation of  $\Delta B_z$

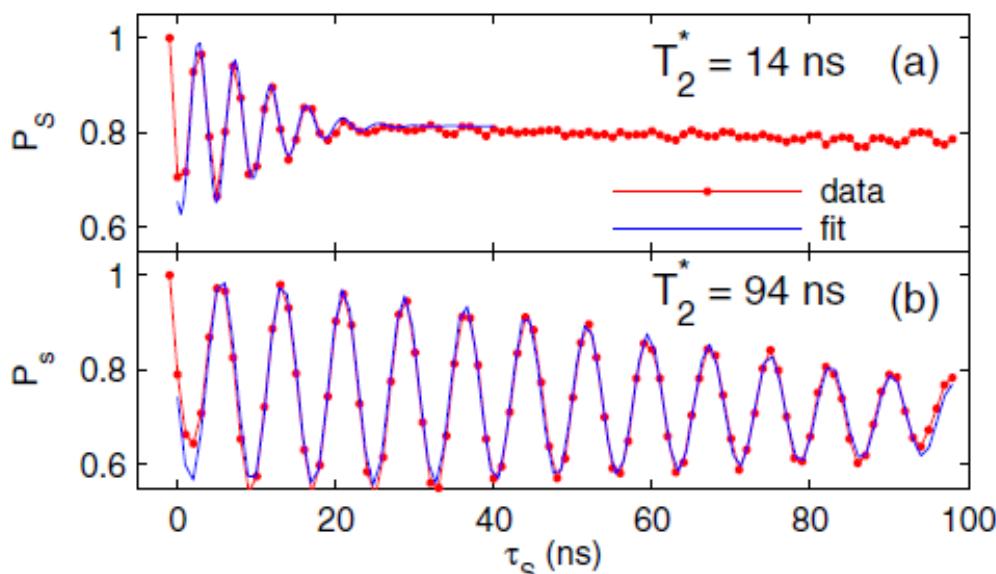
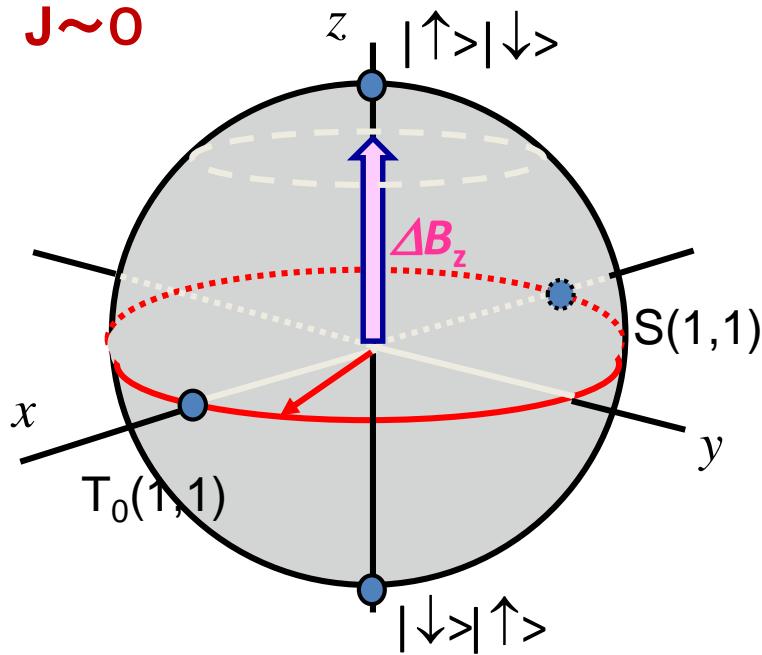
Feedback pulse to compensate the  
environment change

Measure the actual data



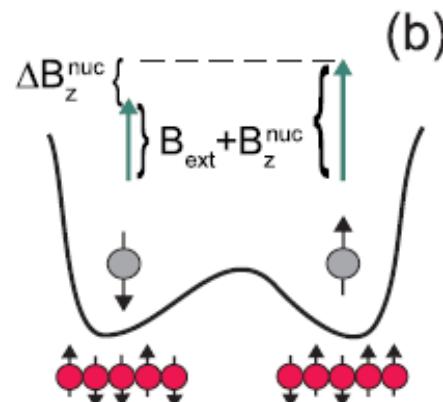
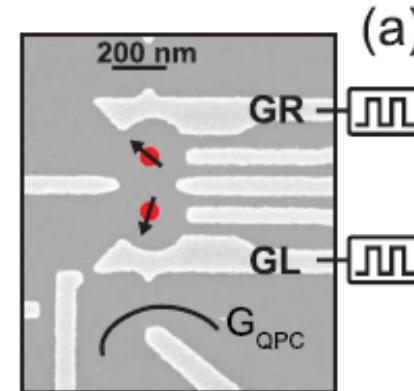
# 核スピン環境の揺らぎ(動的核スピン分極の揺らぎ)補正

$J \sim 0$



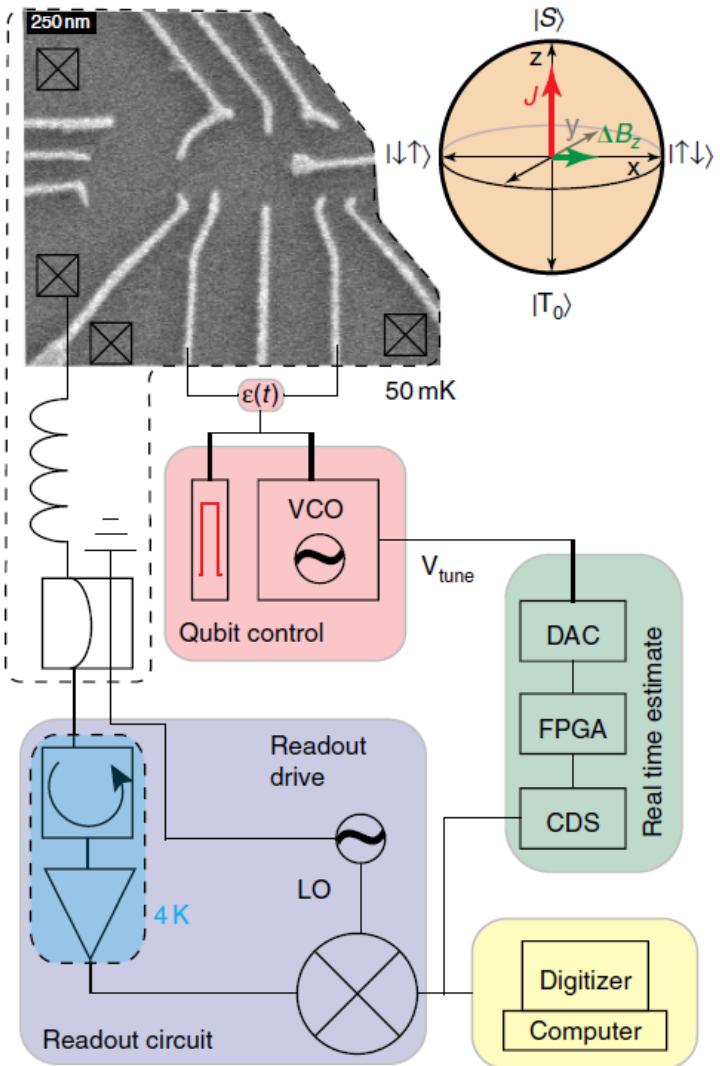
H. Bluhm et al. PRL 105, 216803 (2010).

: feedback response time to  $\approx 1$  s.

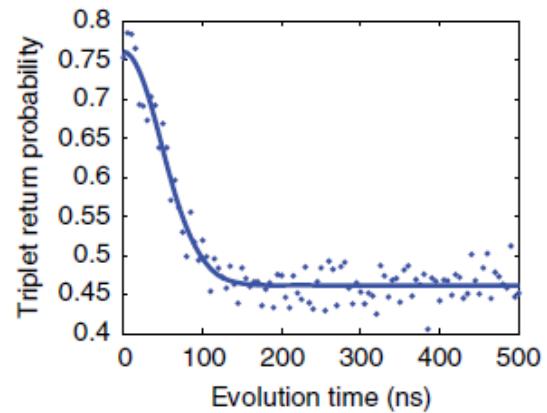
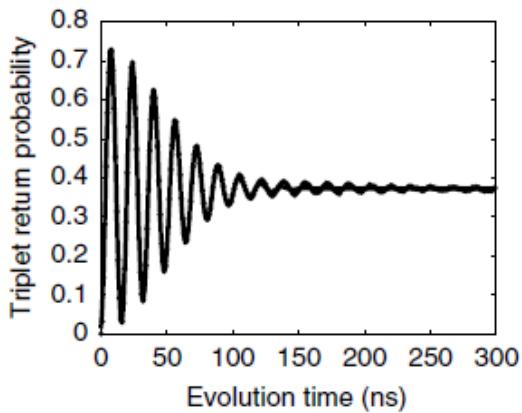


# 核スピン環境の揺らぎの高速補正

M.D. Shulman, Nat, Commu. 5, 5156 (2014)



Ramsey fringes in rotation about z-axis of T0 and S



Bayesian estimation

$$P(m_k | \Delta B_z) = \frac{1}{2} [1 + r_k(\alpha + \beta \cos(2\pi \Delta B_z t_k))]$$

$$r_k = 1 \text{ (}-1\text{)} \text{ for } m_k = |S\rangle(|T_0\rangle)$$

Determine  $\alpha$  and  $\beta$  from the test measurement

$$\alpha = 0.25 \text{ and } \beta = 0.67$$

# 核スピン環境の揺らぎの高速補正

M.D. Shulman, Nat. Commu. 5, 5156 (2014)

Measure N~100 time within 0.1 msec to estimate  $\Delta B_z$  using the Bayesian estimation.

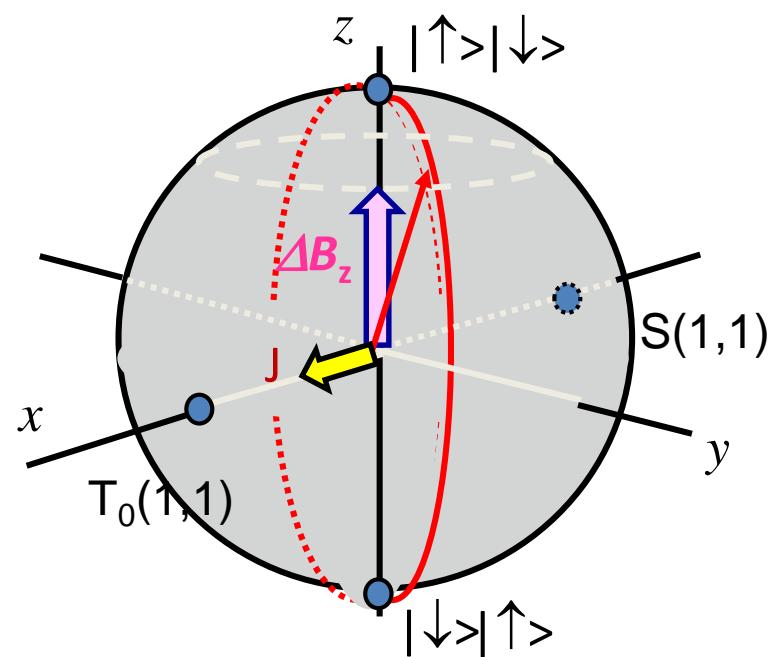
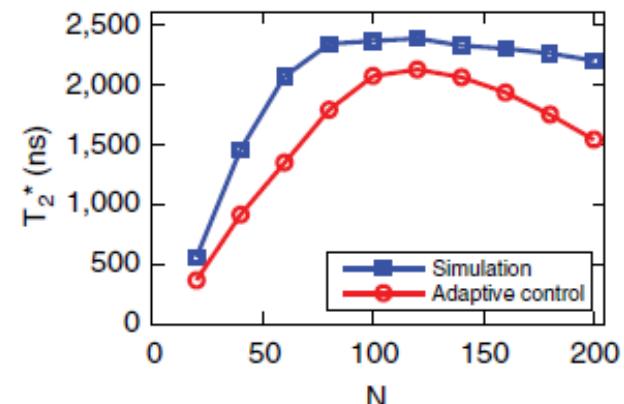
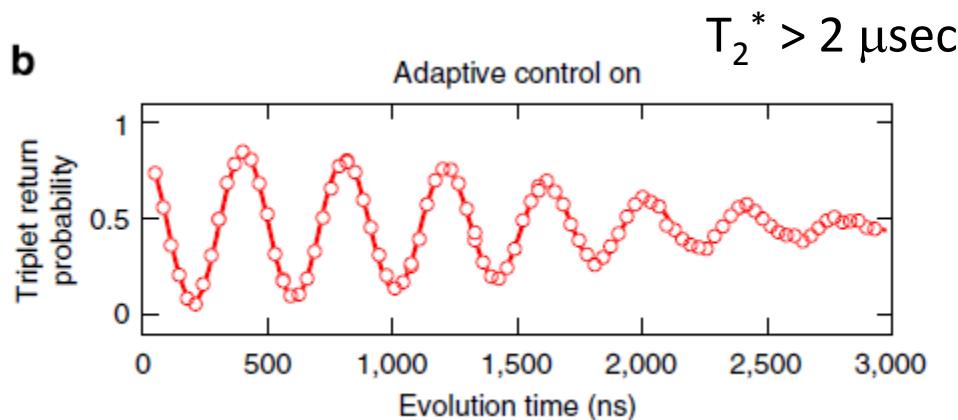
Apply resonance AC voltage to tune J whose frequency is consistent with the measured  $\Delta B_z$ .

$$J = J_0 \cos(\Omega_J t)$$

$$\frac{\Omega_J}{2\pi} = \Delta \tilde{B}_z$$



Control  $| \uparrow \rangle | \downarrow \rangle \leftrightarrow | \downarrow \rangle | \uparrow \rangle$  oscillation in the rotational frame about x-axis



# 内容

## 1. 量子コヒーレンスと量子情報

量子ドットの電子状態と量子ゲートへの応用

单一電荷、单一生スピンの検出

量子ビットと量子もつれの操作と射影測定

多重ドットへの拡張

## 2. 環境とデコヒーレンス

電気的雑音と磁気的雑音

スピン緩和(スピン軌道相互作用と超微細相互作用)

デコヒーレンスのフィードバック制御

非エルゴート領域での不均一デコヒーレンス時間の拡張

核スピン環境の除去

## 3. 位相制御と計測

位相の時空間発展

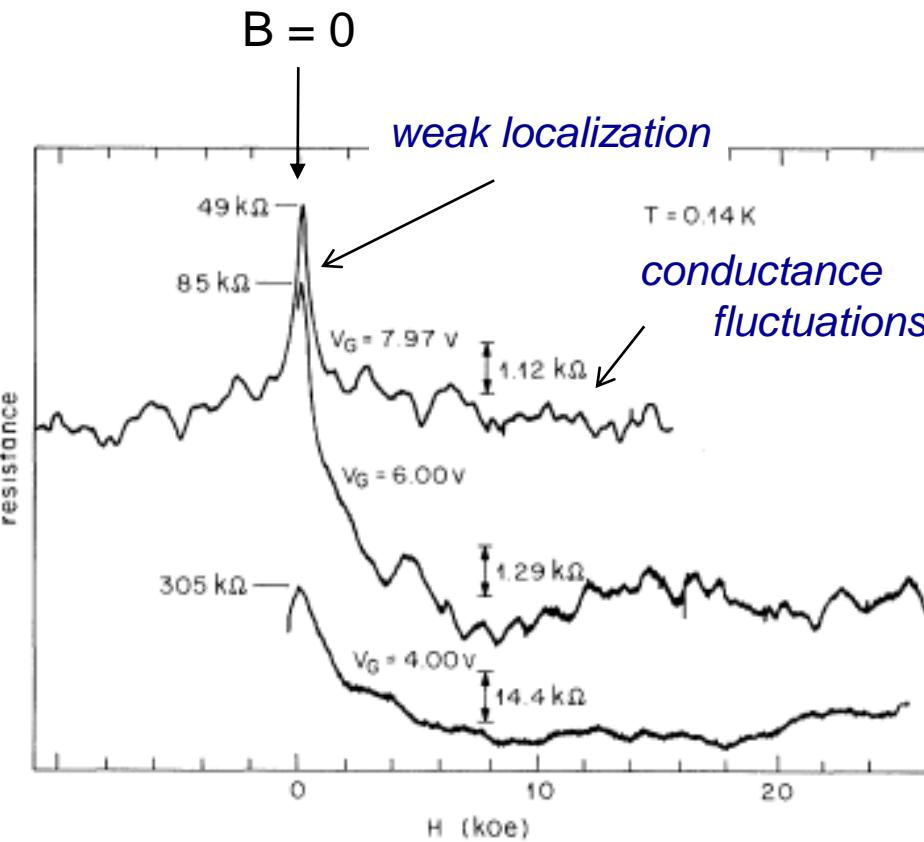
2経路干渉計による位相制御と検出

量子ドットのフリーデル総和則

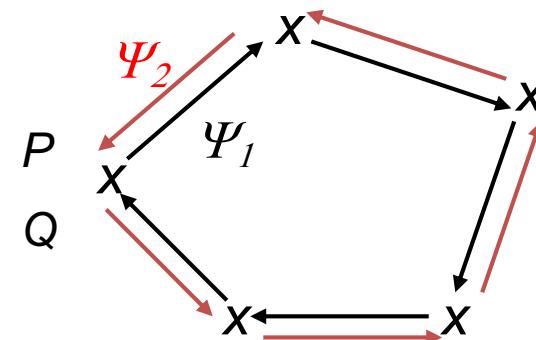
近藤位相の実証実験

非局所量子もつれの生成と検証

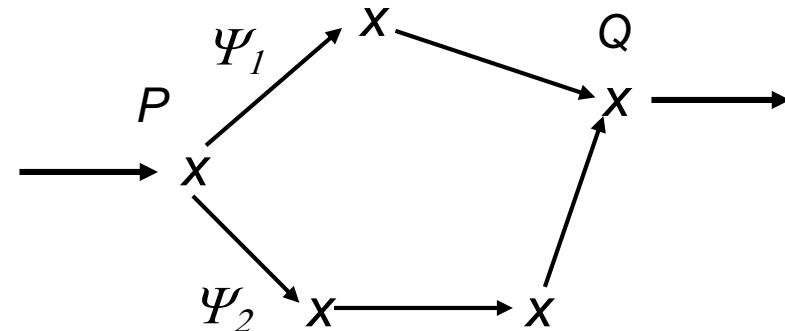
# 干渉と位相



*Weak localization:*  $P \longleftrightarrow P$



*Conductance fluctuation*

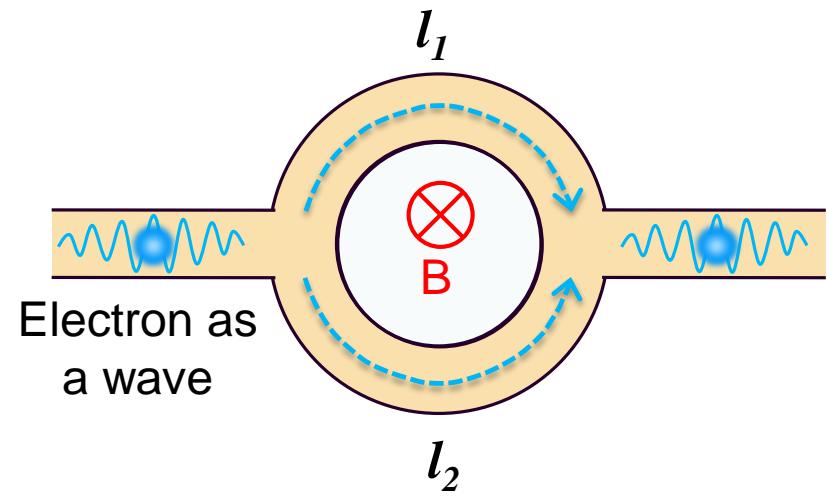
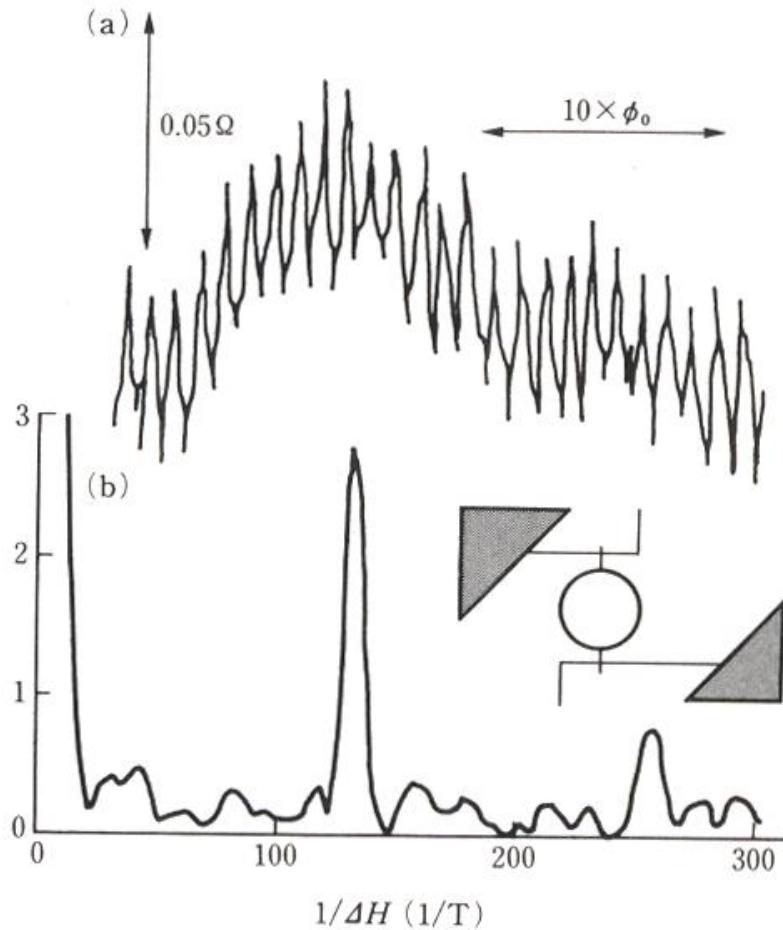


Narrow Si inversion layer

J.C. Licini et al. Phys. Rev. Lett. 55, 2987 (1985).

# 干涉と位相

AB ring for phase control with B



Phase difference between two paths

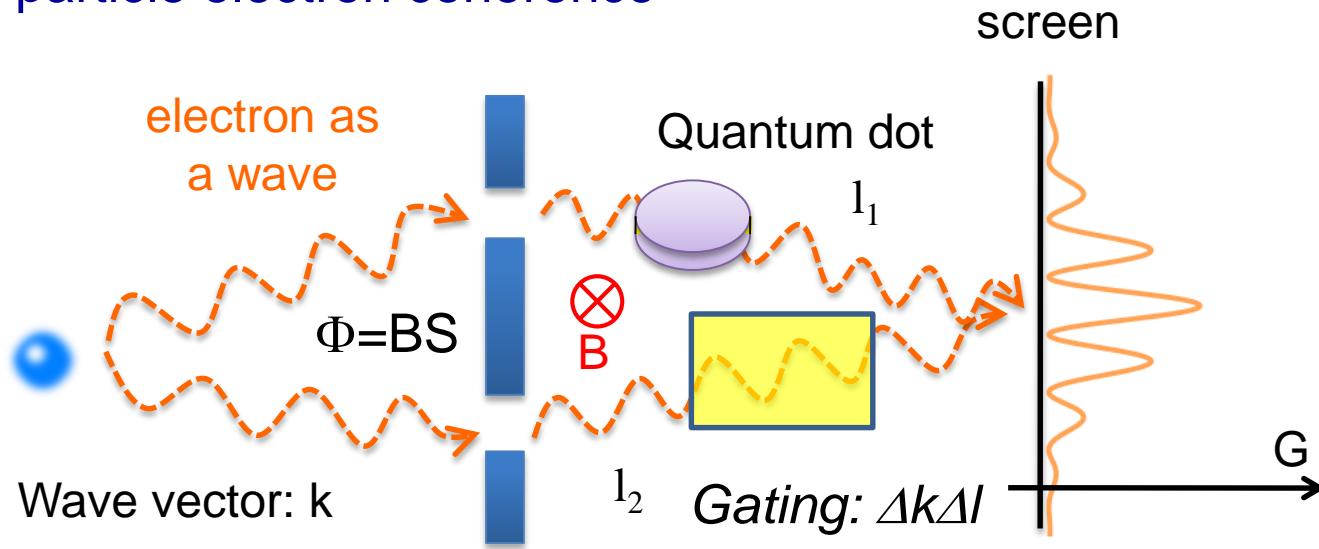
$$\Delta\varphi = k \cdot (l_1 - l_2) + \frac{e}{\hbar} BS$$

Normal-metal AB ring

R. Webb et al. Phys. Rev. Lett. 54, 2696 (1985).

# 位相の制御と検出

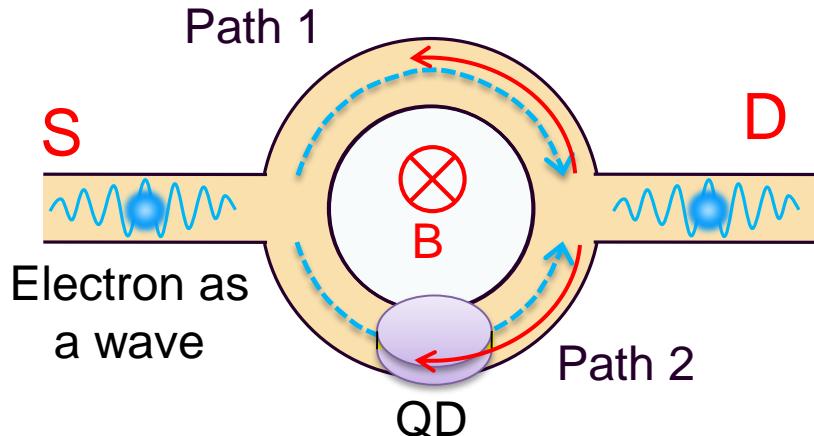
Single-particle electron coherence



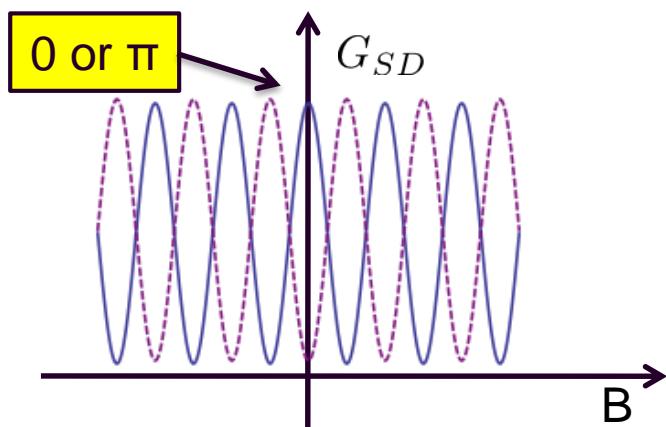
$$\Delta\varphi = \underbrace{k \cdot (l_1 - l_2)}_{\text{Path difference phase}} + \underbrace{\frac{e}{\hbar} BS}_{\text{AB phase}}$$

Path difference phase AB phase

# 位相剛性の問題



Two-terminal AB Ring with a QD



$G_{SD}$  : Linear conductance from S to D

Boundary conditions between contacts

**Unitarity:**  $G_{SD}(B) = G_{DS}(B)$

**Time reversal symmetry:**

$$G_{SD}(B) = G_{DS}(-B)$$

→  $G_{SD}(B) = G_{SD}(-B)$

Phase rigidity 0 or  $\pi$  ... only maintained by back scattering and multiple interference within the ring although expected phase shift is smooth:

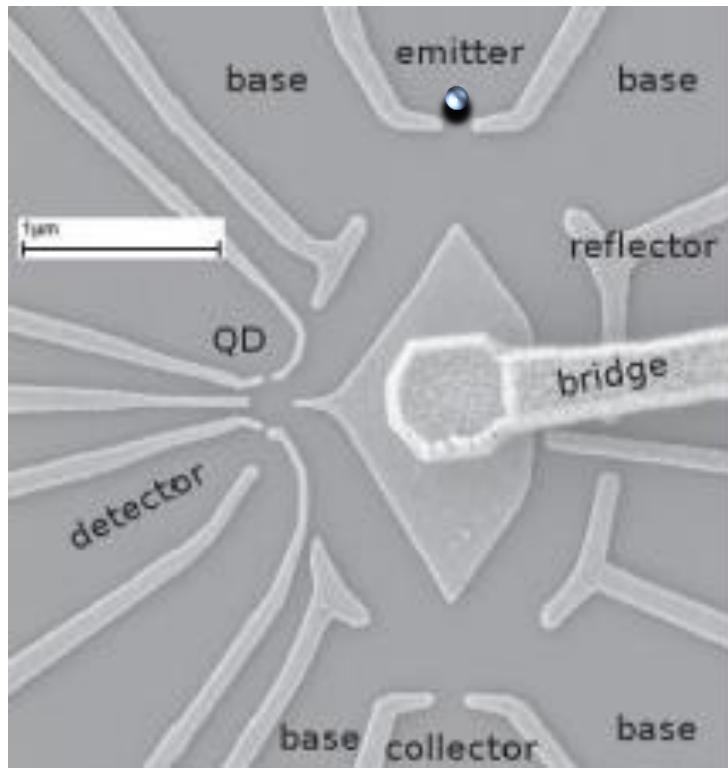
$$\Delta\varphi = k \cdot (l_1 - l_2) + \frac{e}{\hbar} BS + \Delta\varphi_{\text{dot}}$$



Difficult to measure transmission phase

# 位相剛性の問題の問題の解消の試み

Weizmann group

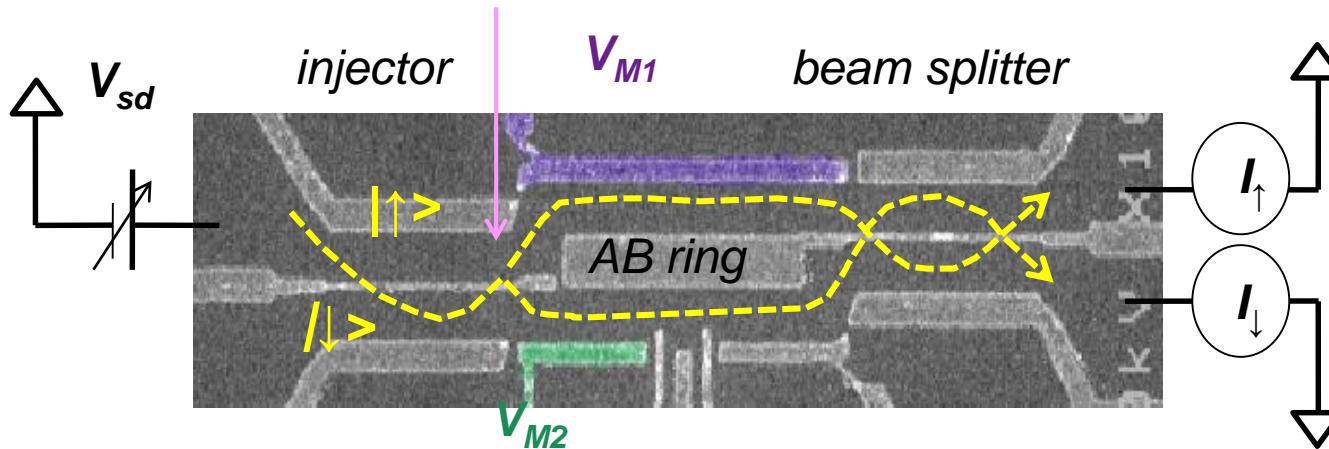


Multi-terminal AB ring

# 新型2経路干渉計

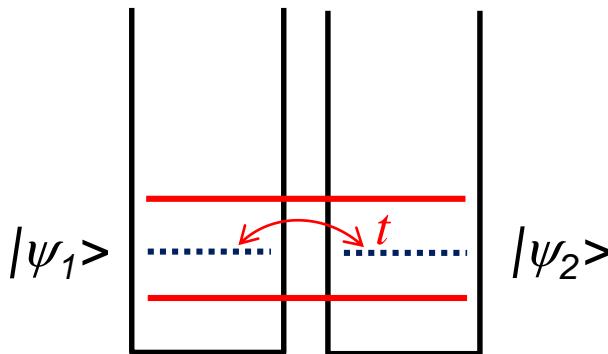
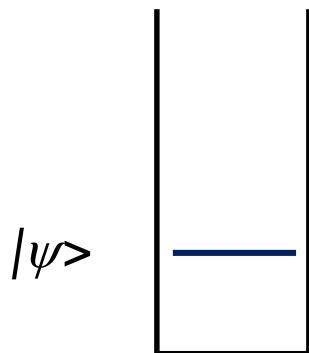
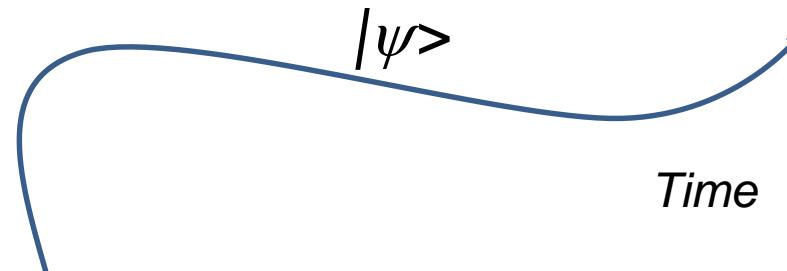
M. Yamamoto et al. Nature Nano. 7, 247 (2012).

$$\text{Input to AB ring} = \frac{| \uparrow \rangle - i | \downarrow \rangle}{\sqrt{2}}$$



# 固有エネルギー $E$ の状態ベクトルの位相発展

$$|\psi\rangle = |\psi\rangle e^{-i\int \frac{E}{\hbar} dt}$$



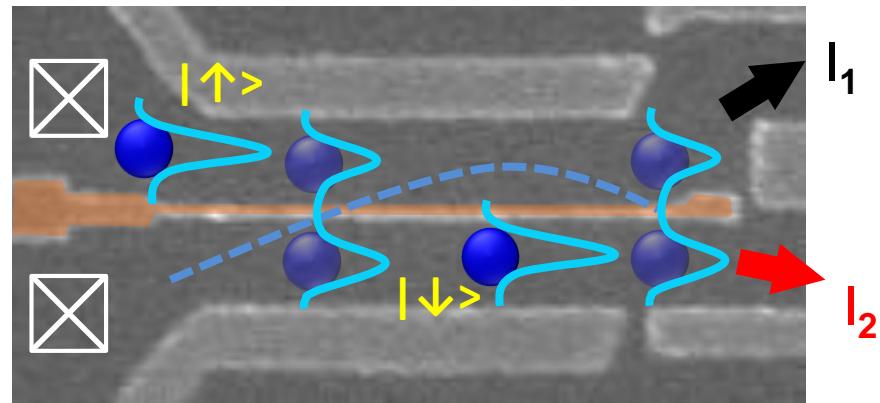
$$E_{AS} - E_S = 2t$$

$$|\psi\rangle_{AS} = \frac{1}{\sqrt{2}}(|\psi_1\rangle - |\psi_2\rangle)$$

$$|\psi\rangle_S = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)$$

$|\psi_i\rangle = \frac{1}{\sqrt{2}}(|\psi_S\rangle + |\psi_{AS}\rangle) \rightarrow |\psi\rangle_f = \frac{1}{\sqrt{2}}|\psi\rangle_S e^{-i\int \frac{E_S}{\hbar} dt} + \frac{1}{\sqrt{2}}|\psi\rangle_{AS} e^{-i\int \frac{E_{AS}}{\hbar} dt}$

# 結合量子細線を伝搬する電子波



**Input**  $|\uparrow\rangle = \frac{1}{\sqrt{2}}(|\psi_S\rangle + |\psi_{AS}\rangle)$

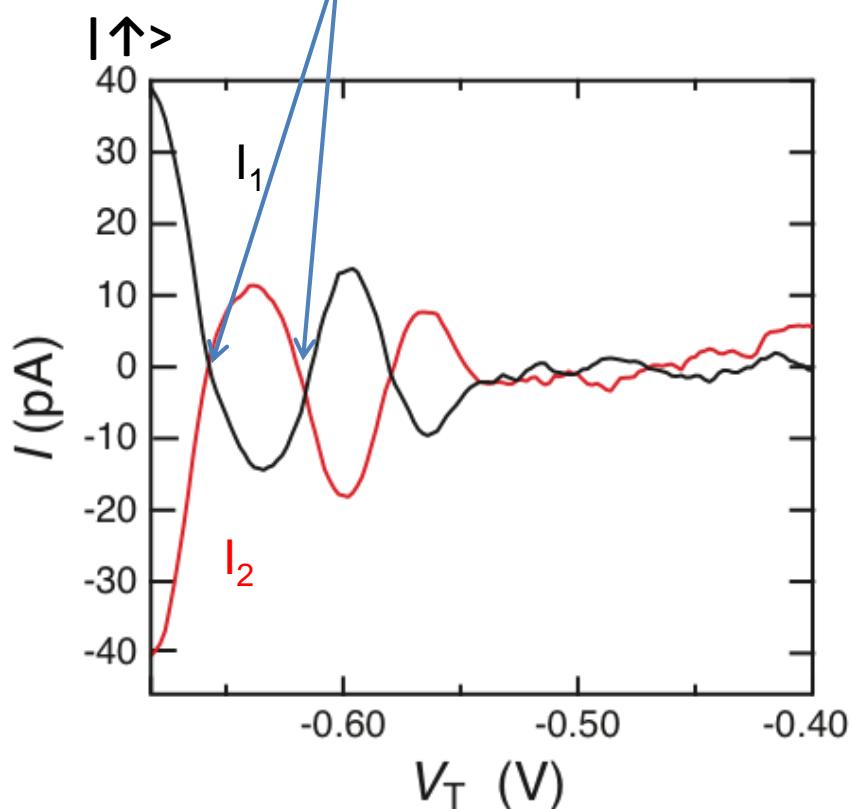
**Output**

$$|\psi\rangle_f = \frac{1}{\sqrt{2}}|\psi\rangle_S e^{-i\int \frac{E_S}{\hbar} dt} + \frac{1}{\sqrt{2}}|\psi\rangle_{AS} e^{-i\int \frac{E_{AS}}{\hbar} dt}$$

$$\propto \frac{1}{2}[|\uparrow\rangle(1+e^{-i\theta}) + |\downarrow\rangle(1-e^{-i\theta})]$$

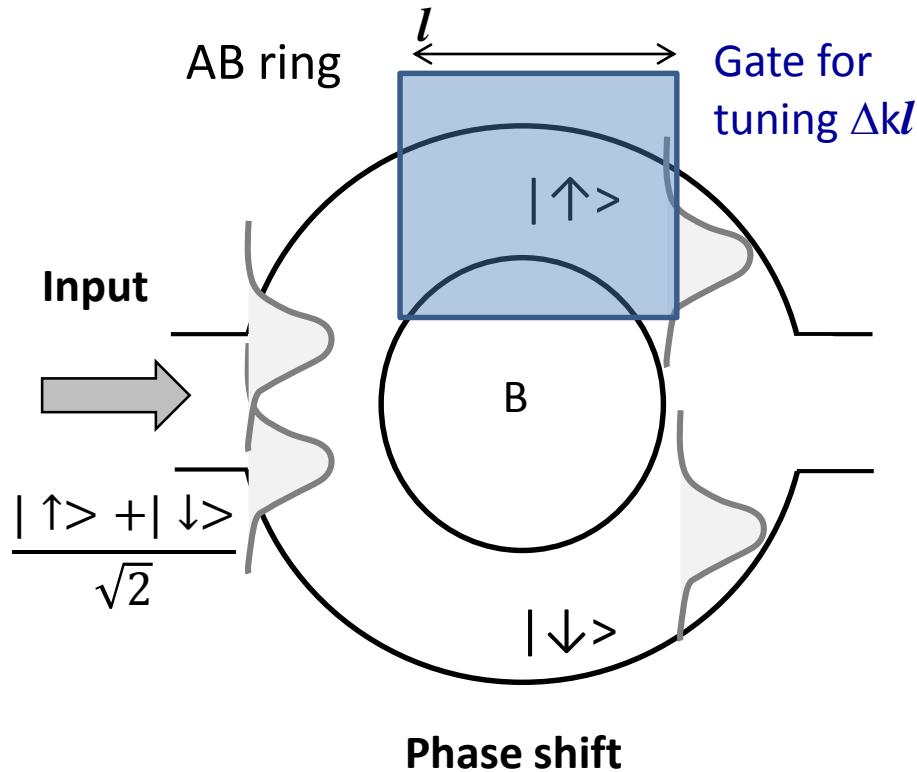
$$\theta(t) = \int \frac{\Delta E}{\hbar} dt$$

Beam splitter when  $\theta=(1+2n)(\pi/2)$   
50% (upper) : 50% (lower)



→  $I_{1,2} = \frac{|1 \pm \cos\theta|}{2}$

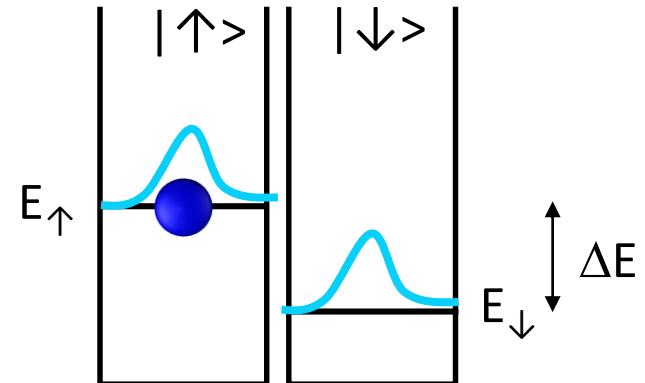
# ABリングでの位相制御



$$|\uparrow\rangle + e^{i\Delta\varphi} |\downarrow\rangle$$

$$\varphi = \varphi_{\uparrow} - \varphi_{\downarrow} = \frac{e}{\hbar} BS + \Delta k l$$

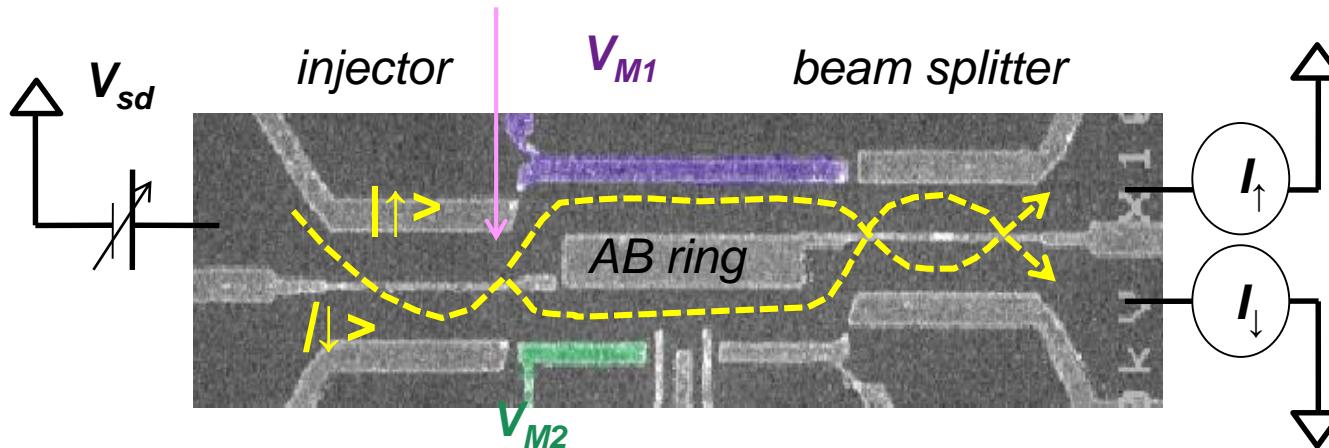
$$R_z(\varphi) \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}} = \frac{|\uparrow\rangle + e^{i\Delta\varphi} |\downarrow\rangle}{\sqrt{2}}$$



$$\begin{aligned} t=0 & \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}} \\ & \downarrow \\ & \frac{e^{-i\frac{E_{\uparrow}}{\hbar}t} |\uparrow\rangle + e^{i\frac{E_{\downarrow}}{\hbar}t} |\downarrow\rangle}{\sqrt{2}} \\ & = \frac{e^{-i\frac{E_{\uparrow}}{\hbar}t} (|\uparrow\rangle + e^{i\frac{\Delta E}{\hbar}t} |\downarrow\rangle)}{\sqrt{2}} \end{aligned}$$

# 2経路干渉計の出力

$$\text{Input to AB ring} = \frac{| \uparrow \rangle - i | \downarrow \rangle}{\sqrt{2}} \quad \text{for } \theta = 3\pi/2$$



*Right tunnel-coupled wire as a beam splitter*

*AB ring output*

$$\frac{| \uparrow \rangle + e^{-i\varphi} | \downarrow \rangle}{\sqrt{2}}$$

$$| 1 \rangle \rightarrow \frac{| 1 \rangle + i | 2 \rangle}{\sqrt{2}}$$

for  $\theta = \pi/2$

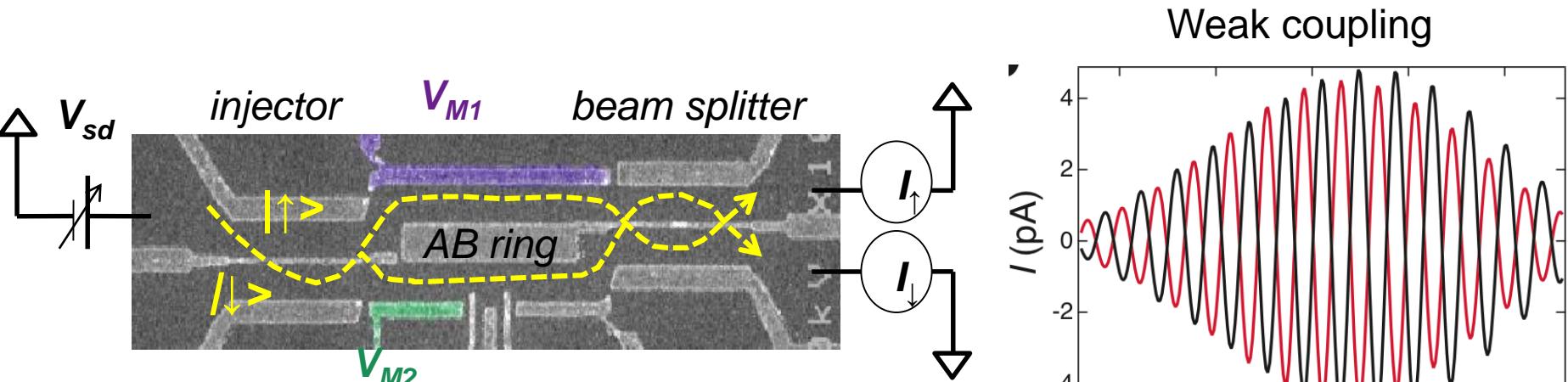
*Coupled wire output*

$$\frac{(1 + ie^{i\varphi}) | \uparrow \rangle + (i + e^{i\varphi}) | \downarrow \rangle}{2}$$

$$I_{\uparrow\downarrow} \propto \left| \frac{1 \pm ie^{i\varphi}}{2} \right|^2 = \frac{1 \mp \sin \varphi}{2}$$

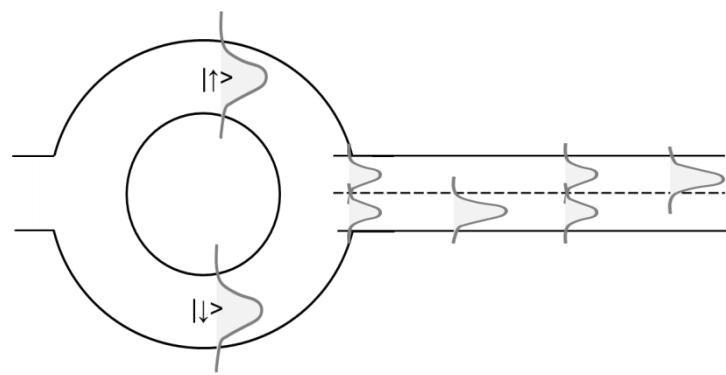
*Anti-phase with  $\varphi$   
between two outputs  
No phase rigidity in  
either output*

# 弱結合量子細線の出力:逆位相

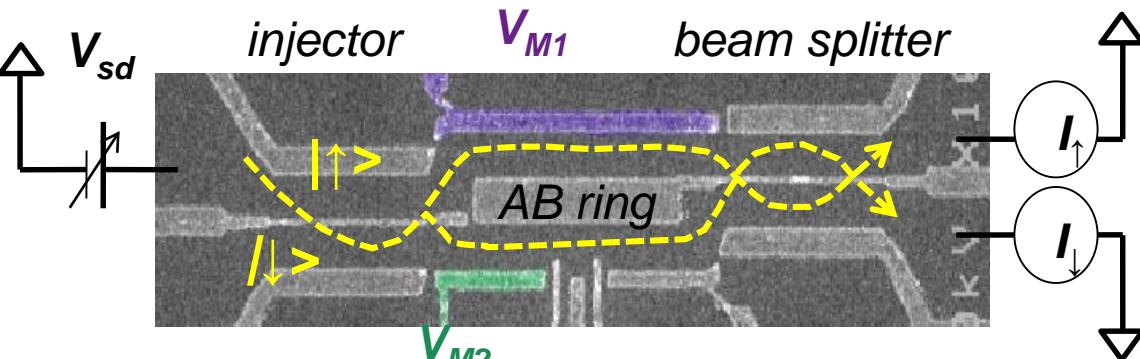


$$\frac{|\uparrow\rangle + e^{-i\varphi} |\downarrow\rangle}{\sqrt{2}} \xrightarrow{\text{Yellow arrow}} \frac{(1 + ie^{i\varphi})|\uparrow\rangle + (i + e^{i\varphi})|\downarrow\rangle}{2}$$

$$I_{\uparrow\downarrow} \propto \left| \frac{1 \pm ie^{i\varphi}}{2} \right|^2 = \frac{1 \mp \sin \varphi}{2} \quad \text{Anti-phase with } \varphi$$

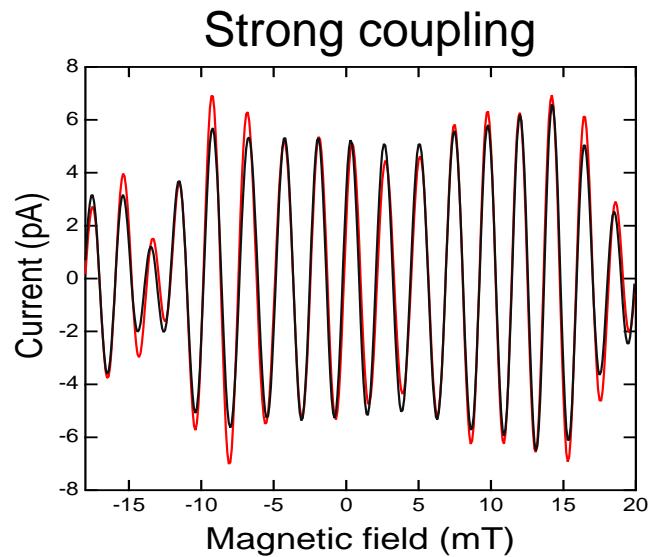
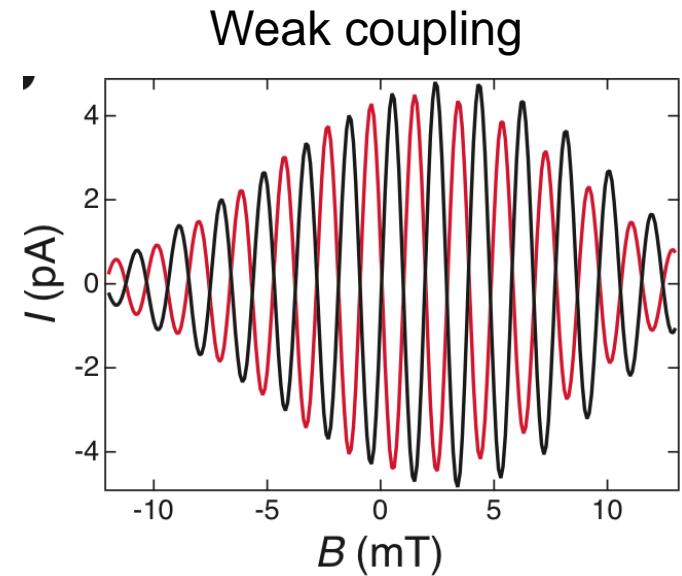


# 結合量子細線の出力

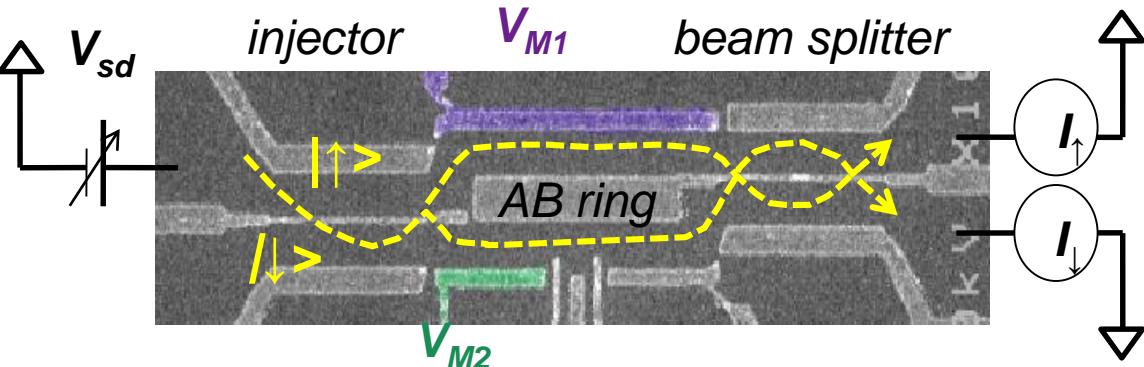


$$\frac{|\uparrow\rangle + e^{-i\varphi} |\downarrow\rangle}{\sqrt{2}} \xrightarrow{\text{Yellow arrow}} \frac{(1 + ie^{i\varphi})|\uparrow\rangle + (i + e^{i\varphi})|\downarrow\rangle}{2}$$

$$I_{\uparrow\downarrow} \propto \left| \frac{1 \pm ie^{i\varphi}}{2} \right|^2 = \frac{1 \mp \sin \varphi}{2} \quad \text{Anti-phase with } \varphi$$

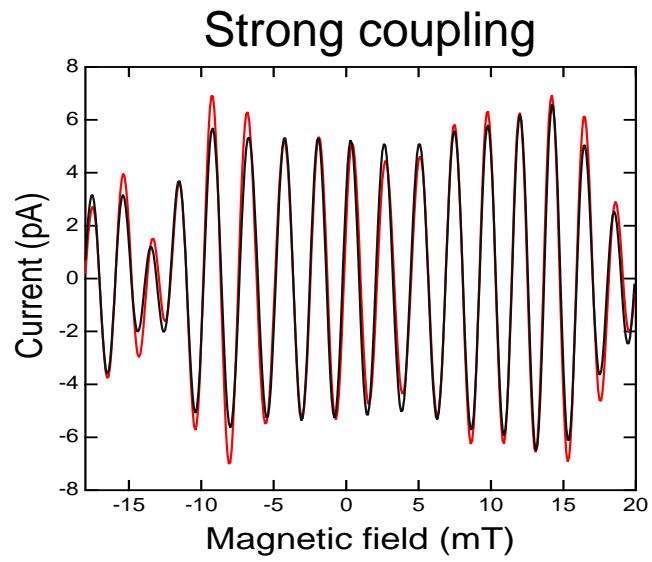
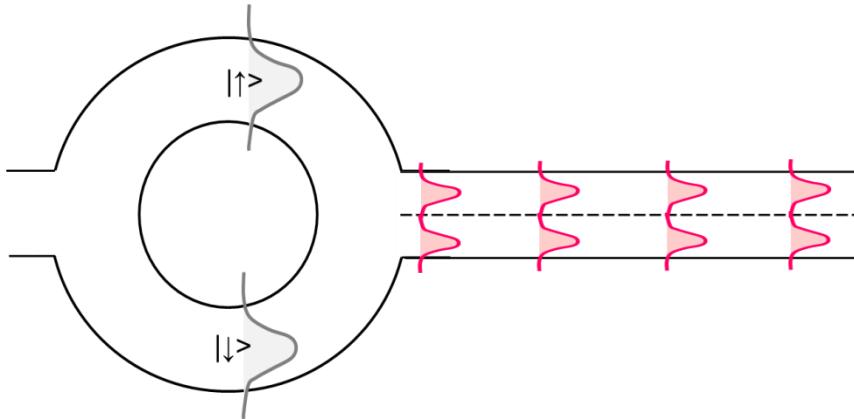


# 強結合量子細線の出力: 同位相



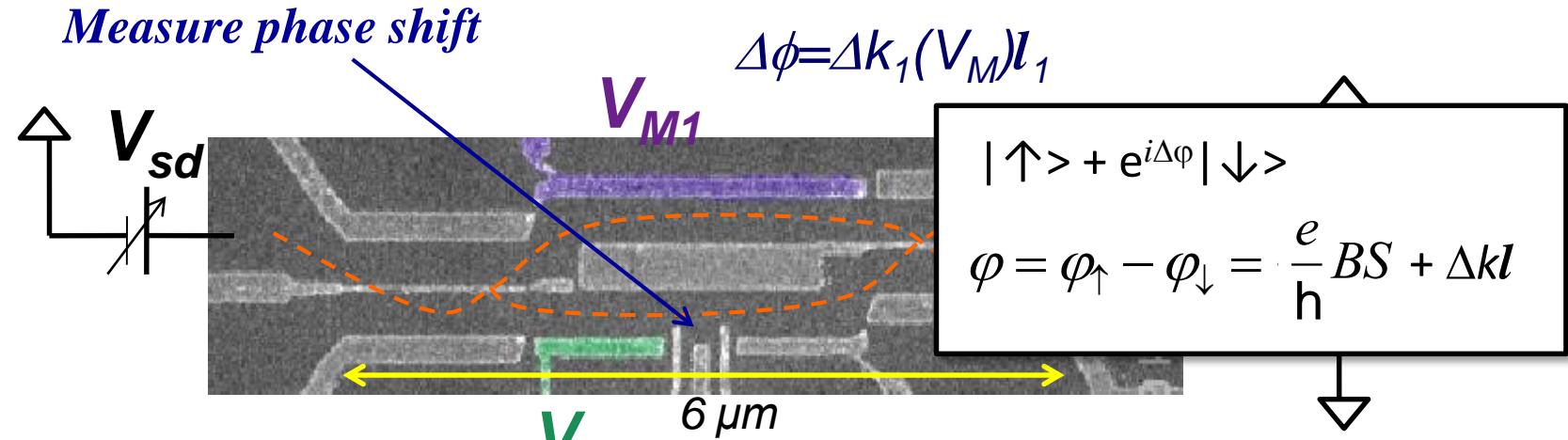
$$\frac{|\uparrow\rangle + e^{-i\varphi} |\downarrow\rangle}{\sqrt{2}} \xrightarrow{\text{Yellow arrow}} \frac{(1 + ie^{i\varphi})|\uparrow\rangle + (i + e^{i\varphi})|\downarrow\rangle}{2}$$

$$I_{\uparrow\downarrow} \propto \left| \frac{1 \pm ie^{i\varphi}}{2} \right|^2 = \frac{1 \mp \sin \varphi}{2} \quad \text{Anti-phase with } \varphi$$

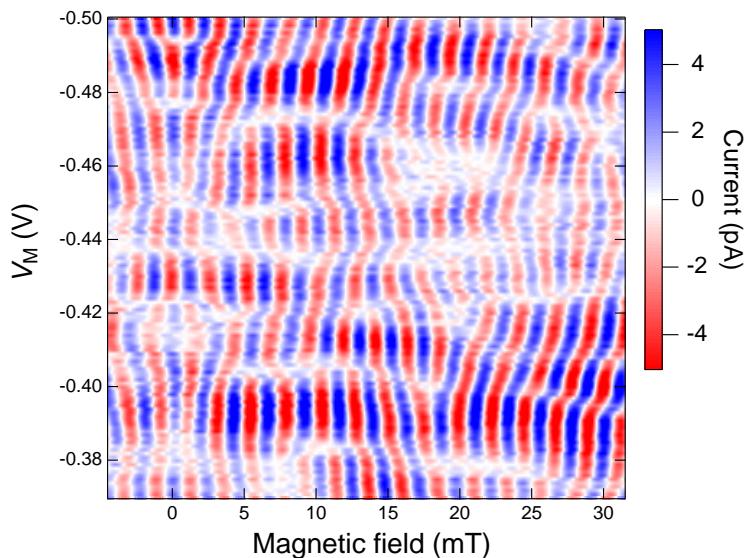


# 位相検出器(位相剛性の有無)

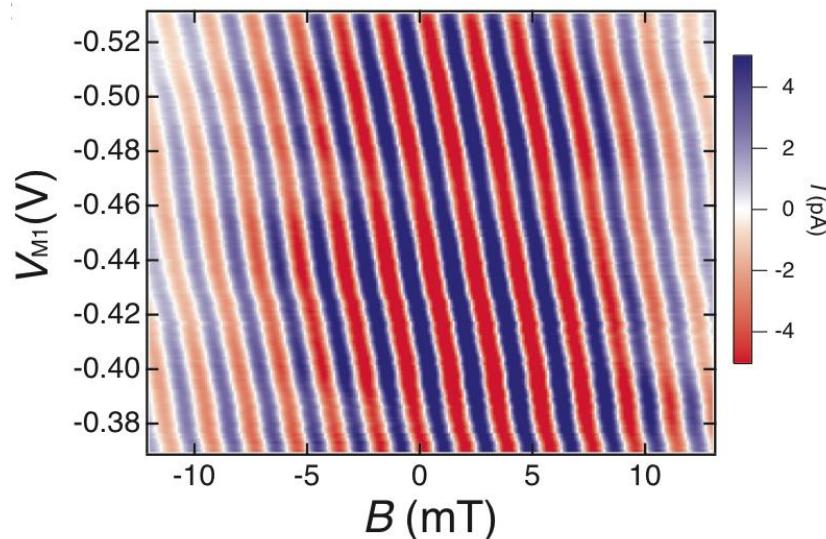
S. Takada et al. APL 2015



*Strong coupling*



*Weak coupling*



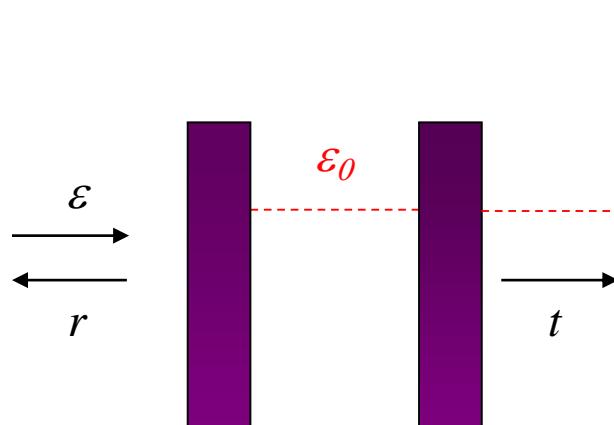
Multi-path interferometer

Phase rigidity:  $G(B) = G(-B)$

Two-path interferometer

Unambiguous measurement of phase

# 量子ドットを透過する電子波の位相と電子数の関係

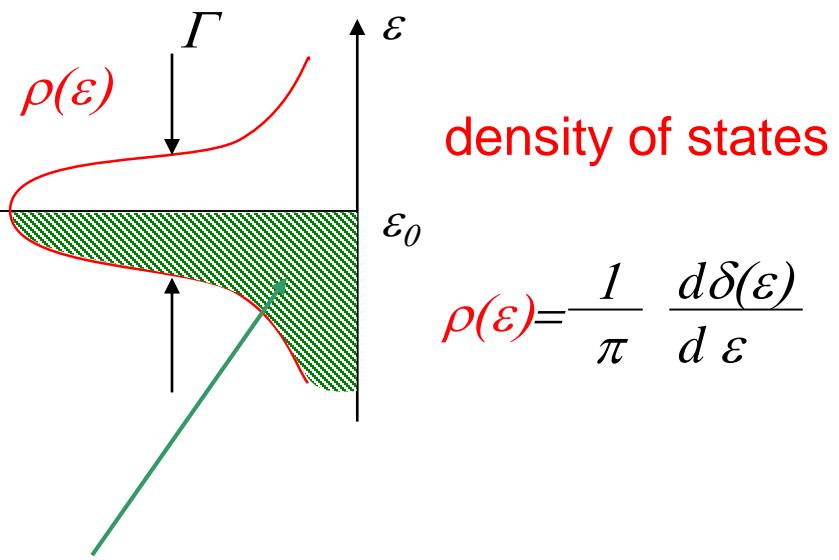
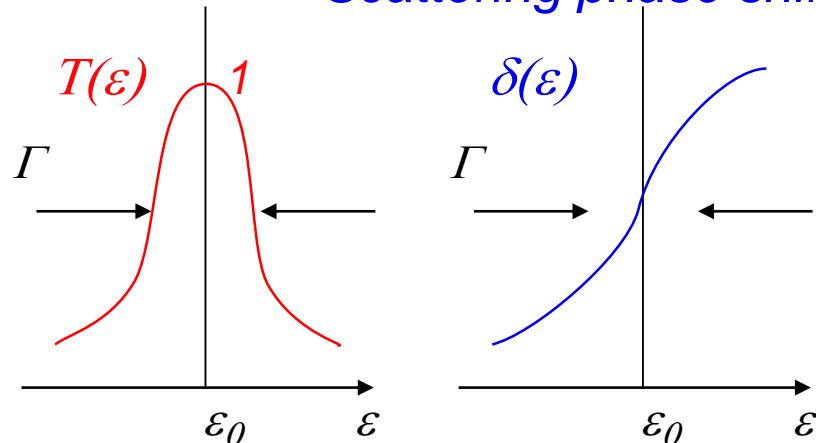


Breit - Wigner

$$T(\epsilon) = |t|^2 = \frac{\Gamma^2}{(\epsilon - \epsilon_0)^2 + \Gamma^2}$$

$$= \sin^2 \delta(\epsilon)$$

Scattering phase shift



Localized level: half-occupied  
for resonance  $\epsilon_F = \epsilon_0$

$$N = \int \rho(\omega) d\omega = \frac{1}{2}$$

$$T(\epsilon_F) = 1 \quad \text{for} \quad \delta(\epsilon_F) = \frac{\pi}{2}$$

$N = \frac{1}{\pi} \delta(\epsilon_F)$  : Friedel sum rule

# 量子ドットを透過する電子波の位相と電子数の関係

$N - \delta(\varepsilon_F)$  related  
Friedel sum rule

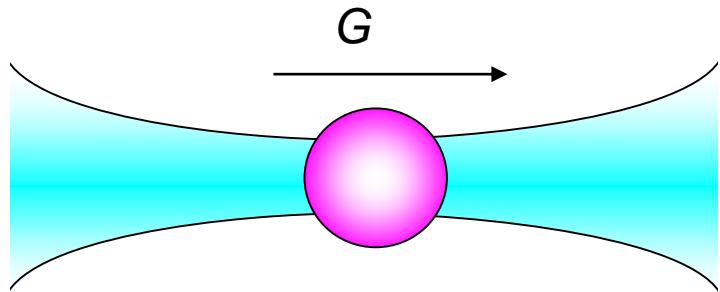
$$N = \frac{1}{\pi} \delta(\varepsilon_F) \quad \text{for spinless fermions}$$

$$N = \frac{1}{\pi} [\delta_{\uparrow}(\varepsilon_F) + \delta_{\downarrow}(\varepsilon_F)] \quad \text{for spinful electrons}$$

Landauer formula

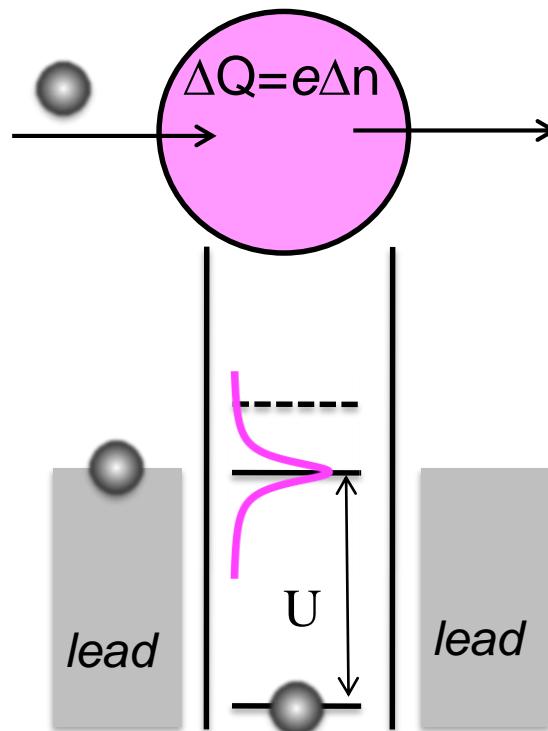
$$G = \frac{e^2}{h} (T_{\uparrow} + T_{\downarrow})$$

$$T_{\uparrow\downarrow}(\varepsilon_F) = \sin^2 \delta_{\uparrow\downarrow}(\varepsilon_F)$$
$$= 1$$



# 量子ドットを透過する電子波の位相

Excess charge due to tunneling

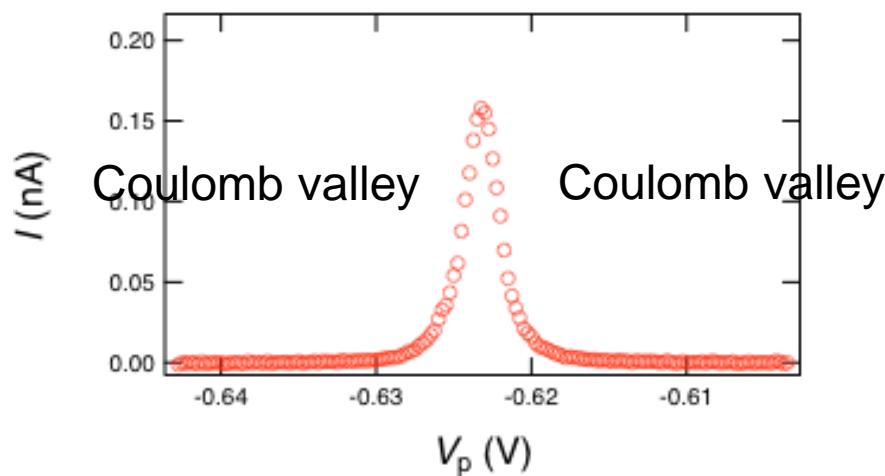


Phase shift  $\Delta\varphi_{\text{dot}}$

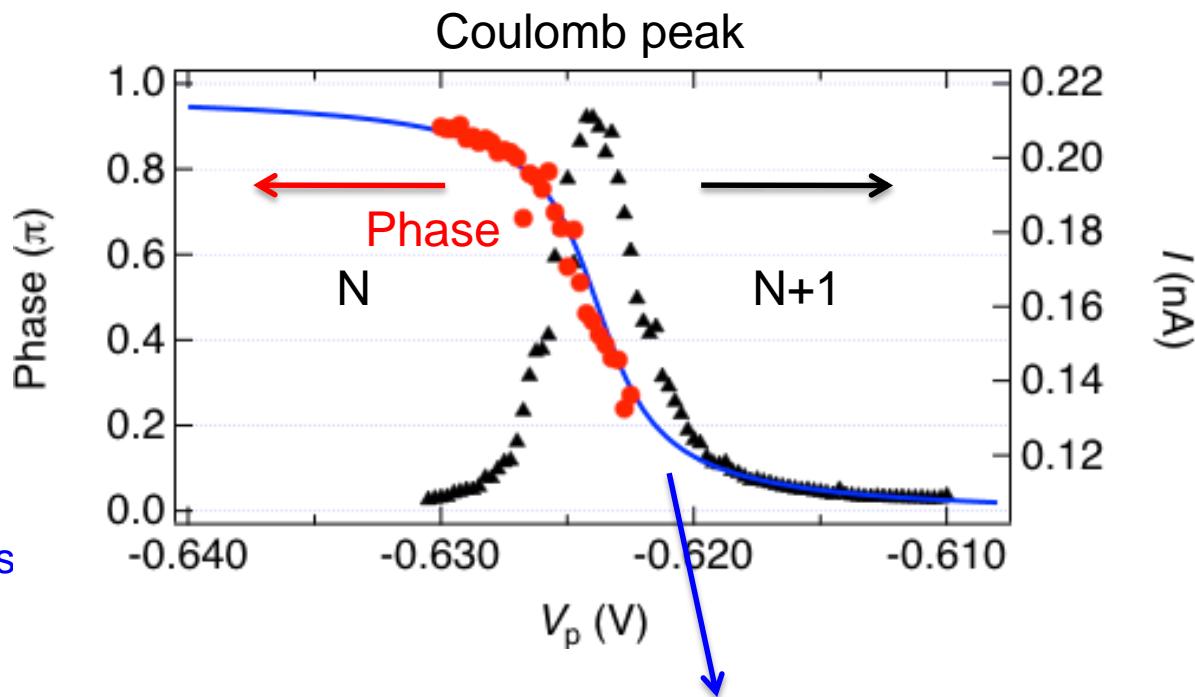
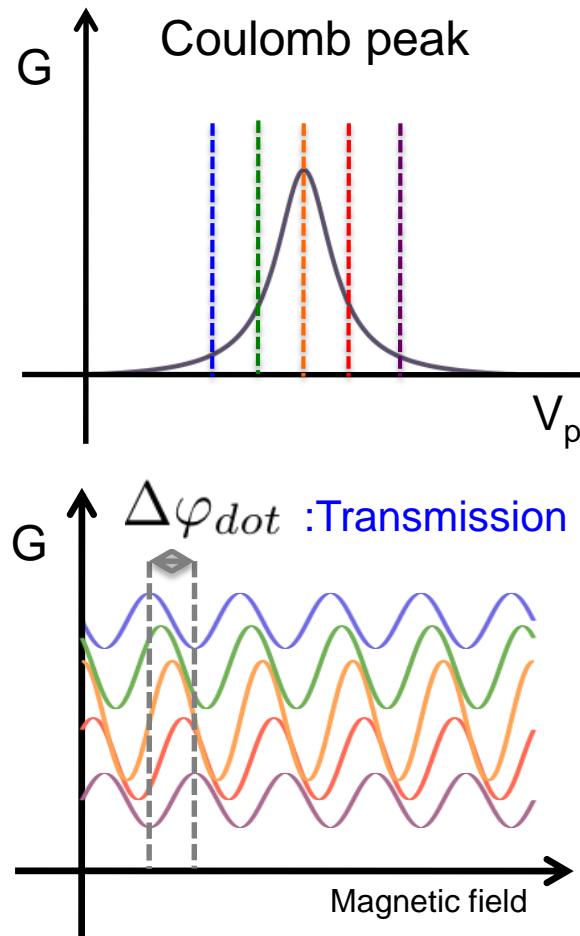
Friedel's sum rule:  $\Delta\varphi_{\text{dot}} = \pi\Delta n$

One-by-one tunneling due to Coulomb blockade,  
then  $\Delta n$  across a Coulomb peak between two  
Coulomb valleys

$\Delta n = 1$  then  $\Delta\varphi_{\text{dot}} = \pi$



# クーロンピークを透過する単一電子の位相

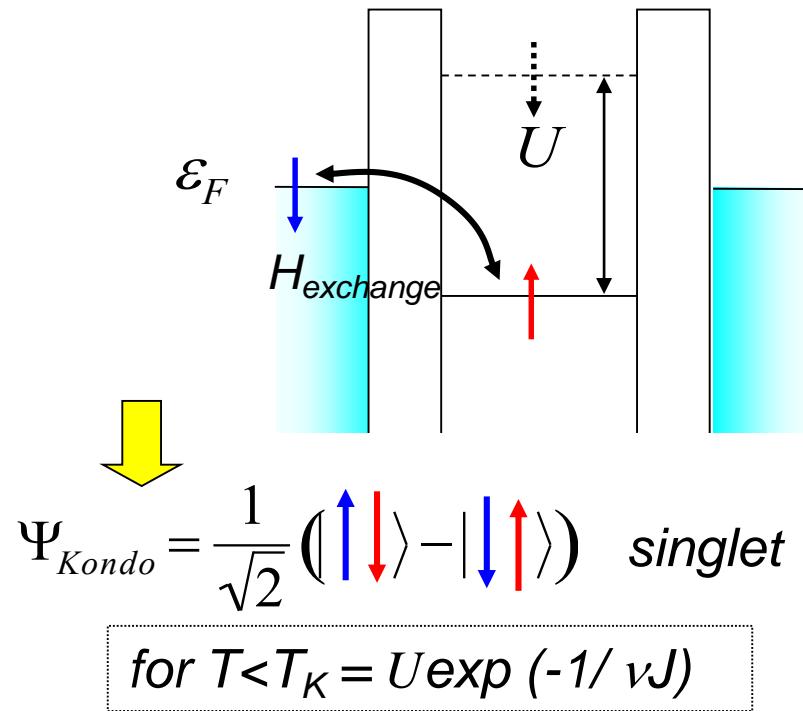
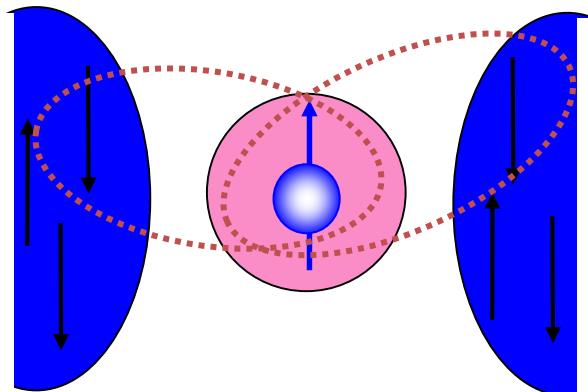


Phase deduced from  $I$  line shape by Friedel's sum rule

$$\text{Friedel's sum rule: } \Delta\varphi_{dot} = \pi \frac{\Delta Q}{e}$$

Phase shifts  $\pi$  across the Coulomb peak

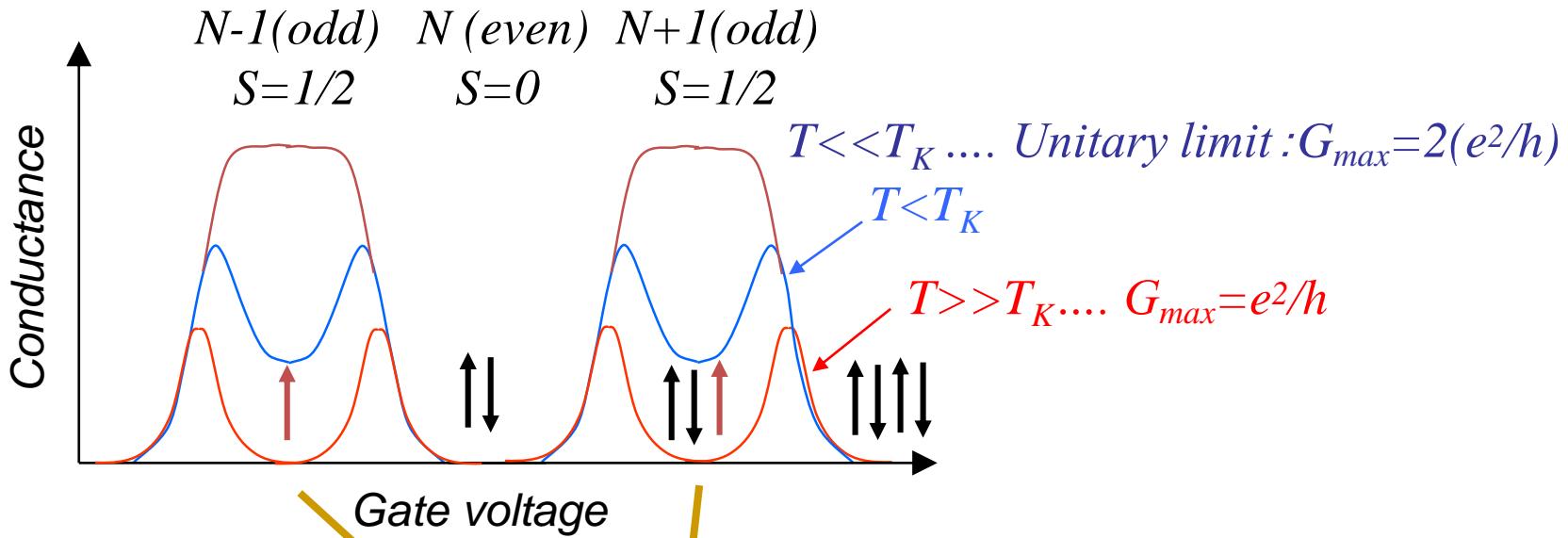
# ドット中の局在スピンによる近藤効果



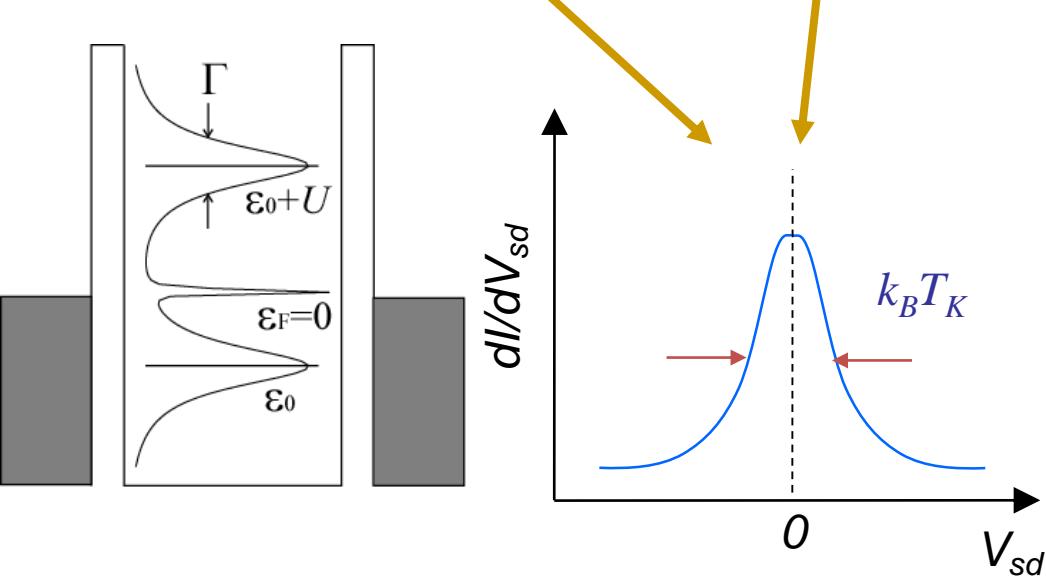
Screening of local spin by exchange coupling with electrons at Fermi sea in the leads.

# 量子ドットの近藤効果の特徴

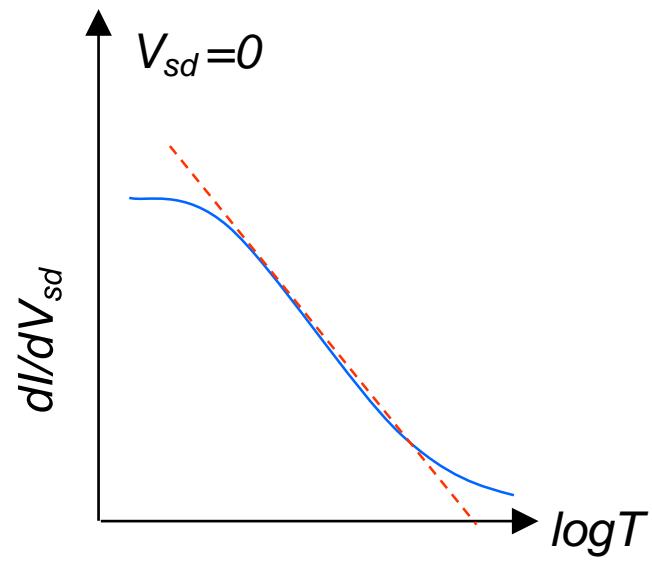
**Lifting Coulomb blockade at low temperatures**



**Zero-bias ( $dI/dV$ ) peak in density of states**

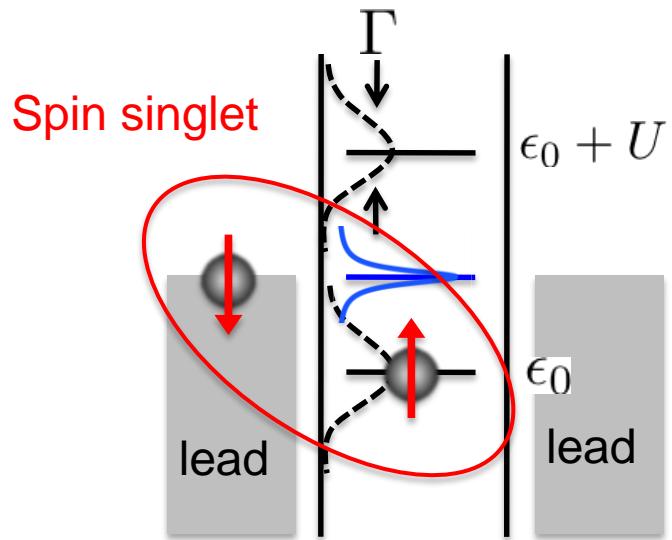


**LogT dep. of  $dI/dV$  peak**

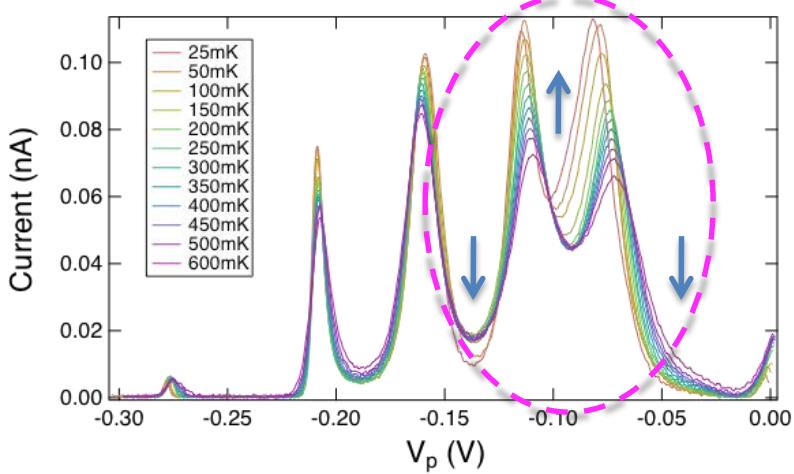


# 近藤谷での位相シフト

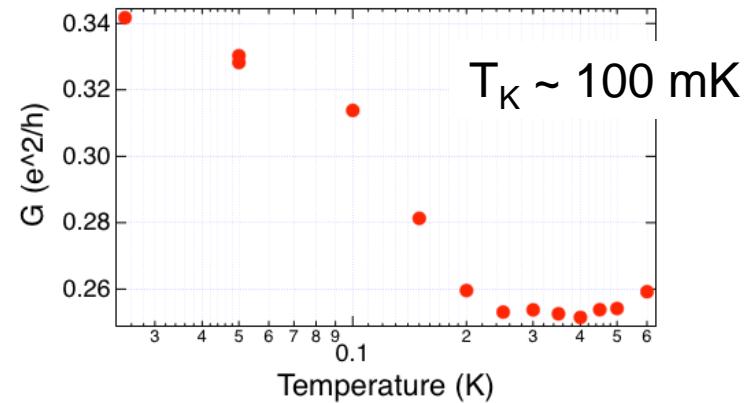
$$\Delta\phi_{\text{dot}} = \pi/2 \text{ for } \Delta n = 1/2$$



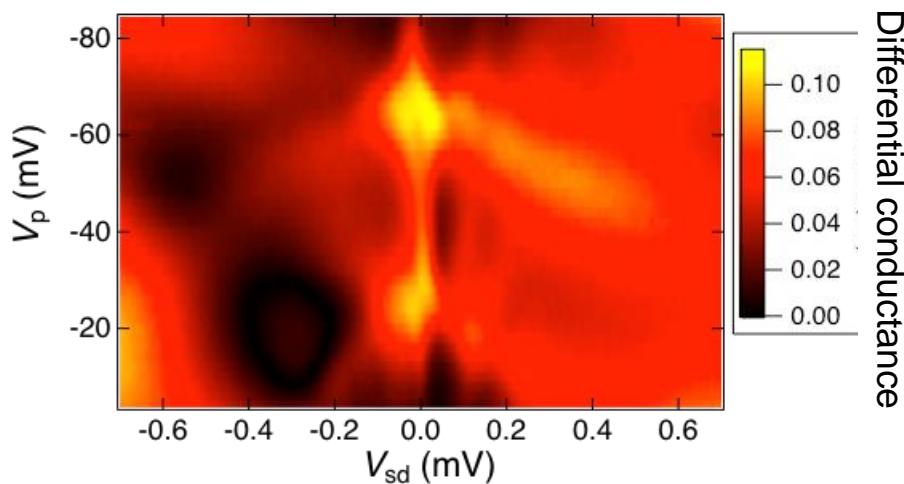
Peak appears at low temperatures ( $T \ll T_K$ )



Temperature dependence of Coulomb peaks



Temperature dependence of the conductance at the Kondo valley



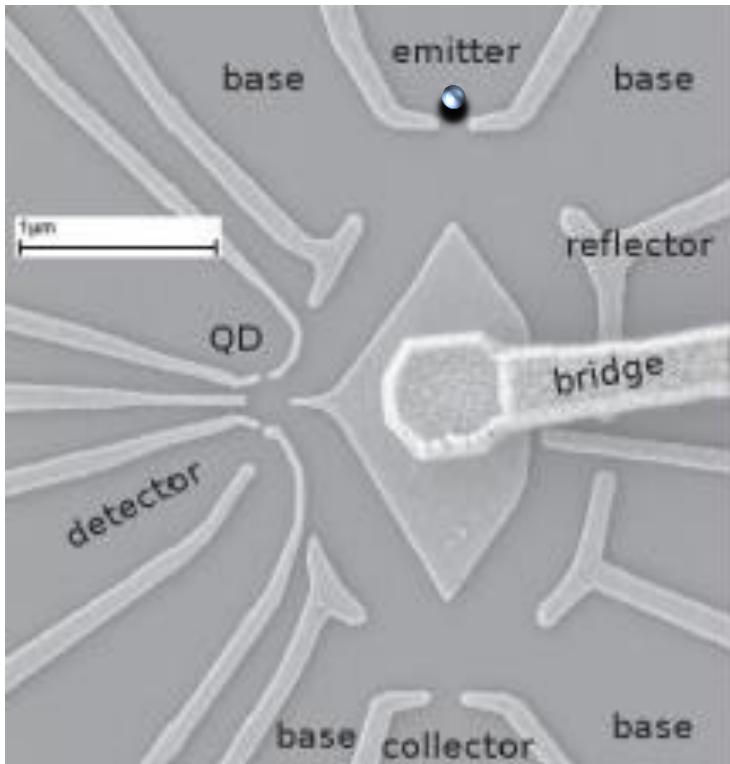
Coulomb diamond

# 多重干渉の除去の試み

Weizmann group

## Theory

- $\pi / 2$  – shift in the Kondo regime ( $T < T_K$ )
- $\pi$  – shift in the Coulomb blockade regime ( $T \gg T_K$ )



Multi-terminal AB ring

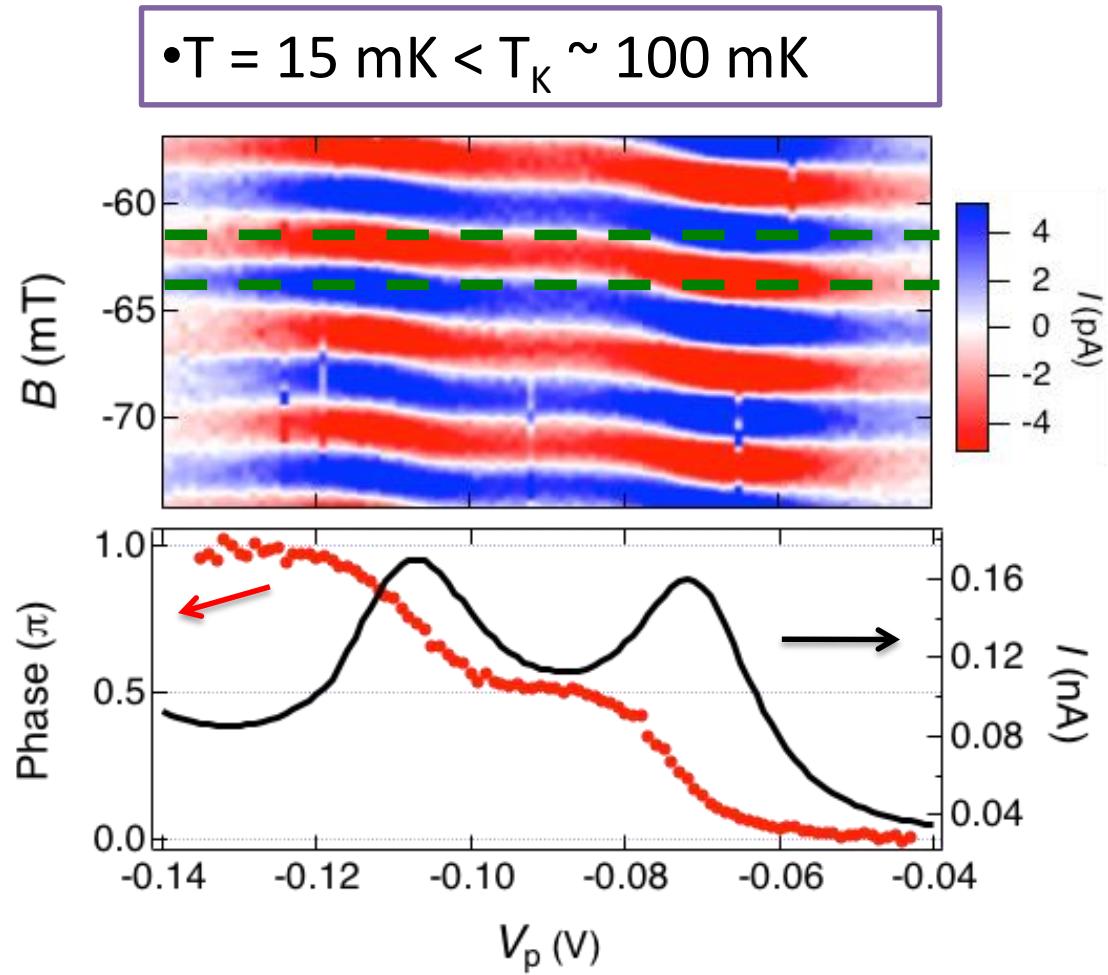
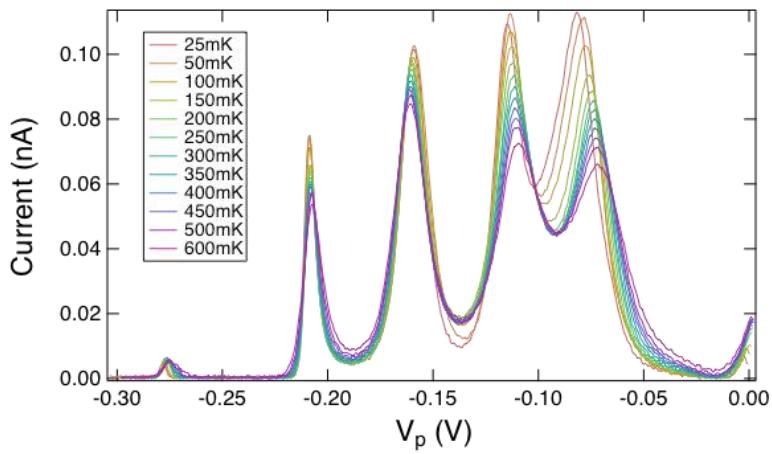
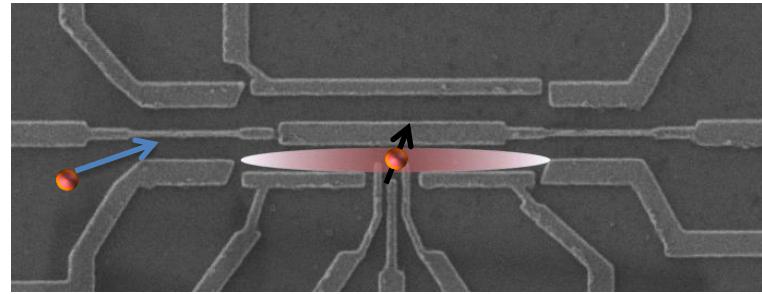
- Large QD ( $\delta < \Gamma$ )
  - $\pi$  - shift in the Kondo valley ( $\neq \pi/2$ )  
 $T (= 90 \text{ mK}) < T_K (= 1 \text{ K})$
- Y. Ji et al., Science **290**, 779 (2000)
- Small QD ( $\delta > \Gamma$ )
  - $\pi/2$  - shift across each Coulomb peak  
 $T (= 30 \text{ mK}) > T_K (= 1 \text{ mK})$

M. Zaffalon et al., PRL **100**, 226601 (2008)



Inconsistent with theory

# 近藤谷での位相シフトの観測 ( $T < T_K$ )



•  $\pi/2$  plateau at the Kondo valley

Direct evidence of Kondo singlet state