Unified picture of non-geometric fluxes and geometry of double field theory via supergeometry

Noriaki Ikeda
Ritsumeikan University

JHEP 1702 (2017) 078, arXiv:1611.08346 [hep-th]
with M.A.Heller and S.Watamura

Geometry, Duality and Strings, YITP 2017

§1. Introduction

Motivation

T-duality chain of NS fluxes Shelton-Taylor-Wecht '05

$$H_{abc} \overset{T_a}{\longleftrightarrow} F^a_{bc} \overset{T_b}{\longleftrightarrow} Q^{ab}_c \overset{T_c}{\longleftrightarrow} R^{abc},$$

Fluxes are considered to consist of the Kaloper-Myers algebra,

$$\begin{aligned}
[e_a, e_b] &= F^c_{ab} e_c + H_{abc} e^c_\sharp, \\
[e_a, e^b_\sharp] &= Q^{bc}_a e_c - F^b_{ac} e^c_\sharp, \\
[e^a_\sharp, e^b_\sharp] &= R^{abc} e_c + Q^{ab}_c e^c_\sharp.
\end{aligned}$$

1, $H = dB$, where B is an NS B-field, and

$$F_{ab}^{c} = \cdots \,, \qquad Q_{a}^{bc} = \cdots \,, \qquad R^{abc} = \cdots \,.$$

- What is the background geometric and algebraic structure of fluxes?
- Can we formulate a simple method of complicated equations?

2, A manifest T-duality invariant formulation and double field theory

Siegel '93, Hull-Zwiebach '09

- What is the background geometric structure of a double field theory?
- Can we obtain nontrivial new solutions?

Plan of Talk

Supergeometric construction of Lie algebra

Supergeometric formulation of geometry of H-, F-, Q-, R-fluxes

Supergeometric formulation of geometry of double field theory

§2. Supergeometric construction of algebraic and geometric structures : Lie algebras

motivated by a BRST-BV formalism.

BRST charge of Lie Algebra

$[p_a, p_b] = f^c{}_{ab} p_c$ with basis p_a

1, Introduce the canonical conjugate basis q^a.

Take p_a and q^a as Grassmann odd.

degree (ghost number): $|q^a| = |p_a| = 1$.

2, $\{q^a, p_b\} = \delta^a{}_b$.

3, The BRST charge of a Lie algebra such that $Q(-) = \{\Theta, -\}$ is

$$\Theta = \frac{1}{2} f^{a}{}_{bc} p_a q^b q^c,$$

$|Q| = 1$ and $Q^2 = 0$ since $f^{d}_{e[a} f^{e}_{bc]} = 0$.

The Hamiltonian function Θ satisfies the classical master equation

$$\{\Theta, \Theta\} = 0.$$

Note: The Lie bracket is recovered by the **derived bracket**,

$$[p_a, p_b] = -\{\{p_a, \Theta\}, p_b\}.$$

Note: Q defines a Chevalley-Eilenberg complex and the CE coboundary operator of a Lie algebra.

The corresponding gauge theory is a three-dimensional BF theory or a Chern-Simons theory.

QP-manifold (differential graded symplectic manifold)

1, local coordinates $(q^a, p_a, \cdots)$ are 'super' coordinates.

2, $\{-,-\}$: graded Poisson brackets, $|\{-,-\}| = -2$.

3, Θ: BRST charge (Hamiltonian) such that $Q = \{\Theta, -\}$ satisfies $Q^2 = 0$ and $|Q| = 1$.

Note: $|\Theta| = 3$.

The derived bracket, $[-,-] = -\{\{-,\Theta\},-\}$, defines a bracket of an algebra.

§3. (Super)Geometry of fluxes

Heller-NI-Watamura '16

QP-manifold

We introduce a spacetime coordinate x^i.

Introduce the canonical conjugate ξ_i, such that $\{x^i, \xi_j\} = \delta^i{}_j$ and $|x^i| = 0$, $|\xi_i| = 2$.

(q^i, p_i) such that $\{q^i, p_j\} = \delta^i{}_j$ and $|q^i| = |p_i| = 1$,

Total coordinates (x^i, ξ_i, q^i, p_i), and $|\{-,-\}| = -2$.

A general form of Θ is of degree 3,

$$\begin{aligned}\Theta &= \rho^j{}_i(x)q^i\xi_j + \pi^{ij}(x)\xi_i p_i + \frac{1}{3!}H_{ijk}(x)q^iq^jq^k \\ &\quad + \frac{1}{2}F_{ij}^k(x)q^iq^jp_k + \frac{1}{2}Q_i^{jk}(x)q^ip_jp_k + \frac{1}{3!}R^{ijk}(x)p_ip_jp_k,\end{aligned}$$

where H, F, Q, R are defined by this equation.

Note: In fact, a structure is a Courant algebroid.

Q without flux

We start the simplest Hamiltonian function Θ of degree 3 without fluxes (and backgrounds),

$$\Theta_{\mathsf{S},0} = q^i \xi_i,$$

which satisfies $Q^2 = 0$.

We introduce fluxes by the next technique.

Twisting

Twisting is the exponential adjoint action,

$$e^{\delta_\alpha} f = f + \{f, \alpha\} + \frac{1}{2}\{\{f, \alpha\}, \alpha\} + \cdots ,$$

where $f(x, q, p, \xi)$ is any function. If $|\alpha| = 2$, the adjoint action is degree-preserving and obeys

$$\{e^{\delta_\alpha} f, e^{\delta_\alpha} g\} = e^{\delta_\alpha}\{f, g\}.$$

B-transformation

$$B \equiv \frac{1}{2} B_{ij}(x) q^i q^j.$$

Twisting

$$\exp(-\delta_B)\Theta_{\mathsf{S},0} = q^i \xi_i + \frac{1}{2} \partial_i B_{jk} q^i q^j q^k,$$

so that $H_{ijk} = 3\partial_{[i} B_{jk]}$ or $H = dB$.

$Q^2 = 0$ is equivalent to $dH = 0$. This is the condition of an NS H-flux.

β-transformation

$$\beta \equiv \frac{1}{2}\beta^{ij}(x)p_i p_j.$$

Twisting

$$\exp(-\delta_\beta)\Theta_{\mathsf{S},0} = \xi_i q^i - \beta^{mi}\xi_m p_i + \frac{1}{2}\partial_i \beta^{jk} q^i p_j p_k + \frac{1}{2}\beta^{im}\partial_m \beta^{jk} p_i p_j p_k.$$

The twist induces $Q_i^{jk} \equiv \partial_i \beta^{jk}$ and $R^{ijk} \equiv 3\beta^{[i|m|}\partial_m \beta^{jk]}$.

$Q = d\beta$ and $R = \frac{1}{2}[\beta, \beta]_S$, where $[-,-]_S$ is a Schouten bracket.

The classical master equation, $\{\Theta_\beta, \Theta_\beta\} = 0$, then implies the

Bianchi identities

$$\partial_{[m} Q_{i]}^{[jk]} = 0,$$
$$3\beta^{[i|m|}\partial_m Q_n^{jk]} - \partial_n R^{[ijk]} + 3Q_n^{[i|m|} Q_m^{jk]} = 0,$$
$$\beta^{[i|m|}\partial_m R^{jkl]} - \frac{3}{2} R^{[ij|m|} Q_m^{kl]} = 0.$$

These equations are relations in the β-supergravity.

Aldazabal-Baron-Marques-Nunez, '11

R-flux under Poisson manifold background (Poisson Courant algebroid)

Asakawa-Muraki-Sasa-Watamura '15

$$[\pi, \pi]_S = 0, \qquad R = \frac{1}{2}[\pi, \beta]_S.$$

satisfies $[\pi, R]_S = 0$. These equations are

$$\partial_l \pi^{[ij} \pi^{k]l} = 0, \qquad R^{ijk} = \frac{1}{2}\pi^{[i|l}\partial_l \beta^{jk]},$$
$$\pi^{[i|l}\partial_l R^{jkl]} - 2R^{[ij|l}\partial_l \pi^{kl]} = 0.$$

We take functions of degree 2, $\pi = \frac{1}{2}\pi^{ij}(x)p_i p_j$ and $\pi^{-1} = \frac{1}{2}\pi^{-1}_{ij}(x)q^i q^j$ and consider the twisting,

$$e^{\delta_\beta}e^{\delta_\pi}e^{-\delta_{\pi^{-1}}}e^{\delta_\pi}\Theta_{\mathsf{S},0}$$

This gives

$$\Theta_{\pi,R} = e^{\delta_\beta}e^{\delta_\pi}e^{-\delta_{\pi^{-1}}}e^{\delta_\pi}\Theta_{\mathsf{S},0} = \pi^{ij}\xi_j p_j - \partial_k\pi^{ij}q^k p_i p_j + \frac{1}{3!}R^{ijk}p_i p_j p_k.$$

where $R^{ijk} = \frac{1}{2}\pi^{[i|l}\partial_l\beta^{jk]}$. $Q^2 = 0$ gives correct relations.

Fluxes with metric and Bianchi identity

Blumenhagen-Deser-Plauschinn-Rennecke '12

We introduce a vielbein $e^a{}_i$, where $a, b, \cdots$ are local Lorentz indices.

$$H_{abc} = 3\nabla_{[a} B_{bc]}, \qquad H_{mns} = 3\partial_{[m} B_{ns]},$$

$$F^a_{bc} = f^a_{bc} - H_{mns}\beta^{si} e^a{}_i e_b{}^m e_c{}^n, \qquad f^a_{bc} = 2 e_{[b}{}^m \partial_m e_{c]}{}^j e^a{}_j,$$

$$Q^{bc}_a = \partial_a \beta^{bc} + f^b_{ad}\beta^{dc} - f^c_{ad}\beta^{db} + H_{isr}\beta^{sh}\beta^{rk} e_a{}^i e^b{}_h e^c{}_k,$$

$$R^{abc} = 3(\beta^{[a|m|}\partial_m \beta^{bc]} + f^{[a}_{mn}\beta^{b|m|}\beta^{c]n}) - H_{mns}\beta^{mi}\beta^{nh}\beta^{sk} e^a{}_i e^b{}_h e^c{}_k.$$

The Bianchi identity

$$
\begin{aligned}
&e_{[a}{}^{m}\partial_{|m|}H_{bcd]} - \frac{3}{2}F^{e}_{[ab}H_{|e|cd]} = 0, \\
&e^{[a}{}_{l}\beta^{|lm|}\partial_{m}R^{bcd]} - \frac{3}{2}Q^{[ab}_{e}R^{|e|cd]} = 0, \\
&e^{d}{}_{l}\beta^{ln}\partial_{n}H_{[abc]} - 3e_{[a}{}^{n}\partial_{n}F^{d}_{bc]} - 3H_{e[ab}Q^{ed}_{c]} + 3F^{d}_{e[a}F^{e}_{bc]} = 0, \\
&-2e^{[c}{}_{l}\beta^{|ln|}\partial_{n}F^{d]}_{[ab]} - 2e_{[a}{}^{n}\partial_{n}Q^{[cd]}_{b]} + H_{e[ab]}R^{e[cd]} \\
&+Q^{[cd]}_{e}F^{e}_{[ab]} + F^{[c}_{e[a}Q^{|e|d]}_{b]} = 0, \\
&3e^{[b}{}_{l}\beta^{|ln|}\partial_{n}Q^{cd]}_{a} - e_{a}{}^{n}\partial_{n}R^{[bcd]} + 3F^{[b}_{ea}R^{|e|cd]} - 3Q^{[bc}_{e}Q^{|e|d]}_{a} = 0.
\end{aligned}
$$

Introduction of vielbein

We introduce local Lorentz supercoordinates (q^a, p_a) such that $\{q^a, p_b\} = \delta^a_b$ and $|q^a| = |p_a| = 1$.

All basis are $(x^i, \xi_i, q^i, p_i, q^a, p_a)$. Let

$$e \equiv e_a{}^i(x) q^a p_i,$$
$$e^{-1} \equiv e^a{}_i(x) q^i p_a.$$

Twisting

$$\Theta_{B\beta e} = \exp(-\delta_e)\exp(\delta_{e^{-1}})\exp(-\delta_e)\exp(-\delta_\beta)\exp(-\delta_B)\Theta_{\mathsf{S},0}$$

$$\Theta_{B\beta e} = e_b{}^i q^b \xi_i + e^b{}_l \beta^{lm} p_b \xi_m - e^b{}_l \beta^{lm} \partial_m e_a{}^j e^a{}_i q^i p_j p_b$$
$$+ e_b{}^m \partial_m e_a{}^j e^a{}_i q^i q^b p_j$$
$$+ \frac{1}{3!} H_{abc} q^a q^b q^c + \frac{1}{2} F^a_{bc} p_a q^b q^c + \frac{1}{2} Q^{bc}_a q^a p_b p_c + \frac{1}{3!} R^{abc} p_a p_b p_c,$$

where H, F, Q, R are proposed forms.

$\{\Theta_{B\beta e}, \Theta_{B\beta e}\} = 0$ gives the correct Bianchi identity.

Summary

Start at the simplest form $\Theta_{S,0} = q^i \xi_i$ and twist. It gives known geometric structures of fluxes.

Other many twistings are possible.

Conjecture

Flux geometries obtained by T-duality have this structure.

T-duality is described by a (discrete) canonical transformation and twisting on a differential graded symplectic manifold (QP-manifold).

§4. Geometry of double field theory

Hull-Zwiebach '09

Double field theory is a manifestly T-duality invariant formulation of the effective theory of string theory.

The D-dimensional spacetime is doubled $x^M = (x^i, \tilde{x}_i)$ in $2D$-dimensions.

The metric g_{ij} is generalized to the generalized metric,

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik}B_{kj} \\ B_{ik}g^{kj} & g_{ij} - B_{ik}g^{kl}B_{lj} \end{pmatrix}.$$

The $O(D,D)$-invariance of the theory is required. The $O(D,D)$-

invariant metric is denoted by

$$\eta_{MN} = \begin{pmatrix} 0 & \delta^i{}_j \\ \delta_i{}^j & 0 \end{pmatrix}.$$

The reduction to the physical spacetime is provided by the so called the strong constraint (the section condition),

$$\eta^{MN}\partial_M\phi\partial_N\psi = 0,$$

for any field ψ. It can be rewritten as

$$\tilde{\partial}^i\phi\partial_i\psi + \partial_i\phi\tilde{\partial}^i\psi = 0.$$

One more condition is

$$\eta^{MN}\partial_M\partial_N\psi = 0,$$

We can decompose the generalized metric to generalized vielbeins,

$$\mathcal{H}_{MN} = E^A{}_M S_{AB} E^B{}_N,$$

where

$$S_{AB} = \begin{pmatrix} \eta^{ab} & 0 \\ 0 & \eta_{ab} \end{pmatrix},$$

$$E^A{}_M = \begin{pmatrix} e_a{}^i & e_a{}^j B_{ji} \\ e^a{}_j \beta^{ji} & e^a{}_i + e^a{}_j \beta^{jk} B_{ki} \end{pmatrix}.$$

§5. Supergeometry of double field theory

Deser-Stasheff '15, Deser-Saemann '16, Heller-NI-Watamura '16

Doubled graded manifold and Hamiltonian

Super coordinates are doubled,
$(x^M = (x^i, \tilde{x}_i),\ q^M = (q^i, \tilde{q}_i),\ p_M = (p_i, \tilde{p}^i),\ \xi_M = (\xi_i, \tilde{\xi}^i))$ of degrees $(0,\ 1,\ 1,\ 2)$ with

$$\{x^M, \xi_N\} = \{q^M, p_N\} = \delta^M{}_N.$$

An $O(D, D)$-invariant Hamiltonian function without flux is

$$\Theta_{\text{DFT},0} \;=\; \xi_M(q^M + \eta^{MN} p_M) = \xi_i(q^i + \tilde{p}^i) + \tilde{\xi}^i(p_i + \tilde{q}_i).$$

The D-bracket and the C-bracket are

$$\begin{aligned} [U, V]_D &= -\{\{U, \Theta_{\text{DFT},0}\}, V\}, \\ [U, V]_C &= [U, V]_D - [V, U]_D. \end{aligned}$$

However $Q^2_{\text{DFT},0} \neq 0$. In fact,

$$\{\Theta_{\text{DFT},0}, \Theta_{\text{DFT},0}\} = 4\xi_i \tilde{\xi}^i.$$

The variables ξ_i and $\tilde{\xi}^i$ induce the derivatives ∂_i and $\tilde{\partial}^i$. $\xi_i\tilde{\xi}^i = 0$ is equivalent to the section condition, $\partial_i f\tilde{\partial}^i g + \tilde{\partial}^i f\partial_i g = 0$. In fact,

$$\{\{f, \{\Theta_{\mathsf{DFT},0}, \Theta_{\mathsf{DFT},0}\}, g\} = -4(\partial_i f\tilde{\partial}^i g + \tilde{\partial}^i f\partial_i g).$$

The strong constraint is equivalent to $Q^2_{\mathbf{DFT},0} = 0$.

We choose a subspace by $Q^2_{\mathsf{DFT},0} = 0$.

Two typical solutions

The supergravity frame Hamiltonian is defined by $\tilde{\xi}^i = 0$,

whereas the winding frame Hamiltonian by $\xi_i = 0$.

Then, $\Theta_{\mathsf{DFT},0}$ reduces to

$$\Theta_{\mathsf{S},0} = q^i \xi_i,$$
$$\Theta_{\mathsf{W},0} = \tilde{q}_i \tilde{\xi}^i.$$

$Q^2_{\mathsf{S},0} = 0$ and $Q^2_{\mathsf{W},0} = 0$.

Twisting

Applying the B-, β- and vielbein e- twists to the untwisted double field theory, fluxes are introduced,

$$\tilde{\Theta}_{B\beta e} = \exp(-\delta_e)\exp(\delta_{e^{-1}})\exp(-\delta_e)\exp(-\delta_\beta)\exp(-\delta_B)\Theta_{\mathsf{DFT},0}.$$

$$\begin{aligned}\tilde{\Theta}_{B\beta e} = {} & e_d{}^i \xi_i q^d - e_d{}^i B_{mi}\tilde{\xi}^m q^d + e^c{}_l \tilde{\xi}^l p_c - \beta^{ml} e^c{}_l \xi_m p_c + e^c{}_l B_{nm}\beta^{ml}\tilde{\xi}^n p_c \\ & + e_d{}^i(\partial_i + B_{im}\tilde{\partial}^m) e_a{}^j e^a{}_k p_j q^k q^d \\ & + e^c{}_l(\tilde{\partial}^l + \beta^{lm}\partial_m + \beta^{lm}B_{mn}\tilde{\partial}^n) e_a{}^j e^a{}_k p_j q^k p_c \\ & + (\xi_i + \partial_i e_a{}^j e^a{}_k p_j q^k + \partial_i e_a{}^j e^b{}_j q^a p_b)\tilde{p}^i\end{aligned}$$

$$+ (\tilde{\xi}^i + \tilde{\partial}^i e_a{}^j e^a{}_k p_j q^k + \tilde{\partial}^i e_a{}^j e^b{}_j q^a p_b)\tilde{q}_i$$

$$+ \frac{1}{2}(\partial_i B_{jk}\tilde{p}^i + \tilde{\partial}^i B_{jk}\tilde{q}_i) e_a{}^j e_b{}^k q^a q^b$$

$$+ \frac{1}{2}(\partial_i \beta^{jk}\tilde{p}^i + \tilde{\partial}^i \beta^{jk}\tilde{q}_i) e^b{}_j e^c{}_k p_b p_c$$

$$- \partial_i B_{jk}\beta^{km} e_b{}^j e^c{}_m \tilde{p}^i q^b p_c - \tilde{\partial}^i B_{jk}\beta^{km} e_b{}^j e^c{}_m \tilde{q}_i q^b p_c$$

$$+ \frac{1}{2}\partial_i B_{jk}\beta^{jm}\beta^{kn} e^b{}_m e^c{}_n \tilde{p}^i p_b p_c + \frac{1}{2}\tilde{\partial}^i B_{jk}\beta^{jm}\beta^{kn} e^b{}_m e^c{}_n \tilde{q}_i p_b p_c$$

$$+ \frac{1}{3!}H_{abc} q^a q^b q^c + \frac{1}{2}F^a_{bc} p_a q^b q^c + \frac{1}{2}Q^{bc}_a q^a p_b p_c + \frac{1}{3!}R^{abc} p_a p_b p_c.$$

$$
\begin{aligned}
H_{abc} &= 3(\nabla_{[a}B_{bc]} + B_{[a|m|}\tilde{\partial}^m B_{bc]} + \tilde{f}^{mn}_{[a}B_{b|m|}B_{c]n}),\\
F^a_{bc} &= f^a_{bc} - H_{mns}\beta^{si}e^a{}_i e_b{}^m e_c{}^n + \tilde{\partial}^a B_{bc} + \tilde{f}^{ad}_b B_{dc} - \tilde{f}^{ad}_c B_{db},\\
Q^{bc}_a &= \tilde{f}^{bc}_a + \partial_a\beta^{bc} + f^b_{ad}\beta^{dc} - f^c_{ad}\beta^{db} + H_{isr}\beta^{sh}\beta^{rk}e_a{}^i e^b{}_h e^c{}_k\\
&\quad + B_{am}\tilde{\partial}^m\beta^{bc} + \tilde{\partial}^{[b}B_{ae}\beta^{e|c]} + 2B_{[a|e}\tilde{f}^{be}_{d]}\beta^{dc} - 2B_{[a|e}\tilde{f}^{ce}_{d]}\beta^{db},\\
R^{abc} &= 3(\beta^{[a|m|}\partial_m\beta^{bc]} + f^{[a}_{mn}\beta^{b|m|}\beta^{c]n} + \tilde{\partial}^{[a}\beta^{bc]} - \tilde{f}^{[ab}_d\beta^{|d|c]}\\
&\quad + B_{ln}\tilde{\partial}^l\beta^{[ab}\beta^{|n|c]} + \tilde{\partial}^{[a}B_{ed}\beta^{|e|b}\beta^{|d|c]} + \tilde{f}^{[a|e|}_n B_{ed}\beta^{|n|b|}\beta^{|d|c]})\\
&\quad - H_{mns}\beta^{mi}\beta^{nh}\beta^{sk}e^a{}_i e^b{}_h e^c{}_k,\\
H_{mns} &= 3(\partial_{[m}B_{ns]} + B_{[m|l|}\tilde{\partial}^l B_{ns]}),\\
\tilde{f}^{ab}_c &= 2e^{[a}{}_m\tilde{\partial}^m e^{b]}{}_j e_c{}^j.
\end{aligned}
$$

The classical master equation then

$$\{\tilde{\Theta}_{B\beta e}, \tilde{\Theta}_{B\beta e}\} = 0.$$

leads to relations between the fluxes in the double space. Under the section condition, these equations reduce to the Bianchi identity of fluxes.

Nontrivial reduction

Start with the fundamental form $\Theta_{\mathsf{DFT},0} = \xi_M(q^M + \eta^{MN} p_M) = \xi_i(q^i + \tilde{p}^i) + \tilde{\xi}^i(p_i + \tilde{q}_i)$,

Twisting, $\pi = \frac{1}{2}\pi^{ij}(x) p_i p_j$.

$$\begin{aligned} \Theta'_{\mathsf{DFT},0} &= e^{\pi}\Theta_{\mathsf{DFT},0} \\ &= \xi_i(q^i + \tilde{p}^i) + \tilde{\xi}^i(p_i + \tilde{q}_i) + \pi^{ij}\xi_i p_j - \frac{1}{2}\frac{\partial \pi^{jk}}{\partial x^i}(x)(q^i + \tilde{p}^i) p_j p_k. \end{aligned}$$

The section condition is deformed to

$$\tilde{\xi}^i \left(4\xi_i - \frac{1}{2}\frac{\partial \pi^{jk}}{\partial x^i}(x) p_j p_k\right) = 0.$$

By the projectio n to the standard frame $\tilde{\xi}^i = \tilde{q}_i = \tilde{p}^i = 0$, Θ becomes the Poisson Courant algebroid with a standard Courant algebroid part without fluxes,

$$\begin{aligned}\Theta'_{\mathsf{DFT},0}|_{\tilde{x}=0} &= \Theta_{H=0} + \Theta_{\pi,R=0} \\ &= (\xi_i q^i) + \left(\pi^{ij}\xi_i p_j - \frac{1}{2}\frac{\partial \pi^{jk}}{\partial x^i}(x) q^i p_j p_k\right).\end{aligned}$$

Note that, since $\{\Theta_{H=0}, \Theta_{\pi,R=0}\} = 0$, the projected Hamiltonian $\Theta'_{\mathsf{DFT},0}|_{\tilde{x}=0}$ defines a double complex.

Moreover by taking the β-transformation $\beta = \frac{1}{2}\beta^{ij}(x)p_i p_j$, an R-flux on a Poisson manifold, $R = [\pi, \beta]_S$ is obtained.

Note: The Courant bracket is closed.

§6. T-duality as canonical transformation

T-duality as an $O(D,D)$-transformation relates the fluxes and vielbeins associated to different backgrounds. A T-duality in x^k-direction is the transformation

$$x^k \leftrightarrow \tilde{x}_k, \quad \xi_k \leftrightarrow \tilde{\xi}^k, \quad q^k \leftrightarrow \tilde{q}_k, \quad p_k \leftrightarrow \tilde{p}^k.$$

$\{-,-\}$ and Θ_{DFT} are invariant under this transformation, i.e.
the transformation is a canonical transformation.

Example: S^1 isometry

We consider a generalized vielbein,

$$E^A{}_M = \begin{pmatrix} E_a{}^i & E_{ai} \\ E^{ai} & E^a{}_i \end{pmatrix} = \begin{pmatrix} e_a{}^i & e_a{}^l B_{li} \\ e^a{}_l \beta^{li} & e^a{}_i + e^a{}_l B_{im} \beta^{ml} \end{pmatrix}.$$

Here we introduce odd local Lorentz basis $q_A \equiv (q^a, p_a)$ and $\tilde{p}_A \equiv (\tilde{p}^a, \tilde{q}_a)$. The Hamiltonian function without fluxes is

$$\Theta_{\mathsf{DFT,A}} = E^A{}_M \xi^M (q_A + \tilde{p}_A).$$

The easiest example concerns T-duality on an S^1-isometry background without B- and β-fields, where the circle has radius R. It is well known, that T-duality maps the radius $R \mapsto R' = \frac{1}{R}$. The corresponding Hamiltonian is given by

$$\Theta_R = e_1{}^1 \xi_1 (q^1 + \tilde{p}^1) + e^1{}_1 \tilde{\xi}^1 (p_1 + \tilde{q}_1) = R \xi_1 (q^1 + \tilde{p}^1) + R^{-1} \tilde{\xi}^1 (p_1 + \tilde{q}_1).$$

The canonical transformation is

$$x^1 \leftrightarrow \tilde{x}_1, \quad \xi_1 \leftrightarrow \tilde{\xi}^1, \quad q^1 \leftrightarrow \tilde{q}_1, \quad p_1 \leftrightarrow \tilde{p}^1.$$

Under this transformation, Θ_R is invariant,

$$\Theta_R \leftrightarrow \Theta_R.$$

We can project into the supergravity frame by taking ($\tilde{\xi}^1 = 0$, $\tilde{q}_1 = 0$, $\tilde{p}^1 = 0$) leading to

$$\Theta_R = R\xi_1 q^1.$$

Applying the transformation described above, the Hamiltonian, which models the T-dual background, is given by

$$\Theta'_{R^{-1}} = R^{-1}\xi_1(q^1 + \tilde{p}^1) + R\tilde{\xi}^1(p_1 + \tilde{q}_1).$$

In this case, the projection into the supergravity frame gives

$$\Theta'_{R^{-1}} = R^{-1}\xi_1 q^1.$$

Another derivation is that we project into the winding frame directly by ($\xi_1 = 0$, $q^1 = 0$, $p_1 = 0$) to get the result

$$\Theta_R = R^{-1}\tilde{\xi}^1 \tilde{q}_1.$$

- This formalism works for the case with fulxes.

§7. Conclusions

• We have formulated geometry of fluxes by a supermanifold with a Poisson bracket and a BRST charge called a QP-manifold.

• All known proposals of nongeometric fluxes and R-fluxes, and their identities are obtained by twisting of the simple form $\Theta_{\mathrm{S},0}$.

• The section conditions in double field theory is formulated as the nilpotent condition of the $O(D,D)$-invariant BRST charge on the doubled space.

• T-duality transformations are reformulated as canonical transformations on a supermanifold.

Future Outlook

- New flux solutions
- Actions and dynamics
- α'-corrections
- Quantizations
- Exceptional field theory

Thank you for your attention!