Towards the Cosmological Constant Problem

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1 Cosmological Constant Problem

Dark Clouds hanging over the two well-established theories

Quantum Field Theory $\Longleftrightarrow$ Einstein Gravity Theory

We know the recently observed Dark Energy Λ_0, which looks like a small Cosmological Constant (CC):

$$\text{Present observed CC} \quad 10^{-29}\text{gr/cm}^3 \sim 10^{-47}\text{GeV}^4 \equiv \Lambda_0 \tag{1}$$

We do not mind this tiny CC, which will be explained after our CC problem is solved. However, we use it as the scale unit Λ_0 of our discussion.

Essential point: multiple mass scales are involved!

There are several dynamical symmetry breakings and they are necessarily accompanied by vacuum condensation energy:

In particular, we are confident from the success of the Standard Model of the existence of at least two symmetry breakings:

$$\text{Higgs Condensation} \sim \quad (\,200\,\text{GeV}\,)^4 \ \sim 10^9\text{GeV}^4 \sim 10^{56}\Lambda_0$$

$$\text{QCD Chiral Condensation } \langle\bar{q}q\rangle^{4/3} \sim \quad (\,200\,\text{MeV}\,)^4 \sim 10^{-3}\text{GeV}^4 \sim 10^{44}\Lambda_0$$

Nevertheless, these seem not contributing to the Cosmological Constant!

It is a Super fine tuning problem:

$$c \ : \ \text{initially prepared CC } (>0)$$

$$c - 10^{56}\Lambda_0 \ : \ \text{should cancell, but leaving 1 part per } 10^{12}\text{; i.e., } \sim 10^{44}\Lambda_0$$

$$c - 10^{56}\Lambda_0 - 10^{44}\Lambda_0 \ : \ \text{should cancell, but leaving 1 part per } 10^{44}\text{; i.e., } \sim \Lambda_0$$

$$c - 10^{56}\Lambda_0 - 10^{44}\Lambda_0 \sim \Lambda_0 \ : \text{present Dark Energy}$$

Note that the vacuum energy is almost totally cancelled at each stage of spontaneous breaking as far as the the relevant energy scale order.

2 Vacuum Energy $\simeq$ vacuum condensation energy

Vacuum Energy in QFT:

$$\sum_{\boldsymbol{k},s} \frac{1}{2}\hbar\omega_{\boldsymbol{k}} - \sum_{\boldsymbol{k},s} \hbar E_{\boldsymbol{k}} \tag{2}$$

Vacuum Condensation Energy:

$$V(\phi_c) : \text{potential} \tag{3}$$

They are separately stored in our (or my, at least) memory, but actually, almost the same object, as we see now.

Consider the chiral quark condensation in QCD. For simplicity, consider NJL model as a parallel model for the realistic QCD:

$$\begin{aligned} \mathcal{L}_{\rm NJL} &= \bar{q}i\gamma^\mu\partial_\mu q + \frac{G}{4}\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5 q)^2\right] \\ &\to \quad \bar{q}(i\gamma^\mu\partial_\mu - \sigma - i\gamma_5\pi)q - \frac{1}{G}(\sigma^2 + \pi^2) \end{aligned}$$

The effective potential $V(\sigma,\pi)$ is a function of $\sigma^2+\pi^2$ and can be computed at the $\pi=0$ section $V(\sigma)=V(\sigma,\pi=0)$:

$$V(\sigma)=\frac{1}{G}\sigma^2-\int\frac{d^4p}{i(2\pi)^4}\ln\det(\not{p}-\sigma)$$

But the second term is nothing but the vacuum energy

$$-\int\frac{d^4k}{i(2\pi)^4}\ln\det(\not{k}-\sigma)=-\sum_{\boldsymbol{k},s}\hbar\sqrt{\boldsymbol{k}^2+\sigma^2}+(\sigma\text{-independent const})$$

implying that

$$\langle\bar{q}q\rangle \text{ condensation energy } \simeq \text{ Dirac sea vacuum energy} \tag{4}$$

Moreover, in a Shwinger-Dyson approach to realistic QCD, the quark mass is calculated as a function $\Sigma(p)$ possessing the support only $\lesssim\Lambda_{\mathrm{QCD}}$, and the condensation energy is computed finite.

3 Does Vacuum Energy really work as Cosmological Constant?

As far as Einstein Gravity is correct, it does :
For the *usual* symmetry breaking,

$$\text{potential } V(\phi) \quad \rightarrow \quad S \simeq \int d^4x \sqrt{-g}\, (-V(\phi))$$

$$\rightarrow \quad S \simeq \int d^4x \sqrt{-g}\, (\, -c\,) \qquad (\langle V(\phi) \rangle = V(\phi_c) = c)$$

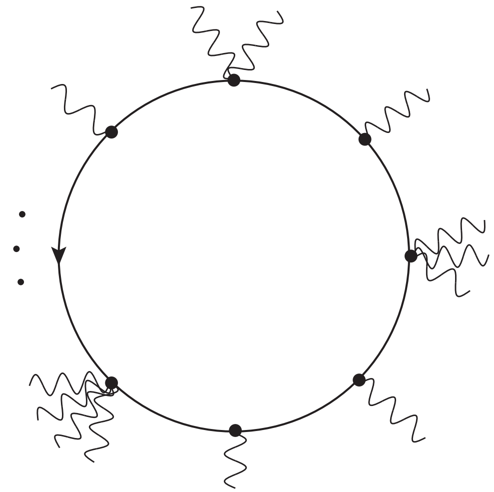
Also in the dynamical symmetry breaking case, the potential $V(\phi)$, and hence the vacuum energy, is dynamically generated, so that it works as the cosmological constant.

Let us show this more explicitly, taking the previous NJL model: In the presence of gravity in that model, we have the kinetic term for the quark field:

$$\int d^4x\, e(x)\; \bar{q}(x)\left(i{e_a}^{\mu}(x)\gamma^a\partial_\mu - \sigma(x)\right)q(x)$$

Needs ${e_\mu}^a$ the same gravity field as our macroscopic one!

with which we have the vacuum bubble diagrams. Sum of them give an effective action



$$\Gamma[e] = -i\ln \mathrm{Det}\left[e(x)\left(i{e_a}^{\mu}(x)\gamma^a\partial_\mu - \sigma(x)\right)\right]$$

The lowest order term in the derivative expansion in the background gravity field $e_\mu^a(x)$ and $\sigma(x)$, (i.e., the no derivative term in $e_\mu^a(x)$ and $\sigma(x)$), can be calculated by treating $e_\mu^a(x)$ and $\sigma(x)$ as if they are x-independent constant:

$$\mathcal{L}_{\rm eff}(x) = \int \frac{d^4k}{i(2\pi)^4} \ln\det\left[e(x)\left(e_a{}^\mu(x)\gamma^a k_\mu - \sigma(x)\right)\right] \tag{6}$$

Perform change of variable

$$k_\mu \quad \to \quad p_a = e_a{}^\mu k_\mu, \qquad \text{or,} \qquad k_\mu = e_\mu{}^a p_a \tag{7}$$

Then the Jacobian yields

$$\left|\frac{\partial(k)}{\partial(p)}\right| = \det(e_\mu{}^a) = e \quad \to \quad d^4k = \left|\frac{\partial(k)}{\partial(p)}\right| d^4p = e\, d^4p \tag{8}$$

so that

$$\mathcal{L}_{\rm eff}(x) = e \times \left(-V(\sigma)\right), \qquad V(\sigma) = -\int \frac{d^4p}{i(2\pi)^4} \ln\det(\not{p} - \sigma)$$

That is, the dynamical potential $V(\sigma)$ previously obtained actually couples to the gravity $e = \sqrt{-g}$ as CC term!

4 Quantum Gravity is irrelevant

CC problem is to be considered in Einstein Gravity theory.

$$\text{Einstein gravity is a unique Low Energy Effective Theory} \tag{9}$$

Just like Chiral Lagrangian

$$\mathcal{L} = f_\pi \, \mathrm{tr} \left(\partial_\mu U^\dagger \partial^\mu U \right)$$
$$U = \exp(i\pi/f_\pi), \qquad \pi = \pi^a(x) T^a$$

is a unique Effective Theory in the low energy region $E \lesssim f_\pi$, i.e., in the lowest (second) order in the derivative. We know that the fundamental theory describing the strong interaction is QCD. But, whatever the dynamical theory is beyond $E > f_\pi$, the sysytem is described by the the Nambu-Goldstone (NG) bosons $\boldsymbol{\pi}$ based on the coset $SU(3)_L \times SU(3)_R / SU(3)_V$, and the dynamics is uniquely described by this non-linear sigam model. The non-linearly realized chiral symmetry uniquely determines the dynamics of the NG bosons, self-coupling and coupling to other matters in the low energy regime. Moreover, even the quantum correction in this system can be computed by this Lagrangian in the sense of Weinberg.

In exactly the same manner, the general coordinate (GC) invariance uniquely determine the Lagrangian in the lowest (second) order in the derivative; that is, it is the Einstein-Hilbert action. In this analogy, it is worth noticing

Graviton is a NG tensor boson corresponding to GL(4) $\to$ SO(3,1)

Nakanishi-Ojima (1979)

So the Einstein-Hilbert action is exactly analogous to the chiral Lagrangian, and $M_{\rm Pl}$ is the counterpart of the pion decay constant f_π:

$$S_{\rm eff} = \int d^4x \sqrt{-g}\Big\{ M_{\rm Pl}^4 c_0 + M_{\rm Pl}^2 R + c_2 R^2 + c_3 R_{\mu\nu}R^{\mu\nu} + \cdots \Big\}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_{\rm Pl}$$

The CC term (with no derivatives) is consistent with GC invariance and its natural scale is $O(M_{\rm Pl}^4)$.

Below the Planck energy scale $M_{\rm Pl}$, the dynamics is uniquely described by the E-H action plus interaction terms with matter fields. The quantum gravity is quite irrelevant to any problem in much lower energy region than Planck sacale, $E \ll M_{\rm Pl}$, in particular, to the CC problem associated with the spontaneous breaking of Electro-weak symmetry and chiral symmetry.

5 Running Cosmological constant: Is Gravity field the same in microscopic 1fm and in macro 1m?

Once Prof. Maskawa said to me that

> the gravity field at microscopic scale
> should be different from our macroscopic one.

But I think that they are the same. Indeed, In the case of electro-magnetic interaction, the same vector potential is working from macroscopic scale 1m to the atomic scale or even to nucleus scale 1 fm. So the gravity field will also be the same from 1m to 1fm.

However, here, I only mention to the following fact:

The coupling constant may run with energy scale. In the Wilsonian renormalization scheme, the cosmological constant, in particular, runs quartically in energy scale, since it diverges quartically. So the effective CC may drastically be different scale by scale. The CC observed in cosmology is the one at super-low energy! We should investigate the RGE towards IR direction.

6 Conformal (Scale) Invariance may solve the problem

Our world is almost scale invariant: that is, the standard model Lagrangian is scale invariant except for the Higgs mass term!

If the Higgs mass term comes from the spontaneous breaking of scale invariance at higher energy scale physics, the total system may really be scale invariant.

Suppose that the (effective) potential V of the total system looks like

$$\begin{array}{ccccccc} V = & V_0(\Phi) & + & V_1(\Phi, \varphi) & + & V_3(\varphi, \phi) \\ & \downarrow & & \downarrow & + & \downarrow \\ & M & \gg & \mu & \gg & m \end{array} \tag{10}$$

and it is scale invariant. Then, it satisfies the scale invariance relation:

$$\sum_i \phi^i \frac{\partial}{\partial \phi^i} V(\phi) = 4V(\phi), \tag{11}$$

so that the vacuum energy vanishes at any stationary point $\langle \phi^i \rangle = \phi_0^i$:

$$V(\phi_0) = 0. \tag{12}$$

Important point is that this holds at every stages of spontaneous symmetry breaking.

In the above potential V, we can retain only $V_0(\Phi)$ when discussing the physics at scale M, since φ and ϕ are expected to get VEVs of order μ or lower. Then the scale invariance guarantees $V_0(\Phi_0) = 0$.

If we discuss the next stage spontaneous breaking at energy scale μ, we should take $V_0(\Phi) + V_1(\Phi, \varphi)$, and can conclude $V_0(\Phi_0') + V_1(\Phi_0', \varphi_0) = 0$ (with $\Phi_0' - \Phi_0 = O(\mu)$).

Similarly, at scale m, we have the potential $V_0(\Phi) + V_1(\Phi, \varphi) + V_3(\varphi, \phi)$, and can coclude $V_0(\Phi_0'') + V_1(\Phi_0'', \varphi_0') + V_3(\varphi_0', \phi_0) = 0$.

However, we have neglected the scale invariance anomaly in quantum field theory. Actually, if we take account of the renormalization point μ, we have the RGE

$$\left(\mu\frac{\partial}{\partial\mu} + \sum_a \beta_a(g)\frac{\partial}{\partial g_a} + \sum_i \gamma_i(g)\phi_i\frac{\partial}{\partial\phi_i}\right)V(\phi) = 0 \qquad (13)$$

and the dimension counting identity

$$\left(\mu\frac{\partial}{\partial\mu}+\sum_i \phi_i\frac{\partial}{\partial\phi_i}\right)V(\phi)=4V(\phi) \tag{14}$$

From these we obtain

$$\left(\sum_i(1-\gamma_i(g))\phi_i\frac{\partial}{\partial\phi_i}-\sum_a\beta_a(g)\frac{\partial}{\partial g_a}\right)V(\phi)=4V(\phi) \tag{15}$$

This is the correct equation in place of the above naive one:

$$\sum_i \phi_i\frac{\partial}{\partial\phi_i}V(\phi)=4V(\phi) \tag{16}$$

The anomalous dimension $\gamma_i(g)$ is not the problem.

$\beta_a(g)$ terms are problematic:

$$\longrightarrow \qquad V(\phi_0)=-\frac{1}{4}\sum_a\beta_a(g)\frac{\partial}{\partial g_a}V(\phi_0) \tag{17}$$

So, an obvious possibility is that all the coupling constants go to the Infrared Fixed Points: $\beta_a(g_{\rm IF})=0$.