Codimension-2 Solutions in Five-Dimensional Supergravity

based on arXiv:1505.05169 and
work in progress with Shigemori and Fernandez Melgarejo

Minkyu Park

YITP, Kyoto University

Geometry, Duality and Strings '17

YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Outline

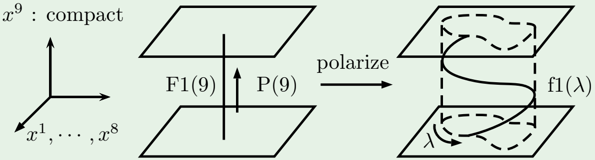
Supertubes [Mateos–Townsend '01]

Supertube

- a tubular D2-brane with angular momentum [Mateos–Townsend '01]
- F1 and D0-brane charges are resolved in D2-brane
- Supergravity solution is known [Emparan–Mateos–Townsend '01]

F1-P system

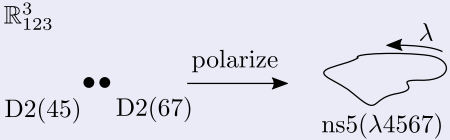
- $\mathrm{F1}(9) + \mathrm{P}(9) \rightarrow \mathrm{f1}(\lambda)$



- Supergravity solution is known [Lunin–Mathur '01]

Supertube effect [de Boer–Shigemori '10, '12]

- a spontaneous polarization phenomenon
- branes blow up into a new dipole charges



- conjectured to be relevant to black hole microstate

Supertubes in various frame

$$
\begin{array}{llllll}
F1(9) & + & P(9) & \to & f1(\lambda) \\
F1(9) & + & D0 & \to & d2(\lambda 9) \\
D2(45) & + & D2(67) & \to & ns5(\lambda 4567) \\
 & & \vdots & &
\end{array}
$$

Setup

$D = 5, \mathcal{N} = 1$ ungauged SUGRA

- with two vector multiplets
- M-theory on $T^6 = T^2_{45} \times T^2_{67} \times T^2_{89}$
- action

$$S = \frac{1}{16\pi G_5} \int \Big(- R * 1 + Q_{IJ} * F^I \wedge F^J + Q_{IJ} * \mathrm{d}X^I \wedge \mathrm{d}X^J - \frac{1}{6} C_{IJK} F^I \wedge F^J \wedge A^K \Big) \,,$$

$$C_{IJK} = |\epsilon_{IJK}| \,, \qquad Q_{IJ} = \frac{1}{2} \,\mathrm{diag}\left((X^1)^{-2}, (X^2)^{-2}, (X^3)^{-2} \right)$$

The 4D/5D solution

[Bates–Denef '03, Gutowski–Reall '04, Bena–Warner '04, Gauntlett–Gutowski '04]

BPS solutions

- Require SUSY
- Assume $\mathrm{U}(1)$ symmetry (M-theory direction)
  $\rightarrow$ 4D theory
- 8 harmonic functions $H = \{V, K^{I=1,2,3}, L_I, M\}$ on $\mathbb{R}^3$

$$\nabla^2 H(\mathbf{x}) = 0$$

- Integrability condition

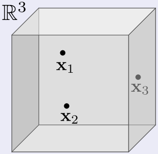
$$0 = V\nabla^2 M - M\nabla^2 V + \frac{1}{2}\left(K^I \nabla^2 L_I - L_I \nabla^2 K^I\right)$$

The 4D/5D solution

[Bates–Denef '03, Gutowski–Reall '04, Bena–Warner '04, Gauntlett–Gutowski '04]

Codimension-3 solutions

- Multi-center solutions with pointlike sources



$$H(\mathbf{x}) = h + \sum_{p=1}^{N} \frac{\Gamma_p}{|\mathbf{x} - \mathbf{x}_p|}\,, \qquad \Gamma_p : \text{ charge}$$

- Brane interpretation in type IIA theory

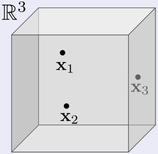
$$V \leftrightarrow \mathrm{D6}(456789)\,, \quad \begin{matrix} K^1 \leftrightarrow \mathrm{D4}(6789) \\ K^2 \leftrightarrow \mathrm{D4}(4589) \\ K^3 \leftrightarrow \mathrm{D4}(4567) \end{matrix}\,, \quad \begin{matrix} L_1 \leftrightarrow \mathrm{D2}(45) \\ L_2 \leftrightarrow \mathrm{D2}(67) \\ L_3 \leftrightarrow \mathrm{D2}(89) \end{matrix}\,, \quad M \leftrightarrow \mathrm{D0}$$

- Integrability condition: force between branes has to be in balance

Microstate geometry program

Microstate geometry program

- Attempt to explain BH entropy from smooth supergravity solutions [Bena–Warner '05, Berglund–Gimon–Levi '05]
- Are the above solutions able to explain BH entropy?



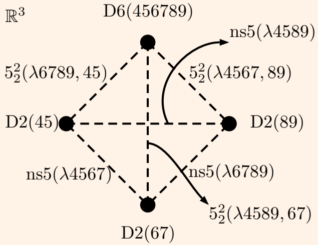
$$H(\mathbf{x}) = h + \sum_{p=1}^{N} \frac{\Gamma_p}{|\mathbf{x} - \mathbf{x}_p|}$$

- For 3-charge BH case, $S_{\text{4D/5D}} \sim N^{5/4} \ll S_{\text{BH}} \sim N^{3/2}$ [de Boer–El-Showk–Messamah–Van den Bleeken '09, Bena–Bobev–Giusto–Ruef–Warner '10]
- Not enough to reproduce BH entropy!

Microstate geometry program

Bring supertube effect into the game

- There should be more general configurations because of the supertube effect [de Boer–Shigemori '10, '12]



$$\text{D6}(456789) \quad \begin{matrix} \text{D2}(45) \\ \text{D2}(67) \\ \text{D2}(89) \end{matrix} \quad \rightarrow \quad \begin{matrix} 5^2_2(\lambda 6789, 45) & \text{ns5}(\lambda 4567) \\ 5^2_2(\lambda 4589, 67) & \text{ns5}(\lambda 6789) \\ 5^2_2(\lambda 4567, 89) & \text{ns5}(\lambda 4589) \end{matrix} \quad \rightarrow \quad \cdots$$

Supertubes in 4D/5D solution

$\mathrm{D2}(45) + \mathrm{D2}(67) \to \mathrm{ns5}(\lambda 4567)$

NS5 supertube

- Harmonic functions

$$V = 1\,, \qquad K^1 = 0\,, \qquad K^2 = 0\,, \qquad K^3 = \gamma\,,$$
$$L_1 = f_2\,, \qquad L_2 = f_1\,, \qquad L_3 = 1\,, \qquad M = -\frac{\gamma}{2}$$

where

$$f_1 = 1 + \frac{Q_1}{L}\int_0^L \frac{\mathrm{d}\lambda}{|\mathbf{x} - \mathbf{F}(\lambda)|}\,, \quad f_2 = 1 + \frac{Q_1}{L}\int_0^L \frac{|\dot{\mathbf{F}}(\lambda)|^2 \mathrm{d}\lambda}{|\mathbf{x} - \mathbf{F}(\lambda)|}$$

and

$$\mathrm{d}\gamma = *_3 \mathrm{d}\alpha\,, \quad \alpha_i = \frac{Q_1}{L}\int_0^L \frac{\dot{F}_i(\lambda)\,\mathrm{d}\lambda}{|\mathbf{x} - \mathbf{F}(\lambda)|}\,.$$

Supertubes in 4D/5D solution

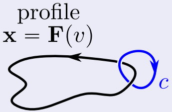
$\mathrm{D2}(45) + \mathrm{D2}(67) \to \mathrm{ns5}(\lambda 4567)$

Monodromy

- Complexified Kähler moduli are

$$\tau^1 = i\sqrt{f_1/f_2}\,, \quad \tau^2 = i\sqrt{f_2/f_1}\,, \quad \tau^3 = \gamma + i\sqrt{f_1 f_2}$$

- γ has monodromy



$$\int_c \mathrm{d}\gamma = \int_c *_3 \mathrm{d}\alpha = 1$$

- $\tau^3 \to \tau^3 + 1$

Supertubes in 4D/5D solution

$\mathrm{D2}(45) + \mathrm{D2}(67) \to \mathrm{ns5}(\lambda 4567)$

Type IIA uplift

- All quantities are written in terms of 8 harmonic functions

$$ds^2_{10} = -(f_1 f_2)^{-1/2}(\mathrm{d}t - \alpha)^2 + (f_1 f_2)^{1/2}\mathrm{d}x^i \mathrm{d}x^i + (f_1/f_2)^{1/2} dx^2_{45} + (f_2/f_1)^{1/2} dx^2_{67} + (f_1 f_2)^{1/2} dx^2_{89}\,,$$

$$e^{2\Phi} = (f_1 f_2)^{1/2}\,, \qquad B_2 = \gamma\, \mathrm{d}x^8 \wedge \mathrm{d}x^9\,, \quad \cdots$$

- Metric is sIngle-valued (geometric)

Summary so far

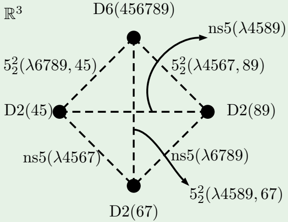
We have seen

- 4D/5D solutions are one of the framework of black hole research
- supertube effect is important in black hole physics
- a example of supertube in the 4D/5D solutions

General configurations for black holes

4-charge black hole

- In general,



- There can be further supertube effects

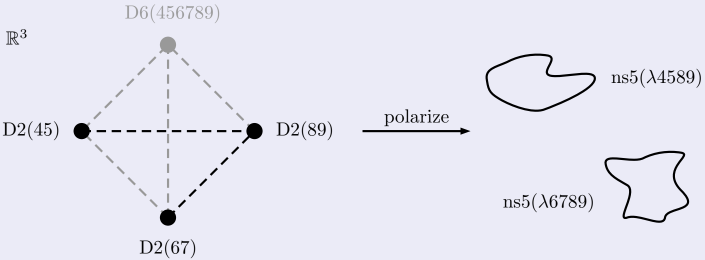
$$\mathrm{D6}(456789) \quad \begin{matrix} \mathrm{D2}(45) \\ \mathrm{D2}(67) \\ \mathrm{D2}(89) \end{matrix} \quad \rightarrow \quad \begin{matrix} 5^2_2(\lambda 6789, 45) \\ 5^2_2(\lambda 4589, 67) \\ 5^2_2(\lambda 4567, 89) \end{matrix} \quad \begin{matrix} \mathrm{ns5}(\lambda 4567) \\ \mathrm{ns5}(\lambda 6789) \\ \mathrm{ns5}(\lambda 4589) \end{matrix} \quad \rightarrow \quad \cdots$$

General configurations for black holes

Simplest possible configuration

- 2-dipole solution

$$\begin{array}{ccccc} D2(45) & + & D2(89) & \to & ns5(\lambda 4589) \\ D2(67) & + & D2(89) & \to & ns5(\lambda 6789) \end{array}$$



Find 2-dipole solutions

Naive attempt

- Superpose two 1-dipole solutions

$$\mathrm{D2}(45) \quad + \quad \mathrm{D2}(89) \quad \to \quad \mathrm{ns5}(\lambda 4589)$$

- Harmonic functions are

$$\begin{aligned} V = 1\,, \qquad K^1 = 0\,, \qquad K^2 = \gamma\,, \qquad K^3 = 0\,, \\ L_1 = f_1\,, \qquad L_2 = 0\,, \qquad L_3 = f_2\,, \qquad M = -\frac{\gamma}{2} \end{aligned}$$

Find 2-dipole solutions

Naive attempt

- Superpose two 1-dipole solutions

$$\begin{array}{ccccc} \mathrm{D2}(45) & + & \mathrm{D2}(89) & \to & \mathrm{ns5}(\lambda 4589) \\ \mathrm{D2}(67) & + & \mathrm{D2}(89) & \to & \mathrm{ns5}(\lambda 6789) \end{array}$$

- Harmonic functions are

$$V = 1\,, \quad K^1 = \gamma'\,, \quad K^2 = \gamma\,, \quad K^3 = 0\,,$$

$$L_1 = f_1\,, \quad L_2 = f_1'\,, \quad L_3 = f_2 + f_2'\,, \quad M = -\frac{\gamma}{2} - \frac{\gamma'}{2}$$

- This does not work. Integrability condition cannot be satisfied

2-dipole solutions [MP–Shigemori '15]

Superthread [Niehoff–Vasilakis–Warner '12]

- Solutions in 6D SUGRA
- D1 and D5-branes with traveling waves on them

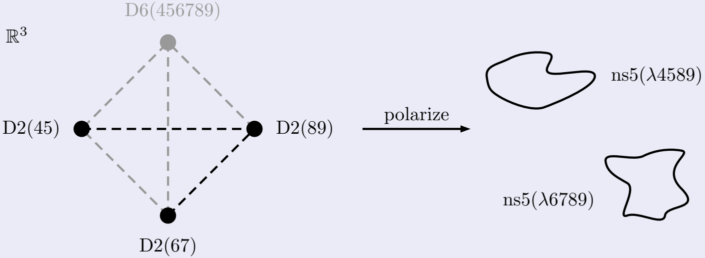
$$\begin{array}{lclcl} \mathrm{D1}(5) & + & \mathrm{P}(5) & \rightarrow & \mathrm{d1}(\lambda) \\ \mathrm{D5}(56789) & + & \mathrm{P}(5) & \rightarrow & \mathrm{d5}(\lambda 6789) \end{array}$$

- Supertubes interact with each other → it solves the problem before

2-dipole solutions [MP–Shigemori '15]

$D2(45) + D2(67) + D2(89) \to ns5(\lambda 4589) + ns5(\lambda 6789)$

- Smear and dualize superthread
- After some messy calculations, we get 2-dipole solutions



- This can be described in 4D/5D solutions
- Multi-valued harmonic functions on $\mathbb{R}^3$

2-dipole solutions [MP–Shigemori '15]

$\mathrm{D2}(45) + \mathrm{D2}(67) + \mathrm{D2}(89) \to \mathrm{ns5}(\lambda 4589) + \mathrm{ns5}(\lambda 6789)$

- Harmonic functions

$$V = 1\,, \qquad K^1 = \gamma_2\,, \quad K^2 = \gamma_1\,, \quad K^3 = 0\,,$$

$$L_I = 1 + \sum_p Q_{pI} \int_p \frac{1}{R_p} = Z_I\,, \qquad I = 1, 2\,,$$

$$L_3 = 1 + \sum_p \int_p \frac{\rho_p}{R_p} - K^1 K^2$$

$$+ \sum_{p,q} Q_{pq} \iint_{p,q} \left[\frac{\dot{\mathbf{F}}_p \cdot \dot{\mathbf{F}}_q}{2R_p R_q} - \frac{\dot{F}_{pi}\dot{F}_{qj}(R_{pi}R_{qj} - R_{pj}R_{qi})}{F_{pq}R_p R_q(F_{pq} + R_p + R_q)} \right],$$

$$M = \frac{1}{2}\sum_{p,q} Q_{pq} \iint_{p,q} \frac{\epsilon_{ijk}\dot{F}_{pqi}R_{pj}R_{qk}}{F_{pq}R_p R_q(F_{pq} + R_p + R_q)} - \frac{1}{2}(K^1 L_1 + K^2 L_2)$$

2-dipole solutions [MP–Shigemori '15]

$\mathrm{D2}(45) + \mathrm{D2}(67) + \mathrm{D2}(89) \to \mathrm{ns5}(\lambda 4589) + \mathrm{ns5}(\lambda 6789)$

- $\gamma_{1,2}$ is multi-valued as before

$$\mathrm{d}\gamma_I = *_3 \mathrm{d}\alpha_I \,, \quad \alpha_I = \sum_p Q_{pI} \int_p \frac{\dot{F}_p \cdot \mathrm{d}\mathbf{x}}{R_p} \,.$$

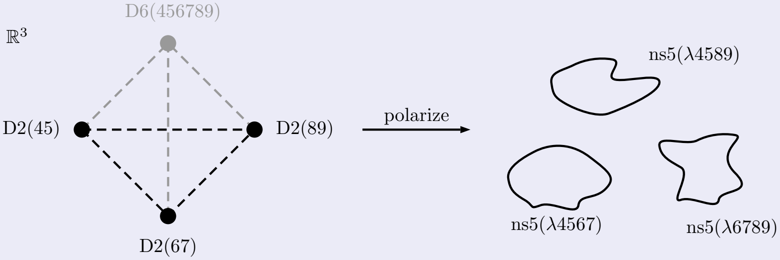
- But metric is single-valued
- Proper monodromies for two NS5-branes

$$\tau^1 \to \tau^1 + 1 \,, \quad \tau^2 \to \tau^2 + 1$$

Other configurations [MP–Shigemori '15]

3-dipole solutions

- $\mathrm{D2}(45) + \mathrm{D2}(67) + \mathrm{D2}(89)$
  $\to \mathrm{ns5}(\lambda 4567) + \mathrm{ns5}(\lambda 4589) + \mathrm{ns5}(\lambda 6789)$

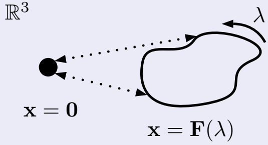


- Although this is not explicit as 2-dipole solutions, it satisfies all conditions for 3-dipole solutions

Other configurations [MP–Shigemori '15]

Mixed configurations

- Arbitrary codim-3 and NS5-dipole sources



Limit of found solutions

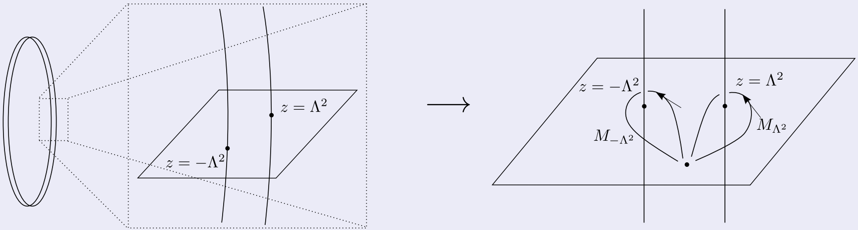
- They are not bound state
- Difficult to regard those as (geometric) microstates of black hole
- We have to approach in a different way

2D analysis (work in progress)

Seiberg–Witten solution

2D approximation

- If we zoom into the near-ring region



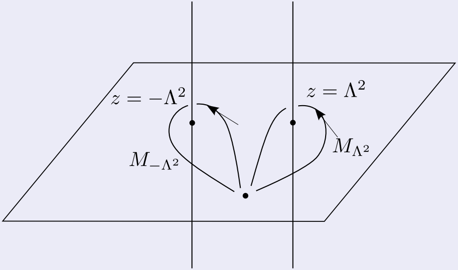
- Monodromy is an important feature of having supertubes

2D analysis (work in progress)

Seiberg–Witten solution

Similarity between SW solution and 2-dipole solution

- SW solutions exhibit monodromy



$$\begin{pmatrix} a_D \\ a \end{pmatrix} \to M \begin{pmatrix} a_D \\ a \end{pmatrix}$$

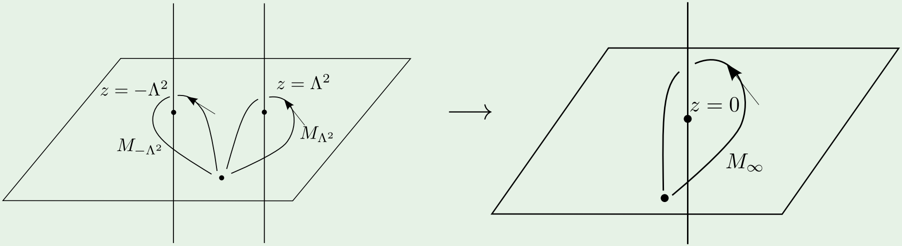
- Take mathematical structure of SW solution
- Can we obtain corresponding 3D harmonic functions?

2D analysis (work in progress)

Seiberg–Witten solution

Simplified configuration

- distance between tubes $(\Lambda^2) \ll$ radius of supertube
  $\rightarrow$ two tubes can be regarded as one tube

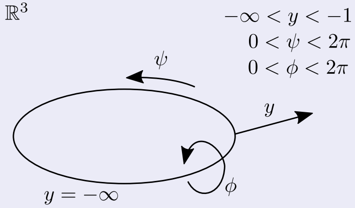


- 3D harmonic functions$\big|_{\text{near ring}} = z \sim \infty$ behaviour of a' and a'_D

3D harmonic functions with M_∞ monodromy

Coordinate system

- Take a ring-suited coordinate



- Make an ansatz for 3D harmonic functions

$$H = \sqrt{\cos\phi - y}\, e^{im\phi} f(y) \,,$$

$$H = \sqrt{\cos\phi - y}\, e^{im\phi} \left(f(y) + i\phi\, g(y)\right) \,.$$

3D harmonic functions with M_∞ monodromy

Harmonic functions

- Reproduce SW-like behaviour when it close to the tube

$$H_1 = \sqrt{\cos\phi - y}\,\sin\frac{\phi}{2}\,p(y)\,, \qquad H_2 = \sqrt{\cos\phi - y}\,\cos\frac{\phi}{2}\,p(y)\,,$$

$$H_3 = \sqrt{\cos\phi - y}\left(\sin\frac{\phi}{2}\,q(y) + \phi\cos\frac{\phi}{2}\,p(y)\right)\,,$$

$$H_4 = \sqrt{\cos\phi - y}\left(\cos\frac{\phi}{2}\,q(y) - \phi\sin\frac{\phi}{2}\,p(y)\right)\,,$$

where

$$p(y) = A\,, \qquad q(y) = C + A\ln(1-y)\,,$$

with constant A, C.

3D harmonic functions with M_∞ monodromy

- Relation between H_i and 3D harmonic functions

$$\begin{aligned} V = L_3 = \mathrm{Re}\, a' &= H_1 \,, \\ K^{1,2} = -\,\mathrm{Im}\, a' &= H_2 \,, \\ L_{1,2} = \mathrm{Re}\, a'_D &= H_3 \,, \\ K^3 = -2M = \mathrm{Im}\, a'_D &= H_4 \,. \end{aligned}$$

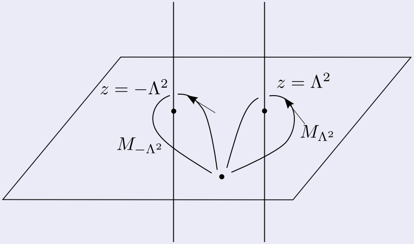
- Monodromy

$$\begin{aligned} & H_1 \to -H_1 \,, \qquad H_2 \to -H_2 \,, \\ & H_3 \to -H_3 - 2\pi H_2 \,, \\ & H_4 \to -H_4 + 2\pi H_1 \,. \end{aligned}$$

3D harmonic functions with M_∞ monodromy

Go further

- Trying to increase resolution and playing with two monodromy matrices $M_{-\Lambda^2}$ and M_{Λ^2}



- What are 3D harmonic functions with these monodromies?
- We expect that this has non-abelian anyonic properties $\because [M_{\Lambda^2}, M_{-\Lambda^2}] \neq 0$

Conclusions

- 4D/5D solution: a framework for BH research
- Only codim-3 solutions are studied so far
- Codim-2 solutions should be included because of supertube effect

Future directions

- Can we find a solution with monodromy only?
- Find a systematic way to construct general multi-dipole solutions