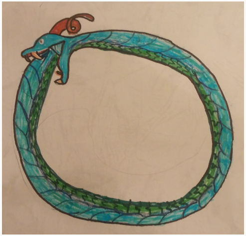
# Double Field Theory : Stringy Extension of General Relativity



*– 'Uroboros' solution to the Dark Matter problem –*

**Jeong-Hyuck Park** 박 정 혁 (朴 廷 爀) **Sogang University**

**9th March 2017, Workshop on GEOMETRY, DUALITY AND STRINGS @ YITP, Kyoto University**

# Prologue

# General Relativity, Riemannian Geometry & String Theory

- Ever since Einstein formulated his theory of gravity *i.e.* GR, by employing the mathematics of Riemannian geometry, the Riemannian metric, $g_{\mu\nu}$, has been privileged to be the only geometric and hence gravitational field:
  - Diffeomorphism: $\partial_\mu \longrightarrow \nabla_\mu = \partial_\mu + \Gamma_\mu$
  - $\nabla_\lambda g_{\mu\nu} = 0$, $\Gamma^\lambda_{[\mu\nu]} = 0 \longrightarrow \Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\rho}(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu})$
  - Curvature: $[\nabla_\mu, \nabla_\nu] \longrightarrow R^\kappa{}_{\lambda\mu\nu} \longrightarrow R$

- On the other hand, string theory puts the metric, $g_{\mu\nu}$, two-form gauge potential, $B_{\mu\nu}$, and scalar dilaton, $\phi$, on an equal footing, as they, so called the massless NS-NS sector, form a 'multiplet of T-duality' (this string theory symmetry mixes them).

- Namely, string theory suggests to view **the whole massless NS-NS sector as the gravitational unity.**
  - Riemannian geometry is for **particle** theory.
  - **String** theory requires a novel differential geometry for the NS-NS sector.

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- Namely, string theory suggests to view **the whole massless NS-NS sector as the gravitational unity.**
  - Riemannian geometry is for **particle** theory.
  - **String** theory requires a novel differential geometry for the NS-NS sector.

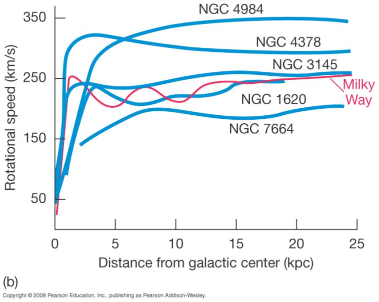
- However, in the conventional treatment of the NS-NS sector, the effective action describing its dynamics is 'organized' in terms of Riemannian geometry,

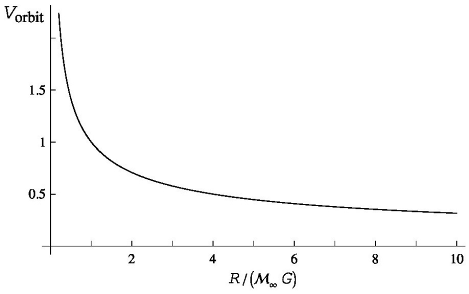
$$\int \mathrm{d}^D x \, \sqrt{-|g|}\, e^{-2\phi} \left(R + 4\,|\mathrm{d}\phi|^2 - \tfrac{1}{12}\,|\mathrm{d}B|^2\right).$$

  - In this conventional description, the Riemannian metric provides the background geometry, while the dilaton and the $B$-field are viewed as *'matter'* living on it.
  - Further, the $\mathbf{O}(D,D)$ T-duality symmetry mixing the NS-NS sector is not manifest at all, while it is secretly hidden there.
  - There is also much ambiguity to occur, when we try to couple the NS-NS sector, especially $\phi$ and $B_{\mu\nu}$, to other matters, e.g. the Standard Model.

- **Thus, Riemannian geometry fails to provide the unifying geometric description of the massless NS-NS sector.**

# Dark Matter Problem




Galaxy rotation curves : observation

Keplerain $1/\sqrt{R}$ fall-off : GR or Newton

- The galaxy rotation curve is a plot of the orbital velocities of visible stars versus their radial distance from the galactic center.
- While Einstein gravity (GR), with Schwarzschild solution, predicts the Keplerian (inverse square root) monotonic fall-off of the velocities, observations however show rather 'flat' ($\sim 200$ km/s) curves after a fairly rapid rise.
- The resolution of the discrepancy may call for 'dark matter', or modifications of the law of gravity, or perhaps both as is the case with Double Field Theory.

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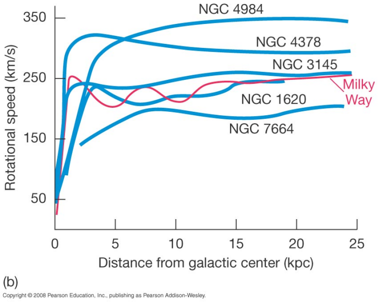


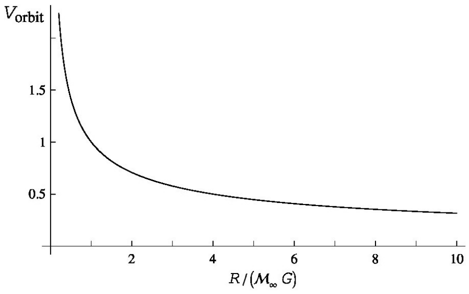


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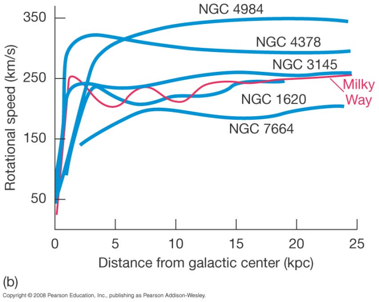


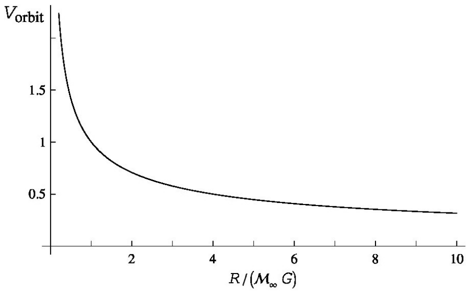


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- The resolution of the discrepancy may call for 'dark matter', or modifications of the law of gravity, or perhaps both as is the case with Double Field Theory.

# Talk Abstract: *Double Field Theory in a nutshell*

- Double Field Theory (DFT) formally uses doubled, $(D+D)$-dimensonal coordinates, to manifest $\mathbf{O}(D,D)$ symmetry and to unify diffeomorphisms and $B$-field gauge symmetry.
- It looks like a $(D+D)$-dimensonal theory, but the theory is required to satisfy so-called 'section condition' such that it lives on a $D$-dimensional hyperspace, *i.e.* 'section'.
- DFT assumes the whole massless NS-NS sector as the gravitational unity.
  - The underlying differential geometry is genuinely 'stringy' beyond Riemann.
- DFT is formulated in terms of its own field variables, $V_{Ap}$, $\bar{V}_{A\bar{p}}$, $d$, which are strictly $\mathbf{O}(D,D)$ covariant. The connection to GR can be only established after parametrizing them by the conventional Riemannian variables, such as $g_{\mu\nu}$, ${e_\mu}^a$, $B_{\mu\nu}$, $\phi$.
- Covariant derivatives and curvatures have been constructed, and successfully applied
  - to identify the expression, $R+4\,|\mathrm{d}\phi|^2-\frac{1}{12}\,|\mathrm{d}B|^2$, as a 'scalar curvature' of DFT;
  - to unify IIA and IIB SUGRAs into $D=10$ maximally supersymmetric DFT;
  - to couple $D=4$ DFT to the Standard Model unambiguously.
- Each term in every formula is manifestly covariant for the Fundamental Symmetries:
  * $\mathbf{O}(D,D)$ T-duality
  * DFT-diffeomorphisms (diffeomorphisms plus $B$-field gauge symmetry)
  * A pair of local Lorentz symmetries, $\mathbf{Spin}(1,D{-}1)_L\times\mathbf{Spin}(D{-}1,1)_R$
  * 'Coordinate gauge symmetry' (section condition)

\# The self-interaction of the NS-NS sector modifies GR at 'short' distance ( $R/MG$ ), and may solve the DM problem in 'uroboros' manner.

\# **Superstring theory itself is better formulated in terms of doubled geometry.**

# Talk Organization

I. **Geometric formulation of DFT & Coupling to the Standard Model**

II. **Doubled-yet-gauged coordinates**

III. **'Uroboros' solution to the Dark Matter Problem via DFT**

∗ Based on works in collaborations with Imtak Jeon, Kanghoon Lee, Yoonji Suh, Wonyoung Cho, Jose Fernández-Melgarejo, Soo-Jong Rey, Woohyun Rim, Yuho Sakatani, Sung Moon Ko, Minwoo Suh, Kang-Sin Choi, Rene Meyér, Charles Melby-Thompson, Chris Blair, Emanuel Malek, and Xavier Bekaert.

# I. Geometric Formulation

- **Stringy differential geometry, beyond Riemann**
  Imtak Jeon, Kanghoon Lee, JHP 1105.6294

- **Stringy Unification of IIA and IIB Supergravities under $\mathcal{N}=2$ $D=10$ Supersymmetric Double Field Theory**
  Imtak Jeon, Kanghoon Lee, JHP, Yoonji Suh 1210.5078

- **Supersymmetric gauged Double Field Theory: Systematic derivation by virtue of Twist**
  Wonyoung Cho, Jose J. Fernández-Melgarejo, Imtak Jeon, JHP 1505.01301

- **Standard Model as a Double Field Theory**
  Kang-Sin Choi, JHP 1506.05277

- *Notation*

| Index | Representation | Metric (raising/lowering indices) |
| --- | --- | --- |
| $A, B, \cdots$ | $\mathbf{O}(D, D)$ & Diffeomorphism vector | $\mathcal{J}_{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ |
| $p, q, \cdots$ | $\mathbf{Spin}(1, D-1)_L$ vector | $\eta_{pq} = \mathrm{diag}(-++\cdots+)$ |
| $\alpha, \beta, \cdots$ | $\mathbf{Spin}(1, D-1)_L$ spinor | $C_{\alpha\beta}, \qquad (\gamma^p)^T = C\gamma^p C^{-1}$ |
| $\bar{p}, \bar{q}, \cdots$ | $\mathbf{Spin}(D-1, 1)_R$ vector | $\bar{\eta}_{\bar{p}\bar{q}} = \mathrm{diag}(+--\cdots-)$ |
| $\bar{\alpha}, \bar{\beta}, \cdots$ | $\mathbf{Spin}(D-1, 1)_R$ spinor | $\bar{C}_{\bar{\alpha}\bar{\beta}}, \qquad (\bar{\gamma}^{\bar{p}})^T = \bar{C}\bar{\gamma}^{\bar{p}}\bar{C}^{-1}$ |

- Here $D$ denotes the dimenison of the physical spacetime. In this talk, $D \equiv 4$ or $10$.
- The constant $\mathbf{O}(D, D)$ metric, $\mathcal{J}_{AB}$, naturally decomposes the doubled coordinates of DFT into two parts,

$$x^A = (\tilde{x}_\mu, x^\nu)\,, \qquad\qquad \partial_A = (\tilde{\partial}^\mu, \partial_\nu)\,,$$

where $\mu$, $\nu$ are $D$-diemensional curved indices.

- *Fundamental fields of DFT*

$$V_{Ap}\,,\qquad \bar{V}_{A\bar{p}}\,,\qquad d$$ **: Geometric and hence Gravitational**

These represent the massless NS-NS sector in string theory, *c.f.* R-R sector, $\mathcal{C}^{\alpha}{}_{\bar{\alpha}}$ .

- The pair of vielbeins satisfy four defining properties,

$$V_{Ap}V^{A}{}_{q}=\eta_{pq}\,,\qquad \bar{V}_{A\bar{p}}\bar{V}^{A}{}_{\bar{q}}=\bar{\eta}_{\bar{p}\bar{q}}\qquad V_{Ap}\bar{V}^{A}{}_{\bar{q}}=0\,,\qquad V_{Ap}V_{B}{}^{p}+\bar{V}_{A\bar{p}}\bar{V}_{B}{}^{\bar{p}}=\mathcal{J}_{AB}\,,$$

such that they are the "square-roots" of projectors,

$$P_{A}{}^{B}=V_{Ap}V^{Bp}\,,\qquad \bar{P}_{A}{}^{B}=\bar{V}_{A\bar{p}}\bar{V}^{B\bar{p}}$$

satisfying

$$P^{2}=P\,,\qquad \bar{P}^{2}=\bar{P}\,,\qquad P\bar{P}=0\,,\qquad P+\bar{P}=\mathbf{1}\,.$$

- The dilaton gives rise to the $\mathbf{O}(D,D)$ invariant integral measure with weight one, after exponentiation:

$$e^{-2d}$$

Naturally the cosmological constant term in DFT should be given by $e^{-2d}\Lambda_{\rm DFT}$ which differs from the conventional one in Riemannian GR, and hence reformulates the 'cosmological constant problem'.

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- *Semi-covariant derivative :* Jeon-Lee-JHP 2010, 2011

$$\nabla_C T_{A_1A_2\cdots A_n} := \partial_C T_{A_1A_2\cdots A_n} - \omega_T \Gamma^B{}_{BC} T_{A_1A_2\cdots A_n} + \sum_{i=1}^{n} \Gamma_{CA_i}{}^B T_{A_1\cdots A_{i-1}BA_{i+1}\cdots A_n}\,.$$

The DFT-version of the Christoffel connection has been uniquely determined,

$$\Gamma_{CAB}=2\big(P\partial_C P\bar{P}\big)_{[AB]}+2\Big(\bar{P}_{[A}{}^D\bar{P}_{B]}{}^E-P_{[A}{}^D P_{B]}{}^E\Big)\partial_D P_{EC}-\tfrac{4}{D-1}\Big(\bar{P}_{C[A}\bar{P}_{B]}{}^D+P_{C[A}P_{B]}{}^D\Big)\Big(\partial_D d+(P\partial^E P\bar{P})_{[ED]}\Big)$$

by demanding the compatibility with the NS-NS sector, $\nabla_A P_{BC} = \nabla_A \bar{P}_{BC} = \nabla_A d = 0$, plus some extra 'torsionless' conditions.

- *Semi-covariant Riemann-like curvature :*

$$S_{ABCD} := \tfrac{1}{2}\Big(R_{ABCD} + R_{CDAB} - \Gamma^E{}_{AB}\Gamma_{ECD}\Big)$$

where $R_{ABCD}$ denotes the ordinary "field strength" of a connection,

$$R_{CDAB}=\partial_A\Gamma_{BCD}-\partial_B\Gamma_{ACD}+\Gamma_{AC}{}^E\Gamma_{BED}-\Gamma_{BC}{}^E\Gamma_{AED} \quad \Leftarrow \mathrm{d}\Gamma+\Gamma\wedge\Gamma\,.$$

Under arbitrary transformation of the connection, it transforms as 'total derivative',

$$\delta S_{ABCD} = \nabla_{[A}\delta\Gamma_{B]CD} + \nabla_{[C}\delta\Gamma_{D]AB}\,,$$

and further satisfies

$$S_{ABCD}=S_{[AB][CD]}=S_{CDAB}\,, \qquad S_{[ABC]D}=0\,, \qquad P_I{}^A P_J{}^B \bar{P}_K{}^C \bar{P}_L{}^D S_{ABCD}=0\,, \qquad P_I{}^A \bar{P}_J{}^B P_K{}^C \bar{P}_L{}^D S_{ABCD}=0\,.$$

- *Semi-covariant 'Master' derivative :*

$$\mathcal{D}_A := \partial_A + \Gamma_A + \Phi_A + \bar{\Phi}_A = \nabla_A + \Phi_A + \bar{\Phi}_A \,.$$

where the spin connections are determined in terms of the Christoffel-like connection by requiring the compatibility with the vielbeins,

$$\mathcal{D}_A V_{Bp} = \nabla_A V_{Bp} + \Phi_{Ap}{}^{q} V_{Bq} = 0 \,, \qquad \mathcal{D}_A \bar{V}_{B\bar{p}} = \nabla_A \bar{V}_{B\bar{p}} + \bar{\Phi}_{A\bar{p}}{}^{\bar{q}} \bar{V}_{B\bar{q}} = 0 \,.$$

- *Complete covariatizations :* ⟨ divergences, Laplacians, Dirac operators and curvatures ⟩

$$P_C{}^D \bar{P}_{A_1}{}^{B_1} \cdots \bar{P}_{A_n}{}^{B_n} \nabla_D T_{B_1 \cdots B_n} \quad \Longrightarrow \quad \mathcal{D}_p T_{\bar{q}_1 \bar{q}_2 \cdots \bar{q}_n} \,,$$

$$\bar{P}_C{}^D P_{A_1}{}^{B_1} \cdots P_{A_n}{}^{B_n} \nabla_D T_{B_1 \cdots B_n} \quad \Longrightarrow \quad \mathcal{D}_{\bar{p}} T_{q_1 q_2 \cdots q_n} \,,$$

$$\mathcal{D}^p T_{p\bar{q}_1 \bar{q}_2 \cdots \bar{q}_n} \,, \qquad \mathcal{D}^{\bar{p}} T_{\bar{p} q_1 q_2 \cdots q_n} \,, \qquad \mathcal{D}_p \mathcal{D}^p T_{\bar{q}_1 \bar{q}_2 \cdots \bar{q}_n} \,, \qquad \mathcal{D}_{\bar{p}} \mathcal{D}^{\bar{p}} T_{q_1 q_2 \cdots q_n} \,,$$

$$\gamma^p \mathcal{D}_p \rho \,, \quad \bar{\gamma}^{\bar{p}} \mathcal{D}_{\bar{p}} \rho' \,, \quad \mathcal{D}_{\bar{p}} \rho \,, \quad \mathcal{D}_p \rho' \,, \quad \gamma^p \mathcal{D}_p \psi_{\bar{q}} \,, \quad \bar{\gamma}^{\bar{p}} \mathcal{D}_{\bar{p}} \psi'_q \,, \quad \mathcal{D}_{\bar{p}} \psi^{\bar{p}} \,, \quad \mathcal{D}_p \psi'^p \,,$$

$$\mathcal{D}_\pm \mathcal{C} := \gamma^p \mathcal{D}_p \mathcal{C} \pm \gamma^{(D+1)} \mathcal{D}_{\bar{p}} \mathcal{C} \bar{\gamma}^{\bar{p}} \,, \quad (\mathcal{D}_\pm)^2 = 0 \quad \Longrightarrow \quad \mathcal{F} := \mathcal{D}_+ \mathcal{C} \quad (\text{RR field strength}) \,,$$

$$P_A{}^C \bar{P}_B{}^D S_{CD} \quad (\text{Ricci-like}) \,, \qquad (P^{AC} P^{BD} - \bar{P}^{AC} \bar{P}^{BD}) S_{ABCD} \quad (\text{scalar}) \,.$$

\# Combining the curvatures, we also have the 'conserved' Einstein-like curvature:

$$\nabla_A G^{AB} = 0 \,, \qquad G_{AB} := 2(P_{AC} \bar{P}_{BD} - \bar{P}_{AC} P_{BD}) S^{CD} - \tfrac{1}{2} \mathcal{J}_{AB} (S_{pq}{}^{pq} - S_{\bar{p}\bar{q}}{}^{\bar{p}\bar{q}}) \,.$$

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$$\gamma^p\mathcal{D}_p\rho\,, \qquad \bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho'\,, \qquad \mathcal{D}_{\bar{p}}\rho\,, \qquad \mathcal{D}_p\rho'\,, \qquad \gamma^p\mathcal{D}_p\psi_{\bar{q}}\,, \qquad \bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\psi'_q\,, \qquad \mathcal{D}_{\bar{p}}\psi^{\bar{p}}\,, \qquad \mathcal{D}_p\psi'^p\,,$$

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- *Type II $D=10$ Maximally Supersymmetric Double Field Theory :* Jeon-Lee-JHP-Suh 2012

$$\mathcal{L}_{\rm Max}=e^{-2d}\Big[\tfrac{1}{8}(P^{AB}P^{CD}-\bar{P}^{AB}\bar{P}^{CD})S_{ACBD}+\tfrac{1}{2}{\rm Tr}(\mathcal{F}\bar{\mathcal{F}})+i\bar{\rho}\mathcal{F}\rho'+i\bar{\psi}_{\bar{p}}\gamma_{q}\mathcal{F}\bar{\gamma}^{\bar{p}}\psi'^{q}$$
$$+i\tfrac{1}{2}\bar{\rho}\gamma^{p}\mathcal{D}_{p}\rho-i\bar{\psi}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho-i\tfrac{1}{2}\bar{\psi}^{\bar{p}}\gamma^{q}\mathcal{D}_{q}\psi_{\bar{p}}-i\tfrac{1}{2}\bar{\rho}'\bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho'+i\bar{\psi}'^{p}\mathcal{D}_{p}\rho'+i\tfrac{1}{2}\bar{\psi}'^{p}\bar{\gamma}^{\bar{q}}\mathcal{D}_{\bar{q}}\psi'_{p}\Big]$$

  - Due to the twofold spin groups, $\mathbf{Spin}(1,9)_{L}\times\mathbf{Spin}(9,1)_{R}$, the theory unifies the conventional IIA and IIB SUGRAs. Namely the theory is chiral w.r.t. both spin groups and hence unique. IIA and IIB appear as two distinct types of solutions.

  - Maximal $16+16$ local SUSY (*full order construction realizing '1.5 formalism'*).

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# Coupling to the Standard Model

- $D = 4$ **DFT naturally, or minimally, couples to the Standard Model**, in a completely covariant and unambiguous manner:
  - $\mathbf{O}(4,4)$ T-duality
  - Twofold local Lorentz symmetry, $\mathbf{Spin}(1,3)_L \times \mathbf{Spin}(3,1)_R$
  - DFT-diffeomorphisms
  - $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ gauge symmetry

$$\mathcal{L}_{\mathrm{SM-DFT}} = e^{-2d}\left[\begin{array}{l} \frac{1}{16\pi G_N}(P^{AB}P^{CD} - \bar{P}^{AB}\bar{P}^{CD})S_{ACBD} \\ +\sum_{\mathcal{A}} P^{AB}\bar{P}^{CD}\mathrm{Tr}(\mathcal{F}_{AC}\mathcal{F}_{BD}) + \sum_{\psi}\bar{\psi}\gamma^{a}\mathcal{D}_{a}\psi + \sum_{\psi'}\bar{\psi}'\bar{\gamma}^{\bar{a}}\mathcal{D}_{\bar{a}}\psi' \\ -\mathcal{H}^{AB}(\mathcal{D}_A\phi)^{\dagger}\mathcal{D}_B\phi - V(\phi) + y_d\,\bar{q}\cdot\phi\, d + y_u\,\bar{q}\cdot\tilde{\phi}\, u + y_e\,\bar{l}'\cdot\phi\, e' \end{array}\right]$$

which reduces to the 'standard' SM on trivial flat background after gauge fixings.

Choi-JHP 2015 [PRL]

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  - The doubled coordinates, $x^A$, $A = 1, 2, \cdots, D + D$, and the associated doubled derivatives, $\partial_A$, need to be 'halved'.

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$$\partial_A \partial^A \textbf{ anything } = 0 \,.$$

  - Explicitly, for arbitrary functions, $\Phi$, $\hat{\Phi}$, the section condition means

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Jeon-Lee-JHP-Suh 2012

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# II. Doubled-yet-gauged coordinates

- **Comments on double field theory and diffeomorphisms** JHP 1304.5946

- The section condition can be easily shown to be equivalent to a particular type of translational invariance, in a self-consistent way:

$$\partial_A \partial^A \equiv 0 \qquad \Longleftrightarrow \qquad \left[\hat{\Phi}(x+\Delta) = \hat{\Phi}(x)\,, \quad \Delta^A = \tilde{\Phi}\partial^A\Phi\right]$$

where $\hat{\Phi}$, $\tilde{\Phi}$, $\Phi$ denote arbitrary functions in DFT, such that $\Delta^A = \tilde{\Phi}\partial^A\Phi$ generates the most general form of a 'derivative-index-valued' vector, to satisfy $\Delta^A\partial_A = 0$.

- This equivalence suggests that **the doubled coordinates in DFT are actually gauged**: the doubled coordinate space is equipped with an 'equivalence relation', JHP 2013

$$x^A \sim x^A + \Delta^A \qquad \text{where} \qquad \Delta^A\partial_A = 0\,.$$

which we call **'Coordinate Gauge Symmetry'**.

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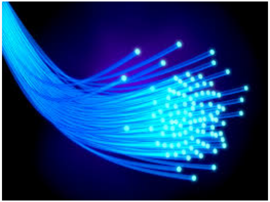
- *Doubled-yet-gauged coordinates*



**Each equivalence class, or gauge orbit in $\mathbb{R}^{D+D}$, represents a single physical point in $\mathbb{R}^D$.**

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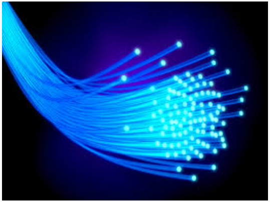

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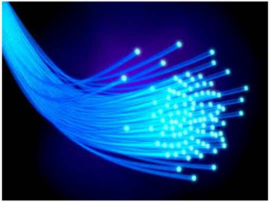

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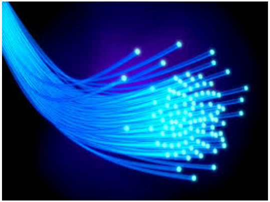
$$x^A \sim x^A + \Delta^A \qquad \text{where} \qquad \Delta^A \partial_A = 0\,.$$

which we call **'Coordinate Gauge Symmetry'**.

# For example, w.r.t. $\frac{\partial}{\partial \tilde{x}_\mu} \equiv 0$, we have explicitly $(\tilde{x}_\mu\,,\, x^\nu) \sim \left(\tilde{x}_\mu + \tilde{\Phi}\partial_\mu \Phi\,,\, x^\nu\right)$.

- *Doubled-yet-gauged coordinates*



**Each equivalence class, or gauge orbit in $\mathbb{R}^{D+D}$, represents a single physical point in $\mathbb{R}^D$.**

The claim is that, spacetime physics can be better understood in terms of the doubled-yet-gauged coordinate system.

- *Diffeomorphisms in doubled-yet-gauged spacetime.*

  – Diffeomorphisms in doubled-yet-gauged spacetime are generated by a generalized Lie derivative,

$$\hat{\mathcal{L}}_{\mathcal{V}} T_{M_1\cdots M_n} := \mathcal{V}^N\partial_N T_{M_1\cdots M_n} + \omega_T\,\partial_N\mathcal{V}^N T_{M_1\cdots M_n} + \sum_{i=1}^{n}(\partial_{M_i}\mathcal{V}_N - \partial_N\mathcal{V}_{M_i})\,T_{M_1\cdots M_{i-1}}{}^N{}_{M_{i+1}\cdots M_n}$$

  where $\omega_T$ denotes the weight. Siegel, *c.f.* Courant

  – In particular, the generalized Lie derivative of the $\mathbf{O}(D,D)$ invariant metric vanishes

$$\hat{\mathcal{L}}_{\mathcal{V}}\mathcal{J}_{AB} = 0\,.$$

  – The commutator is closed by C-bracket,

$$\left[\hat{\mathcal{L}}_{\mathcal{U}},\hat{\mathcal{L}}_{\mathcal{V}}\right] = \hat{\mathcal{L}}_{[\mathcal{U},\mathcal{V}]_{\mathrm{C}}}\,,\qquad [\mathcal{U},\mathcal{V}]^M_{\mathrm{C}} := \mathcal{U}^N\partial_N\mathcal{V}^M - \mathcal{V}^N\partial_N\mathcal{U}^M + \tfrac{1}{2}\mathcal{V}^N\partial^M\mathcal{U}_N - \tfrac{1}{2}\mathcal{U}^N\partial^M\mathcal{V}_N$$

  Hull-Zwiebach

  # DFT-diffeomorphisms decompose into undoubled Riemannian diffeomorphisms and $B$-field gauge symmetry,

$$\mathcal{V}^M = (\lambda_\mu\,,\,\xi^\nu)\quad\Longrightarrow\quad \delta B_{\mu\nu} = \partial_\mu\lambda_\nu - \partial_\nu\lambda_\mu\,,\qquad \delta x^\mu = \xi^\mu\,.$$

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## Problems with the usual infinitesimal one-form, $\mathrm{d}x^M$

- In the doubled-yet-gauged spacetime, the usual infinitesimal one-form, $\mathrm{d}x^M$, is neither covariant under DFT-diffeomorphisms,

$$\delta x^M = \mathcal{V}^M \,, \qquad\qquad \delta(\mathrm{d}x^M) = \mathrm{d}x^N \partial_N \mathcal{V}^M \neq (\partial_N \mathcal{V}^M - \partial^M \mathcal{V}_N)\mathrm{d}x^N \,,$$

nor invariant under coordinate gauge symmetry,

$$\mathrm{d}x^M \quad \longrightarrow \quad \mathrm{d}\big(x^M + \tilde{\Phi}\partial^M \Phi\big) \;\neq\; \mathrm{d}x^M \,.$$

$\Longrightarrow$ The naive contraction with the DFT-metric, $\mathrm{d}x^M \mathrm{d}x^N \mathcal{H}_{MN}$, is not a scalar, and thus cannot be used to define a 'proper length' in DFT.

# Doubled-yet-gauged coordinates & Gauged infinitesimal one-form, $Dx^M$

- **These problems can be all cured by gauging the infinitesimal one-form,**

$$Dx^M := \mathrm{d}x^M - \mathcal{A}^M \,.$$

  - The gauge potential should satisfy the same property as the coordinate gauge symmetry generator (derivative-index-valued vector, $\Delta^M = \tilde{\Phi}\partial^M\Phi$), such that

$$\mathcal{A}^M\partial_M = 0\,, \qquad \mathcal{A}_M\mathcal{A}^M = 0\,.$$

    Essentially, half of the components are trivial, for example w.r.t. $\frac{\partial}{\partial\tilde{x}_\mu} \equiv 0$,

$$\mathcal{A}^M = A_\lambda\partial^M x^\lambda = (A_\mu\,,\,0)\;, \qquad Dx^M = (\mathrm{d}\tilde{x}_\mu - A_\mu\,,\,\mathrm{d}x^\nu)\;.$$

- With the appropriate transformations of the gauge potential, the coordinate gauge symmetry invariance and the DFT-diffeomorphism covariance of $Dx^M$ can be assured:

$$\delta_{\mathrm{C.G.}}\,x^M = \tilde{\Phi}\partial^M\Phi\,, \quad \delta_{\mathrm{C.G.}}\,\mathcal{A}^M = \mathrm{d}\left(\tilde{\Phi}\partial^M\Phi\right)\,, \quad \delta_{\mathrm{C.G.}}\,(Dx^M) = 0\,;$$

$$\delta x^M = \mathcal{V}^M\,, \quad \delta\mathcal{A}^M = -\partial^M\mathcal{V}_N\mathcal{A}^N + \partial^M\mathcal{V}_N\mathrm{d}x^N\,, \quad \delta(Dx^M) = (\partial_N\mathcal{V}^M - \partial^M\mathcal{V}_N)Dx^N\,.$$

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# Length in the doubled-yet-gauged spacetime

- **The proper length is defined through a path integral,**

$$\text{Length} := -\ln\left[\int \mathcal{D}\mathcal{A}\ \exp\left(-\int\sqrt{Dx^M Dx^N \mathcal{H}_{MN}}\right)\right].$$

- For the Riemannian DFT-metric, we have

$$Dx^M Dx^N \mathcal{H}_{MN} \equiv \mathrm{d}x^\mu \mathrm{d}x^\nu g_{\mu\nu} + (\mathrm{d}\tilde{x}_\mu - A_\mu + \mathrm{d}x^\rho B_{\rho\mu})(\mathrm{d}\tilde{x}_\nu - A_\nu + \mathrm{d}x^\sigma B_{\sigma\nu})\, g^{\mu\nu},$$

and hence, after integrating out the gauge potential, $A_\mu$, the above $\mathbf{O}(D,D)$ covariant path integral definition of the length reduces to the conventional one,

$$\text{Length} \implies \int\sqrt{\mathrm{d}x^\mu \mathrm{d}x^\nu g_{\mu\nu}}\,.$$

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# Point particle propagating in the doubled-yet-gauged spacetime

- Point particle action on doubled-yet-gauged spacetime coupled to the NS-NS sector:

$$\mathcal{S}_{\rm particle} = \int \mathrm{d}\tau \left[ e^{-1} D_\tau X^M D_\tau X^N \mathcal{H}_{MN}(X) - \tfrac{1}{4} m^2 e \right],$$

Ko-JHP-Suh 2016

where $e$ is an einbein and $m$ is the mass of the particle.

- With Riemannian DFT-metric, after integrating out $e$ and $\mathcal{A}^M$, the above action reduces to the conventional one for a relativistic point particle now coupled to the string frame metric only:

$$S_{\rm particle} \equiv \int \mathrm{d}\tau \;\; -m\sqrt{-\dot{X}^\mu \dot{X}^\nu g_{\mu\nu}}\,.$$

- This implies that **the particle follows the geodesic path defined in the string frame.**
- This preferred choice of the frame, *i.e.* String frame over Einstein frame, is due to the fundamental symmetries of DFT: $\mathbf{O}(D,D)$ symmetry , DFT-diffeomorphisms and the coordinate gauge symmetry

# Newton mechanics can be also formulated in the doubled-yet-gauged Euclidean space,

$$\mathcal{L}_{\rm Newton} = \tfrac{1}{2} m\, D_t X^M D_t X^N \delta_{MN} - V(X)\,,$$

where $M, N = 1, 2, \cdots, 6$ and the potential, $V(X)$, satisfies the section condition.

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# String theory itself is better formulated

# on doubled-yet-gauged spacetime:

- **Covariant action for a string in doubled-yet-gauged spacetime**
  **Kanghoon Lee, JHP 1307.8377**
- **Green-Schwarz superstring on doubled-yet-gauged spacetime** **JHP 1609.04265**

# String probes the doubled-yet-gauged spacetime

- **The doubled-yet-gagued string action** is, with $D_i X^M = \partial_i X^M - \mathcal{A}_i^M$ ,

$$\frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\ \mathcal{L}_{\mathrm{string}}\,, \qquad \mathcal{L}_{\mathrm{string}} = -\tfrac{1}{2}\sqrt{-h}\,h^{ij}D_iX^M D_jX^N \mathcal{H}_{MN}(X) - \epsilon^{ij}D_iX^M\mathcal{A}_{jM}\,.$$

JHP-Lee 2013 (*c.f.* Hull 2006)

- The action is **fully symmetric** for an arbitrary curved DFT-metric, $\mathcal{H}_{MN}(X)$, essentially due to the auxiliary coordinate gauge potential, $\mathcal{A}_i^M$,
  - worldsheet diffeomorphisms plus Weyl symmetry
  - $\mathbf{O}(D,D)$ T-duality
  - target spacetime DFT-diffeomorphisms
  - the coordinate gauge symmetry : $X^M \ \sim\ X^M + \tilde{\Phi}\partial^M\Phi$

- With the Riemannian DFT-metric, after integrating out $\mathcal{A}^M$, the doubled-yet-gauged string action reduces to the conventional one,

$$\frac{1}{4\pi\alpha'}\mathcal{L}_{\rm string} \equiv \frac{1}{2\pi\alpha'}\left[-\frac{1}{2}\sqrt{-h}h^{ij}\partial_i X^\mu \partial_j X^\nu g_{\mu\nu}(X) + \frac{1}{2}\epsilon^{ij}\partial_i X^\mu \partial_j X^\nu B_{\mu\nu}(X) + \frac{1}{2}\epsilon^{ij}\partial_i \tilde{X}_\mu \partial_j X^\mu\right],$$

with the bonus of the topological term introduced by Giveon-Rocek; Hull.

  - The EOM of $\mathcal{A}_i^M$ implies **self-duality** in the full doubled spacetime,

$$\mathcal{H}^M{}_N D^i X^N + \frac{1}{\sqrt{-h}}\epsilon^{ij} D_j X^M = 0\,,$$

    which relates $X^\mu$ and $\tilde{X}_\mu$.

  - The EOM of $X^M$ is identified as the **Stringy Geodesic Equation:**

$$\frac{1}{\sqrt{-h}}\partial_i\left(\sqrt{-h}\mathcal{H}_{LM}D^i X^M\right) + \Gamma_{LMN}\,(\bar{P}^M{}_A D_j X^A)(P^N{}_B D^i X^B) = 0\,.$$

- On the other hand, upon non-Riemannian backbrounds, the doubled-yet-gauged string action leads to chiral or non-Relativistic string theory *a la* Gomis-Ooguri.
  Lee-JHP 2013, Ko-Melby-Thompson-Meyer-JHP 2015

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# Supersymmetric extension

- **The doubled-yet-gagued Green-Schwarz superstring action is**

$$\mathcal{S}_{\rm superstring}=\frac{1}{4\pi\alpha'}\int{\rm d}^2\sigma\ \mathcal{L}_{\rm superstring}\,,$$

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**JHP 1609.04265**

- Here, with a pair of Majorana-Weyl spinors, $\theta^\alpha$ for $\mathbf{Spin}(1,9)_L$ and $\theta'^{\bar{\alpha}}$ for $\mathbf{Spin}(9,1)_R$, we set

$$\Pi_i^M:=D_iX^M-i\Sigma_i^M\,,\qquad\qquad \Sigma_i^M:=\bar{\theta}\gamma^M\partial_i\theta+\bar{\theta}'\bar{\gamma}^M\partial_i\theta'\,.$$

- Symmetries:
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  - target spacetime DFT-diffeomorphisms
  - coordinate gauge symmetry : $X^M\ \sim\ X^M+\tilde{\Phi}\partial^M\Phi$
  - **twofold Lorentz symmetry,** $\mathbf{Spin}(1,9)_L\times\mathbf{Spin}(9,1)_R$ $\Rightarrow$ **Unification of IIA & IIB**
  - **Maximal 16+16 SUSY & kappa symmetries**

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  - coordinate gauge symmetry : $X^M\ \sim\ X^M+\tilde{\Phi}\partial^M\Phi$
  - **twofold Lorentz symmetry, Spin**$(1,9)_L\times$ **Spin**$(9,1)_R$ $\Rightarrow$ **Unification of IIA & IIB**
  - **Maximal 16+16 SUSY & kappa symmetries**

# III. Solution to the Dark Matter Problem

- **The rotation curve of a point particle in stringy gravity**

**Sungmoon Ko, JHP, Minwoo Suh 1606.09307**

- The master derivative, $\mathcal{D}_A$, naturally provides the minimal coupling of the Stadard Model to DFT, or the massless NS-NS sector.

*"Symmetry dictates interaction", C. N. Yang*

i) Each SM fermion couples to the massless NS-NS sector as

$$\begin{aligned} e^{-2d}\,\bar{\psi}\gamma^A\mathcal{D}_A\psi &= e^{-2d}\,\bar{\psi}\gamma^A(\partial_A\psi + \tfrac{1}{4}\Phi_{Apq}\gamma^{pq}\psi) \\ &\equiv \tfrac{1}{\sqrt{2}}\sqrt{-g}e^{-2\phi}\,\bar{\psi}\gamma^\mu\left(\partial_\mu\psi + \tfrac{1}{4}\omega_{\mu pq}\gamma^{pq}\psi + \tfrac{1}{24}H_{\mu pq}\gamma^{pq}\psi - \partial_\mu\phi\psi\right) \\ &\equiv \tfrac{1}{\sqrt{2}}\sqrt{-g}\,\bar{\chi}\gamma^\mu\left(\partial_\mu\chi + \tfrac{1}{4}\omega_{\mu pq}\gamma^{pq}\chi + \tfrac{1}{24}H_{\mu pq}\gamma^{pq}\chi\right) \end{aligned}$$

*c.f.* Coimbra-Strickland-Constable-Waldram

where the field redefinition of the fermion, $\chi \equiv e^{-\phi}\psi$, has been performed which removes the scalar dilaton completely. This result shows that

- the scalar dilaton is transparent or 'dark' to the SM fermions;
- (not only F1 but also) the SM fermions can source the $H$-flux!

ii) On the other hand, each SM gauge boson couples to the massless NS-NS sector as

$$e^{-2d}\,\mathrm{Tr}\left(P^{AB}\bar{P}^{CD}\mathcal{F}_{AC}\mathcal{F}_{BD}\right) \equiv -\tfrac{1}{4}\sqrt{-g}e^{-2\phi}\,\mathrm{Tr}\left(g^{\kappa\lambda}g^{\mu\nu}F_{\kappa\mu}F_{\lambda\nu}\right)$$

- $B$-field, or 'axion' (dual scalar), is dark to the gauge bosons;
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# Spherical symmetry in DFT

- The coupling of DFT to the Standard Model motivated us to look for spherically symmetric DFT-vacua .

  - Such spherically symmetric solutions should admit three Killing vectors in doubled-yet-gauged spacetime, $V_a^A$, $a = 1, 2, 3$,

$$\hat{\mathcal{L}}_{V_a}\mathcal{H}_{MN} = 0 \qquad \Longleftrightarrow \qquad (P\nabla)_M(\bar{P}V_a)_N - (\bar{P}\nabla)_N(PV_a)_M = 0$$

$$\hat{\mathcal{L}}_{V_a}(e^{-2d}) = 0 \qquad \Longleftrightarrow \qquad \nabla_M V_a^M = 0$$

  which form an $\mathbf{so}(3)$ algebra in terms of the $\mathbf{C}$-bracket,

$$[V_a, V_b]_{\mathbf{C}} = \sum_c \epsilon_{abc} V_c \,.$$

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- The most general spherically symmetric, asymptotically flat DFT-vacuum solution is

$$e^{2\phi} = \gamma_+ \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{b}{\sqrt{a^2+b^2}}} + \gamma_- \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{-b}{\sqrt{a^2+b^2}}} , \qquad B_{(2)} = h\cos\vartheta\, \mathrm{d}t \wedge \mathrm{d}\varphi ,$$

$$\mathrm{d}s^2 = e^{2\phi}\left[-\left(\frac{r-\alpha}{r+\beta}\right)^{\frac{a}{\sqrt{a^2+b^2}}} \mathrm{d}t^2 + \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{-a}{\sqrt{a^2+b^2}}} \left(\mathrm{d}r^2 + (r-\alpha)(r+\beta)\mathrm{d}\Omega^2\right)\right] ,$$

where $a, b, h$ $(h^2 \leq b^2)$ are three free parameters and

$$\alpha = \frac{a}{a+b}\sqrt{a^2+b^2} , \qquad \beta = \frac{b}{a+b}\sqrt{a^2+b^2} , \qquad \gamma_\pm = \frac{1}{2}\left(1 \pm \sqrt{1-h^2/b^2}\right) .$$

  - This is a rederivation of the solution by Burgess-Myers-Quevedo (1994) who generated the above solution by applying S-duality to the scalar-gravity solution of Fischer (1948), Janis-Newman-Winicour (1968). It solves the familiar action,

$$\int \mathrm{d}^4x \sqrt{-|g|}\, e^{-2\phi}\left(R + 4\,|\mathrm{d}\phi|^2 - \tfrac{1}{12}\,|\mathrm{d}B|^2\right) .$$

  - Equivalently, it solves the EOMs of $D=4$ DFT (*i.e.* pure Stringy Gravity):

$$(P^{AB}P^{CD} - \bar{P}^{AB}\bar{P}^{CD})S_{ACBD} \equiv 0 , \qquad P_A{}^C \bar{P}_B{}^D S_{CD} \equiv 0 .$$

  - Thus, within the DFT framework, it should be identified as **the DFT-vacuum solution** in analogy with the Schwarzschild solution in Einstein gravity.

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- We define the 'proper' radius,

$$R := \sqrt{g_{\vartheta\vartheta}(r)} = \left[(r-\alpha)(r+\beta)\left(\gamma_{+}\left(\tfrac{r-\alpha}{r+\beta}\right)^{\frac{-a+b}{\sqrt{a^2+b^2}}} + \gamma_{-}\left(\tfrac{r-\alpha}{r+\beta}\right)^{\frac{-a-b}{\sqrt{a^2+b^2}}}\right)\right]^{\frac{1}{2}} ,$$

which converts the metric into a canonical form where the angular part is 'properly' normalized (hence comparable to observations, *e.g.* galaxy rotation curves):

$$\mathrm{d}s^2 = g_{tt}\mathrm{d}t^2 + g_{RR}\mathrm{d}R^2 + R^2\mathrm{d}\Omega^2 .$$

- After solving the circular geodesic motion of a point particle (with the string frame metric), the orbital velocity is given by the proper radius times the angular velocity,

$$V_{\mathrm{orbit}} = \left|R\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right| = \left[-\tfrac{1}{2}R\frac{\mathrm{d}g_{tt}}{\mathrm{d}R}\right]^{\frac{1}{2}} .$$

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# Physical observables of the spherically symmetric DFT-vacuum

- There are three physical observables, on account of the three free parameters, $a, b, h$,

$$\mathcal{M}_\infty G := \lim_{R\to\infty}(RV^2_{\mathrm{orbit}}) = \tfrac{1}{2}\left(a + b\sqrt{1-h^2/b^2}\right),$$
$$R_{\mathrm{photon}} = R(r_{\mathrm{photon}}), \qquad r_{\mathrm{photon}} = a + \tfrac{1}{2}\left(\tfrac{a-b}{a+b}\right)\sqrt{a^2+b^2},$$
$$\mathcal{Q}_{\mathrm{Noether}}[\partial_t] = \tfrac{1}{4}\left[a + \left(\tfrac{a-b}{a+b}\right)\sqrt{a^2+b^2}\right].$$

  - The first defines the asymptotic or Newtonian mass, $\mathcal{M}_\infty$, from the Keplerian fall-off of the orbital velocity which eventually takes place at spatial infinity:

$$g_{tt} \rightarrow -1 + \tfrac{2\mathcal{M}_\infty G}{R} \qquad \text{as} \quad R \rightarrow \infty .$$

    # Hence, the rotation curve can be non-Keplerian only over a finite range.
    **Namely, DFT modifies GR at short-distance.**

  - The second gives the radius of a photon sphere (if positive).

  - The last is the conserved Noether charge for the time translational symmetry, computable from the DFT-generalization of the Wald prescription in GR.

JHP-Rey-Rim-Sakatani, *c.f.* Blair

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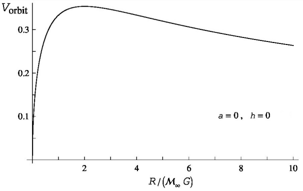
# Various choices of the parameters, $a, b, h$, $(h^2 \leq b^2)$

- If we set $b = h = 0$, the solution reduces to the Schwarzschild metric, with the Keplerian orbital velocity, $V_{\rm orbit} = \sqrt{\frac{\mathcal{M}_\infty G}{R}}$.

- If $a = h = 0$, we reproduce the renowned orbital velocity formula by Hernquist,




Rotation curve of Hernquist Model, $V_{\rm orbit} = \sqrt{\frac{\mathcal{M}_\infty R}{(R+2\mathcal{M}_\infty G)^2}}$.

Remarkably, the orbital velocity is not monotonic; it assumes its maximum value, $\frac{1}{2\sqrt{2}}$, about 35% of the speed of light, at $R = 2\mathcal{M}_\infty G$.

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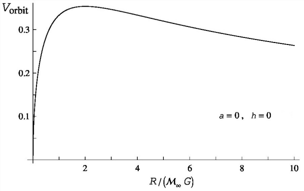


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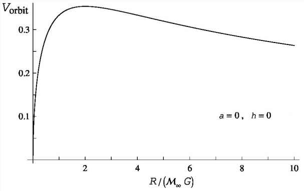


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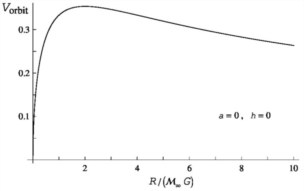


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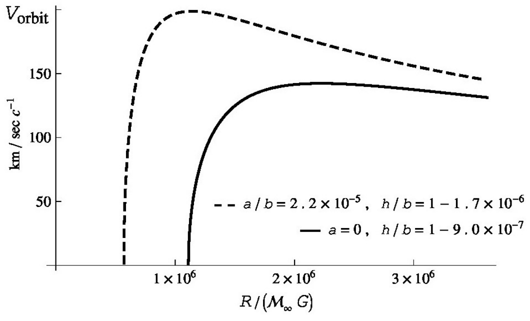


**Rotation curves in DFT** (dimensionless, nonexhaustive).

  - The curves feature a maximum of the orbital velocity after a fairly rapid rise. It is roughly about $150\,\mathrm{km/s}\,c^{-1}$ which is comparable to observations.

  - Further, if we let $R$ and $\mathcal{M}_\infty$ assume the radius and the mass of the visible matter in the Milky Way, *i.e.* $15\,\mathrm{kpc}$ and $2 \times 10^{11} M_\odot$, we have as an order of magnitude, $R/(\mathcal{M}_\infty G) \simeq 1.5 \times 10^6$. This number fits the scale of the horizontal axis.

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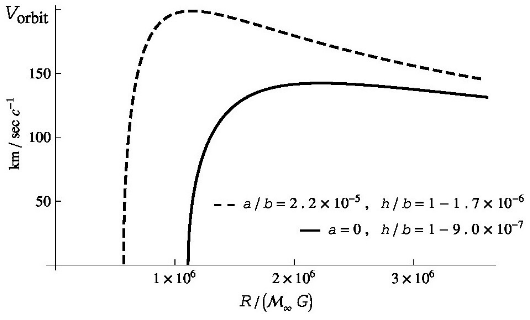


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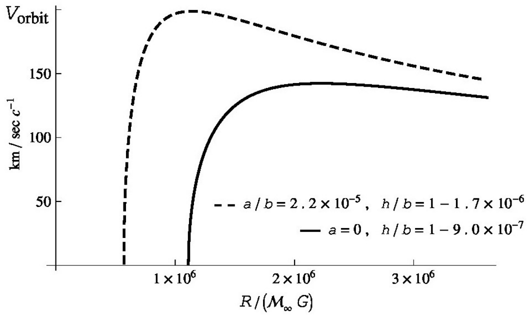


**Rotation curves in DFT** (dimensionless, nonexhaustive).

  - The curves feature a maximum of the orbital velocity after a fairly rapid rise. It is roughly about $150\,\mathrm{km/s}\,c^{-1}$ which is comparable to observations.

  - Further, if we let $R$ and $\mathcal{M}_\infty$ assume the radius and the mass of the visible matter in the Milky Way, *i.e.* $15\,\mathrm{kpc}$ and $2\times10^{11}M_\odot$, we have as an order of magnitude, $R/(\mathcal{M}_\infty G)\simeq 1.5\times10^6$. This number fits the scale of the horizontal axis.

  - For sufficiently small $R/(\mathcal{M}_\infty G)$, the gravitational force becomes repulsive.

# Discussion

- We have attempted to view DFT as the stringy extension of, and hence potentially an alternative to, Einstein gravity.
  - The fundamental symmetries of DFT unambiguously fix the theory itself as well as the couplings to the Standard Model and to a point-like particle.

- The circular geodesic motion around the most general, spherically symmetric, asymptotically flat $D=4$ DFT-vacuum reveals that
  i) its rotation curve features generically a maximum and thus non-Keplerian over a finite range (short-distance), while becoming asymptotically Keplerian/Newtonian at infinity (long-distance) as $g_{tt} \rightarrow -1+\frac{2M_{\infty}G}{R}$.
  ii) Furthermore, the gravitational force can be even repulsive very close to the origin (far-short-distance).

- DFT is, by nature, Stringy Gravity, which is compatible with GR: it still includes GR.
  - Yet, the self-interaction of the massless NS-NS sector can 'modify' GR.
  - From the conventional GR point of view, the scalar dilaton and the $B$-field may well be regarded as 'dark matter' (*c.f.* axion) or 'dark graviy'.

- Deeper understanding of the three free parameters of the DFT-vacuum, perhaps as the intrinsic properties of matter or an elementary particle, would be desirable.

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- While the proper radius, $R$, is the dimensionful physical radius, the normalized radius, $R/(\mathcal{M}_\infty G)$, is the mathematically natural dimensionless variable which essentially probes the theoretical nature of the gravitational force.

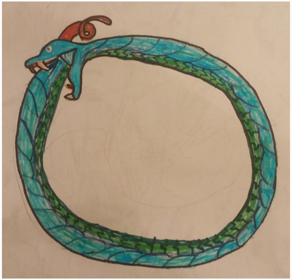
  - Intriguingly, $R/(\mathcal{M}_\infty G)$ is thousand times smaller for the Milky Way compared to the Earth at each surface (of the visible matter): $1.5 \times 10^6$ versus $1.4 \times 10^9$.

  - Generically, if the mass density is constant, $R/(\mathcal{M}_\infty G)$ becomes smaller as the physical radius, $R$, grows.

**Cosmic Uroboros:**

**– The observations of stars and galaxies far away, or the dark matter and the dark energy problems, are revealing the short-distance nature of gravity!**

**– The repulsive gravitational force at very short-distance, $R/(\mathcal{M}_\infty G) \to 0^+$, may be responsible for the acceleration of the Universe.**



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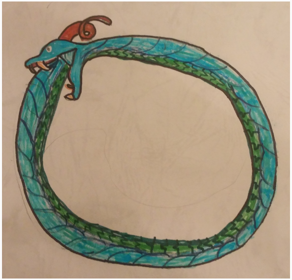

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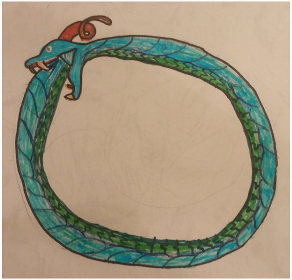

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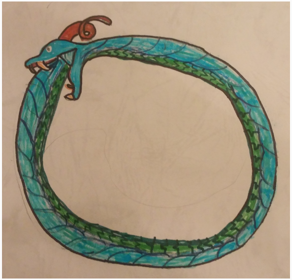

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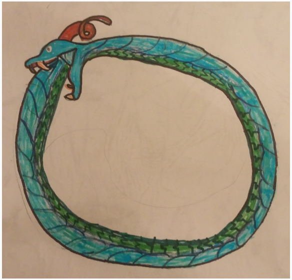

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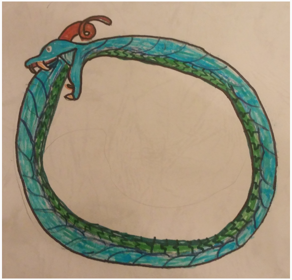

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**Talk based on collaborations with Imtak Jeon (8 papers), Kanghoon Lee (8 papers), Yoonji Suh (3 papers), Chris Blair, Emanuel Malek, Wonyoung Cho, Jose Fernández-Melgarejo, Soo-Jong Rey, Woohyun Rim, Yuho Sakatani, Sung Moon Ko, Charles Melby-Thompson, Rene Meyér, Minwoo Suh, Kang-Sin Choi and Xavier Bekaert.**

- *Differential geometry with a projection: Application to double field theory* 1011.1324 JHEP
- **Stringy differential geometry, beyond Riemann** **1105.6294 PRD**
- *Incorporation of fermions into double field theory* 1109.2035 JHEP
- *Ramond-Ramond Cohomology and O(D,D) T-duality* 1206.3478 JHEP
- *Supersymmetric Double Field Theory: Stringy Reformulation of Supergravity* 1112.0069 PRD
- **Stringy Unification of IIA and IIB Supergravities under $\mathcal{N}=2$ $D=10$ Supersymmetric Double Field Theory** **1210.5078 PLB**
- **Supersymmetric gauged Double Field Theory: Systematic derivation by virtue of 'Twist'** **1505.01301 JHEP**
- *Comments on double field theory and diffeomorphisms* 1304.5946 JHEP
- *Covariant action for a string in doubled yet gauged spacetime* 1307.8377 NPB
- **Green-Schwarz superstring on doubled-yet-gauged spacetime** **1609.04265 JHEP**
- *Double field formulation of Yang-Mills theory* 1102.0419 PLB
- **Standard Model as a Double Field Theory** **1506.05277 PRL**
- **The rotation curve of a point particle in stringy gravity** **1606.09307**
- **O**$(D,D)$ *Covariant Noether Currents and Global Charges in Double Field Theory* 1507.07545 JHEP
- *Dynamics of Perturbations in Double Field Theory & Non-Relativistic String Theory* 1508.01121 JHEP
- *Higher Spin Double Field Theory: A Proposal* 1605.00403 JHEP
- *U-geometry: SL(5)* $\Rightarrow$ **U-gravity: SL($N$)** 1302.1652 JHEP/**1402.5027 JHEP**
- *M-theory and Type IIB from a Duality Manifest Action* 1311.5109 JHEP

# The End