Brane worldvolume theory based on duality symmetry

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based on a collaboration with **Shozo Uehara** (KPUM)

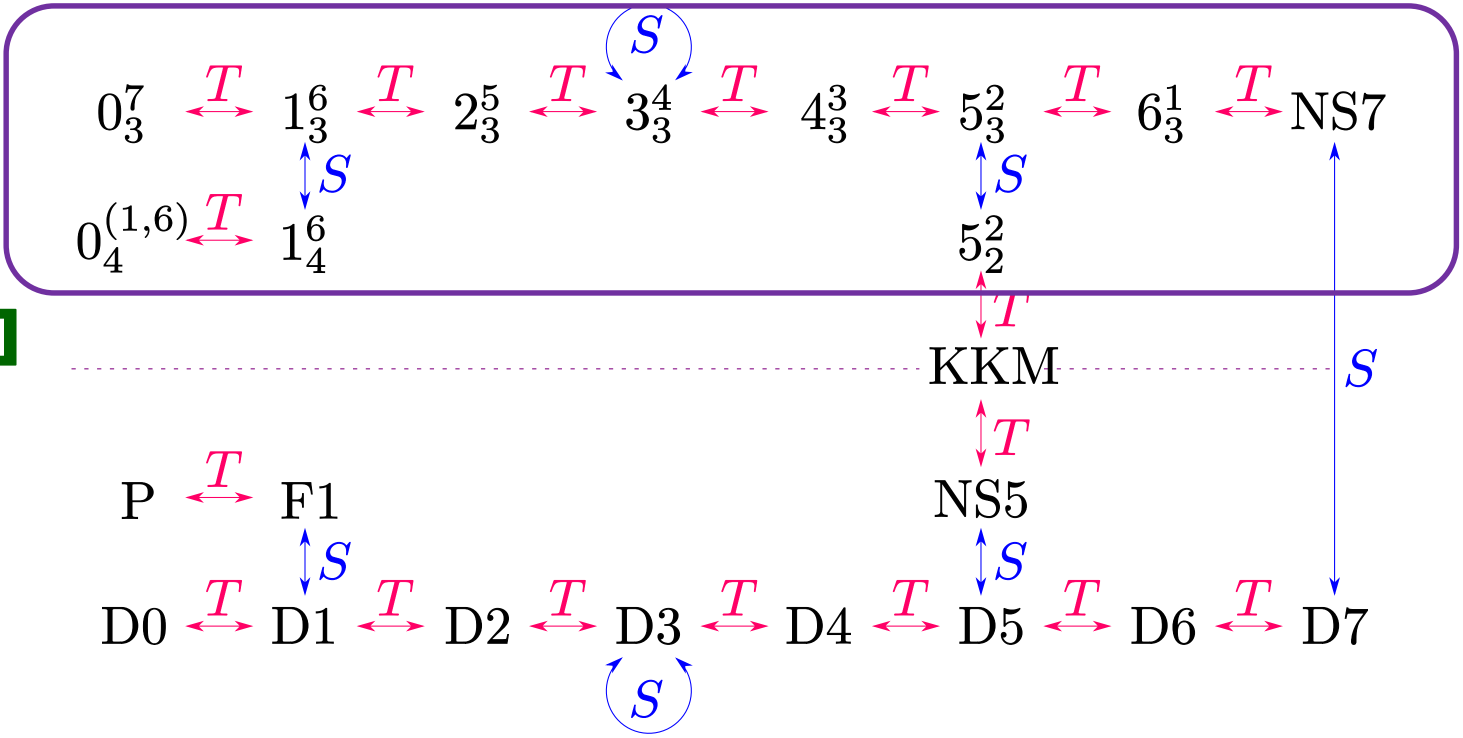
- Phys. Rev. Lett. 117, 191601 [arXiv:1607.04265]
- \+ ongoing work.

GEOMETRY, DUALITY AND STRINGS
March 10th, YITP

Motivation

In **type II string / T^d**, there are various branes which are related by ***U*-duality transformations**.

E.g.; Type II string / T^7



Exotic branes

[Kimura-san's talk]

$$\begin{tikzcd}
0^7_3 \arrow[r, leftrightarrow, "T"] & 1^6_3 \arrow[r, leftrightarrow, "T"] \arrow[d, leftrightarrow, "S"] & 2^5_3 \arrow[r, leftrightarrow, "T"] & 3^4_3 \arrow[r, leftrightarrow, "T"] \arrow[loop above, "S"] & 4^3_3 \arrow[r, leftrightarrow, "T"] & 5^2_3 \arrow[r, leftrightarrow, "T"] \arrow[d, leftrightarrow, "S"] & 6^1_3 \arrow[r, leftrightarrow, "T"] & \mathrm{NS7} \arrow[dddd, leftrightarrow, "S"] \\
0^{(1,6)}_4 \arrow[r, leftrightarrow, "T"] & 1^6_4 & & & & 5^2_2 \arrow[d, leftrightarrow, "T"] & & \\
& & & & & \mathrm{KKM} \arrow[d, leftrightarrow, "T"] & & \\
\mathrm{P} \arrow[r, leftrightarrow, "T"] & \mathrm{F1} \arrow[d, leftrightarrow, "S"] & & & & \mathrm{NS5} \arrow[d, leftrightarrow, "S"] & & \\
\mathrm{D0} \arrow[r, leftrightarrow, "T"] & \mathrm{D1} \arrow[r, leftrightarrow, "T"] & \mathrm{D2} \arrow[r, leftrightarrow, "T"] & \mathrm{D3} \arrow[r, leftrightarrow, "T"] \arrow[loop below, "S"] & \mathrm{D4} \arrow[r, leftrightarrow, "T"] & \mathrm{D5} \arrow[r, leftrightarrow, "T"] & \mathrm{D6} \arrow[r, leftrightarrow, "T"] & \mathrm{D7}
\end{tikzcd}$$

Motivation

Also in M-theory / T^d, there are various branes which are related by *U*-duality transformations:

e.g.; M-theory / T^8

$$\begin{tikzcd}
0^{(1,7)} \arrow[r, leftrightarrow] & 2^6 \arrow[r, leftrightarrow] & 5^3 \arrow[d, leftrightarrow] \\
 & & \mathrm{KKM} \arrow[d, leftrightarrow] \\
\mathrm{P} \arrow[r, leftrightarrow] & \mathrm{M2} \arrow[r, leftrightarrow] & \mathrm{M5}
\end{tikzcd}$$

$$M_{0^{(1,7)}} = \frac{\left(R_{m_1} \cdots R_{m_7}\right)^2 R^3}{l_{11}^{18}}$$

$$M_{2^6} = \frac{R_{n_1} R_{n_2} \left(R_{m_1} \cdots R_{m_6}\right)^2}{l_{11}^{15}}$$

$$M_{5^3} = \frac{R_{n_1} \cdots R_{n_5} \left(R_{m_1} \cdots R_{m_3}\right)^2}{l_{11}^{12}}$$

$$M_{\mathrm{KKM}} = \frac{R_{n_1} \cdots R_{n_6} R^2}{l_{11}^{9}}$$

Motivation

Worldvolume actions for these branes have very different forms:

$$
\left\{
\begin{aligned}
&S_{\mathrm{F1}} = \frac{1}{2}\int_{\Sigma_2} G_{ij}\,\mathrm{d}X^i \wedge *\mathrm{d}X^j + \int_{\Sigma_2} B_2 \,. \\
&S_{\mathrm{D}p} = -\int_{\Sigma_{p+1}} \mathrm{d}^{p+1}\sigma\ \mathrm{e}^{-\Phi}\sqrt{-\det(G + B_2 - F_2)} + \int_{\Sigma_{p+1}} \mathrm{e}^{B_2 - F_2} \wedge C \,. \\
&S_{\mathrm{M2}} = \frac{1}{2}\int_{\Sigma_3} \big(G_{ij}\,\mathrm{d}X^i \wedge *_\gamma \mathrm{d}X^j - *_\gamma 1\big) + \int_{\Sigma_3} C_3 \,. \\
&S_{\mathrm{KKM}} = -\int_{\Sigma} \mathrm{d}^7\sigma\, k^2\sqrt{-\det(G_{\mu\nu}D_\alpha X^\mu D_\beta X^\nu) + \cdots} + \cdots \,. \\
&S_{\mathrm{M5}} = \cdots \text{PST action} \cdots \,. \\
&S_{5^2_2\,/\,5^3} = \cdots [\textbf{Kimura-Sasaki-Yata actions}] \cdots \,.
\end{aligned}
\right.
$$

Motivation

In this talk, I will propose
simple worldvolume actions:

$$S_{p\text{-brane}} = \overbrace{\frac{1}{2}\int_{\Sigma_{p+1}} \mathcal{M}_{IJ}\,\mathcal{P}^{I}\wedge *_{\gamma}\mathcal{P}^{J}}^{\substack{\text{independent of branes} \\ \text{duality invariant}}} - \overbrace{\int_{\Sigma_{p+1}} \Omega_{p+1}}^{\substack{\text{depends} \\ \text{on the brane}}} \,.$$

information about the target space
is contained only here

contains the
worldvolume
gauge fields

generalized metric
in the doubled/exceptional space

⇨ I will first review the geometry of the extended space.

Plan

- **Review:** Double Field Theory (DFT)
- **Review:** Exceptional Field Theory (EFT)

  + report some results on DFT/EFT
  (not related to the main topic)

- **Review:** Double Sigma Model
  (based on doubled geometry)

- Our action for a *p*-brane
  (based on exceptional geometry)
- Comparison with the known worldvolume theories for

  M2-brane, M5-brane [PRL 117, 191601],
  KKM (partial result) [ongoing work
  F1/D1-string, D3-brane, … with S. Uehara]

T-duality

String theory / T^d has the *T*-duality symmetry:

$$p_i \Longleftrightarrow w^i$$

momenta windings

$$[x^i,\, p_j] = i\,\delta^i_j \qquad [\tilde{x}_i,\, w^j] = i\,\delta^j_i$$

$$x^i \updownarrow \quad \textbf{conjugate} \quad \updownarrow \tilde{x}_i$$

winding coordinates

People have noticed that
in order to make
the *T*-duality covariance manifest,
it is efficient to introduce
winding coordinates.

**[Duff '90;
Tseytlin '91;
Kugo-Zwiebach '92;
Siegel '93;...]**

Doubled space

We consider *2d* dimensional doubled space, which has the generalized coordinates,

$$(x^I) = (x^i,\ \tilde{x}_i)$$

winding coordinates

There is a natural metric on the doubled space:

$$(\mathcal{H}_{IJ}) = \begin{pmatrix} (G - B\,G^{-1}\,B)_{ij} & B_{ik}\,G^{kj} \\ -G^{ik}\,B_{kj} & G^{ij} \end{pmatrix} .$$

generalized metric

<u>mass of a string:</u>

$$M^2 = \frac{2}{\alpha'}\left(z^I\,\mathcal{H}_{IJ}\,z^J + N + \tilde{N} - 2\right)$$

$$(z^I) \equiv \begin{pmatrix} w^i \\ p_i \end{pmatrix}$$

winding

momenta

Doubled space

There is another metric on the doubled space:

signature

$$(\underbrace{+1,\cdots,+1}_{d},\underbrace{-1,\cdots,-1}_{d})$$

O(*d*,*d*) metric $$(\eta_{IJ}) \equiv \begin{pmatrix} 0 & \delta_i^j \\ \delta_j^i & 0 \end{pmatrix} .$$

Level-matching condition: $$N - \tilde{N} = \frac{1}{2} z^I \eta_{IJ} z^J .$$

O(*d*,*d*) *T*-duality transformations

$$z^I \to \Lambda^I{}_J z^J , \quad \mathcal{H}_{IJ} \to (\Lambda^{-\mathrm{T}})_I{}^K (\Lambda^{-\mathrm{T}})_J{}^L \mathcal{H}_{KL} ,$$

$$\eta_{IJ} \to (\Lambda^{-\mathrm{T}})_I{}^K (\Lambda^{-\mathrm{T}})_J{}^L \eta_{KL} = \eta_{IJ} ,$$

keep the mass spectrum/level-matching cond. invariant.

Double Field Theory

Gravitational theory on the doubled space

**[Siegel '93;
Hull, Zwiebach '09]**

Fundamental fields:

$$\begin{cases} \mathcal{H}_{IJ}(x) & \text{generalized metric} \\ d(x) & \text{DFT dilaton} \end{cases}$$

2-derivative Lagrangian:

**[Hohm, Hull, Zwiebach '10;
I. Jeon, K. Lee, J.-H. Park '11]**

$$\mathcal{L}_{\text{DFT}} = \mathrm{e}^{-2d}\,\mathcal{S}\,,$$

← generalization of the Einstein-Hilbert action

$$\mathcal{S} \equiv \mathcal{H}^{IJ}\,\partial_I\partial_J d - \partial_I\partial_J\mathcal{H}^{IJ} - 4\mathcal{H}^{IJ}\,\partial_I d\,\partial_J d + 4\partial_I\mathcal{H}^{IJ}\,\partial_J d$$
$$+\frac{1}{8}\,\mathcal{H}^{IJ}\,\partial_I\mathcal{H}^{KL}\,\partial_J\mathcal{H}_{KL} - \frac{1}{2}\,\mathcal{H}^{IJ}\,\partial_I\mathcal{H}^{KL}\,\partial_K\mathcal{H}_{JL}\,.$$

$$\boxed{\tilde{\partial}^i = 0}$$

parametrization:

$$(\mathcal{H}_{IJ}) = \begin{pmatrix} (G - B\,G^{-1}\,B)_{ij} & B_{ik}\,G^{kj} \\ -G^{ik}\,B_{kj} & G^{ij} \end{pmatrix}, \qquad \mathrm{e}^{-2d} = \mathrm{e}^{-2\Phi}\,\sqrt{|G|}$$

equivalent

$$\Downarrow$$

$$\mathcal{L} = \sqrt{|G|}\,\mathrm{e}^{-2\Phi}\left(R + 4\,|\partial\Phi|^2 - \frac{1}{12}\,|H_3|^2\right) + \partial_i\bigl(4\sqrt{|G|}\,\mathrm{e}^{-2\Phi}\,G^{ij}\partial_j\Phi\bigr)\,.$$

Strong constraint

In fact, $\boxed{\tilde{\partial}^i = 0}$ is always required (up to O(*d*,*d*) rotation) by the consistency of the theory.

Generalized diffeo. in the doubled space is generated by generalized Lie derivative:

[Siegel '93;
Hull, Zwiebach '09]

$$\hat{\pounds}_V W^I = V^K \partial_K W^I - W^K \partial_K V^I + W^K \partial^I V_K \,.$$

The strong constraint must be satisfied if we require

1. generalized diffeo. is the gauge symmetry of DFT
2. gauge algebra is closed

strong constraint $\eta^{IJ}\,\partial_I\,\partial_J\,(\text{anything}) = 0$ $\quad (\eta^{IJ}) = \begin{pmatrix} 0 & \delta^i_j \\ \delta^j_i & 0 \end{pmatrix}$

From this, the fields/gauge parameters can depend only on a half of the doubled coordinates.

Generalized Diffeomorphism

Let us recall the meaning of the generalized diffeo.

$$\delta_V \mathcal{H}_{IJ} = \hat{\pounds}_V \mathcal{H}_{IJ} = V^K \partial_K \mathcal{H}_{IJ} + (\partial_I V^K - \partial^K V_I)\,\mathcal{H}_{KJ} + (\partial_J V^K - \partial^K V_J)\,\mathcal{H}_{IK}\,.$$

$$\boxed{\tilde{\partial}^i = 0}$$

$$(\mathcal{H}_{IJ}) \equiv \begin{pmatrix} (G - B\,G^{-1}\,B)_{ij} & B_{ik}\,G^{kj} \\ -G^{ik}\,B_{kj} & G^{ij} \end{pmatrix}\,.$$

$$V^I = (v^i,\ \tilde{v}_i)$$

$$\boxed{\begin{cases} \delta_V G_{ij} & = \pounds_v G_{ij} \\ \delta_V B_{ij} & = \pounds_v B_{ij} + (\partial_i \tilde{v}_j - \partial_j \tilde{v}_i)\,. \end{cases}}$$

Generalized diffeo. = **conventional diffeo.**
B-field gauge transf.

$$\boxed{\tilde{\partial}^i = 0}$$

Short summary: DFT

Gravitational theory
on the doubled space $(x^I) = (x^i,\, \tilde{x}_i)$

Fund. fields:

$$(\mathcal{H}_{IJ}) = \begin{pmatrix} (G - B\,G^{-1}\,B)_{ij} & B_{ik}\,G^{kj} \\ -G^{ik}\,B_{kj} & G^{ij} \end{pmatrix}, \qquad \mathrm{e}^{-2d} = \mathrm{e}^{-2\Phi}\,\sqrt{|G|}\,.$$

$$\mathcal{L}_{\rm DFT} \xrightarrow[\boxed{\tilde{\partial}^i = 0}]{\text{strong constraint}} \mathcal{L} = \sqrt{|G|}\,\mathrm{e}^{-2\Phi}\left(R + 4\,|\partial\Phi|^2 - \frac{1}{12}\,|H_3|^2\right).$$

$$2d \text{ dim. generalized diffeo. } \underset{\boxed{\tilde{\partial}^i = 0}}{=} \left\{\begin{array}{l} d \text{ dim. diffeo.} \\ \text{gauge sym. of } B_2 \end{array}\right.$$

Double Field Theory is Necessary

in order to describe

- **Non-Riemannian background** **[J.-H. Park, K. Lee '13]**
  obtained by performing *T*-dualities
  in the F-string background:

$$(\mathcal{H}_{IJ}) = \begin{pmatrix} (G - B\,G^{-1}\,B)_{ij} & B_{ik}\,G^{kj} \\ -G^{ik}\,B_{kj} & \boxed{G^{ij}} \end{pmatrix}$$

= 0

  $\mathcal{H}_{IJ}$ is non-singular but $(G_{ij},\, B_{ij})$ are singular.

  ⇨ conventional supergravity doesn't work
  and **DFT is necessary**.

- **Non-geometric backgrounds**

  [related to Minkyu and Kimura-san's talk]

- **Solutions of generalized supergravity**

  [Yoshida-san's talk]

Non-geometric backgrounds

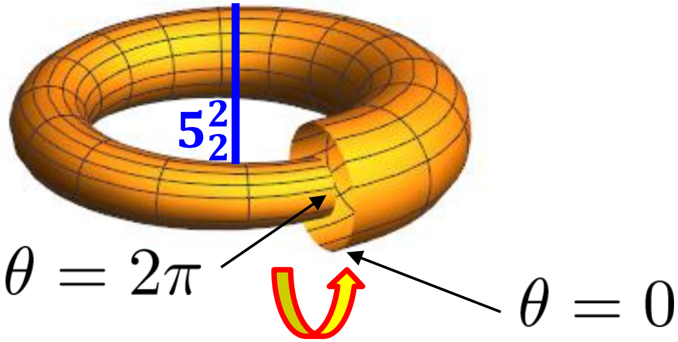
$5_2^2\,(34567,89)$

[Lozano-Tellechea, Ortin (2000)]

$$ds^2 = H(r)(dr + r^2 d\theta^2) + dx^2_{034567} + \frac{H(r)}{K(r,\theta)}\,dx^2_{89}\,,$$

$$B_2 = -\frac{\sigma\,\theta}{K(r,\theta)}\,dx^8 \wedge dx^9\,, \quad e^{2\Phi} = \frac{H(r)}{K(r,\theta)}\,, \quad K(r,\theta) \equiv H^2(r) + \sigma^2\,\theta^2\,.$$

metric and B-field on the 8-9 torus are not single-valued!



This background cannot be described globally in the SUGRA (non-geometric).

We can patch these tori with *T*-duality

[de Boer, Shigemori '10; '12]

⇨ 5_2^2 **background is a *T*-fold!**

Non-geometric *T*-folds are well-described in DFT.

Solutions of generalized supergravity

Yang-Baxter deformation

[Yoshida-san's talk]

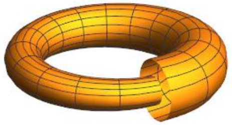
Solutions of generalized SUGRA

Solution of modified DFT
[arXiv:1611.05856,
YS, S. Uehara, K. Yoshida]

Solutions of DFT (with a non-standard section)
[arXiv:1703.xxxxx, J. Sakamoto, YS, S. Uehara, K. Yoshida]

These are also *T*-folds *!*

RR-fields are also twisted by the monodromy*!*



Solutions of generalized SUGRA are solutions of DFT *!*

There are various non-trivial backgrounds in string theory:

- **Non-Riemannian background**
- **Non-geometric backgrounds**
- **Solutions of generalized SUGRA**

These backgrounds cannot be described in the conventional SUGRA, but **these are well-described in DFT**.

U-duality covariant generalization

NS-NS sector of supergravity on T^d

$$(x^I) = (x^i,\ \tilde{x}_i)$$

winding coordinates

P ⟺ F1

***T*-dual**

M-theory/11D supergravity on T^d

$$(x^I) = (x^i,\ y_{i_1 i_2},\ y_{i_1\cdots i_5},\ \cdots)$$

[Duff, Lu '90; West '03]

P ⟺ M2 ⟺ M5 ⟺ …

***U*-dual *U*-dual *U*-dual**

We introduce winding coordinates for all branes which are related by *U*-duality transformations

Exceptional space

$$(x^I) = (\overset{\text{P}}{x^i},\ \overset{\text{M2}}{y_{i_1 i_2}},\ \overset{\text{M5}}{y_{i_1\cdots i_5}},\ \overset{\text{KKM}}{y_{i_1\cdots i_7,i}},\ \cdots)$$

$\underline{d=4}$	$4+{}_4\mathrm{C}_2$	✖	✖	$=\ 10$
$\underline{d=5}$	$5+{}_5\mathrm{C}_2$	$+\ {}_5\mathrm{C}_5$	✖	$=\ 16$
$\underline{d=6}$	$6+{}_6\mathrm{C}_2$	$+\ {}_6\mathrm{C}_5$	✖	$=\ 27$
$\underline{d=7}$	$7+{}_7\mathrm{C}_2$	$+\ {}_7\mathrm{C}_5$	$+\ \ 7$	$=\ 56$
$\underline{d=8}$	$8+{}_8\mathrm{C}_2$	$+\ {}_8\mathrm{C}_5$	$+\ \boxed{\cdots}$	$=\ 248$

exotic!

We introduce an "exceptional space"
whose dimensions are those of the fund. reps. of E_d .

***U*-duality group**

Exceptional Field Theory

Generalized metric
of the exceptional space :

[Duff, Lu '90; Berman, Perry '11; Berman, Godazgar, Perry, West '12; ….]

$$\mathcal{M}_{IJ} = E^A{}_I \, E^B{}_J \, \delta_{AB} \,, \qquad (E^A{}_I) = \begin{pmatrix} e_i^a & 0 & \cdots \\ \frac{1}{\sqrt{2}}(e^{-1})_a^{i_1 i_2} \, C_{i_1 i_2 j} & (e^{-1})^{i_1 i_2}_{a_1 a_2} & \cdots \\ \cdots C_{i_1 \cdots i_5 j} \cdots & \cdots & \cdots \end{pmatrix} .$$

$$\mathcal{L}_{\rm EFT} \longrightarrow \mathcal{L}_{\rm 11d\text{-}SUGRA}$$

(*U*-duality inv. 2-derivative action)

[Hohm, Samtleben '13]

$$\frac{\partial}{\partial y_{i_1 i_2}} = 0 \,, \qquad \frac{\partial}{\partial y_{i_1 \cdots i_5}} = 0$$

★ **generalized diffeo. =**

- ***d*-dim. diffeo.**
- **Gauge sym. of C_3 , C_6 .**

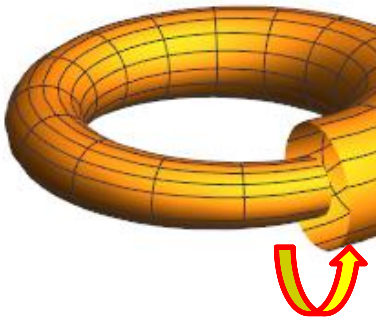
$$(V^I) = \begin{pmatrix} v^i \\ \frac{1}{\sqrt{2}} \, \tilde{v}_{i_1 i_2} \\ \frac{1}{\sqrt{5!}} \, \tilde{v}_{i_1 \cdots i_5} \\ \vdots \end{pmatrix}$$

$$\frac{\partial}{\partial y_{i_1 i_2}} = 0 \,, \qquad \frac{\partial}{\partial y_{i_1 \cdots i_5}} = 0$$

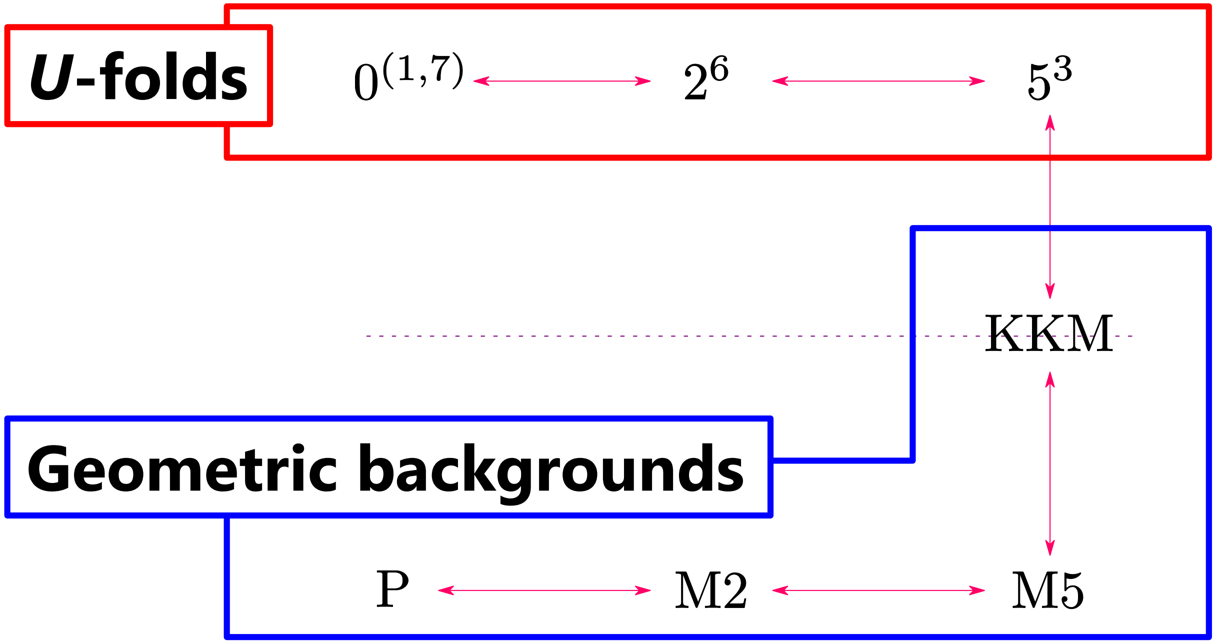
***U*-folds**

In M-theory, there are many non-geometric *U*-folds.

[de Boer, Shigemori '12]



***U*-duality monodromy**



***U*-folds**

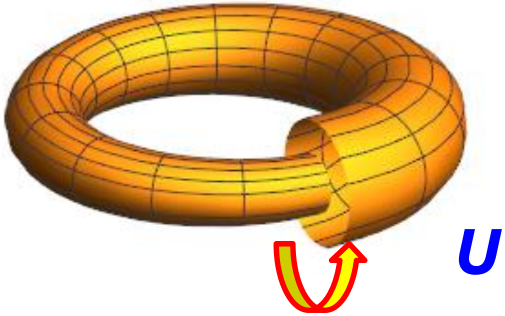
$0^{(1,7)} \longleftrightarrow 2^6 \longleftrightarrow 5^3$

KKM

Geometric backgrounds

$\mathrm{P} \longleftrightarrow \mathrm{M2} \longleftrightarrow \mathrm{M5}$

U-folds



U-duality monodromy
∈ gauge symmetry of EFT

From EFT point of view,
background fields are single-valued
(up to gauge transf.)

[K. Lee, S.-J. Rey, YS, arXiv:1612.08738]

We studied monodromies of exotic-brane backgrounds
from EFT point of view.

Double/Exceptional Field Theory

supergravity

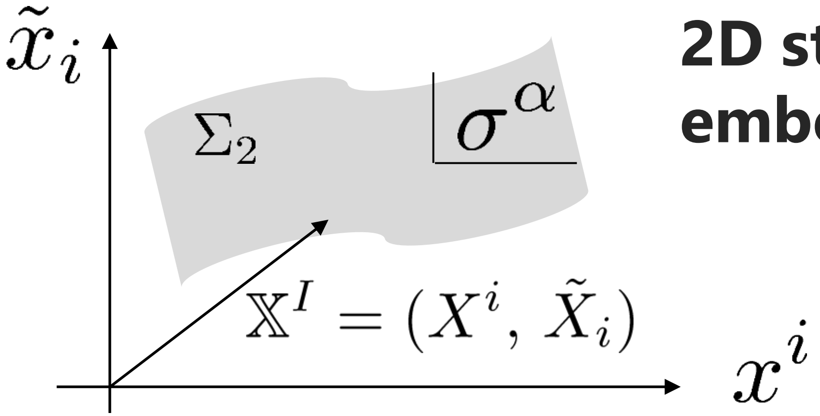
↓

double sigma model/our model

string sigma model/
worldvolume theory for a *p*-brane

Double Sigma Model

[Hull '04; '06; Lee, Park '13]



2D string **worldsheet** embedded into the **doubled space**

$$(\mathcal{P}^I(\sigma)) = \begin{pmatrix} \mathrm{d}X^i(\sigma) \\ \mathrm{d}\tilde{X}_i(\sigma) + C_i(\sigma) \end{pmatrix} .$$

$$\updownarrow$$

$$DX^I = \mathrm{d}X^I - \mathcal{A}^I$$

$$S = \frac{1}{4}\int_{\Sigma_2}\left[\mathcal{H}_{IJ}\,\mathcal{P}^I \wedge *_\gamma \mathcal{P}^J + \Omega_{IJ}\,\mathcal{P}^I \wedge \mathcal{P}^J\right] \qquad (\Omega_{IJ}) = \begin{pmatrix} 0 & \delta^j_i \\ -\delta^i_j & 0 \end{pmatrix}$$

$$= \frac{1}{4}\int_{\Sigma_2}\left[\mathcal{H}_{IJ}\,\mathcal{P}^I \wedge *_\gamma \mathcal{P}^J - 2\,(\mathrm{d}\tilde{X}_i + C_i) \wedge \mathrm{d}X^i\right] .$$

local symmetry

$$\left\{ \begin{array}{l} \tilde{X}_i(\sigma) \to \tilde{X}_i(\sigma) + \tilde{v}_i(\sigma) \\ C_i(\sigma) \to C_i(\sigma) - \mathrm{d}\tilde{v}_i(\sigma) \end{array} \right.$$

coordinate gauge symmetry

[Jeong-Hyuck's talk]

Double Sigma Model

[Hull '04; '06; Lee, Park '13]

$$S = \frac{1}{4}\int_{\Sigma_2}\Big[\mathcal{H}_{IJ}\,\mathcal{P}^I \wedge *_\gamma \mathcal{P}^J - 2\,(\mathrm{d}\tilde{X}_i + C_i) \wedge \mathrm{d}X^i\Big]\,.$$

e.o.m. for C_i

$$\mathrm{d}\tilde{X}_i + C_i = G_{ik} *_\gamma \mathrm{d}X^k + B_{ik}\,\mathrm{d}X^k$$

$$S = \frac{1}{2}\int_{\Sigma_2}\big(G_{ij}\,\mathrm{d}X^i \wedge *_\gamma \mathrm{d}X^j + B_{ij}\,\mathrm{d}X^i \wedge \mathrm{d}X^j\big)\,.$$

(classically) equivalent to the conventional sigma model

Winding coordinates $\tilde{X}_i$ disappeared from the action

not independent of X^i

Gauge fixing

In fact, we can always set $\tilde{X}_i(\sigma) = 0$
by using the local symmetry,

$$\left\{\begin{aligned} &\tilde{X}_i(\sigma) \to \tilde{X}_i(\sigma) + \tilde{v}_i(\sigma) \\ &C_i(\sigma) \to C_i(\sigma) - \mathrm{d}\tilde{v}_i(\sigma) \end{aligned}\right.$$

$$(\mathcal{P}^I) = \begin{pmatrix} \mathrm{d}X^i \\ \cancel{\mathrm{d}\tilde{X}_i} + C_i \end{pmatrix} = \begin{pmatrix} \mathrm{d}X^i \\ \mathcal{P}_i \end{pmatrix} .$$

$$S = \frac{1}{4} \int_{\Sigma_2} \left[\mathcal{H}_{IJ}\, \mathcal{P}^I \wedge *_\gamma \mathcal{P}^J - \Omega_{IJ}\, \mathcal{P}^I \wedge \mathcal{P}^J \right] .$$

Jeong-Hyuck's DSM [Lee, Park '13]

Hull's action ("doubled everything")

$$S = \frac{1}{4}\int_{\Sigma_2}\Big[\mathcal{H}_{IJ}\,\mathcal{P}^I \wedge *_\gamma \mathcal{P}^J - 2\,(\mathrm{d}\tilde{X}_i + C_i) \wedge \mathrm{d}X^i\Big]\,.$$

$$+\frac{1}{2}\int_{\Sigma_2}\mathrm{d}\tilde{X}_i \wedge \mathrm{d}X^i$$

Jeong-Hyuck's action

$$S = \frac{1}{4}\int_{\Sigma_2}\Big[\mathcal{H}_{IJ}\,DX^I \wedge *_\gamma DX^J - 2\,C_i \wedge \mathrm{d}X^i\Big]$$

$$= \frac{1}{4}\int_{\Sigma_2}\Big[\mathcal{H}_{IJ}\,DX^I \wedge *_\gamma DX^J - 2\,DX^I \wedge \mathcal{A}_I\Big]\,.$$

$$\big(\mathcal{P}^I(\sigma)\big) = \begin{pmatrix}\mathrm{d}X^i(\sigma) + & 0\\ \mathrm{d}\tilde{X}_i(\sigma) + & C_i(\sigma)\end{pmatrix} = \mathrm{d}X^I - \mathcal{A}^I \equiv DX^I\,.$$

Symmetry of the action

$$S = \frac{1}{4}\int_{\Sigma_2}\Big[\underbrace{\mathcal{H}_{IJ}\,\mathcal{P}^I\wedge *_\gamma \mathcal{P}^J}_{\text{O}(d,d)\text{ inv.}} + \underbrace{\Omega_{IJ}\,\mathcal{P}^I\wedge\mathcal{P}^J}_{\cancel{\text{O}(d,d)\text{ inv.}}}\Big] \qquad (\mathcal{P}^I) = \begin{pmatrix} \mathrm{d}X^i \\ \mathcal{P}_i \end{pmatrix}$$

$$(\Omega_{IJ}) = \begin{pmatrix} 0 & \delta_i^j \\ -\delta_j^i & 0 \end{pmatrix}$$

this constant matrix is NOT O(*d*,*d*) invariant

We modify the action such that
the action is inv. under the generalized diffeo.
on the target doubled space.

Untwisted vector

We prepare a 2-form $b_{ij}(x)$ in the target doubled space, which transforms in the same way as the B-field under generalized diffeo.:

$$\begin{cases} \delta_V B_{ij} &= \pounds_v B_{ij} + (\partial_i \tilde{v}_j - \partial_j \tilde{v}_i)\,. \\ \delta_V b_{ij} &= \pounds_v b_{ij} + (\partial_i \tilde{v}_j - \partial_j \tilde{v}_i)\,. \end{cases}$$

Untwisted vector **[Hull '14]**

$$(\hat{\mathcal{P}}^I) \equiv \begin{pmatrix} \mathrm{d}X^i \\ \hat{\mathcal{P}}_i \end{pmatrix} \equiv \begin{pmatrix} \delta_i^j & 0 \\ -b_{ij} & \delta_j^i \end{pmatrix} \begin{pmatrix} \mathrm{d}X^j \\ \mathcal{P}_j \end{pmatrix} = \begin{pmatrix} \mathrm{d}X^i \\ \mathcal{P}_i - b_{ij}\,\mathrm{d}X^j \end{pmatrix}$$

⇨ $\Omega_{IJ}\,\hat{\mathcal{P}}^I \wedge \hat{\mathcal{P}}^J$ **is invariant under generalized diffeo.**

Our DSM action

$$S = \frac{1}{4}\int_{\Sigma_2}\Big[\mathcal{H}_{IJ}\,\mathcal{P}^I\wedge *_\gamma\mathcal{P}^J + \Omega_{IJ}\,\mathcal{P}^I\wedge\mathcal{P}^J\Big]$$

⇓

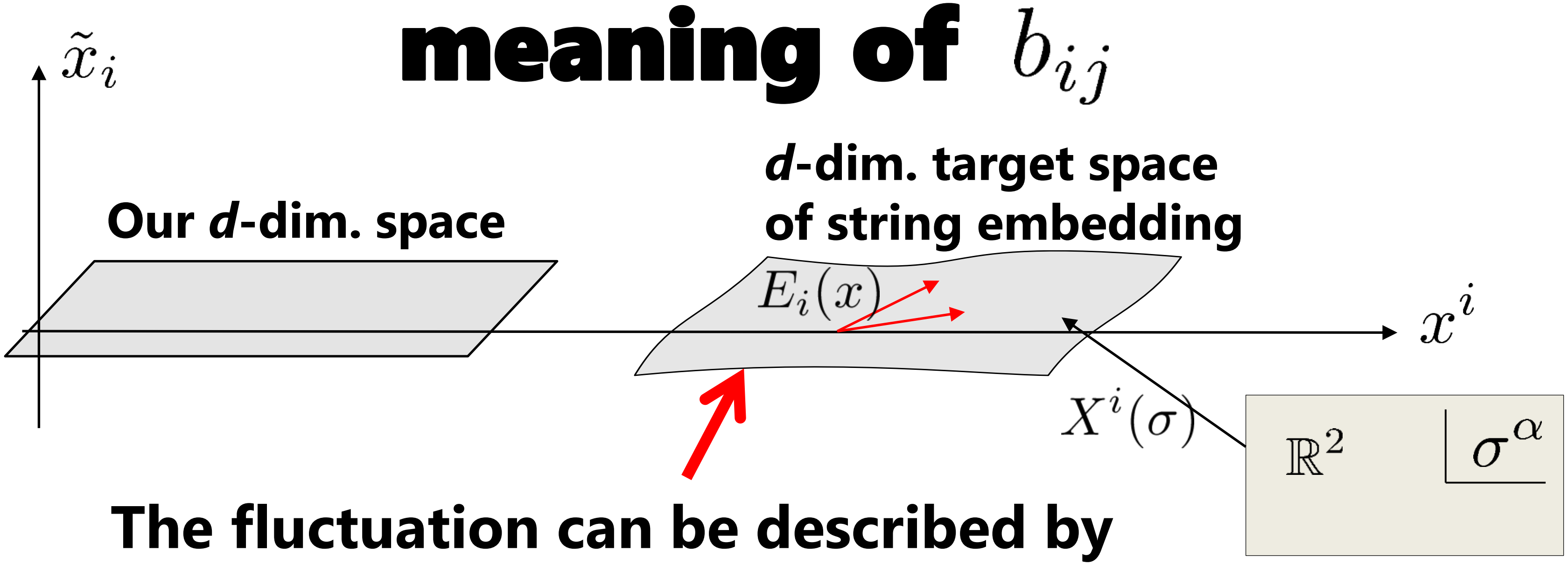
$$S = \frac{1}{4}\int_{\Sigma_2}\Big[\mathcal{H}_{IJ}\,\mathcal{P}^I\wedge *_\gamma\mathcal{P}^J + \Omega_{IJ}\,\hat{\mathcal{P}}^I\wedge\hat{\mathcal{P}}^J\Big]$$

inv. under generalized diffeo.
- **diffeo.**
- **B-field gauge transf.**

$$\begin{aligned}\frac{1}{2}\Omega_{IJ}\,\hat{\mathcal{P}}^I\wedge\hat{\mathcal{P}}^J &= \mathrm{d}X^i\wedge(\mathcal{P}_i - b_{ij}\,\mathrm{d}X^j)\\ &= \mathrm{d}X^i\wedge\mathcal{P}_i - 2\times\underbrace{\frac{1}{2}b_{ij}\,\mathrm{d}X^i\wedge\mathrm{d}X^j}_{F_2(\sigma)}\end{aligned}$$

new

meaning of b_{ij}



The fluctuation can be described by

frame fields : $E_i(x) = \partial_i + {\color{red} b_{ij}(x)}\, \tilde{\partial}^j\,.$

b_{ij} **has to satisfy**
some conditions
in order to define
a "good" *d*-dim. space

[S.-J. Rey, YS '15]

$$\left\{\begin{array}{l} b_{ij}(x) = b_{[ij]}(x)\,, \quad \mathrm{d}b_2 = 0(x)\,, \\ \delta_V b_{ij}(x) = \pounds_v b_{ij}(x) + \partial_i \tilde{v}_j - \partial_j \tilde{v}_i\,. \end{array}\right.$$

$$F_2(\sigma) = \frac{1}{2}\, b_{ij}(X(\sigma))\, \mathrm{d}X^i \wedge \mathrm{d}X^j\,,$$

$$F_2(\sigma) = \mathrm{d}A_1(\sigma)$$

short summary

auxiliary fields

$$F_2(\sigma) = \mathrm{d}A_1(\sigma)\,.$$

Fund. fields: $\{X^i(\sigma)\,,\ \mathcal{P}_i(\sigma)\,,\ \gamma_{\alpha\beta}(\sigma)\,,\ A_1(\sigma)\}$

Conventional
embedding function

intrinsic
metric

fluctuation along
winding directions

Our DSM:

$$S = \frac{1}{2}\int_{\Sigma_2}\left[\frac{1}{2}\mathcal{H}_{IJ}(X(\sigma))\,\mathcal{P}^I(\sigma)\wedge *_\gamma \mathcal{P}^J(\sigma) - \Omega_2(\sigma)\right].$$

$$(\mathcal{P}^I) = \begin{pmatrix} \mathrm{d}X^i \\ \mathcal{P}_i \end{pmatrix}\,, \qquad \Omega_2(\sigma) = \mathcal{P}_i(\sigma)\wedge \mathrm{d}X^i(\sigma) + 2\,F_2(\sigma)\,.$$

$$F_2(\sigma) = \mathrm{d}A_1(\sigma)\,.$$

Reproducing the conventional action

$$S = \frac{1}{2}\int_{\Sigma_2}\Big[\,\frac{1}{2}\,\mathcal{H}_{IJ}(X(\sigma))\,\mathcal{P}^I(\sigma)\wedge *_\gamma\mathcal{P}^J(\sigma) - \Omega_2(\sigma)\Big]\,.$$

$$\boxed{\Omega_2(\sigma) = \mathcal{P}_i(\sigma)\wedge \mathrm{d}X^i(\sigma) + 2\,F_2(\sigma)}$$

eliminating the auxiliary field $\mathcal{P}_i$

invariant under B-field gauge transf.

$$\begin{aligned} S &= \frac{1}{2}\int_{\Sigma_2} G_{ij}\,\mathrm{d}X^i\wedge *_\gamma\mathrm{d}X^j + \int_{\Sigma_2}\overbrace{(B_2 - F_2)} \\ &= \frac{1}{2}\int_{\Sigma_2} G_{ij}\,\mathrm{d}X^i\wedge *_\gamma\mathrm{d}X^j + \int_{\Sigma_2} B_2 + \int_{\partial\Sigma_2} A_1\,. \end{aligned}$$

Generalization to other branes

Our action for a *p*-brane:

$$S = \int_{\Sigma_{p+1}} \left[\frac{1}{2} \mathcal{M}_{IJ}(X(\sigma))\, \mathcal{P}^{I}(\sigma) \wedge *\mathcal{P}^{J}(\sigma) - \Omega_{p+1}(\sigma) \right] .$$

$$\Omega_{p+1}(\sigma) = \frac{1}{p!} \mathcal{P}_{i_1 \cdots i_p}(\sigma) \wedge \mathrm{d}X^{i_1}(\sigma) \wedge \cdots \wedge \mathrm{d}X^{i_p}(\sigma) + \cdots .$$

equivalent to known worldvolume theory

for
$\begin{cases} \textbf{membrane} \\ \textbf{M5-brane} \end{cases}$

gauge fields
are introduced
(fluctuations along the dual directions)

M-theory branes in Exceptional space

(for simplicity, let's consider E_6 case)

generalized coordinates

$$(x^I) = (x^i,\, y_{i_1 i_2},\, y_{i_1\cdots i_5})\,. \quad \Longleftrightarrow \quad (\mathcal{P}^I) = \begin{pmatrix} \mathrm{d}X^i \\ \boxed{\begin{array}{c} \frac{1}{\sqrt{2}}\,\mathcal{P}_{i_1 i_2} \\ \frac{1}{\sqrt{5!}}\,\mathcal{P}_{i_1\cdots i_5} \end{array}} \end{pmatrix}$$

27 dim = 6 + 15 + 6

15+6 auxiliary fields

M2 and M5 can wrap the torus

$$(\mathcal{P}^I) = \begin{pmatrix} \mathrm{d}X^i \\ \mathcal{P}_i \end{pmatrix}$$

DSM

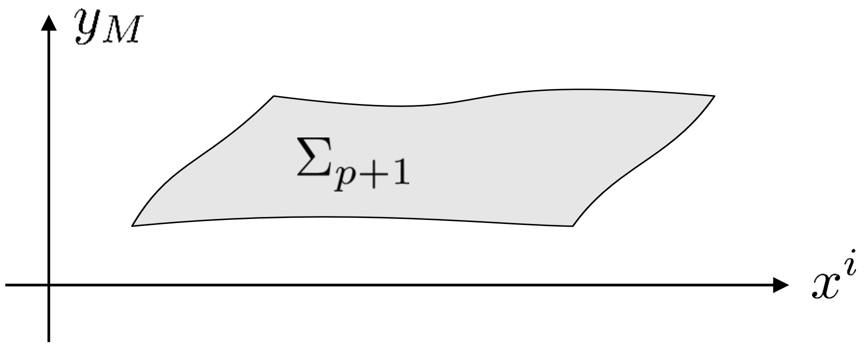
Generalized metric : $\mathcal{M}_{IJ} = L^K{}_I\, \hat{\mathcal{M}}_{KL}\, L^L{}_J$

$$(\hat{\mathcal{M}}_{IJ}) \equiv \begin{pmatrix} G_{ij} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & G^{i_1 i_2, j_1 j_2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & G^{i_1\cdots i_5, j_1\cdots j_5} \end{pmatrix},$$

$$(L^I{}_J) \equiv \begin{pmatrix} \delta^i_j & \mathbf{0} & \mathbf{0} \\ \frac{1}{\sqrt{2}}\, C_{i_1 i_2 j} & \delta^{j_1 j_2}_{i_1 i_2} & \mathbf{0} \\ -\frac{1}{\sqrt{5!}}\,(C_{i_1\cdots i_5 j} - 5\, C_{[i_1 i_2 i_3} C_{i_4 i_5] j}) & \frac{10\sqrt{2}}{\sqrt{5!}}\, \delta^{j_1 j_2}_{[i_1 i_2}\, C_{i_3 i_4 i_5]} & \delta^{j_1\cdots j_5}_{i_1\cdots i_5} \end{pmatrix}.$$

$$\delta^{j_1\cdots j_q}_{i_1\cdots i_q} \equiv \delta^{j_1}_{[i_1} \cdots \delta^{j_q}_{i_q]}\,, \qquad G^{i_1\cdots i_q, j_1\cdots j_q} \equiv \delta^{i_1\cdots i_q}_{k_1\cdots k_q}\, G^{k_1 j_1} \cdots G^{k_q j_q}\,.$$

Brane action



Consider a *p*-brane (*p*=2 or 5) embedded into the E_6 exceptional space

$$S = \frac{1}{p+1}\int_{\Sigma_{p+1}}\left(\underbrace{\frac{1}{2}\,\mathcal{M}_{IJ}\,\mathcal{P}^I\wedge *_\gamma\mathcal{P}^J}_{\text{U-duality inv.}} - \Omega_{p+1}\right). \qquad (\mathcal{P}^I) = \begin{pmatrix} \mathrm{d}X^i \\ \frac{1}{\sqrt{2}}\,\mathcal{P}_{i_1 i_2} \\ \frac{1}{\sqrt{5!}}\,\mathcal{P}_{i_1\cdots i_5} \end{pmatrix}\begin{array}{l} \\ \textbf{M2} \\ \textbf{M5} \end{array}$$

$$\begin{array}{ll} \textbf{M2} & \Omega_3 \equiv \frac{1}{2}\,\mathcal{P}_{i_1 i_2}\wedge \mathrm{d}X^{i_1 i_2} + 3\,F_3\,, \\ \textbf{M5} & \Omega_6 \equiv \frac{1}{5!}\,\mathcal{P}_{i_1\cdots i_5}\wedge \mathrm{d}X^{i_1\cdots i_5} + \frac{1}{2}\,\mathcal{P}_{i_1 i_2}\wedge \mathrm{d}X^{i_1 i_2}\wedge F_3 + 6\,F_6\,. \end{array}$$

$$\mathrm{d}X^{i_1\cdots i_p} \equiv \mathrm{d}X^{i_1}\wedge\cdots\wedge \mathrm{d}X^{i_p}$$

$$F_3 = \mathrm{d}A_2 \qquad F_6 = \mathrm{d}A_5$$

Fund. fields: $\{\gamma_{\alpha\beta}(\sigma)\,,\ X^i(\sigma)\,,\ \mathcal{P}_{i_1 i_2}(\sigma)\,,\ \mathcal{P}_{i_1\cdots i_5}(\sigma)\,,\ A_2(\sigma)\,,\ A_5(\sigma)\}$

***p*=2 (1/2)**

Our action for a membrane :

$$S=\frac{1}{3}\int_{\Sigma}\left(\frac{1}{2}\,\underbrace{\mathcal{M}_{IJ}\,\mathcal{P}^{I}\wedge *_{\gamma}\mathcal{P}^{J}}\;-\;\Omega_3\right)$$

$$\Omega_3=\frac{1}{2}\,\mathcal{P}_{i_1 i_2}\wedge \mathrm{d}X^{i_1}\wedge \mathrm{d}X^{i_2}+3\,F_3$$

$$G_{ij}\,\mathrm{d}X^{i}\wedge *\mathrm{d}X^{j}+\frac{1}{2!}\,G^{i_1 i_2,j_1 j_2}\,(\mathcal{P}_{i_1 i_2}+\star_{i_1 i_2 k}\,\mathrm{d}X^{k})\wedge *_{\gamma}(\mathcal{P}_{j_1 j_2}+\star_{j_1 j_2 l}\,\mathrm{d}X^{l})$$

$$+\frac{1}{5!}\,G^{i_1\cdots i_5,j_1\cdots j_5}\,(\mathcal{P}_{i_1\cdots i_5}+\star_{[i_1 i_2 i_3}\,\mathcal{P}_{i_4 i_5]}+\star_{i_1\cdots i_5 k}\,\mathrm{d}X^{k})\wedge *_{\gamma}(\mathcal{P}_{j_1\cdots j_5}+\star_{[j_1 j_2 j_3}\,\mathcal{P}_{j_4 j_5]}+\star_{j_1\cdots j_5 l}\,\mathrm{d}X^{l})$$

$\Downarrow$ **eliminate the auxiliary fields** $\mathcal{P}_{i_1 i_2}$, $\mathcal{P}_{i_1\cdots i_5}$

C_6 **appears only here (⇨ disappears !)**

$$S=\frac{1}{6}\int_{\Sigma_3}\left[G_{ij}\,\mathrm{d}X^{i}\wedge *_{\gamma}\mathrm{d}X^{j}+\frac{1}{2}\,G_{i_1 i_2,j_1 j_2}\,\mathrm{d}X^{i_1}\wedge \mathrm{d}X^{i_2}\wedge *_{\gamma}(\mathrm{d}X^{j_1}\wedge \mathrm{d}X^{j_2})\right]$$

$$+\int_{\Sigma_3}(C_3-F_3)\,.$$

p=2 (2/2)

$$S = \frac{1}{6}\int_{\Sigma_3}\Big[G_{ij}\,\mathrm{d}X^i\wedge *_\gamma\mathrm{d}X^j + \frac{1}{2}G_{i_1i_2,j_1j_2}\,\mathrm{d}X^{i_1}\wedge\mathrm{d}X^{i_2}\wedge *_\gamma(\mathrm{d}X^{j_1}\wedge\mathrm{d}X^{j_2})\Big] + \int_{\Sigma_3}(C_3 - F_3)\,.$$

⇕ **apparently different from the well-known action**

membrane action **[Bergshoeff, Sezgin, Townsend '87]**

$$\boxed{S = \frac{1}{2}\int_{\Sigma_3}\left(G_{ij}\,\mathrm{d}X^i\wedge *_\gamma\mathrm{d}X^j - *_\gamma 1\right) + \int_{\Sigma_3} C_3\,.}$$

e.o.m. **for** $\gamma_{\alpha\beta}$ **,**

$$h_{\alpha\beta} \equiv G_{ij}\,\partial_\alpha X^i\,\partial_\beta X^j = \frac{\det h}{\det\gamma}(\gamma h^{-1}\gamma)_{\alpha\beta} \quad\Rightarrow\quad \gamma_{\alpha\beta} = h_{\alpha\beta}\,.$$

eliminate $\gamma_{\alpha\beta}$ ⇒

$$\boxed{S = -\int_{\Sigma_3}\mathrm{d}^3\sigma\sqrt{-\det h} + \int_{\Sigma_3} C_3 - \int_{\partial\Sigma_3} A_2\,.}$$

(classically) equivalent!

Comment

We here considered E_6 case, but for, E_7 or E_8 ,

$$(\mathcal{P}^I) = \begin{pmatrix} \mathrm{d}X^i \\ \frac{1}{\sqrt{2}}\,\mathcal{P}_{i_1 i_2} \\ \frac{1}{\sqrt{5!}}\,\mathcal{P}_{i_1\cdots i_5} \\ \vdots \end{pmatrix}$$

additional auxiliary fields appear !

$$S = \frac{1}{3}\int_\Sigma \left(\frac{1}{2}\,\mathcal{M}_{IJ}\,\mathcal{P}^I \wedge *_\gamma \mathcal{P}^J - \Omega_3 \right)$$

$$\Omega_3 = \frac{1}{2}\,\mathcal{P}_{i_1 i_2} \wedge \mathrm{d}X^{i_1} \wedge \mathrm{d}X^{i_2} + 3\,F_3$$

However, Ω_3 does not contain the new auxiliary fields

So, the additional auxiliary fields are simply integrated out, and the resulting action for X^i does not change.

We can reproduce the membrane action also for E_7 / E_8 .

p=5 (1/2)

$$(\mathcal{P}^I) = \begin{pmatrix} \mathrm{d}X^i \\ \frac{1}{\sqrt{2}}\,\mathcal{P}_{i_1 i_2} \\ \frac{1}{\sqrt{5!}}\,\mathcal{P}_{i_1\cdots i_5} \end{pmatrix}$$

Our action for a 5-brane :

$$S = \frac{1}{6}\int_{\Sigma_6}\Bigl(\frac{1}{2}\,\mathcal{M}_{IJ}\,\mathcal{P}^I \wedge *_\gamma \mathcal{P}^J - \Omega_6\Bigr)\,.$$

$$\Omega_6 = \frac{1}{5!}\,\mathcal{P}_{i_1\cdots i_5}\wedge \mathrm{d}X^{i_1\cdots i_5} + \frac{1}{2}\,\mathcal{P}_{i_1 i_2}\wedge \mathrm{d}X^{i_1 i_2}\wedge F_3 + 6\,F_6\,.$$

$$F_3 = \mathrm{d}A_2 \qquad F_6 = \mathrm{d}A_5$$

eliminate auxiliary rields

$$\mathcal{P}_{i_1 i_2},\ \mathcal{P}_{i_1\cdots i_5}\,.$$

$$S = -\frac{1}{12}\int_{\Sigma_6}\mathrm{d}^6\sigma\Bigl[\sqrt{-\gamma}\,\gamma^{\alpha\beta}\,h_{\alpha\beta} - \frac{\det h}{\sqrt{-\gamma}}\,\theta^\alpha{}_\beta\,(h^{-1}\gamma)^\beta{}_\alpha\Bigr]$$

$$+\int_{\Sigma_6}\Bigl(C_6 - \frac{1}{2}\,H_3\wedge C_3 - F_6\Bigr)\,,$$

gauge field appears in a non-trivial manner

$$h_{\alpha\beta} \equiv G_{ij}\,\partial_\alpha X^i\,\partial_\beta X^j\,,\qquad H_3 \equiv F_3 - C_3\,,$$

$$\theta^\alpha{}_\beta \equiv \Bigl[1 + \frac{\mathrm{tr}(H^2)}{6}\Bigr]\,\delta^\alpha_\beta - \frac{1}{2}\,(H^2)^\alpha{}_\beta\,.$$

p=5 (2/2)

eliminate $\gamma_{\alpha\beta}$ $\left(\gamma_{\alpha\beta} \neq h_{\alpha\beta} \textbf{!}\right)$

$$S = -\int_{\Sigma} \mathrm{d}^6\sigma \sqrt{-h}\, \frac{\mathrm{tr}(\theta^{\frac{1}{2}})}{6} + \int_{\Sigma} \left(C_6 - \frac{1}{2} H_3 \wedge C_3 - F_6\right).$$

This is not a known action for M5-brane.

However, if we consider
a weak-field approximation for H_3

$$S \sim -\int_{\Sigma} \mathrm{d}^6\sigma \sqrt{-h} + \frac{1}{4}\int_{\Sigma} H_3 \wedge *_h H_3 + \int_{\Sigma} \left(C_6 - \frac{1}{2} H_3 \wedge C_3 - F_6\right).$$

[Bergshoeff, de Roo, Ortin '96]

e.o.m. for A_2 ⟹ $\mathrm{d}(*_h H_3 - C_3) = \mathrm{d}(*_h H_3 + H_3) = 0\,.$

Consistent with the linearized self-duality relation:

$$H_3 = - *_h\, H_3\,.$$

Non-linear case?

Without the approximation, e.o.m. for A_2 becomes

$$\partial_\alpha \mathcal{E}^{\alpha\beta\gamma} = 0\,, \quad \mathcal{E}^{\alpha\beta\gamma} \equiv \frac{\partial \mathcal{L}}{\partial H_{\alpha\beta\gamma}}\,.$$

$$S = \int_{\Sigma_6} \mathrm{d}^6\sigma \mathcal{L} = \int_{\Sigma_6} \left[-\mathrm{d}^6\sigma \sqrt{-h}\, \frac{\mathrm{tr}(\theta^{\frac{1}{2}})}{6} + C_6 - \frac{1}{2} H_3 \wedge C_3 - F_6 \right].$$

$$\mathcal{E}^{\alpha\beta\gamma} \equiv \frac{\partial \mathcal{L}}{\partial H_{\alpha\beta\gamma}} = -\frac{1}{12}\left[\mathcal{C}^{[\alpha}{}_{\delta}\, H^{\beta\gamma]\delta} - (*_h C_3)^{\alpha\beta\gamma} \right].$$

indices raised/ lowered with $h_{\alpha\beta}$

$$\mathcal{C}_\alpha{}^\beta \equiv \frac{\mathrm{tr}(\theta^{-\frac{1}{2}})}{3}\, \delta_\alpha^\beta - (\theta^{-\frac{1}{2}})_\alpha{}^\beta\,.$$

Consistent with "non-linear self-duality relation"

$$\mathcal{C}_{[\alpha_1}{}^{\alpha}\, H_{\alpha_2\alpha_3]\alpha} = -(*_h H_3)_{\alpha_1\alpha_2\alpha_3}\,.$$

$\delta^\alpha_{\alpha_1}$ **weak field**

Known results

Our result:

$$\mathcal{C}_{[\alpha_1}{}^{\alpha}\, H_{\alpha_2\alpha_3]\alpha} = -(*_h H_3)_{\alpha_1\alpha_2\alpha_3}\,.$$

$$\mathcal{C}_\alpha{}^\beta \equiv \frac{\mathrm{tr}(\theta^{-\frac{1}{2}})}{3}\,\delta_\alpha^\beta - (\theta^{-\frac{1}{2}})_\alpha{}^\beta\,.$$

Known result: **[Howe, Sezgin '97; Howe, Sezgin, West '97; Sezgin, Sundell '98]**

$$C_{[\alpha_1}{}^{\alpha}\, H_{\alpha_2\alpha_3]\alpha} = -(*_h H_3)_{\alpha_1\alpha_2\alpha_3}$$

$$C_\alpha{}^\beta = K^{-1}\left\{\left[1+\frac{1}{12}\,\mathrm{tr}(H^2)\right]\delta_\alpha^\beta - \frac{1}{4}\,(H^2)_\alpha{}^\beta\right\}, \quad K \equiv \sqrt{1+\frac{\mathrm{tr}(H^2)}{24}}\,.$$

Apparently different, but in fact the same:

$$C_\alpha{}^\beta = \mathcal{C}_\alpha{}^\beta\,.$$ ⇨ **Consistent!**

e.o.m. for X^i also looks the same: $$\partial_\alpha\left(\sqrt{-h}\,C^{\alpha\beta}\,G_{ij}\partial_\beta X^j\right) = 0\,.$$

checking details...

YS, S. Uehara,
PRL 117, 191601 [arXiv:1607.04265]

YS, S. Uehara, ongoing work.

Kaluza-Klein Monopole

If we consider E_8 exceptional space,
we can also consider the Kaluza-Klein Monopole
wrapped on the 8-torus.

Kaluza-Klein Monopole requires
a Killing vector (Taub-NUT direction) k^i.

$$(\mathcal{P}^I) = \begin{pmatrix} DX^i \\ \frac{\mathcal{P}_{i_1 i_2}}{\sqrt{2!}} \\ \frac{\mathcal{P}_{i_1\cdots i_5}}{\sqrt{5!}} \\ \frac{\mathcal{P}_{i_1\cdots i_7, i}}{\sqrt{7!}} \\ \vdots \end{pmatrix}, \qquad DX^i \equiv \mathrm{d}X^i - a_1 k^i \, .$$

Kaluza-Klein Monopole

Our action:

worldvolume
gauge fields

$$S = \int_{\Sigma_7} \Big(\frac{1}{2} \mathcal{M}_{IJ} \mathcal{P}^I \wedge *\mathcal{P}^J + \frac{1}{6!} \mathcal{P}_{i_1\cdots i_7,\, i} \wedge DX^{i_1\cdots i_6} k^{i_7} k^i \boxed{+\cdots} \Big) .$$

very complicated !

Let's consider the metric only.

eliminating auxiliary fields

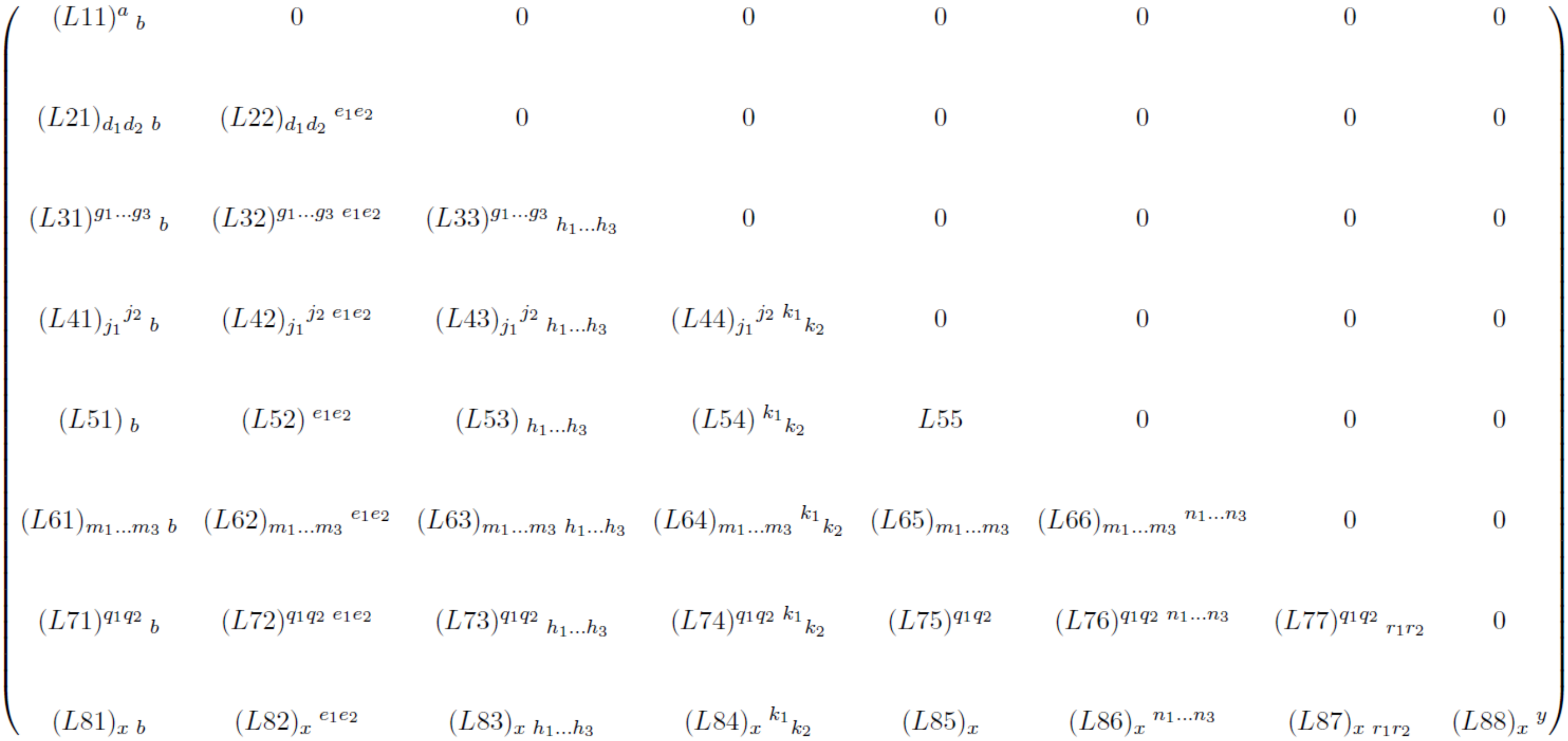
$$S = \int_{\Sigma_7} k^2 \sqrt{\left|\det(G_{ij} D_\alpha X^i D_\beta X^j)\right|}$$

**[Bergshoeff,
Janssen, Ortin '97]**

Part of the known action for KKM.

Generalized metric for E_8

[H. Godazgar, M. Godazgar, M. Perry '13]

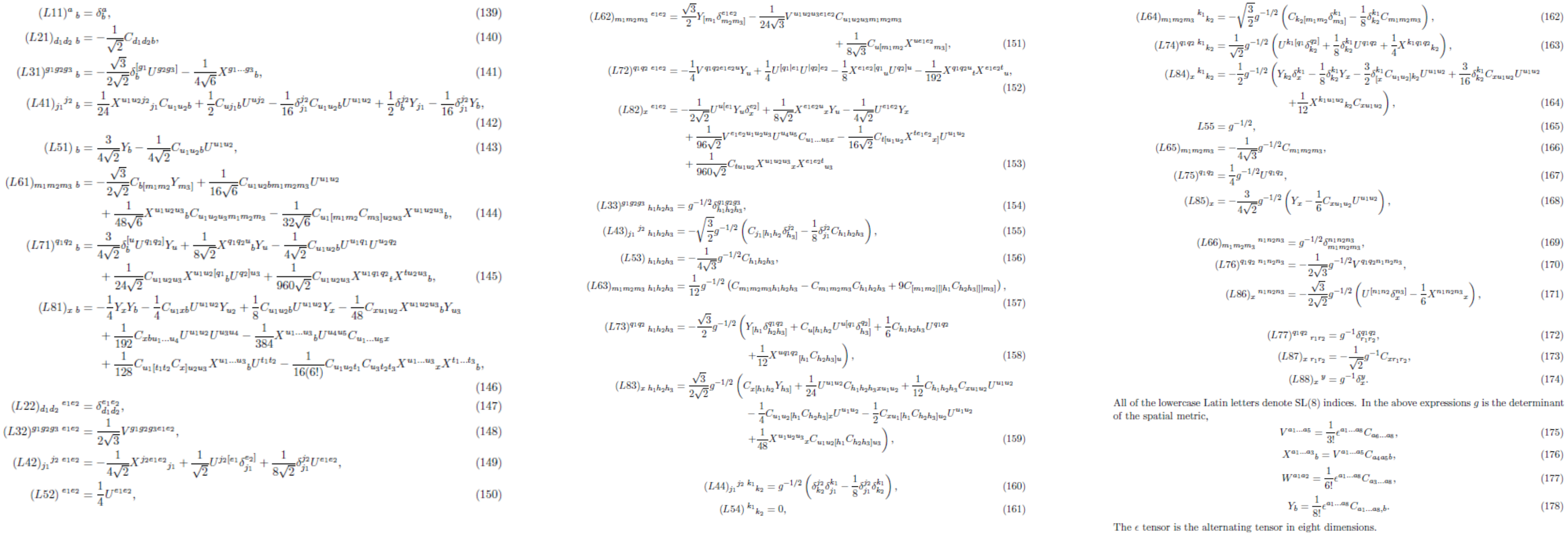


$$\begin{pmatrix}
(L11)^a{}_b & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(L21)_{d_1d_2\,b} & (L22)_{d_1d_2}{}^{e_1e_2} & 0 & 0 & 0 & 0 & 0 & 0 \\
(L31)^{g_1\ldots g_3}{}_b & (L32)^{g_1\ldots g_3\,e_1e_2} & (L33)^{g_1\ldots g_3}{}_{h_1\ldots h_3} & 0 & 0 & 0 & 0 & 0 \\
(L41)_{j_1}{}^{j_2}{}_b & (L42)_{j_1}{}^{j_2\,e_1e_2} & (L43)_{j_1}{}^{j_2}{}_{h_1\ldots h_3} & (L44)_{j_1}{}^{j_2\,k_1}{}_{k_2} & 0 & 0 & 0 & 0 \\
(L51)_b & (L52)^{e_1e_2} & (L53)_{h_1\ldots h_3} & (L54)^{k_1}{}_{k_2} & L55 & 0 & 0 & 0 \\
(L61)_{m_1\ldots m_3\,b} & (L62)_{m_1\ldots m_3}{}^{e_1e_2} & (L63)_{m_1\ldots m_3\,h_1\ldots h_3} & (L64)_{m_1\ldots m_3}{}^{k_1}{}_{k_2} & (L65)_{m_1\ldots m_3} & (L66)_{m_1\ldots m_3}{}^{n_1\ldots n_3} & 0 & 0 \\
(L71)^{q_1q_2}{}_b & (L72)^{q_1q_2\,e_1e_2} & (L73)^{q_1q_2}{}_{h_1\ldots h_3} & (L74)^{q_1q_2\,k_1}{}_{k_2} & (L75)^{q_1q_2} & (L76)^{q_1q_2\,n_1\ldots n_3} & (L77)^{q_1q_2}{}_{r_1r_2} & 0 \\
(L81)_{x\,b} & (L82)_x{}^{e_1e_2} & (L83)_{x\,h_1\ldots h_3} & (L84)_x{}^{k_1}{}_{k_2} & (L85)_x & (L86)_x{}^{n_1\ldots n_3} & (L87)_{x\,r_1r_2} & (L88)_x{}^y
\end{pmatrix}$$

Figure 1: The generalised vielbein

Generalized metric for E_8

[H. Godazgar, M. Godazgar, M. Perry '13]



$$(L11)^{a}{}_{b} = \delta^{a}_{b}, \tag{139}$$

$$(L21)_{d_1 d_2}{}_{b} = -\frac{1}{\sqrt{2}} C_{d_1 d_2 b}, \tag{140}$$

$$(L31)^{g_1 g_2 g_3}{}_{b} = -\frac{\sqrt{3}}{2\sqrt{2}} \delta^{[g_1}_{b} U^{g_2 g_3]} - \frac{1}{4\sqrt{6}} X^{g_1 \cdots g_3}{}_{b}, \tag{141}$$

$$(L41)_{j_1}{}^{j_2}{}_{b} = \frac{1}{24} X^{u_1 u_2 j_2}{}_{j_1} C_{u_1 u_2 b} + \frac{1}{2} C_{u j_1 b} U^{u j_2} - \frac{1}{16} \delta^{j_2}_{j_1} C_{u_1 u_2 b} U^{u_1 u_2} + \frac{1}{2} \delta^{j_2}_{b} Y_{j_1} - \frac{1}{16} \delta^{j_2}_{j_1} Y_{b}, \tag{142}$$

$$(L51)\,{}_{b} = \frac{3}{4\sqrt{2}} Y_{b} - \frac{1}{4\sqrt{2}} C_{u_1 u_2 b} U^{u_1 u_2}, \tag{143}$$

$$\begin{aligned}(L61)_{m_1 m_2 m_3}{}_{b} = {} & -\frac{\sqrt{3}}{2\sqrt{2}} C_{b[m_1 m_2} Y_{m_3]} + \frac{1}{16\sqrt{6}} C_{u_1 u_2 b m_1 m_2 m_3} U^{u_1 u_2} \\ & + \frac{1}{48\sqrt{6}} X^{u_1 u_2 u_3}{}_{b} C_{u_1 u_2 u_3 m_1 m_2 m_3} - \frac{1}{32\sqrt{6}} C_{u_1 [m_1 m_2} C_{m_3] u_2 u_3} X^{u_1 u_2 u_3}{}_{b},\end{aligned} \tag{144}$$

$$\begin{aligned}(L71)^{q_1 q_2}{}_{b} = {} & \frac{3}{4\sqrt{2}} \delta^{[u}_{b} U^{q_1 q_2]} Y_{u} + \frac{1}{8\sqrt{2}} X^{q_1 q_2 u}{}_{b} Y_{u} - \frac{1}{4\sqrt{2}} C_{u_1 u_2 b} U^{u_1 q_1} U^{u_2 q_2} \\ & + \frac{1}{24\sqrt{2}} C_{u_1 u_2 u_3} X^{u_1 u_2 [q_1}{}_{b} U^{q_2] u_3} + \frac{1}{960\sqrt{2}} C_{u_1 u_2 u_3} X^{u_1 q_1 q_2}{}_{t} X^{t u_2 u_3}{}_{b},\end{aligned} \tag{145}$$

$$\begin{aligned}(L81)_{x}{}_{b} = {} & -\frac{1}{4} Y_{x} Y_{b} - \frac{1}{4} C_{u_1 x b} U^{u_1 u_2} Y_{u_2} + \frac{1}{8} C_{u_1 u_2 b} U^{u_1 u_2} Y_{x} - \frac{1}{48} C_{x u_1 u_2} X^{u_1 u_2 u_3}{}_{b} Y_{u_3} \\ & + \frac{1}{192} C_{x b u_1 \ldots u_4} U^{u_1 u_2} U^{u_3 u_4} - \frac{1}{384} X^{u_1 \ldots u_3}{}_{b} U^{u_4 u_5} C_{u_1 \ldots u_5 x} \\ & + \frac{1}{128} C_{u_1 [t_1 t_2} C_{x] u_2 u_3} X^{u_1 \ldots u_3}{}_{b} U^{t_1 t_2} - \frac{1}{16(6!)} C_{u_1 u_2 t_1} C_{u_3 t_2 t_3} X^{u_1 \ldots u_3}{}_{x} X^{t_1 \ldots t_3}{}_{b},\end{aligned} \tag{146}$$

$$(L22)_{d_1 d_2}{}^{e_1 e_2} = \delta^{e_1 e_2}_{d_1 d_2}, \tag{147}$$

$$(L32)^{g_1 g_2 g_3 \, e_1 e_2} = \frac{1}{2\sqrt{3}} V^{g_1 g_2 g_3 e_1 e_2}, \tag{148}$$

$$(L42)_{j_1}{}^{j_2 \, e_1 e_2} = -\frac{1}{4\sqrt{2}} X^{j_2 e_1 e_2}{}_{j_1} + \frac{1}{\sqrt{2}} U^{j_2 [e_1} \delta^{e_2]}_{j_1} + \frac{1}{8\sqrt{2}} \delta^{j_2}_{j_1} U^{e_1 e_2}, \tag{149}$$

$$(L52)\,{}^{e_1 e_2} = \frac{1}{4} U^{e_1 e_2}, \tag{150}$$

$$\begin{aligned}(L62)_{m_1 m_2 m_3}{}^{e_1 e_2} = {} & \frac{\sqrt{3}}{2} Y_{[m_1} \delta^{e_1 e_2}_{m_2 m_3]} - \frac{1}{24\sqrt{3}} V^{u_1 u_2 u_3 e_1 e_2} C_{u_1 u_2 u_3 m_1 m_2 m_3} \\ & + \frac{1}{8\sqrt{3}} C_{u[m_1 m_2} X^{u e_1 e_2}{}_{m_3]},\end{aligned} \tag{151}$$

$$(L72)^{q_1 q_2 \, e_1 e_2} = -\frac{1}{4} V^{q_1 q_2 e_1 e_2 u} Y_{u} + \frac{1}{4} U^{[q_1 | e_1} U^{| q_2] e_2} - \frac{1}{8} X^{e_1 e_2 [q_1}{}_{u} U^{q_2] u} - \frac{1}{192} X^{q_1 q_2 u}{}_{t} X^{e_1 e_2 t}{}_{u}, \tag{152}$$

$$\begin{aligned}(L82)_{x}{}^{e_1 e_2} = {} & -\frac{1}{2\sqrt{2}} U^{u[e_1} Y_{u} \delta^{e_2]}_{x} + \frac{1}{8\sqrt{2}} X^{e_1 e_2 u}{}_{x} Y_{u} - \frac{1}{4\sqrt{2}} U^{e_1 e_2} Y_{x} \\ & + \frac{1}{96\sqrt{2}} V^{e_1 e_2 u_1 u_2 u_3} U^{u_4 u_5} C_{u_1 \ldots u_5 x} - \frac{1}{16\sqrt{2}} C_{t[u_1 u_2} X^{t e_1 e_2}{}_{x]} U^{u_1 u_2} \\ & + \frac{1}{960\sqrt{2}} C_{t u_1 u_2} X^{u_1 u_2 u_3}{}_{x} X^{e_1 e_2 t}{}_{u_3}\end{aligned} \tag{153}$$

$$(L33)^{g_1 g_2 g_3}{}_{h_1 h_2 h_3} = g^{-1/2} \delta^{g_1 g_2 g_3}_{h_1 h_2 h_3}, \tag{154}$$

$$(L43)_{j_1}{}^{j_2}{}_{h_1 h_2 h_3} = -\sqrt{\frac{3}{2}} g^{-1/2} \left(C_{j_1 [h_1 h_2} \delta^{j_2}_{h_3]} - \frac{1}{8} \delta^{j_2}_{j_1} C_{h_1 h_2 h_3} \right), \tag{155}$$

$$(L53)\,{}_{h_1 h_2 h_3} = -\frac{1}{4\sqrt{3}} g^{-1/2} C_{h_1 h_2 h_3}, \tag{156}$$

$$(L63)_{m_1 m_2 m_3}{}_{h_1 h_2 h_3} = \frac{1}{12} g^{-1/2} \left(C_{m_1 m_2 m_3 h_1 h_2 h_3} - C_{m_1 m_2 m_3} C_{h_1 h_2 h_3} + 9 C_{[m_1 m_2 || [h_1} C_{h_2 h_3] || m_3]} \right), \tag{157}$$

$$\begin{aligned}(L73)^{q_1 q_2}{}_{h_1 h_2 h_3} = {} & -\frac{\sqrt{3}}{2} g^{-1/2} \left(Y_{[h_1} \delta^{q_1 q_2}_{h_2 h_3]} + C_{u[h_1 h_2} U^{u[q_1} \delta^{q_2]}_{h_3]} + \frac{1}{6} C_{h_1 h_2 h_3} U^{q_1 q_2} \right. \\ & \left. + \frac{1}{12} X^{u q_1 q_2}{}_{[h_1} C_{h_2 h_3] u} \right),\end{aligned} \tag{158}$$

$$\begin{aligned}(L83)_{x}{}_{h_1 h_2 h_3} = {} & \frac{\sqrt{3}}{2\sqrt{2}} g^{-1/2} \left(C_{x[h_1 h_2} Y_{h_3]} + \frac{1}{24} U^{u_1 u_2} C_{h_1 h_2 h_3 x u_1 u_2} + \frac{1}{12} C_{h_1 h_2 h_3} C_{x u_1 u_2} U^{u_1 u_2} \right. \\ & - \frac{1}{4} C_{u_1 u_2 [h_1} C_{h_2 h_3] x} U^{u_1 u_2} - \frac{1}{2} C_{x u_1 [h_1} C_{h_2 h_3] u_2} U^{u_1 u_2} \\ & \left. + \frac{1}{48} X^{u_1 u_2 u_3}{}_{x} C_{u_1 u_2 [h_1} C_{h_2 h_3] u_3} \right),\end{aligned} \tag{159}$$

$$(L44)_{j_1}{}^{j_2 \, k_1}{}_{k_2} = g^{-1/2} \left(\delta^{j_2}_{k_2} \delta^{k_1}_{j_1} - \frac{1}{8} \delta^{j_2}_{j_1} \delta^{k_1}_{k_2} \right), \tag{160}$$

$$(L54)\,{}^{k_1}{}_{k_2} = 0, \tag{161}$$

$$(L64)_{m_1 m_2 m_3}{}^{k_1}{}_{k_2} = -\sqrt{\frac{3}{2}} g^{-1/2} \left(C_{k_2 [m_1 m_2} \delta^{k_1}_{m_3]} - \frac{1}{8} \delta^{k_1}_{k_2} C_{m_1 m_2 m_3} \right), \tag{162}$$

$$(L74)^{q_1 q_2 \, k_1}{}_{k_2} = \frac{1}{\sqrt{2}} g^{-1/2} \left(U^{k_1 [q_1} \delta^{q_2]}_{k_2} + \frac{1}{8} \delta^{k_1}_{k_2} U^{q_1 q_2} + \frac{1}{4} X^{k_1 q_1 q_2}{}_{k_2} \right), \tag{163}$$

$$\begin{aligned}(L84)_{x}{}^{k_1}{}_{k_2} = {} & -\frac{1}{2} g^{-1/2} \left(Y_{k_2} \delta^{k_1}_{x} - \frac{1}{8} \delta^{k_1}_{k_2} Y_{x} - \frac{3}{2} \delta^{k_1}_{[x} C_{u_1 u_2] k_2} U^{u_1 u_2} + \frac{3}{16} \delta^{k_1}_{k_2} C_{x u_1 u_2} U^{u_1 u_2} \right. \\ & \left. + \frac{1}{12} X^{k_1 u_1 u_2}{}_{k_2} C_{x u_1 u_2} \right),\end{aligned} \tag{164}$$

$$L55 = g^{-1/2}, \tag{165}$$

$$(L65)_{m_1 m_2 m_3} = -\frac{1}{4\sqrt{3}} g^{-1/2} C_{m_1 m_2 m_3}, \tag{166}$$

$$(L75)^{q_1 q_2} = \frac{1}{4} g^{-1/2} U^{q_1 q_2}, \tag{167}$$

$$(L85)_{x} = -\frac{3}{4\sqrt{2}} g^{-1/2} \left(Y_{x} - \frac{1}{6} C_{x u_1 u_2} U^{u_1 u_2} \right), \tag{168}$$

$$(L66)_{m_1 m_2 m_3}{}^{n_1 n_2 n_3} = g^{-1/2} \delta^{n_1 n_2 n_3}_{m_1 m_2 m_3}, \tag{169}$$

$$(L76)^{q_1 q_2 \, n_1 n_2 n_3} = -\frac{1}{2\sqrt{3}} g^{-1/2} V^{q_1 q_2 n_1 n_2 n_3}, \tag{170}$$

$$(L86)_{x}{}^{n_1 n_2 n_3} = -\frac{\sqrt{3}}{2\sqrt{2}} g^{-1/2} \left(U^{[n_1 n_2} \delta^{n_3]}_{x} - \frac{1}{6} X^{n_1 n_2 n_3}{}_{x} \right), \tag{171}$$

$$(L77)^{q_1 q_2}{}_{r_1 r_2} = g^{-1} \delta^{q_1 q_2}_{r_1 r_2}, \tag{172}$$

$$(L87)_{x}{}_{r_1 r_2} = -\frac{1}{\sqrt{2}} g^{-1} C_{x r_1 r_2}, \tag{173}$$

$$(L88)_{x}{}^{y} = g^{-1} \delta^{y}_{x}. \tag{174}$$

All of the lowercase Latin letters denote SL(8) indices. In the above expressions g is the determinant of the spatial metric,

$$V^{a_1 \ldots a_5} = \frac{1}{3!} \epsilon^{a_1 \ldots a_8} C_{a_6 \ldots a_8}, \tag{175}$$

$$X^{a_1 \ldots a_3}{}_{b} = V^{a_1 \ldots a_5} C_{a_4 a_5 b}, \tag{176}$$

$$W^{a_1 a_2} = \frac{1}{6!} \epsilon^{a_1 \ldots a_8} C_{a_3 \ldots a_8}, \tag{177}$$

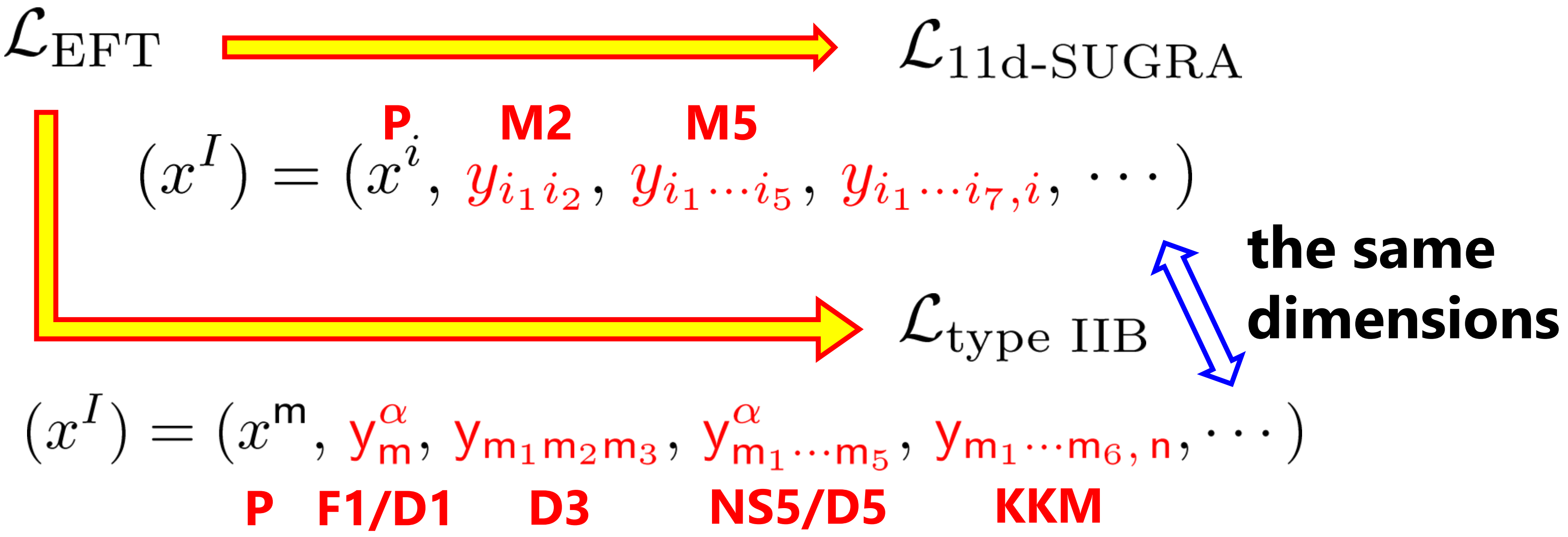
$$Y_{b} = \frac{1}{8!} \epsilon^{a_1 \ldots a_8} C_{a_1 \ldots a_8, b}. \tag{178}$$

The ϵ tensor is the alternating tensor in eight dimensions.

$$\mathcal{M}_{IJ} = E_I{}^A \, E_J{}^B \, \delta_{AB}$$

EFT for type IIB theory

Not only the 11 dim. SUGRA but
10 dim. type IIB SUGRA can be also
reproduced from EFT **[Blair, Malek, J.-H. Park '13; Hohm, Samtleben '13]**



$$\mathcal{L}_{\text{EFT}} \longrightarrow \mathcal{L}_{\text{11d-SUGRA}}$$

$$(x^I) = (\overset{\text{P}}{x^i},\ \overset{\text{M2}}{y_{i_1 i_2}},\ \overset{\text{M5}}{y_{i_1\cdots i_5}},\ y_{i_1\cdots i_7,i},\ \cdots)$$

$$\mathcal{L}_{\text{EFT}} \longrightarrow \mathcal{L}_{\text{type IIB}}$$

$$(x^I) = (\underset{\text{P}}{x^{\mathsf{m}}},\ \underset{\text{F1/D1}}{\mathsf{y}^{\alpha}_{\mathsf{m}}},\ \underset{\text{D3}}{\mathsf{y}_{\mathsf{m}_1\mathsf{m}_2\mathsf{m}_3}},\ \underset{\text{NS5/D5}}{\mathsf{y}^{\alpha}_{\mathsf{m}_1\cdots\mathsf{m}_5}},\ \underset{\text{KKM}}{\mathsf{y}_{\mathsf{m}_1\cdots\mathsf{m}_6,\mathsf{n}}},\ \cdots)$$

the same dimensions

Generalized metric for type IIB

Einstein-frame metric

$$\mathsf{M}_{\mathsf{MN}} = (\mathsf{L}^{T}\,\hat{\mathsf{M}}\,\mathsf{L})_{\mathsf{MN}}$$

$$\hat{\mathsf{M}} \equiv |\mathsf{G}|^{\frac{1}{n-2}} \begin{pmatrix} \mathsf{G}_{\mathsf{mn}} & 0 & 0 & 0 & 0 \\ 0 & m_{\alpha\beta}\,\mathsf{G}^{\mathsf{mn}} & 0 & 0 & 0 \\ 0 & 0 & \mathsf{G}^{\mathsf{m}_1\mathsf{m}_2\mathsf{m}_3,\,\mathsf{n}_1\mathsf{n}_2\mathsf{n}_3} & 0 & 0 \\ 0 & 0 & 0 & m_{\alpha\beta}\,\mathsf{G}^{\mathsf{m}_1\cdots\mathsf{m}_5,\,\mathsf{n}_1\cdots\mathsf{n}_5} & 0 \\ 0 & 0 & 0 & 0 & \mathsf{G}^{\mathsf{m}_1\cdots\mathsf{m}_6,\,\mathsf{n}_1\cdots\mathsf{n}_6}\,\mathsf{G}^{\mathsf{mn}} \end{pmatrix},$$

$$(m_{\alpha\beta}) \equiv \mathrm{e}^{\varphi} \begin{pmatrix} \mathrm{e}^{-2\varphi} + (\mathsf{C}_0)^2 & \mathsf{C}_0 \\ \mathsf{C}_0 & 1 \end{pmatrix}$$

$$\mathsf{L} = \begin{pmatrix} \delta^{\mathsf{m}}_{\mathsf{n}} & 0 & 0 & 0 & 0 \\ -\mathsf{B}^{\alpha}_{\mathsf{nm}} & \delta^{\alpha}_{\beta}\,\delta^{\mathsf{n}}_{\mathsf{m}} & 0 & 0 & 0 \\ -\frac{\mathsf{D}_{\mathsf{nm}_1\mathsf{m}_2\mathsf{m}_3} + \frac{3}{2}\,\epsilon_{\gamma\delta}\,\mathsf{B}^{\gamma}_{\mathsf{n}[\mathsf{m}_1}\,\mathsf{B}^{\delta}_{\mathsf{m}_2\mathsf{m}_3]}}{\sqrt{3!}} & \frac{3!\,\epsilon_{\beta\gamma}\,\delta^{\mathsf{n}}_{[\mathsf{m}_1}\,\mathsf{B}^{\gamma}_{\mathsf{m}_2\mathsf{m}_3]}}{2!\sqrt{3!}} & \delta^{\mathsf{n}_1\mathsf{n}_2\mathsf{n}_3}_{\mathsf{m}_1\mathsf{m}_2\mathsf{m}_3} & 0 & 0 \\ -\frac{\mathsf{D}^{\alpha}_{\mathsf{nm}_1\cdots\mathsf{m}_5} + 10\,\mathsf{D}_{\mathsf{n}[\mathsf{m}_1\mathsf{m}_2\mathsf{m}_3}\,\mathsf{B}^{\alpha}_{\mathsf{m}_4\mathsf{m}_5]} + 5\,\epsilon_{\gamma\delta}\,\mathsf{B}^{\gamma}_{\mathsf{n}[\mathsf{m}_1}\,\mathsf{B}^{\delta}_{\mathsf{m}_2\mathsf{m}_3}\,\mathsf{B}^{\alpha}_{\mathsf{m}_4\mathsf{m}_5]}}{\sqrt{5!}} & -\frac{5\,(\delta^{\alpha}_{\beta}\,\delta^{\mathsf{n}}_{[\mathsf{m}_1}\,\mathsf{D}_{\mathsf{m}_1\cdots\mathsf{m}_4]} - 3\,\epsilon_{\beta\gamma}\,\delta^{\mathsf{n}}_{[\mathsf{m}_1}\,\mathsf{B}^{\gamma}_{\mathsf{m}_2\mathsf{m}_3}\,\mathsf{B}^{\alpha}_{\mathsf{m}_4\mathsf{m}_5]})}{\sqrt{5!}} & \frac{5!\,\delta^{\mathsf{n}_1\mathsf{n}_2\mathsf{n}_3}_{[\mathsf{m}_1\mathsf{m}_2\mathsf{m}_3}\,\mathsf{B}^{\alpha}_{\mathsf{m}_4\mathsf{m}_5]}}{2!\sqrt{3!\,5!}} & \delta^{\alpha}_{\beta}\,\delta^{\mathsf{n}_1\cdots\mathsf{n}_5}_{\mathsf{m}_1\cdots\mathsf{m}_5} & 0 \\ * & * & * & * & * \end{pmatrix}$$

contains $\begin{pmatrix} \mathsf{B}_{\mathsf{mn}} \\ \mathsf{C}_{\mathsf{mn}} \end{pmatrix}$ $\quad \mathsf{C}_{\mathsf{m}_1\cdots\mathsf{m}_4} \quad$ $\begin{pmatrix} \mathsf{C}_{\mathsf{m}_1\cdots\mathsf{m}_6} \\ \mathsf{B}_{\mathsf{m}_1\cdots\mathsf{m}_6} \end{pmatrix}$

[K. Lee, S.-J. Rey, YS, arXiv:1612.08738]

Action for type IIB branes

Our action:

$$S = \frac{1}{p+1} \int_{\Sigma} \Big(\frac{1}{2} \, \mathsf{M}_{\mathsf{MN}} \, \mathcal{P}^{\mathsf{M}} \wedge *_{\gamma} \mathcal{P}^{\mathsf{N}} - \Omega_{p} \Big) \, .$$

$$(\mathcal{P}^{\mathsf{M}}) = \begin{pmatrix} \mathrm{d}X^{\mathsf{m}} \\ \mathcal{P}^{\alpha}_{\mathsf{m}} \\ \frac{\mathcal{P}_{\mathsf{m}_1 \mathsf{m}_2 \mathsf{m}_3}}{\sqrt{3!}} \\ \frac{\mathcal{P}^{\alpha}_{\mathsf{m}_1 \cdots \mathsf{m}_5}}{\sqrt{5!}} \\ \frac{\mathcal{P}_{\mathsf{m}_1 \cdots \mathsf{m}_6 \, , \, \mathsf{m}}}{\sqrt{6!}} \\ \vdots \end{pmatrix} \quad \left. \begin{array}{c} \textbf{F1/D1} \\ \textbf{D3} \\ \textbf{NS5/D5} \\ \textbf{KKM} \\ \textbf{(exotic)} \end{array} \right\} \; \textbf{auxiliary fields}$$

F/D-string

Our action:

$\mathcal{P}^{\alpha}_{\mathsf{m}}$

$$S = \frac{1}{2}\int_{\Sigma}\Big[\frac{1}{2}\,\mathsf{M}_{\mathsf{MN}}\,\mathcal{P}^{\mathsf{M}}\wedge *_{\gamma}\mathcal{P}^{\mathsf{N}} - \Omega_{1}\Big]\,.$$

⇓

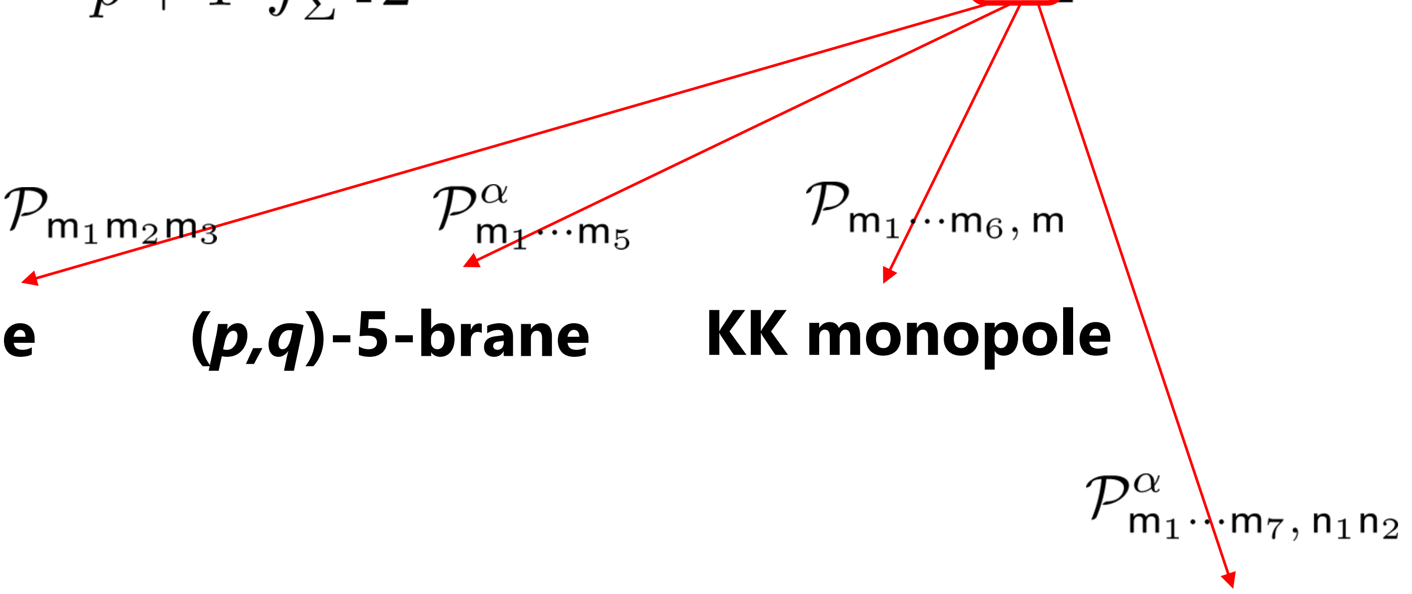
well-known action for *(p,q)*-string [Schwarz '95]:

$$S = -\mu_{1}\int_{\Sigma}\mathrm{d}^{2}\sigma\sqrt{q_{\alpha}\,m^{\alpha\beta}\,q_{\beta}}\,\sqrt{-h} + \mu_{1}\int_{\Sigma}q_{\alpha}\big(\mathsf{B}_{2}^{\alpha} - F_{2}^{\alpha}\big)\,.$$

D3, D5/NS5, KKM, …

Our action:

$$S = \frac{1}{p+1}\int_{\Sigma}\Big[\frac{1}{2}\,\mathsf{M}_{\mathsf{MN}}\,\mathcal{P}^{\mathsf{M}}\wedge *_{\gamma}\mathcal{P}^{\mathsf{N}} - \boxed{\Omega_p}\Big]\ .$$



$\mathcal{P}_{\mathsf{m}_1\mathsf{m}_2\mathsf{m}_3}$ $\mathcal{P}^{\alpha}_{\mathsf{m}_1\cdots\mathsf{m}_5}$ $\mathcal{P}_{\mathsf{m}_1\cdots\mathsf{m}_6,\,\mathsf{m}}$

D3-brane **(*p*,*q*)-5-brane** **KK monopole**

$\mathcal{P}^{\alpha}_{\mathsf{m}_1\cdots\mathsf{m}_7,\,\mathsf{n}_1\mathsf{n}_2}$

Actions for exotic branes will be also derived*!*

Summary

We proposed a simple action for a *p*-brane:

$$S = \int_{\Sigma_{p+1}} \left[\frac{1}{2} \mathcal{M}_{IJ}(X(\sigma))\, \mathcal{P}^I(\sigma) \wedge *_\gamma \mathcal{P}^J(\sigma) - \Omega_{p+1}(\sigma) \right] .$$

M2-brane ... reproduced a known action.
M5-brane ... reproduced a known linearized action.
Even at the non-linear level,
e.o.m. seem to be the same (checking the detail).
KK Monopole ... reproduced (a part of) known action.
(now checking)

1-brane ... reproduced a known (*p*,*q*)-string action.
3-brane, 5-brane, KKM, exotic ... now checking.