▨ localization in Quantum quasi-particle boson (or magnon) Hall systems.

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- Finally, I like to talk about effect of disorders in topological quasi-particle boson systems.

- I want to address myself to the following questions.

- Firstly, the charge conservation is the funtamental symmetry in QHE, which plays vital role in the quantization of Hall conductance.

- If the QH system has an anomalous term ($c_i c_j$), the two terminal electric conductance along the chiral edge mode is no longer quantized because electron can locally diminish by this term.

- Therefore, the Hall conductance obtained from the edge transport will <u>not</u> be quantized anymore.

- From this, we can see that the charge conservation is the vital symmetry, which enables the topological distinction between integer QH states with different topological integers.

- On the one hand, topological quasi-particle boson systems I have described so far usually have no explicit U(1) symmetry at all.
- In the case of magnon, for example, the U(1) symmetry corresponds to a continuous spin rotational symmetry around the ferromagnetic moment.

- In the presence of spin-orbital locking interaction, however, there is no continuous spin rotational symmetry.

- Thus, the key question I should ask to myself is,

◎ whether topological quasi-particle boson systems with different topological integers can be still distinguishable or not in the presence of generic disorders, which breaks the explicit U(1) symmetry of the quadratic boson Hamiltonian.

◎ Or equivalently, whether they can be still distinguished from conventional localized regime or not.

e.g.)

crossover

1

conventional Anderson localized regime

generic disorder strength. g

Quantum Magnon Hall at the clean limit

two-terminal magnon conductance along the chiral edge mode is <u>always zero</u> once $g \neq 0$.

2

Quantum magnon Hall regime

conventional localized regime

clean limit

generic disorder strength

Quantum phase transition

two-terminal Magnon conductance <u>along the chiral edge mode</u> is quantized to be non-zero integer

2-terminal magnon conductance is zero.

bulk wavefunction is delocalized

- From numerical simulations in this paper, we found that the latter is the case.

- The reason is as follows.

- First of all, the quadratic Hamiltonian with disorder always has the generic particle-hole symmetry:

$$\hat{\sigma}_1 \cdot \hat{H} \cdot \hat{\sigma}_1 = \hat{H}^* \quad \text{— ⓒ-1}$$

- Thus, an eigenstate of $\hat{H}$ which corresponds to the chiral edge mode ($|\phi\rangle$) always has its hole-$\boxed{\text{counter part}}$ eigenstate ($\sigma_1 |\phi\rangle^*$)

$$\begin{cases} \hat{H}\,|\phi\rangle = \sigma_3\,|\phi\rangle\,E \\ \hat{H}\,(\sigma_1 |\phi\rangle^*) = \sigma_3\,(\sigma_1 |\phi\rangle^*)\,(-E) \end{cases} \quad \text{— ⓒ-2}$$

- Since both $|\phi\rangle$ and $\sigma_1 |\phi\rangle^*$ are localized at the same side of the system's boundary, there is a finite matrix element of generic disorder potential ($H'_{disorder}$) between these two!

$$\langle \phi_h |\, H'_{disorder}\, |\phi_p\rangle \neq 0. \quad \text{— ⓒ-3}$$

spatially local, but contains $b_i^\dagger b_i^\dagger$

- However, these two states correspond to different number states of the same single particle state. (with finite energy E):

$$\begin{cases} |\phi\rangle = |n+1\rangle & \text{particle-type state} \\ \sigma_1 |\phi\rangle^* = |n-1\rangle & \text{hole-counterpart} \end{cases}$$

- Therefore, the scattering between these two is alway accompanied by an energy absorption or emission of $2E$, where E is an energy of the single-particle state.

$$P_{\phi \to \sigma_1 \phi^*} = |\langle \phi | H' | \sigma_1 \phi^* \rangle|^2 \delta(\omega - 2E).$$

— ⓒ-4

- In other words, there is no elastic scattering between chiral edge modes and their hole-counterpart states, provided that their energies are finite.

- As a result, the two-terminal magnon conductance along chiral edge modes remains quantized up to a certain disorder strength, even though the system has no explicit U(1) symmetry.

- At the critical disorder strength, the bulk wavefunction become delocalized, which mediates a coupling between the edge mode at one side of the boundary and the edge mode at the other side.

- By this coupling, these two edge modes annihilate with each other

- As a result, above this critical disorder strength, the two-terminal magnon conductance along the edge modes reduces to zero completely.

- This region is essentially connected to the conventional magnon localized regime

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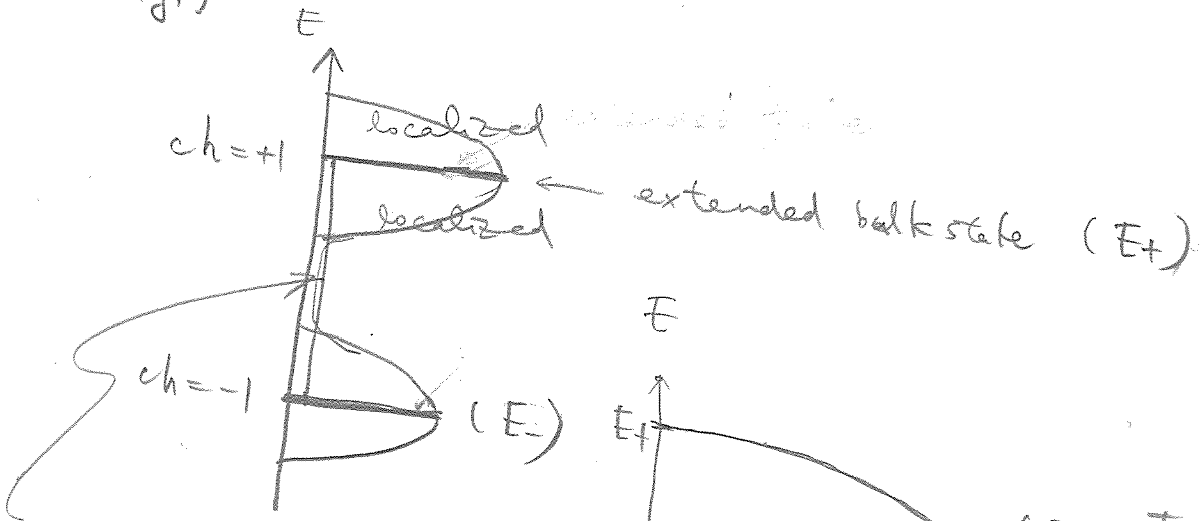
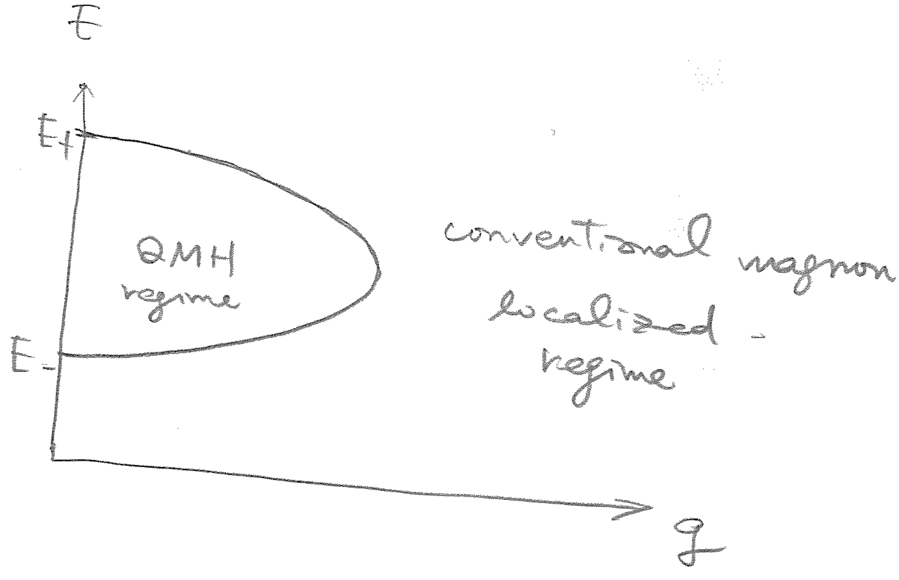
- Our numerical simulation suggests that, all the bulk magnon wavefunctions except for extended states at the centers of those bulk bands with finite topological integer are localized.

e.g.)





chiral edge mode is encompassed by the two entended bulk state.

- Based on this observation, I like to discuss the thermal Hall conductivity κ_{xy} in disordered quantum $\left(\begin{array}{l}\text{magnon} \\ \text{quasi-particle boson}\end{array}\right)$ systems.

- For simplicity, let us consider κ_{xy} in this two band model. (generalize later)

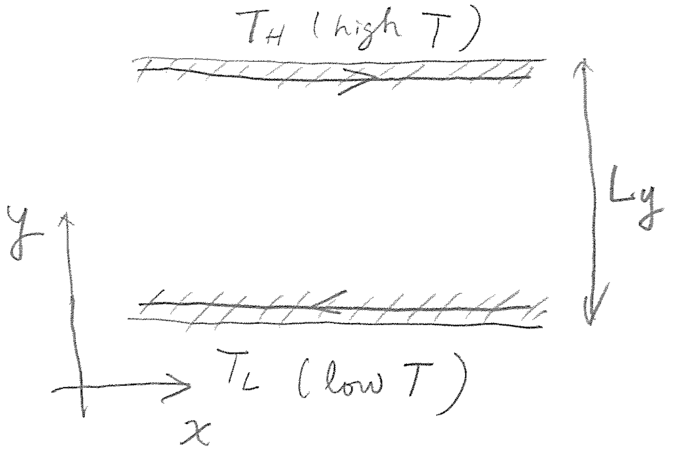
- For a given finite disorder strength, all these single particle states will contribute to κ_{xy} at finite temperature (T).

- However, all the bulk states except for the extended states at $E = E_+$ and at $E = E_-$ are spatially localized; they cannot carry heat and therefore do not contribute to κ_{xy}.

- On the one hand, between $E = E_+$ and $E = E_-$, the system has a (sub)extensive number of chiral edge modes which are delocalized along the 1D boundary of the system; they will contribute to κ_{xy}. 3 -10

- To evaluate this contribution, I introduce a temperature gradient along y-direction, and calculate an energy current carried by the edge modes along the x-direction.

(impose open boundary condition along y,
〃 periodic 〃 〃 x)



- Magnon conductance × dE counts number of those magnon at [E, E+dE], which can transmit through the system

- Therefore, magnon conductance carried by m-number of chiral edge modes is given by m/h ③-11

- In the QMH regime of this 2-bands model, it is quantized to $1/h$ (in QMH regime)

- Therefore, heat current density carried by (energy) the chiral edge mode at one side of the system at energy E is given by

$$\frac{E\,dE}{h\,L_y} \quad \text{— ⓒ – 4}$$

where L_y is a linear dimension of the system size along the y-direction.

- This is true for any edge states from $E = E_+$ to $E = E_-$

• Therefore, the energy current density carried by all the chiral edge mode at higher temperature side is given by

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$$I_H^E = \frac{1}{hL_y}\int_{E_-}^{E_+} g(E, T_H)\, E\, dE$$

$-(©-5)$

• those energy current density carried by all the chiral modes at lower temperature side is given by

$$I_L^E = -\frac{1}{hL_y}\int_{E_-}^{E_+} g(E, T_L)\, E\, dE$$

• where $g(E,T) \equiv \dfrac{1}{e^{\frac{E}{k_B T}}-1}$

• Noting that $T_H - T_L = L_y \nabla_y T$, and regarding this temperature gradient is small enough, we have the total energy current density carried by edge states : as ③−13

$$I_t^E = \frac{1}{hL_y} L_y(\nabla_y T) \int_{E_-}^{E_+} \frac{d}{dT}\Big(g(E,T)\Big) \cdot E \, dE$$

$$= -\frac{(\nabla_y T)}{hT} \int_{E_-}^{E_+} \frac{d}{dE}\Big(g(E,T)\Big) E^2 \, dE$$

$$\overset{x \equiv \frac{E}{k_B T}}{=} -\frac{(\nabla_y T)\, k_B^2 T}{h} \int_{x_-}^{x_+} \frac{dg}{dx} x^2 \, dx$$

$$= -\frac{(\nabla_y T)\, k_B^2 T}{h} \int_{g_-}^{g_+} x^2 \, dg$$

K_{xy}

$$= \underbrace{-\frac{k_B^2 T}{h} \cdot \Big(C_2[g(E_+)] - C_2[g(E_-)] \Big)}_{= K_{xy}} \nabla_y T \qquad \text{—(ⓒ−6)}$$

$$\left(\text{where} \quad C_2[g] \equiv \int_0^g \left[\ln\left(1+\frac{1}{t}\right)\right]^2 dt \right)$$

This leads to

$$\kappa_{xy}^{edge} = -\frac{k_B^2 T}{h}\left(c_2[g(E_+)] - c_2[g(E_-)] \right) \quad \text{— ⓒ-7}$$

- A realistic material may have more than 2 bands, which have non-zero quantized Chern integers.

- Our numerical simulation suggests that even small disorder makes all the bulk bands localized except for delocalized bulk states at respective band center.

- A pair of two delocalized bulk states bound a mobility gap, inside which a chiral edge mode lives.

- We can generalize the argument so far;

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$$K_{xy} = -\frac{k_B^2 T}{h}\sum_j \left\{ C_2[g(\varepsilon_j^+)] - C_2[g(\varepsilon_j^-)] \right\}$$

— ⓒ−8

where the integer j counts chiral edge modes.

- ε_j^+ and ε_j^- stand for a pair of two energies by which the j-th chiral edge mode is bounded.

- We define $\varepsilon_j^+ \gtrless \varepsilon_j^-$ when the j-th mode is $\binom{\text{right}}{\text{left}}$-handed.

- This expression is qualitatively consistent with the thermal Hall conductivity in the clean limit.

$$K_{xy} = -\frac{k_B^2 T}{h}\underbrace{\sum_{n=1}^{N}}_{\text{only over particle bands}}\int \frac{d^2\mathbb{k}}{2\pi}\,\Omega^{xy}_{n,\mathbb{k}}\left(C_2[g(\varepsilon_{n,\mathbb{k}})] - \underbrace{\cancel{\frac{\pi^2}{3}}}_{=0}\right)$$

(due to the sum rule)

- They are identical when the temperature is much larger than the bulk band width.

- In this limit, $\varepsilon_{n,\mathbb{k}}$ in the right hand side can be replaced by its band center energy $\overline{\varepsilon}_n$;

$$\lim_{\Delta \ll k_B T} \kappa_{xy} = -\frac{k_B^2 T}{h} \sum_n \int \frac{d^2\mathbb{k}}{2\pi} \Omega_n(\mathbb{k})\, C_2[g(\overline{\varepsilon}_n)]$$

$$= -\frac{k_B^2 T}{h} \sum_n Ch_n\, C_2[g(\overline{\varepsilon}_n)]$$

$$= \text{eq. ⓒ-8}$$

If we regard this band center energy as the energy for the extended bulk state, at which chiral edge mode is terminated, one can see that this is nothing but eq ⓒ-8.

e.g.) In the two-band case.