# Bell's spin-entangled electron pair generated by local-Fermi-liquid exchange interaction

**Rui Sakano, ISSP**

Collaboration:

Akira Oguri, Yunori Nisikawa (Osaka city university), Eisuke Abe (Keio university),

Meydi Ferrier (University of Paris-Sud), Kensuke Kobayashi (Osaka university)

Discussion:

Shiro Kawabata (AIST)

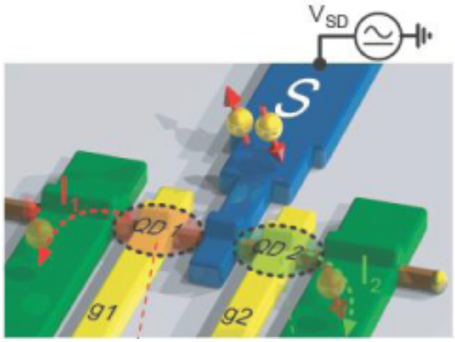
YITP Workshop on Quantum Information Physics, Jan 5th-8th 2016, Yukawa Institute for Theoretical Physics, Kyoto University

# Bell's pair generator by many-body effect

## Cooper pair splitter

Theory: Chtchelkatchev *et al*., PRB **66**, 161320(R) (2002)

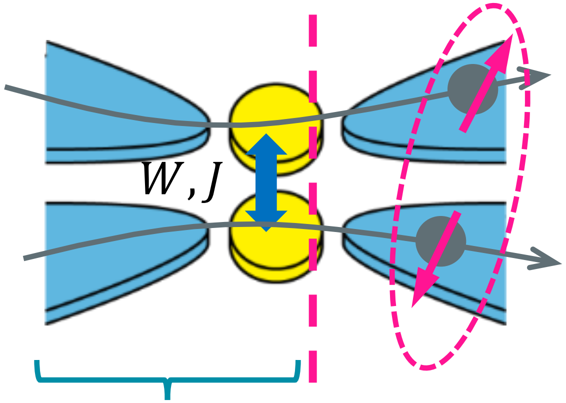
Experiment: Hofstetter *et al*., Nature **461**, 960-963 (2009).




New idea!

## Pair by Local-Fermi-liquid interaction

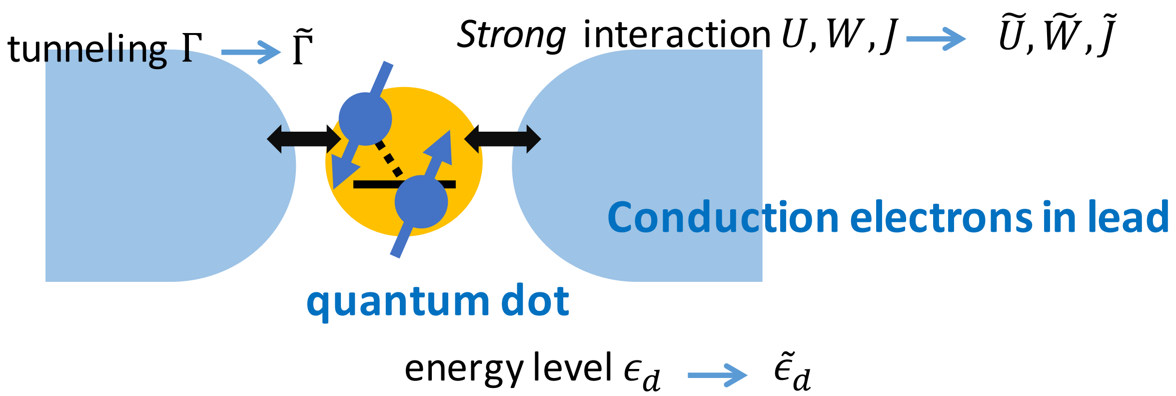
Bell's pair in current through a double quantum dot



# Local Fermi-liquid

Landau's Fermi liquid theory → Quantum dot (impurity) systems

## Ground state of "the Kondo effect"




Nozieres, J. Low Temp. Phys. **17**, 31 (1974); Wilson, Rev. Mod. Phys. **47**, 773 (1975); Yamada, PTP **53**, 970 (1975)

# Outline

## Introduction

## Electric transport and Kondo effect in quantum dot

## Charge pair creation mechanism

shot noise measurement for Kondo effect in quantum dot

Theory: Sela *et al.*, PRL (2006), Gogolin and Komnik PRL (2006), RS *et al.*, PRL (2012)
Experiment: Ferrier, RS *et al.*, Nat. Phys. (2015)

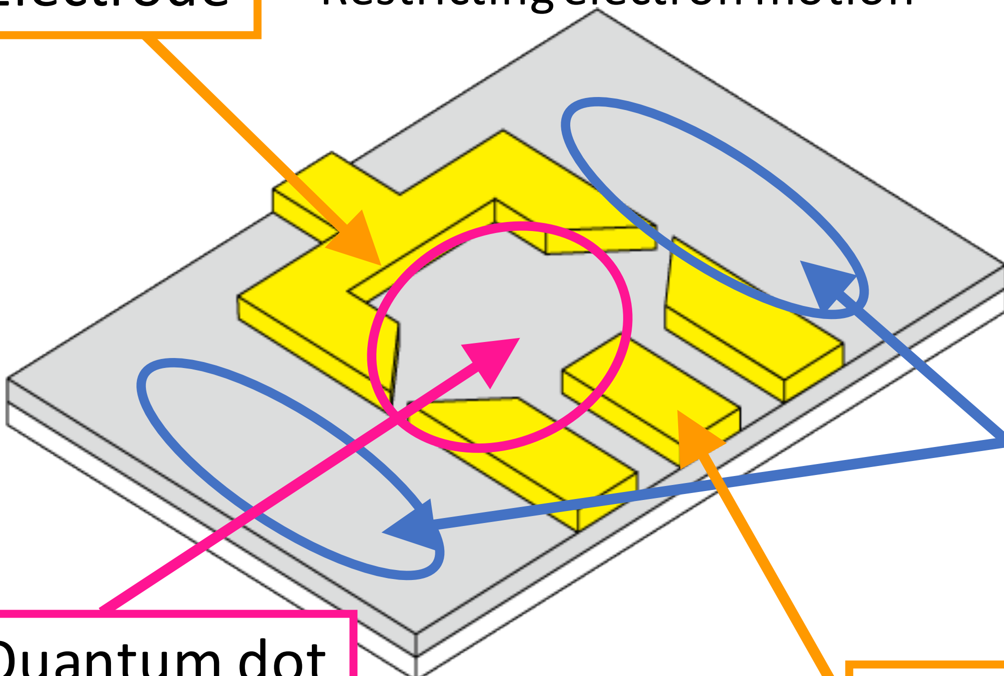
## Bell's pair creation in double quantum dot

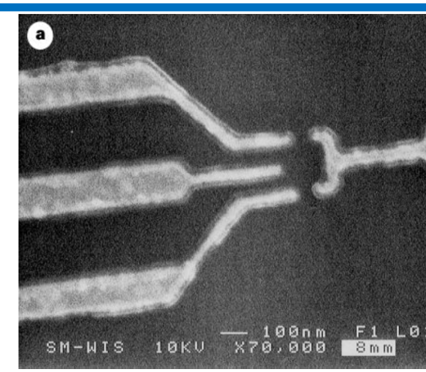
# Electric transport and Kondo effect in quantum dot

# Quantum dot

**Substrate** 2D electron gas

**Electrode** Restricting electron motion






D. Goldhaber-Gordon, *et al.*,
Nature (London) **391**, 156 (1998)

**Lead electrodes**

Conduction electrons tunnel to dot.
Bias voltage → electric current

**Quantum dot**

Electron confinement
to small region(~10nm)

➡ Discrete energy levels
"atom-like properties"

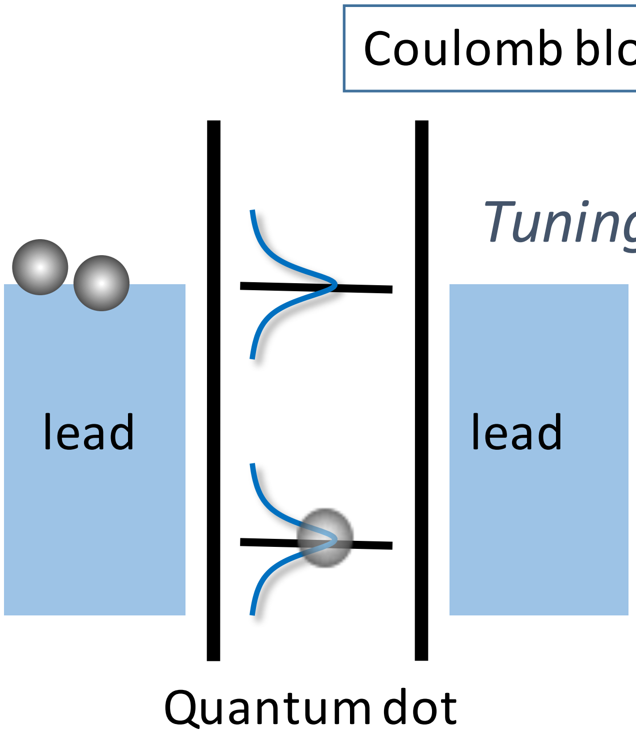
**Gate electrode**

To tune dot energy-level
by electric potential.

# Transport through quantum dot

Tunnel effect → Electron transport

〈**Energy schematic of quantum dot**〉




*Tuning gate electrode*

**One-body effect**

→ **Many-body effect**

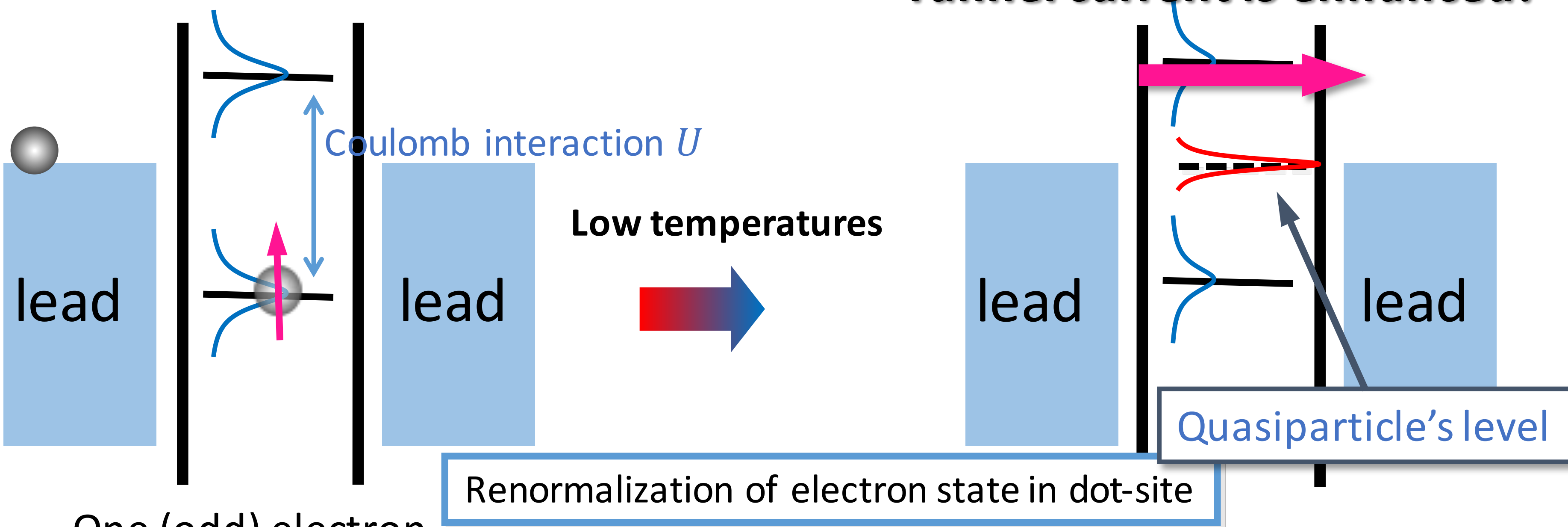
# Kondo effect in quantum dot

## Coulomb blockade

No transport occurs.

## “Kondo effect”

***Tunnel current is enhanced!***




One (odd) electron in dot

Transport at low bias-voltages

**Free quasiparticles + Fermi-liquid interaction**

local-Fermi-liquid state

# Detection of pair charge creation in local Fermi-liquid

## Shot noise measurement

# Current generated by rare event (Poissonian process)

**At zero temperature** where no thermal noise

**Discretized charge state** → **Current noise**

$$\underline{S} = 2e^{*}\underline{\langle I \rangle}$$

**Average current**

**Current-current correlation (current noise)**

$$S \coloneqq \int dt \, \langle \delta I(t)\, \delta I(0) + \delta I(0)\, \delta I(t) \rangle$$

Current fluctuation: $\delta I(t) \coloneqq I(t) - \langle I \rangle$

The proportional constant $\boldsymbol{e}^{*}$

**Effective charge of current carrying states**

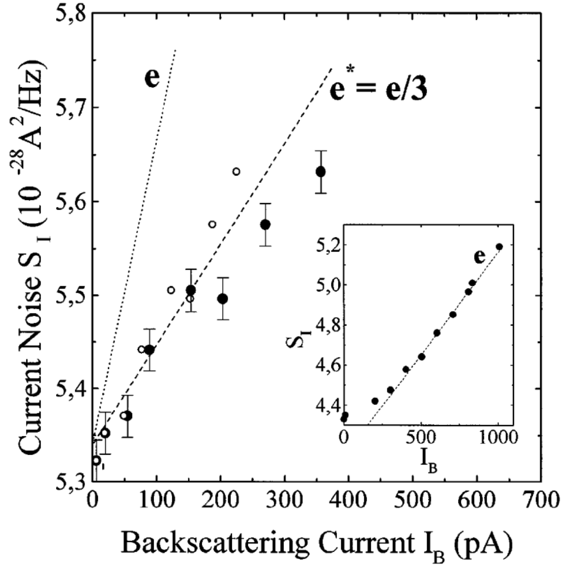
# Effective charges of many-body systems

## Fractional quantum Hall system

Fractional charges of Laughlin's quasi-particle: $e^* = e/3$, etc.

Picciotto, *et al.*, Nature **389**, 162 (1997).

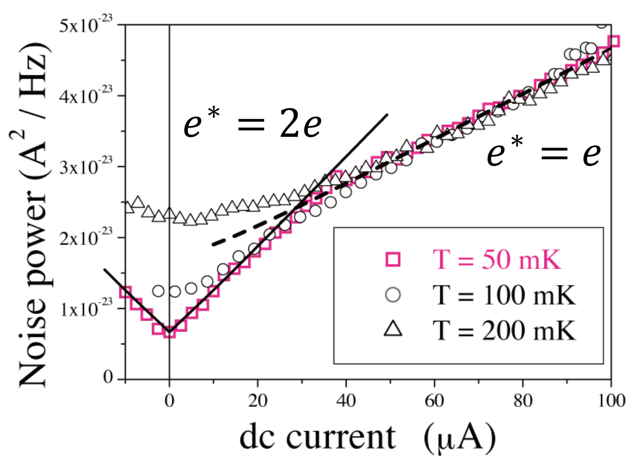
Saminadayar, *et al.*, PRL **79**, 2526 (1997).




## Superconductor/normal metal junction

Cooper pair charge: $e^* = 2e$

Lefloch, *et al.*, PRL **90**, 067002 (2003).



# Question

Effective charge of Kondo-correlated quantum dots?

# Our model

## Impurity Anderson model

$$\mathcal{H}_A = \mathcal{H}_0 + \mathcal{H}_I$$

- lead part $(\alpha = L, R)$ and quantum dot

$$\mathcal{H}_0 = \sum_{k\alpha\sigma} \varepsilon_k c^{+}_{k\alpha\sigma} c_{k\alpha\sigma} + \sum_{\sigma} \epsilon_d d^{+}_{\sigma} d_{\sigma}$$

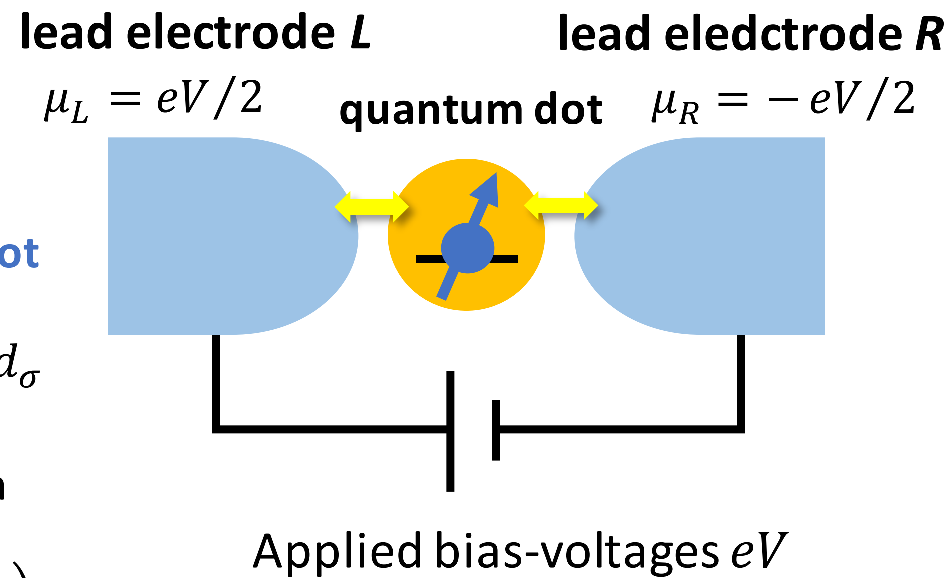
$\sigma$: spin

$$+ \sum_{k\alpha} \left( \frac{v_\alpha}{\sqrt{\mathcal{N}}} d^{+}_{\sigma} c_{k\alpha\sigma} + \mathrm{H.c.} \right)$$

lead-dot tunneling

- Coulomb interaction

$$\mathcal{H}_\mathrm{I} = U d^{+}_{\uparrow} d_{\uparrow} d^{+}_{\downarrow} d_{\downarrow}$$




Dot level linewidth: $\Gamma = \frac{1}{2}(\Gamma_L + \Gamma_R)$

$$\Gamma_\alpha := 2\pi \rho_c v_\alpha^2$$

$\rho_c$: Density of state for conduction electrons in lead electrods

# Reorganized perturbation

## Original model

Perturbation expansion up to *infinite* order in interaction $U$

→ Exact transport quantities up to $V^3$ (Local Fermi liquid)

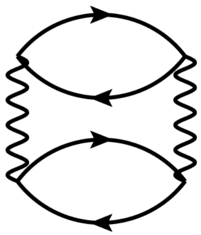
↓ We know how the parameters are renormalized.

## Quasiparticle model (local Fermi liquid)

Perturbation expansion up to *2nd* order in Fermi-liquid interaction $\widetilde{U}$

+ counter term to avoid overcounting

→ Exact transport quantities up to $V^3$ (Local Fermi liquid)



**good point**

Contribution decomposed to perturbation processes in Fermi-liquid interaction $\widetilde{U}$.

Yamada, PTP **53**, 970 (1975); Nozieres, JLTP **17**, 31 (1974); Wilson, RMP **47**, 773 (1975); Hewson, PRL **70**, 4007 (1993)
Oguri, PRB **64**, 153305 (2001)

# Quasiparticle Hamiltonian

**Replacement** **electron in dot** $d_\sigma \rightarrow$ **quasiparticle** $\tilde{d}_\sigma$

interaction $U \rightarrow \tilde{U}$

energy level $\epsilon_d \rightarrow \tilde{\epsilon}_d$

tunneling $v_\alpha \rightarrow \tilde{v}_\alpha$

$$\tilde{\mathcal{H}}_{qp} = \tilde{\mathcal{H}}_0 + \tilde{\mathcal{H}}_I$$

- **lead part** $(\alpha = L, R)$ **and quasiparticle**

$$\tilde{\mathcal{H}}_0 = \sum_{k\alpha\sigma} \varepsilon_k c^+_{k\alpha\sigma} c_{k\alpha\sigma} + \sum_\sigma \tilde{\epsilon}_d \tilde{d}^+_\sigma \tilde{d}_\sigma + \sum_{k\alpha} \left( \frac{\tilde{v}_\alpha}{\sqrt{\mathcal{N}}} \tilde{d}^+_\sigma c_{k\alpha\sigma} + \text{H.c.} \right)$$

$\sigma$: spin

**lead-quasiparticle tunneling**

- **Fermi liquid interaction**

$$\mathcal{H}_\mathrm{I} = \tilde{U} \tilde{d}^+_\uparrow \tilde{d}_\uparrow \tilde{d}^+_\downarrow \tilde{d}_\downarrow$$

quasiparticle's linewidth: $\tilde{\Gamma} = \frac{1}{2}\left(\tilde{\Gamma}_L + \tilde{\Gamma}_R\right)$

$$\tilde{\Gamma}_\alpha := 2\pi \rho_c \tilde{v}_\alpha^2$$

# Electric current

Oguri, PRB **64**, 153305 (2001)

## Low-bias current through spin Kondo dot

$$I = \frac{2e^2}{h} V \left[ \mathbf{1} - \left[ \frac{\mathbf{1}}{\mathbf{12}} + \frac{\mathbf{5}}{\mathbf{12}} \left( \frac{\widetilde{U}}{\pi \widetilde{\Gamma}} \right)^2 \right] \left( \frac{eV}{\widetilde{\Gamma}} \right)^2 \right] + O(V^4)$$

$$=: I_b$$

**$V$-linear term**:

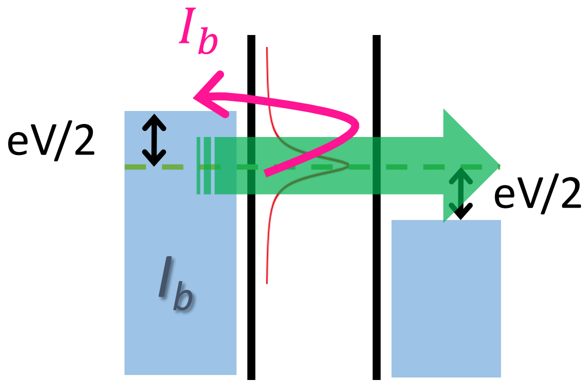
Free quasiparticle contribution

Perfect transmission

**$V^3$ term**:

Fermi-liquid interaction enhances the backscattering current

with small probability **"Poissonian process".**




**Noise-current ratio $\frac{S}{2eI_b}$ for the backscattering current**

# Noise-current ratio

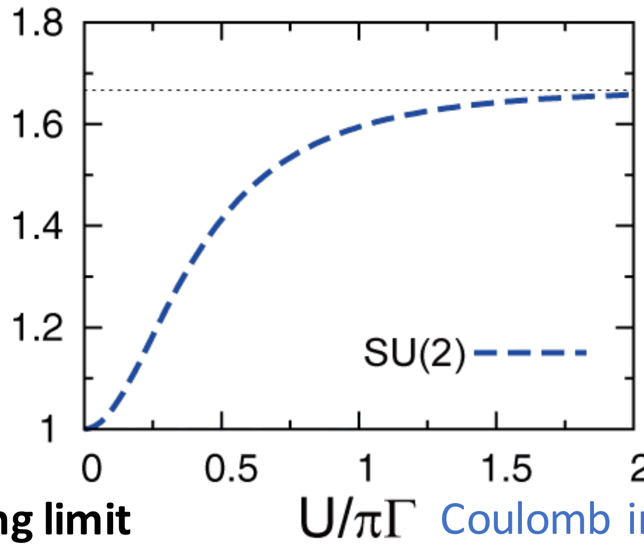
**Local-Fermi-liquid calculation**

$$\frac{S}{2eI_b} = \frac{1+9\left(\frac{\widetilde{U}}{\pi\widetilde{\Gamma}}\right)^2}{1+5\left(\frac{\widetilde{U}}{\pi\widetilde{\Gamma}}\right)^2}$$

Fermi-liquid interaction $\frac{\widetilde{U}}{\pi\widetilde{\Gamma}}$

Exact solution

$\frac{S}{2eI_b}$




**5/3**

**Kondo limit**

**Non-interacting limit**

Coulomb interaction

Gogolin and Komnik, PRL **97,** 016602 (2006); Sela and Malecki, PRB **80**, 233103 (2009); Fujii, JPSJ **79**, 044714 (2010); RS, Fujii, and Oguri, PRB **83**, 075440 (2011)

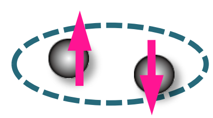
# The meaning of the values of $\frac{S}{2eI_b}$

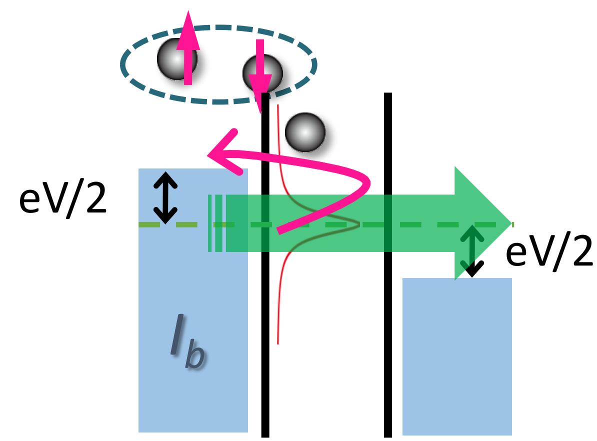
## Analysis of scattering process in quasiparticle picture

### Backscattering current

- Single quasiparticle scattering with charge $\boldsymbol{e}$

- Interacting quasiparticles pair with charge $\mathbf{2}\boldsymbol{e}$





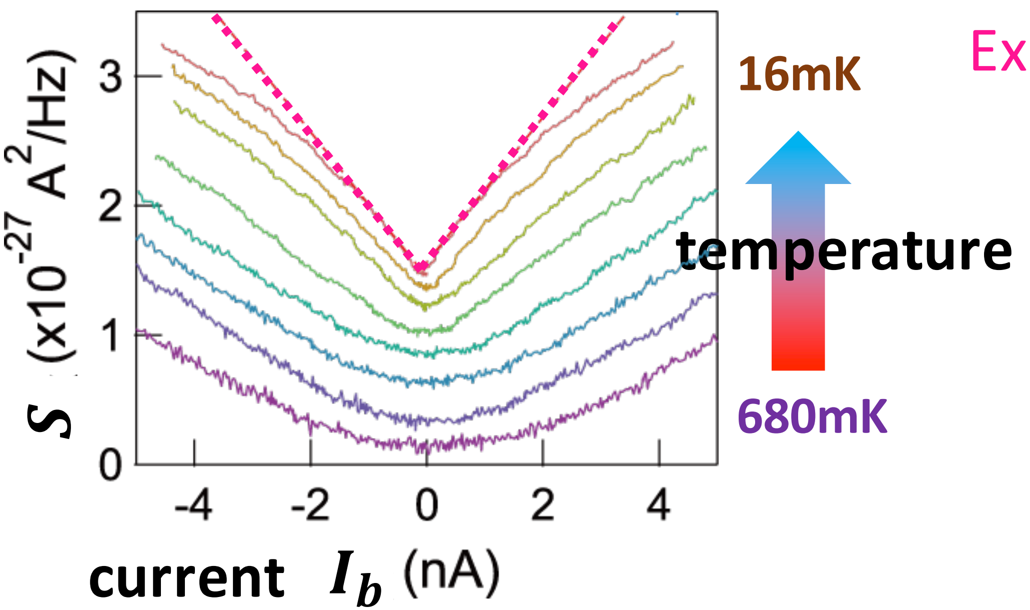

$$1 \leq \frac{S}{2eI_b} < 2$$

**“Not elemental charge of excited states,**
**but evidence of the charge pair creation by the Fermi-liquid interaction.**

# Experiment in SU(2) Kondo dot

M. Ferrier, RS *et al.*, Nat. Phys. (2015)

## Shot noise measurement




Experiment: $\frac{S}{2eI_b} = 1.7 \pm 0.1$

**Good agreement with**

theory: $\frac{S}{2eI_b} = 1.66$ ...

**Direct measurement on "charge pair" creation.**

This is not our goal, yet.

# Bell's pair creation
# by local-Fermi-liquid interaction

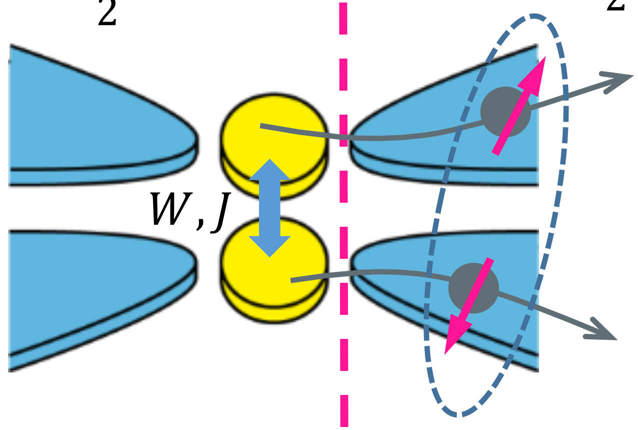
# Double quantum dot device

**Motivation**

**Testing "quantum" correlation of quasiparticle pair**

$$\mu_{1/2L} = -\frac{eV}{2}$$

$$\mu_{1/2R} = +\frac{eV}{2}$$

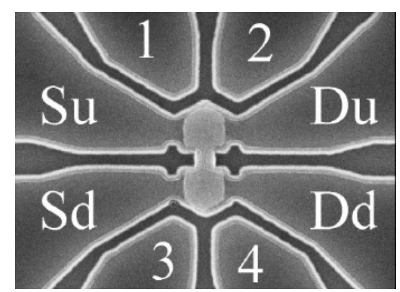



**Generator part**

**Detector part**

- Coulomb interaction
- Exchange coupling

**Correlation between channels**




A. Hübel, et al., PRL 101, 186804 (2008)

# Bell’s correlation in current

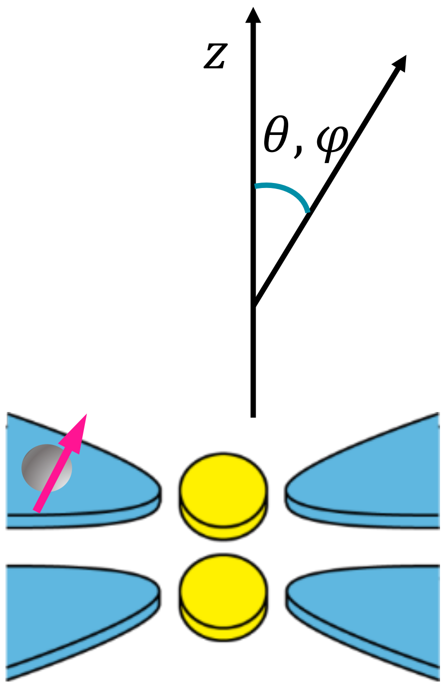
Chtchelkatchev *et al.*, PRB **66**, 161320(R) (2002)

Experimental observable quantity

**Electric current**

## Counting number of electrons

coming from lead $\alpha$ =Left,Right to the dot
with spin orientating $\theta$
through channel $m = 1{,}2$
in time interval $[t, t+\tau]$




$$N_{Rm\theta}(t, t+\tau) = \frac{1}{e}\int_{t}^{t+\tau} dt' I_{Rm\theta}(t')$$

**Generated spin per a particle**

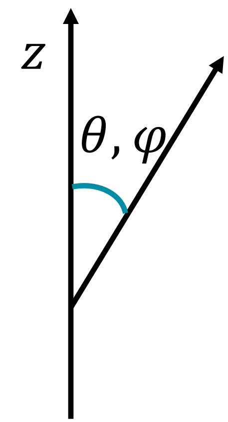
$$\frac{N_{\alpha m\uparrow_\theta} - N_{\alpha m\downarrow_\theta}}{N_{\alpha m\uparrow_\theta} + N_{\alpha m\downarrow_\theta}}$$

**Bell’s correlation**

# CHSH type Bell's correlation

**Current-current correlation with twisted spin orientation**

$$S(\theta,\varphi) \coloneqq \int dt \langle \delta I_{R1\theta}(t)\ \delta I_{R2\varphi}(0) \rangle$$



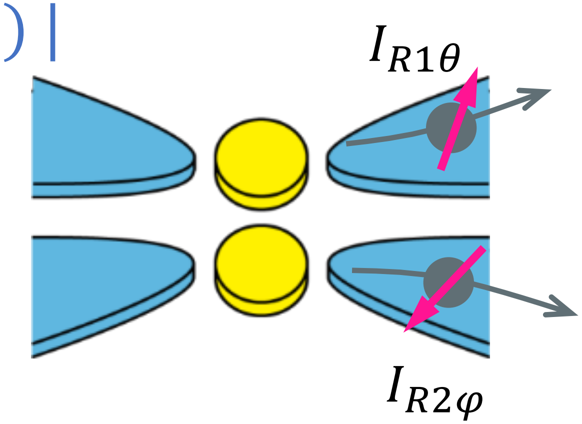

Current fluctuation: $\delta I_{\alpha m \theta}(t) \coloneqq I_{\alpha m \theta}(t) - \langle I_{\alpha m \theta} \rangle$

## CHSH type Bell's correlator

Four spin angle: $\theta, \theta', \varphi, \varphi'$

$$\boldsymbol{C = |F(\theta,\varphi) - F(\theta',\varphi) + F(\theta,\varphi') + F(\theta',\varphi')|}$$

$$F(\theta,\varphi) \coloneqq \frac{S(\theta,\varphi) - S(\theta+\pi,\varphi) - S(\theta,\varphi+\pi) + S(\theta+\pi,\varphi+\pi)}{S(\theta,\varphi) + S(\theta+\pi,\varphi) + S(\theta,\varphi+\pi) + S(\theta+\pi,\varphi+\pi)}$$

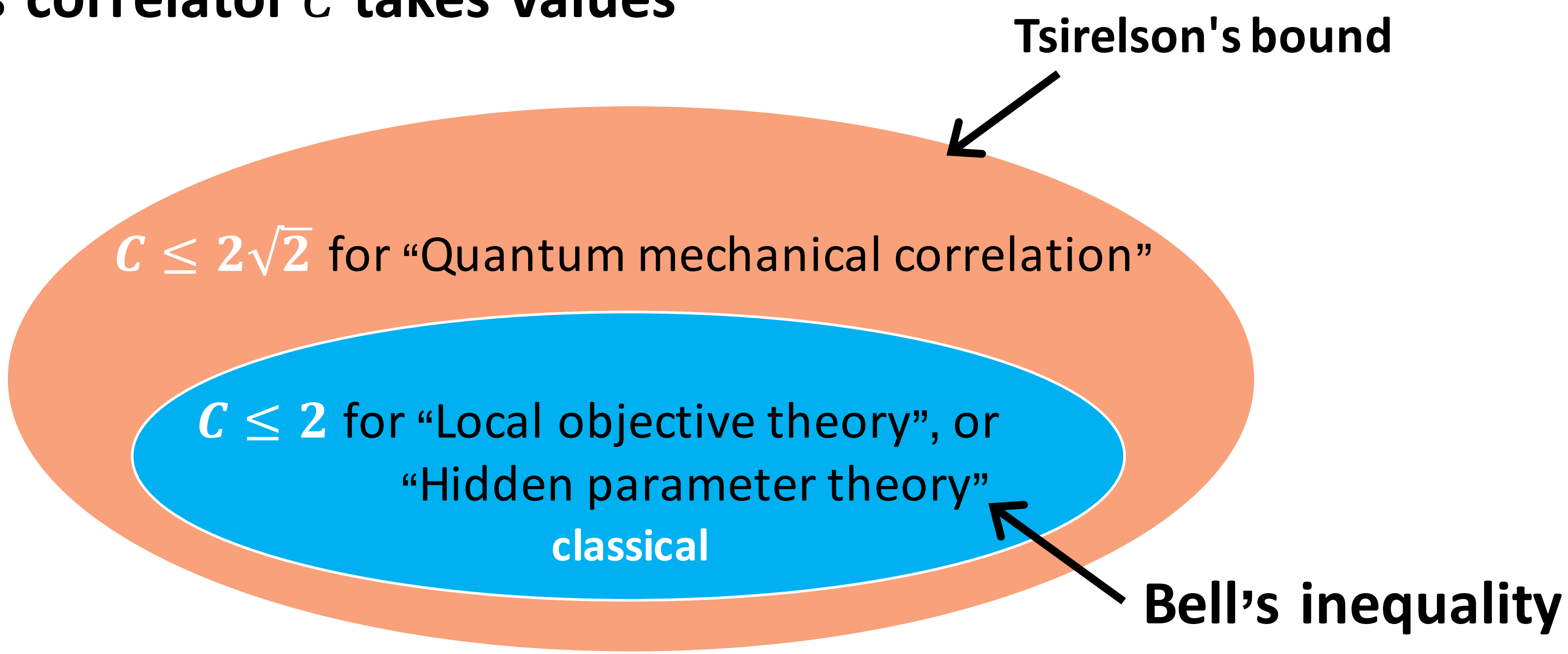




Kawabata, JPSJ **70**, 1210 (2001); Chtchelkatchev *et al.*, PRB **66**, 161320(R) (2002).

# Upper bound of the CHSH Bell's correlator

**Bell's correlator $C$ takes values**

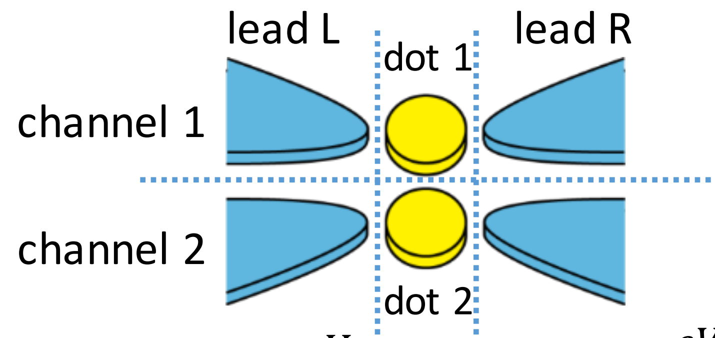



Sufficient condition for quantum correlation

$$2 < C \leq 2\sqrt{2}$$

# Orbital degenerate impurity Anderson model

$$\mathcal{H}_A = \mathcal{H}_0 + \mathcal{H}_T + \mathcal{H}_I$$




$$\mu_{1/2L} = -\frac{eV}{2} \qquad \mu_{1/2R} = +\frac{eV}{2}$$

$$\mathcal{H}_0 = \sum_{k\alpha m\sigma} \varepsilon_k c^+_{k\alpha m\sigma} c_{k\alpha m\sigma} + \sum_m \epsilon_d d^+_{m\sigma} d_{m\sigma}$$

**Electric lead $\alpha = L, R$**

dot

*m=1,2* : channel

$\sigma$ =↑,↓ : spin

$$\mathcal{H}_T = \sum_{k\alpha m} \left( \frac{v_\alpha}{\sqrt{\mathcal{N}}} d^+_{m\sigma} c_{k\alpha m\sigma} + \text{H.c.} \right)$$

**Tunneling between lead and dot**

"Conservation of spin and channel" ➡ ***Local Fermi liquid state***

$$\mathcal{H}_I = \sum_{m=1,2} U d^+_{m\uparrow} d_{m\uparrow} d^+_{m\downarrow} d_{m\downarrow} + \sum_{\sigma\sigma'} W d^+_{1\sigma} d_{1\sigma} d^+_{2\sigma'} d_{2\sigma'}$$

*Intra-* Coulomb repulsion

*Inter*-dot Coulomb repulsion

$$+2J\mathbf{S}_{d1} \cdot \mathbf{S}_{d2'}$$

**"Exchange coupling *J***

- **particle-hole symmetric**: $\epsilon_d = -U/2 - (M-1)W$

**symmetric coupling**: $v_L = v_R$

# Many-body effect

## Quasiparticle Hamiltonian

Replacement

**Electron in dot** $d_{m\sigma}$ → **Quasiparticle** $\tilde{d}_{m\sigma}$

interaction $U, W, J$ → $\tilde{U}, \tilde{W}, \tilde{J}$

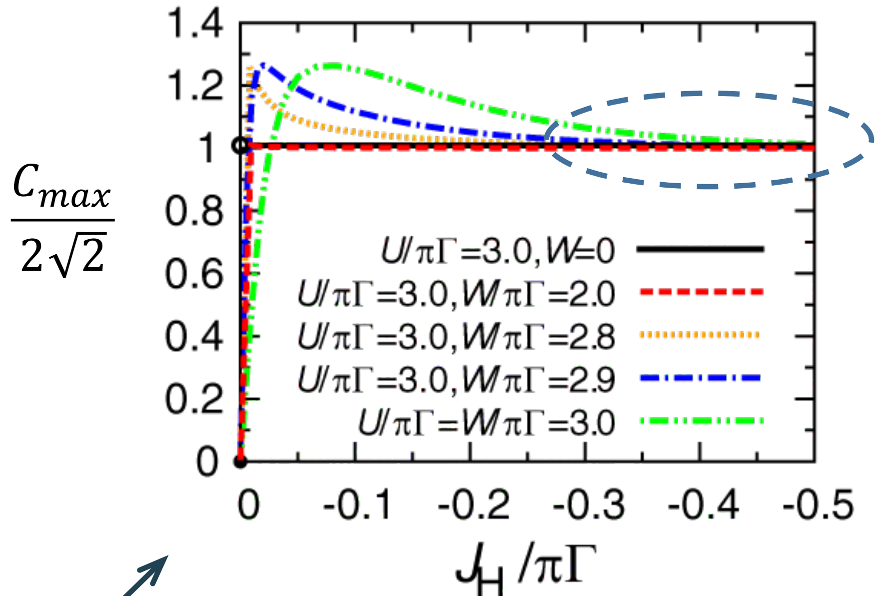
energy level $\epsilon_d$ → $\tilde{\epsilon}_d$

tunneling $v_\alpha$ → $\tilde{v}_\alpha$

Second order perturbation in local-Fermi-liquid interactions $\tilde{U}, \tilde{W}, \tilde{J}$

\+ counter term to avoid overcounting

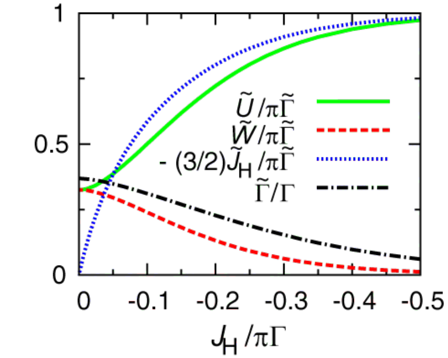
# Maximum value of Bell's correlator



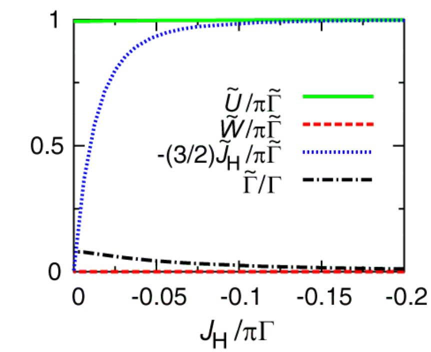

Ferromagnetic coupling

**Numerical renormalization Group for Fermi liquid interactions**

$U/\pi\Gamma = W/\pi\Gamma = 3$



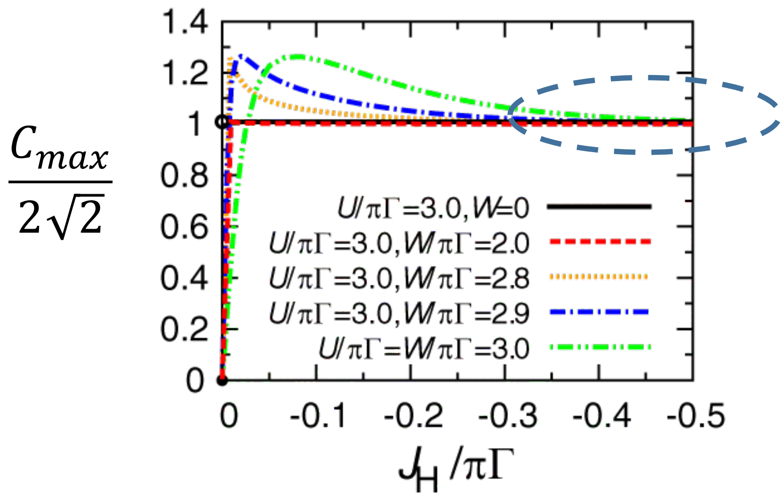

$U/\pi\Gamma = 3, W = 0$




$J = 0$

Coulomb interactions $U, W$ are independent of spin orientation.

# Large ferromagnetic coupling



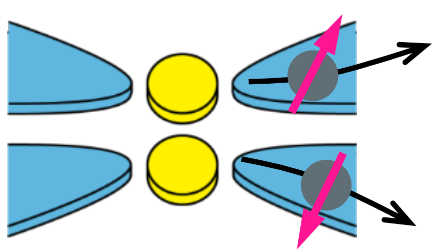

Suppression of charge fluctuation between channels $\widetilde{W} \to 0$

Maximum value of the Bell correlator

$$C_{max} \to 2\sqrt{2}$$

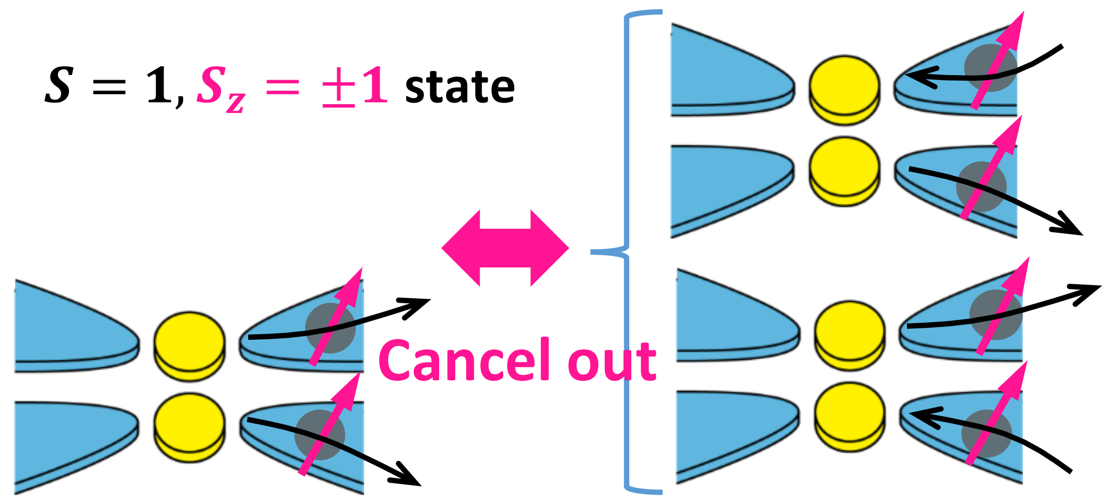
## Spin triplet Bell's pair

**$S = 1, S_z = 0$ state**



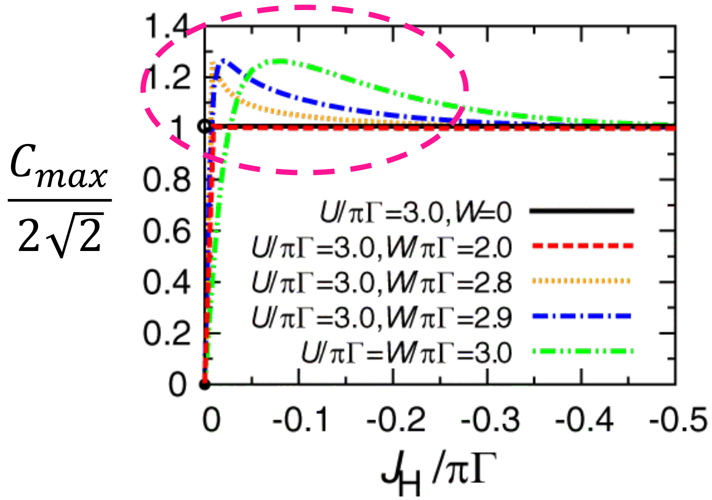
Only this state is observed!

**$S = 1, S_z = \pm 1$ state**




sign of current-current correlation opposite

# Larger than quantum upper bound?

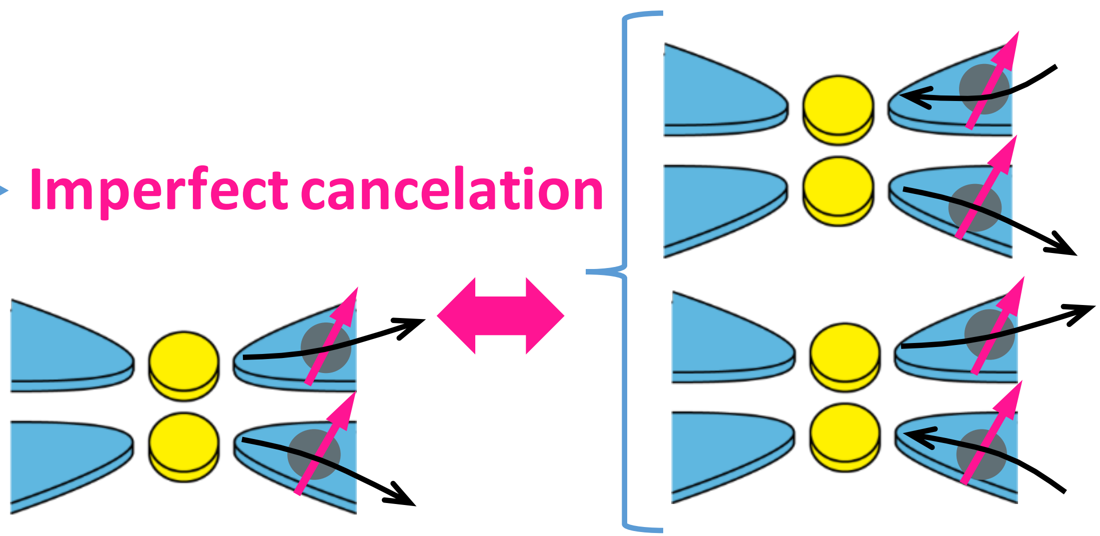



Maximum value of the Bell correlator

$$C_{max} > 2\sqrt{2}$$ **why?**

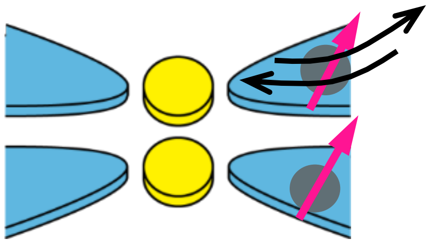
$S = 1, S_z = \pm 1$ state

intermidiate $\widetilde{W}$ → **Imperfect cancelation**



# Triplet pair $S = 1, S_z = \pm 1$ state

Particles are generated in two directions in each channel.



Simple integration

$$N_{Rm\theta}(t, t+\tau) = \frac{1}{e}\int_{t}^{t+\tau} dt' I_{Rm\theta}(t')$$

➡ **Underestimated number of generated particles.**

**Conjecture**

Normalization of Bell's correlator is failed.

➡ $C_{max} > 2\sqrt{2}$ **(Tsirelson's bound)**

# Conclusion

## Double quantum dot in local Fermi liquid state (Kondo state)

Exchange-type interaction

➡ Quasiparticle triplet pair

Observable Bell pair

$$S = 1, S_z = 0$$