Kaon-Nucleon interaction in the Skyrme model

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Contents

2017/05/26 Strangeness and charm in hadrons and dense matter @ YITP, Kyoto Univ.

1. Introduction

Introduction

Kaon nucleon systems are very attractive

- Strong attraction between the anti-kaon($\bar{K}$) and the nucleon(N)

  Y. Akaishi and T. Yamazaki, Phys. Rev. C **65** (2002)
- $\bar{K}$N bound state = $\Lambda(1405)$
- Few body nuclear system with $\bar{K}$ → under debate

$\bar{K}$N interaction is important
to investigate the few body systems with $\bar{K}$

Theoretical studies of $\bar{K}$N interaction

- Phenomenological approach

Y. Akaishi and T. Yamazaki, Phys. Rev. **C 65** (2002) etc

- Chiral theory: based on a 4-point local interaction

T. Hyodo and W. Weise, Phys. Rev. **C 77** (2008)

K. Miyahara and T. Hyodo, Phys. Rev. C **93** (2016) etc

Investigate the $\bar{K}$N system in the Skyrme model
where the nucleon is described as a soliton.

2. Method

The Skyrme model and our ansatz

・ Skyrme model   T.H.R. Skyrme, Nucl. Phys. **31** (1962); Proc. Roy. Soc. A **260** (1961)

- Describe the meson-baryon interaction by mesons
- Baryon emerges as a soliton of meson fields.

$$L = \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 + L_{SB} + L_{WZ}$$

F_π, e: parameter m_π: massless, m_K: massive

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- **Ansatz**

$$U = (3\times 3 \text{ matrix}) \to A(t)\sqrt{U_\pi}U_K\sqrt{U_\pi}A^\dagger(t)$$

C.G. Callan and I. Klebanov, Nucl. Phys. **B 262** (1985)

C .G.Callan, K .Hornbostel and I. Klebanov, Phys. Lett. **B 202** (1988)

$$U_\pi = \begin{pmatrix} U_H & 0 \\ 0 & 1 \end{pmatrix}$$

U_H: Hedgehog soliton (2×2 matrix)

$$U_K = \exp\left[i\frac{2}{F_\pi}\lambda_a K_a\right], \quad a = 4,5,6,7$$

$$\lambda_a K_a = \sqrt{2}\begin{pmatrix} 0_{2\times 2} & K \\ K^\dagger & 0 \end{pmatrix} \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \quad K^\dagger = \begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix}$$

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$$\begin{cases} U_\pi \to A(t)U_\pi A^\dagger(t) \\ U_K = U_K \end{cases}$$

$A(t)$: isospin rotation matrix

$$U = A(t)\sqrt{U_\pi}A^\dagger(t)U_K A(t)\sqrt{U_\pi}A^\dagger(t)$$

T. Ezoe. and A. Hosaka Phys. Rev. D **94**, 034022 (2016)

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- **Ansatz**

Kaon and hedgehog soliton system

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$A(t)$: isospin rotation matrix

Kaon and "rotating" hedgehog soliton system

$$U = A(t)\sqrt{U_\pi}A^\dagger(t)\, U_K\, A(t)\sqrt{U_\pi}A^\dagger(t)$$

T. Ezoe. and A. Hosaka Phys. Rev. D **94**, 034022 (2016)

Derivation 1

・Substitute our ansatz for the Lagrangian

Ansatz

$$U = A(t)\sqrt{U_\pi}A^\dagger(t)U_K A(t)\sqrt{U_\pi}A^\dagger(t)$$

$$U_\pi = \begin{pmatrix} U_H & 0 \\ 0 & 1 \end{pmatrix}$$

U_H: Hedgehog soliton (2×2 matrix)

$$U_K = \exp\left[i\frac{2}{F_\pi}\lambda_a K_a\right], \quad a = 4, 5, 6, 7$$

$$\lambda_a K_a = \sqrt{2}\begin{pmatrix} 0_{2\times 2} & K \\ K^\dagger & 0 \end{pmatrix} \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \quad K^\dagger = \begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix}$$

Lagrangian

$$L = \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 + L_{SB} + L_{WZ}$$

・Expand U_K up to second order of the kaon field K

Obtained Lagrangian

$$L = L_{SU(2)} + L_{KN}$$

$$L_{SU(2)} = \frac{1}{16}{F_\pi}^2 \mathrm{tr}\left[\partial_\mu \tilde{U}^\dagger \partial^\mu \tilde{U}\right] + \frac{1}{32e^2}\mathrm{tr}\left[\partial_\mu \tilde{U}\tilde{U}^\dagger, \partial_\nu \tilde{U}\tilde{U}^\dagger\right]^2$$

$$\begin{aligned} L_{KN} \;=\;& \left(D_\mu K\right)^\dagger D^\mu K - K^\dagger a_\mu^\dagger a^\mu K - m_K^2 K^\dagger K \\ & + \frac{1}{(eF_\pi)^2}\left\{ -K^\dagger K \mathrm{tr}\left[\partial_\mu \tilde{U}\tilde{U}^\dagger, \partial_\nu \tilde{U}\tilde{U}^\dagger\right]^2 - 2\left(D_\mu K\right)^\dagger D_\nu K \mathrm{tr}\left(a^\mu a^\nu\right) \right. \\ & \left. \qquad - \frac{1}{2}\left(D_\mu K\right)^\dagger D^\mu K \mathrm{tr}\left(\partial_\nu \tilde{U}^\dagger \partial^\nu \tilde{U}\right) + 6\left(D_\nu K\right)^\dagger \left[a^\nu, a^\mu\right] D_\mu K \right\} \\ & + \frac{3i}{F_\pi^2} B^\mu \left[\left(D_\mu K\right)^\dagger K - K^\dagger \left(D_\mu K\right)\right] \end{aligned}$$

$$\tilde{U} = A(t) U_H A^\dagger(t), \quad \tilde{\xi} = A(t)\sqrt{U_H} A^\dagger(t) \qquad D_\mu K = \partial_\mu K + v_\mu K$$

$$v_\mu = \frac{1}{2}\left(\tilde{\xi}^\dagger \partial_\mu \tilde{\xi} + \tilde{\xi}\partial_\mu \tilde{\xi}^\dagger\right)$$

$$a_\mu = \frac{1}{2}\left(\tilde{\xi}^\dagger \partial_\mu \tilde{\xi} - \tilde{\xi}\partial_\mu \tilde{\xi}^\dagger\right)$$

$$B^\mu = -\frac{\varepsilon^{\mu\nu\alpha\beta}}{24\pi^2}\mathrm{tr}\left[\left(\tilde{U}^\dagger \partial_\nu \tilde{U}\right)\left(\tilde{U}^\dagger \partial_\alpha \tilde{U}\right)\left(\tilde{U}^\dagger \partial_\beta \tilde{U}\right)\right]$$

G. S. Adkins, C. R. Nappi and E. Witten, Nucl. Phys. B **228** (1983)

Derivation 2

・Decompose the kaon filed

$$\begin{pmatrix} K^+ \\ K^0 \end{pmatrix} = \psi_I K\left(t, \boldsymbol{r}\right) \rightarrow \psi_I \; K\left(\boldsymbol{r}\right) e^{-iEt}$$

Spatial wave function ($K(\boldsymbol{r})$)

Isospin wave function (ψ_I)

・Expand the $K(r)$ by the spherical harmonics

$$K\left(\boldsymbol{r}\right) = \sum_{l,m} C_{lm\alpha} Y_{lm}\left(\theta, \phi\right) k_l^{\alpha}\left(r\right)$$

$Y_{lm}(\theta, \varphi)$: Spherical harmonics

l : orbital angular momentum

m : the 3rd component of l

α : the other quantum numbers

・Take a variation with respect to the kaon radial function

⇒Obtain the equation of motion for the kaon around the nucleon

3. Results and discussions

Equation of motion and potential

・Equation of motion(E.o.M)

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2 h(r)\frac{dk_l^{\alpha}(r)}{dr}\right) - E^2 f(r) k_l^{\alpha}(r) + \left(m_K^2 + V(r)\right) k_l^{\alpha}(r) = 0$$:Klein-Gordon like

Equation of motion and potential

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$$\longrightarrow -\frac{1}{m_K+E}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dk_l^\alpha(r)}{dr}\right) + U(r) k_l^\alpha(r) = \varepsilon k_l^\alpha(r)$$:Schrödinger like

$(E = m_K + \varepsilon)$

$$\begin{aligned} U(r) &= -\frac{1}{m_K+E}\left[\frac{h(r)-1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right) + \frac{dh(r)}{dr}\frac{d}{dr}\right] - \frac{(f(r)-1)E^2}{m_K+E} + \frac{V(r)}{m_K+E} \\ &= U_0^c(r) + U_\tau^c(r)\boldsymbol{\tau}^K\cdot\boldsymbol{\tau}^N + \left(U_0^{LS}(r) + U_\tau^{LS}(r)\boldsymbol{\tau}^K\cdot\boldsymbol{\tau}^N\right)\boldsymbol{L}\cdot\boldsymbol{S} \end{aligned}$$

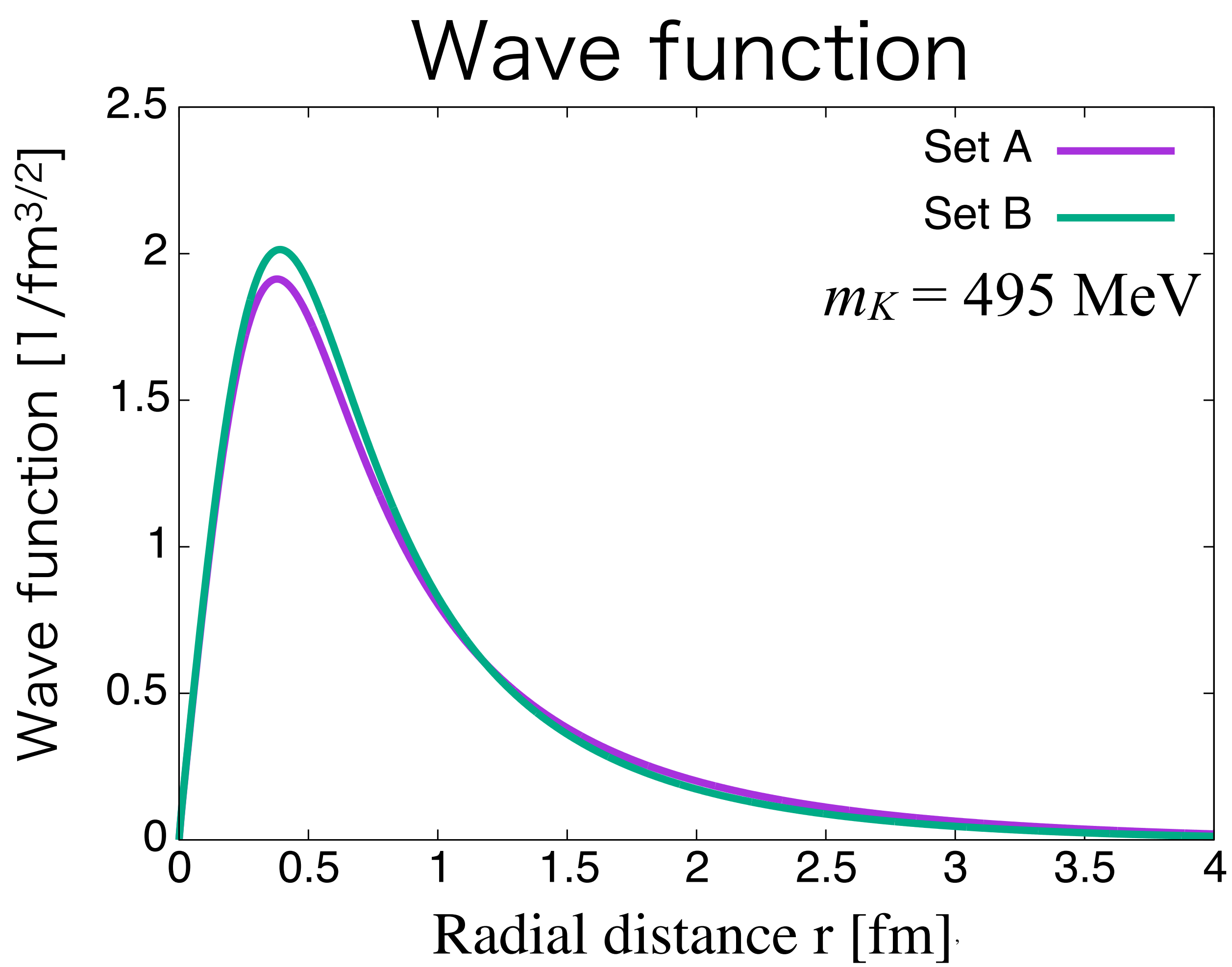
• Properties of resulting potential U

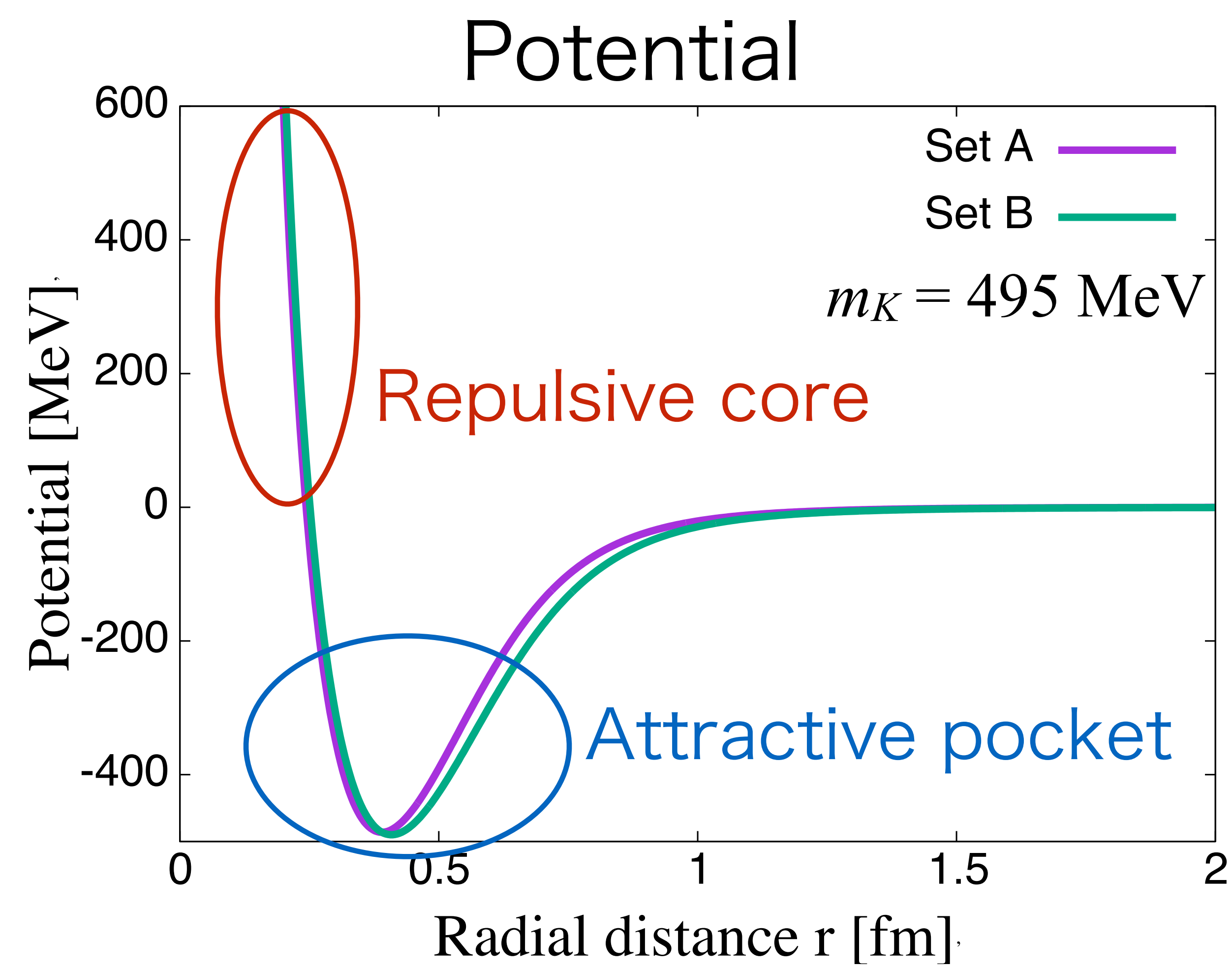
1. Nonlocal and depend on the kaon energy
2. Contain isospin dependent and independent **central forces** and the similar **spin-orbit(LS) forces**
3. A repulsive component is proportional to $1/r^2$ at short distances

• Euivalent local potential:

$$\tilde{U}(r) = \frac{U(r)\,k_l^\alpha(r)}{k_l^\alpha(r)}$$

$\bar{K}N$ ($J^P = 1/2^-$, $L^K = 0$, $I = 0$) Bound state





• Parameter sets and Bound state properties

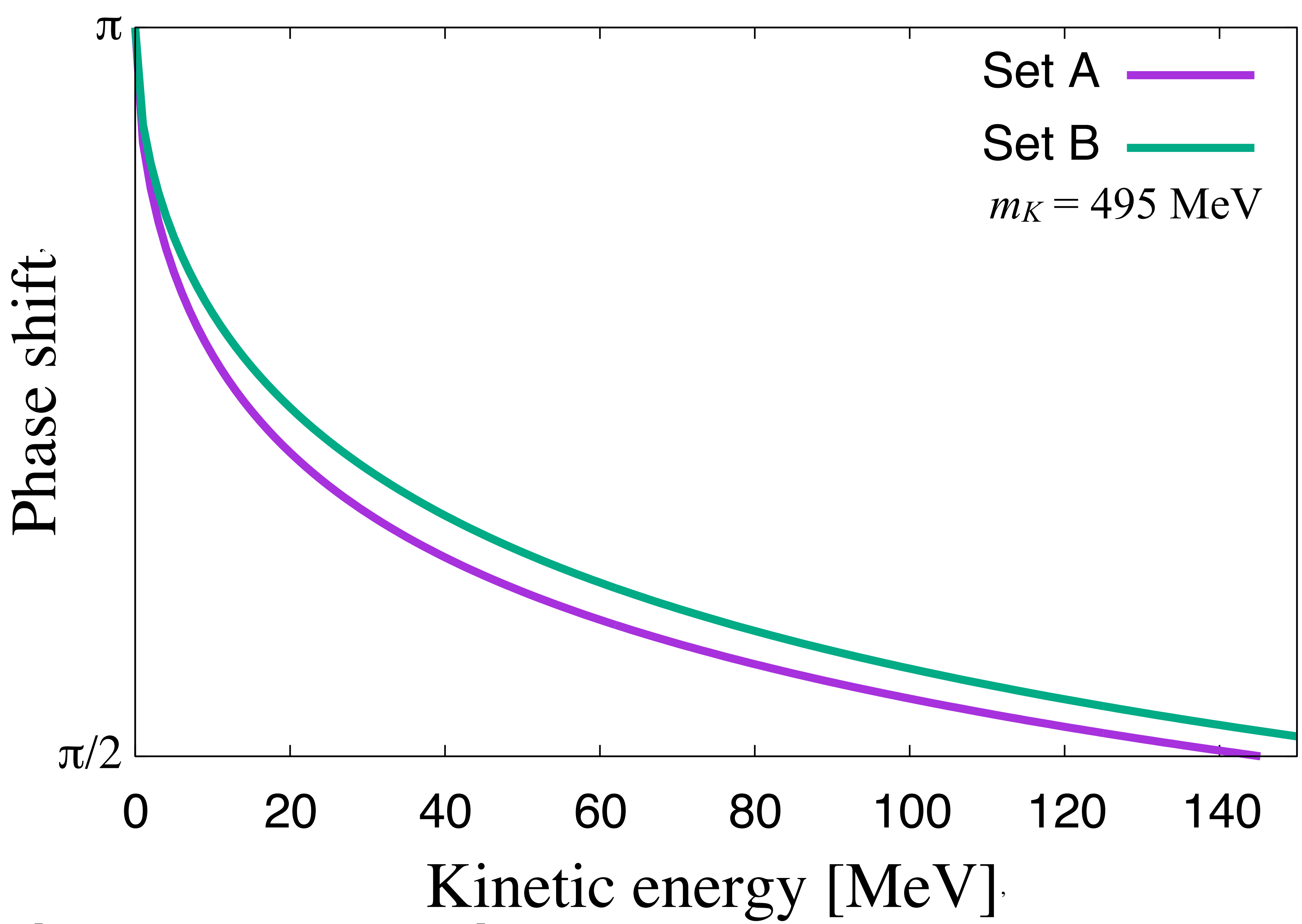
	F_π [MeV]	e	B.E. [MeV]	$\sqrt{\langle r_N^2 \rangle}$	$\sqrt{\langle r_K^2 \rangle}$
Parameter set A	205	4.67	19.9	0.43	1.30
Parameter set B	186	4.82	32.2	0.46	1.15

$$\langle r_N^2 \rangle = \int_0^\infty dr\ r^2 \rho_B(r), \quad \rho_B(r) = -\frac{2}{\pi} \sin^2 F F'$$

G. S. Adkins, C. R. Nappi and E. Witten, Nucl. Phys. B **228** (1983)

$$\langle r_K^2 \rangle = \int dV\ r^2 \left[Y_{00}(\hat{r})\, k_0^0(r) \right]^2 = \int_0^\infty dr\ r^4 k^2(r) \qquad Y_{00} = \frac{1}{\sqrt{4\pi}}$$

$\bar{K}N$ ($J^P = 1/2^-$, $L^K = 0$, $I = 0$) Scattering state



cf) Bound state properties

	F_π [MeV]	e	B.E. [MeV]	$\sqrt{\langle r_N^2 \rangle}$	$\sqrt{\langle r_K^2 \rangle}$
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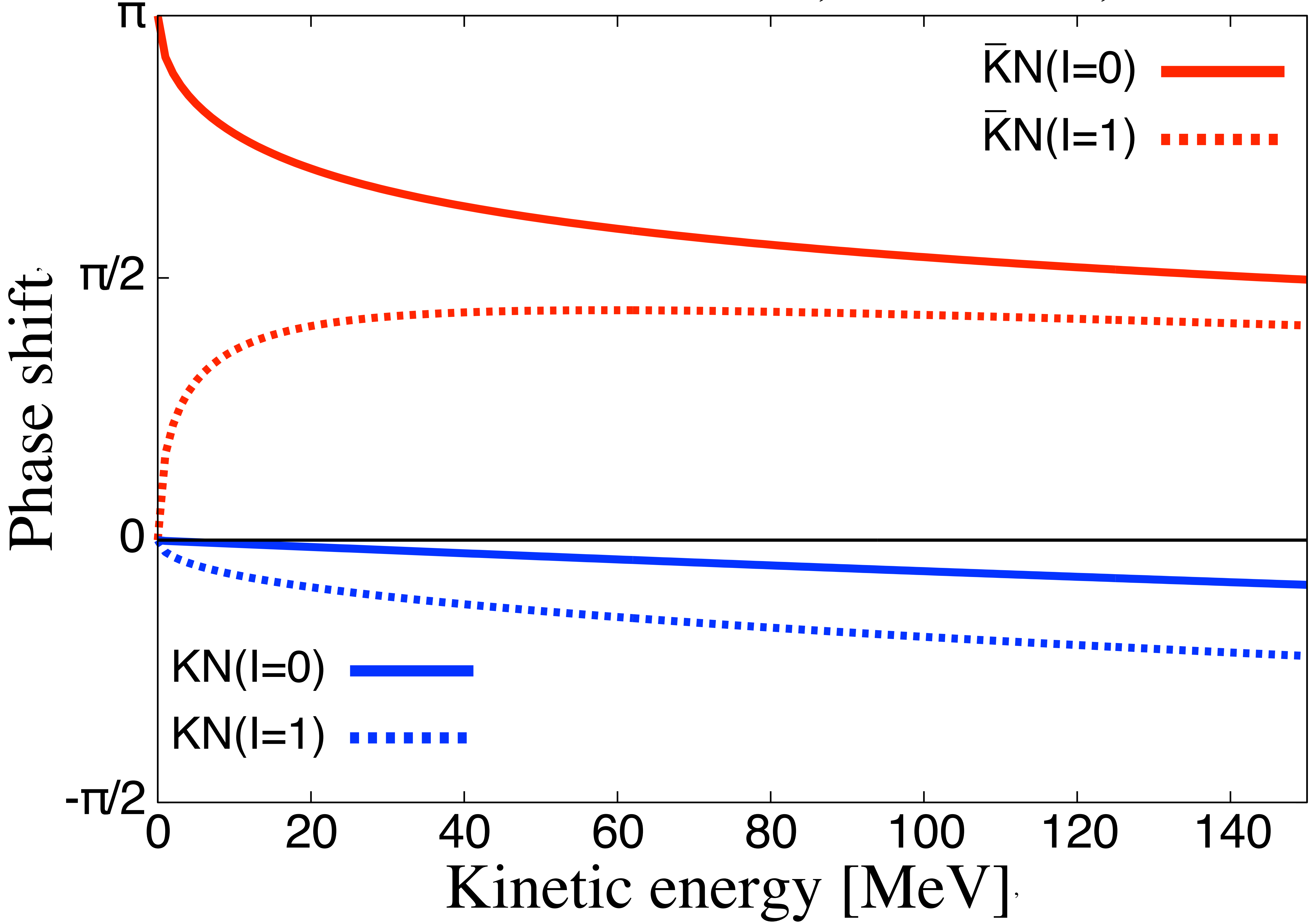
Kaon-Nucleon Scattering states

Channel: $J^P = 1/2^-$, $L^K = 0$

Parameter set A: $m_K = 495$ MeV, $F_\pi = 205$ MeV, $e = 4.67$



4. Summary

Summary

Investigate the kaon-nucleon systems
by a modified bound state approach in the Skyrme model

・Results

1. Properties of the obtained potential
   a. nonlocal and depend on the kaon energy
   b. contain **central and LS terms**
      with and without isospin dependence
   c. **repulsion proportional to $1/r^2$** for small r
2. $\bar{K}N(I=0)$ bound states exist with B.E. of order ten MeV
3. Phases as functions of energy reflect
   a. the properties of the bound state
   b. the quantitative properties of the kaon-nucleon interaction

・Future works

1. Properties of Λ(1405) (on-going)
2. Extension to the charm sector

Thank you for your attention!