Mesonic and nucleon fluctuation effects at finite baryon density

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GF & A. Hosaka, Phys. Rev. D **94**, 036005 (2016)
GF & A. Hosaka, arXiv:1701.03717

Outline

Motivation

Functional Renormalization Group

Chiral effective nucleon-meson theory at finite μ_B

Numerical results

Summary

FLUCTUATION EFFECTS IN FIELD THEORY:

- 2nd order transitions in statistical field theory
  $\longrightarrow$ diverging correlation length invalidates PT
  $\longrightarrow$ solution: Wilson's momentum space RG
  $\longrightarrow$ explanation of universality, critical exponents, etc.

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If one is interested in the finite T and density behavior of the strongly interacting matter, fluctuation effects are important.

Motivation

AXIAL ANOMALY OF QCD:

- $U_A(1)$ anomaly: anomalous breaking of the $U_A(1)$ subgroup of chiral symmetry
  $\longrightarrow$ vacuum-to-vacuum topological fluctuations (instantons)

$$\partial_\mu j_A^{\mu a} = -\frac{g^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left[T^a F_{\mu\nu} F_{\rho\sigma}\right]$$

- Induced $U_A(1)$ breaking interactions depend on instanton density
  $\longrightarrow$ suppressed at high T
  $\longrightarrow$ calculations are trustworthy only beyond T_C
  $\longrightarrow$ is the anomaly present around and below T_C?
- Recent lattice QCD simulations do not seem to have agreement on the issue[1,2]

[1] S. Sharma et al., Nucl. Phys. A **956**, 793 (2016)
[2] A. Tomiya et al., arXiv:1612.01908

Motivation

η' - NUCLEON BOUND STATE:

- Effective models at finite T and/or density:
  $\longrightarrow$ effective models (NJL[3], linear sigma models[4]) predict a drop in $m_{\eta'}$ at finite T and μ_B
- Effective description of the mass drop:
  $\longrightarrow$ attractive potential in medium $\Rightarrow$ $\eta' N$ bound state
  $\longrightarrow$ Analogous to $\Lambda(1405) \sim \bar{K}N$ bound state

[3] P. Costa, M. C. Ruivo & Yu. L. Kalinovsky, Phys. Lett. B 560, 171 (2003).
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- Problem with effective model calculations: they treat model parameters as environment independent constants
  $\longrightarrow$ „$a \cdot v$" type of terms decrease (a-constant, v-decreases)
  $\longrightarrow$ evolution of a at finite T and μ_B?

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CHIRAL EFFECTIVE NUCLEON-MESON MODEL:

- Chiral symmetry of QCD: $U(N_f) \times U(N_f)$ $(\Psi_{L/R} \to U_{L/R}\Psi_{L/R})$
- Effective model of mesons (M) and the nucleon (N):
  [M: π, K, η, η' and a_0, κ, f_0, σ, N: n, p]

$$\begin{aligned}\mathcal{L} &= \mathrm{Tr}\,[\partial_\mu M \partial^\mu M^\dagger] - \mu^2\,\mathrm{Tr}\,(MM^\dagger) - \frac{g_1}{9}[\,\mathrm{Tr}\,(MM^\dagger)]^2 \\ &- \frac{g_2}{3}\,\mathrm{Tr}\,(MM^\dagger)^2 - \mathrm{Tr}\,[H(M+M^\dagger)] - a(\det M + \det M^\dagger) \\ &+ \bar{N}(-\partial_i\gamma_i + \mu_B\gamma_0 - m_N)N - g\bar{N}\tilde{M}_5 N\end{aligned}$$

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\end{aligned}
$$

- Model parameters:
  $\longrightarrow$ meson mass parameter (μ^2), quartic couplings (g_1, g_2),
  $\longrightarrow$ explicit breaking ($H = h_0 T^0 + h_8 T^8$), $U_A(1)$ anomaly (a),
  $\longrightarrow$ nucleon mass parameter (m_N), Yukawa coupling (g)
- Short-range $N - N$ interactions: ω and ρ mesons

Functional Renormalization Group

- Fluctuation effects are included in the partition function and/or in the quantum effective action

$$Z[J] = \int \mathcal{D}\phi e^{-(\mathcal{S}[\phi]+\int J\phi)} \quad \Rightarrow \quad \Gamma[\bar{\phi}] = -\log Z[J] - \int J\bar{\phi}$$

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- Functional RG approach: generalized Wilsonian RG
- Similarities:
  $\longrightarrow$ provides a way to handle IR singularities
  $\longrightarrow$ gradually integrates out high momentum modes
- Differences:
  $\longrightarrow$ flow of the complete effective action is considered
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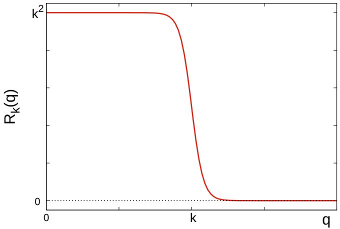
- Functional Renormalization Group provides a non-perturbative approach to resummation of fluctuation effects.

Mathematical implementation:

- Scale dependent partition function:

$$Z_k[J] = \int \mathcal{D}\phi e^{-(\mathcal{S}[\phi]+\int J\phi)} \times e^{-\frac{1}{2}\int \phi R_k \phi}$$



- Scale dependent effective action:

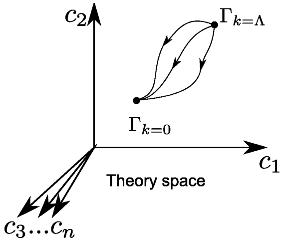
$$\Gamma_k[\bar{\phi}] = -\log Z_k[J] - \int J\bar{\phi} - \frac{1}{2}\int \bar{\phi} R_k \bar{\phi}$$

$\longrightarrow$ $k \approx \Lambda$: no fluctuations included

$\Rightarrow \Gamma_k[\bar{\phi}]|_{k=\Lambda} = \mathcal{S}[\bar{\phi}]$

$\longrightarrow$ $k = 0$: all fluctuations included

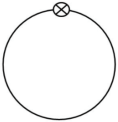
$\Rightarrow \Gamma_k[\bar{\phi}]|_{k=0} = \Gamma[\bar{\phi}]$



Chiral effective nucleon-meson model

- Flow equation of the effective action:

$$\partial_k \Gamma_k = \frac{1}{2}\int_{q,p}^{(T)} \partial_k R_k(q,p)(\Gamma_k^{(2)} + R_k)^{-1}(p,q) = \frac{1}{2}$$



- One-loop structure with dressed and regularized propagators

  $\longrightarrow$ RG change in the n-point vertices are described solely by one-loop diagrams

  $\longrightarrow$ exact relation, approximations are necessary

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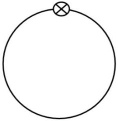

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- Derivative expansion (local potential approximation):

$$\Gamma_k = \int_x \Big[\mathrm{Tr}\,[\partial_\mu M \partial^\mu M^\dagger] + \bar{N}(-\partial_i\gamma_i + \mu_B\gamma_0 - m_N)N - V_k \Big]$$

$$V_k = \mu_k^2[M]\,\mathrm{Tr}\,(MM^\dagger) + \frac{g_{1,k}[M]}{9}[\,\mathrm{Tr}\,(MM^\dagger)]^2 + \frac{g_{2,k}[M]}{3}\,\mathrm{Tr}\,(MM^\dagger)^2$$
$$+ \;\mathrm{Tr}\,[H_k(M+M^\dagger)] + A_k[M](\det M + \det M^\dagger) + g_k[M]\bar{N}\tilde{M}_5 N$$

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- Step I.: solve equations at $T = 0 \Rightarrow$ determine parameters

$\longrightarrow$ μ, g_1, g_2, a: determined by fitting masses of π, K, η, η'

$\longrightarrow$ g: determined by fitting nucleon mass with m_N minimal

$\longrightarrow$ PCAC relations: $(\alpha = \pi, K)$

$$m_\alpha^2 f_\alpha \hat{\pi}_\alpha = \partial_\mu J_\alpha^{5\mu} = -\frac{\partial}{\partial \theta_A^\alpha} \mathrm{Tr}\left(H(M + M^\dagger)\right)$$

$$h_0 \approx (286 MeV)^3 \quad h_8 \approx -(311 MeV)^3$$

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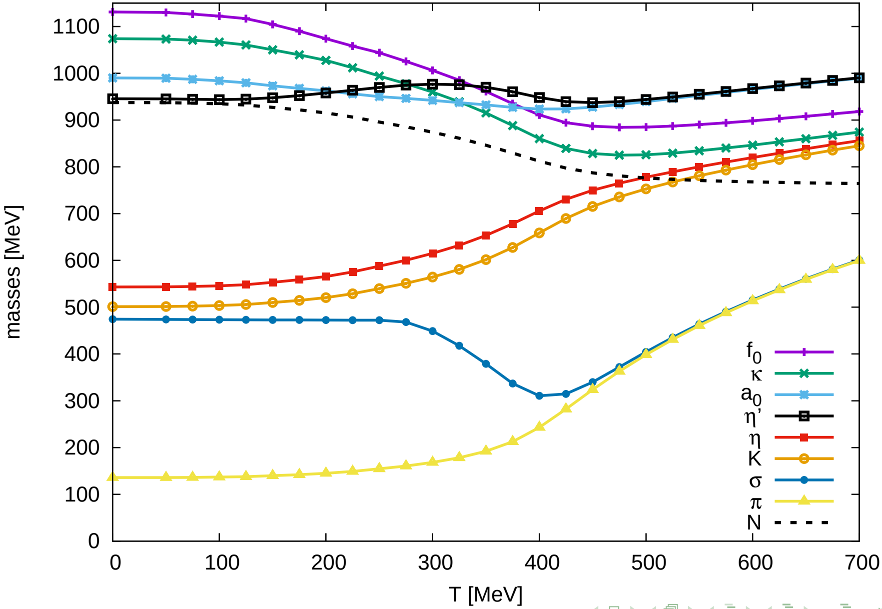
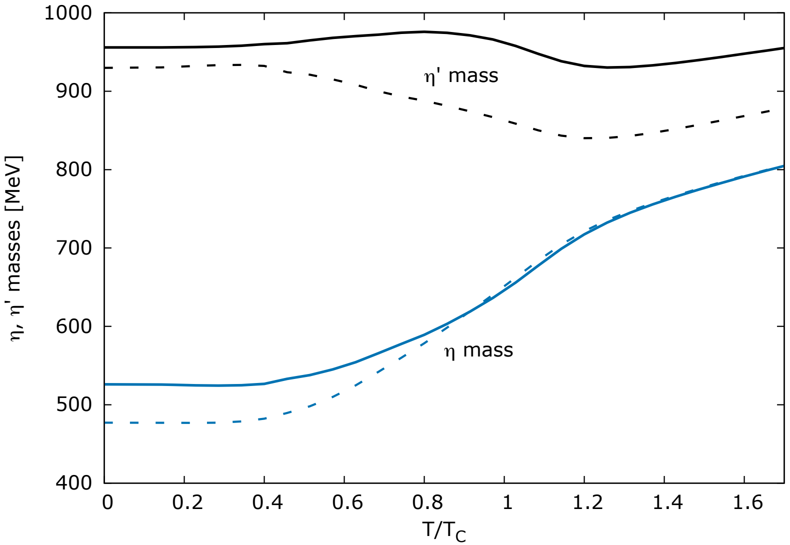
- Step II.: solve the same equations at finite T and μ_B

$\longrightarrow$ spectrum, symmetry restoration, $U_A(1)$ anomaly

Numerical results: mass spectrum at finite T

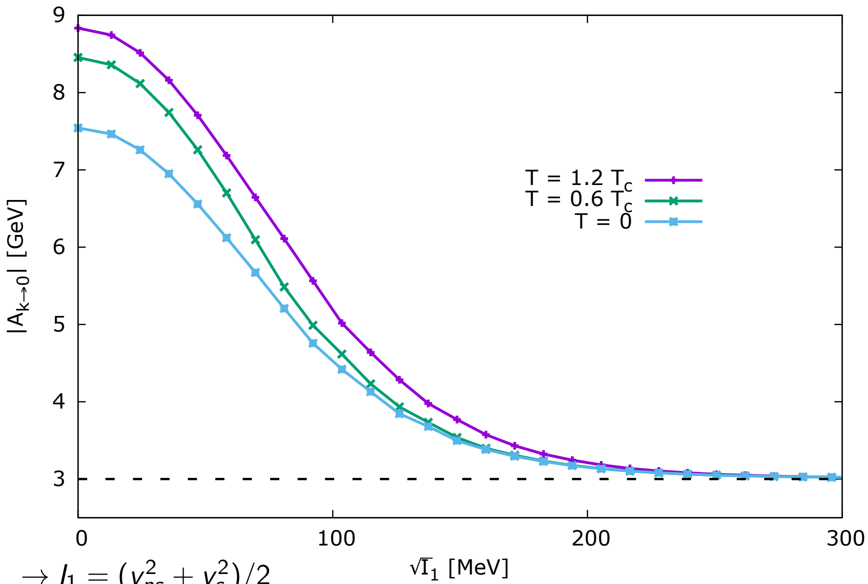


Numerical results: $\eta - \eta'$ system at finite T



$\longrightarrow$ solid: full solution, dashed: field- and T independent anomaly

Numerical results: anomaly at finite T



$\rightarrow I_1 = (v_{ns}^2 + v_s^2)/2$

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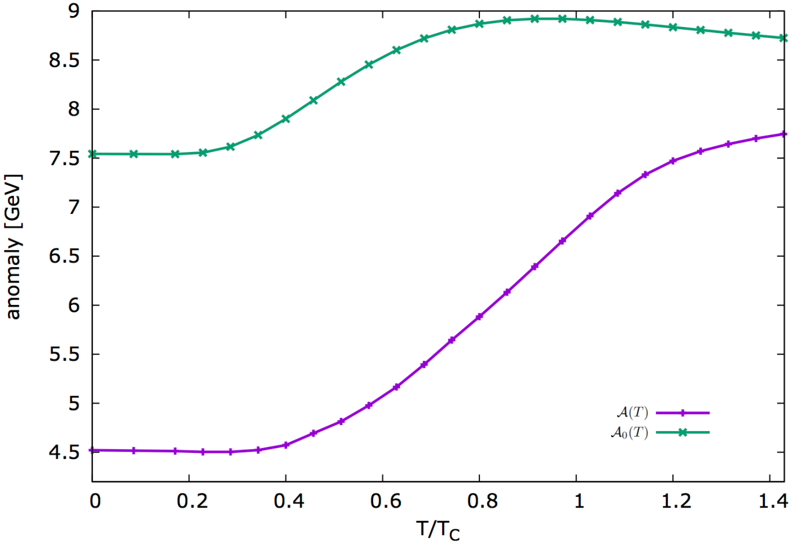
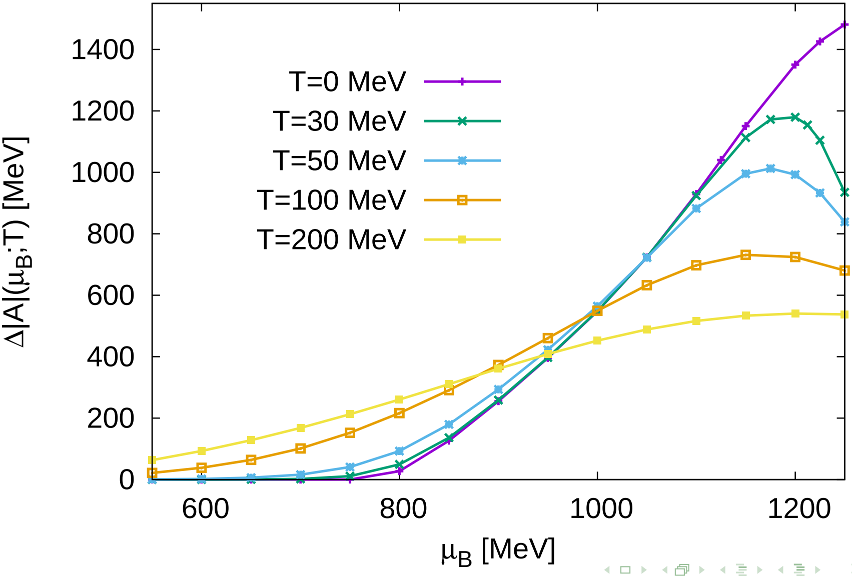
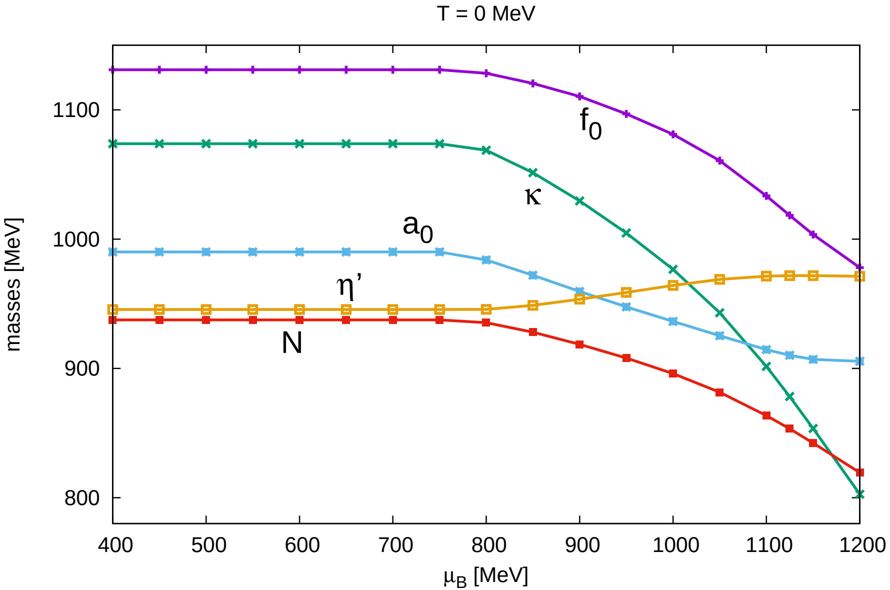


Numerical results: anomaly at finite μ_B



Numerical results: heavy spectrum at finite μ_B



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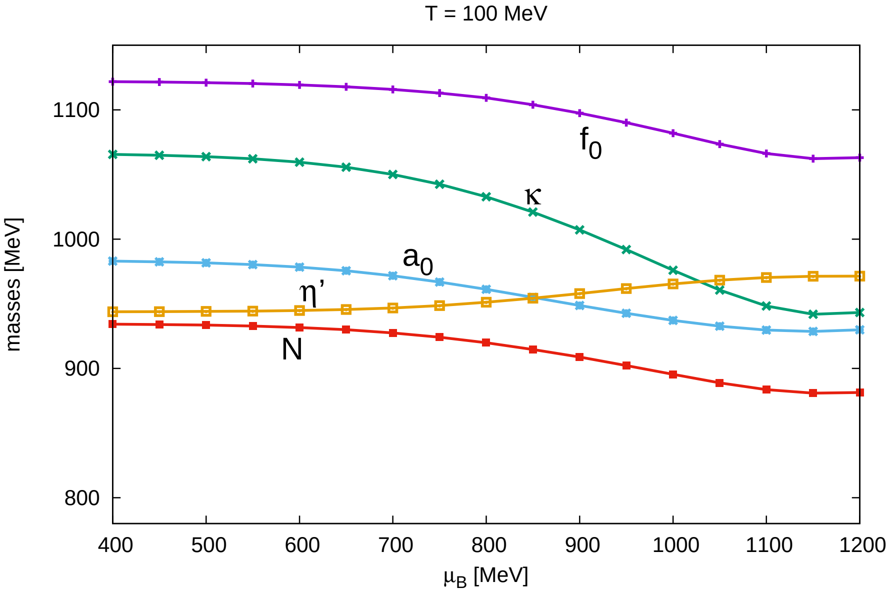

Numerical results

- Mesonic and nucleon fluctuations seems to have a strong effect on the anomaly evolution

  $\longrightarrow$ they cause a relative change of about $\sim 20\%$ at $T \simeq T_C$ and at $\mu_B \simeq \Lambda$ ($\sim 1\,\mathrm{GeV}$)

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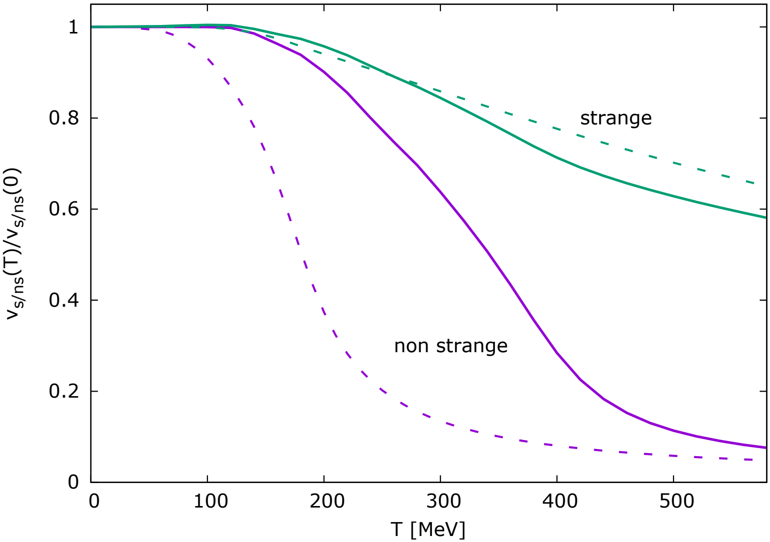

- dashed: without anomaly, solid: with anomaly

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Way out: the bare anomaly parameter a has to depend explicitly on T and μ_B. It should represent the underlying instanton dynamics of QCD.

Summary

- Finite T and density properties of the axial anomaly and mesonic spectra in a chiral effective nucleon-meson model
- Fluctuations (quantum and thermal) via the Functional Renormalization Group (FRG) approach
  $\longrightarrow$ thermal evolution of the mass spectrum and condensates
  $\longrightarrow$ temperature dependence of the $U_A(1)$ anomaly factor
- Findings:
  $\longrightarrow$ meson fluctuations strengthen the anomaly with respect to the temperature $\Rightarrow$ no recovery at T_C
  $\longrightarrow$ putting the system into nuclear medium also affects the anomaly $\Rightarrow$ it increases as μ_B grows
  $\longrightarrow$ η' mass is increasing with μ_B and T
- Important: T_C of chiral restoration and m_N comes out high!
  $\Rightarrow$ T-dependence of the bare anomaly coeff. can be relevant!