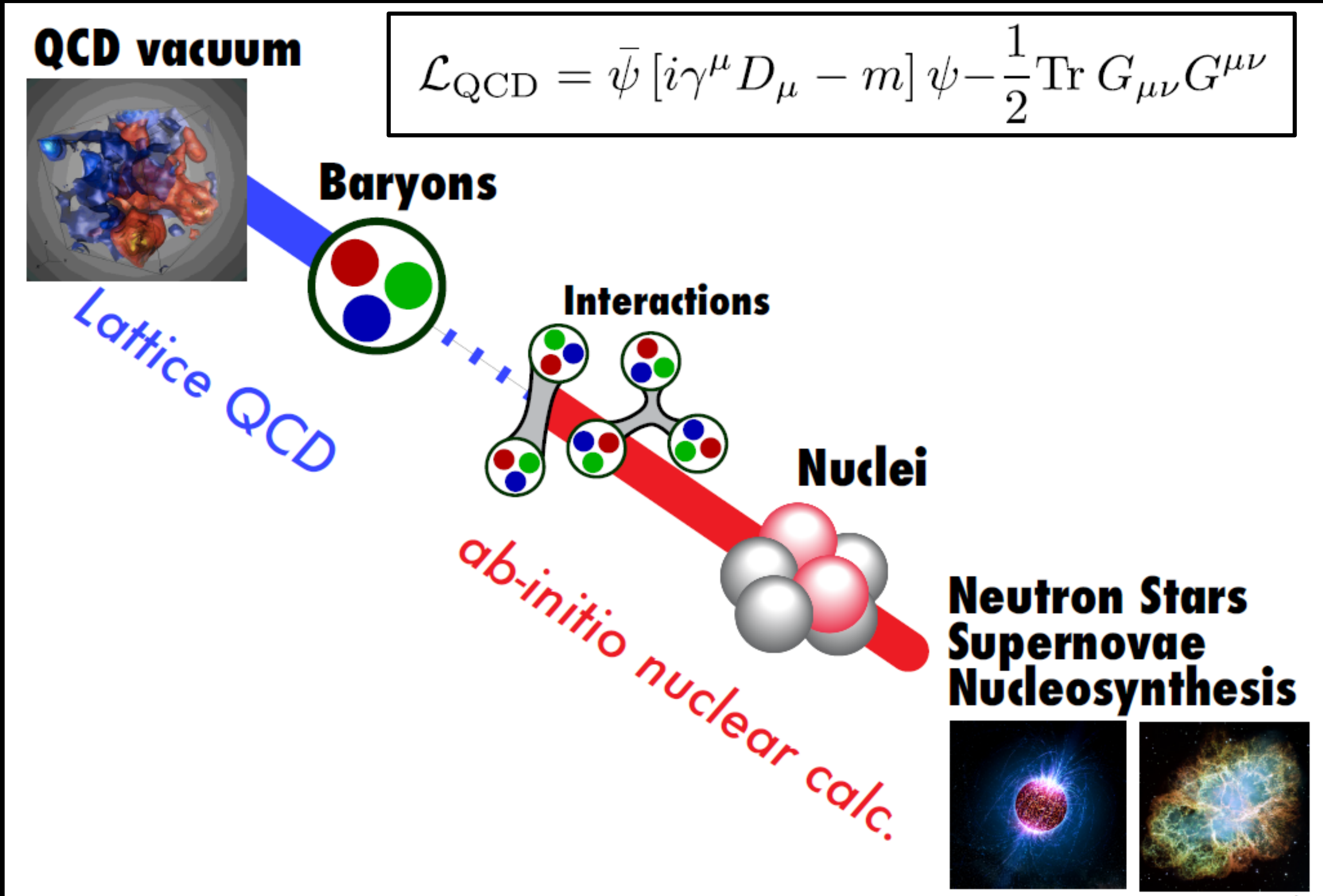
Baryon forces from Lattice QCD

Tetsuo Hatsuda (iTHEMS, RIKEN)

$$\mathcal{L}_{\rm QCD} = \bar{\psi}\left[i\gamma^{\mu} D_{\mu} - m\right]\psi - \frac{1}{2}{\rm Tr}\, G_{\mu\nu}G^{\mu\nu}$$



Contents

1. Introduction on hadronic interactions from LQCD

2. How “fake plateaux” ruin all the previous works (except for HAL QCD)

> ”Mirage in temporal correlation functions for baryon-baryon interactions in lattice QCD”,
> arXiv: 1607.06371 [hep-lat] (JHEP 10 (2016) 101) by HAL QCD Coll.
>
> “Are two nucleons bound in lattice QCD for heavy quark masses ?
> – Sanity check with Lucsher’s finite volume formula –”
> arXiv: 1703.07210 [hep-lat] (submitted to PRD) by HAL QCD Coll.

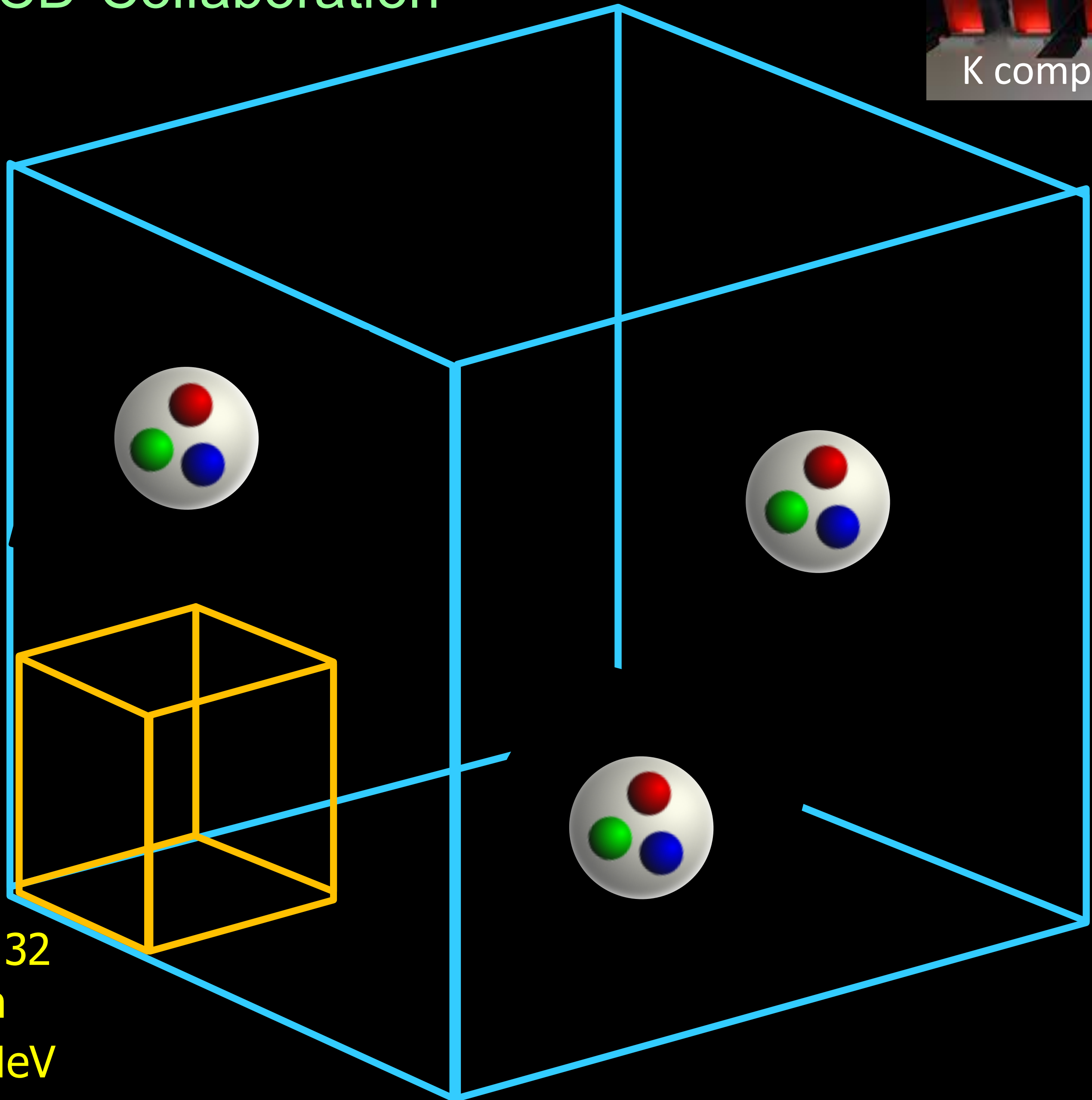
3. Hyperons in dense matter

Hadrons to Atomic nuclei
HAL
from Lattice QCD

S. Aoki, D. Kawai, T. Miyamato, K. Sasaki, T. Aoyama (YITP)
T. Doi, T. M. Doi, S. Gongyo, T.Hatsuda, T. Iritano (RIKEN)
T. Inoue (Nihon Univ.)
Y. Ikeda, N. Ishii, K. Murano (RCNP)
H. Nemura (Univ. of Tsukuba)
F. Etminan (Univ. of Birjand)

LQCD for multi-hadron (2015-)

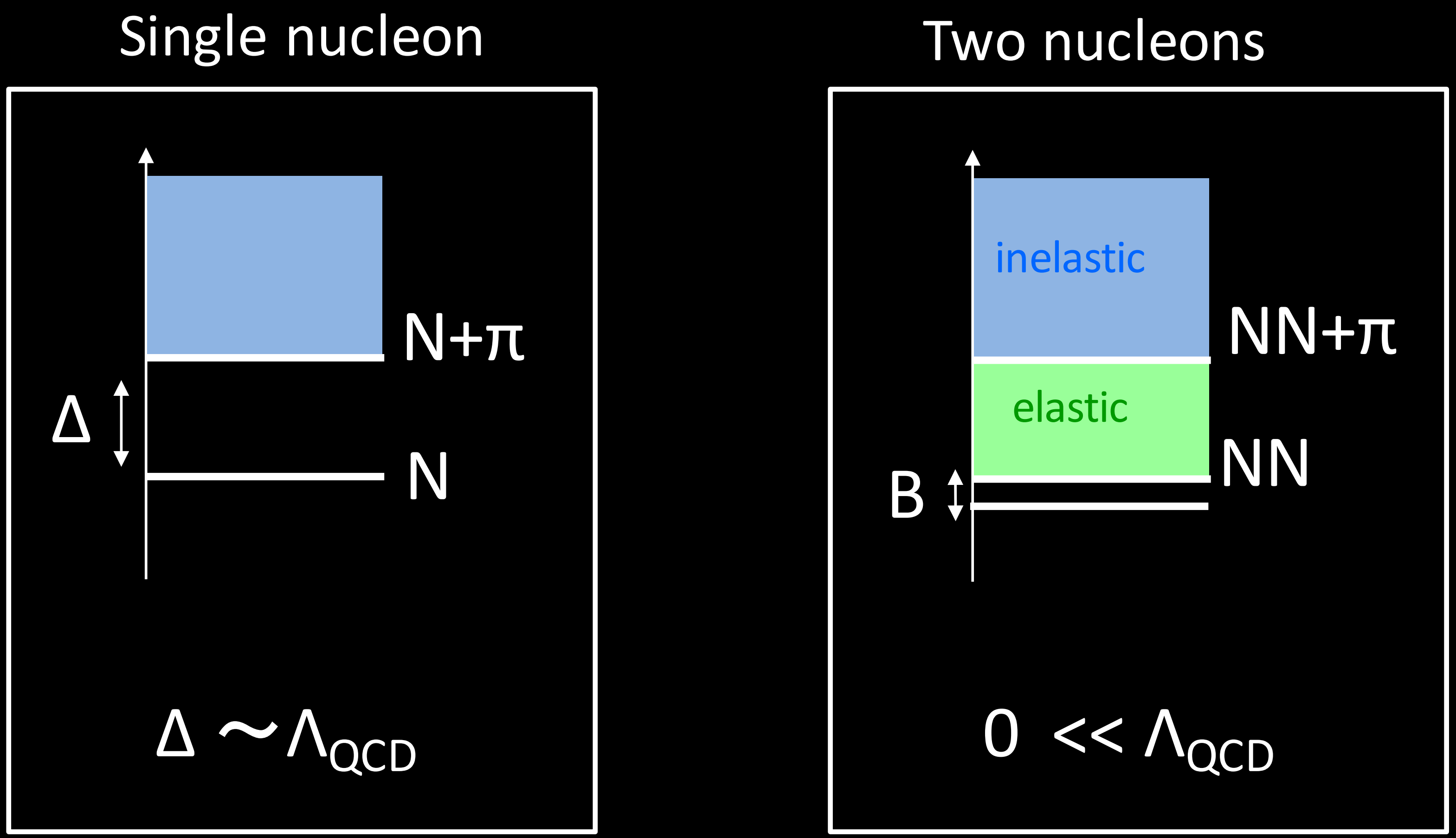
HAL QCD Collaboration



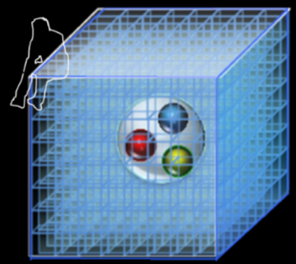
0.121 fm x 32
= 3.9 fm
$M_\pi > 350$ MeV

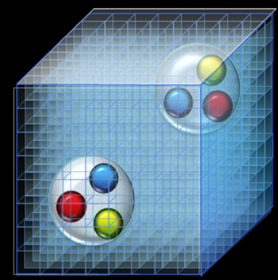
0.085 fm x 96
= 8.2 fm
$M_\pi = 145$ MeV

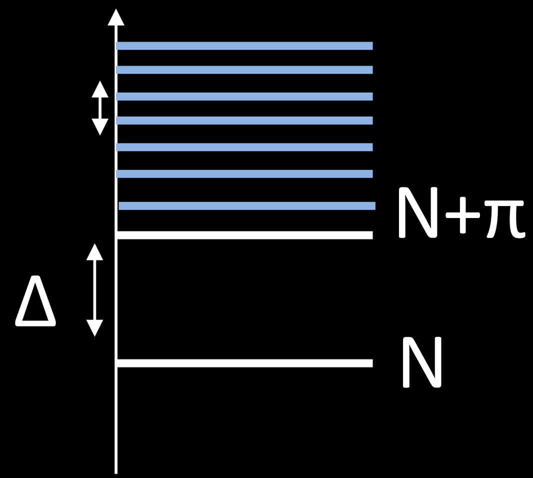
Fundamental difference between A=1 and A > 1



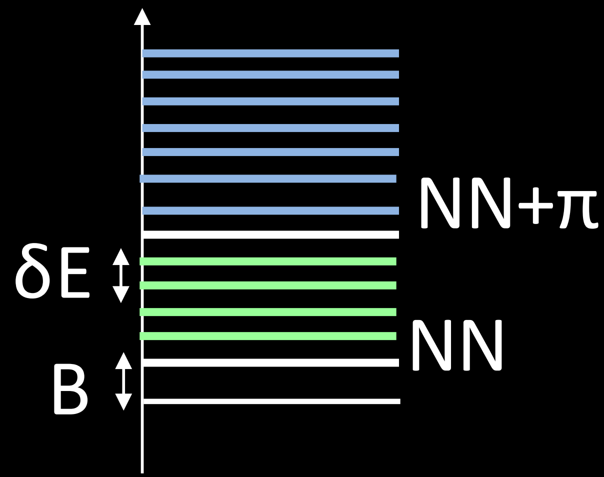
Fundamental difference between A=1 and A > 1





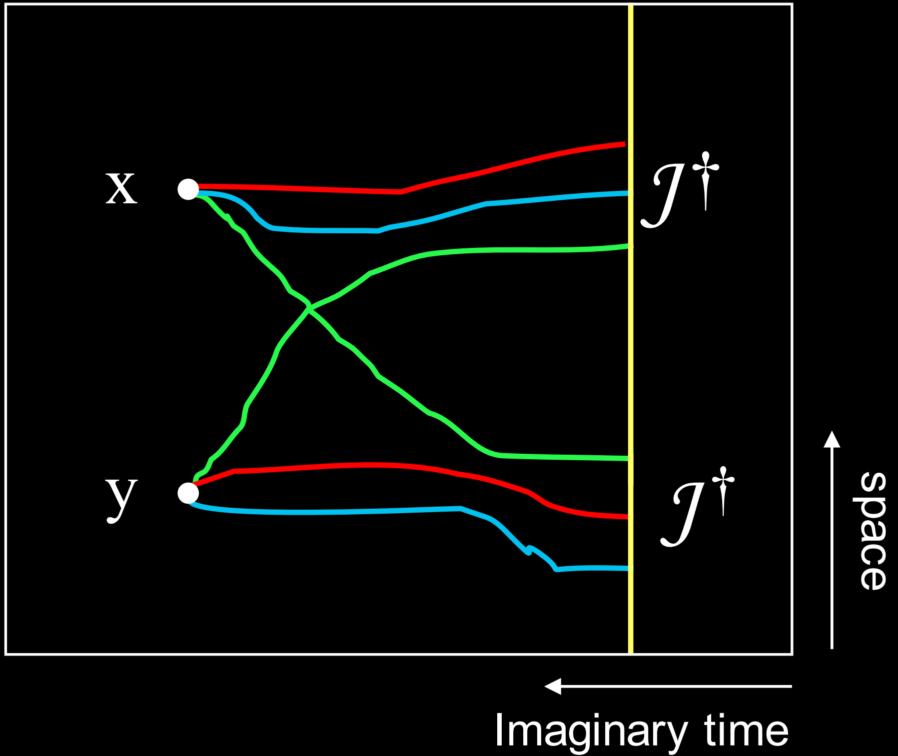


$t^{-1} = \Delta \sim 200$ MeV



$t^{-1} = \delta E \sim 20\,(9/L)^2$ MeV

Scattering observables from LQCD



$$\langle N_1(\mathbf{x},t)N_2(\mathbf{y},t)\mathcal{J}_1^\dagger(0)\mathcal{J}_2^\dagger(0)\rangle$$
$$= \sum_n \langle 0|N_1(\mathbf{x})N_2(\mathbf{y})|n\rangle a_n e^{-E_n t}$$
$$\xrightarrow{t>t^*} \phi(\mathbf{r},t) = \sum_{n<n^*} b_n \phi_n(\mathbf{r}) e^{-E_n t}$$

Finite Volume Method

$E_n(L)$
→ phase shift, binding energy

Luescher, Nucl. Phys. B354 (1991) 531

HAL QCD Method

$\phi(r,t)$ → 2PI kernel (T=U+GUT)
→ phase shift, binding energy

Ishii, Aoki & Hatsuda, PRL 99 (2007) 022001
Ishii et al. [HAL QCD Coll.], PLB 712 (2012) 437

Problem of Signal to Noise Ratio

Parisi, Lepage (1989)

$$G(r,t) = \langle 0|\mathcal{O}(r,t)\overline{\mathcal{O}}(0)|0|\rangle = \sum_n \alpha_n \psi_n(r) e^{-E_n t} \xrightarrow[t\to\infty]{} \alpha_0 \psi_0(r) e^{-E_0 t}$$

Single pion $$\frac{\langle \pi(t)\pi(0)\rangle}{\sqrt{\langle \pi\pi(t)\pi\pi(0)\rangle}} \sim \frac{\exp(-m_\pi t)}{\sqrt{\exp(-2m_\pi t)}} \sim \text{const.}$$ Signal/Noise ~ $\sqrt{N_{conf}}$

Multi pion Signal/Noise ~ $\sqrt{N_{conf}}$

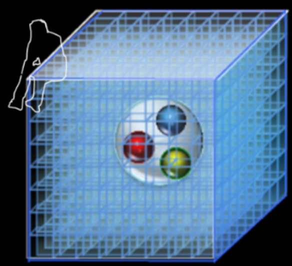
Single nucleon $$\frac{\langle N(t)\bar{N}(0)\rangle}{\sqrt{\langle |N(t)\bar{N}(0)|^2\rangle}} \sim \frac{\exp(-m_N t)}{\sqrt{\exp(-3m_\pi t)}} \sim \exp[-(m_N - 3/2 m_\pi)t]$$

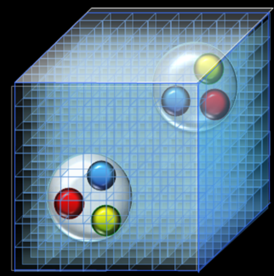
Signal/Noise ~ exp(- m_N t) x $\sqrt{N_{conf}}$

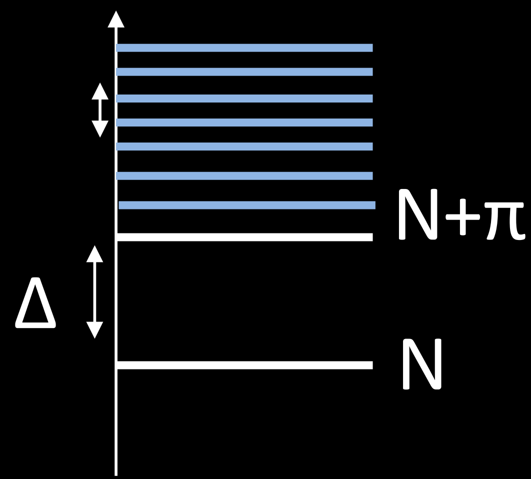
Multi nucleon $$\frac{\langle N^{\mathbf{A}}(t)\bar{N}^{\mathbf{A}}(0)\rangle}{\sqrt{\langle |N^{\mathbf{A}}(t)\bar{N}^{\mathbf{A}}(0)|^2\rangle}} \sim \frac{\exp(-\mathbf{A} m_N t)}{\sqrt{\exp(-3\mathbf{A} m_\pi t)}} \sim \exp[-\mathbf{A}(m_N - 3/2 m_\pi)t]$$

Signal/Noise ~ $\sqrt{\exp(-\mathbf{A}\, m_N\, t)}$ x N_{conf}

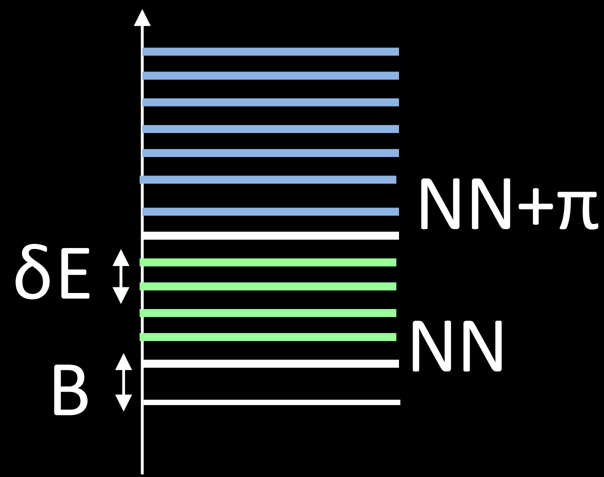
Fundamental difference between A=1 and A > 1







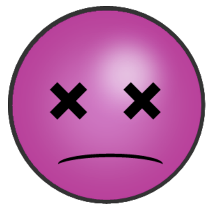
$t^{-1} = \Delta \sim 200$ MeV



$t^{-1} = \delta E \sim 20\,(9/L)^2$ MeV

$S/N \sim \exp(-\; m_N\, t) \times \sqrt{N_{conf}}$
$\sim 10^{-2} \times \sqrt{N_{conf}}$

$S/N \sim \exp(-\; 2m_N\, t) \times \sqrt{N_{conf}}$
$\sim 10^{-41} \times \sqrt{N_{conf}}$



PUBLISHED FOR SISSA BY SPRINGER

RECEIVED: *August 9, 2016*
REVISED: *September 24, 2016*
ACCEPTED: *September 27, 2016*
PUBLISHED: *October 19, 2016*

Mirage in temporal correlation functions for baryon-baryon interactions in lattice QCD

The HAL QCD collaboration

T. Iritani,[a] T. Doi,[b] S. Aoki,[c,d] S. Gongyo,[e] T. Hatsuda,[b,f] Y. Ikeda,[b,g] T. Inoue,[h] N. Ishii,[g] K. Murano,[g] H. Nemura[d] and K. Sasaki[c,d]

JHEP10(2016)101

Demo by Mock-up data @ m_π =0.51GeV, L=4.3fm

Same setup as Yamazaki et al. ('12)

$$R(t) = \frac{G_{BB}(t)}{(G_B(t))^2} = b_1 e^{-\Delta E t} + b_2 e^{-\delta E_{\rm el} t} + c_1 e^{-\delta E_{\rm inel} t}$$

$$\Delta E_{\rm eff}(t) = \ln\left[\frac{R(t)}{R(t+1)}\right] \xrightarrow[t\to\infty]{} \Delta E$$

Ground state energy : ~ 1 MeV precision required

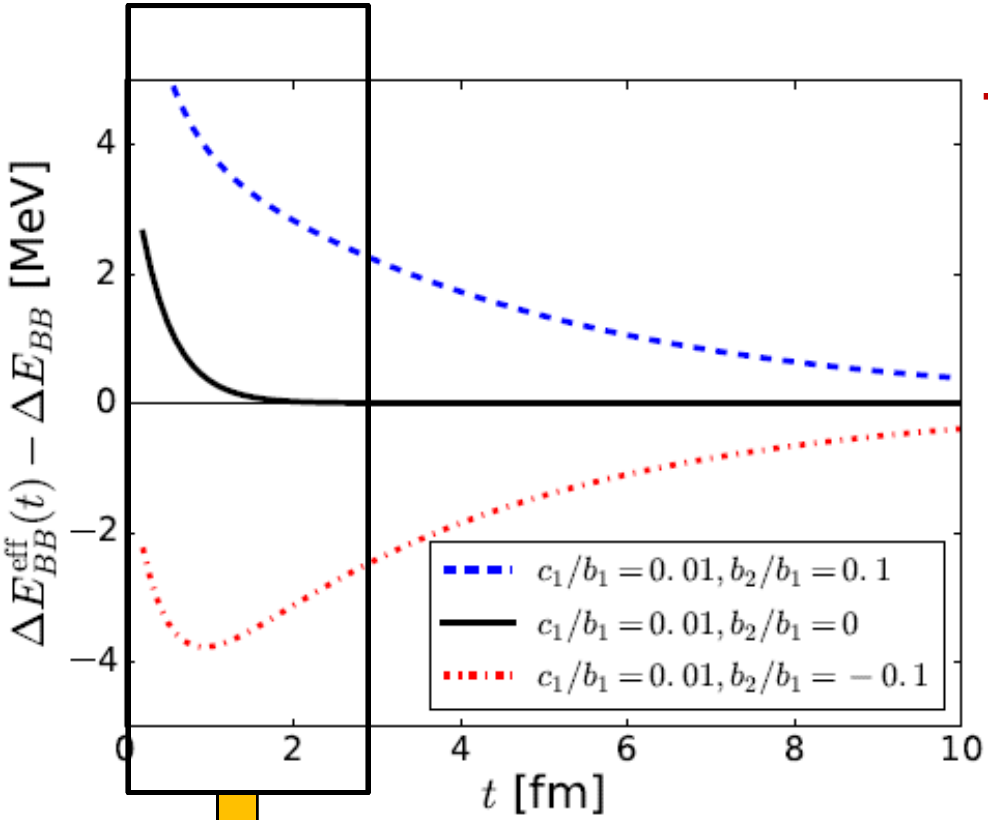
$$\Delta E = E_{BB} - 2m_B$$

Elastic scattering threshold : sensitive to L

$\delta E_{\rm el} = 50{\rm MeV}$ $\quad b_2/b_1 = \pm 0.1 \ \&\ 0$ (10% contamination)

Inelastic threshold : insensitive to L

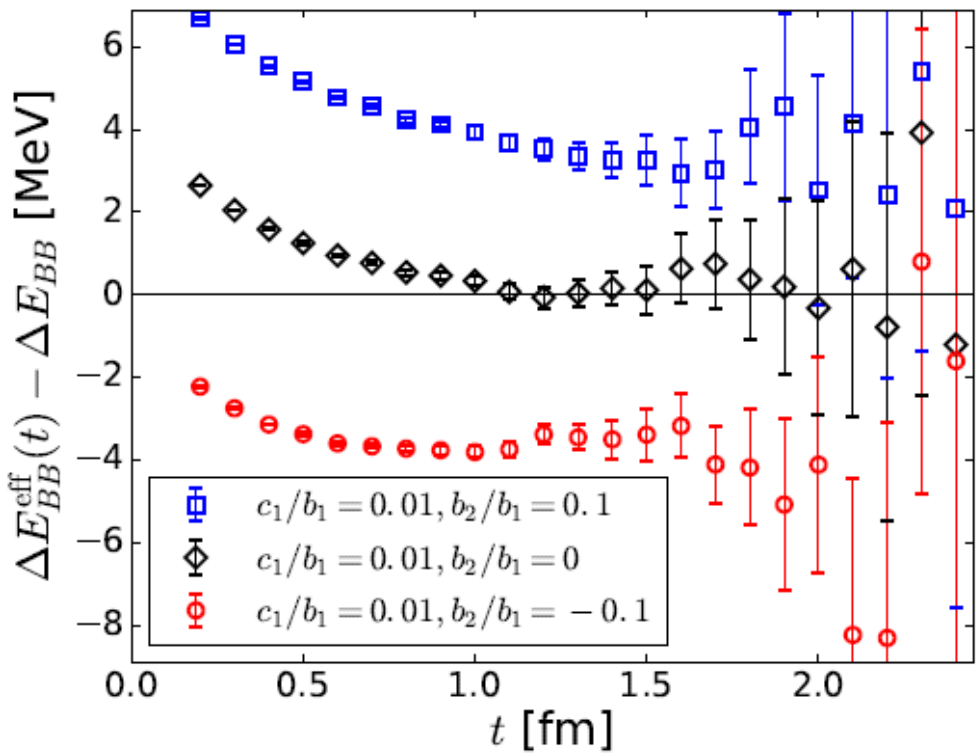
$\delta E_{\rm inel} = 500{\rm MeV}$ $\quad c_1/b_1 = 0.01$ (1% contamination)



True ground state for t > 10 fm

Zoom
+ typical stat error

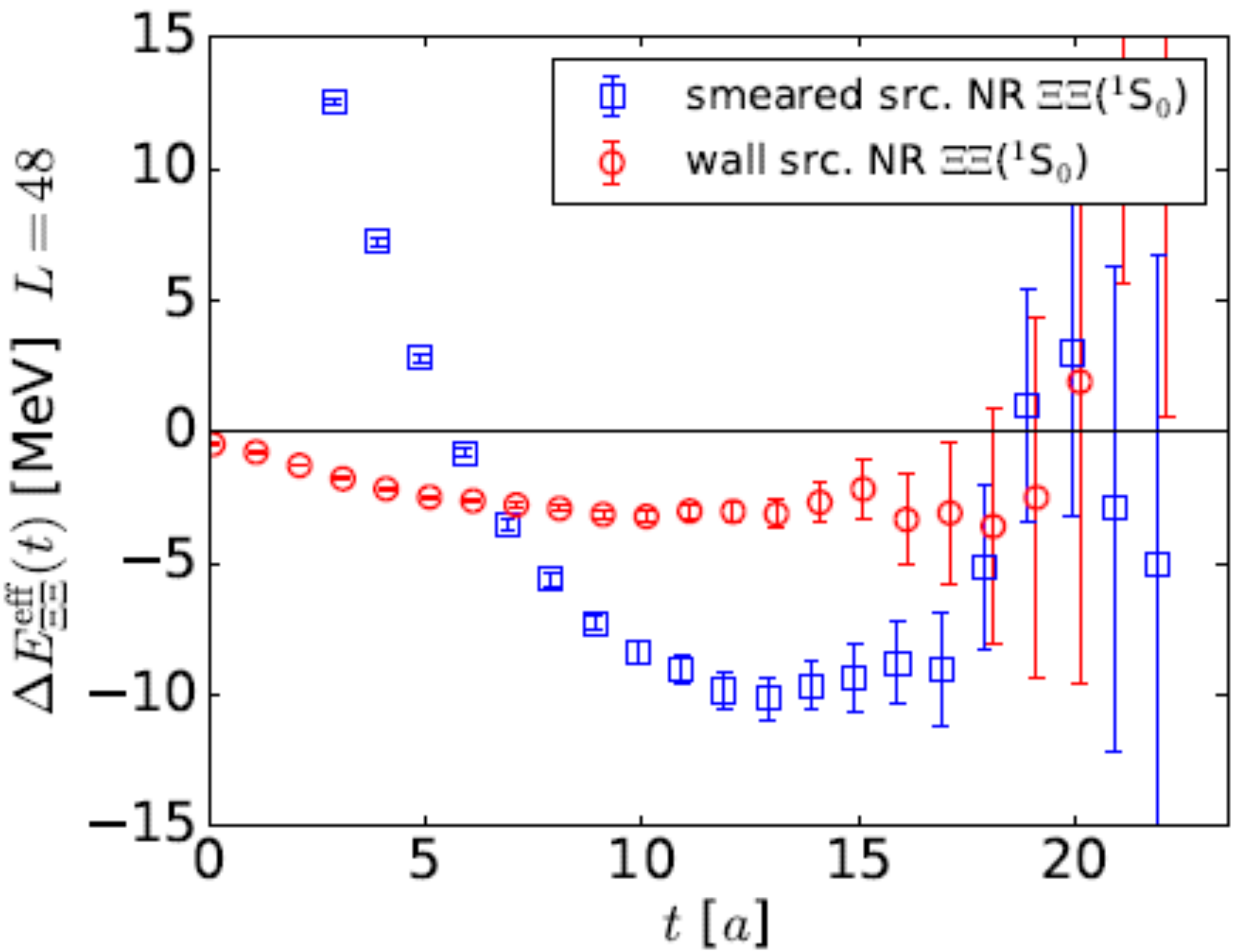
“Fake plateaux” or “Mirage” at t ~ 1 fm

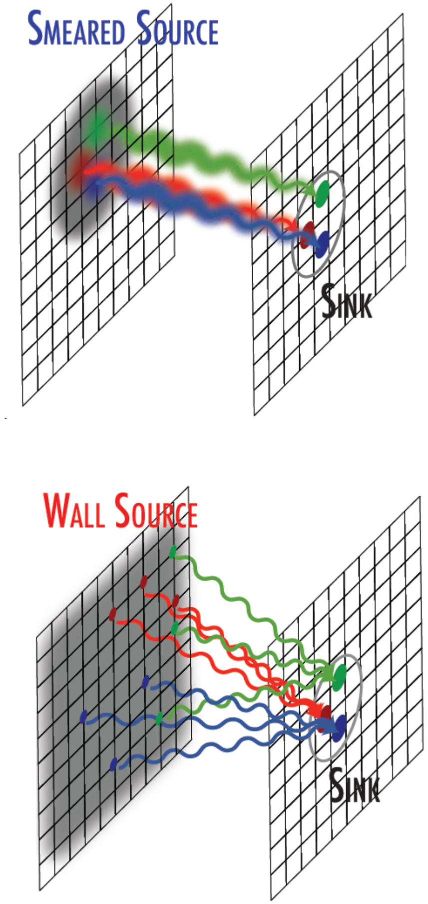


Actual data for ΞΞ ($^1S_0$) @ m_π =0.51GeV, L=4.3fm, a=0.09fm

Source dependence

$$R(t) = \sum_{\vec{r}} \sum_{\vec{x}} \langle 0 | B(\vec{r}+\vec{x},t) B(\vec{x},t) \overline{\mathcal{J}_{\rm src}(0)} | 0 \rangle / \{G_B(t)\}^2$$

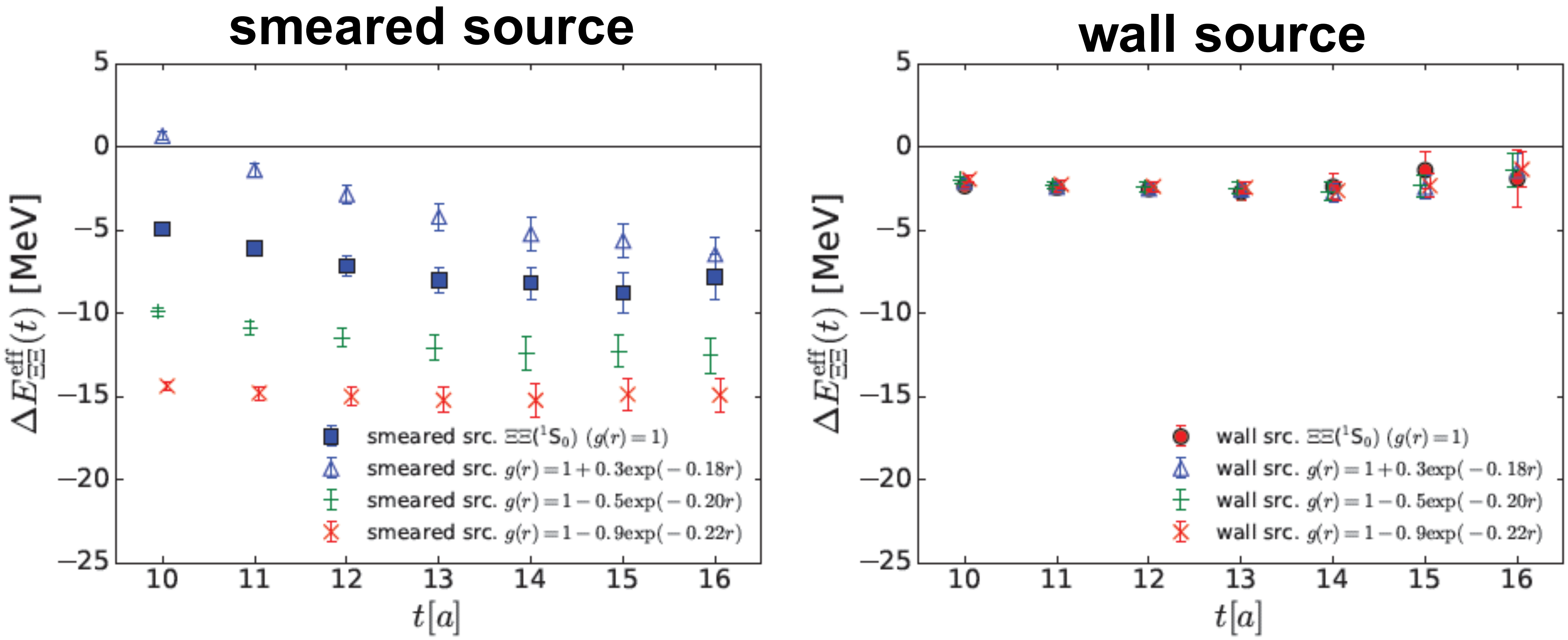




Actual data for ΞΞ ($^1S_0$) @ m_π =0.51GeV, L=4.3fm, a=0.09fm

Sink dependence

$$\tilde{R}^{(f)}(t) = \sum_{\vec{r}} f(\vec{r}) R(\vec{r},t) = \sum_{\vec{r}} f(\vec{r}) \sum_{\vec{x}} \langle 0 | B(\vec{r}+\vec{x},t) B(\vec{x},t) \overline{\mathcal{J}_{\rm src}(0)} | 0 \rangle / \{G_B(t)\}^2$$



All the previous results (Yamazaki et al., NPL QCD, CalLat) using the smeared source were looking at "Fake Plateaux".

T. Iritani et al. (HAL) JHEP1610(2016)101 arXiv:1703.07210v1 [hep-lat] 21 Mar 2017 YITP-17-24, UTCCS-P-102, RIKEN-QHP-301

Are two nucleons bound in lattice QCD for heavy quark masses? – Sanity check with Lüscher's finite volume formula –

Takumi Iritani,[1] Sinya Aoki,[2,3] Takumi Doi,[1] Testuo Hatsuda,[1,4] Yoichi Ikeda,[5] Takashi Inoue,[6] Noriyoshi Ishii,[5] Hidekatsu Nemura,[3] and Kenji Sasaki[2]

(HAL QCD Collaboration)

Analysis of all existing data for Baryon-Baryon interactions using plateau method

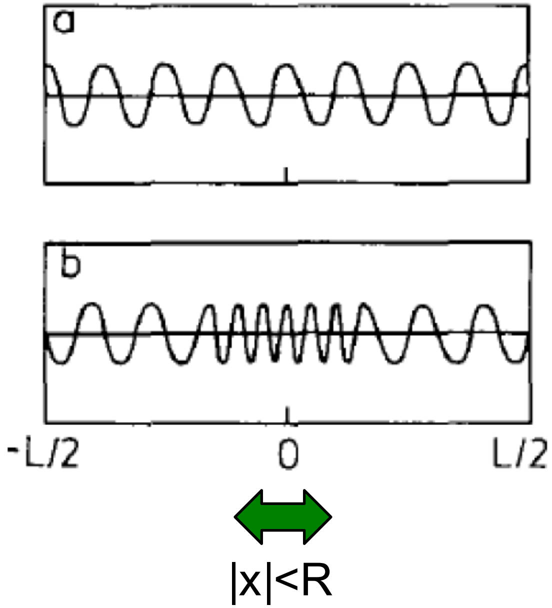
Name	Ref.	N_f	a [fm]	L [fm]	m_π [GeV]	m_N [GeV]	m_Λ [GeV]	m_Ξ [GeV]
YKU2011	[23]	0	0.128	3.1, 4.1, 6.1, 12.3	0.80	1.62	—	—
YIKU2012	[24]	2+1	0.090	2.9, 3.6, 4.3, 5.8	0.51	1.32	—	—
YIKU2015	[25]	2+1	0.090	4.3, 5.8	0.30	1.05	—	—
NPL2012	[26]	2+1	0.123 (aniso.)	2.9, 3.9	0.39	1.17	1.23	1.34
NPL2013	[27, 28]	3	0.145	$3.5^{(*)}$, $4.6^{(*)}$, $7.0^{(*)}$	0.81	1.64	1.64	1.64
NPL2015	[29]	2+1	0.117	2.8, 3.7, 5.6	0.45	1.23	1.31	1.42
CalLat2017	[30]	3	0.145	3.5, 4.6	0.81	1.64	1.64	1.64

Summary Table : At least single “No” implies the result is “Mirage”

Data	$NN(^1S_0)$				$NN(^3S_1)$			
	Source independence	Sanity check (i)	(ii)	(iii)	Source independence	Sanity check (i)	(ii)	(iii)
YKU2011 [23]	†	No	No		†	No	No	
YIKU2012 [24]	No	†	No		No	†	No	
YIKU2015 [25]	†	†	No		†	†	No	No
NPL2012 [26]	†	†	No		†	†		
NPL2013 [27, 28]	No			No	No			No
NPL2015 [29]	†	No		No	†	No		No
CalLat2017 [30]	No	?		No	No	?		No

TABLE IV. A summary of sanity checks (i) consistency between $\mathrm{ERE}_{k^2>0,BE}$ and $\mathrm{ERE}_{k^2<0}$, (ii) non-singular ERE parameters and (iii) physical residue for the bound state pole, together with the source independence of ΔE. Here “No” (blank) means that the source independency/sanity check has failed (passed), while the symbol † implies there is none or only insufficient study on the corresponding item. See appendix B for the meaning of the symbol ? on the Sanity check (i) for CalLat2017.

Luscher's formula: Scatterings on the lattice



Schroedinger eq. in (1+1)-dimension:

$$\left[-\frac{1}{2\mu}\frac{d^2}{dx^2}+V(|x|)\right]\psi(x)=E\psi(x)\equiv\frac{k^2}{2\mu}\psi(x)$$

Wave function at |x|> R
for infinite L:

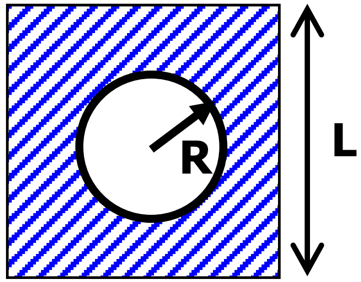
$$\psi(x,k)=A_k\cos(k|x|+\underline{\delta(k)})$$

Quantization condition
for finite L with PBC:

$$\begin{aligned} kL+2\delta(k) &= 0 \quad (\mathrm{mod}\ 2\pi) \\ e^{2i\delta(k)} &= e^{-ikL} \end{aligned}$$

Lucsher's formula in (1+1)-D

Luscher's formula: Scatterings on the lattice



NBS equation in (3+1)-dimension:

$$(\nabla^2 + k^2)\psi_k(\boldsymbol{r}) = mV_k(\boldsymbol{r})\psi_k(\boldsymbol{r})$$

Wave function at |x|> R for infinite L:

$$\psi^k_\infty(r) = A_k \sin(kr + \underline{\delta(k)})/(kr)$$

Wave function at |x|> R for finite L with PBC:

$$\begin{aligned}\psi^k_L(r) &= \frac{1}{L^3}\sum_{\vec{n}\in\mathbb{Z}^3}\frac{e^{i\vec{p}_n\cdot\vec{x}}}{\vec{p}_n^2 - k^2}, \quad \vec{p}_n = 2\pi/L\cdot\vec{n} \\ &= g_{00}(k)\frac{1}{\sqrt{4\pi}}j_0(kr) + \frac{k}{4\pi}n_0(kr) + \cdots (j_{l\geq 1}(kr))\end{aligned}$$

Quantization condition for finite L with PBC:

$$k\cot\delta(\mathbf{k}) = \frac{2}{\sqrt{\pi}L}Z_{00}(1;q^2), \quad q = \frac{kL}{2\pi}$$

$$Z_{00}(s;q^2) = \frac{1}{\sqrt{4\pi}}\sum_{\boldsymbol{n}\in\boldsymbol{Z}^3}\frac{1}{(\boldsymbol{n}^2 - q^2)^s}$$

Lucsher's formula in (3+1)-D

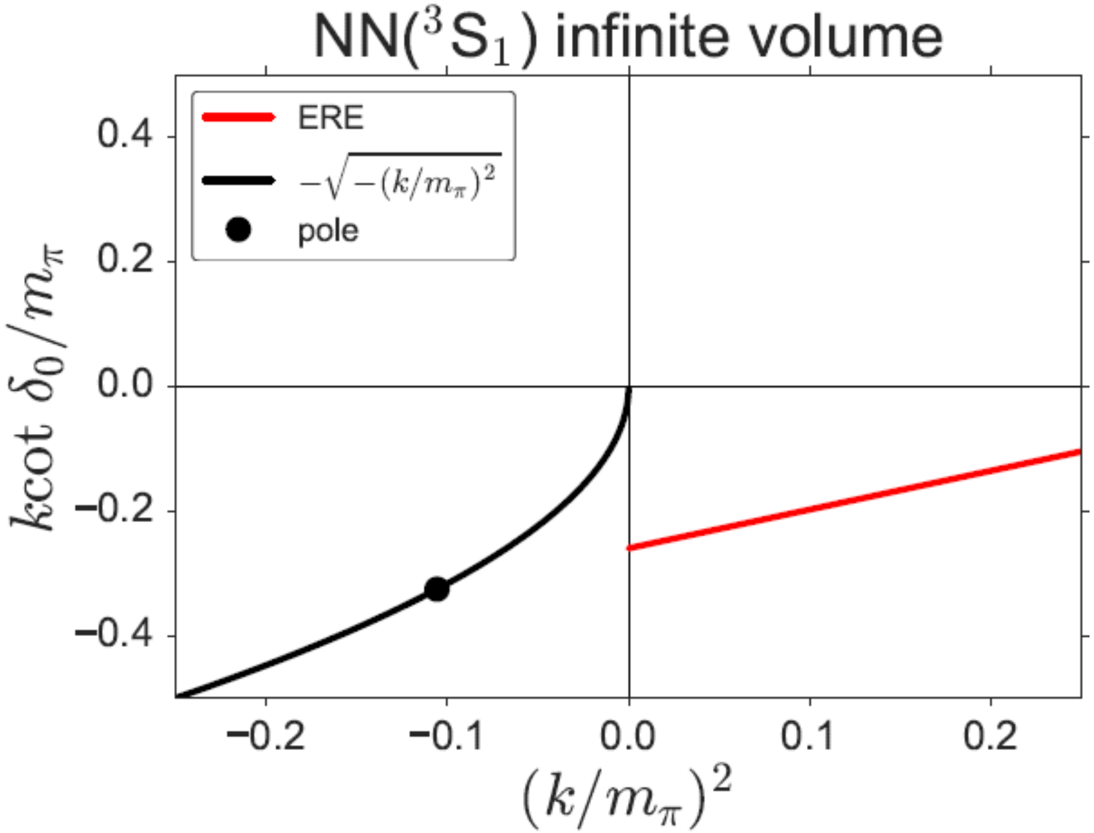
Effective Range Expansion (ERE)

infinite V

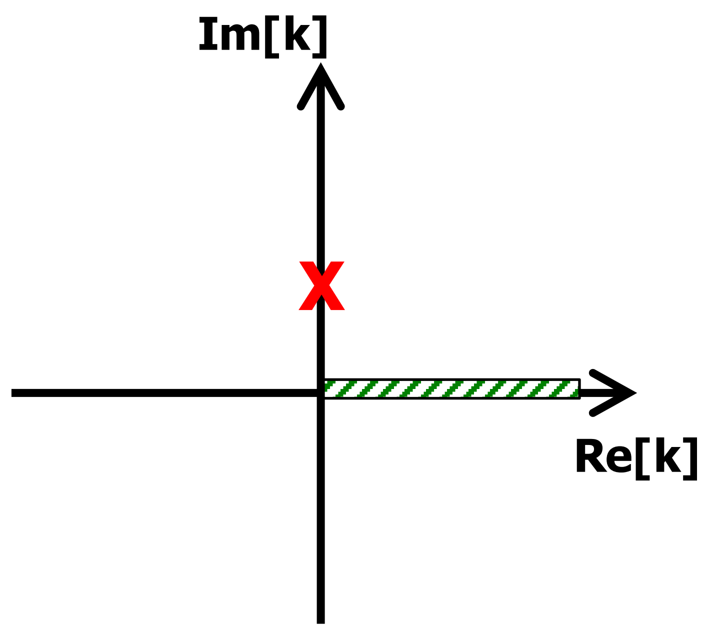
$$k \cot \delta_0(k) = ik \cdot \frac{S(k)+1}{S(k)-1}$$

$$S(k) = e^{2i\delta_0(k)}$$

$$\text{ERE:} \quad k \cot \delta(k) = \frac{1}{\mathbf{a}} + \frac{1}{2} \mathbf{r}\, k^2 + \cdots$$



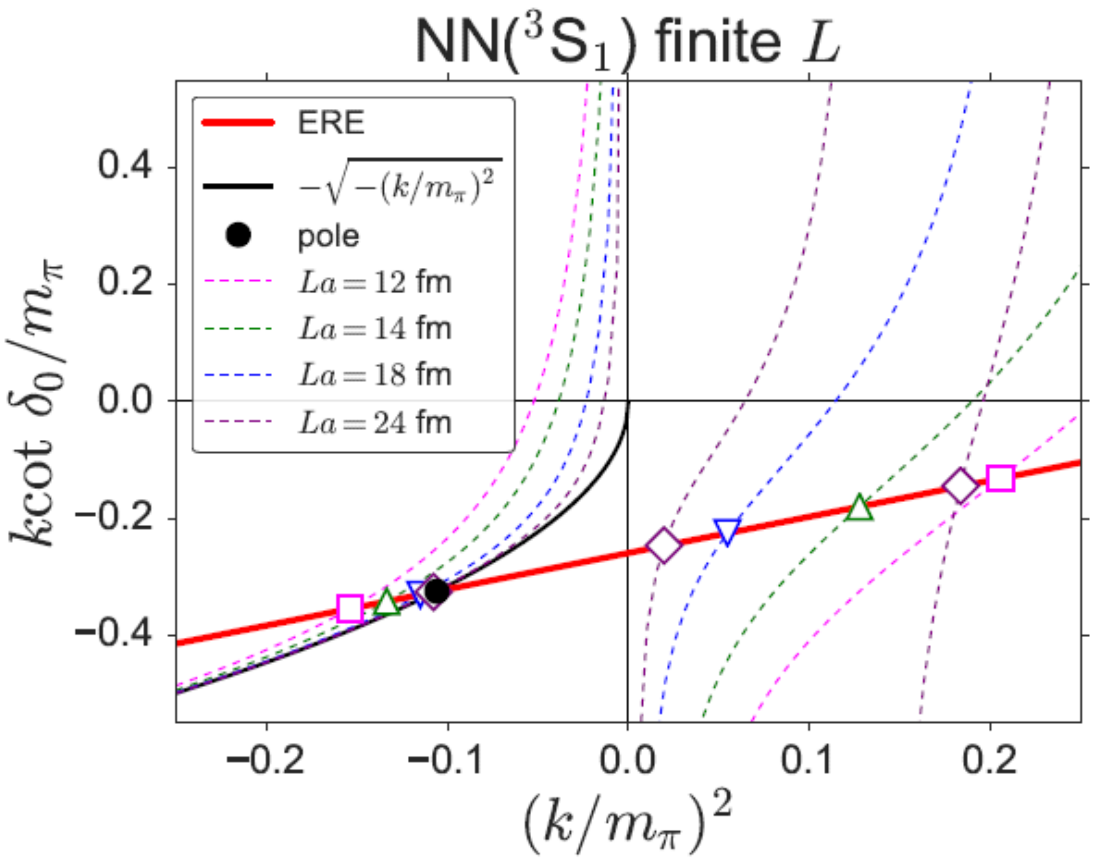
S-matrix

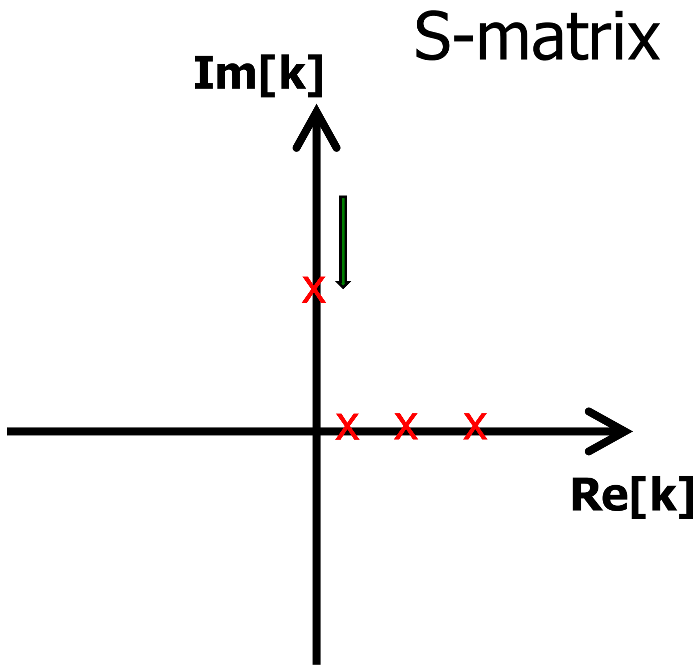


Effective Range Expansion (ERE)

finite V

$$\text{ERE:}\quad k\cot\delta(k) = \frac{1}{\mathbf{a}} + \frac{1}{2}\,\mathbf{r}\,k^2 + \cdots$$





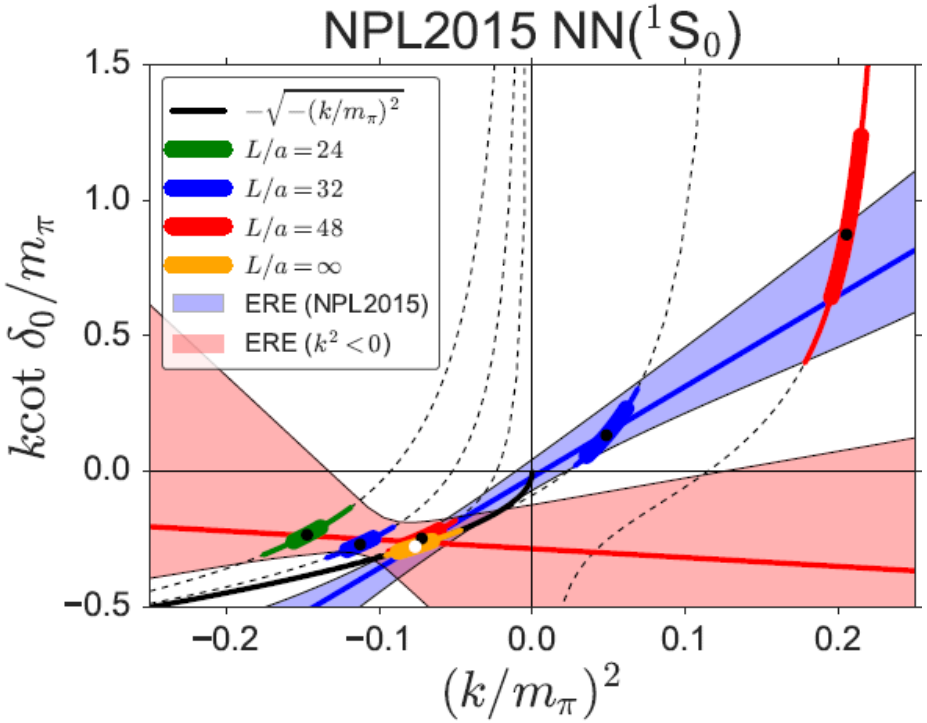
Dotted lines from Luscher's formula

“Sanity Check” for all the existing data

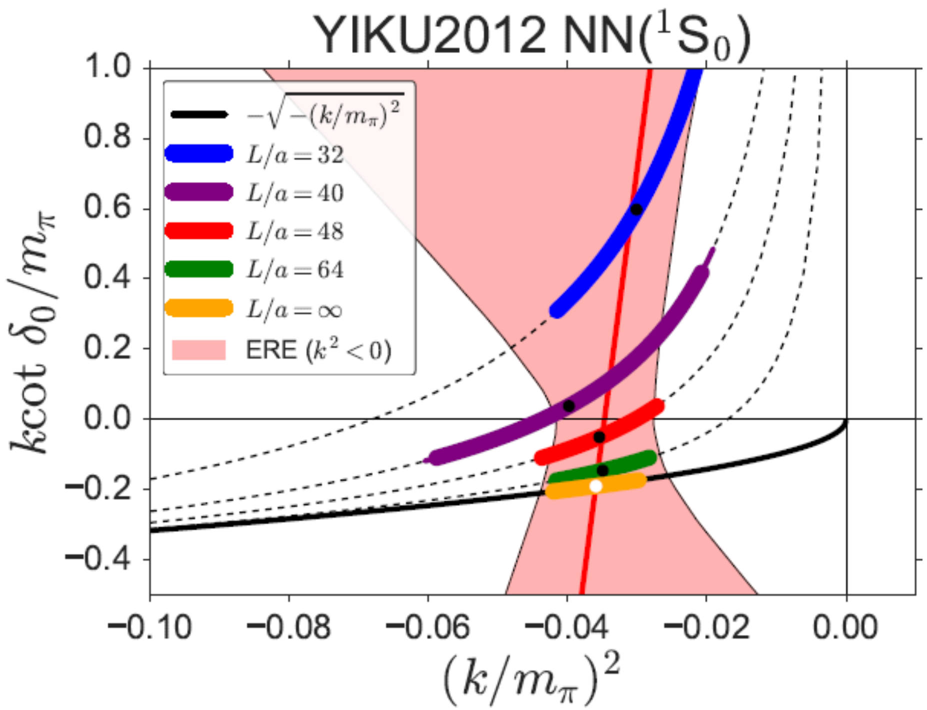
T. Iritani et al. (HAL)
arXiv:1703.07210

$$\text{ERE:}\quad k\cot\delta(k) = \frac{1}{\mathbf{a}} + \frac{1}{2}\,\mathbf{r}\,k^2 + \cdots$$

Data by NPL Coll. (2015)



Data by Yamazaki et al (2012)



(i) Inconsistent ERE

(ii) Singular behavior

(iii) Unphysical pole residue

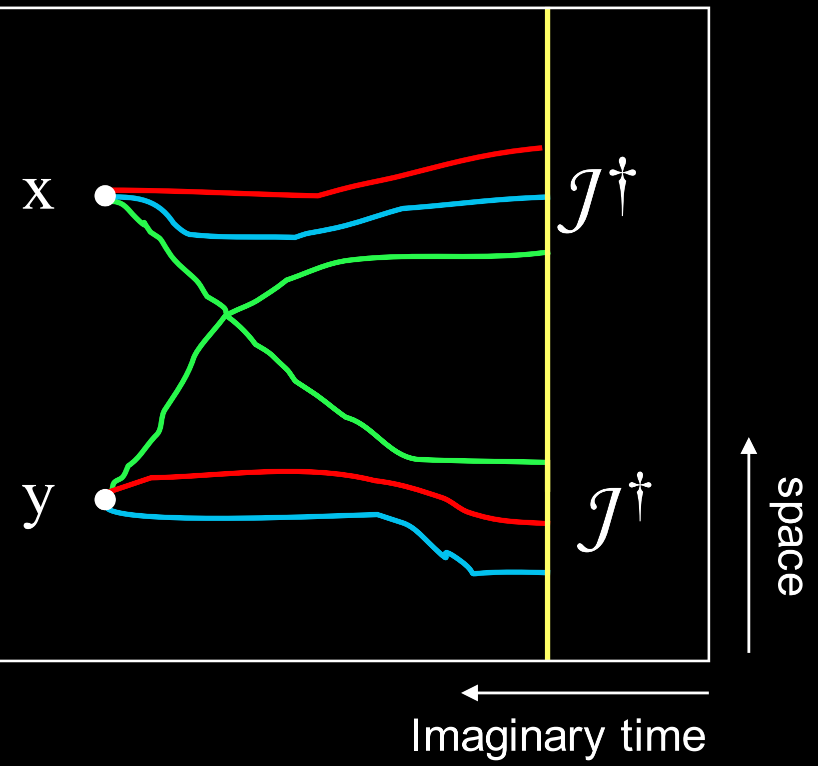
$r \simeq \pm\infty$

Summary Table : At least single "No" implies the result is "Mirage"

Data	$NN(^1S_0)$				$NN(^3S_1)$			
	Source independence	Sanity check (i)	(ii)	(iii)	Source independence	Sanity check (i)	(ii)	(iii)
YKU2011 [23]	†	No	No		†	No	No	
YIKU2012 [24]	No	†	No		No	†	No	
YIKU2015 [25]	†	†	No		†	†	No	No
NPL2012 [26]	†	†	No		†	†		
NPL2013 [27, 28]	No			No	No			No
NPL2015 [29]	†	No		No	†	No		No
CalLat2017 [30]	No	?		No	No	?		No

TABLE IV. A summary of sanity checks (i) consistency between $\text{ERE}_{k^2>0,BE}$ and $\text{ERE}_{k^2<0}$, (ii) non-singular ERE parameters and (iii) physical residue for the bound state pole, together with the source independence of ΔE. Here "No" (blank) means that the source independency/sanity check has failed (passed), while the symbol † implies there is none or only insufficient study on the corresponding item. See appendix B for the meaning of the symbol ? on the Sanity check (i) for CalLat2017.

HAL QCD Method : the Master equation



$$\left\{ \frac{1}{4M_{\rm B}} \frac{\partial^2}{\partial \tau^2} - \frac{\partial}{\partial \tau} - H_0 \right\} \mathscr{R}(\mathbf{r}, \tau) = \int d^3 r' U(\mathbf{r}, \mathbf{r}') \mathscr{R}(\mathbf{r}', \tau)$$

R(r,t) → 2PI kernel (T=U+GUT)→ phase shift, binding energy

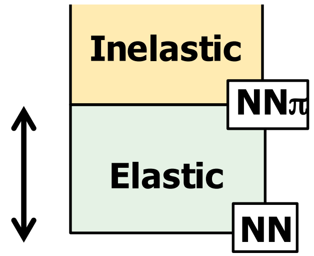
Ishii, Aoki & Hatsuda, PRL 99 (2007) 022001

Ishii et al. [HAL QCD Coll.], PLB 712 (2012) 437

Time-dependent HAL method

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

All the elastic states share the same non-local potential U(r,r')



$$R(\boldsymbol{r},t) \equiv G_{NN}(\boldsymbol{r},t)/G_N(t)^2 = \sum_i A_{W_i}\psi_{W_i}(\boldsymbol{r})e^{-(W_i-2m)t}$$

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\underline{\psi_{W_0}(\boldsymbol{r}')} = (\underline{E_{W_0}} - H_0)\underline{\psi_{W_0}(\boldsymbol{r})}$$

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\underline{\psi_{W_1}(\boldsymbol{r}')} = (\underline{E_{W_1}} - H_0)\underline{\psi_{W_1}(\boldsymbol{r})}$$

...

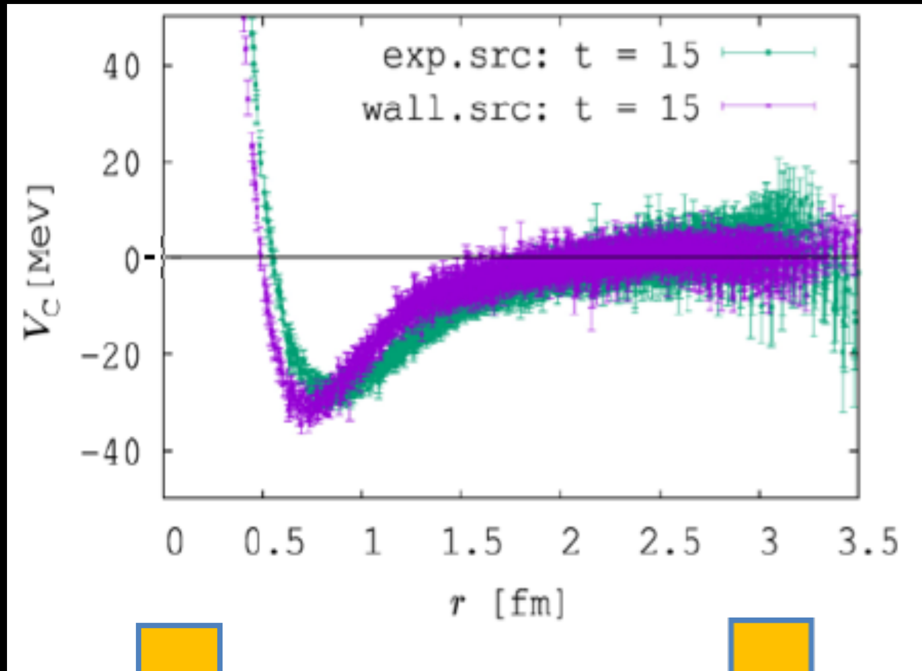
All equations can be combined as

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\underline{R(\boldsymbol{r}',t)} = (\underline{-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2}} - H_0)\underline{R(\boldsymbol{r},t)}$$

	Plateau method	HAL QCD method
Inelastic states	**Noise**	**Noise**
Elastic states	**Noise**	**Signal**
Ground state	**Signal**	**Signal**
necessary t	t > 10 fm	t ~ 1 fm

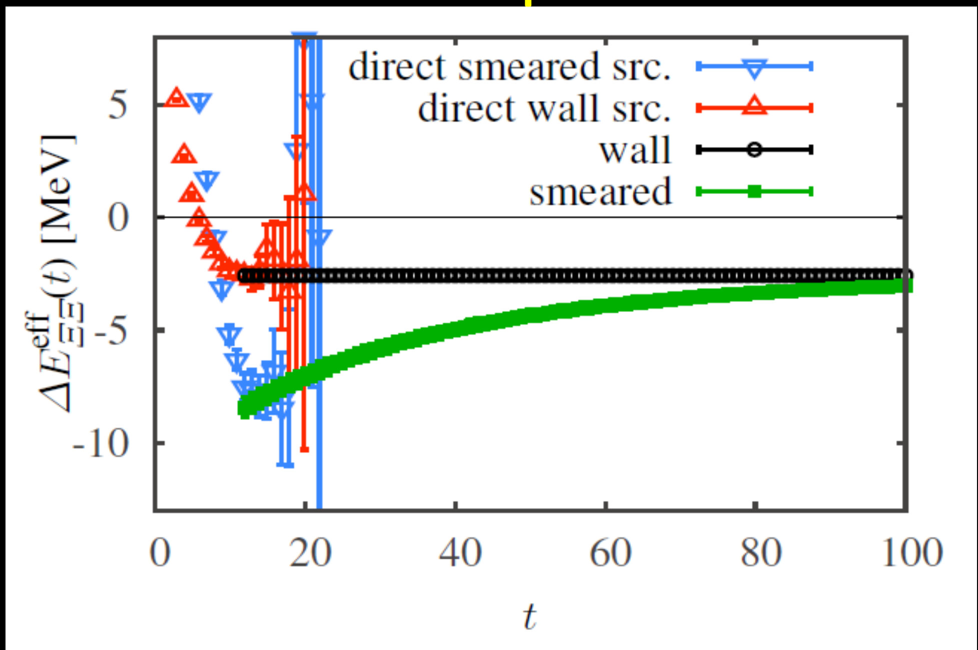
Exponential Improvement!

HAL QCD Method

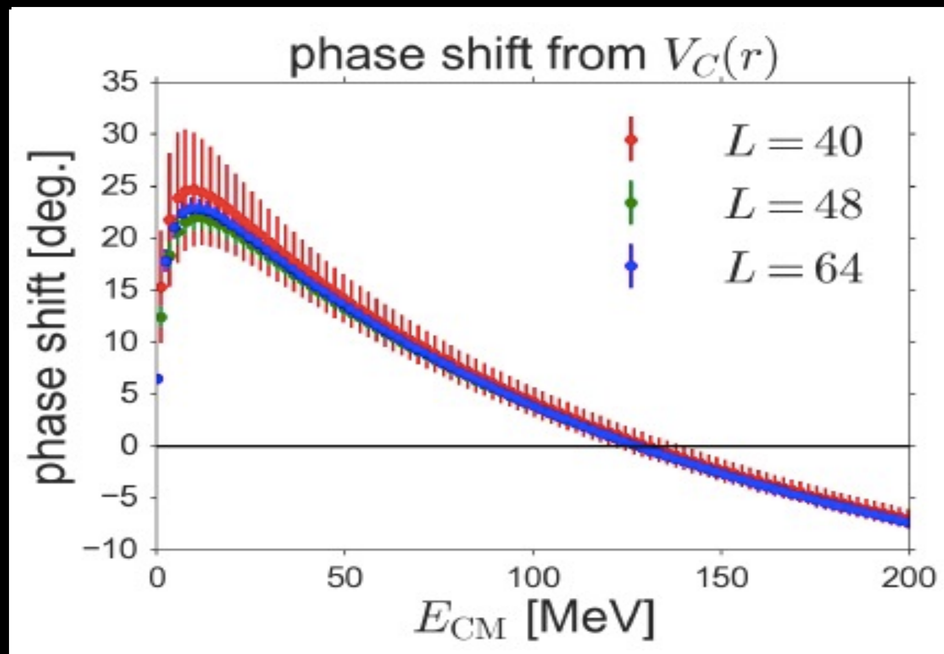


a=0.09 fm
m_π=0.51 GeV
m_K=0.62 GeV

Fate of the fake plateau

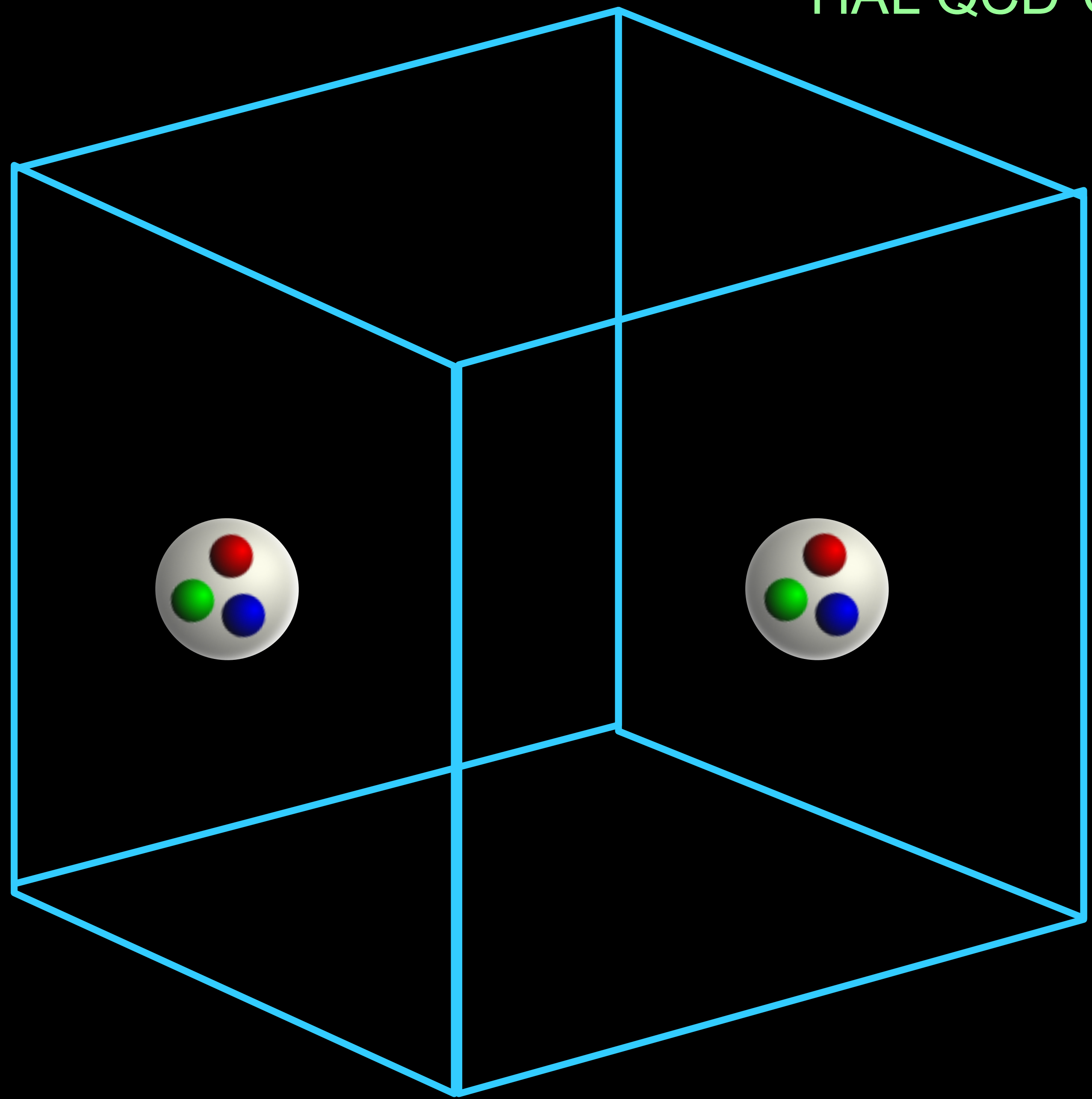


No ΞΞ bound state



Physical point LQCD studies on multi-hadron (2015-)

HAL QCD Collaboration



0.085 fm x 96
= 8.2 fm

M_π = 145 MeV

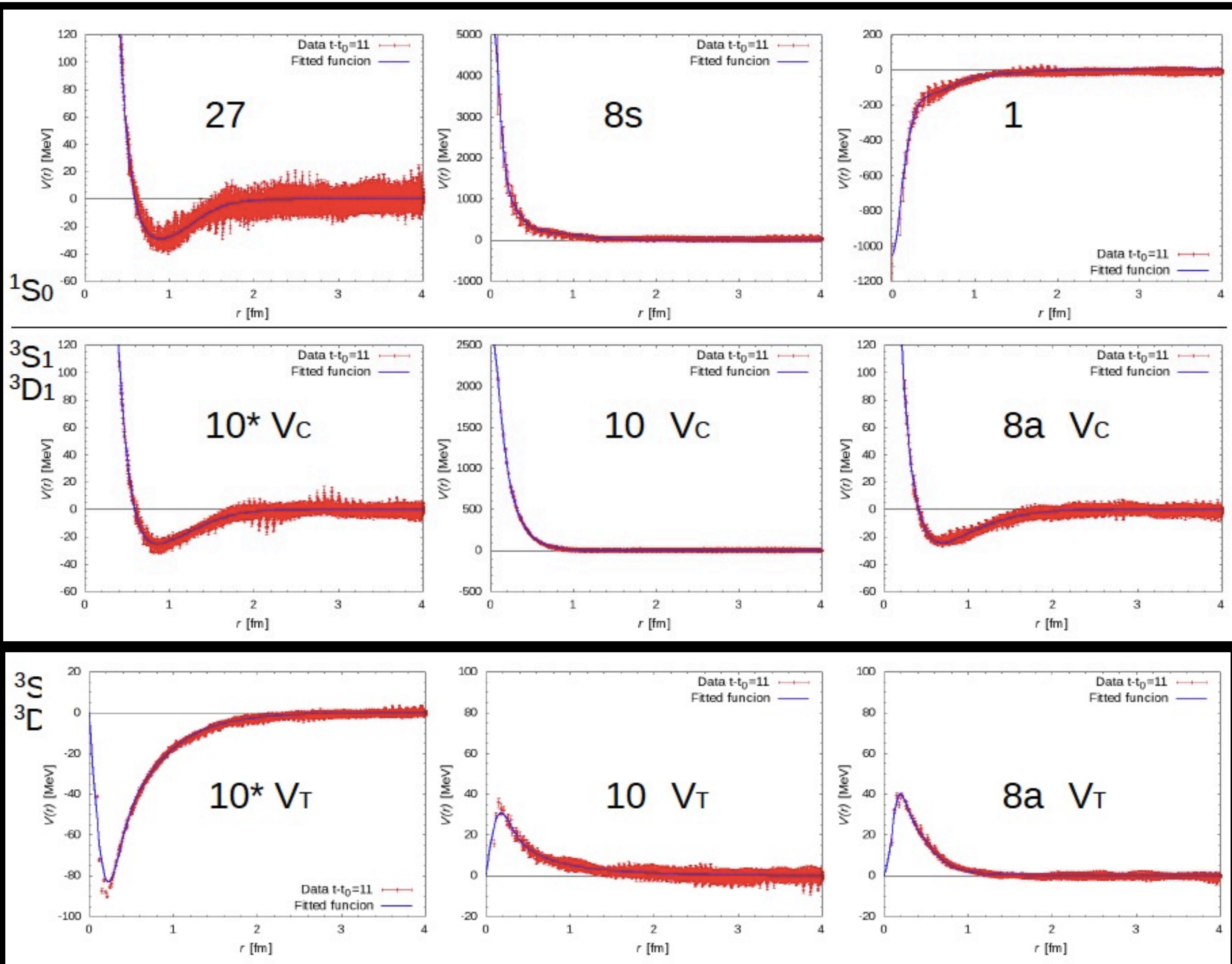
LQCD simulation setup

- Nf = 2+1 full QCD
  - Clover fermion + Iwasaki gauge w/ stout smearing
  - Volume $96^4 \simeq (8\ \mathrm{fm})^4$
  - 1/a = 2333 MeV, a = 0.0845 fm
  - $M_\pi \simeq 146$, $M_K \simeq 525$ MeV
    $M_N \simeq 956$, $M_\Lambda \simeq 1121$, $M_\Sigma \simeq 1201$, $M_\Xi \simeq 1328$ MeV
  - Collaboration in HPCI Strategic Program Field 5 Project 1

K-configuration

almost physical point

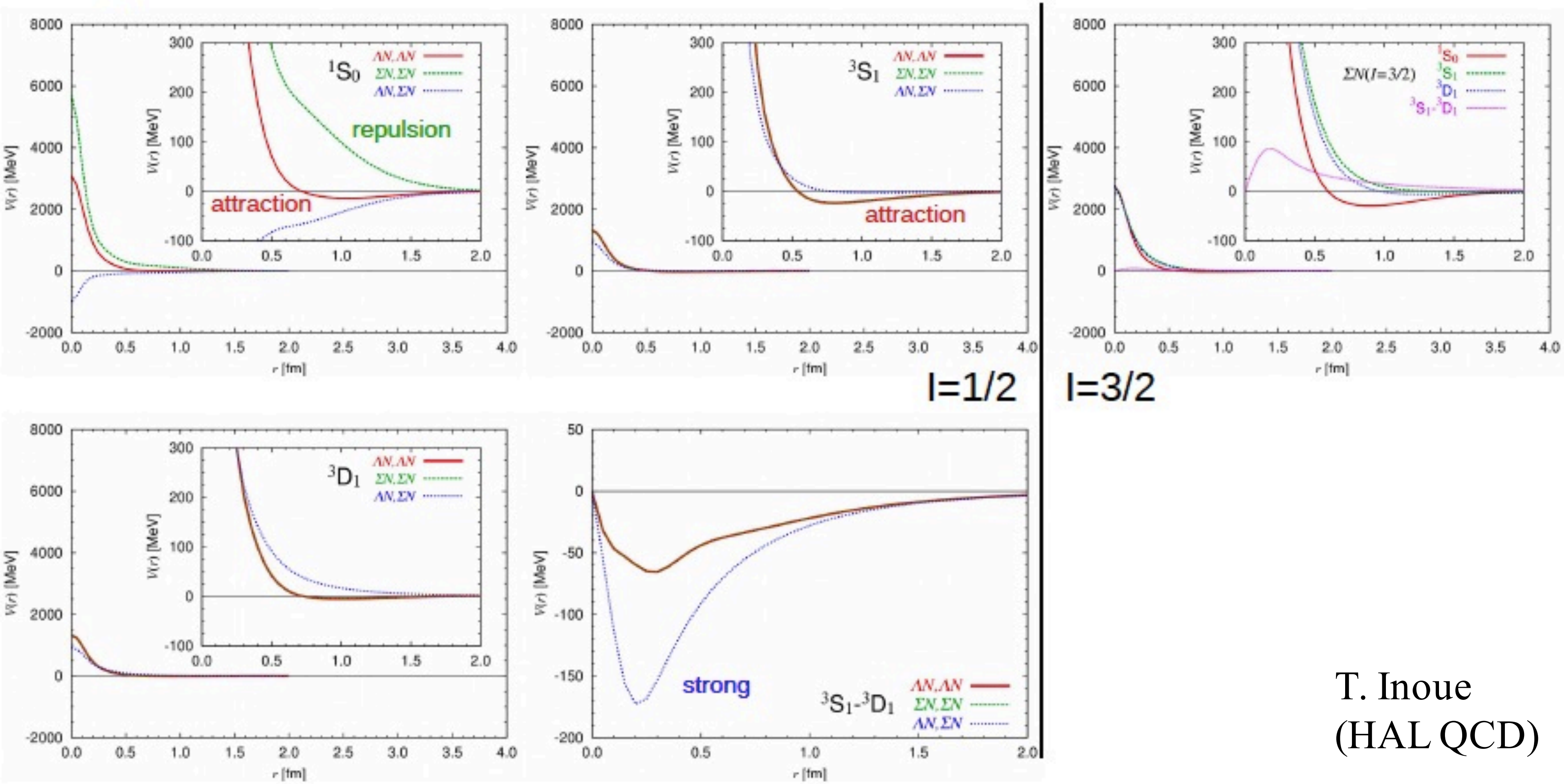
- Measurement
  - 4pt correlators: 52 channels in 2-octet-baryon (+ others)
  - Wall source w/ Coulomb gauge fixing
  - Dirichlet temporal BC to avoid the wrap around artifact
  - #data = 414 confs $\times$ 4 rot $\times$ 28 src.



$$8 \times 8 = \underbrace{27 + 8s + 1}_{^1S_0} + \underbrace{10^* + 10 + 8a}_{^3S_1,\ ^3D_1}$$

LQCD ΛN-ΣN

From K-conf. but rotated from the irr.-rep. base diagonal potentials.

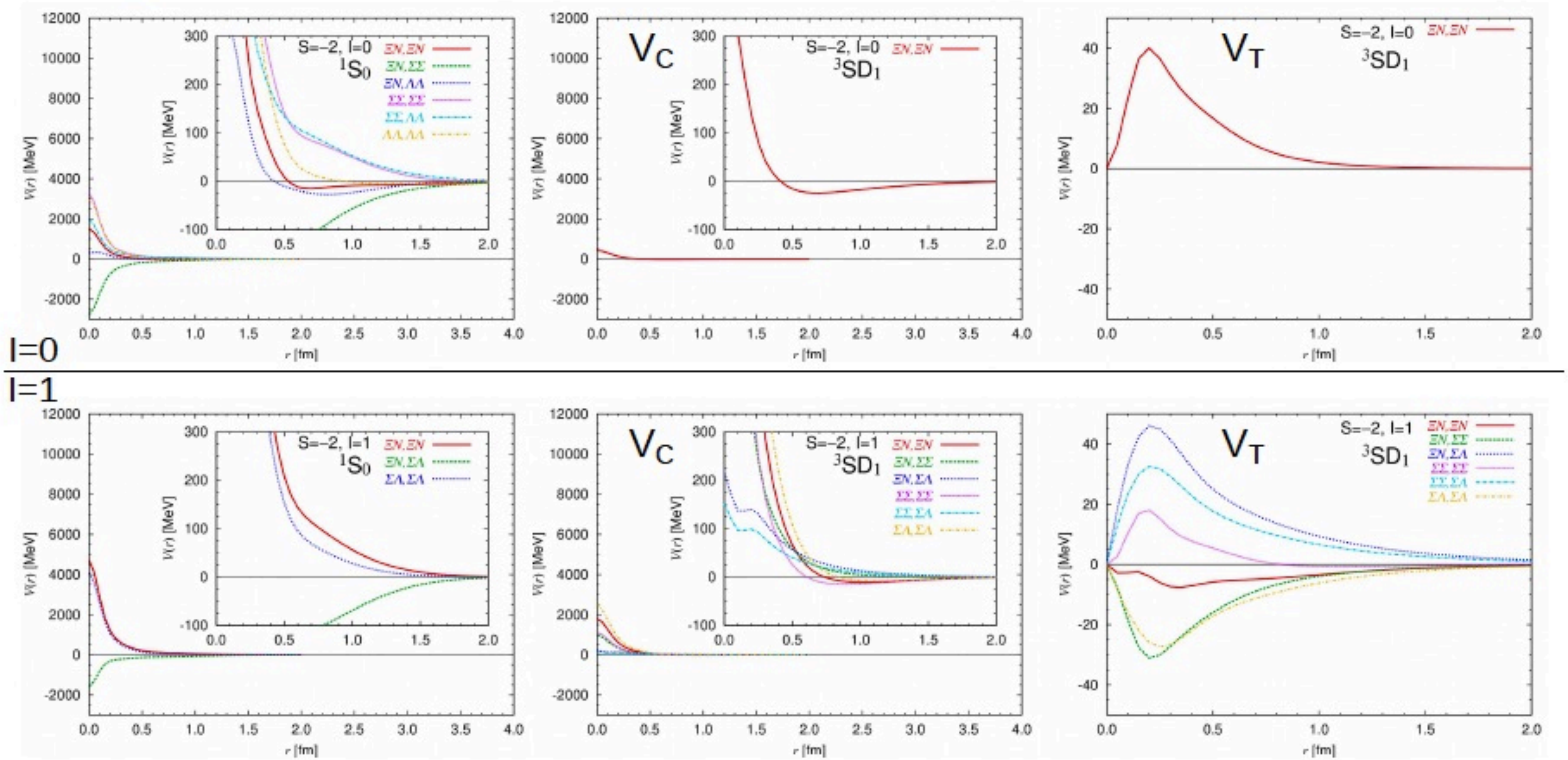


T. Inoue
(HAL QCD)

- In I=1/2, $^1S_0$ channel, ΛN has an attraction, while ΣN is repulsive.
- In I=1/2, $^3S_1$ channel, both ΛN and ΣN have an attraction.
- In I=1/2, strong tensor coupling in flavor off-diagonal.

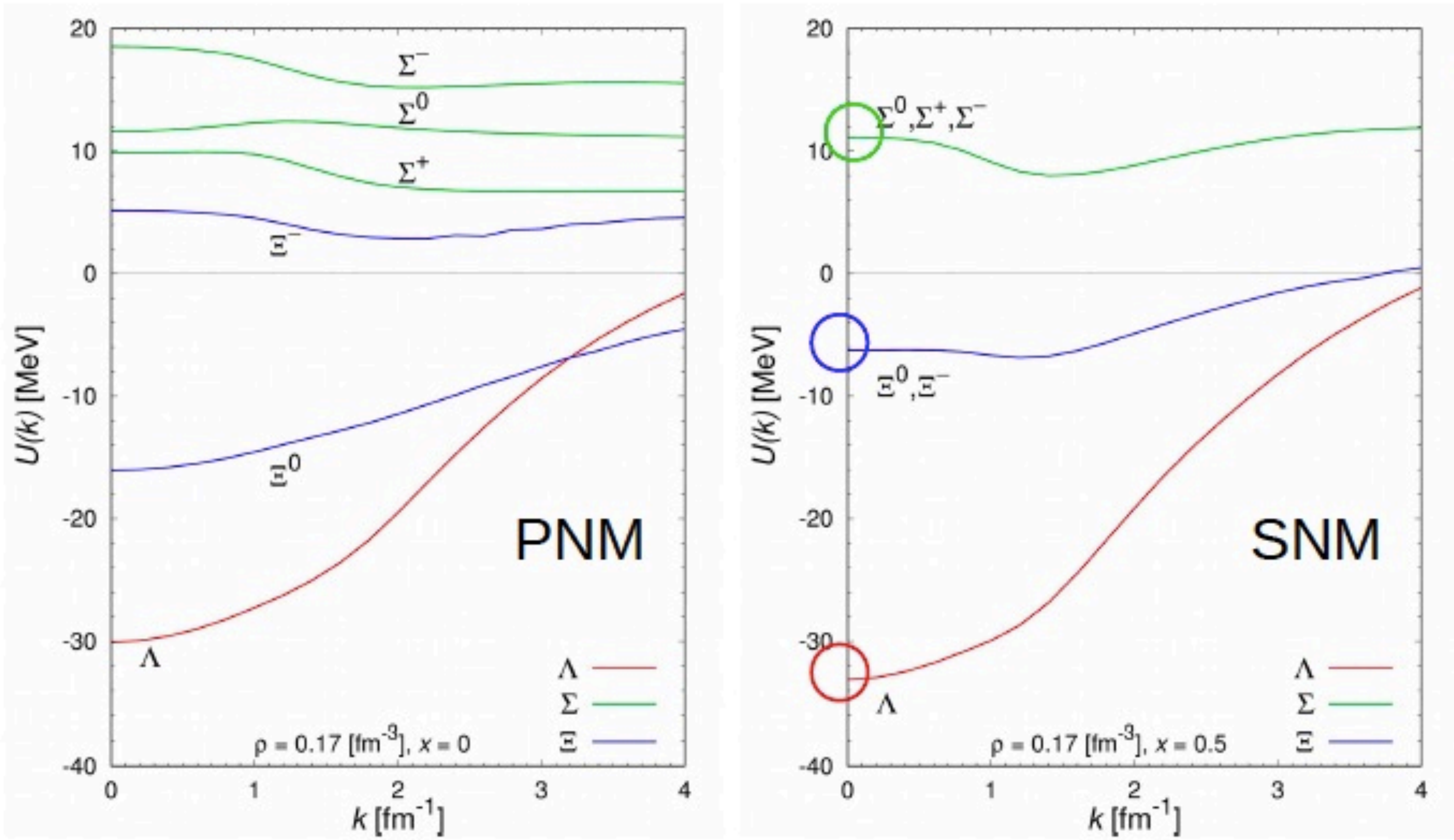
LQCD ΞN-YY

From K-conf. but rotated from the irr.-rep. base diagonal potentials.



- Many experimentally unknown coupled-channel potentials.
- One can see predictive power of the HALQCD method.

Hyperon single-particle potentials (BHF)



@ρ=0.17 [fm^{-3}]

Preliminary

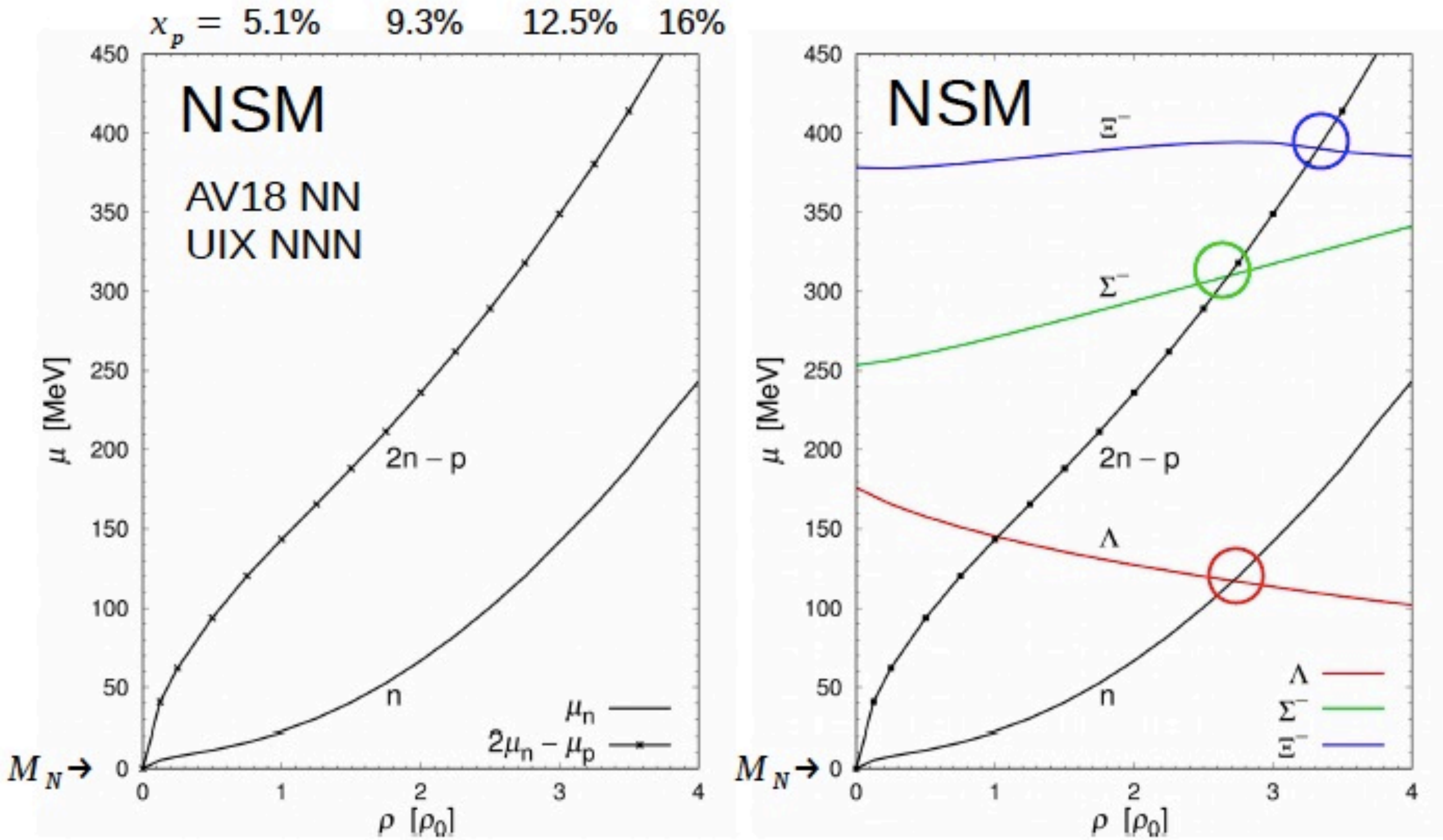
T. Inoue
(HAL QCD)

- obtained by using YN,YY forces form QCD.
- Results are compatible with experimental suggestion.

$$U_\Lambda^{\text{Exp}}(0) \simeq -30, \quad U_\Xi(0)^{\text{Exp}} \simeq -10, \quad U_\Sigma^{\text{Exp}}(0) \geq +20 \quad [\text{MeV}]$$

attraction attraction small repulsion

Hyperon onset in NSM (just for fun)



S-wave YN only

Preliminary

T. Inoue
(HAL QCD)

- We see that Σ^-, Λ, Ξ^- appear at $\rho = 2.5 - 3.5\ \rho_0$

Hadrons to Atomic nuclei HAL from Lattice QCD

Summary

1. Introduction on hadronic interactions from LQCD

Physical point BB data are now available

2. “Fake plateaux” ruin all the previous works except for HAL

”Mirage in temporal correlation functions for baryon-baryon interactions in lattice QCD”,
arXiv: 1607.06371 [hep-lat] (JHEP 10 (2016) 101) by HAL QCD Coll.

“Are two nucleons bound in lattice QCD for heavy quark masses ?
– Sanity check with Lucsher’s finite volume formula –”
arXiv: 1703.07210 [hep-lat] (submitted to PRD) by HAL QCD Coll.

3. Hyperons in dense matter