Few-body calculations for kaonic nuclear and atomic systems

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Outline

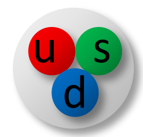
- Precise few-body calculations for kaonic systems
  - Modern $\bar{K}NN$ interaction K. Miyahara, T. Hyodo, PRC93 (2016)
    - Kaonic nuclei: $\bar{K}NN$ to $\bar{K}NNNNNN$ (7-body)

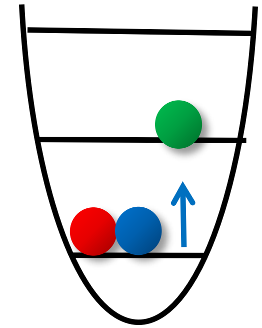
      S. Ohnishi, WH, T. Hoshino, K. Miyahara, T. Hyodo, arXiv:1701.07589
      accepted for publication in Phys. Rev. C, in press.
    - Kaonic deuterium: $\bar{K}NN$ three-body system

      T. Hoshino, S. Ohnishi, WH, T. Hyodo, W. Weise,
      arXiv:1705.06857, submitted to Phys. Rev. C (5/19).
  - Unified approach to atomic and nuclear kaonic systems
    - Nucleus ～few fm
    - Atom ～several hundreds fm

Kaonic nuclei (Nucleus with antikaon)

- $\Lambda(1405)$; $J^{\pi}=1/2^{-}$, S= -1
  - uds constituent quark model
    - Energy is too high

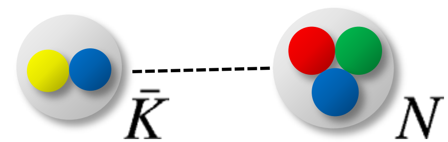


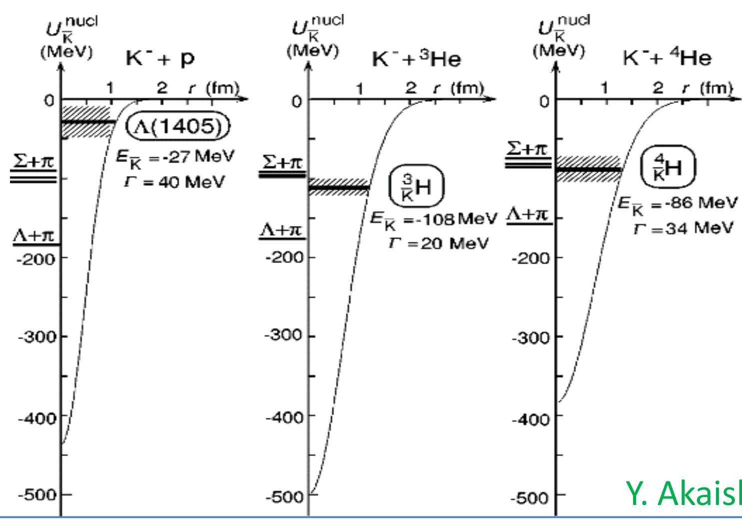


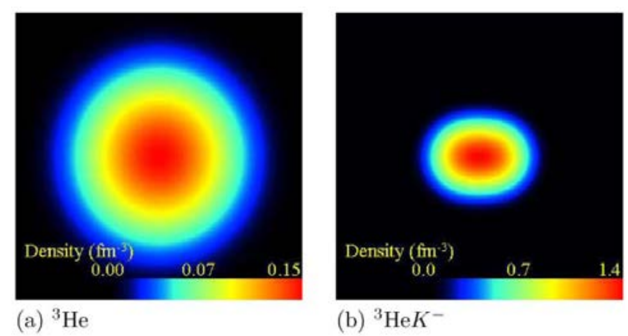
Isgur, Karl, PRD 18, 4187(1978)

  - $\bar{K}N$ quasi-bound state
    → strongly attractive $\bar{K}N$ interaction

Dalitz, Wong, Tajasekaran, PR 153, 1617 (1967)







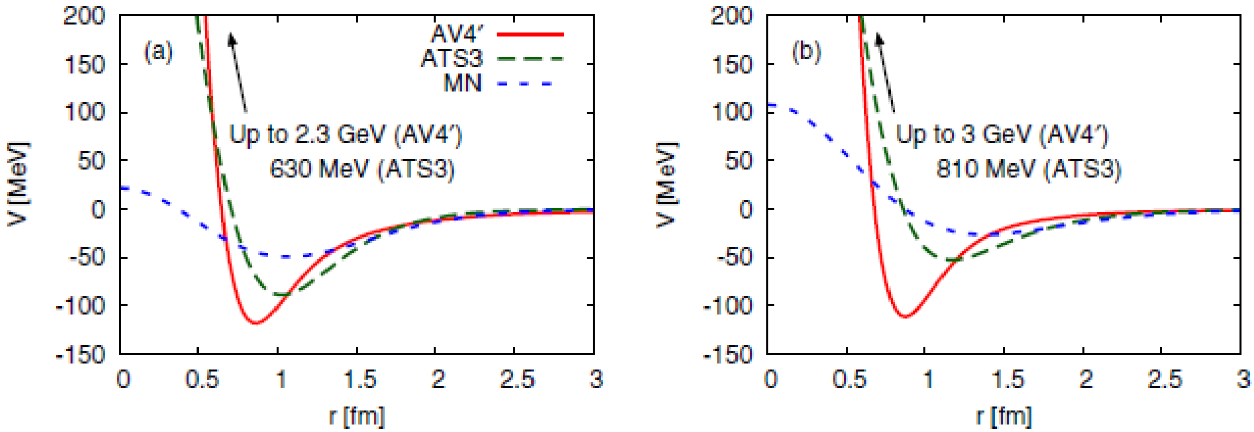
Dote, et. al., PLB590, 51(2004).

Y. Akaishi, T. Yamazaki, PRC 65, 044005 (2002).

Can such a high density system be produced in laboratory?
Does Kaonic nucleus really exist? E15 exp. → $\bar{K}N$ **interaction is essential!**

Kaonic nuclear systems (3 to 7-body)

- Hamiltonian $H = \sum_{i=1}^{\mathcal{N}} T_i - T_{\rm cm} + \sum_{i<j}^{\mathcal{N}-1} V_{ij}^{(NN)} + \sum_{i=1}^{\mathcal{N}-1} V_{i\mathcal{N}}^{(\bar{K}N)} + \sum_{i<j}^{\mathcal{N}} V_{ij}^{\rm Coul.}$
- Correlated Gaussian basis $\Phi_{SM_SM_T}(x, A) = \mathscr{A}\{\exp(-\tilde{x}Ax)\chi_{SM_S}\eta_{M_T}\}$,
  - Many parameters ∼(N-1)(N-2)/2 × (# of basis)
  - → Stochastic variational method K. Varga and Y. Suzuki, PRC52, 2885 (1995).
- Choice of *NN* potential (AV4’, ATS3, MN)



All *NN* interaction models reproduce the binding energy of *s*-shell nuclei

Choice of $\overline{K}N$ interaction

Kyoto $\overline{K}N$ potential

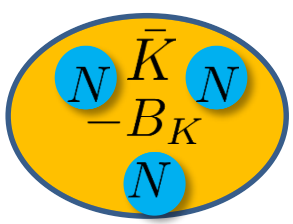
K.Miyahara, T.Hyodo, PRC 93, 015201 (2016)

- Energy-dependent $\overline{K}N$ single-channel potential
- Chiral SU(3) dynamics at NLO
- Pole energy: 1424 - 26i and 1381 – 81i MeV Y.Ikeda, T.Hyodo, W.Weise, NPA881 (2012) 98

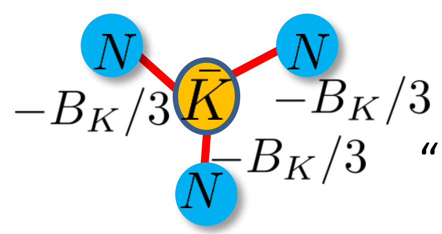
$\overline{K}N$ two-body energy in an *N*-body system are determined as:

$$\sqrt{s} = m_N + m_{\bar{K}} + \delta\sqrt{s} \quad , \quad -B_K \equiv \langle\Psi|H|\Psi\rangle - \langle\Psi|H_N|\Psi\rangle \quad ,$$

Type I: $\delta\sqrt{s} = -B_K$, Type II: $\delta\sqrt{s} = -B_K/(N-1)$, for *N-body*



"Field picture"



"Particle picture"

A. Dote, T. Hyodo, W. Weise, NPA804, 197 (2008).

Akaishi-Yamazaki (AY) potential

Akaishi, Yamazaki, PRC65, 04400(2002).

- Energy-independent
- Reproduce $\Lambda(1405)$ as $\overline{K}N$ quasi-bound state

Variational calculation for many-body quantum system

- Many-body wave function Ψ has all information of the system
- Solve many-body Schoedinger equation

  ⇔ Eigenvalue problem with Hamiltonian matrix

$$H\Psi = E\Psi$$

- Variational principle $<\Psi|H|\Psi> = E \geqq E_0$ ("Exact" energy)

  (Equal holds if Ψ is the "exact" solution)

Many degrees of freedom

→ Expand Ψ with several sets of basis functions

Correlated Gaussian + Global vectors

Explicitly correlated basis approach

Correlated Gaussian with two global vectors

Y. Suzuki, W.H., M. Orabi, K. Arai, FBS42, 33-72 (2008)

$$\phi^{\pi}_{(L_1L_2)LM_L}(A,u_1,u_2)=\exp(-\tilde{\boldsymbol{x}}A\boldsymbol{x})[\mathcal{Y}_{L_1}(\tilde{u}_1\boldsymbol{x})\mathcal{Y}_{L_2}(\tilde{u}_2\boldsymbol{x})]_{LM_L}$$

x: any relative coordinates (cf. Jacobi)

$$\mathcal{Y}_\ell(\boldsymbol{r})=r^\ell Y_\ell(\hat{\boldsymbol{r}})$$

$$\tilde{\boldsymbol{x}}A\boldsymbol{x}=\sum_{i,j=1}^{N-1}A_{ij}\boldsymbol{x}_i\cdot\boldsymbol{x}_j$$

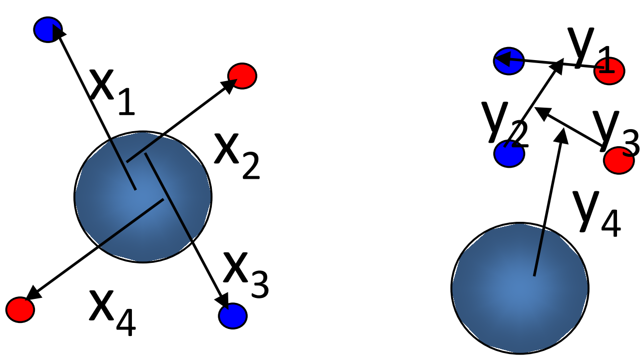
$$\tilde{u}_i\boldsymbol{x}=\sum_{k=1}^{N-1}(u_i)_k\boldsymbol{x}_k$$

Formulation for N-particle system
Analytical expression for matrix elements

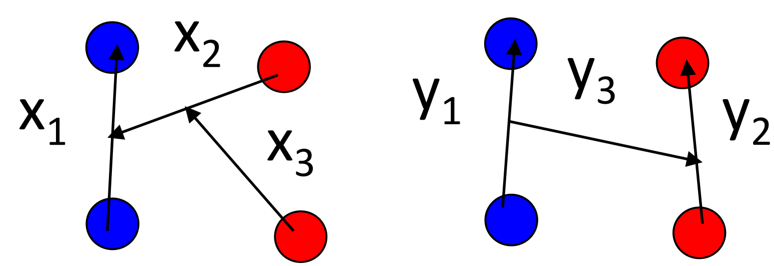
Functional form does not change under any coordinate transformation

$$\boldsymbol{y}=T\boldsymbol{x}\quad\Longrightarrow\quad\tilde{\boldsymbol{y}}B\boldsymbol{y}=\tilde{\boldsymbol{x}}\tilde{T}BT\boldsymbol{x}\qquad\qquad\tilde{v}\boldsymbol{y}=\widetilde{\tilde{T}v}\boldsymbol{x}$$

Shell and cluster structure



Rearrangement channels



See Review: J. Mitroy et al., Rev. Mod. Phys. 85, 693 (2013)

Basis optimization:
Stochastic Variational Method

Possibility of the stochastic optimization

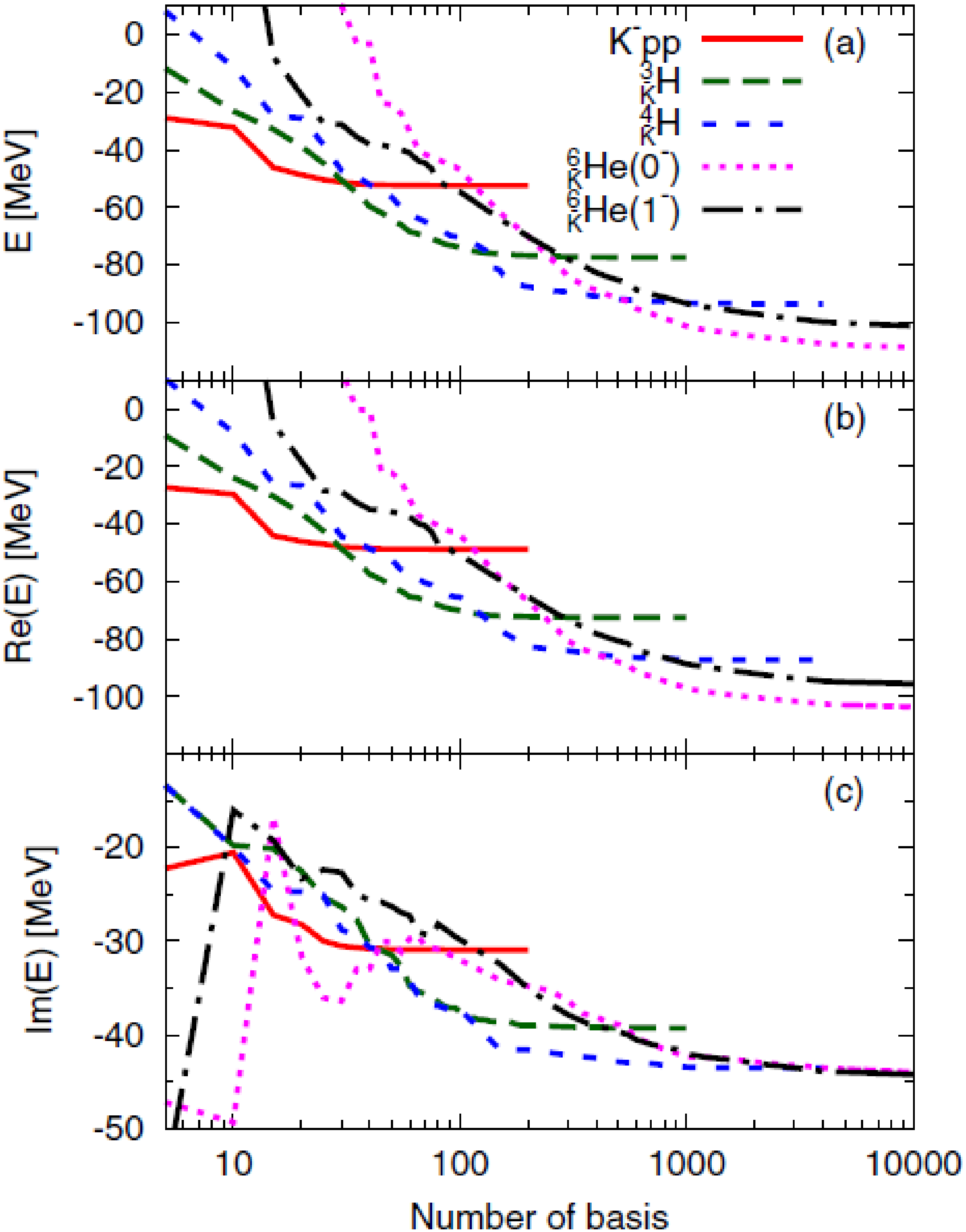
1. increase the basis dimension one by one
2. set up an optimal basis by trial and error procedures
3. fine tune the chosen parameters until convergence

1. **Generate** $(A_k^1, A_k^2, \cdots\cdots, A_k^m)$ **randomly**
2. **Get the eigenvalues** $(E_k^1, E_k^2, \cdots\cdots, E_k^m)$
3. **Select** A_k^n corresponding to the lowest E_k^n and **Include** it in a basis set
4. **k → k+1**

Y. Suzuki and K. Varga, Stochastic variational approach to quantum-mechanical few-body problems, LNP 54 (Springer, 1998).
K. Varga and Y. Suzuki, Phys. Rev. C52, 2885 (1995).

Energy curves

- Optimization only with a real part of the $\overline{K}N$ pot.
- Two-body $\overline{K}N$ energy is self-consistently determined
- AV4' NN pot. is employed



Full energy curves

Validity of this approach is confirmed in the three-body (K-pp) system

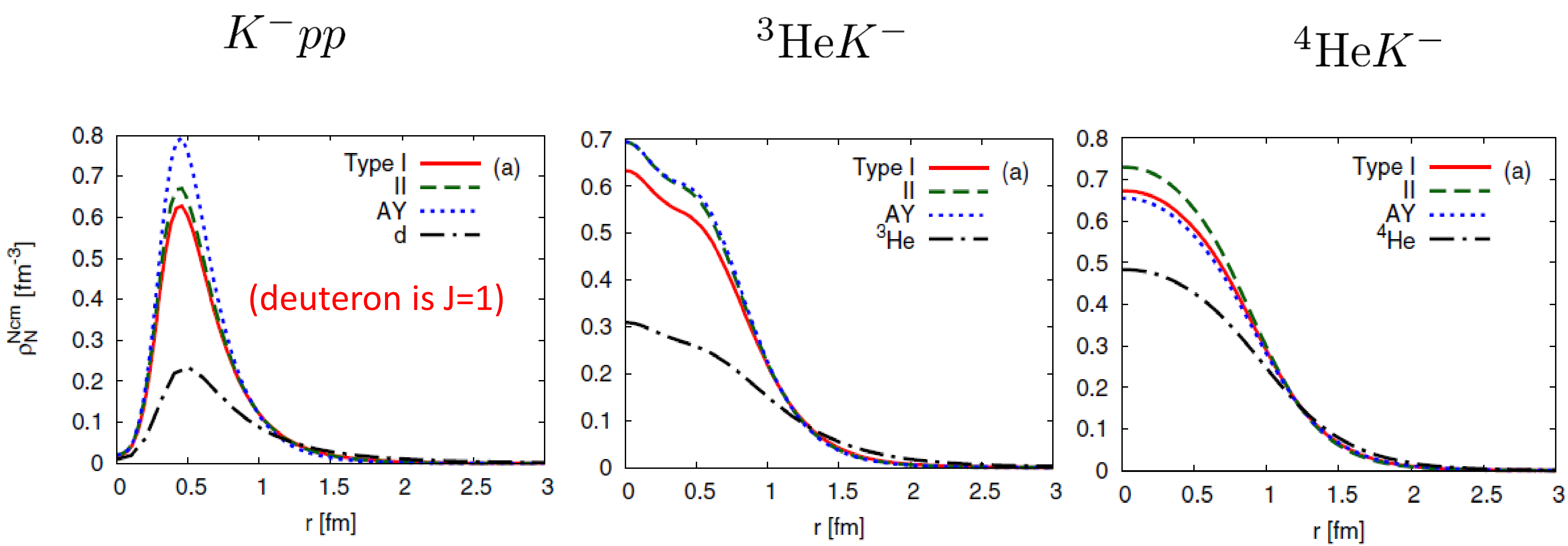
Properties of K^-pp

Model	Kyoto		AY
	Type I	Type II	
B (MeV)	27.9	26.1	48.7
Γ (MeV)	30.9	59.3	61.9
$\delta\sqrt{s}$ (MeV)	$-61.0 - i25.0$	$-30.2 - i23.7$	
P_{K^-}	0.65	0.65	0.64
$P_{\bar{K}^0}$	0.35	0.35	0.36
$\sqrt{\langle r_{NN}^2\rangle}$ (fm)	2.16	2.07	1.84
$\sqrt{\langle r_{\bar{K}N}^2\rangle}$ (fm)	1.80	1.73	1.55
$\sqrt{\langle r_N^2\rangle}$ (fm)	1.12	1.08	0.958
$\sqrt{\langle r_{\bar{K}}^2\rangle}$ (fm)	1.14	1.10	0.988

Kyoto $\overline{K}N$ pot.
Similar binding energies with Types I and II ～27-28 MeV

AY pot.
Deeper binding energy ～49MeV
→ Smaller rms radii

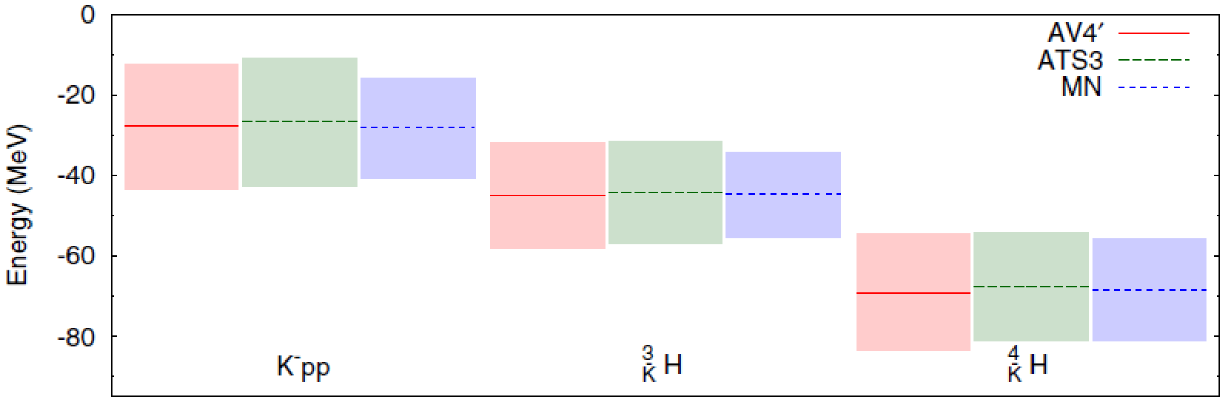
Nucleon Density distributions



- Central nucleon density ρ(0) is enhanced by kaon
- ρ(0)~0.7fm^{-3} at maximum, ∼2 times higher than that without $\overline{K}$

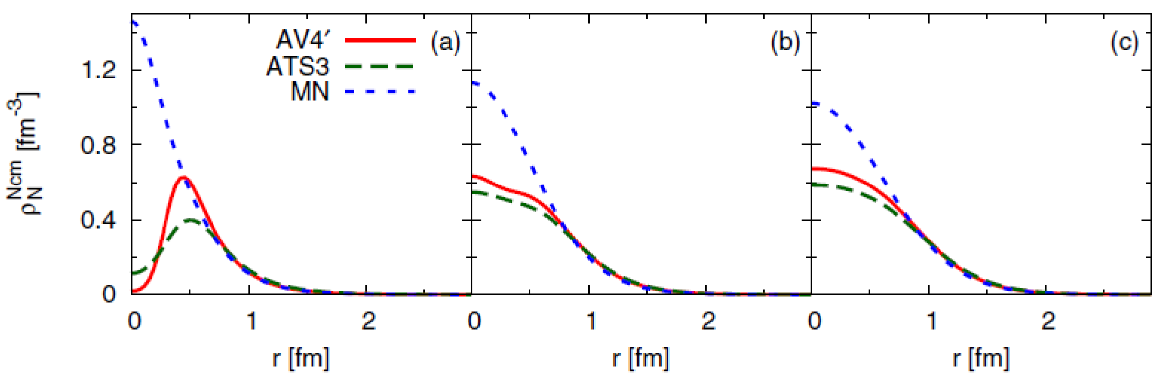
Interaction dependence

Binding energy and decay width with different NN potential models



Not sensitive to the NN interaction models

Nucleon density distributions



AV4’ and ATS3 potential: strong short-range repulsion

MN: weak short-range repulsion

Structure of $\overline{K}NNNNNN$ with $J^{\pi}=0^{-}$ and 1^{-}

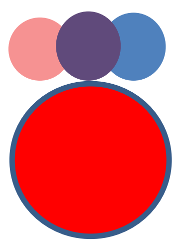
A=2

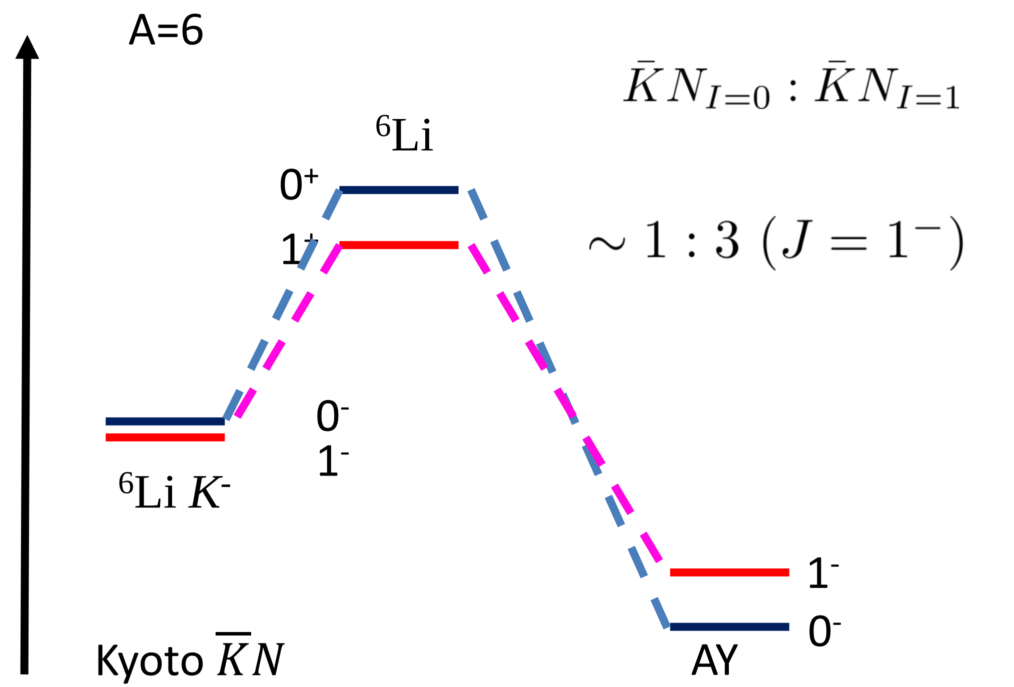
	NN	$\overline{K}NN$
J=0	unbound	Bound
J=1	bound (d)	unbound

$\bar{K}N_{I=0} : \bar{K}N_{I=1}$

$\sim 3:1 \; (J=0^{-})$

$\sim 1:3 \; (J=1^{-})$





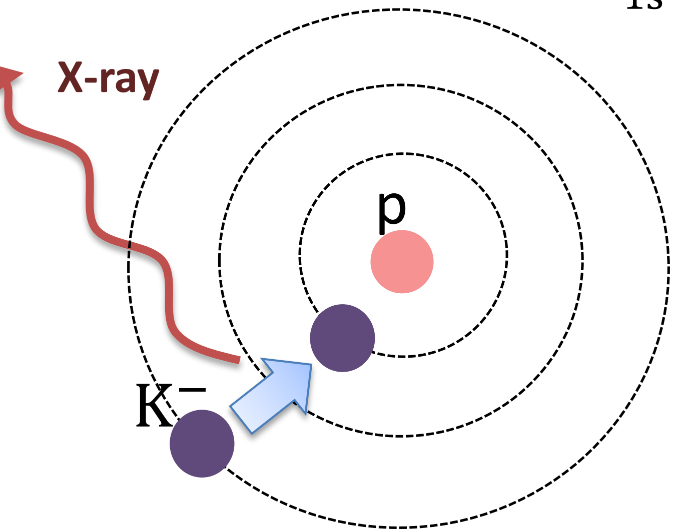
- $\overline{K}N$ interaction in I=0 is more attractive than in I=1, and J=0 state containing more I=0 component than J=1
  → Energy gain in J=0 is larger than J=1 channel
- AY potential in I=0 is strongly attractive
  → J=0 ground state

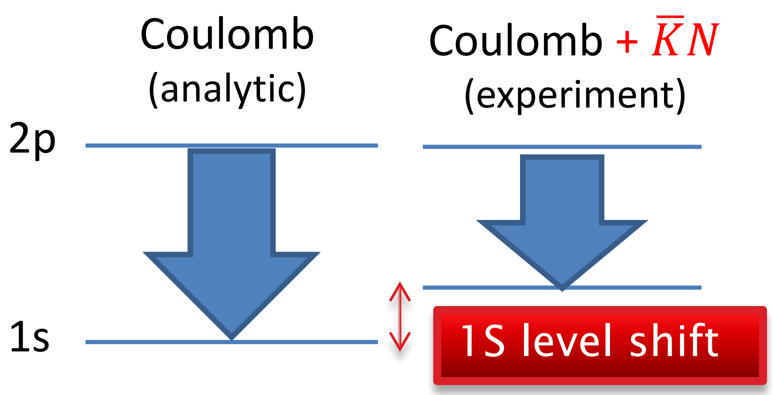
Kaonic hydrogen (atomic system)

- Bound mainly with Coulomb int. (like e^- in H)
- $\bar{K}N$ interaction induces "level shift"
- Precise measurement

(2011, SIDDHARTA experiment , DAΦNE) Bazzi et al., NPA881 (2012)

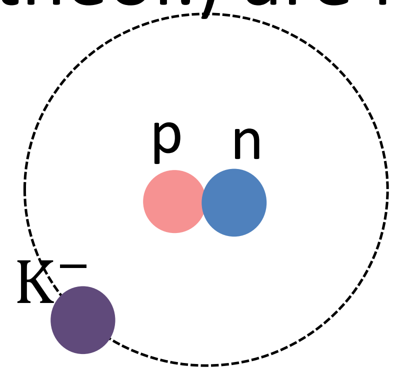
$$\epsilon_{1s} = 283 \pm 36(\text{stat}) \pm 6(\text{syst})\text{eV}$$
$$\Gamma_{1s} = 541 \pm 89(\text{stat}) \pm 22(\text{syst})\text{eV}$$





Constraint for $\overline{K}N$ interaction: Kaonic deuterium

- Isospin dependence of $\overline{K}N$ interactions
  - I=0: well determined by Λ(1405) properties
  - I=1: constraint is weak only with kaonic hydrogen
- Precise kaonic deuterium data (Exp. and theor.) are highly desired



Kaonic deuterium

$$|K^- p\rangle = |\uparrow\downarrow\rangle = |I = 0\rangle + |I = 1\rangle$$

$$|K^- n\rangle = |\uparrow\uparrow\rangle = |I = 1\rangle$$

I=0:I=1=1:3

Three-body calculation for kaonic deuterium

Hamiltonian

$$H = T + V = \sum_{i=1}^{3} \frac{\boldsymbol{p}_i^2}{2m_i} - T_{cm} + V^{NN} + V^{N\bar{K}} + V^{coul}$$

V^{NN} (Minnesota potential) D. R. Thompson, M. Lemere and Y. C. Tang, NPA286 (1977)

$V^{N\bar{K}}$(Kyoto $\bar{K}N$ potential) K.Miyahara, T.Hyodo, PRC93 (2016)

- The Kyoto $\bar{K}N$ pot. simulates scattering amplitude calculated from NLO chiral SU(3) dynamics Y.Ikeda, T.Hyodo, W.Weise, NPA881 (2012)

1. Scattering length extracted from the energy shift measured in SIDDHARTA experiment.
2. Cross section of a $\bar{K}N$ two-body scattering
3. Branching ratio of the $\bar{K}p$ decay

Three-body calculation

<u>Variational Method</u>

- Wave function

Correlated Gaussian basis

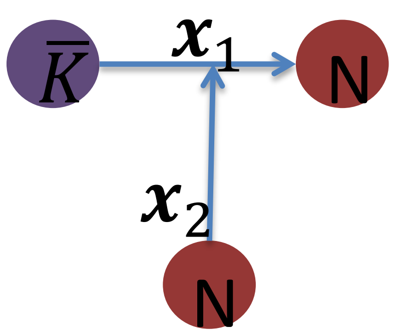
$$\Psi = \sum_{i=1}^{N} c_i \phi_i \,, \qquad \phi = A_{NN}\{e^{-\frac{1}{2}\tilde{x}Ax}[[y_{L_1}(\tilde{\boldsymbol{u}}\boldsymbol{x})y_{L_2}(\tilde{\boldsymbol{v}}\boldsymbol{x})]_L \chi_S]_{JM}\eta_{TM_t}\}$$

A: 2×2 positive definite symmetric matrix

$\boldsymbol{x} = \{\boldsymbol{x}_1, \boldsymbol{x}_2\}$, $\tilde{\boldsymbol{u}}\boldsymbol{x} = u_1\boldsymbol{x}_1 + u_2\boldsymbol{x}_2$, $\tilde{\boldsymbol{v}}\boldsymbol{x} = v_1\boldsymbol{x}_1 + v_2\boldsymbol{x}_2$

$$y_{lm}(\tilde{\boldsymbol{u}}\boldsymbol{x}) = (\tilde{\boldsymbol{u}}\boldsymbol{x})^l Y_{lm}(\widehat{\tilde{\boldsymbol{u}}\boldsymbol{x}})$$

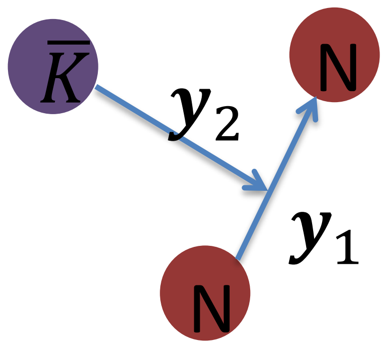
χ_{JM}: spin function, η_{TM_t}: isospin function



Configuration 1

- Geometric progression for Gaussian fall-off parameters
  - Cover 0.1-500 fm
  - $L_1 + L_2 \leq 4$
  - About 8000 basis states
- Channel coupling

$$|K^- pn\rangle = |\downarrow\uparrow\downarrow\rangle, \qquad |\bar{K}^0 nn\rangle = |\uparrow\downarrow\downarrow\rangle$$



Configuration 2

Precise calculation

Generalized eigenvalue problem

$$\sum_{j=1}^{K}(H_{ij}-EB_{ij})C_j=0 \qquad H_{ij}=\langle\Phi_i|H|\Phi_j\rangle \quad B_{ij}=\langle\Phi_i|\Phi_j\rangle$$

New orthonormal set $\phi_\mu=\frac{1}{\sqrt{\mu}}\sum_{i=1}^{K}c_i^{(\mu)}\Phi_i \rightarrow \sum_{j=1}^{N}(H_{ij}-EB_{ij})C_j=0$

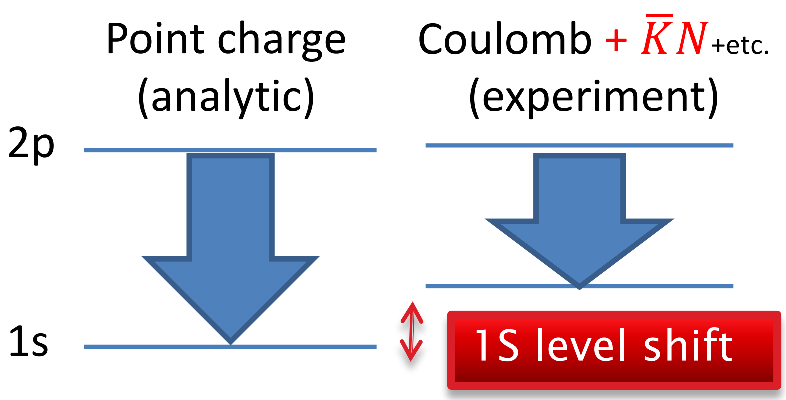
Cutoff parameter $\lambda_{\rm cut}=\mu_{\rm max}/\mu_{\rm min}$

$\log_{10}\lambda_{\rm cut}$	N	Re[E] (MeV)
16	1677	−2.211689436
17	2194	−2.211722964
18	2377	−2.211732072
19	2511	−2.211735493
20	2621	−2.211737242
21	2721	−2.211737609
22	2806	−2.211737677
23	2879	−2.211737682

Energy levels of 1S, 2P, 2S states

	E_{1S}(keV)	E_{2P}(keV)	E_{2S}(keV)
Coulomb	-10.398	-2.602	-2.600
Uniform charge (2-body)	-10.401	-2.602	-2.601
Point charge (2-body)	-10.406	-2.602	-2.602
Coulomb+$\bar{K}N$	$-9.736-i\,0.508$	$-2.602-i\,0.000$	$-2.517-i\,0.067$

- No level shift of 2P state
- Transition energy can directly be used for the extraction of the 1S level shift



Level shift of kaonic deuterium

$$\Delta E - i\frac{\Gamma}{2} = (670 - i\,508)\,\mathrm{eV},$$

Sensitivity of I=1 component

- $Re\, V^{\bar{K}N} = Re\, V_{I=0} + \beta \times Re\, V_{I=1}$
- Factor for the real part of the KN potential within the SIDDHARTA constraint for the level shift of kaonic hydrogen
  - 283 ± 36(stat) $\pm$ 6(syst)eV

Energy shifts of kaonic hydrogen and deuterium

	K^-p		K^-d	
β	ΔE	Γ	ΔE	Γ
1.08	287	648	676	1020
1.00	283	607	670	1016
−0.17	310	430	506	980

～25% uncertainty
Possible constraint for I=1

Conclusions

Unified description of kaonic atom and nuclear systems

– Kaonic nucleus (3- to 7-body)
  - Central density is increased by ～2 times with $\bar{K}N$ int.
    – Soft *NN* interaction induces too high central densities
  - Inverted spin-parity in the g.s, of ^{6}Li $\bar{K}$ is predicted
    – Isospin dependence of $\bar{K}N$ interaction is essential

– Kaonic atom (3-body)
  - Prediction of the energy shift of the kaonic deuterium

$$\Delta E - i\frac{\Gamma}{2} = (670 - i\,508)\,\mathrm{eV},$$

  - I=1 component can be constrained if measurement is performed within 25% uncertainty

(Planned exp. accuracy ～5-10%)