$Z_c(3900)$ from lattice QCD

Yoichi Ikeda (RCNP, Osaka University)

HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

S. Aoki, T. Aoyama, D. Kawai, T. Miyamoto, K. Sasaki (YITP, Kyoto Univ.)
T. Doi, T. M. Doi, S. Gongyo, T. Hatsuda, T. Iritani (RIKEN)
Y. Ikeda, N. Ishii, K. Murano, H. Nemura (RCNP, Osaka Univ.)
T. Inoue (Nihon Univ.)
F. Etminan (Univ. of Birjand)

Strangeness and charm in hadrons and dense matter
@YITP, Kyoto Univ., May 24, 2017.

Contents

✤ Tetraquark candidate Z_c(3900)

Y. Ikeda et al., (HAL QCD), PRL117, 242001 (2016).

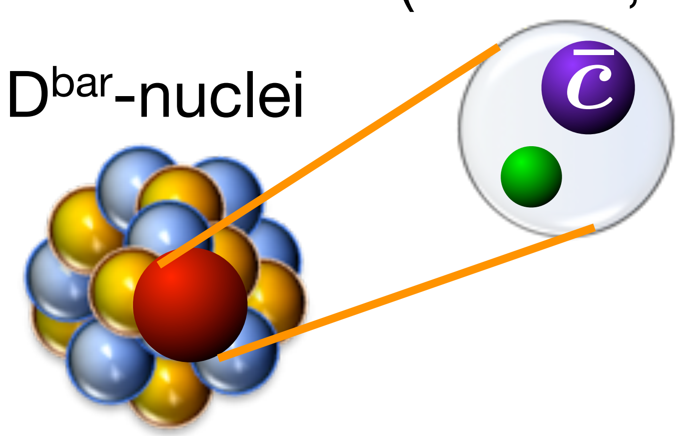
- ✓ Introduction to Z_c(3900)
- ✓ How to analyze exotic hadrons on the lattice
- ✓ Coupled-channel HAL QCD method
- ✓ Nature of Z_c(3900)
- ✓ Comparison with experiments

✤ D^{bar}-nucleon interaction & D^{bar}-nuclei

- ✓ D^{bar}-nucleon interaction from LQCD
- ✓ Possible structure of D^{bar}-nuclei

$D^{bar} = (D^0 = uc^{bar}, D^- = dc^{bar})$

D^{bar}-nuclei

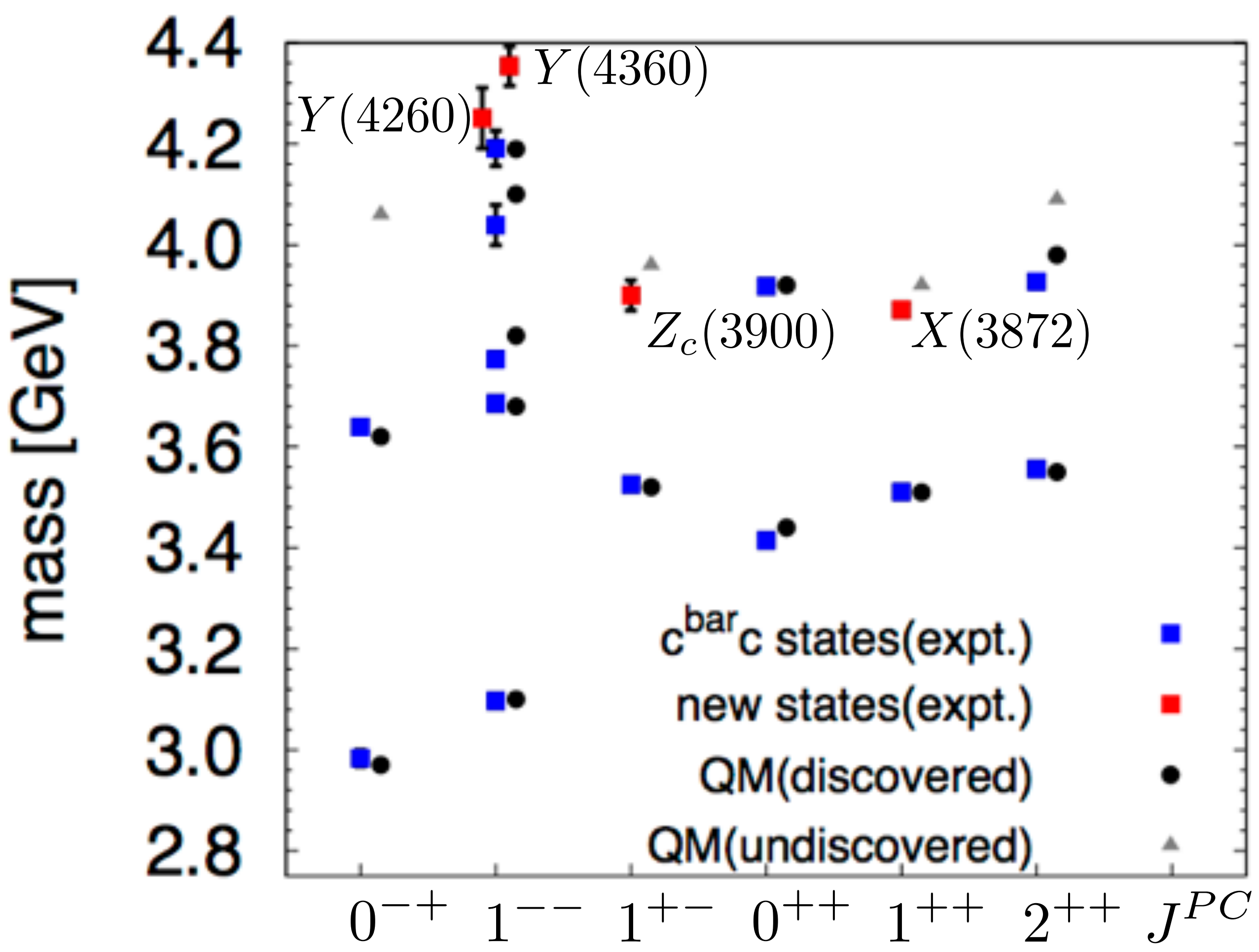


- heavy hadrons as impurity
  (emergence of isospin Kondo effect?)

talk by Yasui (Tue.)

✤ Summary

Charmonium(-like) states



✓ Quark models well describe observed mass spectra at low energies (< 3.8 GeV)

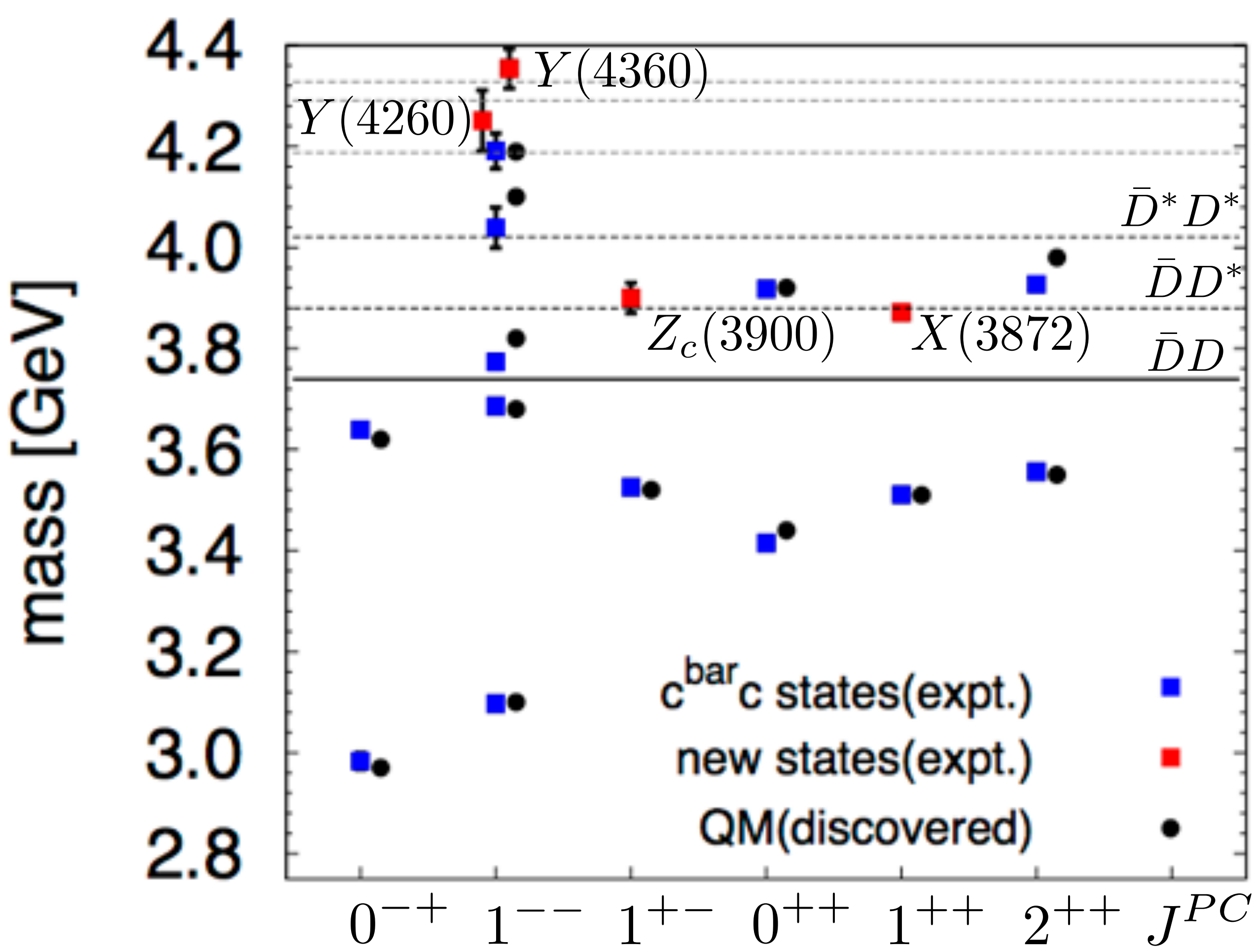
✓ Several states at high energies (> 3.8GeV) are not discovered

Godfrey, Isgur, PRD 32 (1985).
Barnes, Godfrey, Swanson, PRD 72 (2005).

NEW (X, Y, Z) states observed in expt. which are NOT within QM spectrum

➡ Are they exotic?

Charmonium(-like) states



✓ Quark models well describe observed mass spectra at low energies (< 3.8 GeV)

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- **NEW (X, Y, Z) states** observed in expt. which are NOT within QM spectrum

➡ Are they exotic?

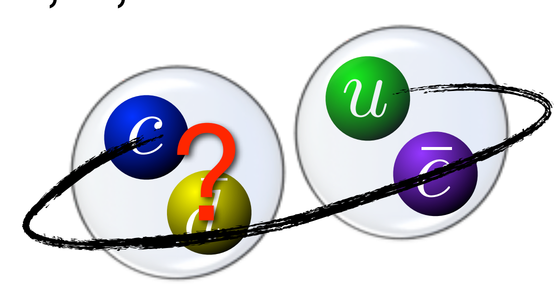
- All new states are found above 3.8 GeV

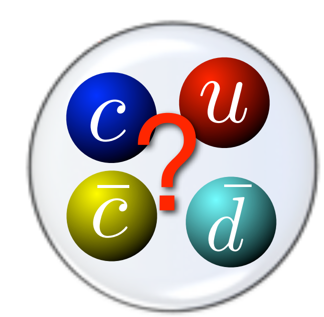
✓ Lowest open charm threshold ($D^{bar}D$) is 3.75 GeV

✓ All new states embedded in two-meson continuum ($D^{bar}D$, $D^{bar}D^*$, $D^{bar*}D^*$,...)

✓ Channel coupling could be a key to investigate X, Y, Z states

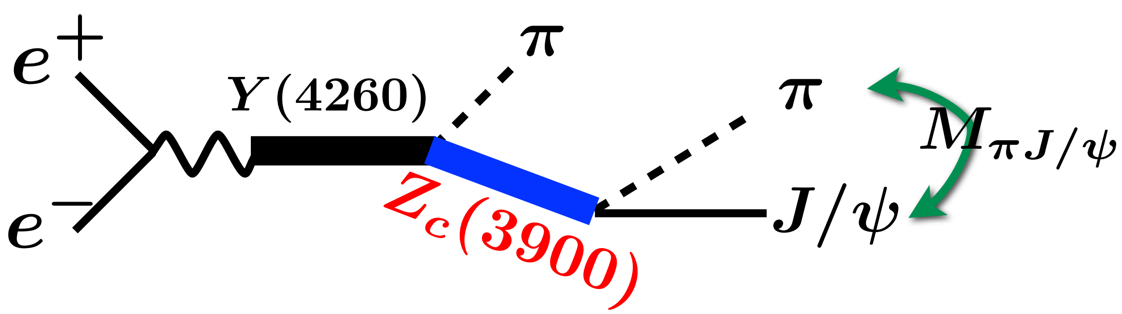
- Our target : tetraquark candidate $Z_c(3900)$



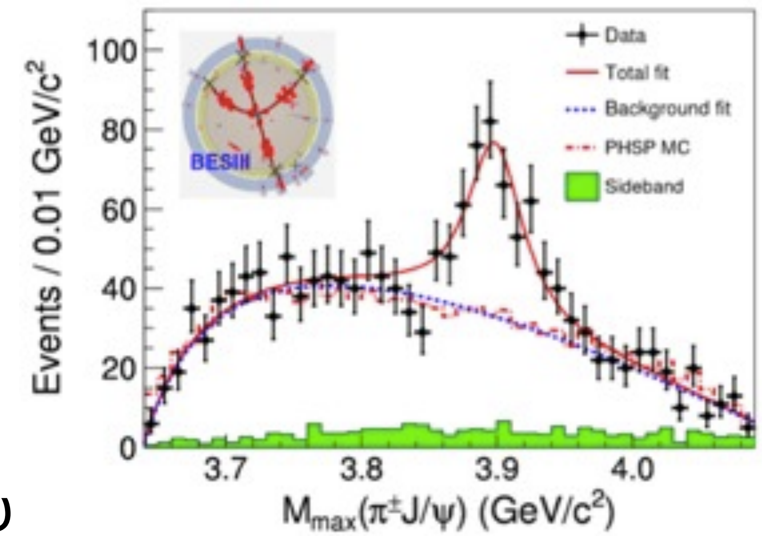


Exotic tetraquark candidate $Z_c(3900)$

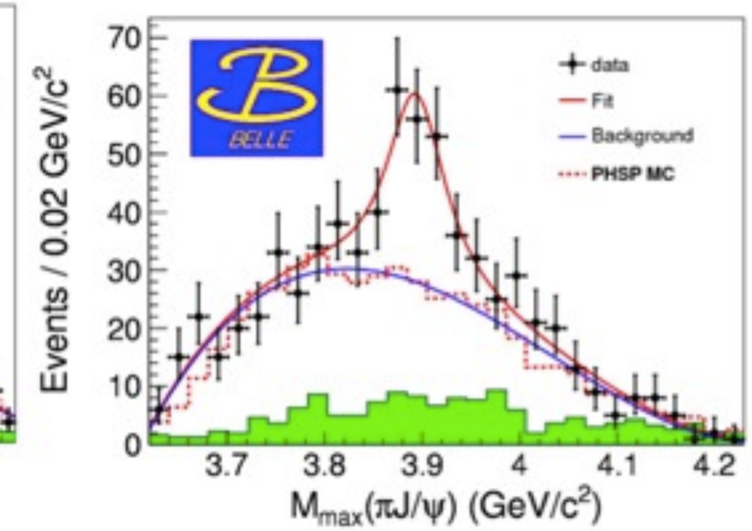
- **Tetraquark $Z_c(3900)$?**



BESIII Coll., PRL110 (2013).



Belle Coll., PRL110 (2013).



$e^+ + e^-$ --> Y(4260) --> π + $Z_c(3900)$

➡ $\pi^{+/-}$ + J/ψ

- peak in **$\pi^{+/-}J/\psi$ invariant mass** (minimal quark content cc^{bar} ud^{bar} <--> tetraquark candidate)
- M ~ 3900, Γ ~ 60 MeV (Breit-Wigner) --> just above $D^{bar}D^*$ threshold

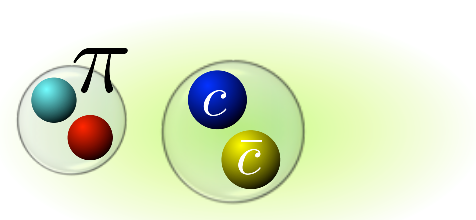
($J^P=1^+$ <--> couple to **s-wave** meson-meson continuum)

★ **structure of $Z_c(3900)$ studied by models**

tetraquark

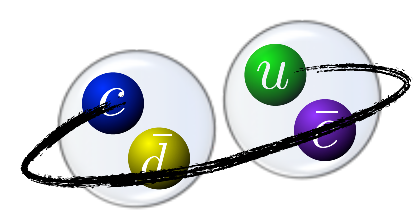
Maiani et al.(‘13)

$J/\psi + \pi$ atom



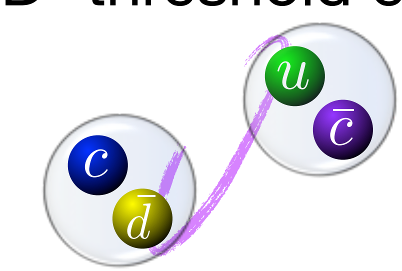
Voloshin(‘08)

$D^{bar}D^*$ molecule



Nieves et al.(‘11) + many others

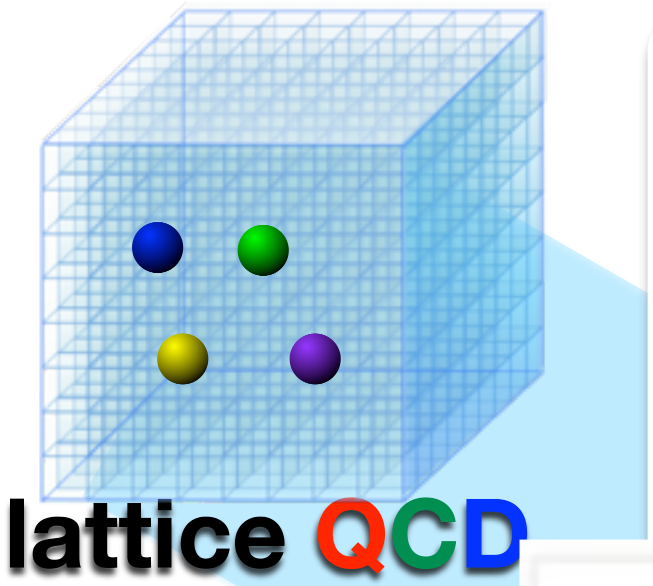
$D^{bar}D^*$ threshold effect



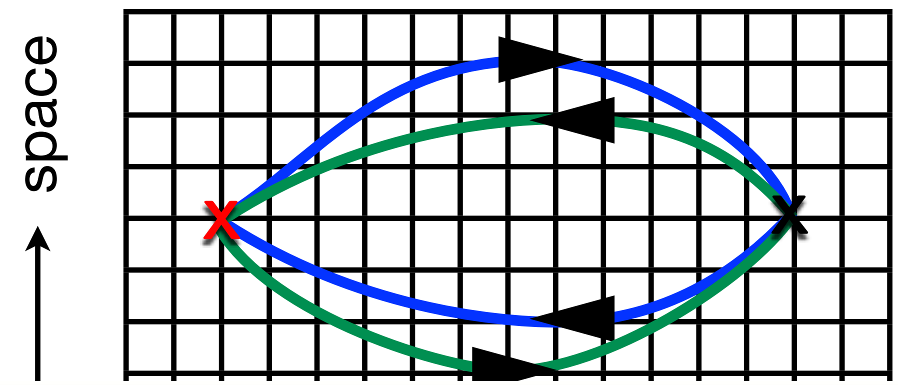
Chen et al.(‘14), Swanson(‘15)

genuine state expected — **kinematical origin**

Exotic hadrons on the lattice



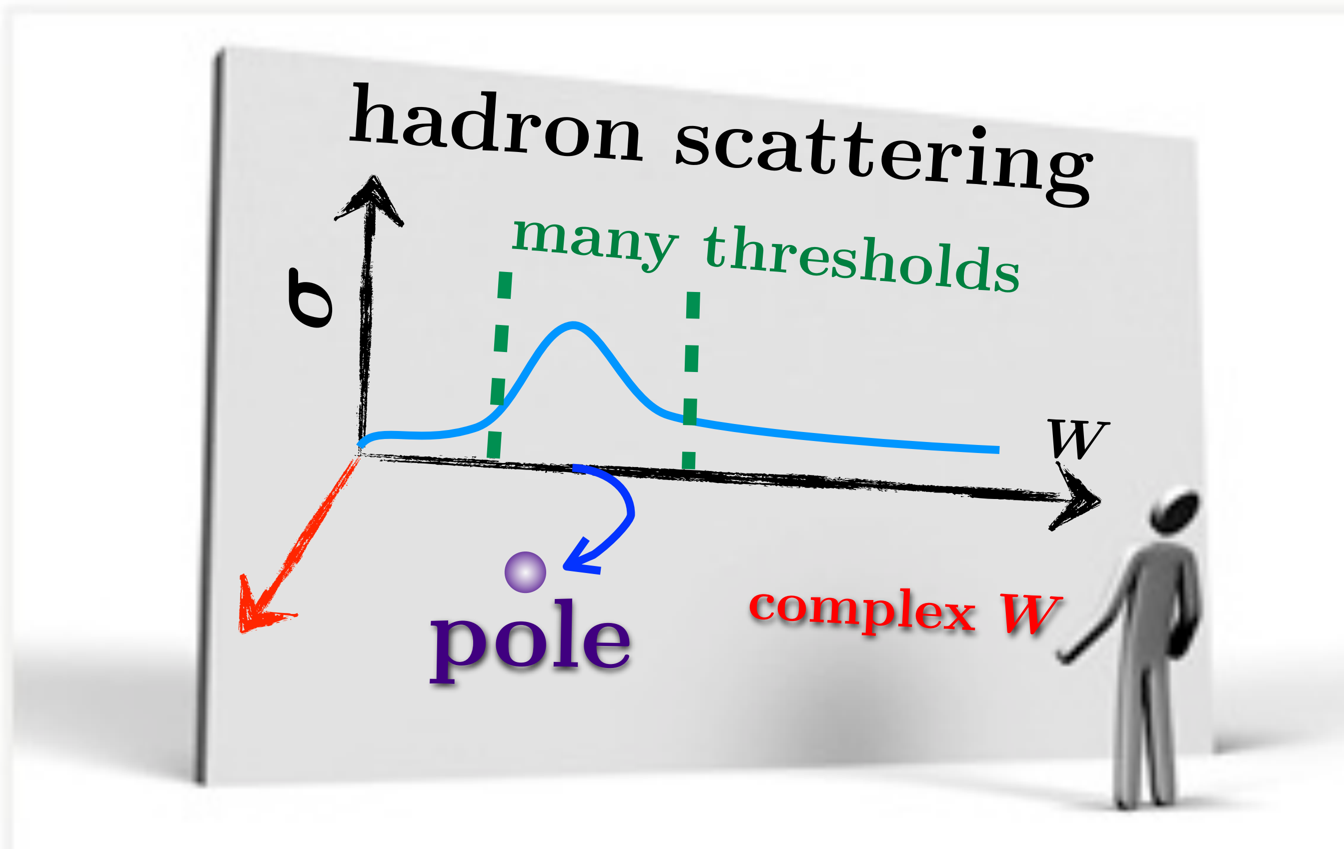
Lattice QCD spectroscopy by conventional approach



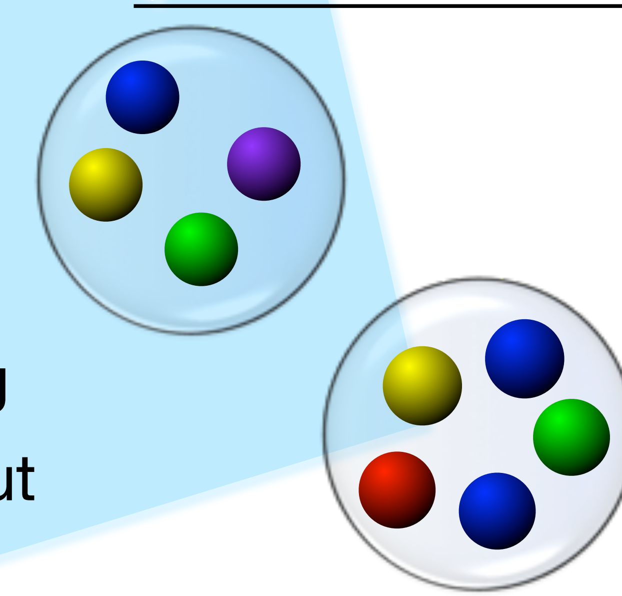
temporal corr. = < 4q(τ) 4q(0) >

$$= A_1 e^{-W_1 \tau} + A_2 e^{-W_2 \tau} + \cdots$$

(W_1, W_2,... ; QCD eigen-energies)



Exotic hadrons



★ Exotic hadron is **resonance in coupled-channel scattering**

★ Resonance energy is **NOT QCD eigen-energy (W_1, W_2, ...)**, but

determined by pole of S-matrix on complex energy plane

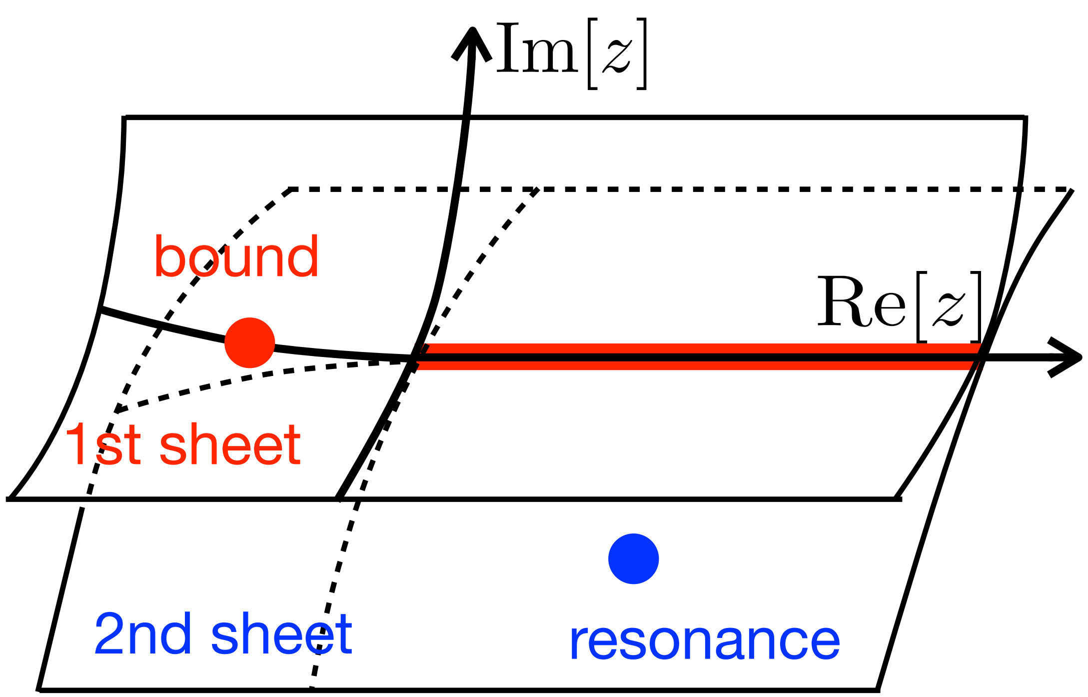
How to search for resonances?

If S-matrix based on QCD obtained. ...

$S(W)$ ⟵ **partial wave analysis of expt. data**

- cross sections (dσ/dΩ)
- spin polarization observables
- etc.

Analyticity of S-matrix is **uniquely** determined **(S-matrix theory)**



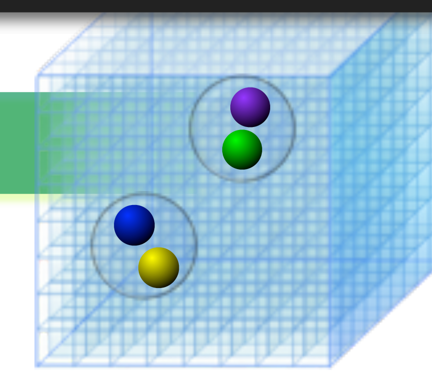
bound state (1st sheet)

- pole position --> binding energy
- residue --> coupling to scattering state

resonance (2nd sheet)

- analytic continuation onto 2nd sheet
- pole position --> resonance energy
- residue --> coupling to scat. state, partial decay

How to search for resonances on the lattice



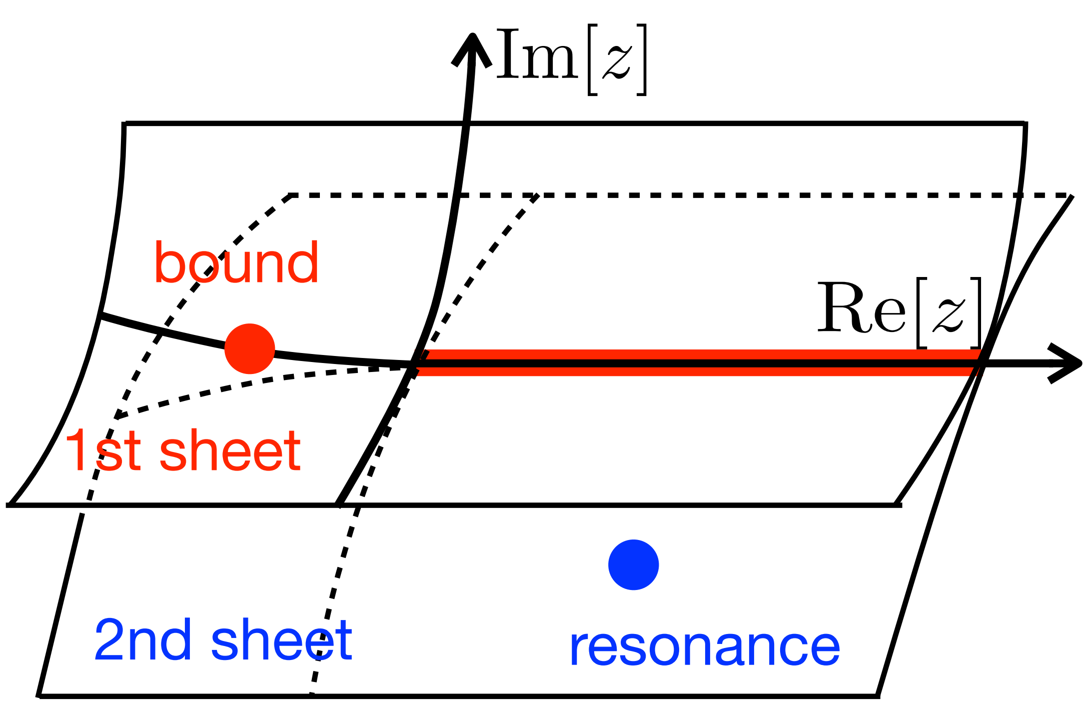
S-matrix based on QCD (lattice QCD simulations)

$S(W)$

two identical methods

- **Lüscher**: temporal correlation --> eigen-energy W_n --> S(W)
  (difficulty in coupled-channel scattering & large volume simulation)
- **HAL QCD**: space-time correlation --> "potentials" --> S(W)

Analyticity of S-matrix is **uniquely** determined **(S-matrix theory)**



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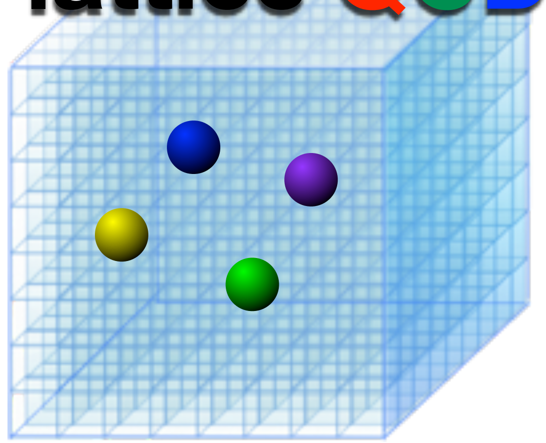
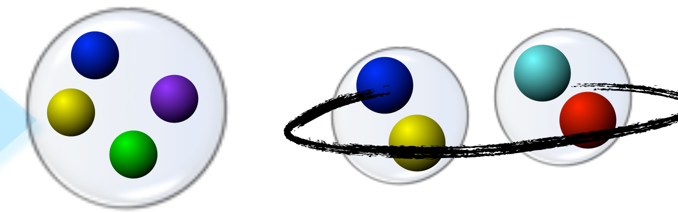
Hadron scattering on the lattice

lattice QCD



hadron interaction
faithful to S-matrix

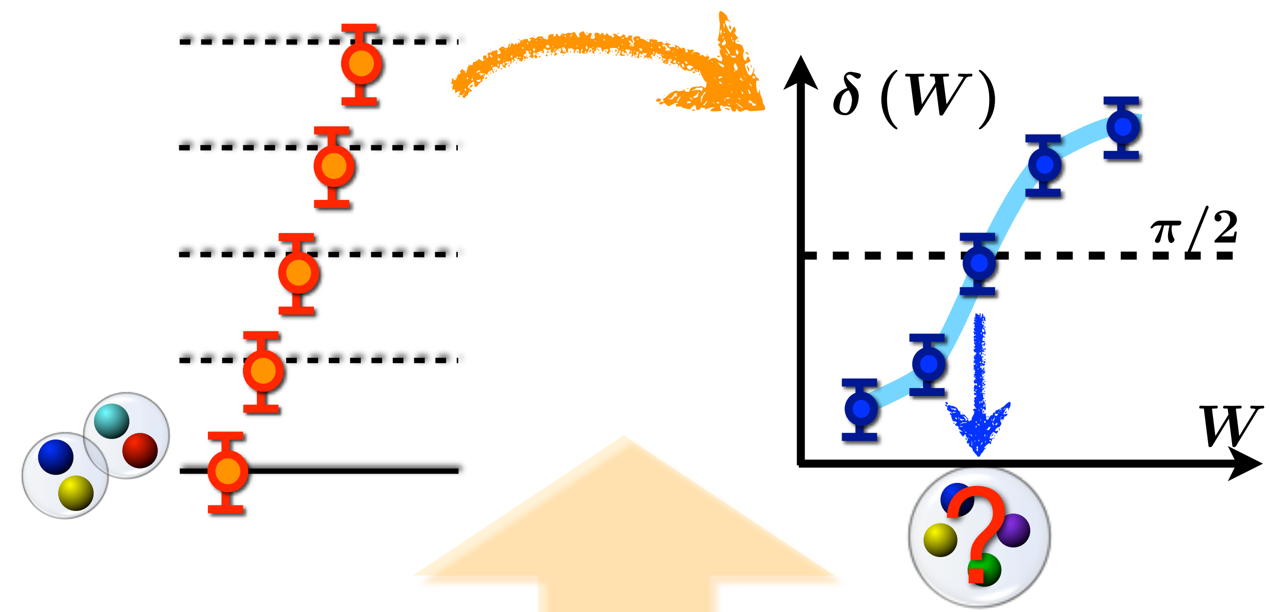
✓ **hadron resonances**

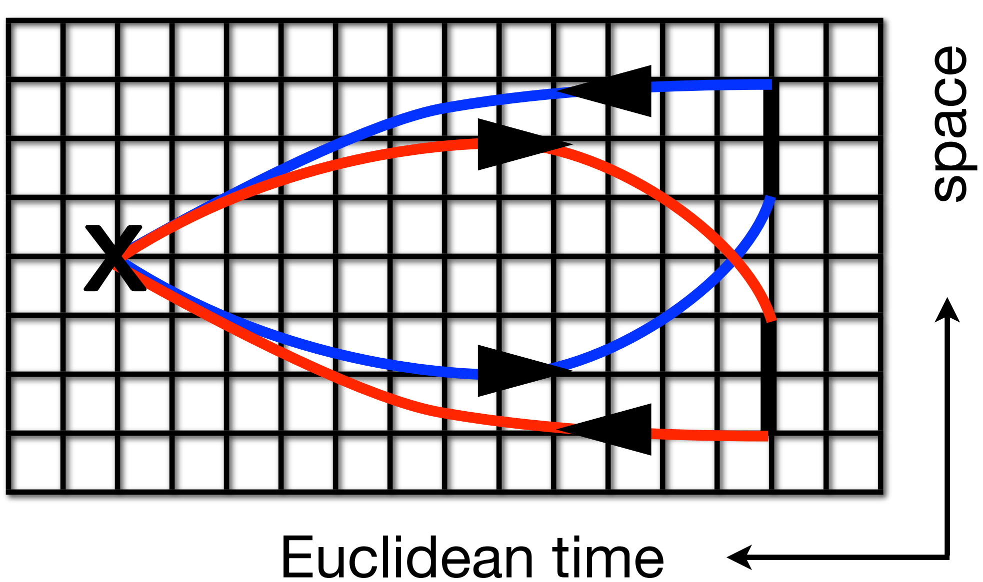


★ single channel scattering

$$\langle 0|\Phi(\tau)\Phi^\dagger(0)|0\rangle = \sum_n A_n e^{-W_n \tau}$$

$$\left(W(k_n) = \sqrt{m_1^2 + k_n^2} + \sqrt{m_2^2 + k_n^2} \right)$$





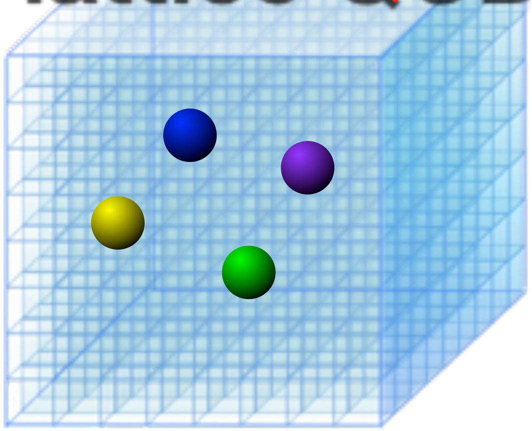
❖ Lüscher's formula Lüscher, NPB354 (1991).

▸ **finite V spectrum --> phase shift δ(W_n)**

$$k_n \cot\delta(k_n) = \frac{4\pi}{L^3} \sum_{m\in\mathbb{Z}^3} \frac{1}{\vec{p}_m^{\,2} - k_n^2}$$

Problem in coupled-channel scattering

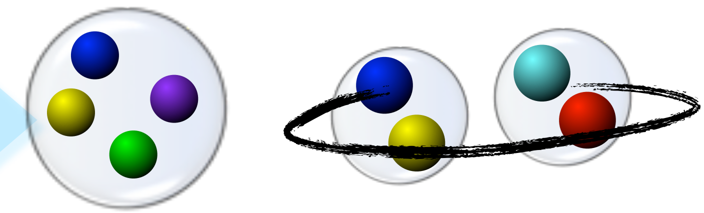
lattice QCD



hadron interaction

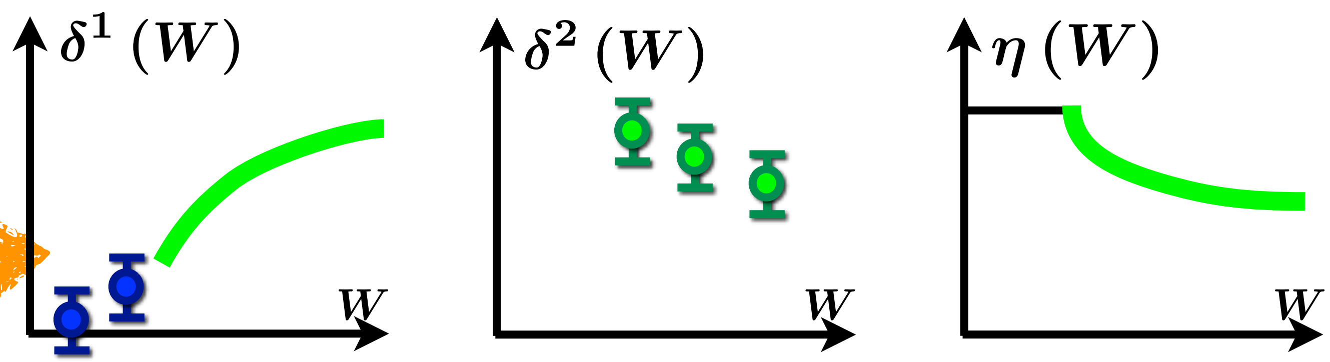
faithful to S-matrix

✓ **hadron resonances**



★ coupled-channel scattering

$$\langle 0|\Phi(\tau)\Phi^\dagger(0)|0\rangle = \sum_n A_n e^{-W_n \tau}$$



➡ coupled-channel Lüscher's formula

elastic region: W --> δ(W)

inelastic region: W --> $\delta^1(W)$, $\delta^2(W)$, $\eta(W)$ **--> find $W(L_1)=W(L_2)=W(L_3)$**

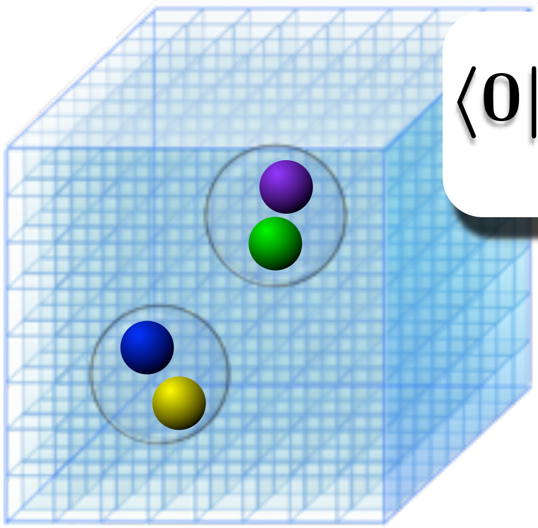
➡ assumptions about interactions necessary...

★ indicate more information mandatory to solve coupled-channel scatterings

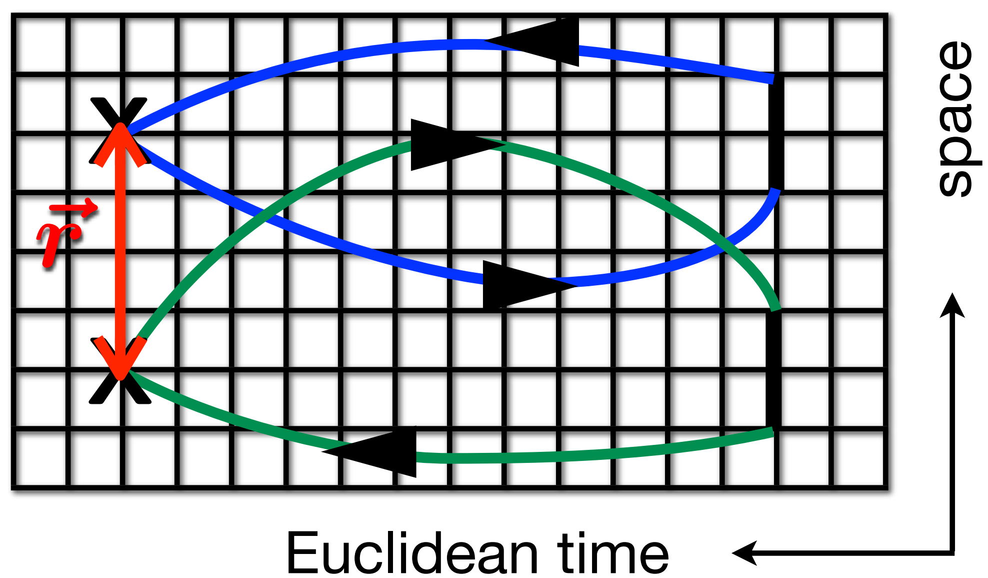
➡ What can we measure in addition to temporal correlations?

HAL QCD method "potentials as representation of S-matrix"

✦ **measure not only temporal but also spatial correlation**

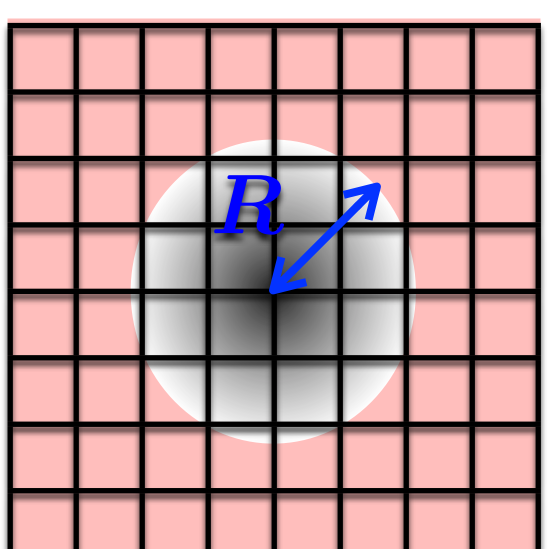


$$\langle 0|\phi_1(\vec{x}+\vec{r},\tau)\phi_2(\vec{x},\tau)\Phi^\dagger(0)|0\rangle = \sqrt{Z_1 Z_2}\sum_n A_n \psi_n(\vec{r}) e^{-W_n \tau}$$



Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al. (HAL QCD), PLB712, 437(2012).

★ **Nambu-Bethe-Salpeter (NBS) wave function: $\psi_n(r)$**



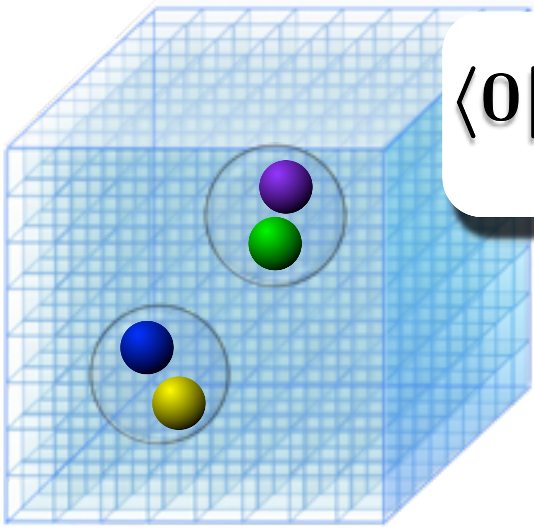
- NBS wave functions (r>|R|) satisfy **Helmholtz equation**

$$\left(\nabla^2 + \vec{k}_n^2\right)\psi_n(\vec{r}) = 0 \quad (|\vec{r}| > R)$$

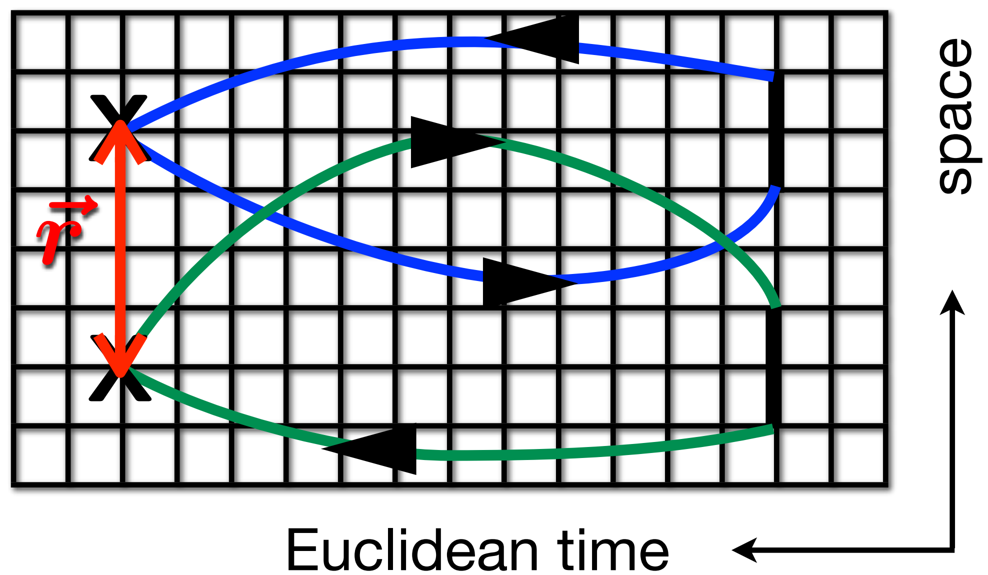
➡ **faithful to S-matrix** (S(W)=$e^{2i\delta(W)}$)

HAL QCD method “potentials as representation of S-matrix”

✦ **measure not only temporal but also spatial correlation**

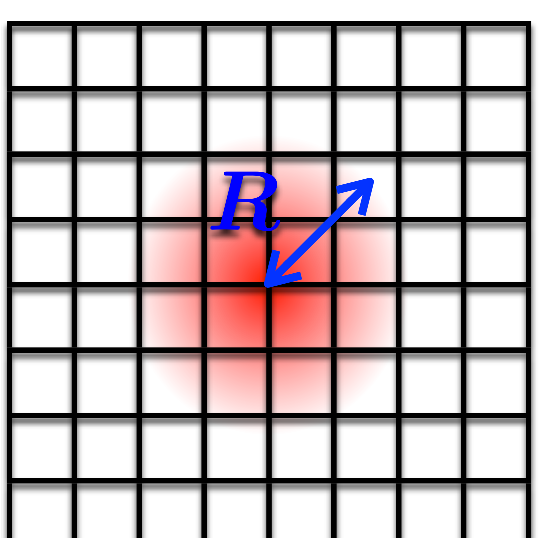


$$\langle 0|\phi_1(\vec{x}+\vec{r},\tau)\phi_2(\vec{x},\tau)\Phi^\dagger(0)|0\rangle = \sqrt{Z_1 Z_2}\sum_n A_n \psi_n(\vec{r}) e^{-W_n \tau}$$



Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al. (HAL QCD), PLB712, 437(2012).

★ **NBS wave function** (r<|R|) **--> “potential U(r,r’)”**



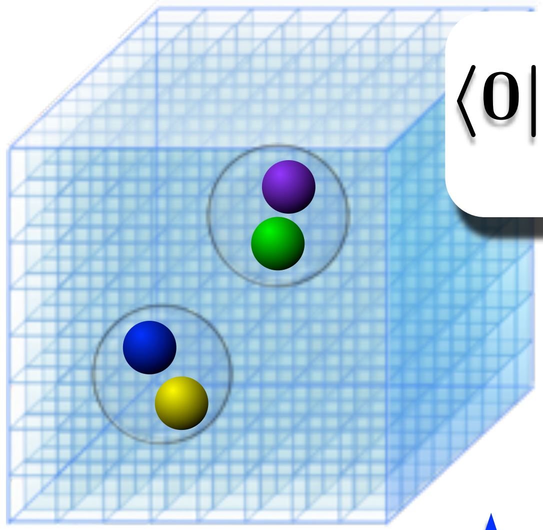
$$\left(\nabla^2 + \vec{k}_n^2\right)\psi_n(\vec{r}) = 2\mu \int d\vec{r}\,' U(\vec{r},\vec{r}\,')\psi_n(\vec{r}\,')$$

- U(r,r’) is faithful to **S-matrix**
- U(r,r’) is **energy independent** (until new threshold opens)
- U(r,r’) contains all 2PI contributions

(Non-relativistic approximation is NOT necessary)

Coupled-channel HAL QCD method

✦ measure not only temporal but also spatial correlation



$$\langle 0|\phi_1^a(\vec{x}+\vec{r},\tau)\phi_2^a(\vec{x},\tau)\Phi^\dagger(0)|0\rangle = \sqrt{Z_1^a Z_2^a}\sum_n A_n \psi_n^a(\vec{r}) e^{-W_n\tau}$$

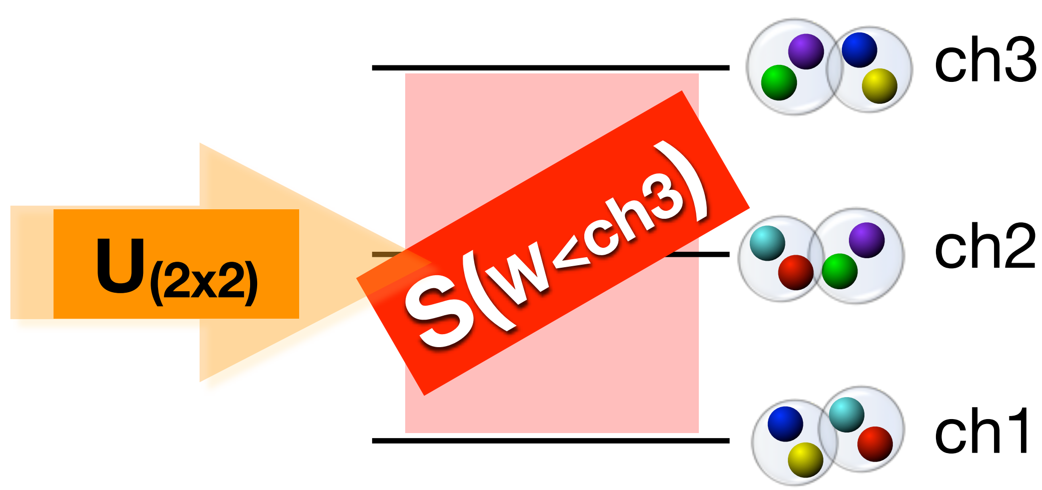
Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al. (HAL QCD), PLB712, 437(2012).

★ spatial correlation --> identify channel wave function

$$\left(\nabla^2 + (\vec{k}_n^a)^2\right)\psi_n^a(\vec{r}) = 2\mu^a \sum_b \int d\vec{r}' U^{ab}(\vec{r},\vec{r}')\psi_n^b(\vec{r}')$$

★ coupled-channel potential $U^{ab}(r,r')$:

- $U^{ab}(r,r')$ is faithful to **coupled-channel S-matrix**
- $U^{ab}(r,r')$ is **energy independent** (until new threshold opens)
- Non-relativistic approximation is not necessary
- $U^{ab}(r,r')$ contains all 2PI contributions



➡ derive potential from time-dependent HAL QCD method

Full details, Aoki et al. (HAL QCD), PTEP 2012, 01A105 (2012); Proc. Jpn. Acad., Ser. B, 87 (2011).

Time-dependent HAL QCD method

✓ **Define energy-independent potential**

$$\psi_n(\vec{r}) = \langle 0|\phi_1(\vec{r}+\vec{x})\phi_2(\vec{x})|W_n; I, J^P\rangle$$

$$\left(\nabla^2 + \vec{k}_n^2\right)\psi_n(\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r},\vec{r}')\psi_n(\vec{r}')$$

$$R(\vec{r},\tau) = A_1 e^{-\Delta W_1 \tau}\psi_1(\vec{r}) + A_2 e^{-\Delta W_2 \tau}\psi_2(\vec{r}) + \cdots$$

1. identify energy eigenstates
2. derive potential

$$\int d\vec{r}' U(\vec{r},\vec{r}')\psi_1(\vec{r}') = \left(\frac{\nabla^2}{2\mu} + \frac{\vec{k}_1^2}{2\mu}\right)\psi_1(\vec{r})$$

$$\int d\vec{r}' U(\vec{r},\vec{r}')\psi_2(\vec{r}') = \left(\frac{\nabla^2}{2\mu} + \frac{\vec{k}_2^2}{2\mu}\right)\psi_2(\vec{r})$$

✓ **Extract energy-independent potential : t-dependent Schrödinger eq.**

$$\left[\nabla^2/2\mu - \partial_\tau + (1/2\mu - 3/2M)\,\partial_\tau^2 + \mathcal{O}(\Delta W^3/M^2)\right] R(\vec{r},\tau) = \int d\vec{r}' U(\vec{r},\vec{r}') R(\vec{r}',\tau)$$

Need NOT identify energy eigenstates

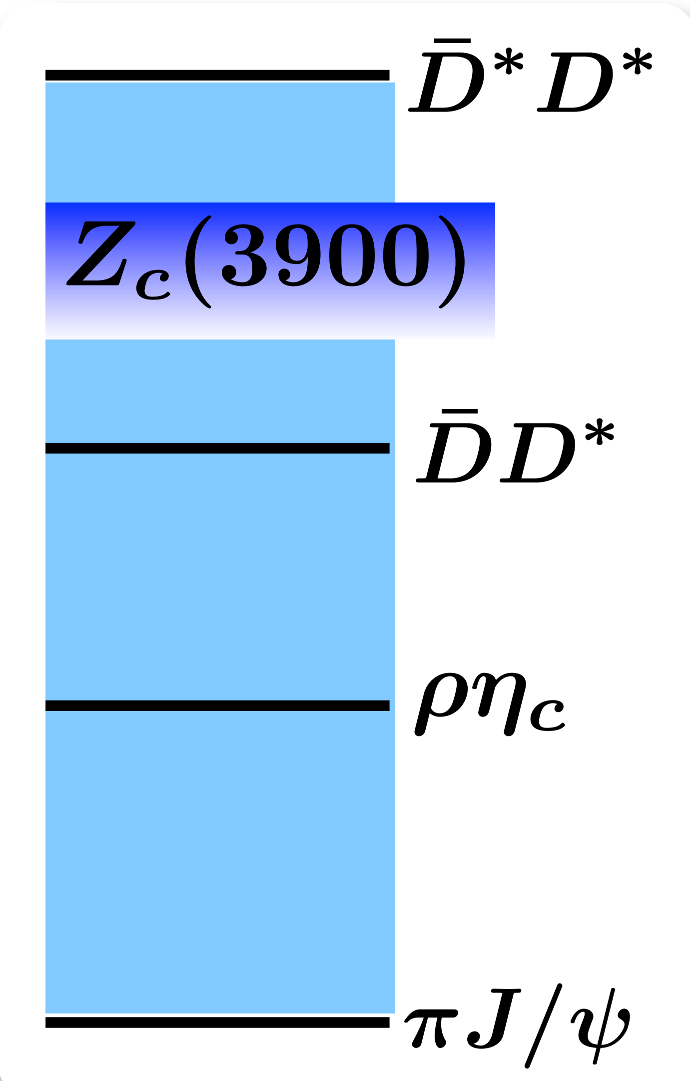
$$M = m_1 + m_2$$

Ishii et al.,(HAL QCD), PLB712, 437(2012).

$Z_c(3900)$ in $I^G(J^{PC})=1^+(1^{+-})$

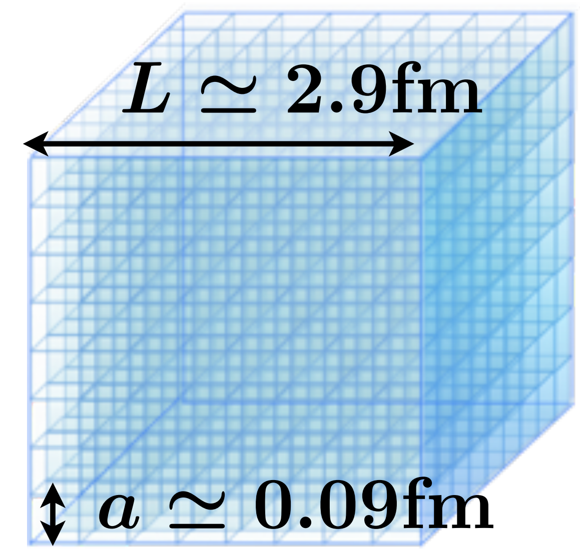
-- $\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$ coupled-channel --

Y. Ikeda et al., [HAL QCD], PRL117, 242001 (2016).



❖ N_f=2+1 full QCD

- Iwasaki gauge
- clover Wilson quark
- 32^3 x 64 lattice



❖ Relativistic Heavy Quark (charm)

- remove leading cutoff errors $O((m_c\, a)^n)$, $O(\Lambda_{QCD}\, a)$, ...

➡ We are left with $O((a\Lambda_{QCD})^2)$ syst. error (~ a few %)

light meson mass (MeV)

m_π= 411(1), 572(1), 701(1)

m_ρ= 896(8), 1000(5), 1097(4)

charm meson mass (MeV)

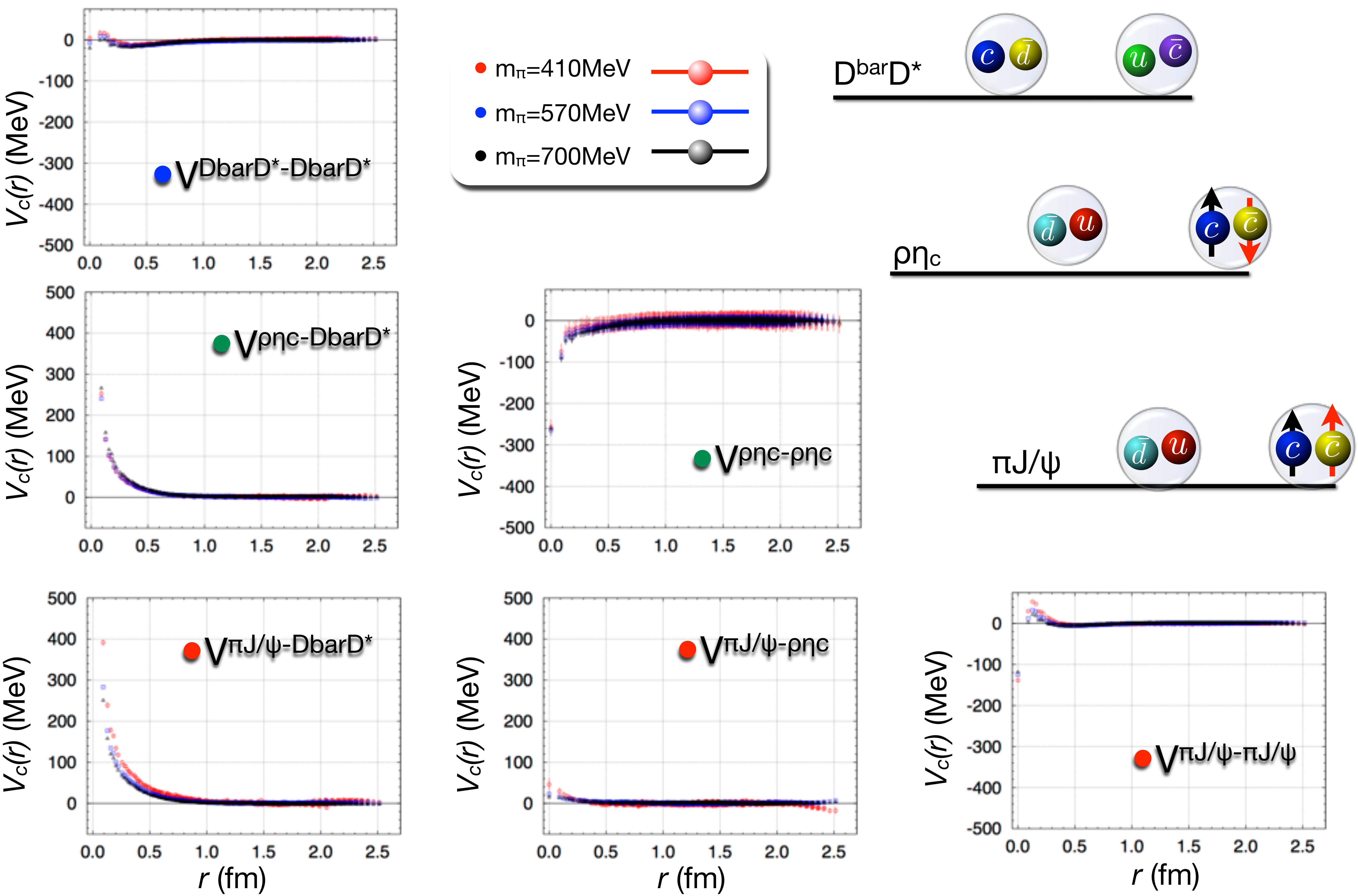
$m_{\eta c}$= 2988(1), 3005(1), 3024(1)

$m_{J/\psi}$= 3097(1), 3118(1), 3143(1)

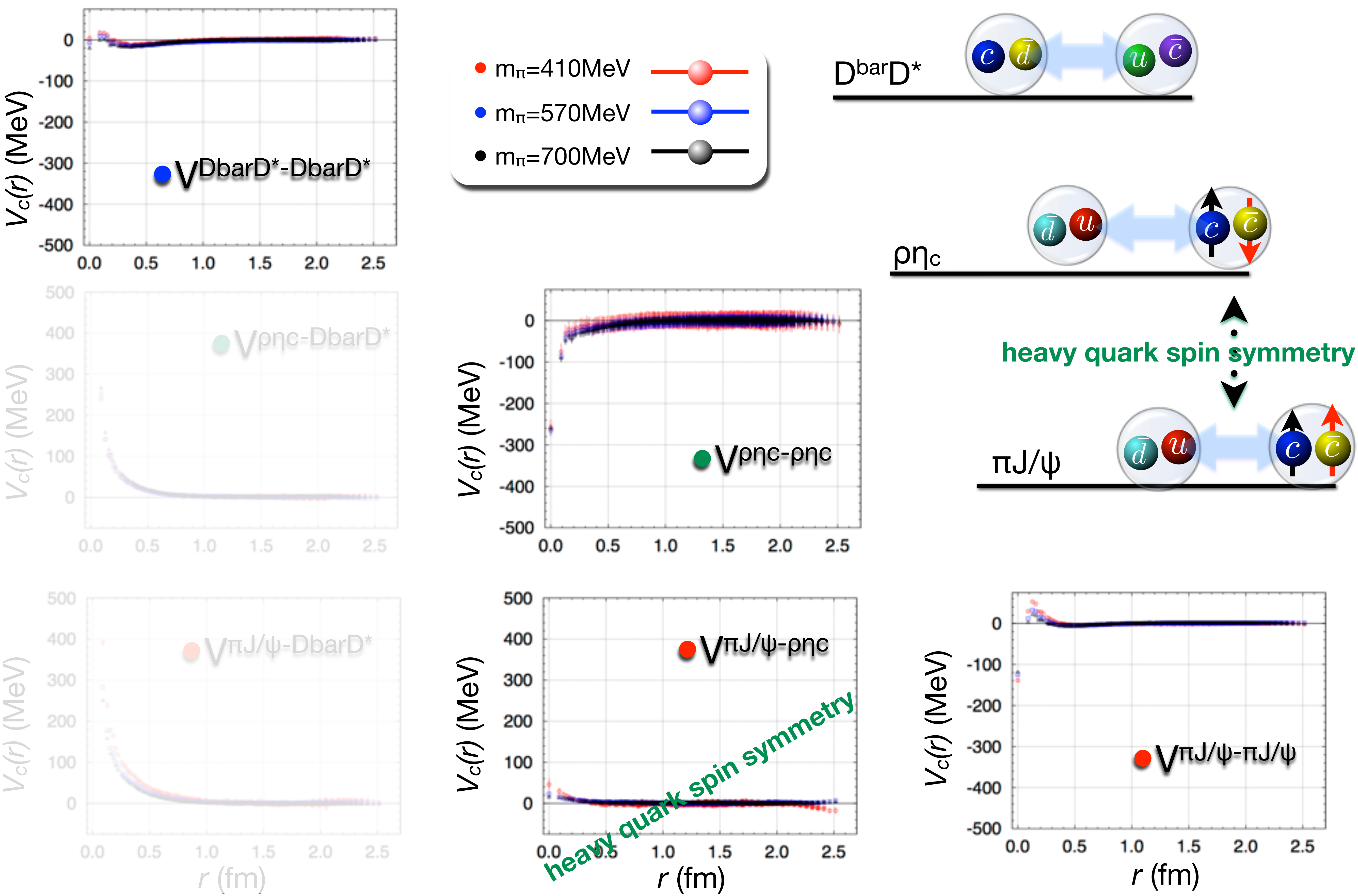
m_D= 1903(1), 1947(1), 2000(1)

m_{D^*}= 2056(3), 2101(2), 2159(2)

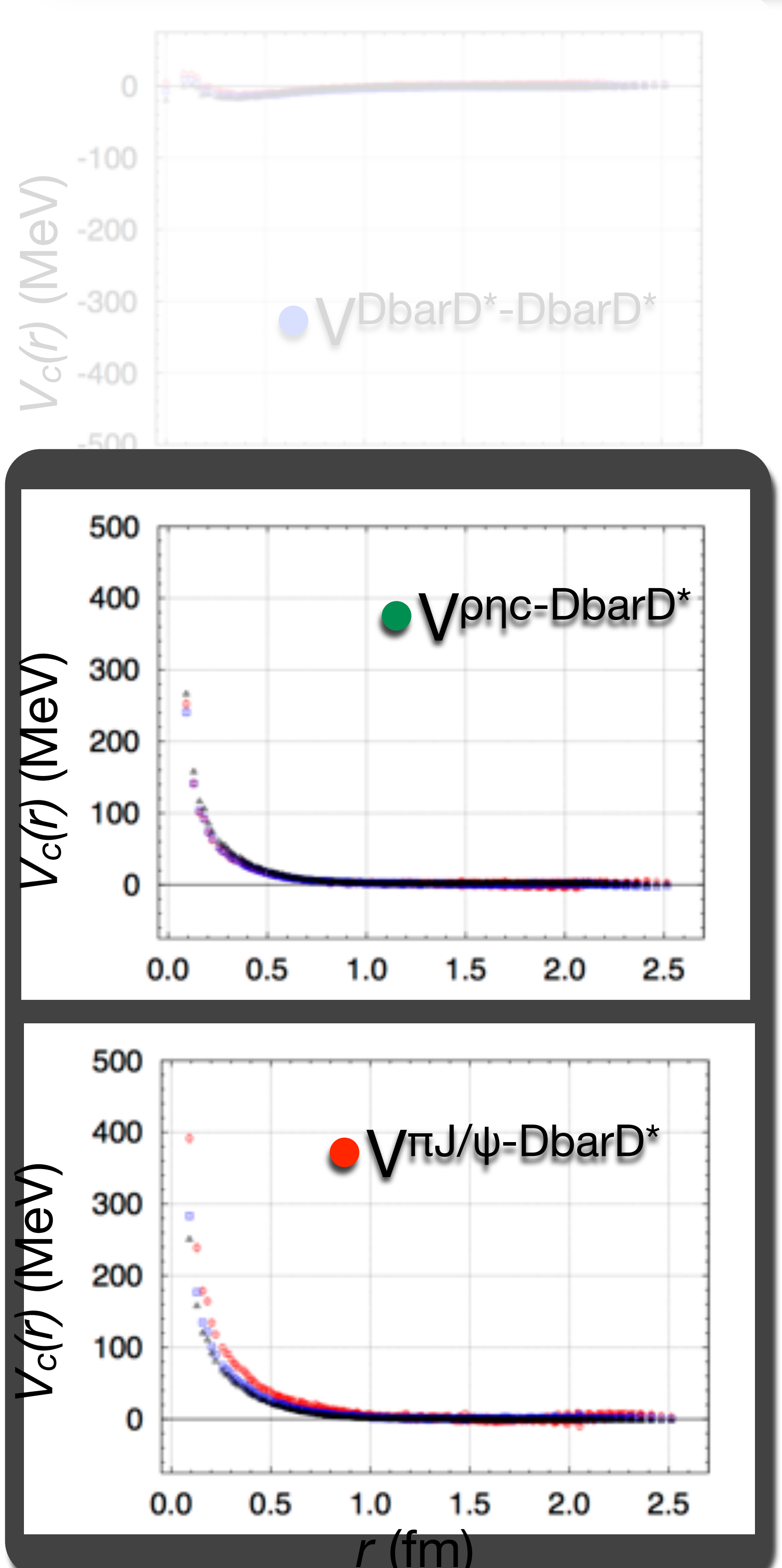
3x3 potential matrix (πJ/ψ - ρη$_c$ - D^{bar}D*)

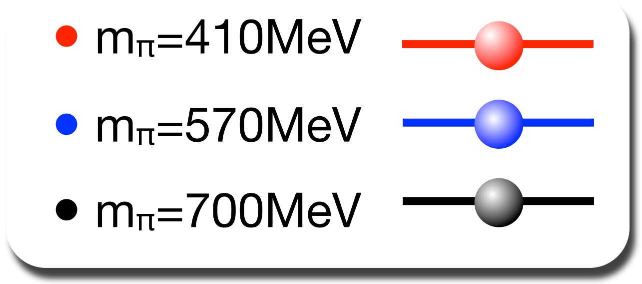


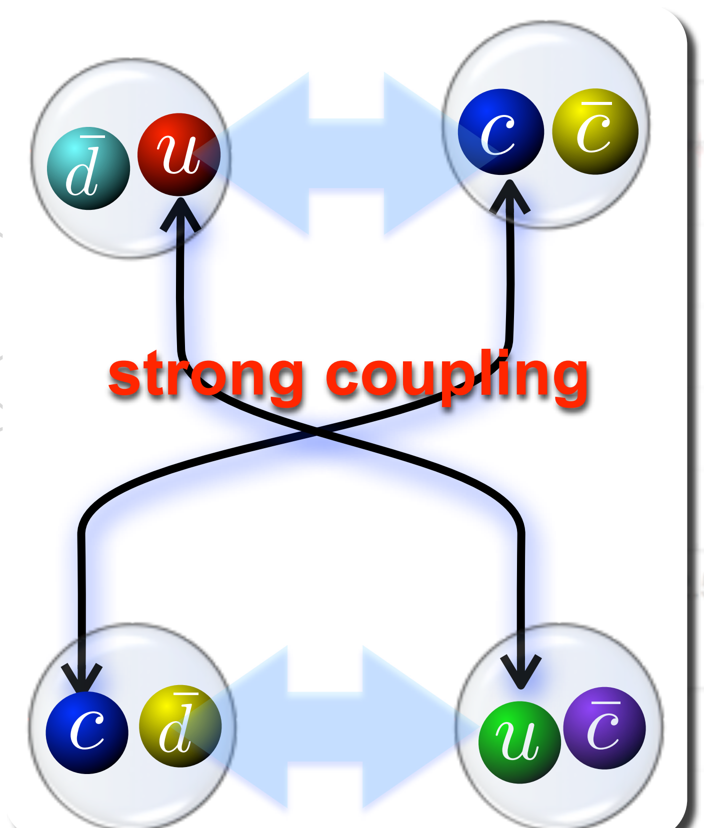
3x3 potential matrix (πJ/ψ - ρηc - DbarD*)

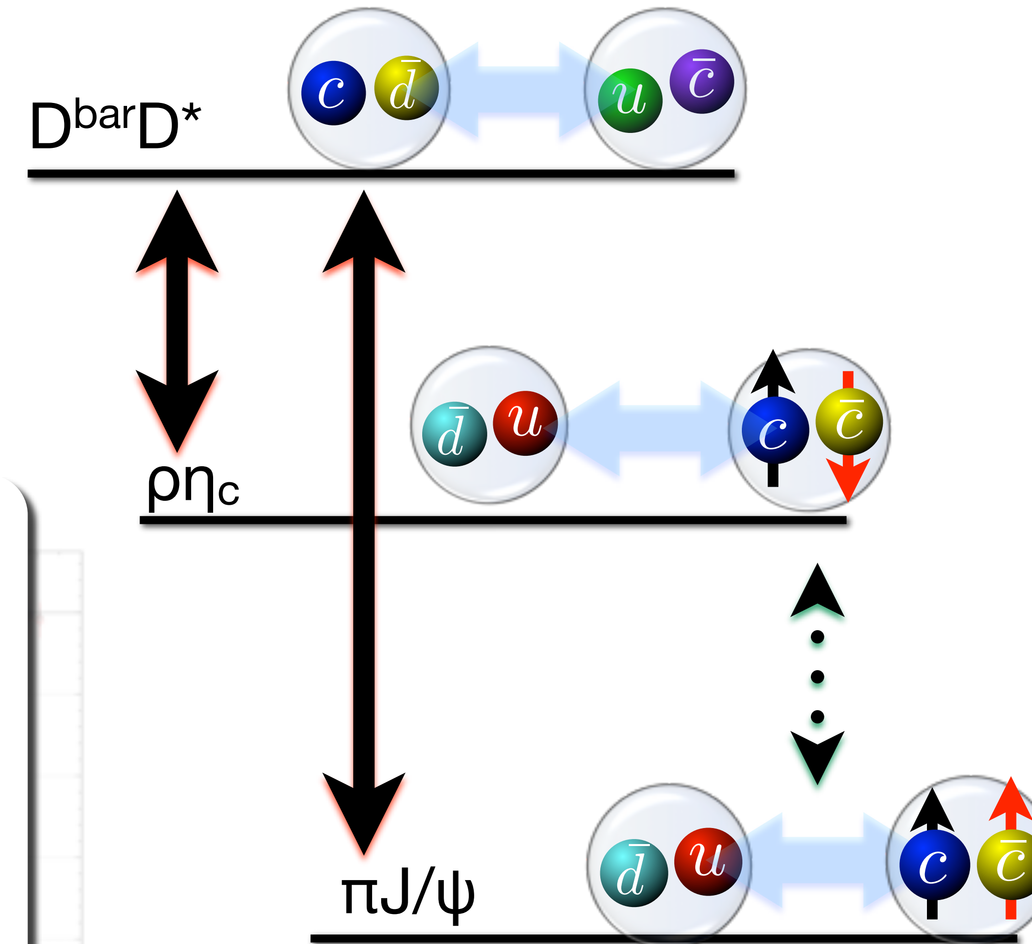


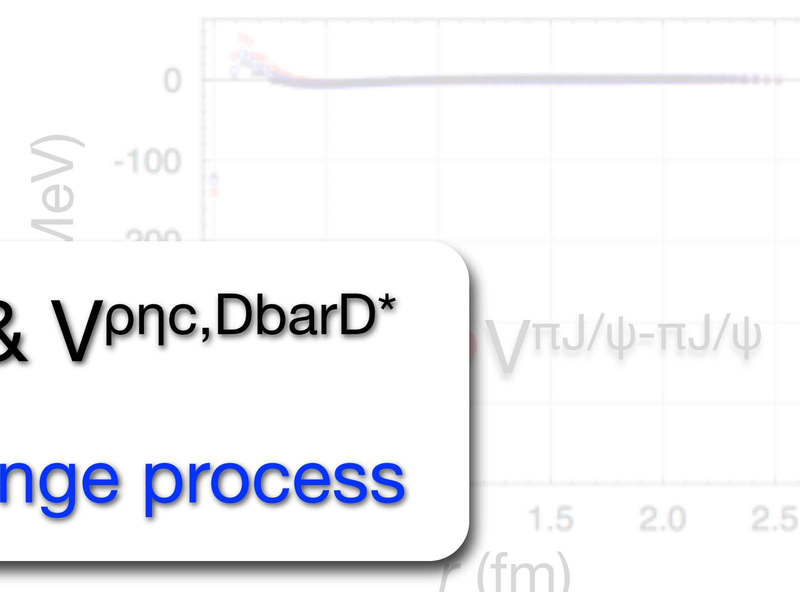
3x3 potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$)











- strong $V^{\pi J/\psi, D^{bar}D^*}$ & $V^{\rho\eta_c, D^{bar}D^*}$

➡ charm quark exchange process

Structure of $Z_c(3900)$

studied by the most ideal scattering process

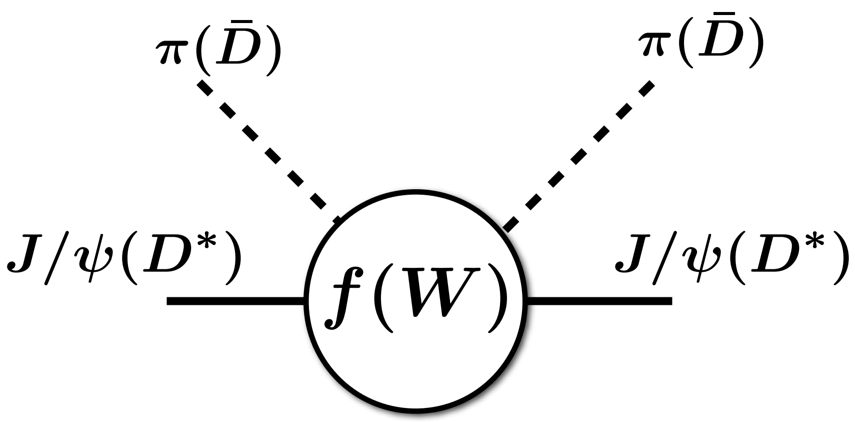
- S-wave $\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$ coupled-channel scattering

➡ **$Z_c(3900)$ is observed in $\pi J/\psi$ & $D^{bar}D^*$ -> 2-body scattering is the most ideal reaction**

1. invariant mass distribution of 2-body scattering

# of scat. particles proportional to imaginary part of amplitude

$$N_{sc} \propto (\text{flax}) \cdot \sigma(W) \propto \text{Im} f(W)$$



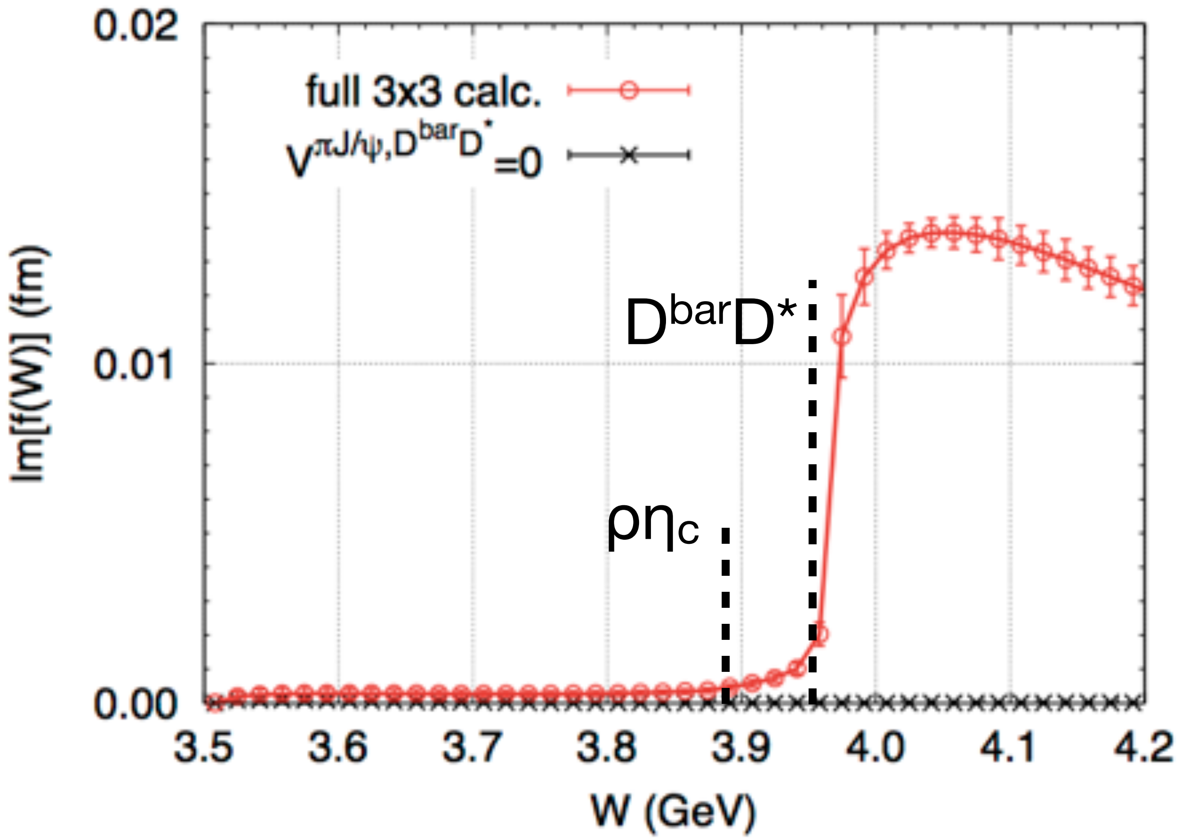
2. pole position of S-matrix

- analytic continuation of c.c. S-matrix onto complex energy plane
- understand nature of $Z_c(3900)$

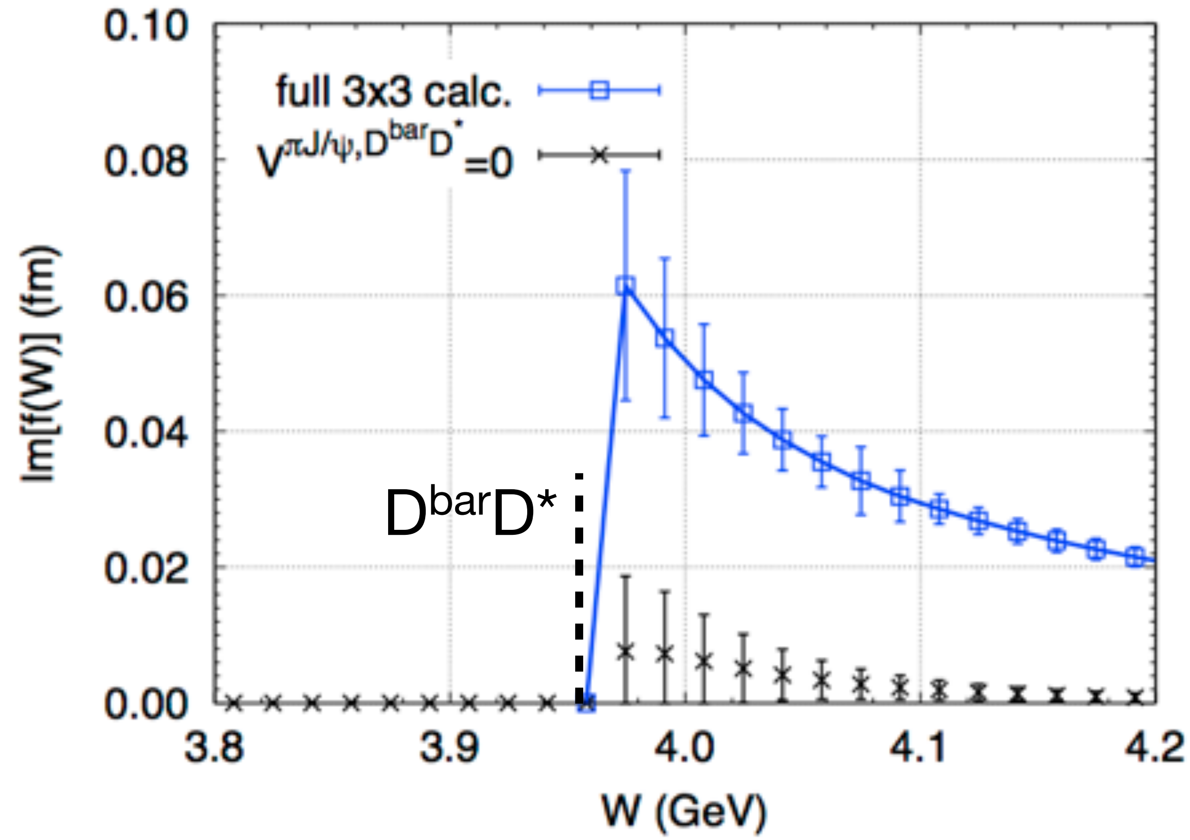
- Results w/ m_π=410MeV are shown. (**weak quark mass dependence observed**)

Invariant mass of πJ/ψ & $D^{bar}D^*$ (2-body scat.)

- πJ/ψ invariant mass



- $D^{bar}D^*$ invariant mass



$$\frac{\Gamma(Zc(3900) \to \bar{D}D^*)}{\Gamma(Zc(3900) \to \pi J/\psi)} = 6.2(1.1)(2.7)$$

BESIII Coll., PRL112 (2014).

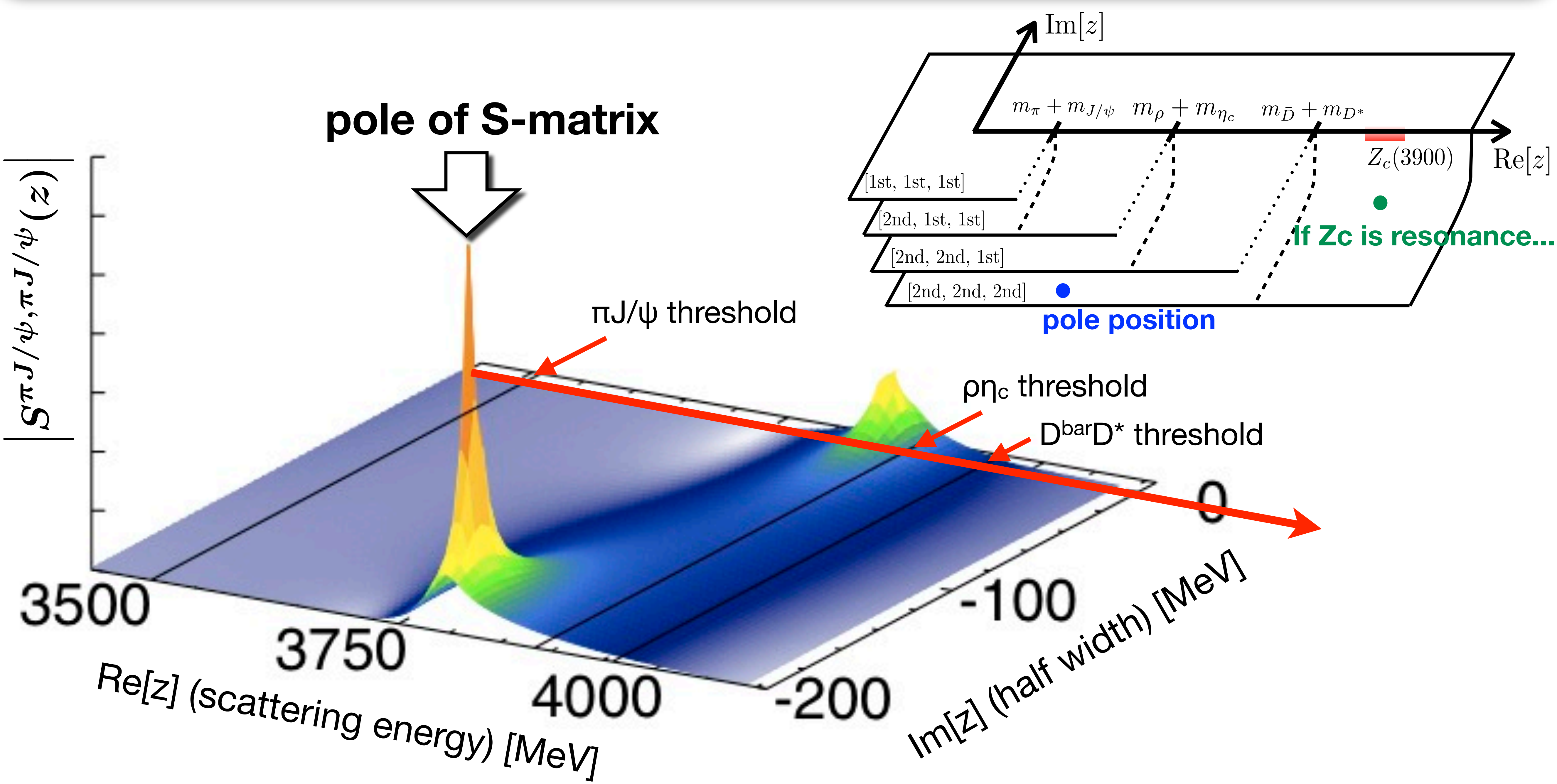
✓ **Enhancement just above $D^{bar}D^*$ threshold** in both amplitudes

➡ **effect of strong $V^{\pi J/\psi, DbarD^*}$** (black --> $V^{\pi J/\psi, DbarD^*}=0$)

- peak in πJ/ψ inv. mass (not Breit-Wigner line shape)

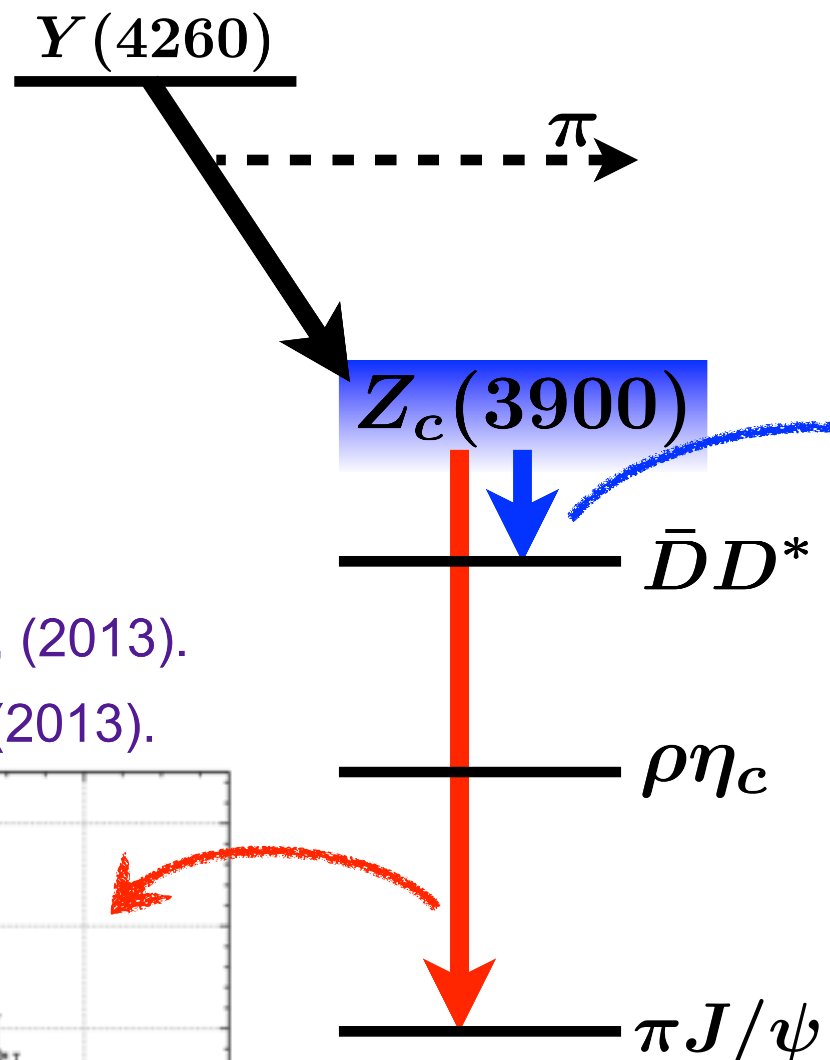
✓ **Is $Z_c(3900)$ a conventional resonance? --> pole of S-matrix**

Pole of S-matrix ($\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $D^{bar}D^*$:2nd)

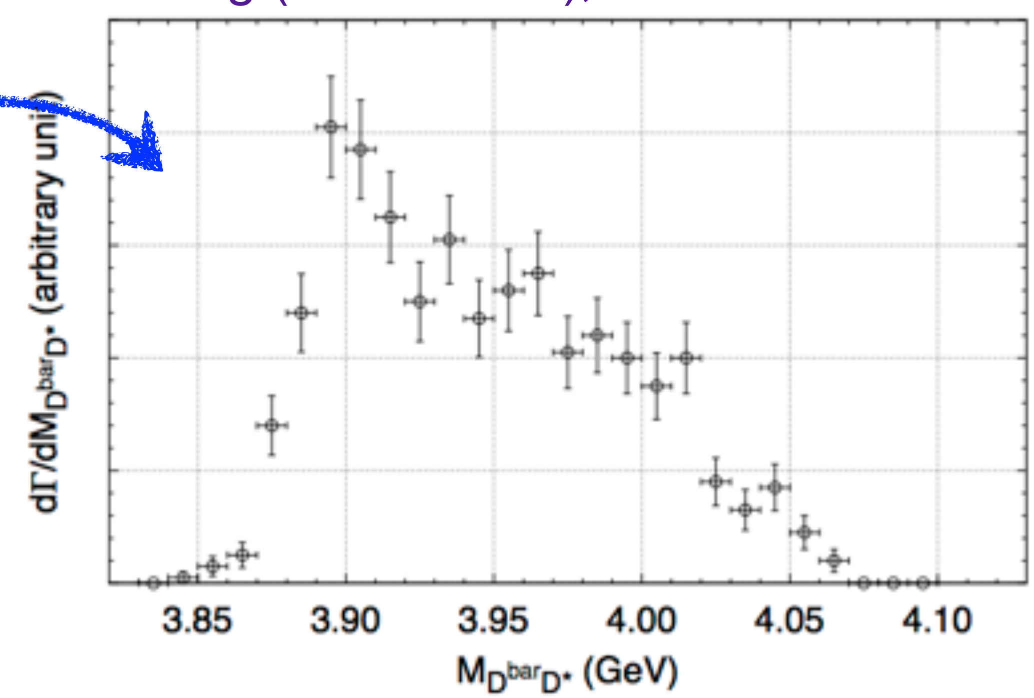


- Pole corresponding to "**virtual state**"
- Pole contribution to scat. observables is small (far from scat. axis)
- **Z_c(3900) is not a resonance but "threshold cusp" induced by strong $V^{\pi J/\psi, DbarD^*}$**

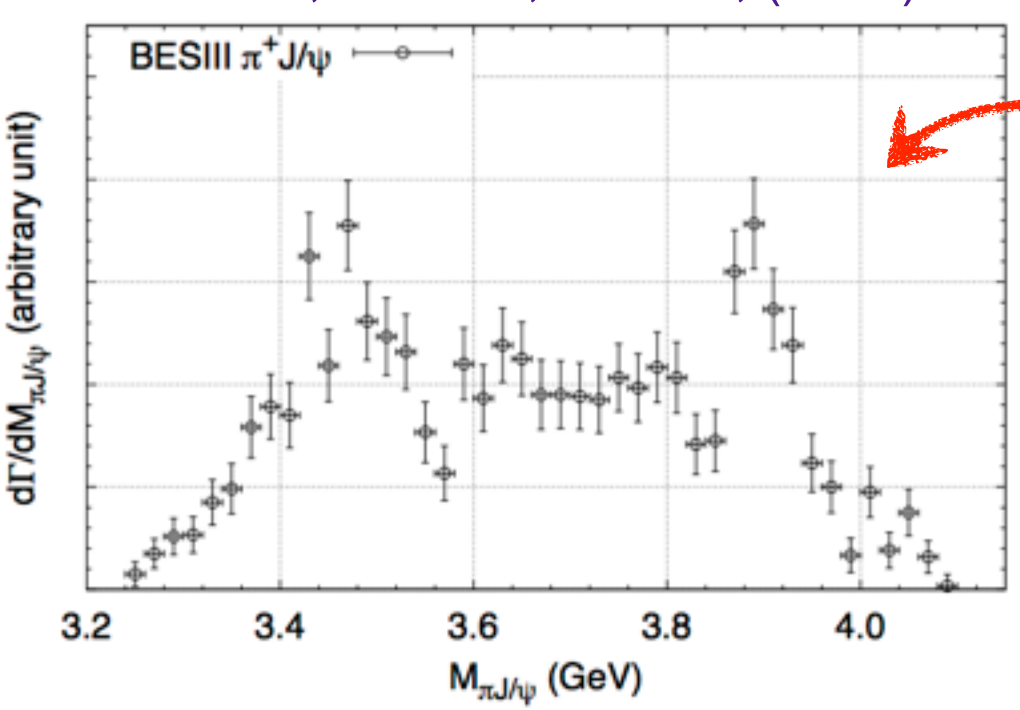
Comparison with expt. data:
-- spectrum of Y(4260) 3-body decay --

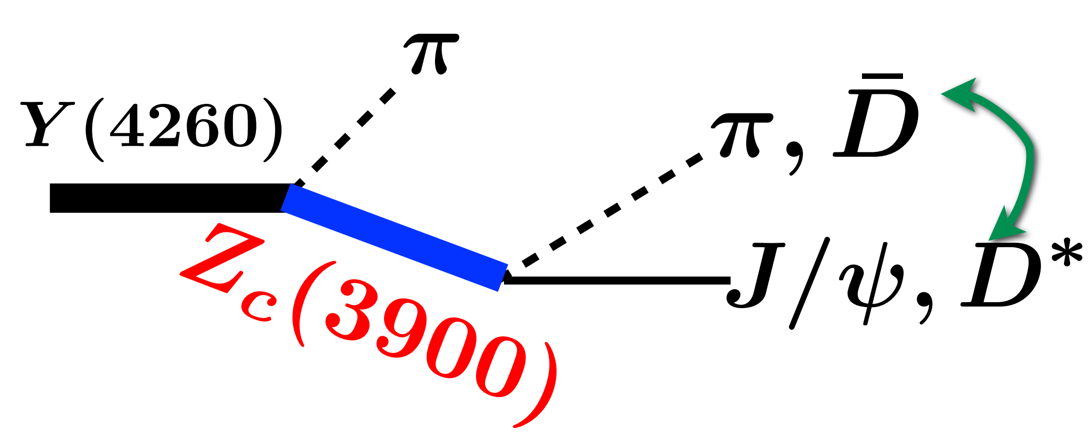


BESIII Coll., PRL112, 022001, (2014).
Wang (BESIII Coll.), MENU2016 talk



BESIII Coll., PRL110, 252001, (2013).
Belle Coll., PRL110, 252002, (2013).



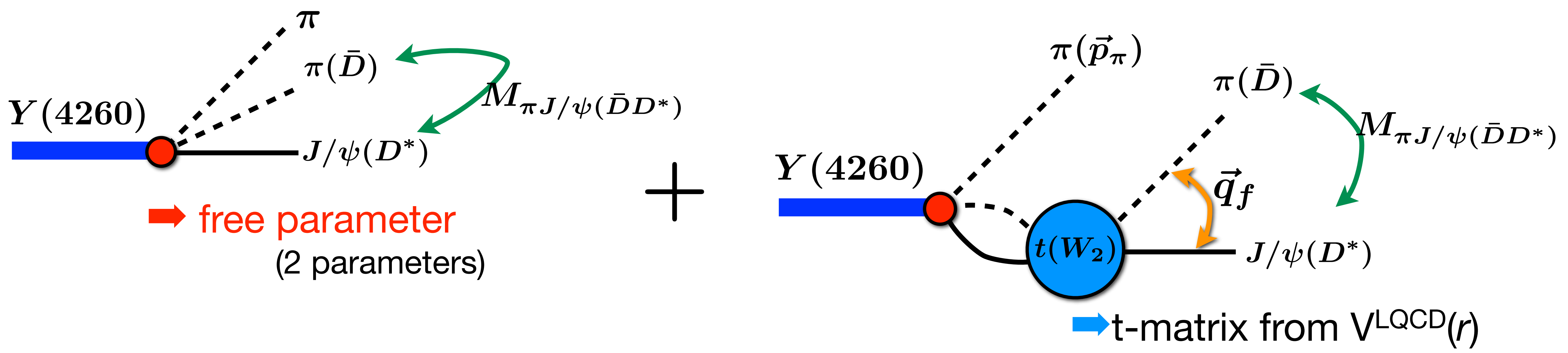


Y(4260) --> ππJ/ψ & πD^bar D*

$$d\Gamma_{Y\to\pi+f} = (2\pi)^4\delta(W_3 - E_\pi(\vec{p}_\pi) - E_f(\vec{q}_f))d^3p_\pi d^3q_f|T_{Y\to\pi+f}(\vec{p}_\pi,\vec{q}_f;W_3)|^2$$

✓ 3-body T-matrix: $T_{Y\text{->}\pi+f}(W_3\text{=}4260\text{MeV})$

$$T_{Y\to\pi+f}(\vec{p}_\pi,\vec{q}_f;W_3) = \sum_{n=\pi J/\psi,\bar{D}D^*} C^{Y\to\pi+n}\left[\delta_{nf} + \int d^3q' \frac{t_{nf}(\vec{q}',\vec{q}_f,\vec{p}_\pi;W_3)}{W_3 - E_\pi(\vec{p}_\pi) - E_n(\vec{q}',\vec{p}_\pi) + i\epsilon}\right]$$



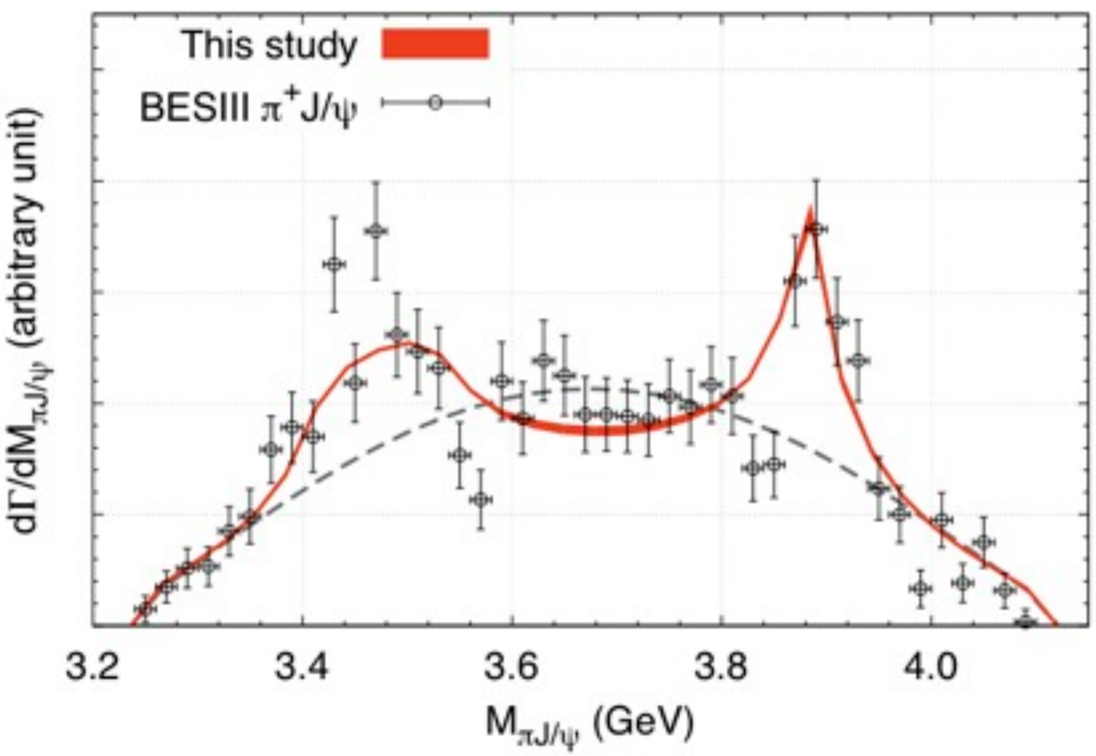
employ physical hadron masses to compare w/ expt. data

✓ $V^{LQCD}(r)$ is taken into account --> calculate t-matrix in subsystem

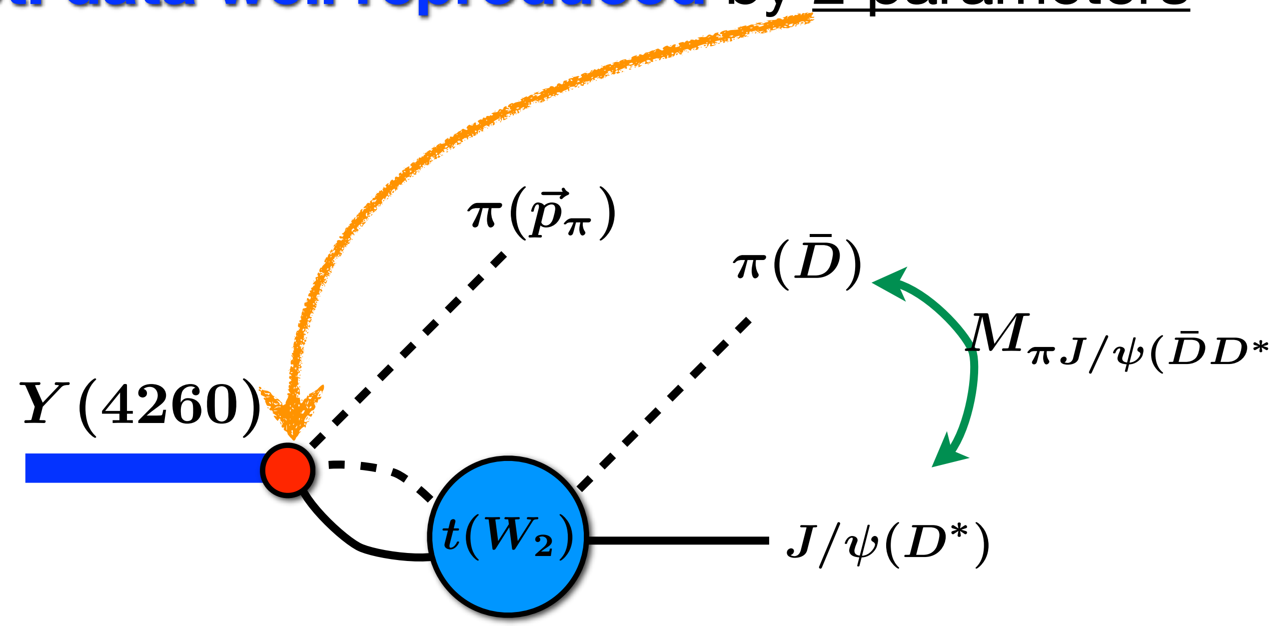
✓ fix free parameters by fitting Y(4260)-->ππJ/ψ expt. data

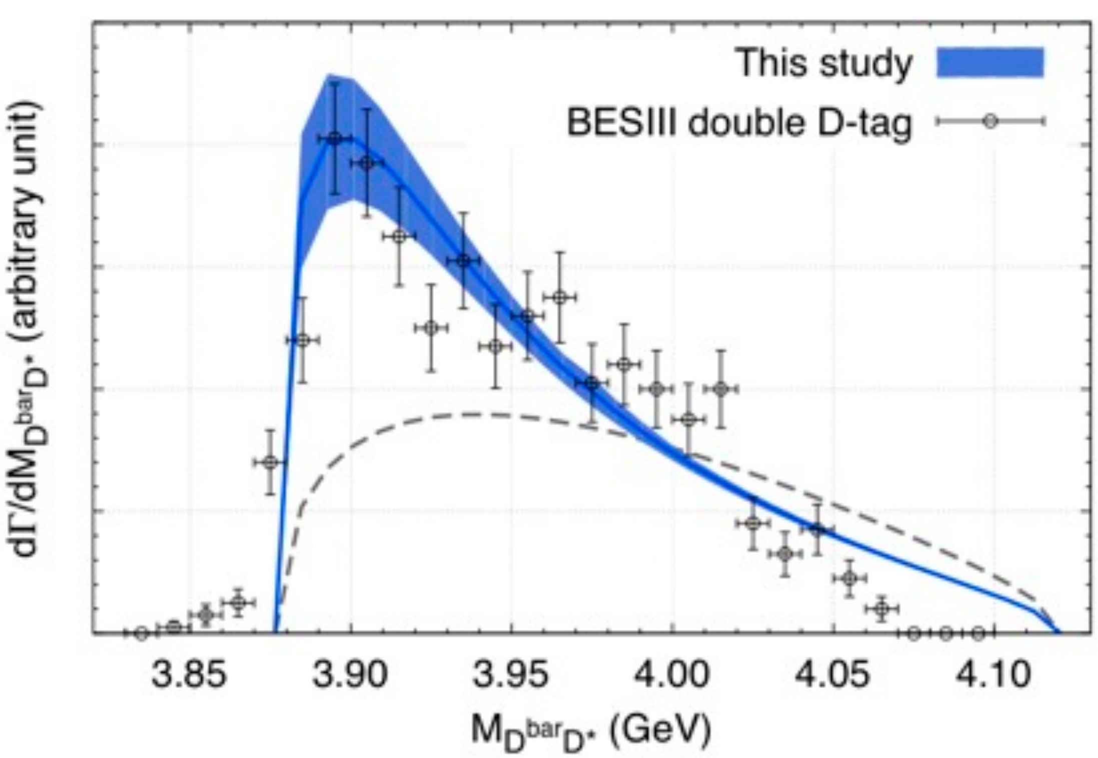
Invariant mass of 3-body decay

Y. Ikeda et al., [HAL QCD], PRL117 & in preparation



- **Expt. data well reproduced** by 2 parameters



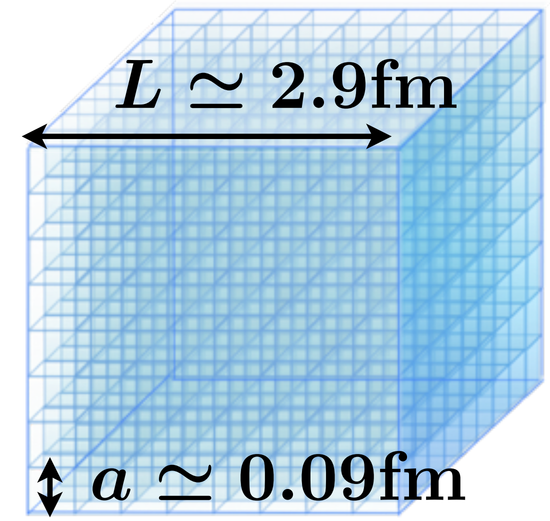


- Without off-diagonal $V^{\pi J/\psi,\ DbarD^*}$ (dashed curves), peak structures are not reproduced.

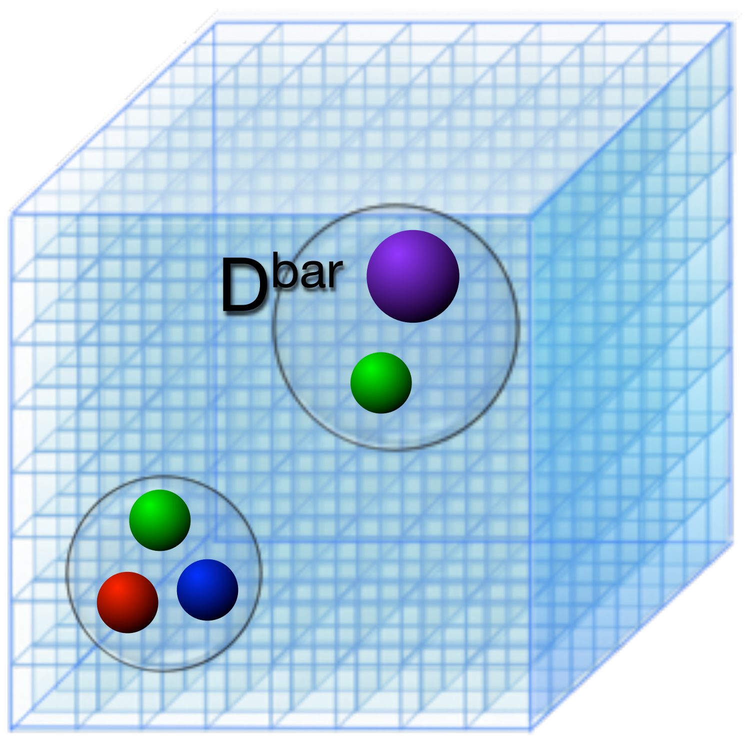
conclusion: $Z_c(3900)$ is threshold cusp caused by strong $V^{\pi J/\psi,\ DbarD^*}$

D^{bar}-nucleon interaction and D^{bar}-nuclei

- N_f=2+1 full QCD configs. (PACS-CS collaboration)
- RHQ action for charm quark

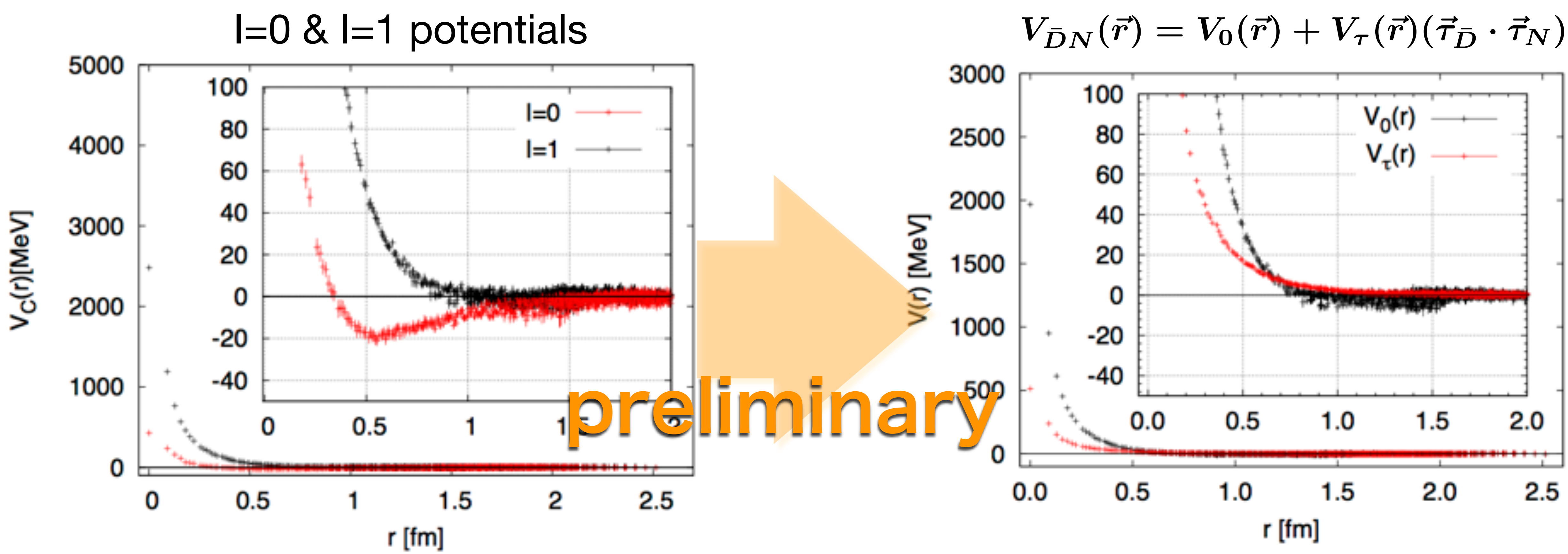


m_π= 411, 572, 701
m_N= 1215, 1411, 1583
m_D= 1903, 1947, 2000
$m_{\Lambda c}$= 2434, 2584, 2710



- D^{bar}N interaction in I=0 & 1 by HAL QCD method
- Naive expectation to D^{bar}-nuclei by simple folding potential model

S-wave $D^{bar}N$ interaction in I=0 & 1 $@m_\pi$=410MeV



$$a_{\bar{D}N} = \begin{cases} 0.79(42) & (I=0) \\ -0.20(1) & (I=1) \end{cases}$$

- $V_0(r)$ is repulsive, $V_\tau(r)$ is positive

Y. Ikeda [HAL QCD], in preparation

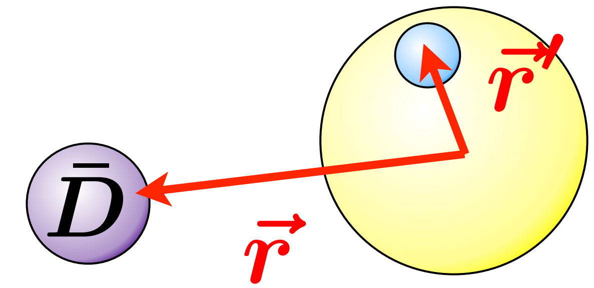
$D^{bar}N$ interaction

(1) attractive in I=0 & repulsive in I=1

(2) $V_0(r)$ is repulsive, $V_\tau(r)$ is positive

D^{bar}-nuclei (naive expectation from $D^{bar}A$ folding potential model)

✓ What is possible structure of D^{bar}-nuclei?



$$V_{\bar{D}N}(\vec{r}) = V_0(\vec{r}) + V_\tau(\vec{r})(\vec{\tau}_{\bar{D}} \cdot \vec{\tau}_N)$$

$$V_{\bar{D}A}(\vec{r}) = \sum_{i=1}^{A} \int d^3r' \rho_{N_i}(\vec{r}') \left[V_0(\vec{r}-\vec{r}') + V_\tau(\vec{r}-\vec{r}')(\vec{\tau}_{\bar{D}} \cdot \vec{\tau}_{N_i})\right]$$

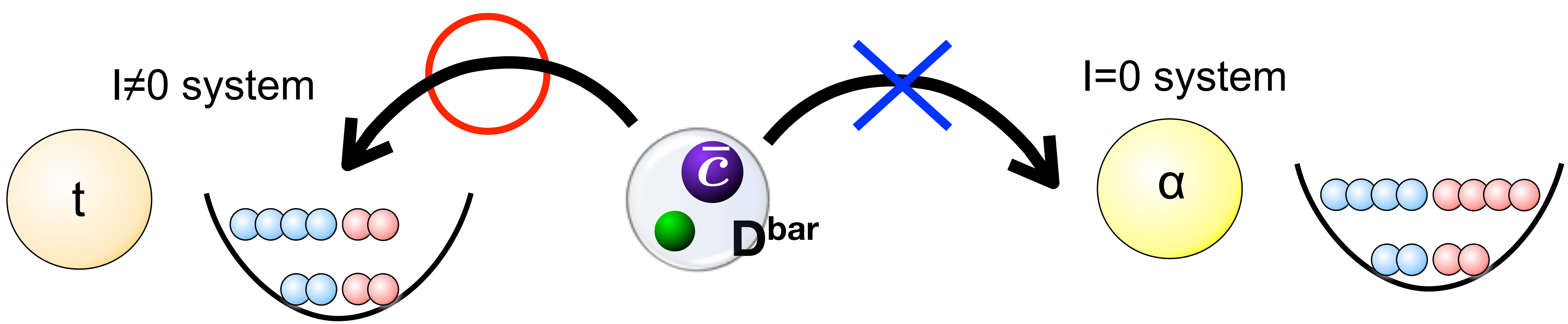
contact interaction : $V_{\bar{D}N}(\vec{r}) \to \{g_0 + g_\tau(\vec{\tau}_{\bar{D}} \cdot \vec{\tau}_N)\}\, \delta(\vec{r})$

g_0 : repulsive, g_τ : positive

Energy shift

$$\Delta E = \int d^3r V_{\bar{D}A}(\vec{r}) = A\left[g_0 + g_\tau(\vec{\tau}_{\bar{D}} \cdot \vec{\tau}_A)\right], \quad \vec{\tau}_A = \sum_{i=1}^{A} \vec{\tau}_{N_i}$$

✓ Isospin correlation plays crucial role to make D^{bar} bound into nuclei



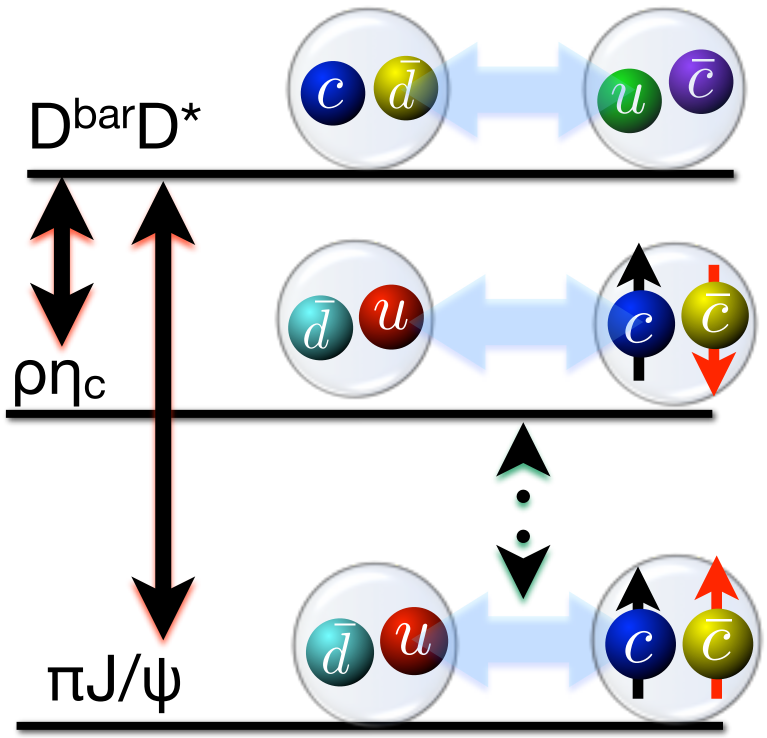
Summary

✦ Coupled-channel HAL QCD method

- derive potential faithful to S-matrix from BS wave function
- a solution to coupled-channel problems from LQCD

Tetra-quark candidate $Z_c(3900)$

- $Z_c(3900)$ is threshold cusp induced by strong $V^{\bar{D}D^*, \pi J/\psi}$
  - pole position very far from scat. axis
  - expt. data of Y(4260) decay well reproduced
  - no peak structure w/o $V^{\bar{D}D^*, \pi J/\psi}$



Dbar-nuclei

- D^{bar} may be bound into neutron or proton rich nuclei
  - repulsive isospin-independent & positive sign of isospin-dependent interactions
  - D^{bar} is good probe to explore isospin correlation inside nuclei

Future: physical point calculations

Thank you for your attention!!