International Molecule-type Workshop "Strangeness and charm in hadrons and dense matter"

Determination of **confinement-deconfinement transition** by **Roberge-Weiss periodicity**

Kouji Kashiwa

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Collaborator: Akira Ohnishi

- **K.K.,** and A. Ohnishi, Phys. Lett. B750 (2015) 282
- **K.K.,** and A. Ohnishi, Phys. Rev D. 93 (2016) 116002
- **K.K.**, and A. Ohnishi, arXiv:1701.04953

2017/05/15

Purpose of this study

To determine

the confinement-deconfinement transition

in the system with dynamical quarks

Heavy quark-mass limit

Polyakov-loop describes the confinement-deconfinement transition

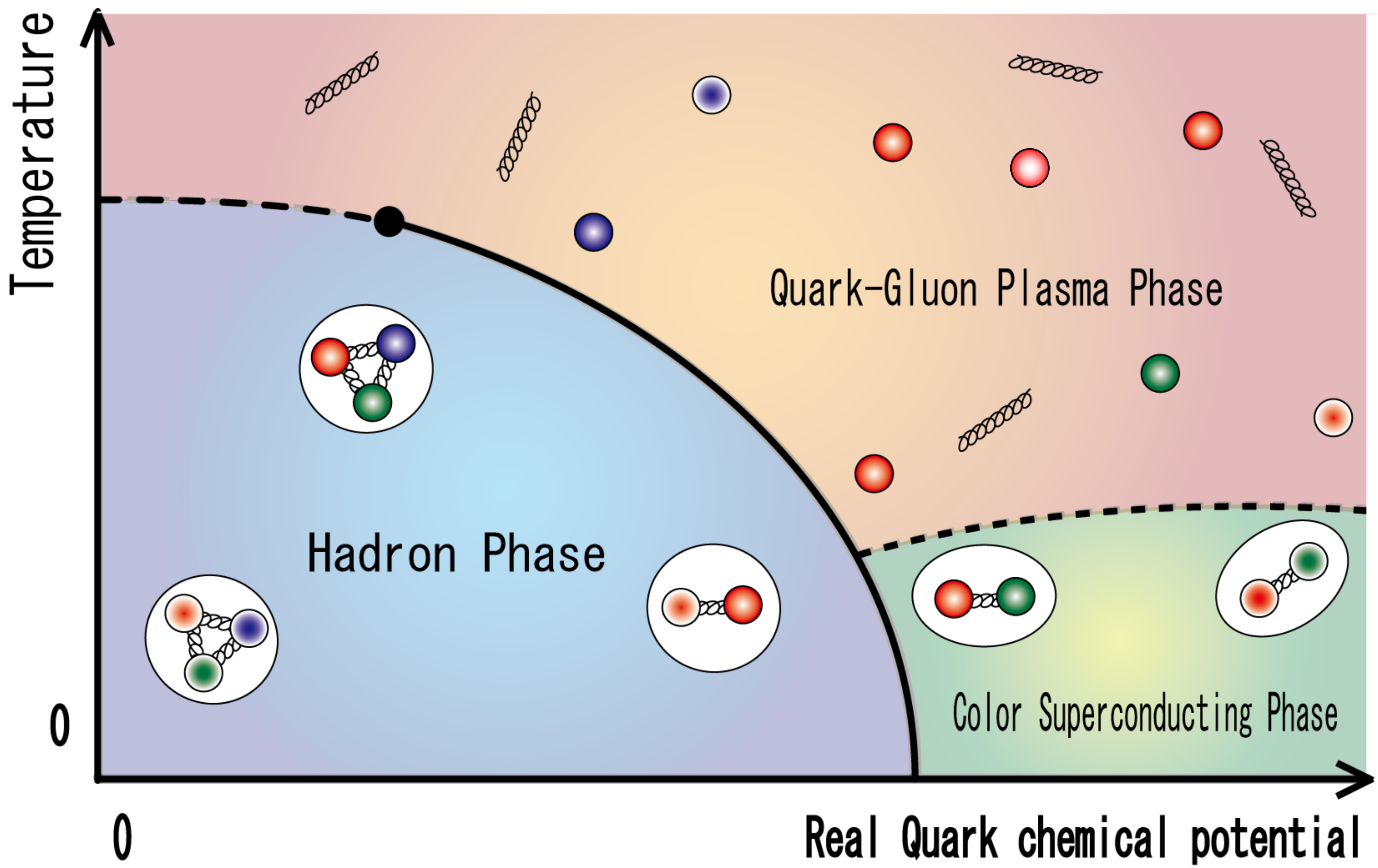
➡ The $\mathbb{Z}_3$ symmetry relates with the deconfinement transition via the free-energy

We can well determine the deconfinement temperature

Finite quark-mass case

Polyakov-loop is **no longer** the order-parameter !

Schematic QCD phase diagram



Important point

Finite quark-mass case :

Polyakov-loop is **no longer** the order-parameter !

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Finite quark-mass case :

Polyakov-loop is **no longer** the order-parameter !

Ordinary phase transition ⟺ Spontaneous symmetry breaking

Phase transition described by the **topological order**

X. G. Wen, Int. J. Mod. Phys. B4 (1990) 239.

⟺ Ground-state degeneracy

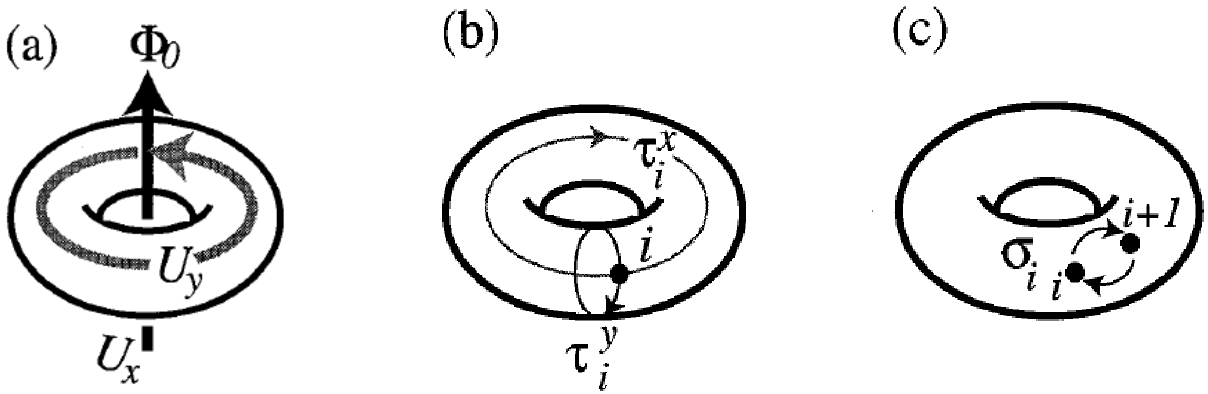
Question

How to see the topological order at $T = 0$?

Tree adiabatic operations

M. Sato, M. Kohmoto and Y.-S. Wu, PRL 97 (2006) 010601.

M. Sato, PRD 77 (2008) 045013.



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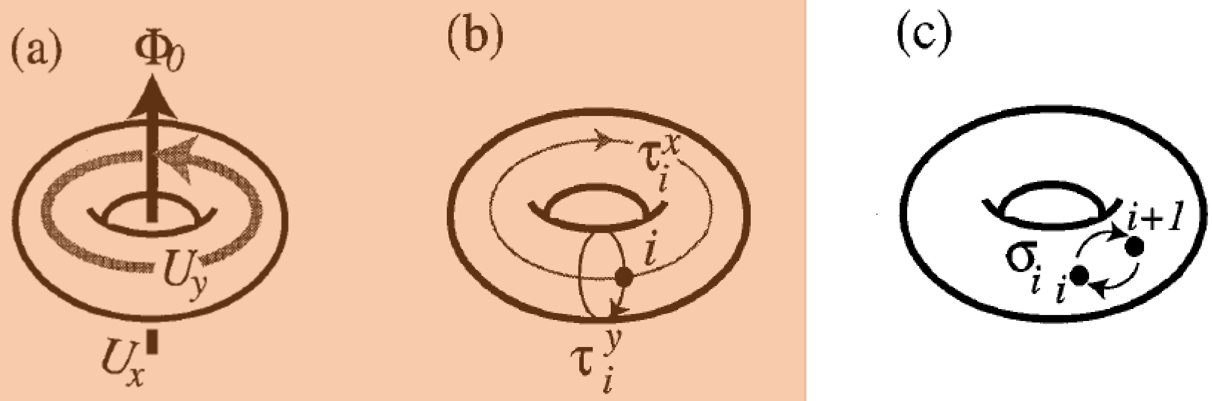

Aharonov-Bohm effect

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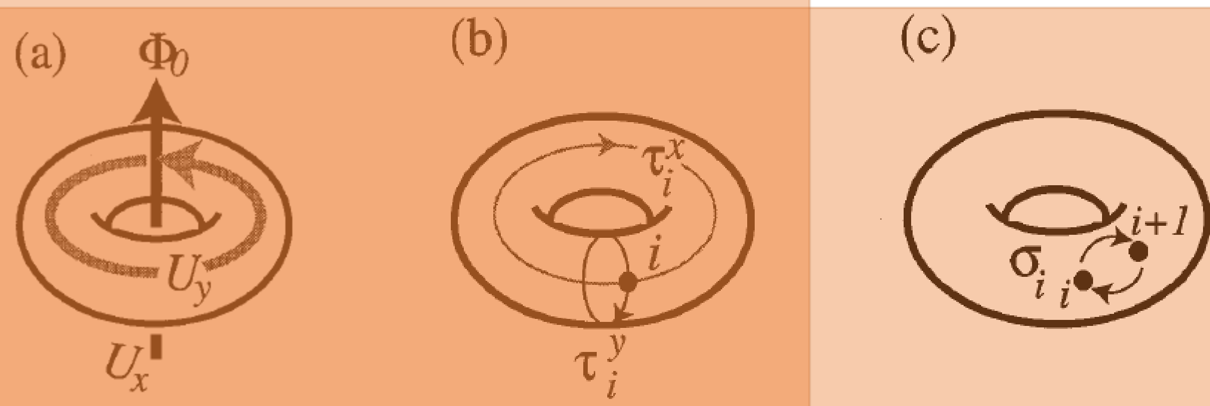

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Braid group

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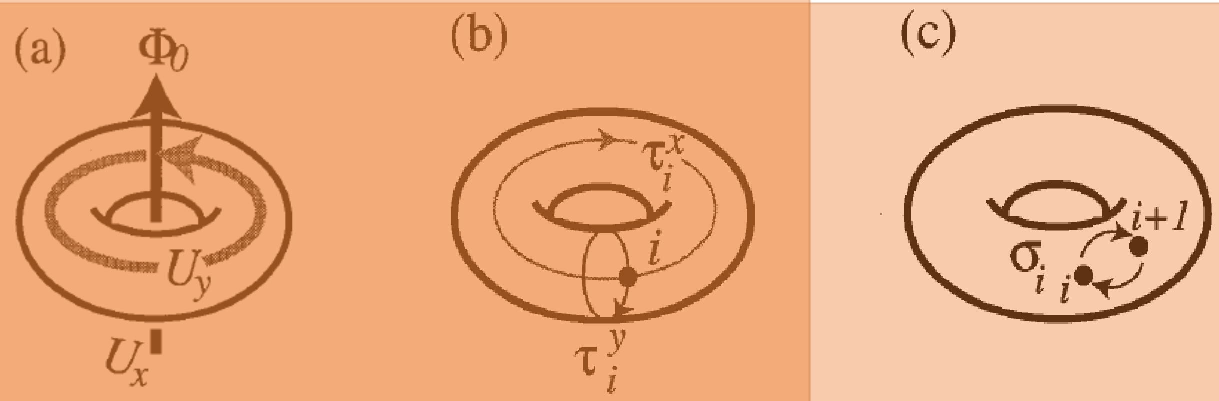


Aharonov-Bohm effect

Braid group

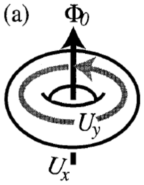
Without fractional charge : Commutable

With fractional charge : Non-commutable → Ground-state degeneracy

We wish to **extend** it to **finite temperature QCD**!

However, direct extension of the ground-state degeneracy is difficult･･･

We consider that the **imaginary chemical potential** is



an probe to determine the deconfinement transition

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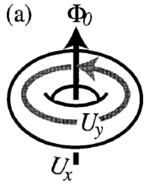
However, direct extension of the ground-state degeneracy is difficult···

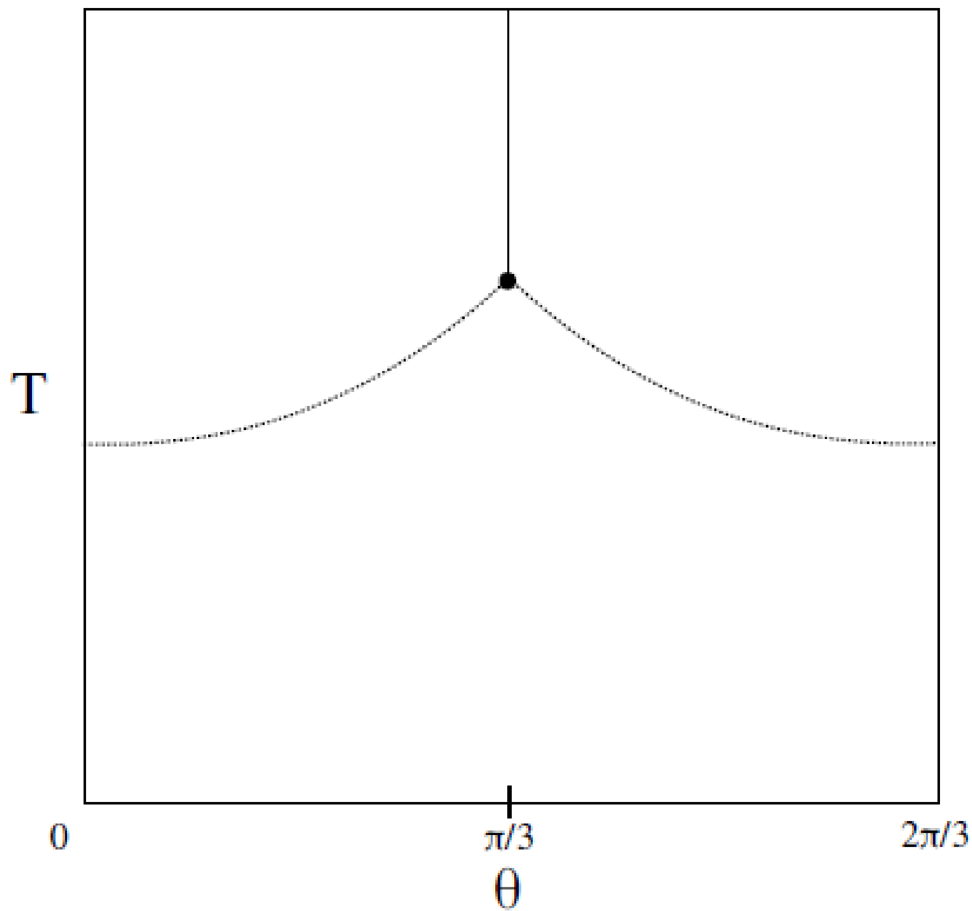


We consider that the **imaginary chemical potential** is

an probe to determine the deconfinement transition

- There is no sign problem
- This region has all information of the region with finite μ_R
- There are topological differences between the low and high T regions
- We can consider similar special operations (Today, I do not explain this point)

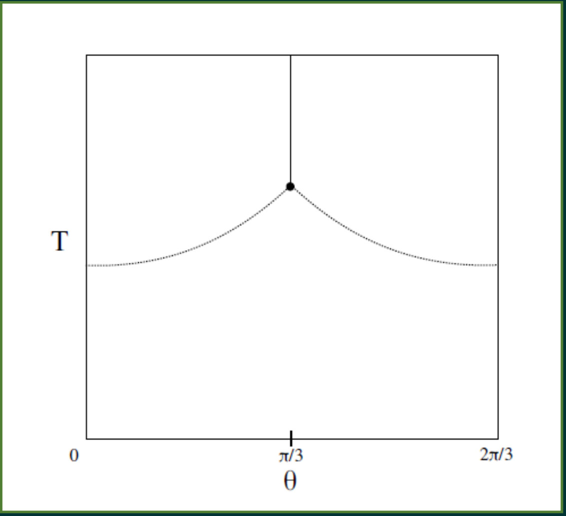
Phase diagram at finite imaginary chemical potential



A. Roberge and N. Weiss, Nucl. Phys. B275 (1986) 734

Important point

A. Roberge and N. Weiss,
Nucl. Phys. B275 (1986) 734



Roberge-Weiss periodicity

Special π/N_c periodicity along θ-direction

It appears at low and high T

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A. Roberge and N. Weiss,
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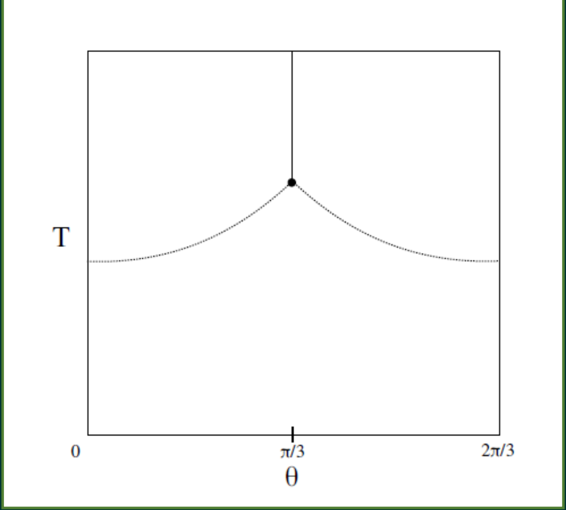

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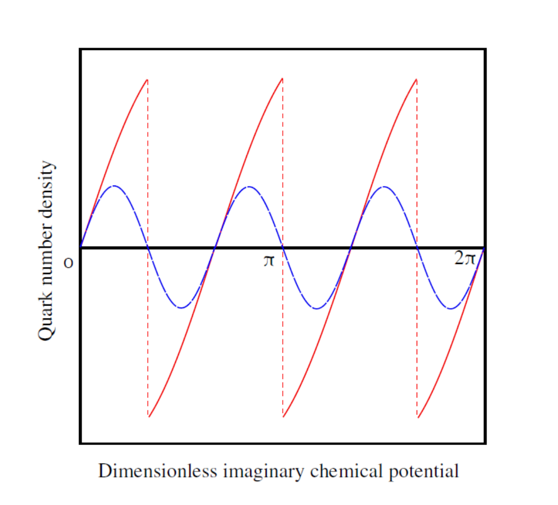

Roberge-Weiss transition

First-order transition along T-direction

It is characterized by the gap
of the quark number density

Question

How to use these properties to determine
the deconfinement transition ?

Result 1 : Free-energy degeneracy

K.K., A. Ohnishi, Phys. Lett. B750 (2015) 282.

It is natural to consider the **free-energy**
since we are interested in the **thermodynamic system**

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since we are interested in the **thermodynamic system**

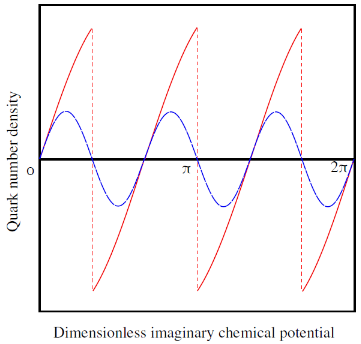
However ···

The topological order is usually difficult to discuss from thermodynamics
(Bulk thermodynamics is insensitive to the topological change)

But

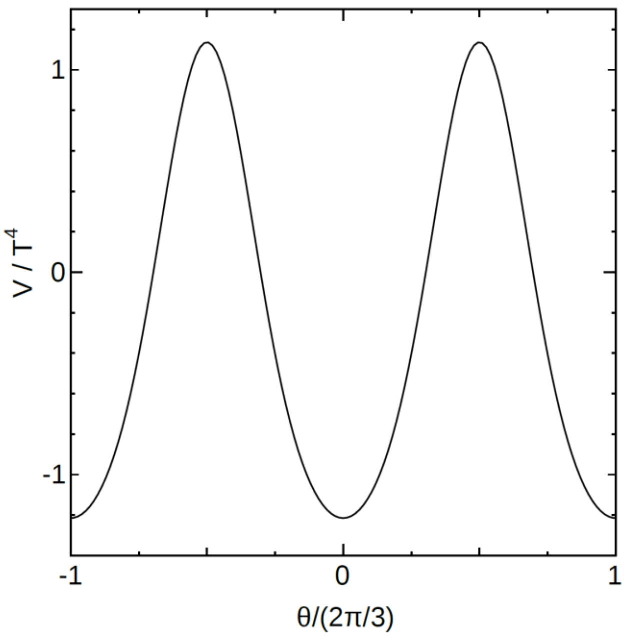
We can discuss topological structures at **finite θ**

(The imaginary chemical potential acts as the extra-dimension)



Result 1 : Free-energy degeneracy

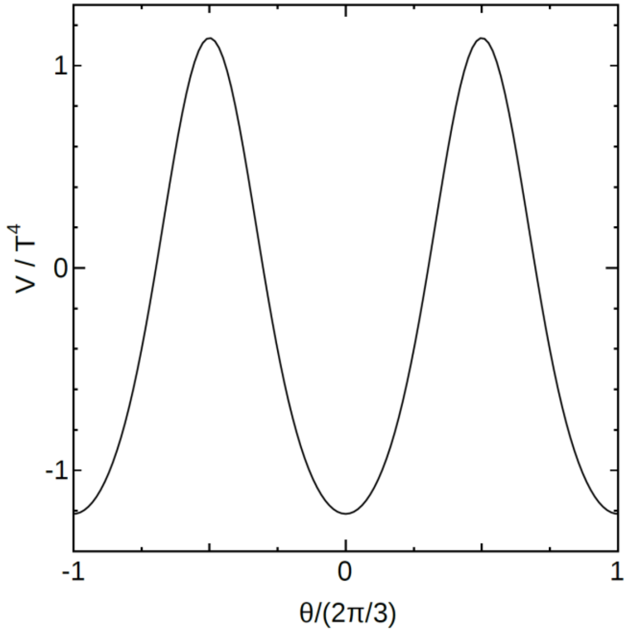
Confined phase



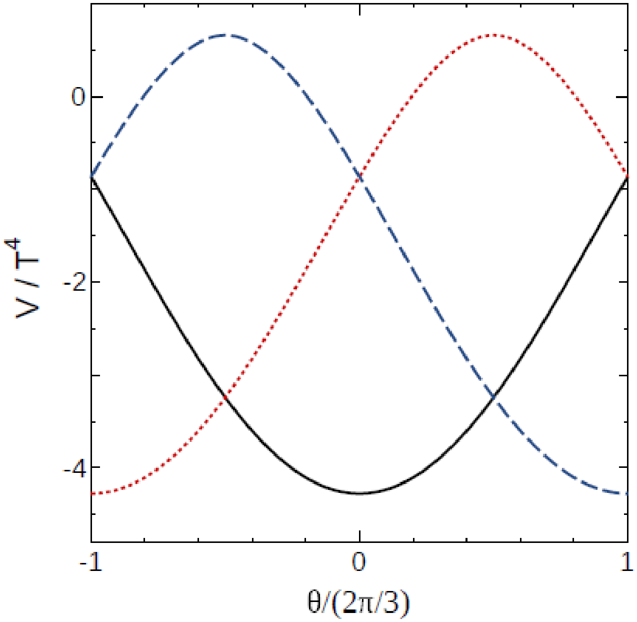
Confined phase : There is no non-trivial degeneracy

Result 1 : Free-energy degeneracy

Confined phase



Deconfined phase



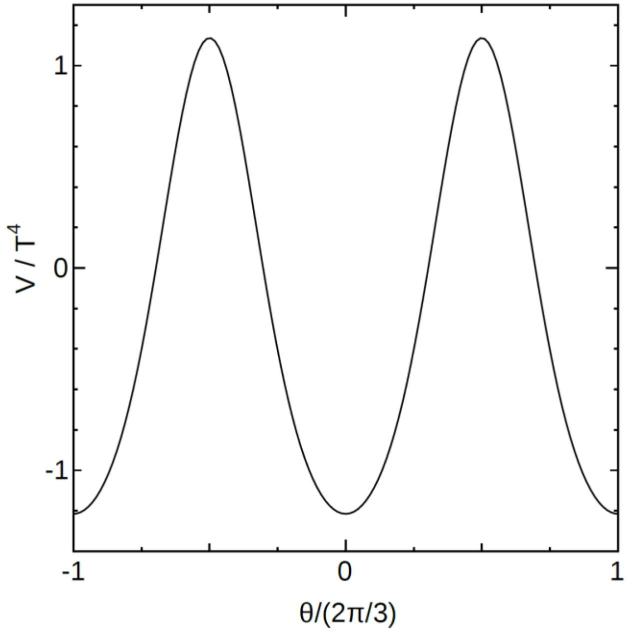
Confined phase : There is no non-trivial degeneracy

Deconfined phase : There is the non-trivial degeneracy!

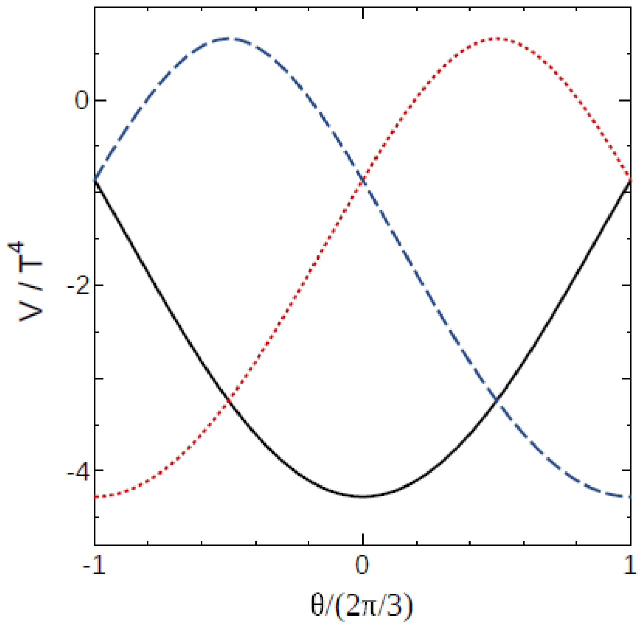
Qualitative differences are already known in A. Roberge and N. Weiss, Nucl. Phys. B275 (1986) 734

Result 1 : Free-energy degeneracy

Confined phase



Deconfined phase

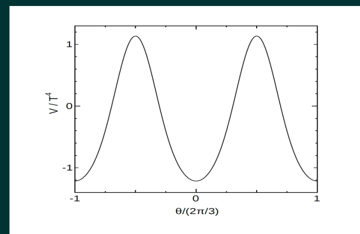


This difference relates to appearance of the quark-gluon dynamics

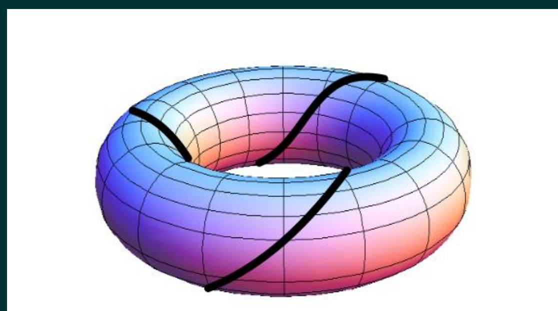
Dominant degree of freedom :   Hadrons → Quarks

Result 1 : Free-energy degeneracy

Confined phase

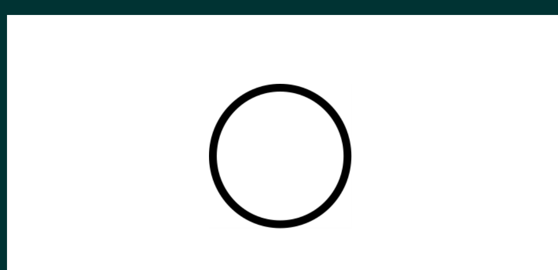


S^1 map

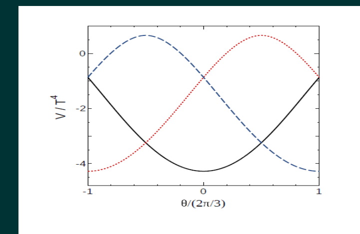


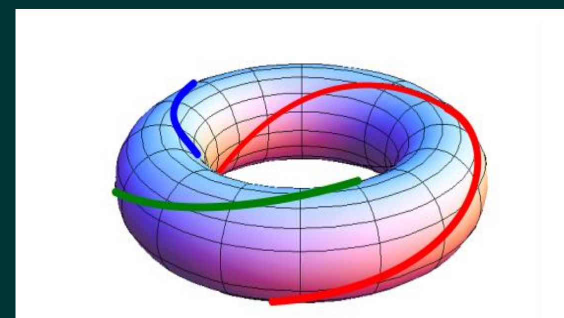
One line winds around the torus

Two dim. map

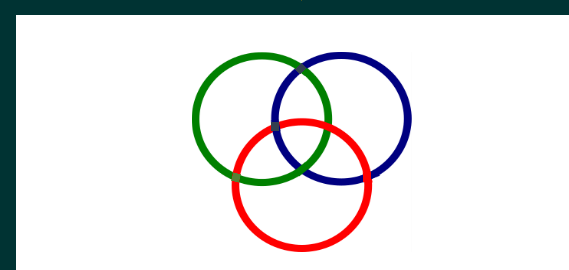


Deconfined phase





Three lines wind around the torus



K.K., A. Ohnishi, Phys. Rev D. 93 (2016) 116002

Based on the topological difference,
we can construct the **quantum order-parameter**

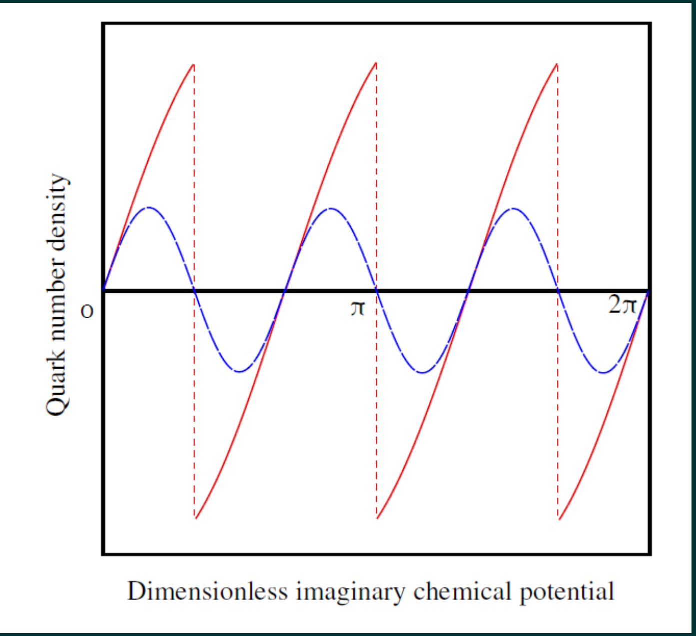
Quark number holonomy

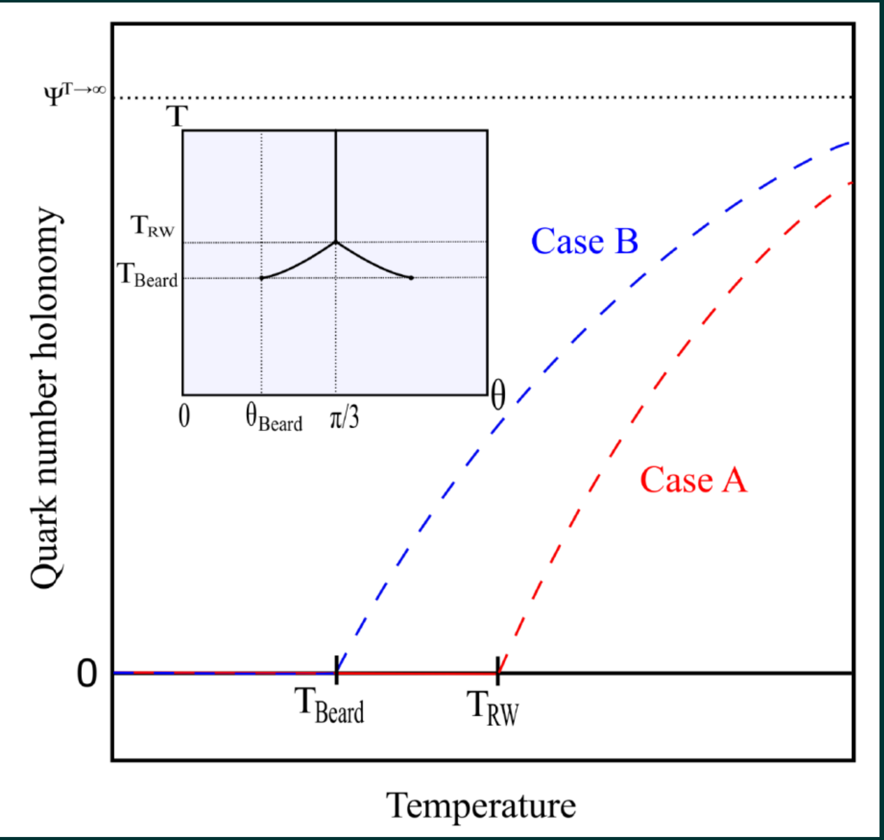
$$\Psi = \left[\oint_0^{2\pi} \left\{ \mathrm{Im}\left(\frac{d\tilde{n}_q}{d\theta}\Big|_T \right) \right\} d\theta \right]$$

$$\tilde{n}_q \equiv C n_q$$

This quantity basically counts the number of gap in
the quark number density along θ-axis

Important point





We can well determine
the deconfinement transition

Result 2 : Estimation of deconfinement transition temperature

Important point

- 2+1 flavor lattice QCD

C. Bonati, M D'Elia, M. Mariti, M. Mesiti and F. Negro, Phys. Rev. D 93 (2016) 074504

T_{RW} = **208(5)** [MeV]

Quark number holonomy becomes nonzero above this temperature

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- Recent 2+1 flavor effective model

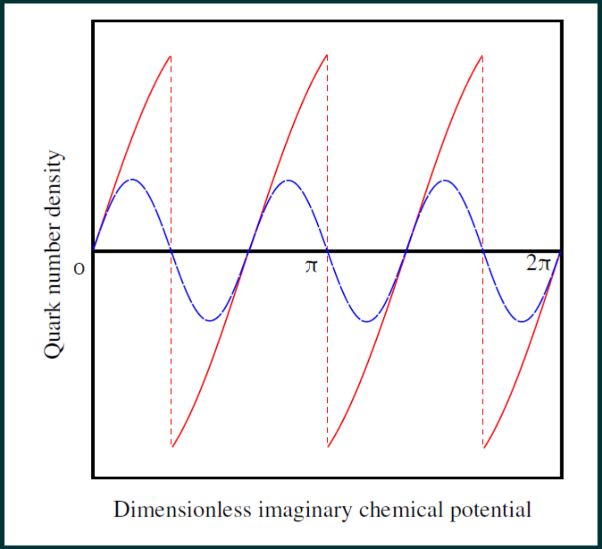
A. Miyahara, Y. Torigoe, H. Kouno, M. Yahiro, Phys. Rev. D 94 (2016) 016003

T_d = **215** [MeV]

Using quantities to determine the deconfinement transition are different, but there is good agreement. (accidental?)

Question

Why the $\theta = \pi k/N_c$ can detect the topological differences?



Result 2 : $\mathbb{Z}_3$ edges

Question

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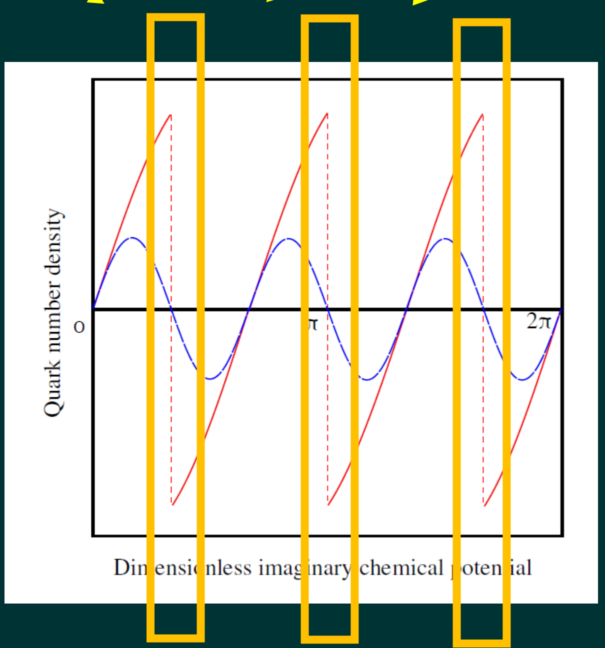
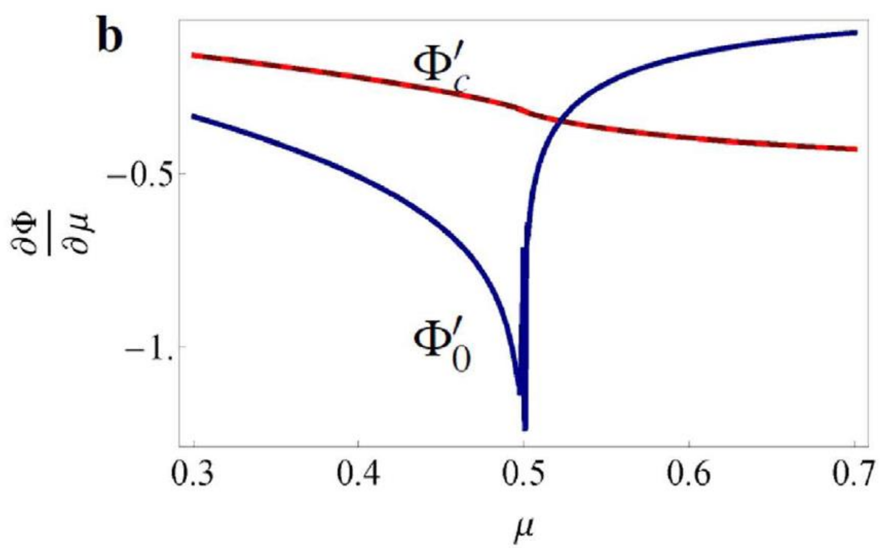


The topological order can be clarify from **thermodynamics (at edges)**

S. Kempkes, A. Quelle, and C. M. Smith, Scientific Reports 6 (2016)

Example : Kitaev chain model

$$\mathcal{H} = -\sum_{j=1}^{N}\left[\kappa(a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j) + \mu a_j^\dagger a_j - \Delta a_j a_{j+1} - \Delta^* a_{j+1}^\dagger a_j^\dagger\right]$$



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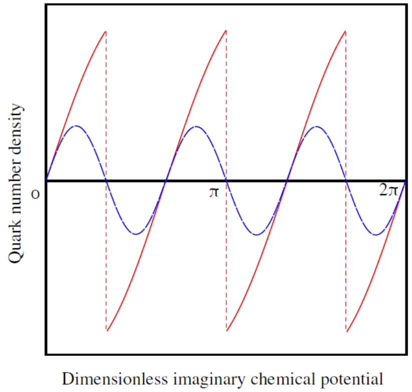


Possible analogy

We treat the **imaginary chemical potential** as the extra-dimension (parameter) and thus the $\theta = \pi k/N_c$ points acts as edges of the extended system

Therefore, thermodynamics (gaps in the quark number density) can describe the deconfinement phase transition

K.K., A. Ohnishi, arXiv:1701.04953

What happen at finite **real chemical potential**?

In our determination, we should consider

the **complex chemical potential**

and thus it is very difficult to discuss it

In this study,
we employ **Polyakov-loop extended Nambu—Jona-Lasinio** model

PNJL Lagrangian density

$$\mathcal{L} = \bar{q}(\not{D} + m_0) - G[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2] + \mathcal{V}_{\mathrm{g}}(\Phi, \bar{\Phi})$$

(Good point)

It can describe the chiral phase transition and
approximately treat the deconfinement transition via the Polyakov-loop

It can reproduce the RW periodicity and transition

(Bad point)

Unfortunately,
this model still has the model sign problem at finite real chemical potential

So, we consider the isospin chemical potential

- **Sign problem free**

$$\tau_2\gamma_5 D\gamma_5\tau_2 = D^\dagger \Rightarrow \det(D) \geq 0$$

- **Orbifold equivalence**

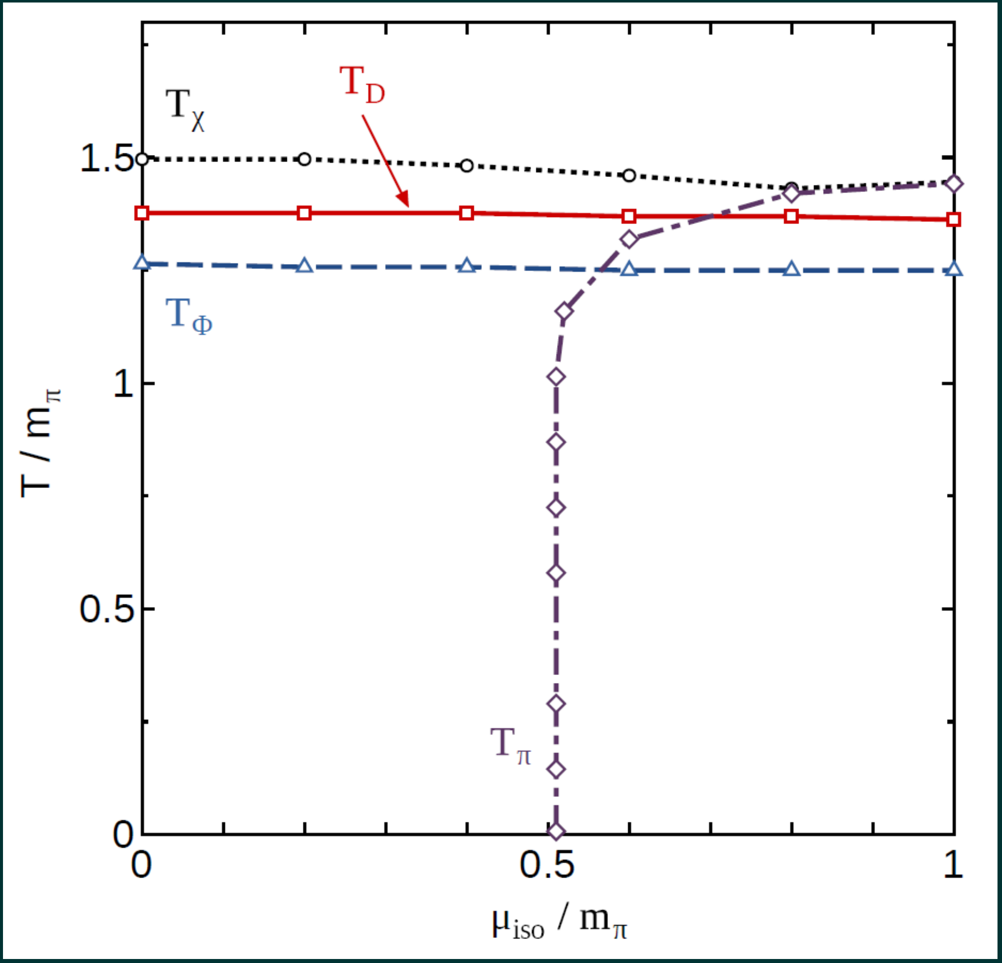
Outside of the pion condensed region,

the phase diagram at finite real μ and
the phase diagram at finite isospin μ

are identical to each other in the large N_c limit

Result 3 : Isospin chemical potential

Phase diagram from PNJL model



How to investigate the deconfinement transition at finite real chemical potential?

We need better ways to handle the **sign problem**

Result 3 ： Isospin chemical potential

How to investigate the deconfinement transition at finite real chemical potential?

We need better ways to handle the **sign problem**

- Lefschetz-thimble path-integral method ?
- Complex Langevin method ?

Result 3 : Isospin chemical potential

How to investigate the deconfinement transition at finite real chemical potential?

We need better ways to handle the **sign problem**

- Lefschetz-thimble path-integral method ?
- Complex Langevin method ?
- **Path optimization method** !

Yuto Mori, K.K., Akira Ohnishi, in preparation

Unfortunately, there is no talk about it …

If you have interesting on it,
please check our forthcoming paper

Summary

We investigate the deconfinement transition from **topological viewpoints**

1. To discuss the deconfinement transition at finite temperature, we use the **nontrivial free-energy degeneracy**

2. We determine the **new order-parameter** of deconfinement transition

$$\Psi = \left[\oint_0^{2\pi} \left\{ \mathrm{Im}\left(\frac{d\tilde{n}_q}{d\theta}\Big|_T \right) \right\} d\theta \right]$$

3. The **density-dependence** of the deconfinement transition is shown by introducing the isospin chemical potential to the PNJL model

What happen in the spatial topology ?

In discussions of the topological order, spatial topology is important

1. Spatial Polyakov-loop with the spatial imaginary chemical potential

2. Entanglement entropy

3. Ulmann phase

   It is a extended quantity of the Berry phase to quantum mixed states

In these calculations, we need heavy numerical computations