rho resonance from the I=1 pipi potential in lattice QCD

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@YITP

Plan of talk

- Introduction
- Operator dependence of potentials
- $\mathrm{I}=1$ $\pi\pi$ potential
  - ✓ ρ resonance signal search
- Pole search of S-matrix in $\mathrm{I}=1$ $\pi\pi$ potential
- Summary and discussion

Introduction

Lattice QCD uncovered a lot of important properties of hadron from the first principle calculation.

HALQDC collaboration has contributed in the field like [S.Aoki, T.Hatsuda, N. Ishii, Prog. Theor.Phys., 123 (2010)] [Ishii, Aoki & Hatsuda, PRL 99 (2007) 022001]

- Negative parity channel
- Heavy quark hadron
- Potential at the physical point
- etc.

However, **all of them** are computed with point-to-all propagators. (∵ Number of inversions)

⟹ Some important channels are yet to be done.

We incorporate **distillation smearing** and use all-to-all propagators in order to overcome this difficulty .

[Michael Peardon, John Bulava et al. Phys.Rev.D80:054506,2009]

Distillation-smeared quark

- Gauge covariant Laplacian

$$\widetilde{\Delta}^{ab}(x,y;U) = \sum_{k=1}^{3} \left\{ \widetilde{U}_k^{ab}(x)\delta(y,x+\hat{k}) + \widetilde{U}_k^{ba}(x)^*\delta(y,x-\hat{k}) - 2\delta(x,y)\delta^{ab} \right\}$$

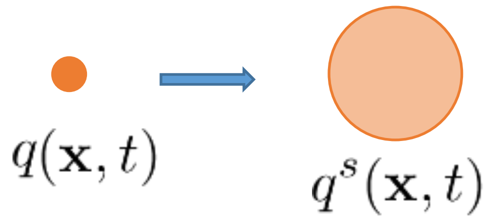
diagonalize

$$\widetilde{\Delta}(t) = V(t)D(t)V(t)^\dagger$$

V is composed of eigenvectors : $V(t) = (\mathbf{e}_1^t, \mathbf{e}_2^t, \ldots, \mathbf{e}_M^t)$ $M = N_c N_x N_y N_z$

Pick up lowest N modes and construct smearing operator

$$\widetilde{V}(t) = (\mathbf{e}_1^t, \mathbf{e}_2^t, \ldots, \mathbf{e}_N^t) \qquad S(t) = \widetilde{V}(t)\widetilde{V}^\dagger(t)$$



Smeared quark : $q^s(\mathbf{x},t) = S_{\mathbf{x},\mathbf{y}}(t)q(\mathbf{y},t)$

(smearing on color is implicitly done)

Local smeared quarks can be created.

Time dependent HAL method [Ishii et al.,PLB712(2012)437]

R-correlator

$$R(\mathbf{r}, t - t_0) = e^{2mt} \sum_{\mathbf{x}} \left\langle 0 \left| T \left\{ N(\mathbf{x}, t) N(\mathbf{x} + \mathbf{r}, t) \right\} \bar{\mathcal{J}}(t_0) \right| 0 \right\rangle$$
$$= \sum_n A_n \psi_{k_n}(\mathbf{r}) e^{-(E_n - 2m)(t - t_0)}$$

time-dependent Schrödinger-like equation

$$\left[\frac{1}{4m} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right] R(\mathbf{r}, t) = \int dr'^3 U(\mathbf{r}, \mathbf{r}') R(\mathbf{r}', t)$$

From the velocity expansion, the potential is given by

$$V(\mathbf{r}) = \frac{1}{4m} \frac{(\partial/\partial t)^2 R(\mathbf{r}, t)}{R(\mathbf{r}, t)} - \frac{(\partial/\partial t) R(\mathbf{r}, t)}{R(\mathbf{r}, t)} - \frac{H_0 R(\mathbf{r}, t)}{R(\mathbf{r}, t)}$$

The operator dependence of potentials

The operator dependence of HAL QCD potentials

Conventional HALQCD method ➡ Wall src. and point sink are used.

$$R(\mathbf{r}, t-t_0) = e^{2mt} \sum_{\mathbf{x}} \langle 0 | T \underbrace{\{B(\mathbf{x},t) B(\mathbf{x}+\mathbf{r},t)\}}_{\text{Point sink}} \underbrace{\bar{\mathcal{J}}(t_0)}_{\text{Wall src}} |0\rangle$$

⬇ Distillation smearing

$$R(\mathbf{r}, t-t_0) = e^{2m(t-t_0)} \sum_{\mathbf{x}} \sum_{y_1, y_2} < 0 | T\{ \underbrace{M^s(\mathbf{x},t) M^s(\mathbf{x}+\mathbf{r},t)}_{\text{smeared sink}} \underbrace{M^s(\mathbf{y_1},t_0) M^s(\mathbf{y_2},t_0)}_{\text{smeared src.}} \} | 0 >$$

➡ Smeared src. and smeared sink are used

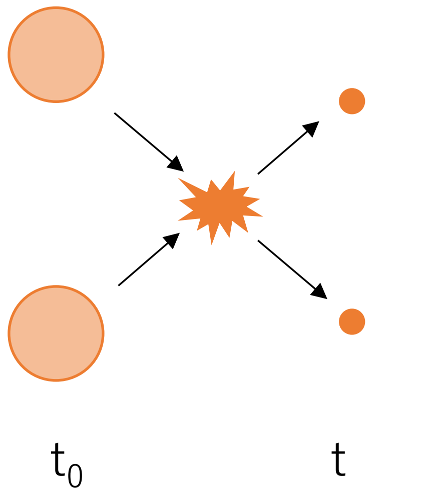
Question : How does the HALQCD potential depend on the sink operator ?

Check the operator dependence of pion-pion interaction potentials with 2 setups

Point sink – Smeared src

- 2-pt correlation $C_M^2(t, t_0) = \sum_{\mathbf{x},\mathbf{y}} \langle 0|\pi^+(\mathbf{x}, t)\pi^{-s}(\mathbf{y}, t_0)|0\rangle$

  $\pi^{-s}(\mathbf{x}, t) = \bar{u}^s\gamma_5 d^s(\mathbf{x}, t)$, $q^s(\mathbf{x}, t) = S_{\mathbf{x},\mathbf{y}}(t)q(\mathbf{y}, t)$
- 4-pt correlation $C_M^4(\mathbf{r}, t; t_0) = \sum_{\mathbf{x}} \sum_{\mathbf{y_1},\mathbf{y_2}} \langle 0|\pi^+(\mathbf{x}, t)\pi^+(\mathbf{x}+\mathbf{r}, t)\pi^{-s}(\mathbf{y_1}, t_0)\pi^{-s}(\mathbf{y_2}, t_0)|0\rangle$



Smeared sink – Smeared src

- 2-pt correlation $C_M^2(t, t_0) = \sum_{\mathbf{x},\mathbf{y}} \langle 0|\pi^{+s}(\mathbf{x}, t)\pi^{-s}(\mathbf{y}, t_0)|0\rangle$

  $\pi^{-s}(\mathbf{x}, t) = \bar{u}^s\gamma_5 d^s(\mathbf{x}, t)$, $q^s(\mathbf{x}, t) = S_{\mathbf{x},\mathbf{y}}(t)q(\mathbf{y}, t)$
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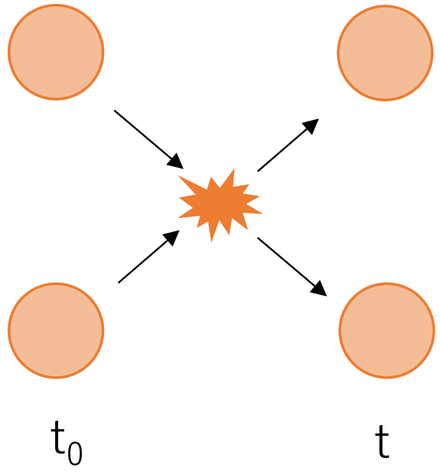
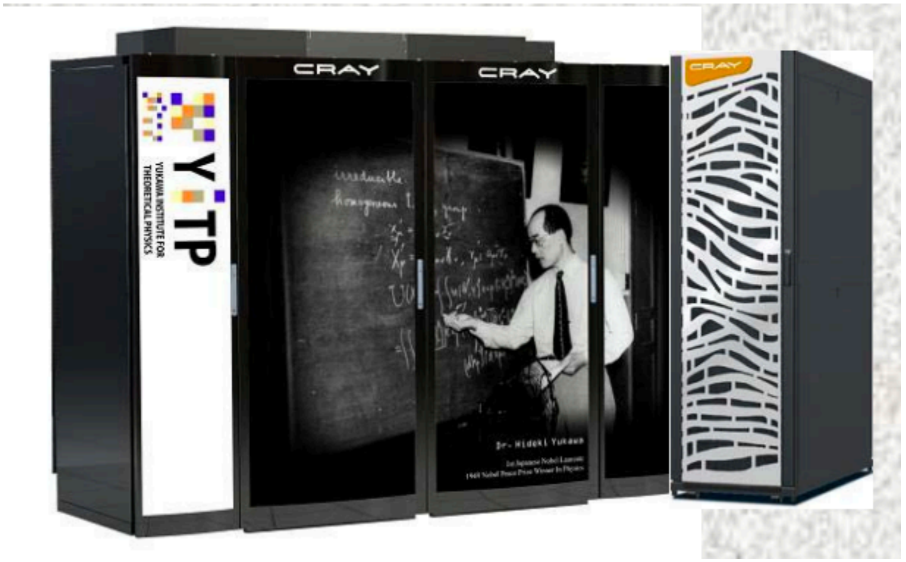


Numerical setup

- 2 + 1 flavor gauge configuration by CP-PACS & JLQCD

  [CP-PACS/JLQCD Collaboration : T.Ishikawa, et al., PRD 78 (2008) 011502(R)]

- Wilson clover fermion and Iwasaki gauge action
- a = 0.1214 fm , $16^3 \times 32$ lattice
- $m_\pi \simeq 870\mathrm{MeV}$
- 60conf × 32 time slice
- Calculated on Cray XC40 in YITP



Cray XC40 in YITP

Remark : the sum over source space improves statistics.

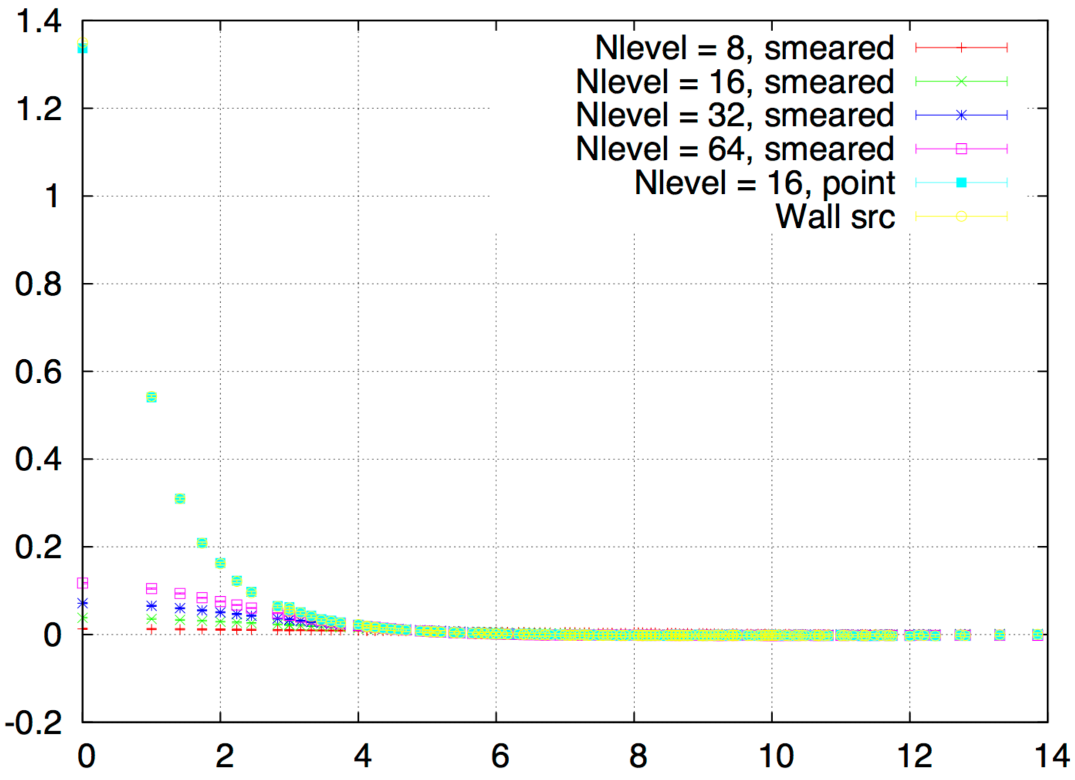
$$C_M^4(\mathbf{r},t;t_0) = \sum_{\mathbf{x}} \sum_{\mathbf{y_1},\mathbf{y_2}} \langle 0|\pi^+(\mathbf{x},t)\pi^+(\mathbf{x}+\mathbf{r},t)\pi^{-s}(\mathbf{y_1},t_0)\pi^{-s}(\mathbf{y_2},t_0)|0\rangle$$

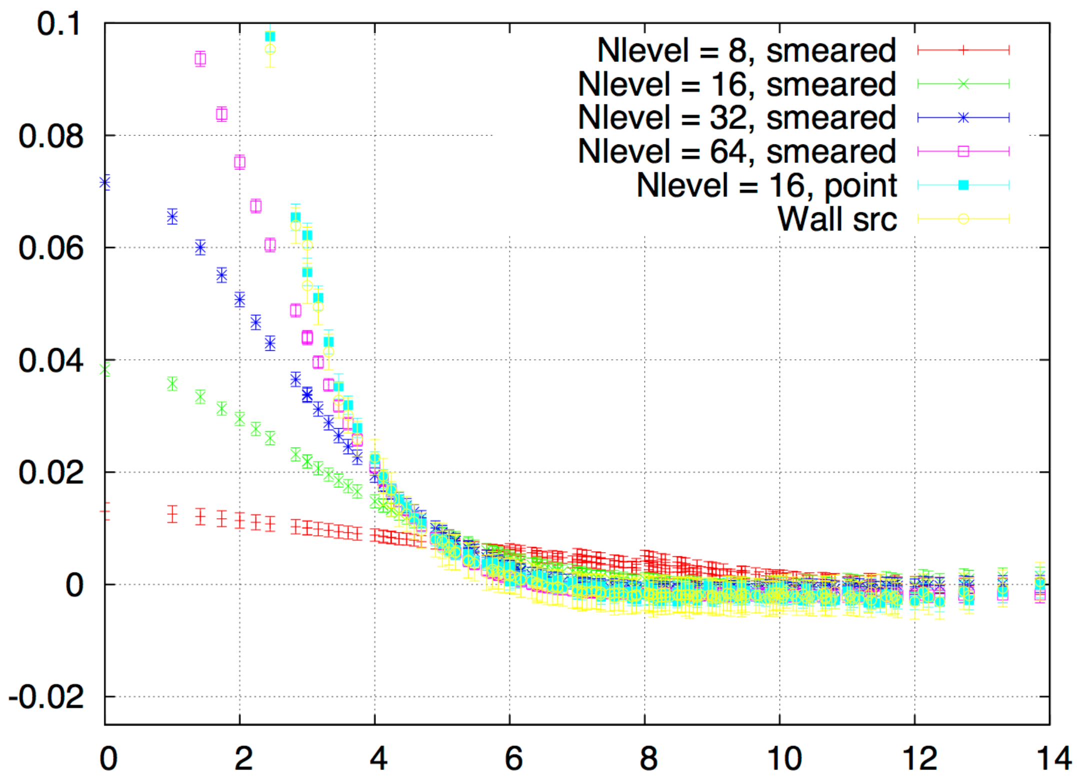
The operator dependence of potentials

Point sink-Smeared src. ➡ Quite similar to Point sink-Wall src

Smeared sink-Smeared src. ➡ Repulsive core is weakened and strong dependence on the number of Laplacian eigenvalue appears.

Strong operator dependence in smeared sink case

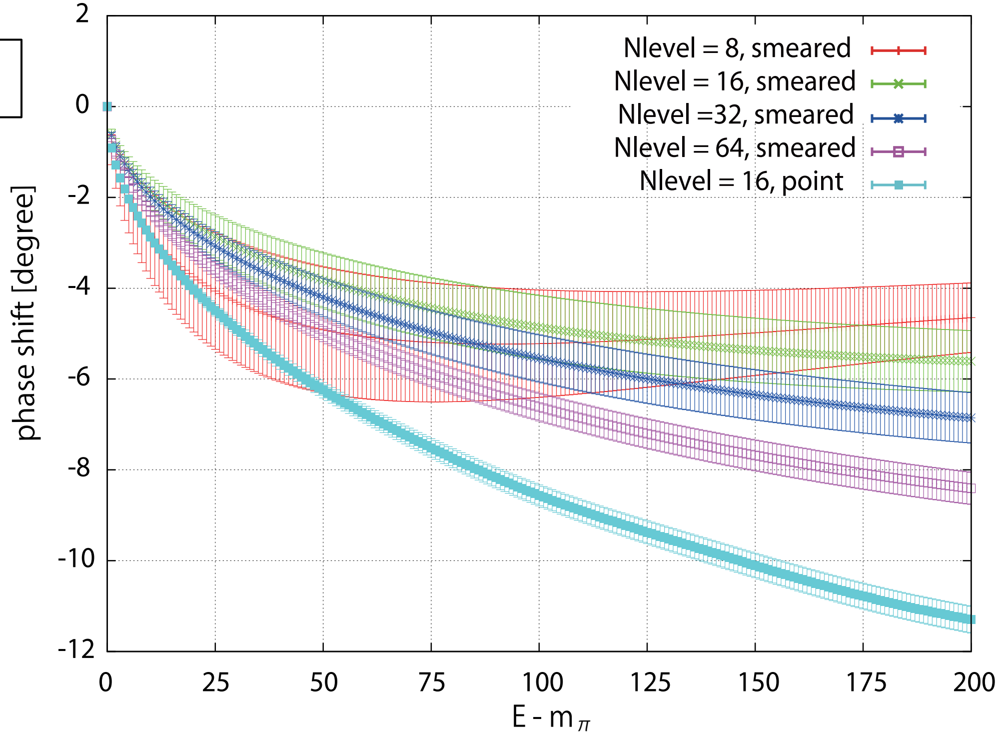




The operator dependence of Phase shift

Phase shift

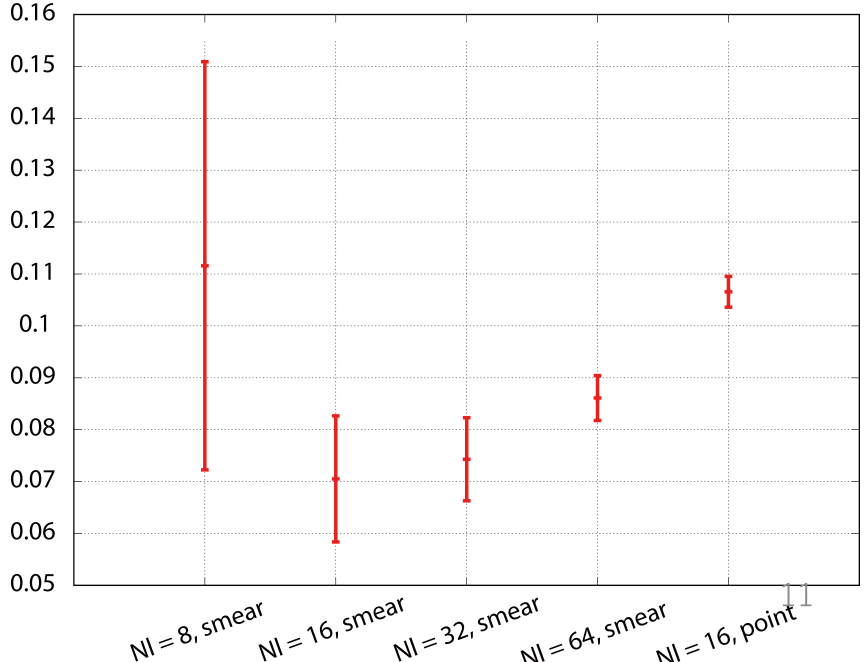
Calculate phase shift based on HAL QCD method



- Point sink-Smeared src. series have smaller phase shift at all energy region

  ➡ It reflects larger repulsive core in point sink case.

Scattering length



- Phase shift by smeared sink approaches to the one by point sink monotonically as the number of Laplacian eigenvalue increases.

Is it impossible to get correct behaviors from smeared sink ?

Phase shift from smeared sink largely deviates from the one with point sink.

→ Unphysical behavior is given because of weak repulsive core.

↓

Is smeared sink useless ? The answer is possibly ***NO*** !!

Currently the leading order(LO) of the derivative expansion is considered.

$$V(\mathbf{r}) = \frac{1}{4m}\frac{(\partial/\partial t)^2 R(\mathbf{r},t)}{R(\mathbf{r},t)} - \frac{(\partial/\partial t)R(\mathbf{r},t)}{R(\mathbf{r},t)} - \frac{H_0 R(\mathbf{r},t)}{R(\mathbf{r},t)}$$

But next-to-leading order(NLO) in the derivative expansion will be needed because of smearing operator.

→ This will improve HAL QCD potential.

Next-to-leading order potential

We consider the potential to next-to-leading order. $\left[\frac{1}{4m}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right] R(\mathbf{r},t) = \left(V_0 + \nabla^2 V_1\right) R(\mathbf{r},t)$

Now we assume V_0 and V_1 is the same regardless of energy in the system.

$$V_0(r) + V_1(r)\frac{\sum_{g\in O_h} R_{stat.}(g^{-1}\mathbf{r},t)^*\nabla^2 R_{stat.}(g^{-1}\mathbf{r},t)}{\sum_{g\in O_h} R_{stat.}(g^{-1}\mathbf{r},t)^* R_{stat.}(g^{-1}\mathbf{r},t)} = V_{stat,tot.}(\mathbf{r},t) \quad \text{(Zero total momentum in src.)}$$

$$V_0(r) + V_1(r)\frac{\sum_{g\in O_h} R_{A1}(g^{-1}\mathbf{r},t)^*\nabla^2 R_{A1}(g^{-1}\mathbf{r},t)}{\sum_{g\in O_h} R_{A1}(g^{-1}\mathbf{r},t)^* R_{A1.}(g^{-1}\mathbf{r},t)} = V_{A1,tot.}(\mathbf{r},t) \quad \text{(1 l.u. total momentum in src.)}$$

Then singular value decomposition (SVD) can be used in order to decompose

leading order and next-to-leading order potential.

$$\begin{pmatrix} 1 & \frac{\sum_{g\in O_h} R_{stat.}(g^{-1}\mathbf{r},t)^*\nabla^2 R_{stat.}(g^{-1}\mathbf{r},t)}{\sum_{g\in O_h} R_{stat.}(g^{-1}\mathbf{r},t)^* R_{stat.}(g^{-1}\mathbf{r},t)} \\ 1 & \frac{\sum_{g\in O_h} R_{A1}(g^{-1}\mathbf{r},t)^*\nabla^2 R_{A1}(g^{-1}\mathbf{r},t)}{\sum_{g\in O_h} R_{A1}(g^{-1}\mathbf{r},t)^* R_{A1}(g^{-1}\mathbf{r},t)} \\ \vdots & \vdots \end{pmatrix} \begin{pmatrix} V_0(r) \\ V_1(r) \end{pmatrix} = \begin{pmatrix} V_{stat,tot.}(\mathbf{r},t) \\ V_{A1,tot.}(\mathbf{r},t) \\ \vdots \end{pmatrix}$$

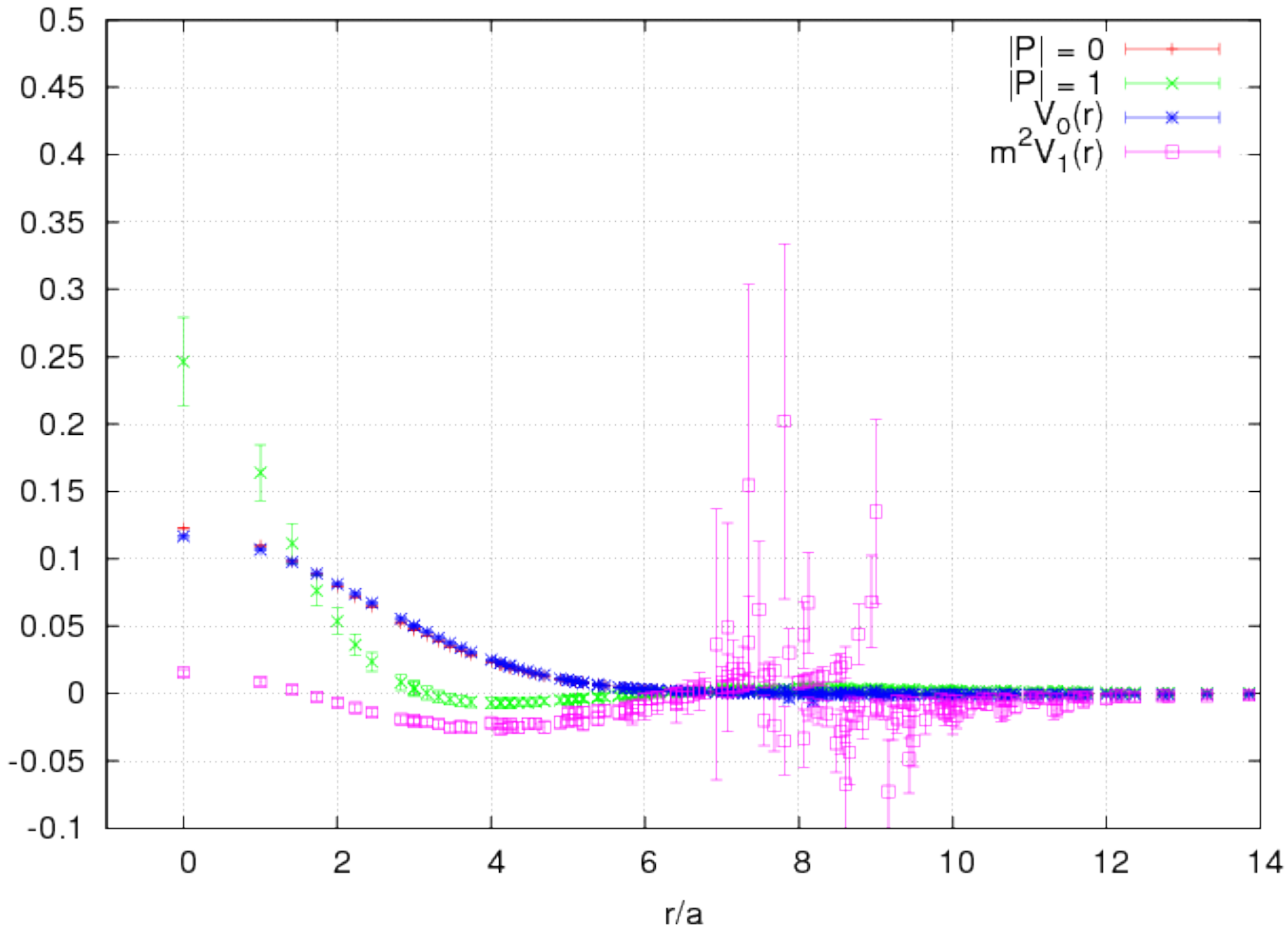
Here is the potentials given by the SVD of Rcorrelators.

Interestingly, the potential from $R_{stat.}$ and leading order potential is quite similar.

The leading order potential is enough for low energy region.

But it's not enough to cover higher energy.

In this case, the behavior at $2\sqrt{s} - 2m_\pi \sim 600\mathrm{MeV}$ is largely modified by next-to-leading order.

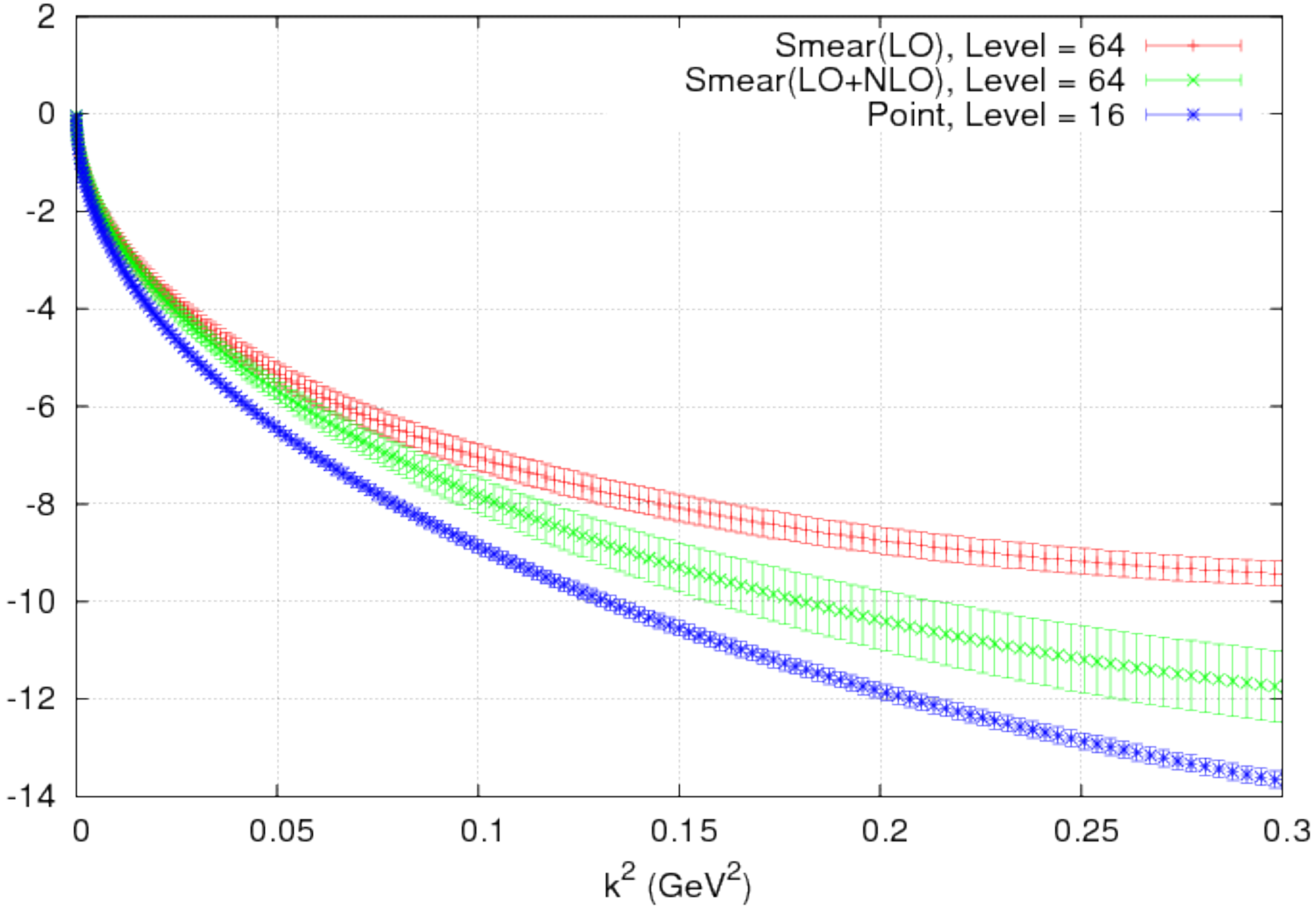


Phase shift from the LO+NLO potential

The comparison of phase shift from the LO, LO+NLO and point sink potential.

LO and LO+NLO potential have similar behavior at low energy.

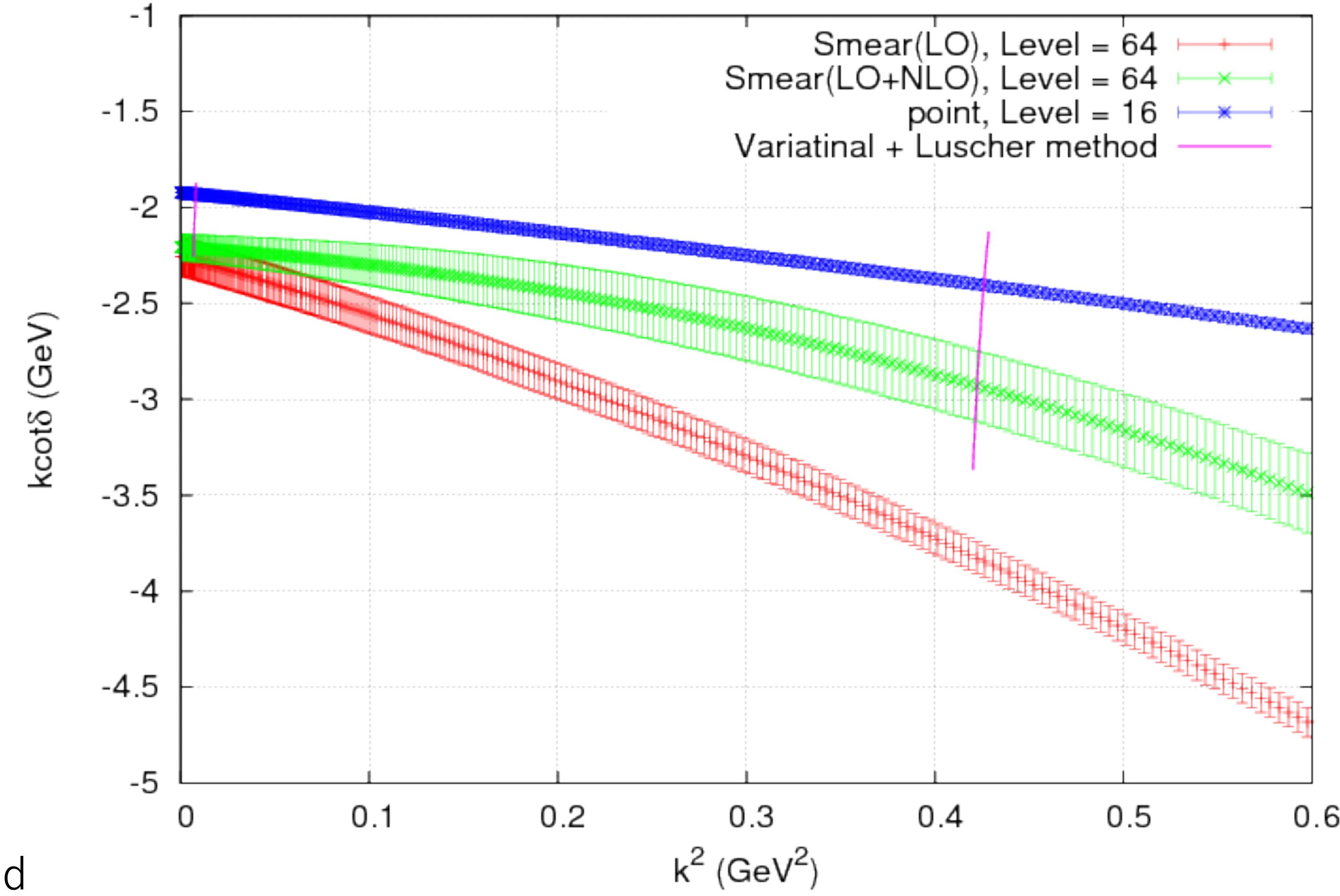
However, as energy rises, phase shift given by the LO+NLO potential deviate from the one given by the LO potential and approach to the one by point sink.



kcot δ from the LO+NLO potential

Phase shifts by point sink and LO+NLO are Consistent with the one by Lüscher method.

- Conventional HAL QCD method draws correct phase shifts.
- Potentials by smeared sink is Improved by considering higher order.



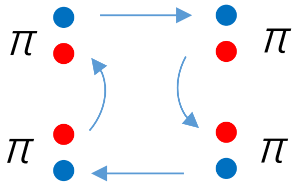
Conclusion for this part

- The potentials given by smeared sink have relatively large operator dependence.
- The deviation from point sink will be recovered by thinking higher order term.

I = 1 $\pi\ \pi$ potential

I=1 channel

Compared with I = 2 channel, I = 1 $\pi\pi$ scattering is very difficult because of so-called box-like diagrams.



⟹ We have to calculate all-to-all correlators to get NBS wave functions.

However, this channel deserves to calculate because ρ resonance is in this channel.

Question : Can the potential correctly generate ρ resonance ?

The significant by-product of getting potentials is the capability to calculate S-matrix in complex energy plane.

⟹ Direct search of pole is possible.

Numerical setup

- 2 + 1 flavor gauge configuration by CP-PACS collaboration

  [PACS-CS Collaboration: S. Aoki, K.-I. Ishikawa, N. Ishizuka, T. Izubuchi, D. Kadoh, K. Kanaya, Y.Kuramashi, Y. Namekawa, M. Okawa, Y. Taniguchi, A. Ukawa, N. Ukita, T. Yoshie Phys. Rev. D. 79 (2009) 034503]

- Wilson clover fermion and Iwasaki gauge action
- a = 0.0907 fm , $32^3 \times 64$ lattice
- m_π = 410MeV, m_ρ = 890 MeV

  ⇒ ρ meson will appear as a resonant state

- 60conf × 64 time slice
- Periodic boundary condition is used for all direction.

κ_{ud}	0.13754
κ_s	0.13640
π	0.18903(79)
	0.002
K	0.29190(67)
	0.002
η_{ss}	0.36870(71)
	0.000
ρ	0.4108(31)
	0.017
K^*	0.4665(23)
	0.007
ϕ	0.5156(21)
	0.002
N	0.5584(53)
	0.358
Λ	0.6208(36)
	0.089
Σ	0.6437(39)
	0.041
Ξ	0.6910(30)
	0.028
Δ	0.6956(66)
	0.102
Σ^*	0.7464(43)
	0.022
Ξ^*	0.7964(41)
	0.005
Ω	0.8456(37)
	0.009

The dimensionless mass spectrum from these configurations

The derivation of the potential

I = 1 channel NBS wave function has node in angular direction

➡ Naïve schrodinger equation is suffered from zero division error.

Prescription

Schrodinger eq : $(E - H_0)R(\mathbf{r}, t) = V(r)R(\mathbf{r}, t)$

We use the O_h Invariance of the potential to obtain

$$\sum_{g \in O_h} R(g\mathbf{r}, t)^\dagger (E - H_0) R(g\mathbf{r}, t) = V(r) \sum_{g \in O_h} R(g\mathbf{r}, t)^\dagger R(g\mathbf{r}, t)$$

Then, we have

$$V(r) = \frac{\sum_{g \in O_h} R(g\mathbf{r}, t)^\dagger (E - H_0) R(g\mathbf{r}, t)}{\sum_{g \in O_h} R(g\mathbf{r}, t)^\dagger R(g\mathbf{r}, t)}$$

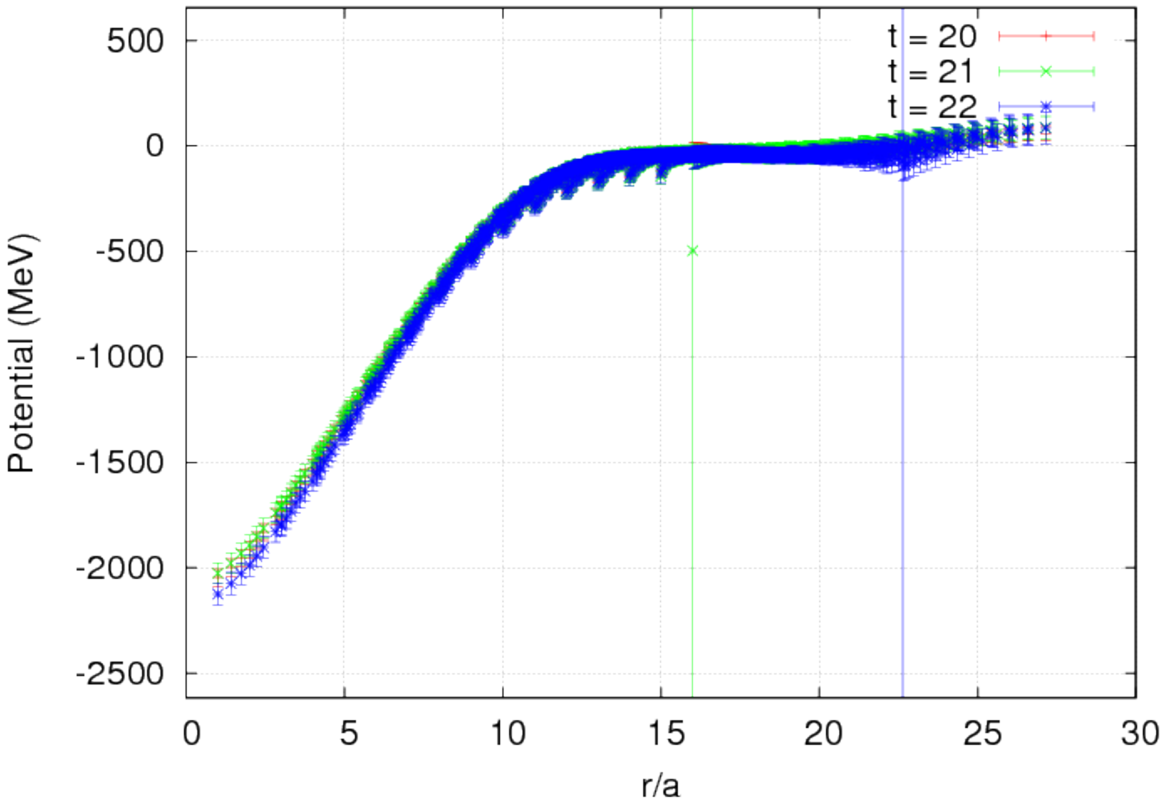
The points where Rcorrelator has small value, which is the main source of noise, has small weight in this expression. [K.Murano et al., Phys.Lett.B735(2014)19]

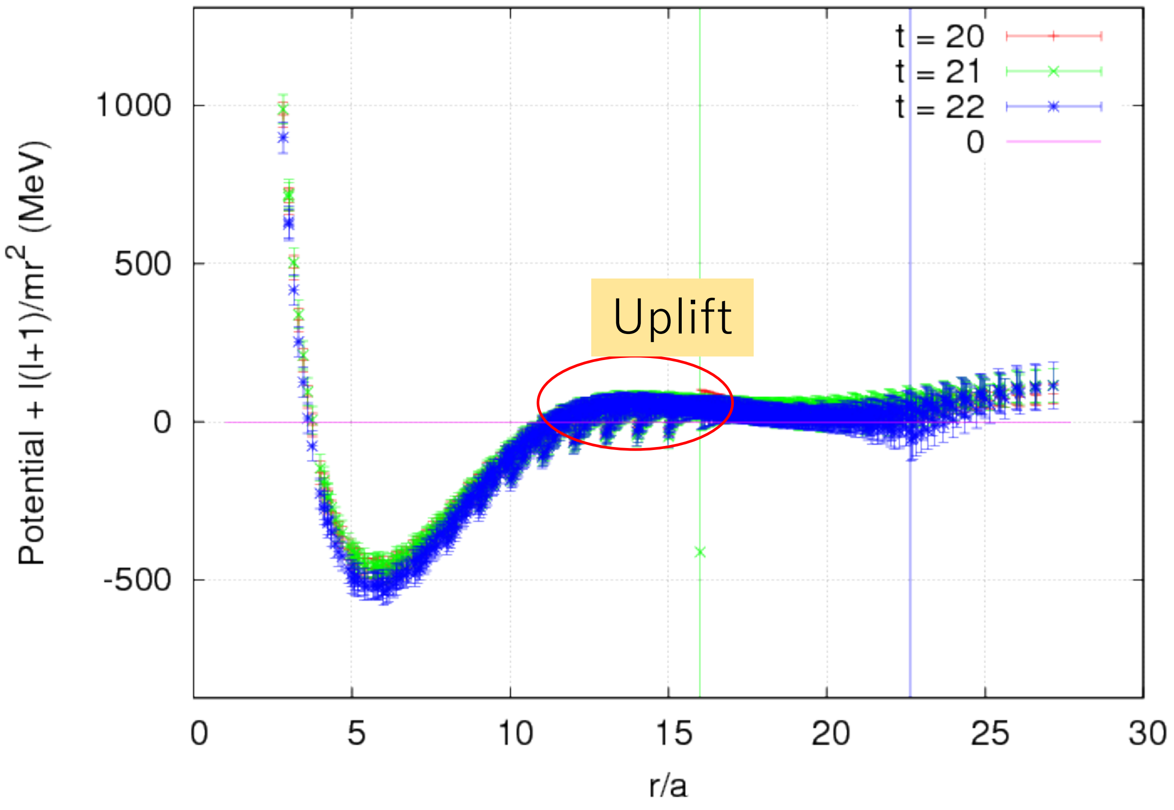
Potential

The plot of l = 1 channel potential ⟹ It has large (~2GeV) attractive behavior in short range.

The sum of the potential and centrifugal force has (shallow) uplift in medium range.

⟹ Sign of resonance appears.





Phase shift

Phase shift given by the potential crosses 90°

at about 70 MeV.

➡ Consistent with configuration data

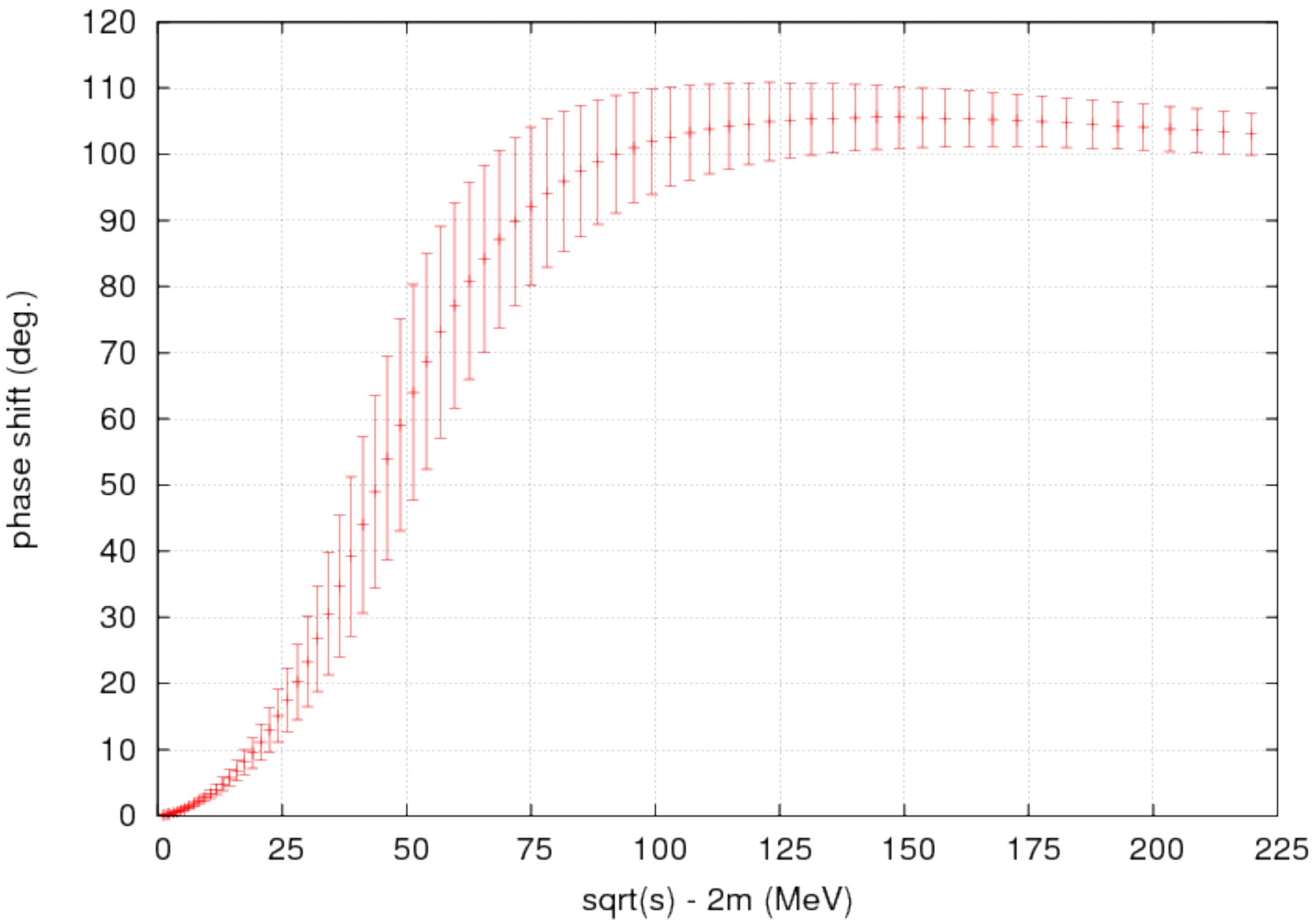
($\because\ m_\pi = 410\mathrm{MeV},\ m_\rho = 890\mathrm{MeV}$)

However, increase of phase shift stops

around 100° .

➡ Contamination from smearing

might be included.



$k^3 \cot \delta$

The phase shift is well fitted with the following effective range expansion.

$$k^3 \cot \delta = -\frac{1}{a_1} + \frac{r_1}{2} k^2 + f_1 k^4$$

where

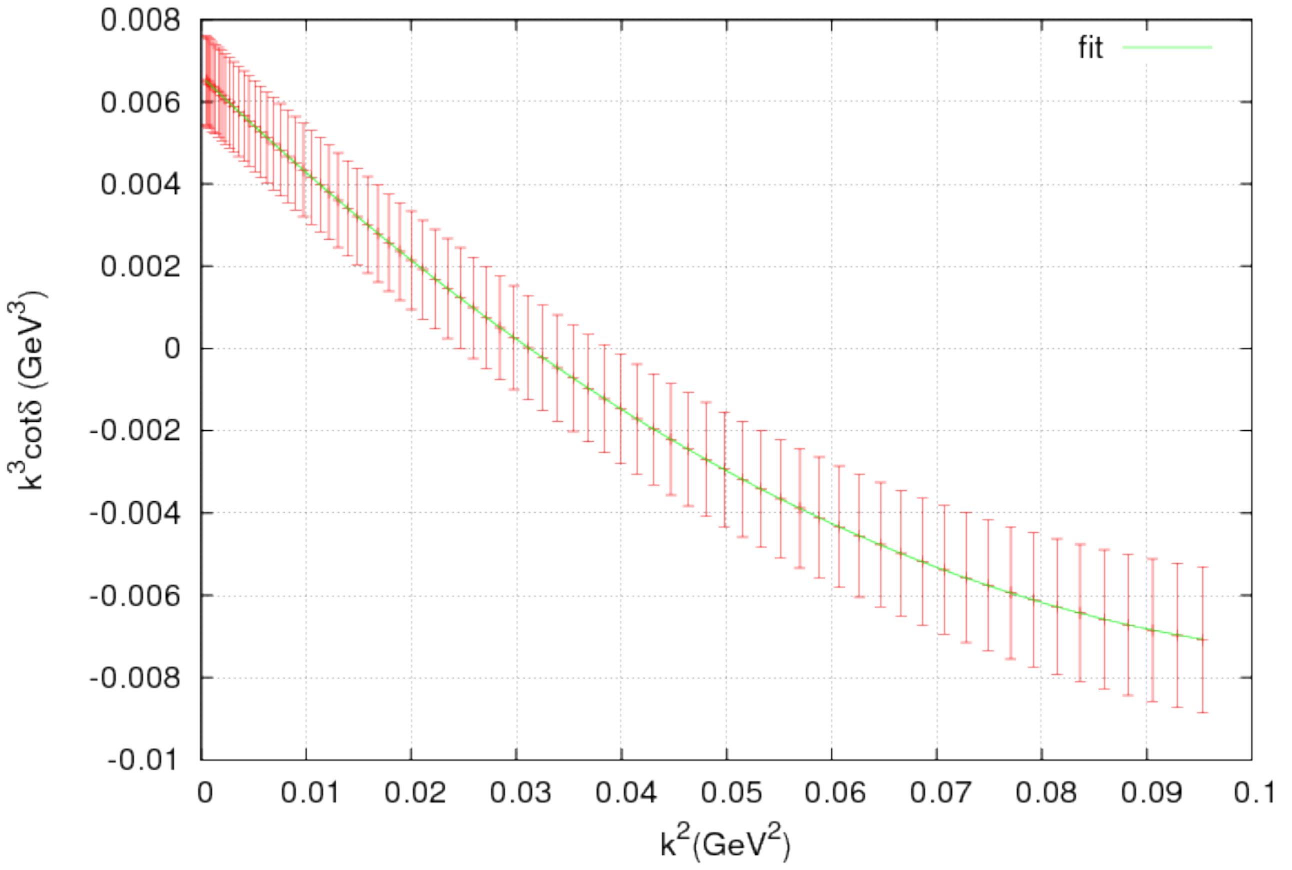
$$a_1 = (1.5101 \pm 1.2837 \times 10^{-6}) \times 10^2 \quad (\mathrm{GeV}^3)$$

$$r_1 = (-1.2250 \pm 2.9855 \times 10^{-4}) \times 10^{-1} \quad (\mathrm{GeV})$$

$$f_1 = 1.06443 \pm 7.2560 \times 10^{-4} \quad (\mathrm{GeV}^{-1})$$

In this channel, k^4 term has a sizable effect

in the effective range expansion.



Pole search of S-matrix in I = 1 $\pi\ \pi$ potential

Complex scaling method

[J.Aguilar and J.M.Combes, Commun. Math. Phys.,22('71)269.]
[E.Balslev and J.M.Combes, Commun. Math. Phys.,22('71)280.]

Rotate momenta and coordinates with $\theta \in R$ simultaneously.

$$k \to k e^{-i\theta} \qquad r \to r e^{i\theta}$$

In this rotated coordinates, the NBS wave function follows

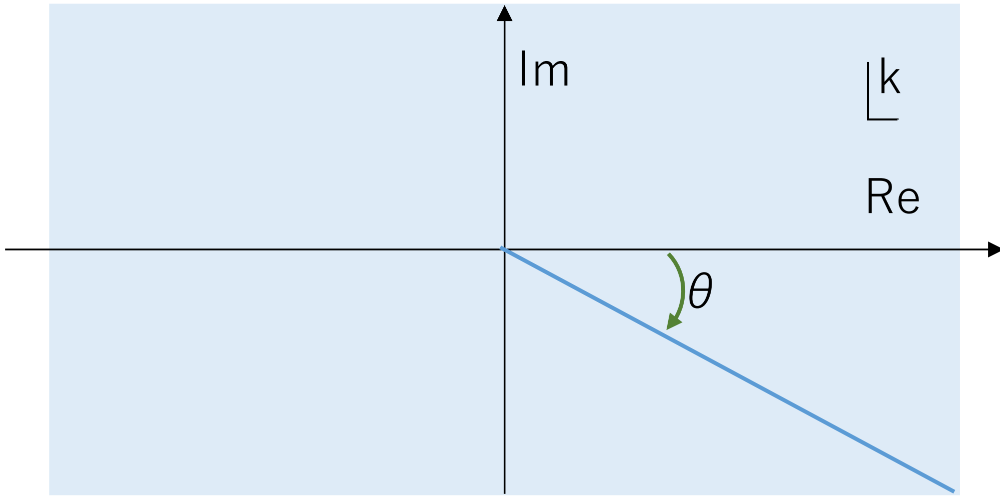
$$\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} - e^{2i\theta} V(e^{i\theta} r) + k^2 \right) \phi(r) = 0$$

At long distance, the solution is approximated by

$$\phi(r) \to \frac{i}{2} \left[\mathcal{J}_l(k e^{-i\theta}) h_l^-(kr) - \mathcal{J}_l^*(k e^{-i\theta}) h_l^+(kr) \right]$$

From this relation, S-matrix in the complex plane is given by

$$\mathcal{S}_l(k e^{-i\theta}) = \frac{\mathcal{J}_l^*(k e^{-i\theta})}{\mathcal{J}_l(k e^{-i\theta})}$$



S-matrix in complex energy plane

Pole-like behavior seems to appear in the second Riemann sheet.

Blue points : average value

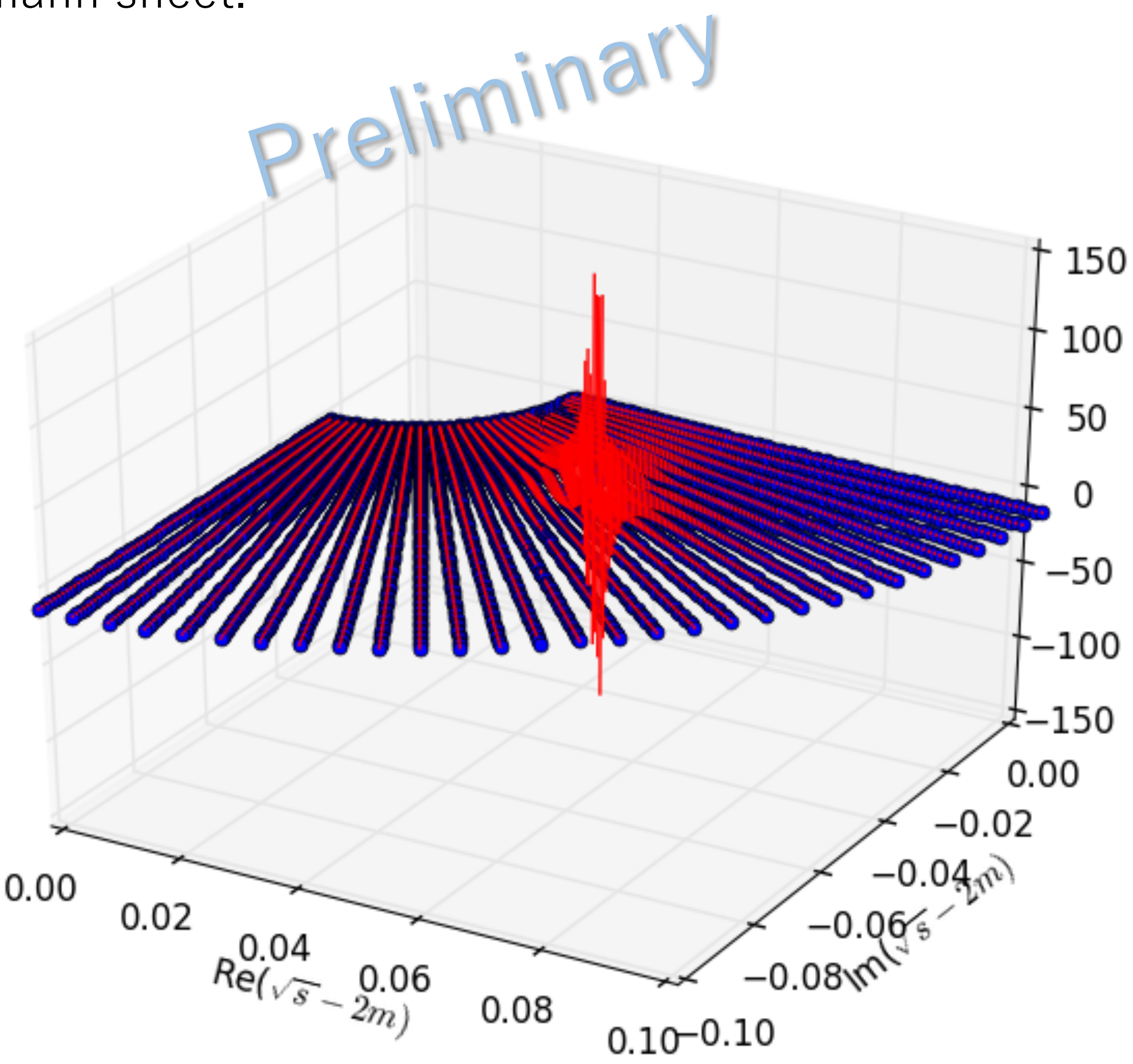
Red lines : statistical error

→ Large statistical error comes from singular behavior around the pole.

The pole like spike is approximately located at

$$\mathrm{Re}[\sqrt{s} - 2m] = 50 \pm 10 \ \mathrm{MeV}$$

$$\mathrm{Im}[\sqrt{s} - 2m] = -30 \pm 10 \ \mathrm{MeV}$$



Summary and discussion

Operator dependence

- The potential with smeared sink have large operator dependence.

  ➡ The height of repulsive core drastically chances.

- Even with the dependence, phase shift can be correctly measured by considering higher order terms.

  ➡ We saw phase shifts in high energy are improved by considering the NLO term.

I=1 $\pi\pi$ scattering

- The potential have the sign of ρ resonance.

  ➡ Peak point is consistent with configuration data.
  However, some improvement might be necessary to get correct behavior in higher energy.

- Complex scaling method will be useful to search for the resonance.