QCD Kondo effect

- Perturbative to Non-perturbative -

Sho Ozaki (Keio Univ.)

Strangeness and charm in hadron and dense mater, YITP, 15-26 May, 2017

Contents

I) Introduction

II) QCD Kondo effect from perturbative RG

K. Hattori, K. Itakura, S. O. and S. Yasui, PRD92 (2015) 065003

S. O., K. Itakura and Y. Kuramoto, PRD94 (2016) 074013

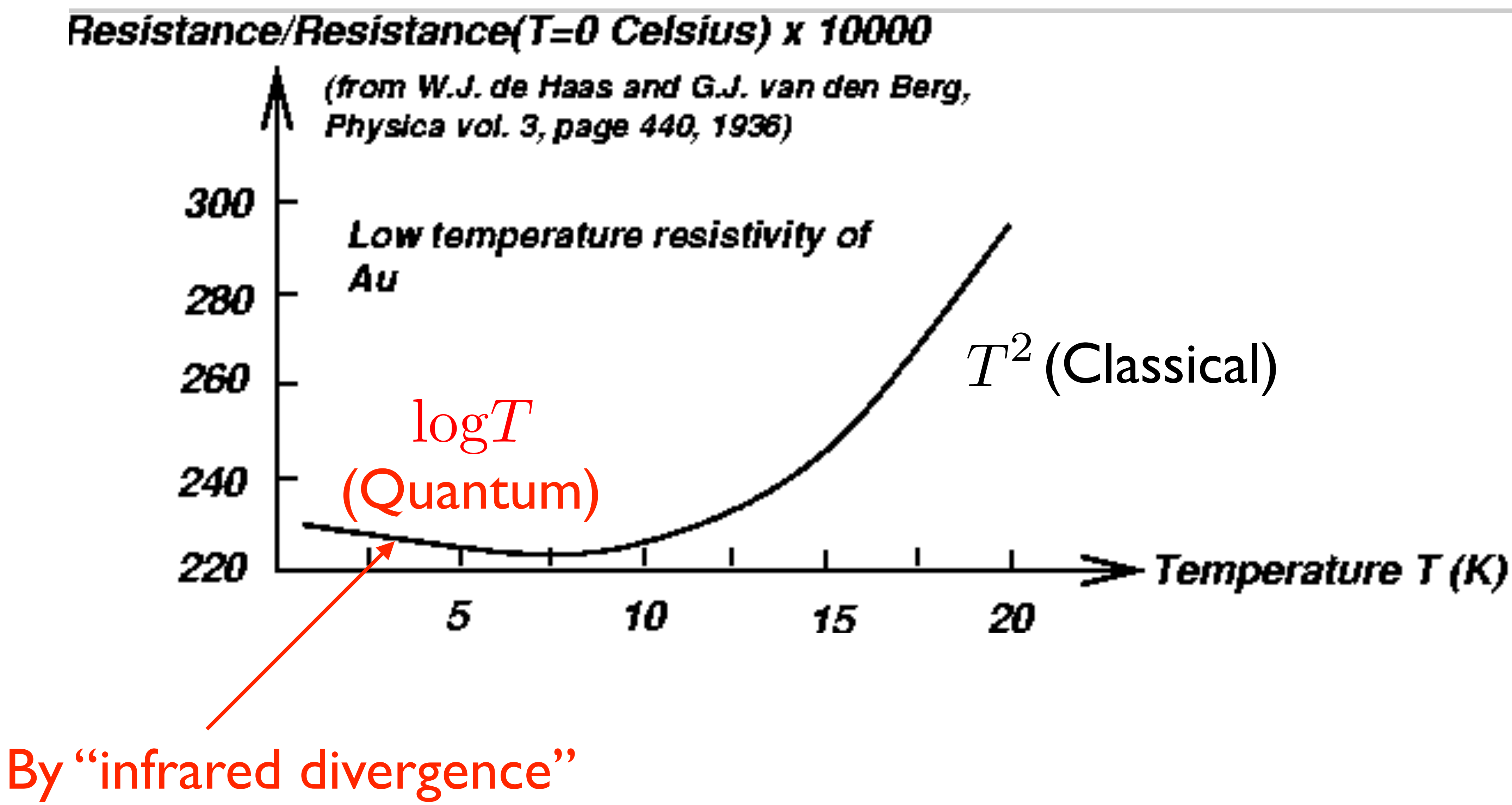
III) QCD Kondo effect from CFT

T. Kimura and S.O., arXiv: 1611.07284

T. Kimura and S. O., in preparation

IV) Summary

Kondo effect



Kondo effect is firstly observed in experiment as an enhancement of electrical resistivity of impure metals.

Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

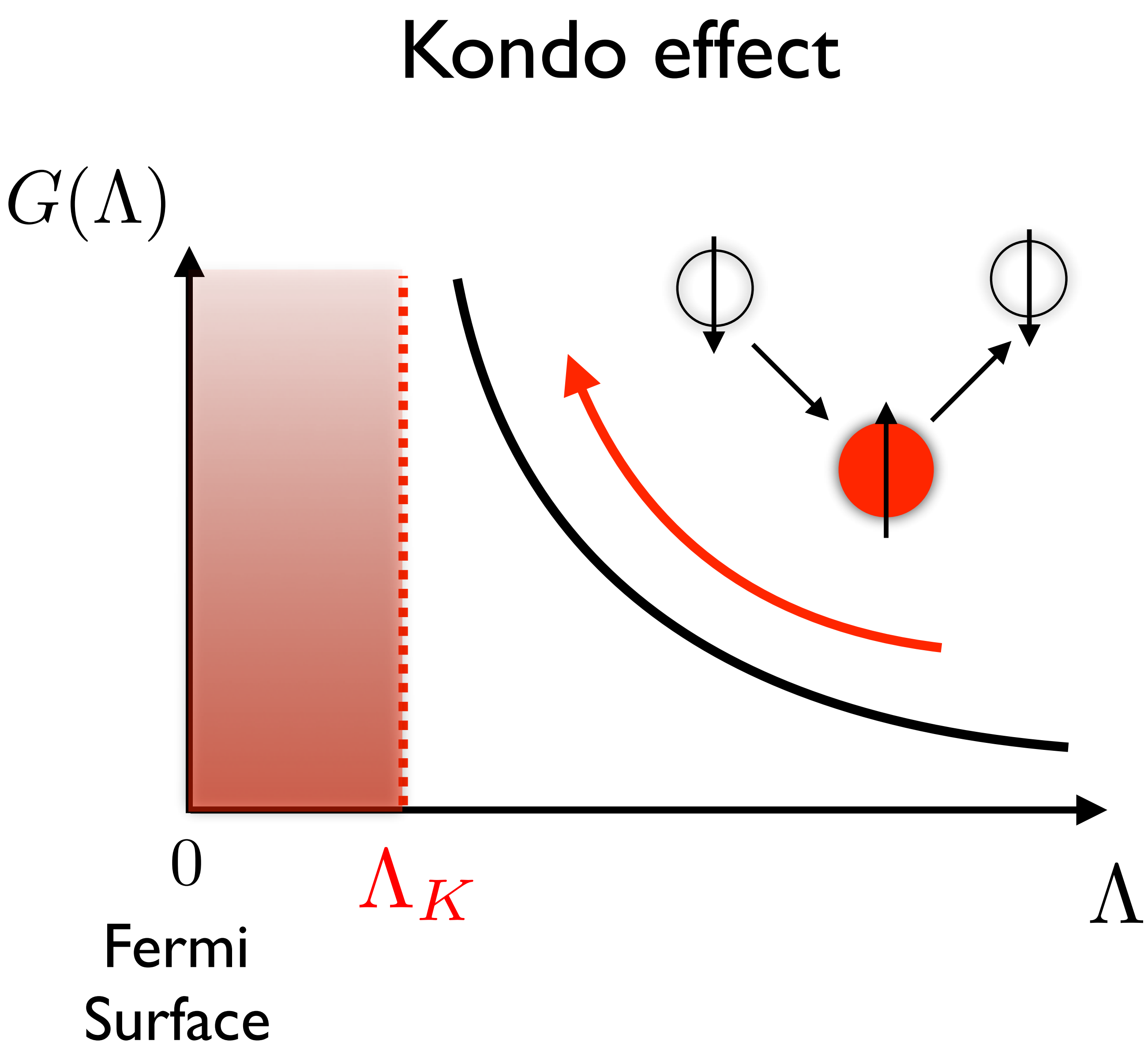
Resistance Minimum in Dilute Magnetic Alloys

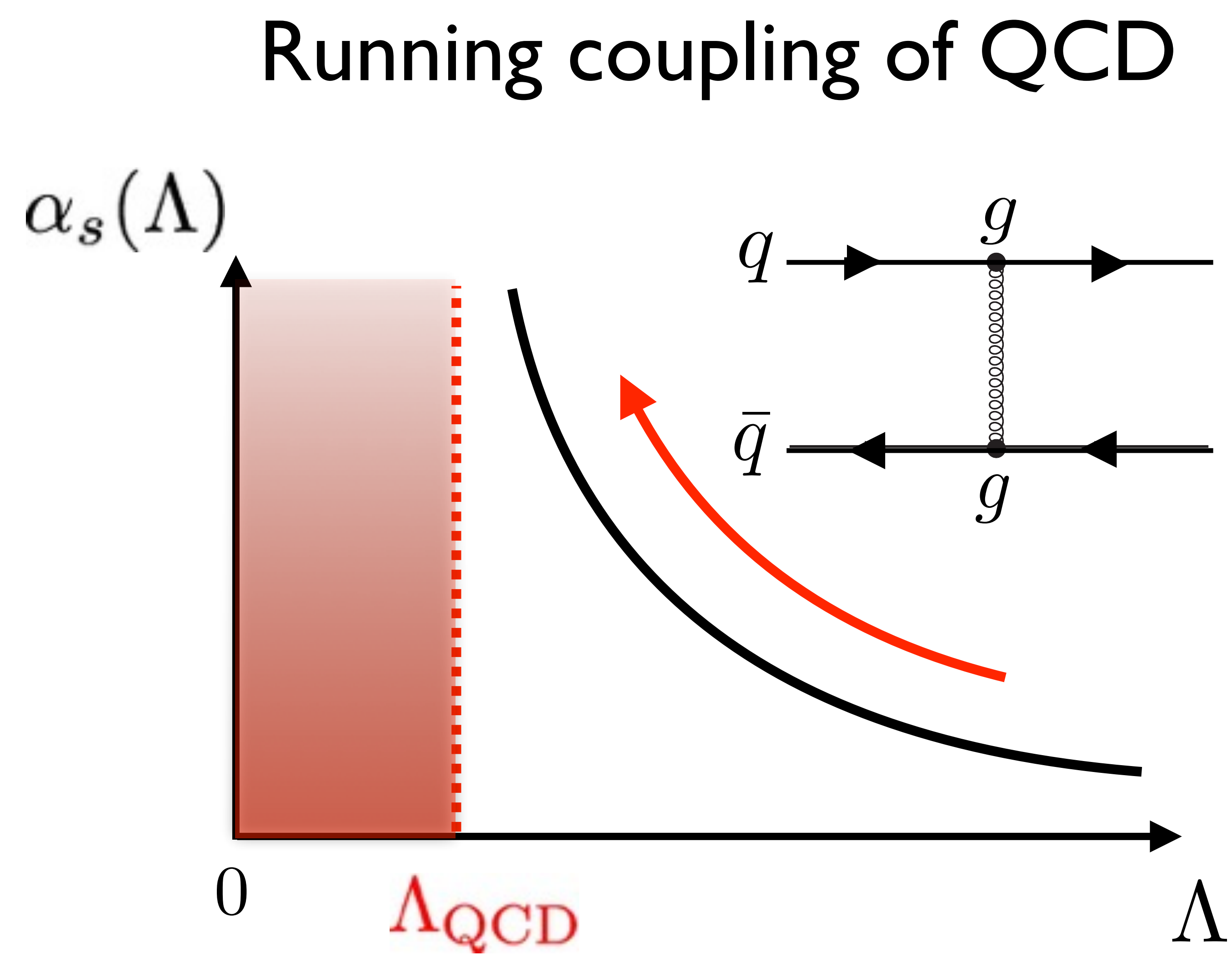
Jun KONDO

Jun Kondo
(1930-)

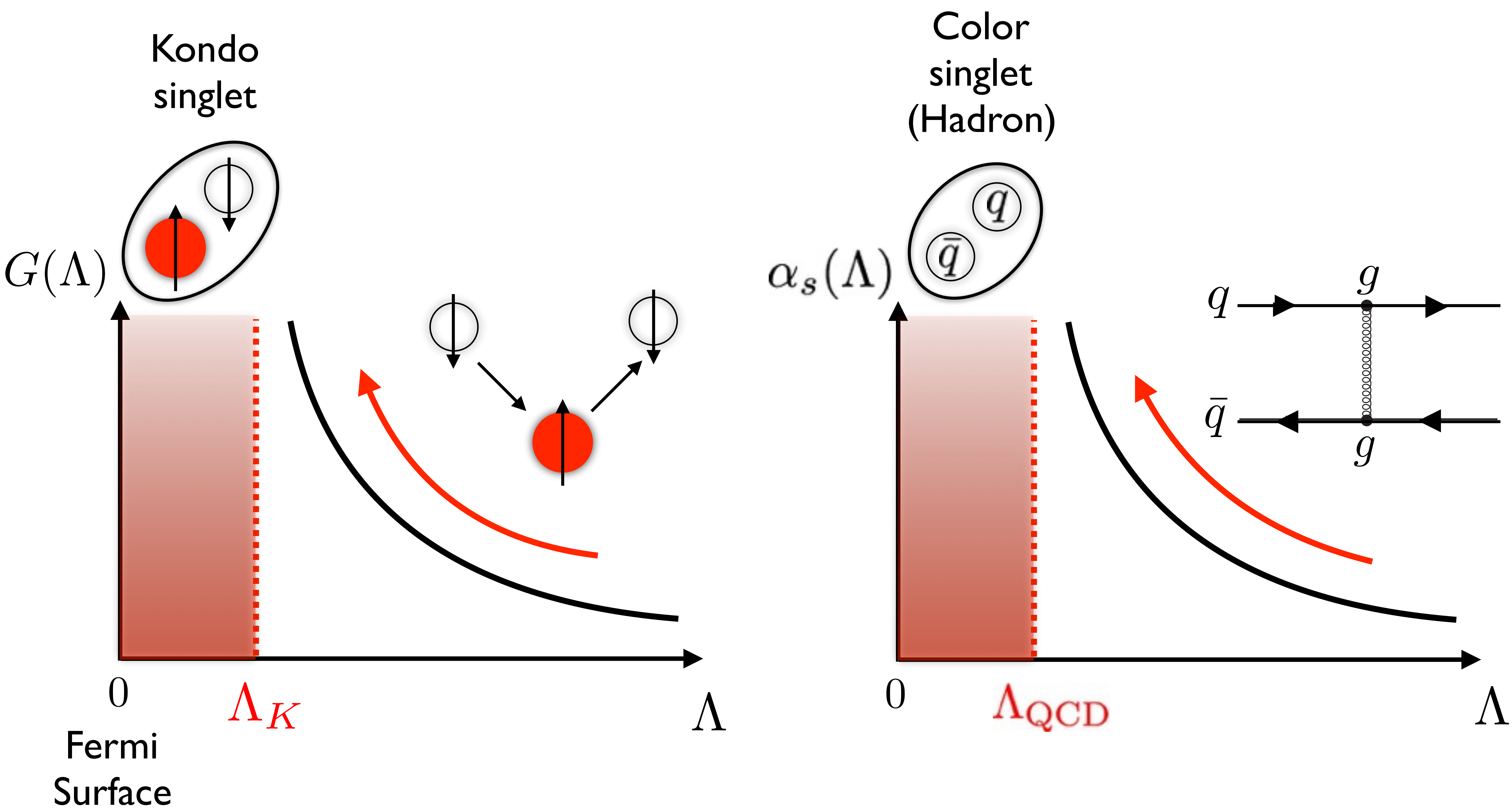
J. Kondo has explained the phenomenon based on the second order perturbation of interaction between conduction electron and impurity.

Asymptotic freedom in Kondo effect and QCD

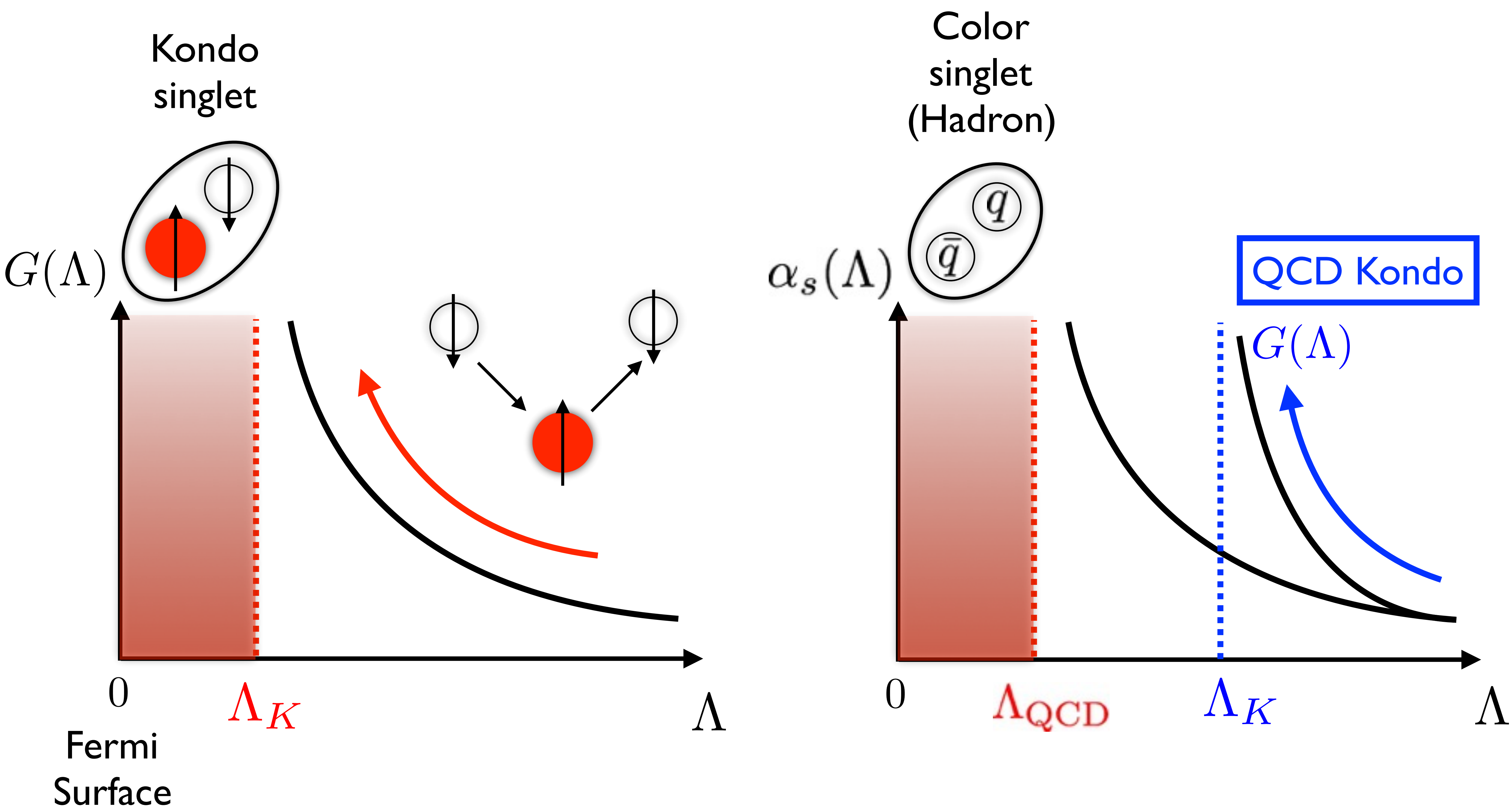




Asymptotic freedom in Kondo effect and QCD



Asymptotic freedom in Kondo effect and QCD



Conditions for the appearance of Kondo effect

0) Heavy impurity

i) Fermi surface

ii) Quantum fluctuation (loop effect)

iii) Non-Abelian property of interaction
(spin-flip int.)

Conditions for the appearance of QCD Kondo effect

0) Heavy quark impurity

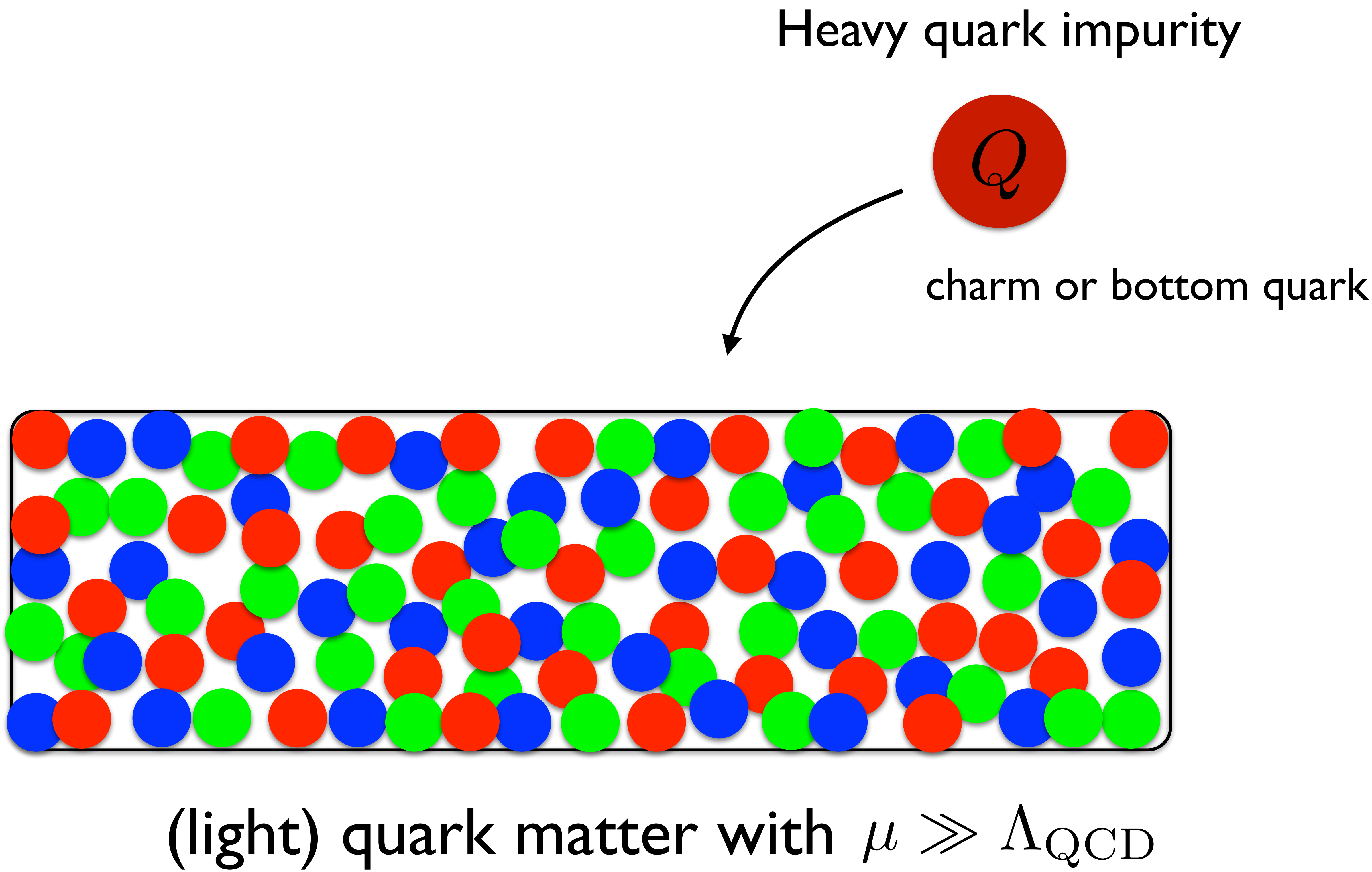
i) Fermi surface of light quarks

ii) Quantum fluctuation (loop effect)

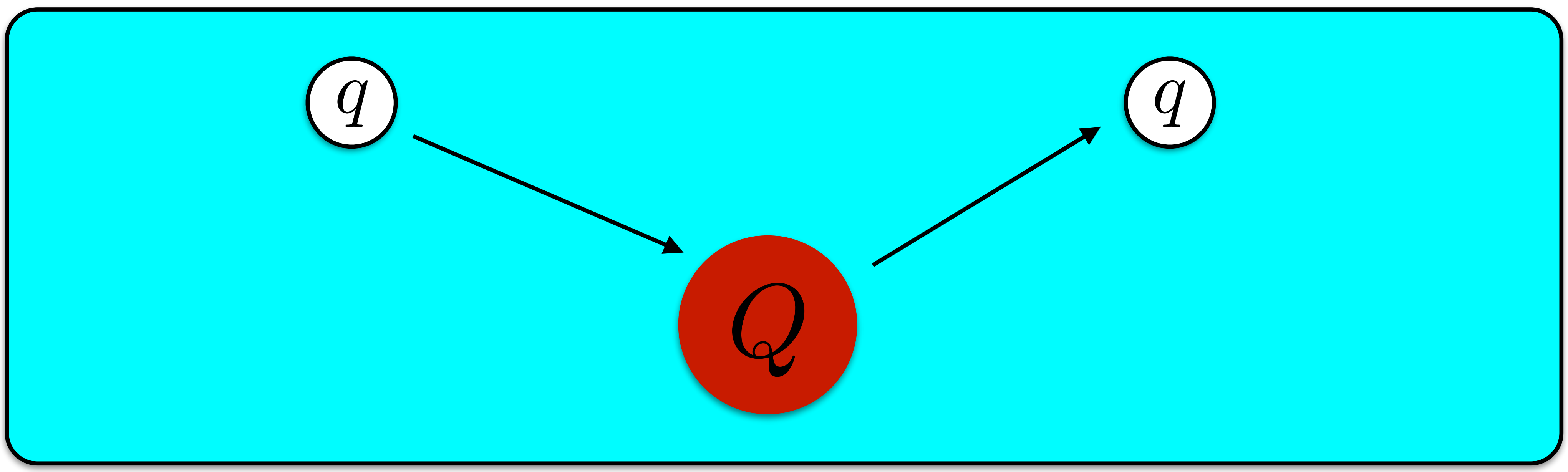
iii) Color exchange interaction in QCD

QCD Kondo effect

K. Hattori, K. Itakura, S. O. and S. Yasui, PRD92 (2015) 065003

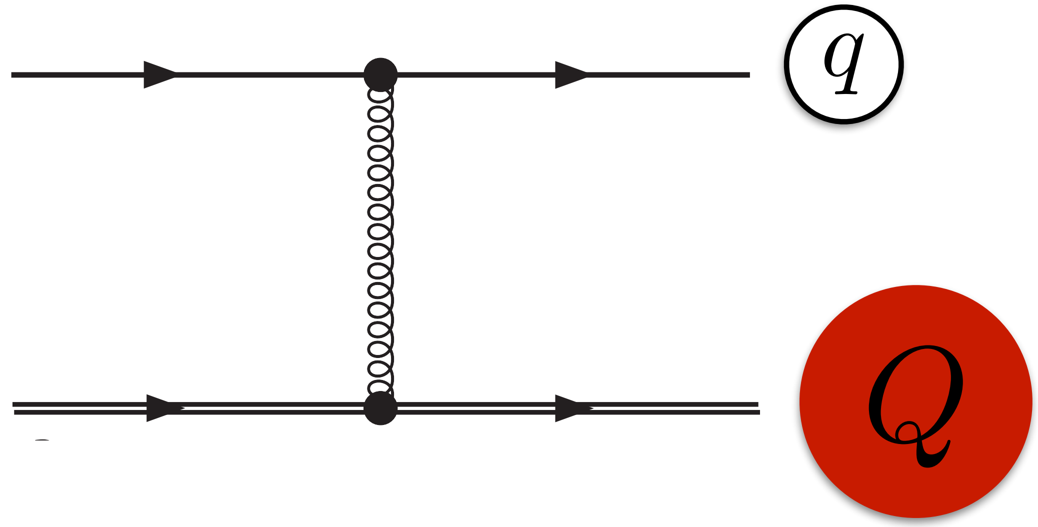


Heavy quark impurity
Q
charm or bottom quark
(light) quark matter with $\mu \gg \Lambda_{\mathrm{QCD}}$

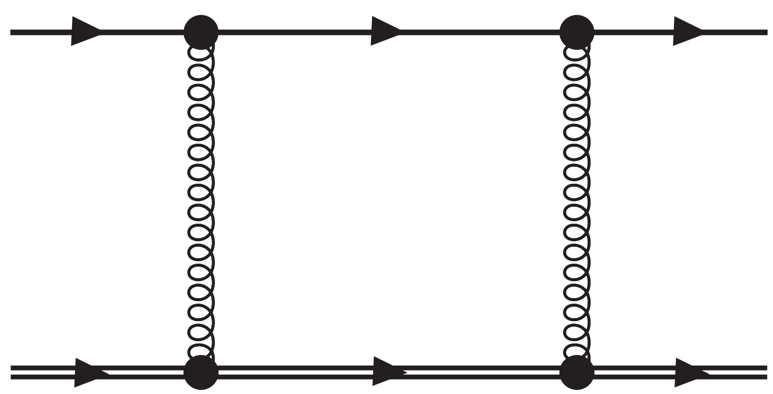


(light) quark matter with $\mu \gg \Lambda_{\mathrm{QCD}}$

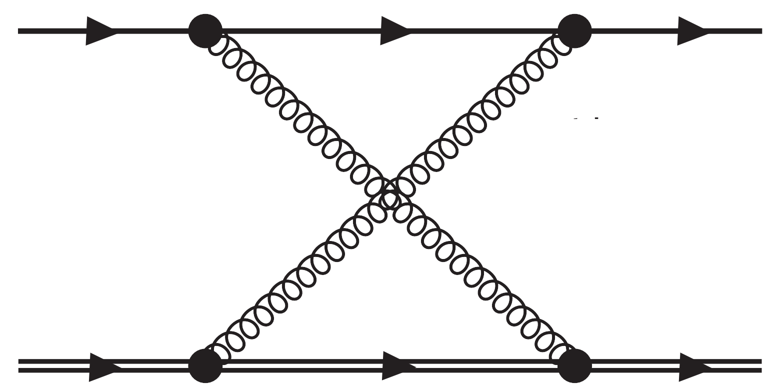
$-i\mathcal{M} =$



$+$



$+$



Heavy quark: $M_Q \to \text{large}$

Quark propagator at finite density (massless quark)

particle — hole

$$iS(q;\mu) = \frac{i\not{q}}{2\epsilon_q}\left(\underbrace{\theta(|\vec{q}| - k_F)\frac{1}{q^0 - \epsilon_q^+ + i\varepsilon}}_{\text{particle}} + \underbrace{\theta(k_F - |\vec{q}|)\frac{1}{q^0 - \epsilon_q^+ - i\varepsilon}}_{\text{hole}}\right.$$

$$\left.\underbrace{-\frac{1}{q^0 - \epsilon_q^- - i\varepsilon}}_{\text{anti-particle}}\right)$$

anti-particle

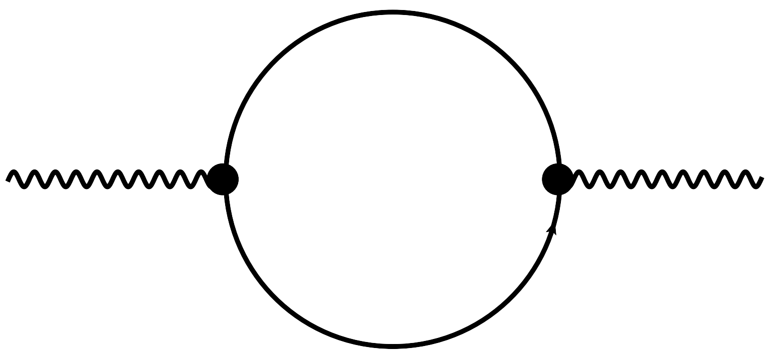
$$\epsilon_q = |\vec{q}|$$

$$\epsilon_q^\pm = \pm|\vec{q}| - \mu$$

Gluon propagator at finite density

- Screening effect

Vacuum polarization



$$= GP_T^{\mu\nu} + FP_L^{\mu\nu}$$

$$P_T^{00} = P_T^{0i} = 0$$

$$P_T^{ij} = \delta^{ij} - q^i q^j / |\vec{q}|^2$$

$$P_L^{\mu\nu} = q^\mu q^\nu / q^2 - g^{\mu\nu} - P_T^{\mu\nu}$$

Screening masses

$$G(q) = i\frac{\pi q^0}{2|\vec{q}|} m_D^2 \,, \qquad F(q) = m_D^2 = \frac{g^2 \mu^2}{2\pi^2}$$

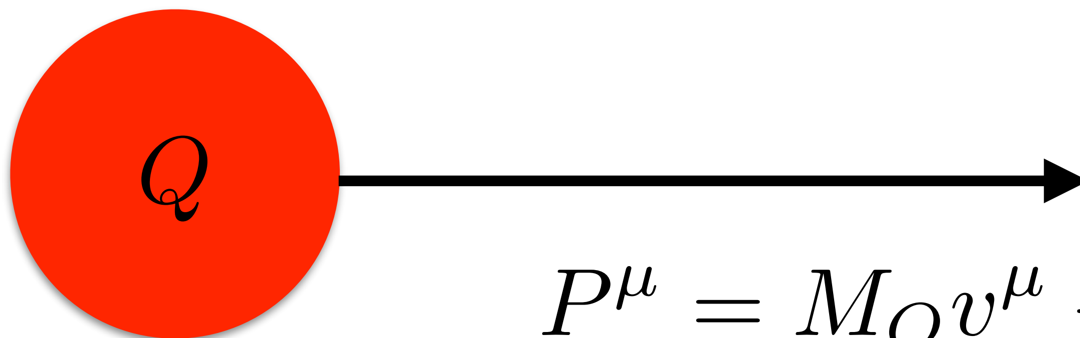
- Gluon propagator

$$i\mathcal{D}^{00} = \frac{i}{q^2 - m_D^2}$$

$$i\mathcal{D}^{ij} = \frac{i(\delta^{ij} - \hat{q}^i \hat{q}^j)}{(q^0)^2 - |\vec{q}|^2 - i\frac{\pi}{2} m_D^2 |q^0|/|\vec{q}|}$$

Heavy quark propagator and vertex

Heavy quark: $M_Q \to \mathrm{large}$

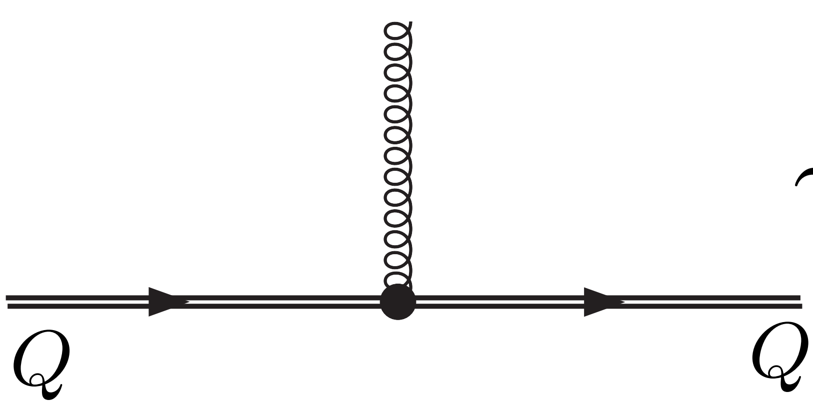


$$P^\mu = \underbrace{M_Q v^\mu}_{\text{on-shell}} + \underbrace{k^\mu}_{\text{off-shell}} \qquad v^\mu = (\sqrt{1+|\vec{v}|^2}, \vec{v})$$

- Heavy quark propagator

$$i\frac{\not{P} + M_Q}{P^2 - M_Q^2} \to i\frac{1}{v\cdot k}\frac{1+\not{v}}{2}$$

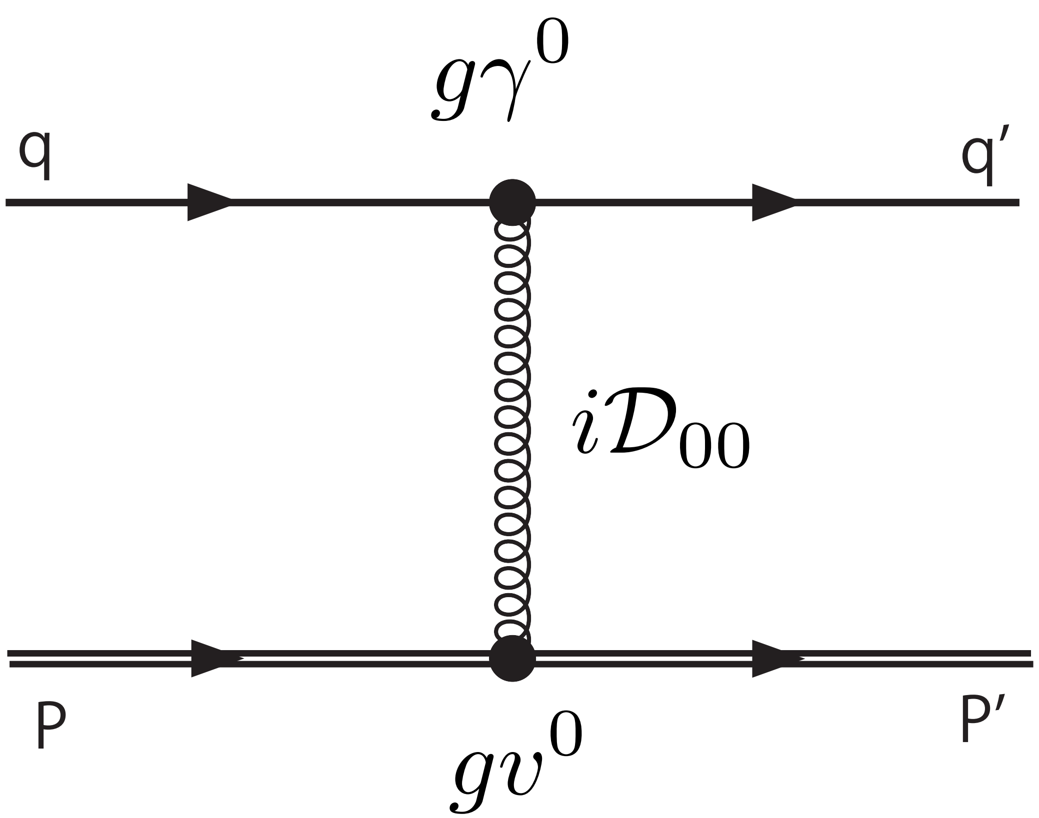
- Vertex



$$\gamma^\mu \to \frac{1+\not{v}}{2}\gamma^\mu\frac{1+\not{v}}{2} = v^\mu\frac{1+\not{v}}{2} \to \underbrace{v^\mu}_{\text{Spin-indep.}}$$

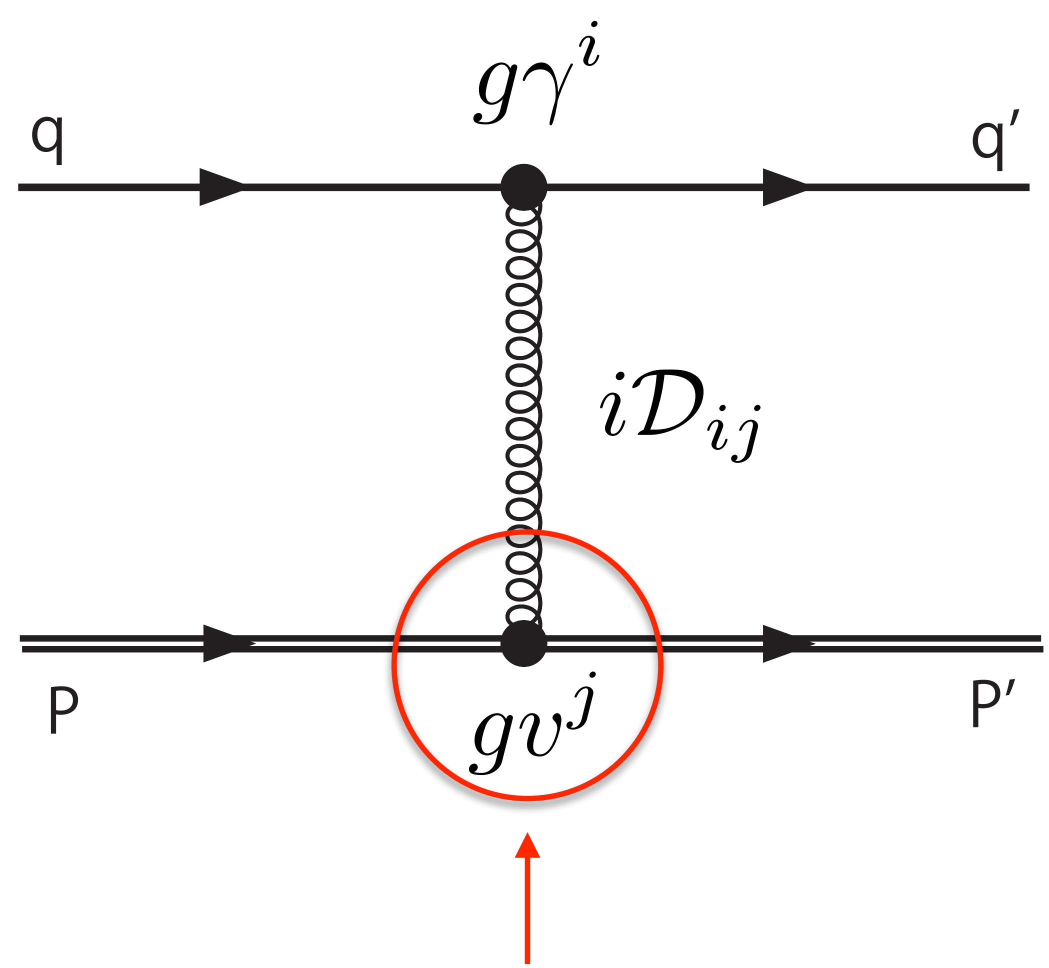
Gluon exchange interactions

Color electric interaction



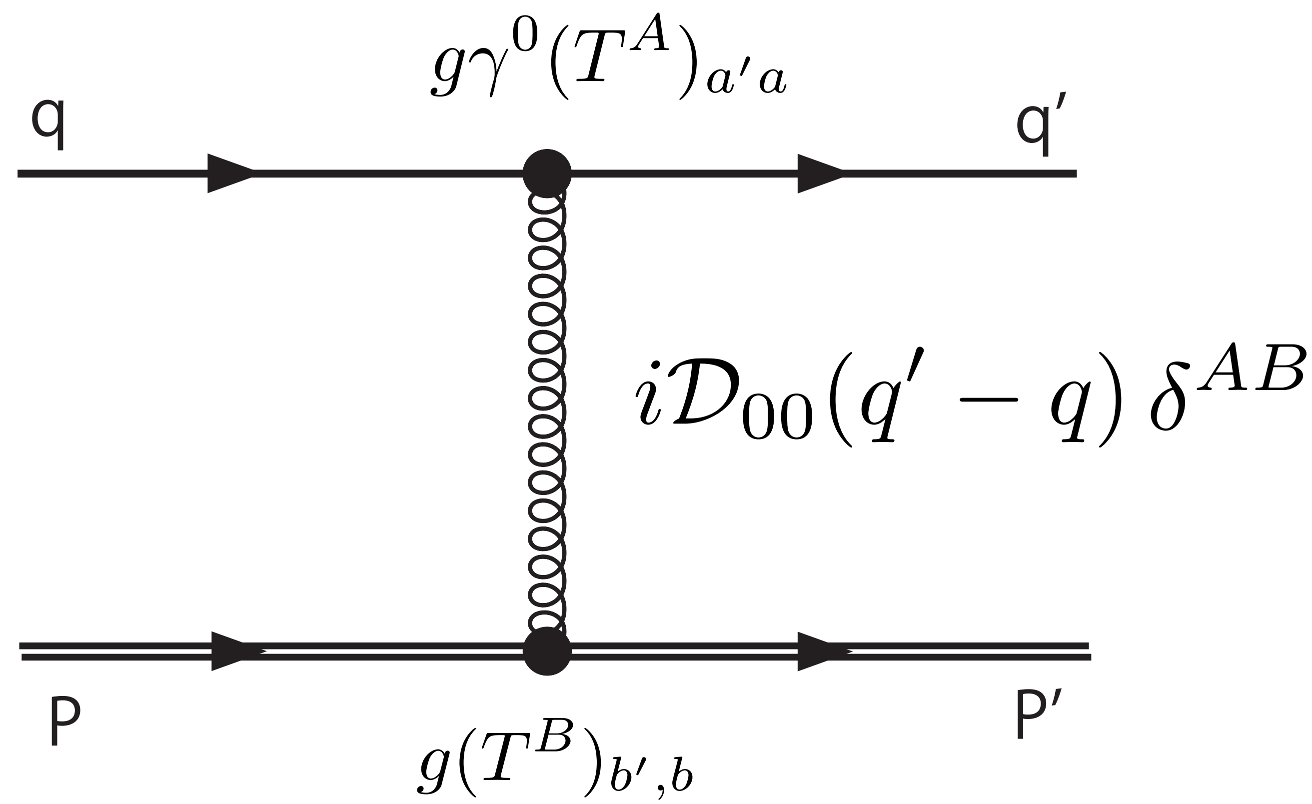
Dominant contribution

Color magnetic interaction



Suppressed by $1/M_Q$

Tree amplitude



$$-i\mathcal{M}_{Born} = -ig^2\mathcal{D}_{00}(q'-q)(T^A)_{a'a}(T^A)_{b'b}\gamma^0 \otimes \frac{1+\gamma^0}{2}$$

S-wave projection (partial wave decomposition)

$$-i\mathcal{M}_{Born}^{S-\text{wave}} = \frac{1}{2}\int_{-1}^{1} d(\cos\theta) P_{l=0}(\cos\theta)\,(-i\mathcal{M}_{Born})$$

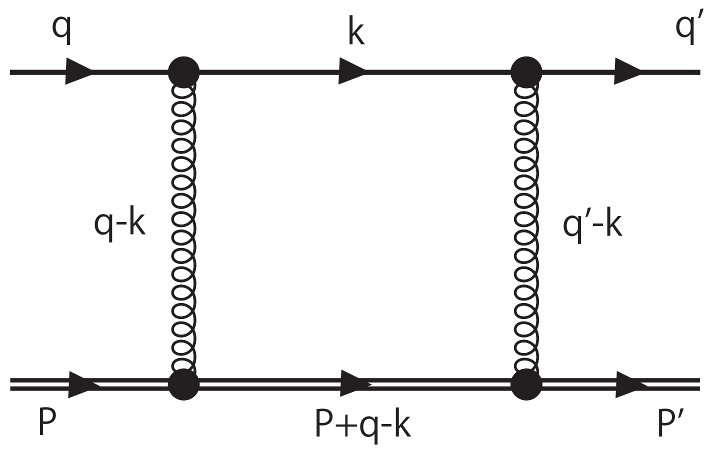
S-wave projected gluon exchange int.

$$\begin{aligned} G &\equiv \frac{1}{2}\int_{-1}^{1} d(\cos\theta) P_{l=0}(\cos\theta) g^2 i\mathcal{D}_{00}(q'-q) \\ &= \frac{1}{2}\int_{-1}^{1} d(\cos\theta) P_{l=0}(\cos\theta) \frac{-g^2}{(q'-q)^2 - m_D^2} \\ &= \frac{g^2}{4\mu^2}\log\frac{4\mu^2}{m_D^2} \end{aligned}$$

$$\underline{-i\mathcal{M}_{Born}^{S-\text{wave}} = -iG(T^A)_{a'a}(T^A)_{b'b}}$$

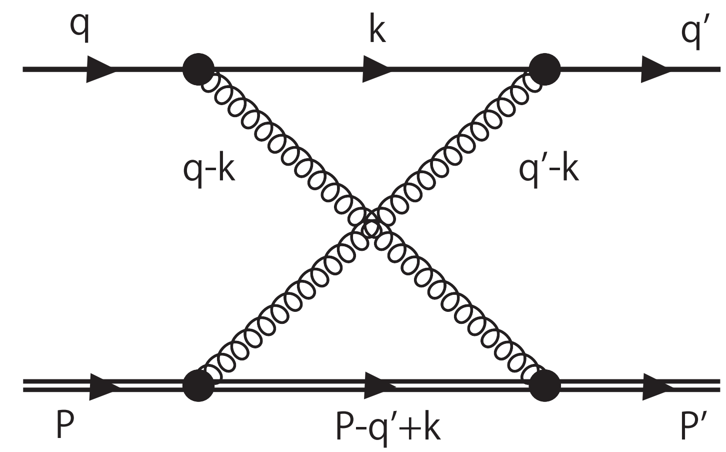
1-loop amplitudes (S-wave projected)

(a) particle



$$i\,G^2\rho_F\mathcal{T}^{(a)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE$$

(b) hole

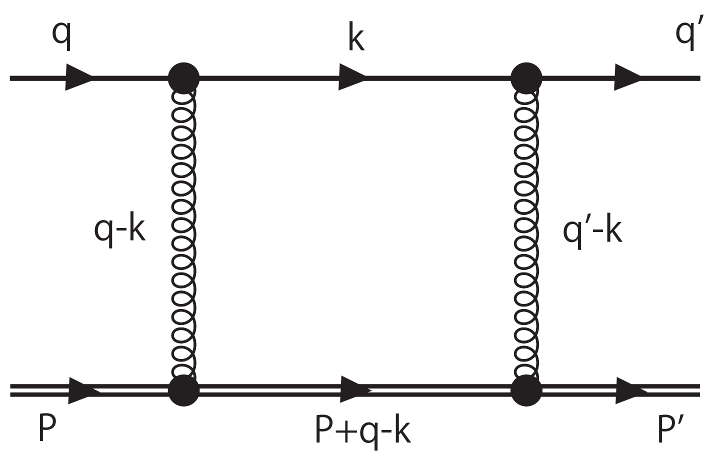


$$-i\,G^2\rho_F\mathcal{T}^{(b)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE$$

$\rho_F = \dfrac{k_F^2}{(2\pi)^2}$: density of state on Fermi surface

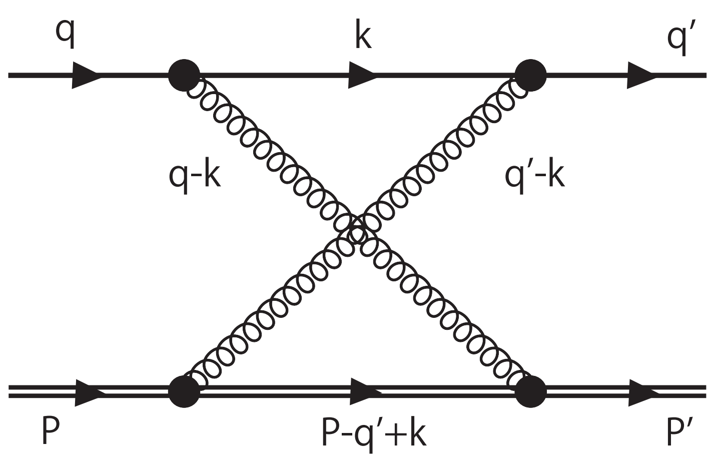
1-loop amplitudes (S-wave projected)

(a) particle



$$i\,G^2\rho_F\mathcal{T}^{(a)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE$$

(b) hole



$$-i\,G^2\rho_F\mathcal{T}^{(b)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE$$

Color factors (Non-abelian property of the QCD interaction)

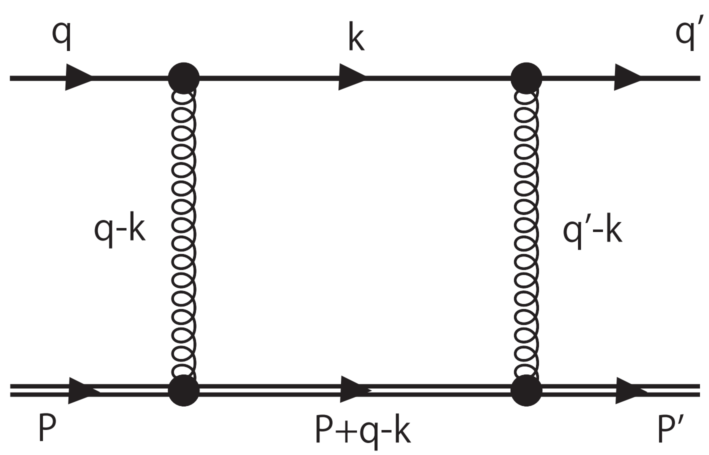
$$\mathcal{T}^{(a)}_{a'a;b'b}=(T^A)_{a'a''}(T^B)_{a''a}(T^A)_{b'b''}(T^B)_{b''b}=\frac{N_c^2-1}{4N_c^2}\delta_{a'a}\delta_{b'b}-\frac{1}{N_c}(T^A)_{a'a}(T^A)_{b'b}$$

$$\mathcal{T}^{(b)}_{a'a;b'b}=(T^A)_{a'a''}(T^B)_{a''a}(T^B)_{b'b''}(T^A)_{b''b}=\frac{N_c^2-1}{4N_c^2}\delta_{a'a}\delta_{b'b}-\left(\frac{1}{N_c}-\underline{\frac{N_c}{2}}\right)(T^A)_{a'a}(T^A)_{b'b}$$

$$\rho_F=\frac{k_F^2}{(2\pi)^2}$$: density of state on Fermi surface

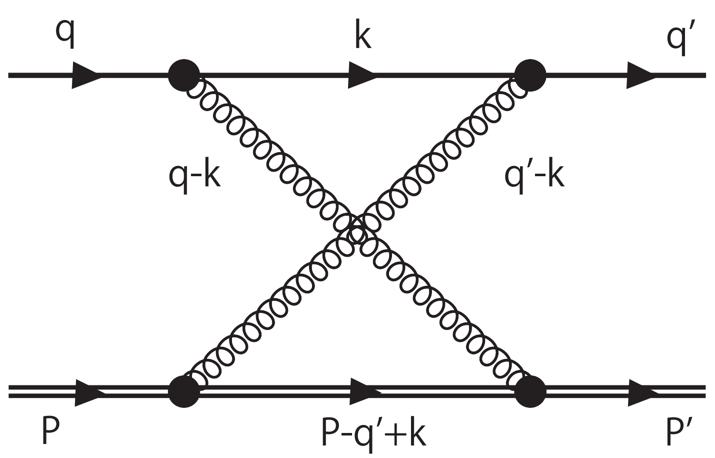
1-loop amplitudes (S-wave projected)

(a) particle



$$i\,G^2\rho_F\mathcal{T}^{(a)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE$$

(b) hole



$$-i\,G^2\rho_F\mathcal{T}^{(b)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE$$

Color factors (Non-abelian property of the QCD interaction)

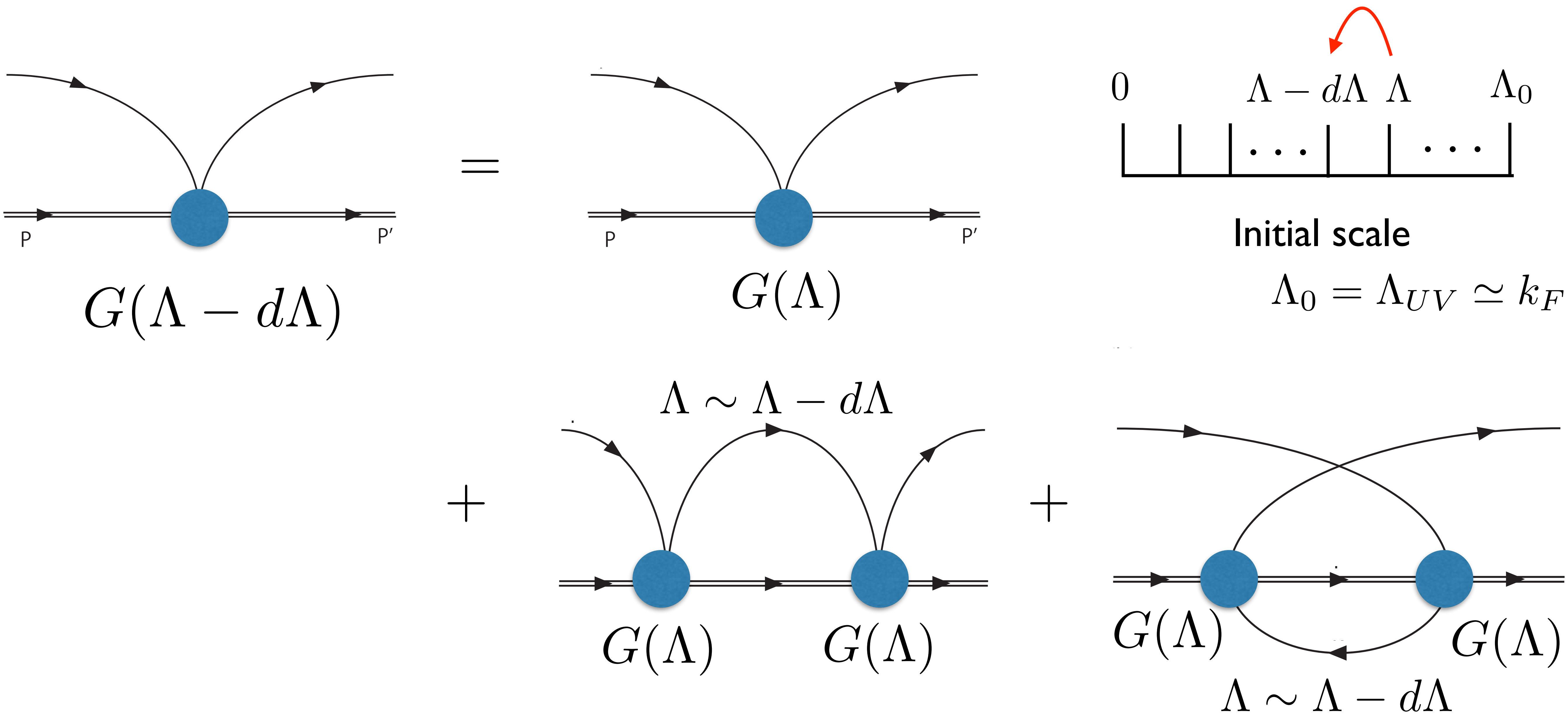
$$\mathcal{T}^{(a)}_{a'a;b'b}=(T^A)_{a'a''}(T^B)_{a''a}(T^A)_{b'b''}(T^B)_{b''b}=\frac{N_c^2-1}{4N_c^2}\delta_{a'a}\delta_{b'b}-\frac{1}{N_c}(T^A)_{a'a}(T^A)_{b'b}$$

$$\mathcal{T}^{(b)}_{a'a;b'b}=(T^A)_{a'a''}(T^B)_{a''a}(T^B)_{b'b''}(T^A)_{b''b}=\frac{N_c^2-1}{4N_c^2}\delta_{a'a}\delta_{b'b}-\left(\frac{1}{N_c}-\frac{N_c}{2}\right)(T^A)_{a'a}(T^A)_{b'b}$$

$$\longrightarrow\ -i\frac{N_c}{2}G^2\rho_F\log\frac{\Lambda_{UV}}{\Lambda}(T^A)_{a'a}(T^A)_{b'b},\qquad \rho_F=\frac{k_F^2}{(2\pi)^2}$$

: density of state on Fermi surface

Renormalization group equation of scattering amplitude
~poor man's scaling~



Renormalization group equation of scattering amplitude

$$\Lambda \frac{dG(\Lambda)}{d\Lambda} = -\frac{N_c}{2}\rho_F G^2(\Lambda)$$

Solution

$$\longrightarrow \quad G(\Lambda) = \frac{G(\Lambda_0)}{1 + \frac{N_c}{2}\rho_F G(\Lambda_0)\log(\Lambda/\Lambda_0)}$$

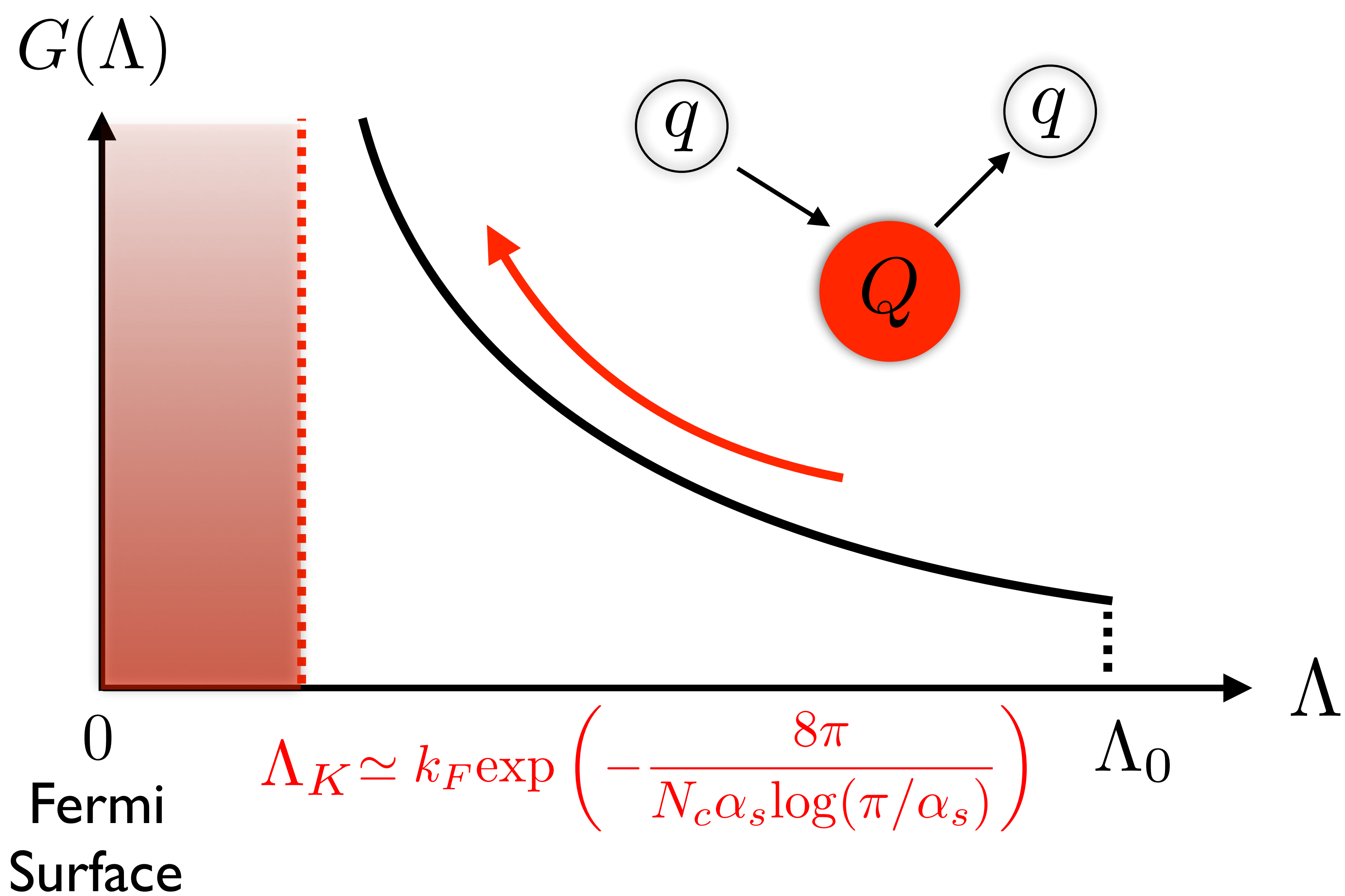
Initial scale

$$\Lambda_0 = \Lambda_{UV} \simeq k_F$$

Kondo scale (from the Landau pole)

$$\Lambda_K \simeq k_F \exp\left(-\frac{8\pi}{N_c \alpha_s \log(\pi/\alpha_s)}\right)$$

QCD Kondo effect

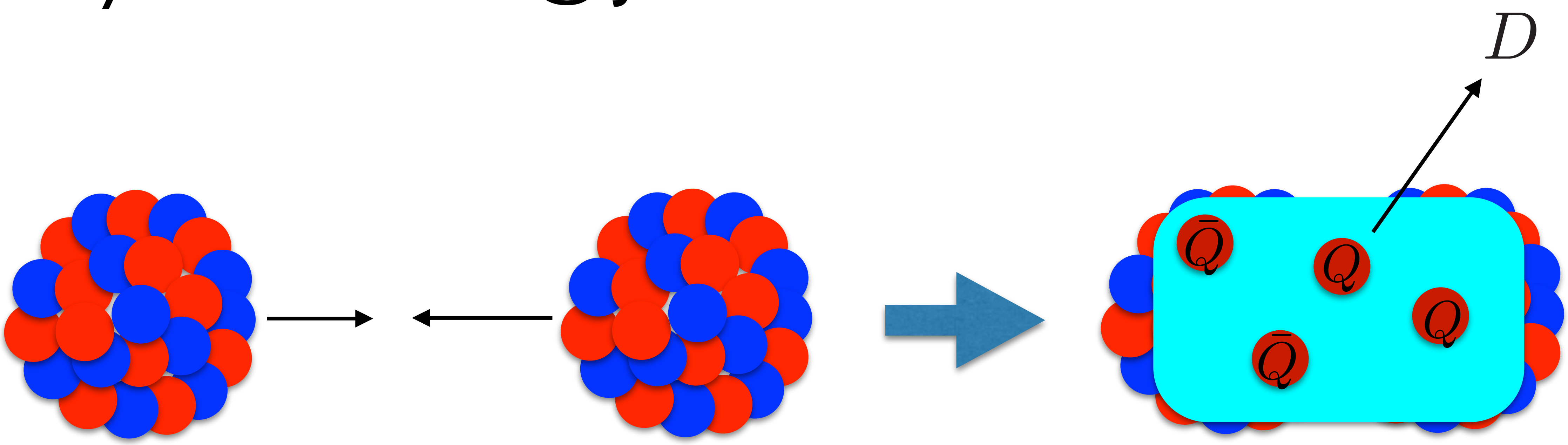


- The strength of the q-Q interaction increases as the energy scale decreases, and the system becomes non-perturbative one below the Kondo scale.

- This indicates a change of mobility of light quarks.

⟶ Several transport coefficients will be largely affected by QCD Konde effect.

Heavy ion collisions @ J-PARC, GSI-FAIR

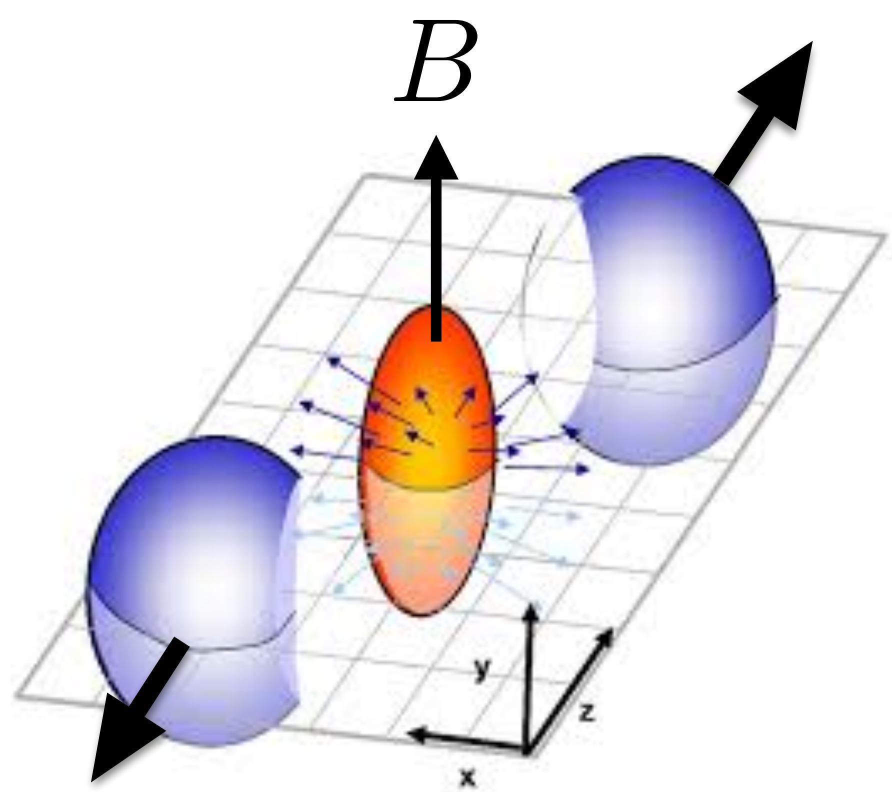


QCD Kondo effect would affect properties of QGP at high density, and also production rate of D mesons.

Magnetically induced QCD Kondo effect

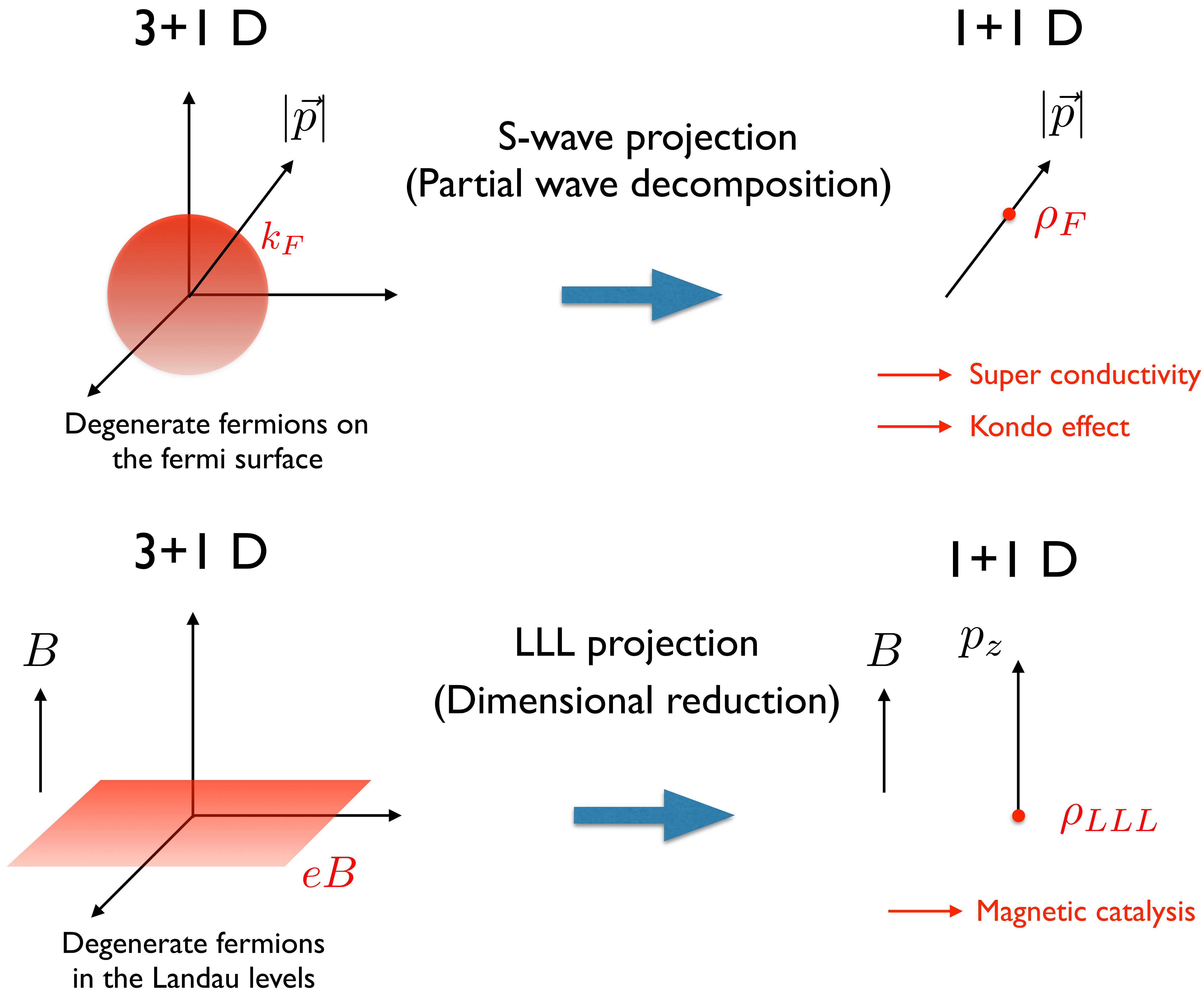
S. O., K. Itakura and Y. Kuramoto, PRD94 (2016) 074013

Non-central heavy ion collisions @ RHIC, LHC



$$|eB| \gtrsim \Lambda_{\mathrm{QCD}}^2$$

- Extremely strong magnetic fields are generated in high energy heavy ion collisions.
- Charm quarks are also produced.



3+1 D
1+1 D
$|\vec{p}|$
S-wave projection
(Partial wave decomposition)
$|\vec{p}|$
k_F
ρ_F
Super conductivity
Kondo effect
Degenerate fermions on
the fermi surface
3+1 D
1+1 D
LLL projection
(Dimensional reduction)
B
B
p_z
ρ_{LLL}
eB
Magnetic catalysis
Degenerate fermions
in the Landau levels

Conditions for the appearance of QCD Kondo effect

0) Heavy quark impurity

i) Fermi surface of light quarks

ii) Quantum fluctuation (loop effect)

iii) Color exchange interaction in QCD

Conditions for the appearance of

"Magnetically induced QCD Kondo effect"

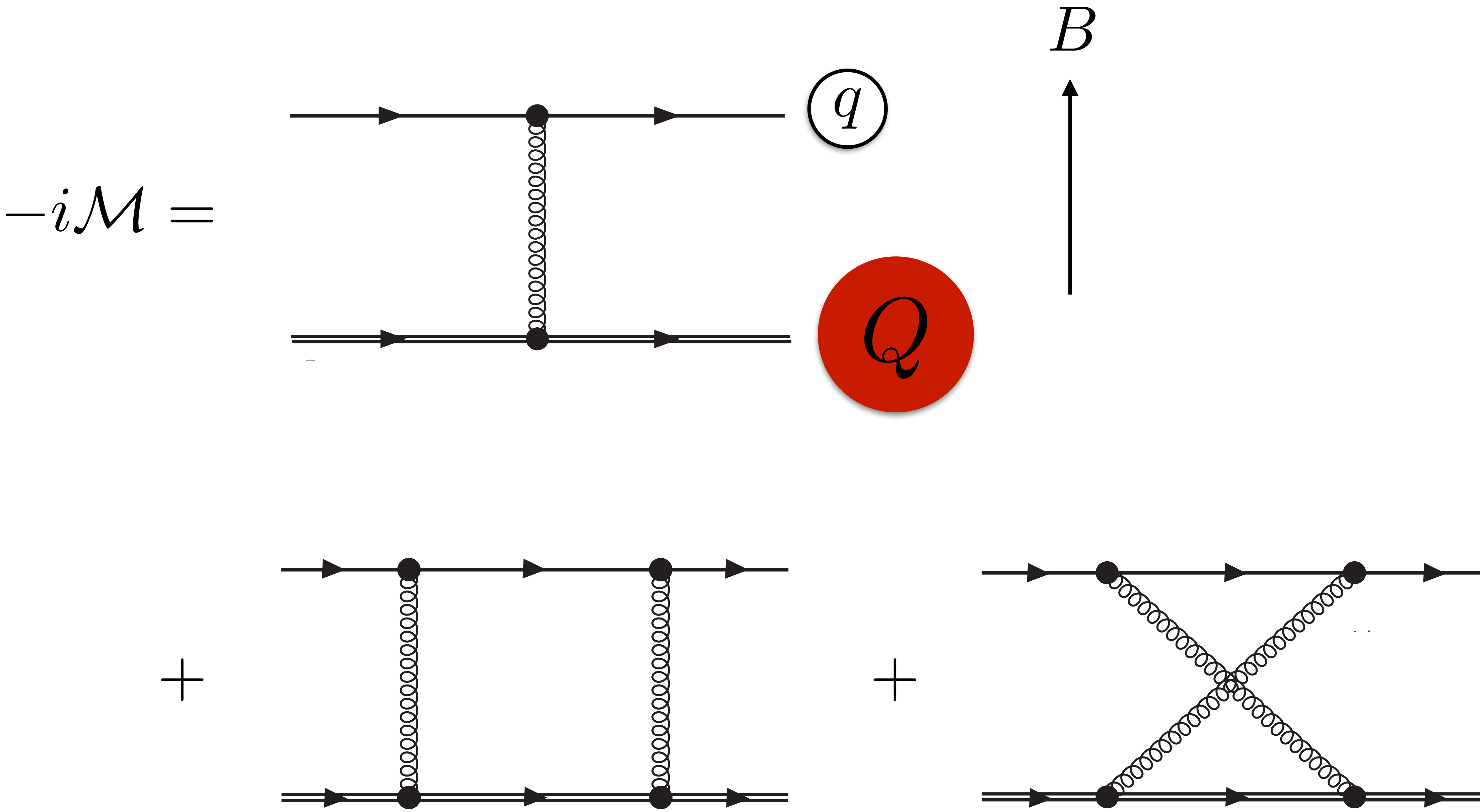
0) Heavy quark impurity

i) Strong magnetic field

ii) Quantum fluctuation (loop effect)

iii) Color exchange interaction in QCD

The magnetic field does not affect color degrees of freedom.


$-i\mathcal{M} =$
q
Q
B
$+$
$+$

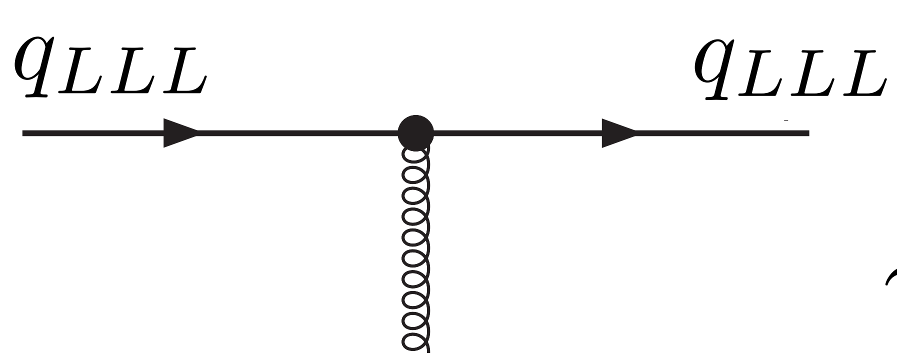
Quark propagator of the Lowest Landau Level (LLL)

$$iS_{LLL}(k;\mu|e_qB) = e^{-\frac{k_\perp^2}{e_qB}}\frac{i}{\epsilon_k}\left\{\frac{\theta(k^3-k_F)}{k^0-\epsilon_k^+ + i\varepsilon} + \frac{\theta(k_F-k^3)\theta(k^3)}{k^0-\epsilon_k^+ - i\varepsilon} - \frac{\theta(-k^3)}{k^0-\epsilon_k^- - i\varepsilon}\right\}\left(k^0\gamma^0 - k^3\gamma^3\right)\mathcal{P}_0$$

with spin projection operator

$$\mathcal{P}_0 = \frac{1+i\gamma^1\gamma^2}{2}$$

Vertex

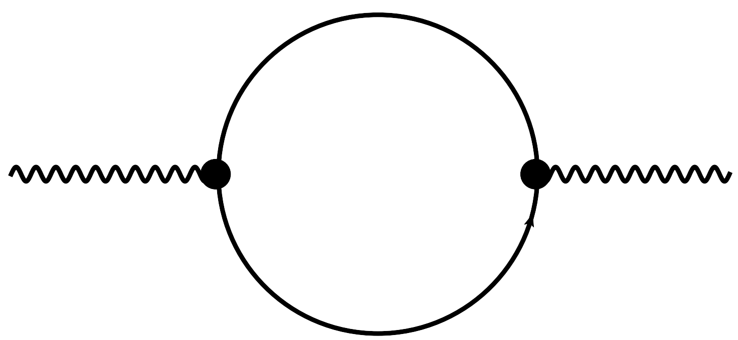


$$\gamma^\mu \to \mathcal{P}_0\gamma^\mu\mathcal{P}_0 = \mathcal{P}_0\gamma^{\bar{\mu}}\mathcal{P}_0\,, \qquad \bar{\mu} = 0,3$$

Gluon propagator in strong magnetic fields

- Magnetic screening effect

Vacuum polarization



$$= (p_{\parallel}^2 g_{\mu\nu}^{\parallel} - p_{\mu}^{\parallel} p_{\nu}^{\parallel}) \Pi(p_{\perp}^2, p_{\parallel}^2)$$

with

$$\Pi(p_{\perp}^2, p_{\parallel}^2) = -\exp\left(-\frac{p_{\perp}^2}{2e_q B}\right) \frac{m_g^2}{p_{\parallel}^2}$$

Gluon mass

$$m_g^2 = \frac{\alpha_s}{\pi} e_q B$$

- Gluon propagator V. P. Gusynin, V. A. Miransky and I. A. Shovkovy, NPB 563 (1999)

$$i\mathcal{D}_{\mu\nu}^{AB}(p) = -i\left(\frac{g_{\mu\nu}^{\parallel}}{p^2 + p_{\parallel}^2 \Pi(p_{\perp}^2, p_{\parallel}^2)} + \frac{g_{\mu\nu}^{\perp}}{p^2} - \frac{p_{\mu}^{\perp} p_{\nu}^{\perp} + p_{\mu}^{\perp} p_{\nu}^{\parallel} + p_{\mu}^{\parallel} p_{\nu}^{\perp}}{p^4}\right) \delta^{AB}$$

The 1+1 dimensional gluon exchange int.

$$G \equiv \frac{1}{e_q B \pi} \int d^2(q'_\perp - q_\perp) e^{-(q'_\perp - q_\perp)^2/e_q B} \left[(ig)^2 i\mathcal{D}(q' - q; e_q B)\right]$$

$$\simeq \frac{g^2}{e_q B \pi} \int d^2(q'_\perp - q_\perp) \frac{e^{-(q'_\perp - q_\perp)^2/e_q B}}{(q'_\perp - q_\perp)^2 + m_g^2}$$

After integrating the transverse momentum, we get

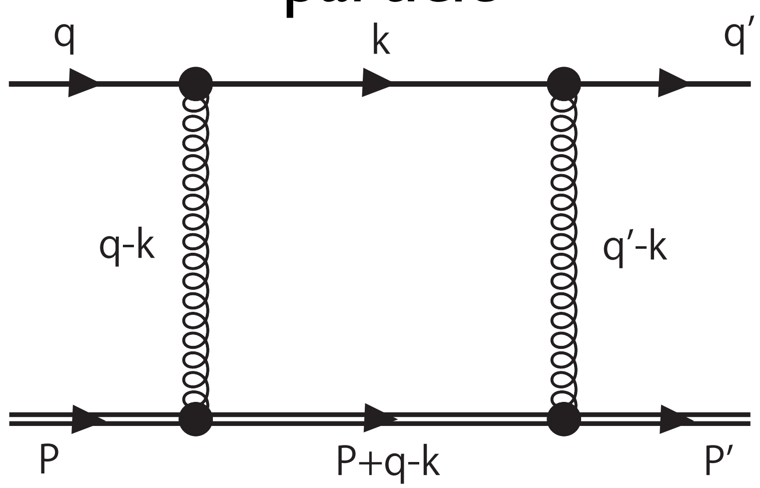
$$G \simeq \frac{g^2}{e_q B} \log\left(\frac{e_q B}{m_g^2}\right)$$

Leading order amplitude

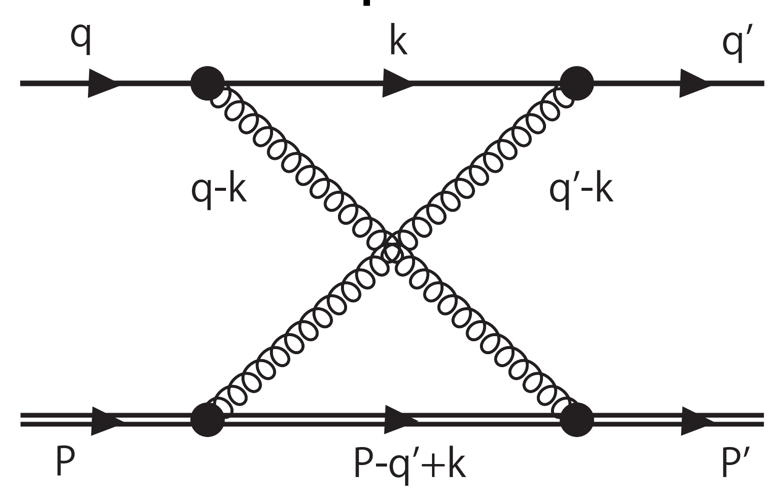
$$\longrightarrow \quad \underline{-i\, G (T^A)_{a'a} (T^A)_{b'b} \, (1 + \mathrm{sgn}(q'_z))}$$

NLO (LLL projected 1-loop amplitudes)

particle



anti-particle



$$iG^2\rho_{LLL}\mathcal{T}^{(a)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE \qquad -iG^2\rho_{LLL}\mathcal{T}^{(b)}_{a'a;b'b}\int_{\Lambda}^{\Lambda_{UV}}\frac{1}{E}dE$$

$$\longrightarrow \quad -iG^2\rho_{LLL}\frac{N_c}{2}\log\frac{\Lambda_{UV}}{\Lambda}(1+\mathrm{sgn}(q'_z))(T^A)_{a'a}(T^A)_{b'b}$$

independent of the chemical potential

$$\rho_{LLL}=\frac{e_qB}{(2\pi)^2}$$: density of state of the LLL

Renormalization group equation

$$\Lambda\frac{dG(\Lambda)}{d\Lambda} = -\frac{N_c}{2}\rho_{LLL}G^2(\Lambda)$$

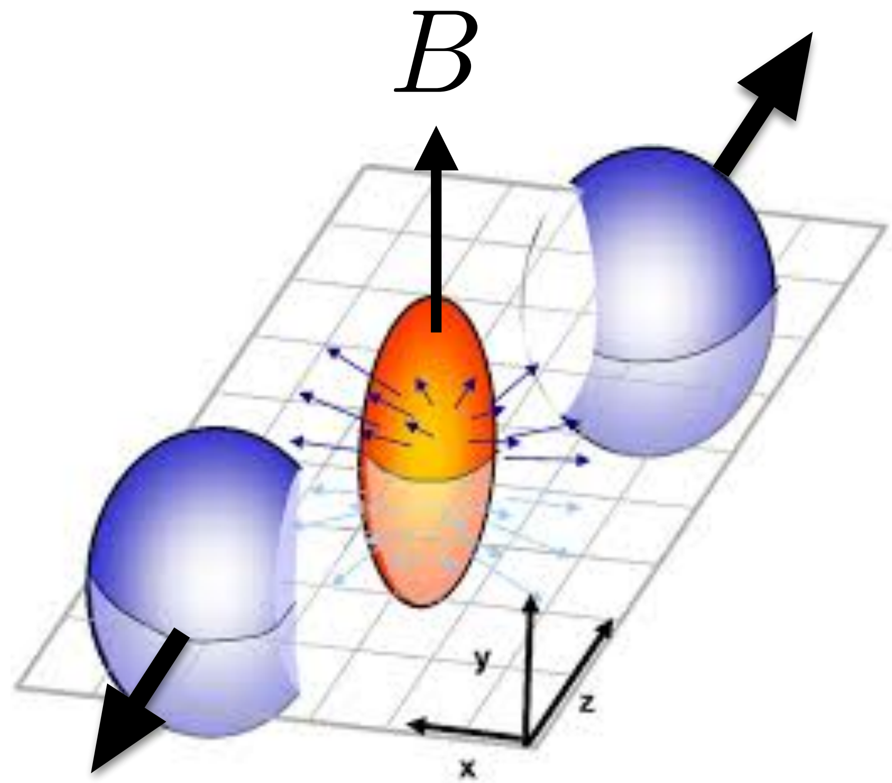
solution $\longrightarrow$

$$G(\Lambda) = \frac{G(\Lambda_0)}{1+\frac{N_c}{2}\rho_{LLL}G(\Lambda_0)\log(\Lambda/\Lambda_0)}$$

Kondo scale (from the Landau pole)

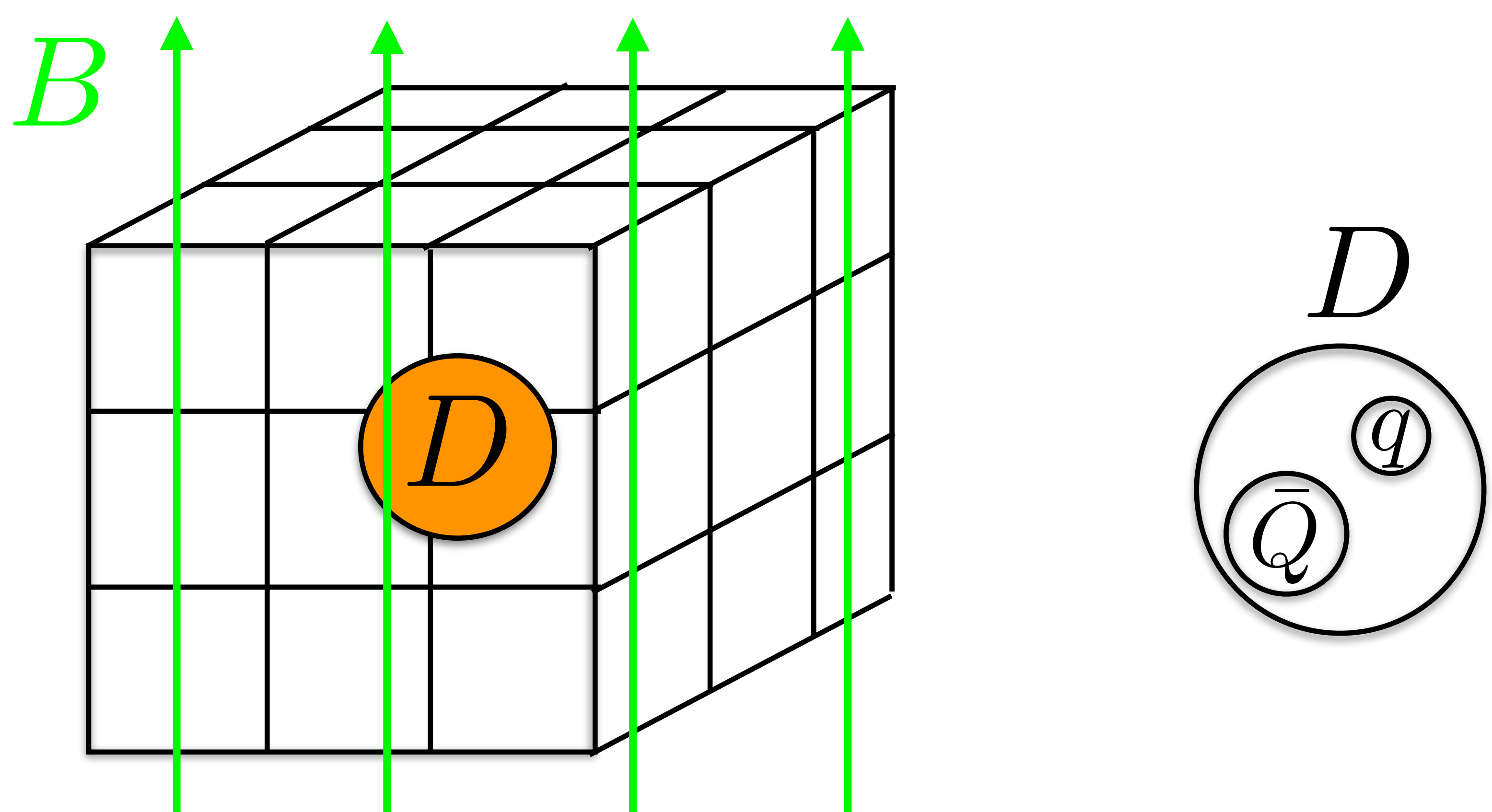
$$\Lambda_K \simeq \sqrt{e_qB}\alpha_s^{1/3}\exp\left\{-\frac{2\pi}{N_c\alpha_s\log(\pi/\alpha_s)}\right\}$$

- Non-central heavy ion collisions @ RHIC, LHC



$$|eB| \gtrsim \Lambda_{\rm QCD}^2$$

- Lattice QCD simulation (Numerical experiment of QCD)

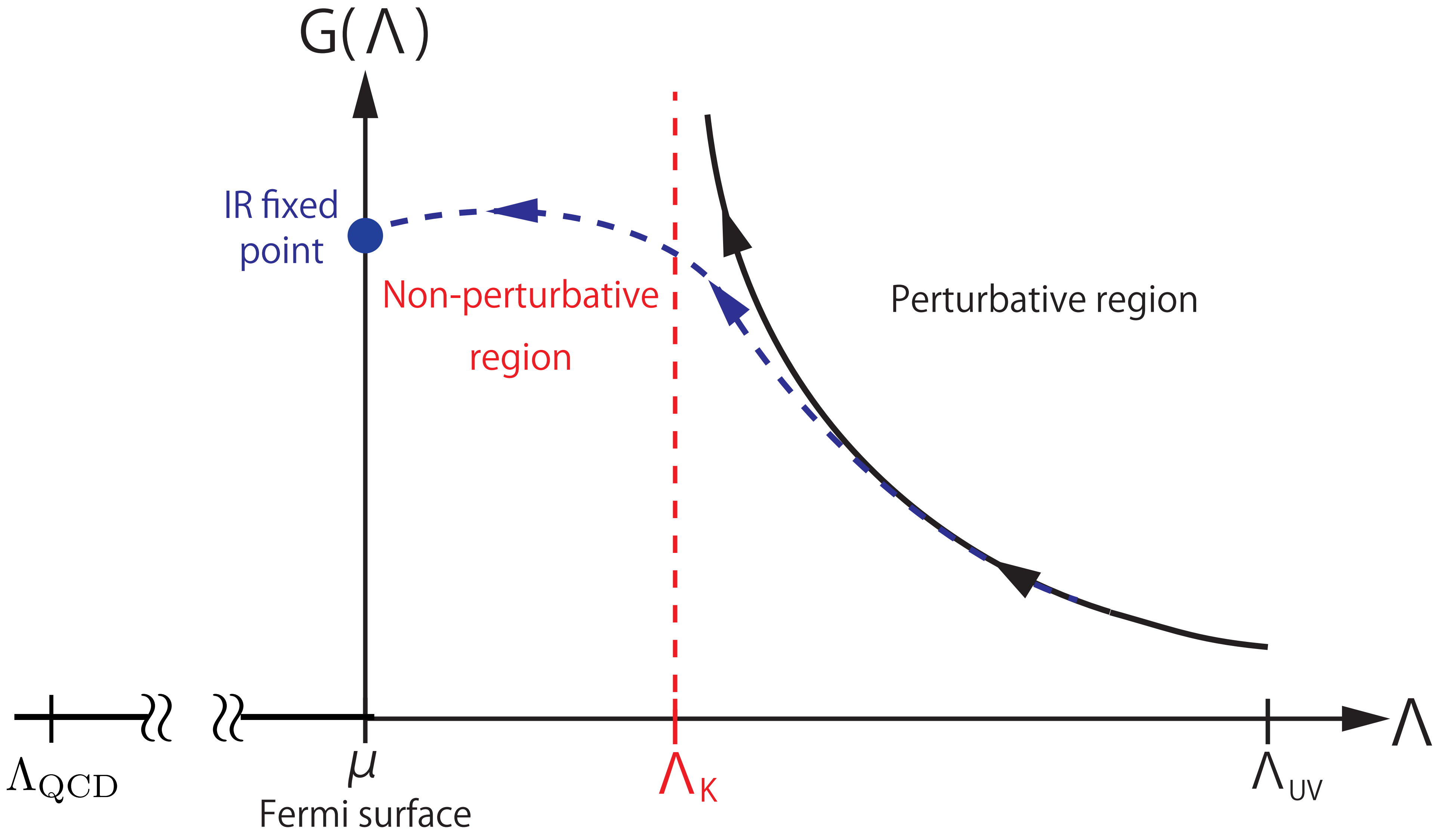


QCD Kondo effect from CFT

T. Kimura and S. O, arXiv:1611.07284

T. Kimura and S. O, in preparation

QCD Kondo effect



In order to investigate QCD Kondo effect in IR region below Kondo scale, we have to rely on non-perturbative method.

Non-perturbative approach for Kondo effect

- Numerical renormalization group [Wilson]

  ⟶ lattice QCD

- Bethe ansatz [Andrei] [Wiegmann] ….

- 1+1 dim. conformal field theory (CFT) approach

  [Affleck-Ludwig]

  k(multi)-channel SU(2) Kondo effect

Non-perturbative approach for Kondo effect

- Numerical renormalization group [Wilson]

  ⟶ lattice QCD

- Bethe ansatz [Andrei] [Wiegmann] ….

- 1+1 dim. conformal field theory (CFT) approach

  [Affleck-Ludwig]

  k(multi)-channel SU(2) Kondo effect

  k-channel SU(N) Kondo effect

T. Kimura and S. O, arXiv:1611.07284

Non-perturbative approach for Kondo effect

- Numerical renormalization group [Wilson]

  ⟶ lattice QCD

- Bethe ansatz [Andrei] [Wiegmann] ….

- 1+1 dim. conformal field theory (CFT) approach

  [Affleck-Ludwig]

  k(multi)-channel SU(2) Kondo effect

  k-channel SU(N) Kondo effect

  T. Kimura and S. O, arXiv:1611.07284

Effective 1+1 dim. theory at high density

High density QCD in the presence of the heavy quark

$\xrightarrow{\text{s-wave}}$ 1+1 dim. (Dimensional reduction)

[E. Shuster & D.T. Son, and T. Kojo et al.]

$$S_{eff}^{1+1} = \int d^2x \, \bar{\Psi} \left[i\Gamma^\mu \partial_\mu \right] \Psi - G\Psi^\dagger t^a \Psi Q^\dagger t^a Q$$

Dimensionless coupling G is obtained from gluon exchange

$$G = \int_{k_F} \frac{d^2q}{(2\pi)^2} \frac{(ig)^2}{q^2 - m_D^2}$$

$$= \rho_F^{2D} \int \frac{d\Omega_q}{4\pi} \frac{(ig)^2}{q^2 - m_D^2}$$

$$= \alpha_s \log \frac{4\mu^2}{m_D^2} = \alpha_s \log \frac{4\pi}{\alpha_s} \ll 1$$

$$\rho_F^{2D} = \frac{k_F^2}{\pi} = \frac{\mu^2}{\pi}$$ density of state on the Fermi surface

Effective 1+1 dim. theory at high density

High density QCD in the presence of the heavy quark

$\xrightarrow{\text{s-wave}}$ 1+1 dim. (Dimensional reduction)

[E. Shuster & D.T. Son, and T. Kojo et al.]

$$S_{eff}^{1+1} = \int d^2x\, \bar{\Psi}\left[i\Gamma^\mu \partial_\mu\right]\Psi - G\Psi^\dagger t^a \Psi Q^\dagger t^a Q$$

with $G = \alpha_s \log \frac{4\mu^2}{m_g^2} = \alpha_s \log \frac{4\pi}{\alpha_s} \ll 1$

Effective 1+1 dim. theory at high density

High density QCD in the presence of the heavy quark

$\xrightarrow{\text{s-wave}}$ 1+1 dim. (Dimensional reduction)

[E. Shuster & D.T. Son, and T. Kojo et al.]

$$S^{1+1}_{eff} = \int d^2x\, \bar{\Psi}\left[i\Gamma^\mu \partial_\mu\right]\Psi - G\Psi^\dagger t^a \Psi Q^\dagger t^a Q$$

with $G = \alpha_s \log\frac{4\mu^2}{m_g^2} = \alpha_s \log\frac{4\pi}{\alpha_s} \ll 1$

- Ψ is light quark fields with 2Nf components of flavor and Nc colors. The 2 comes from spin d.o.f. in 4 dim.

Effective 1+1 dim. theory at high density

High density QCD in the presence of the heavy quark

$\xrightarrow{\text{s-wave}}$ 1+1 dim. (Dimensional reduction)

[E. Shuster & D.T. Son, and T. Kojo et al.]

$$S_{eff}^{1+1} = \int d^2x\, \bar{\Psi}\left[i\Gamma^\mu \partial_\mu\right]\Psi - G\Psi^\dagger t^a \Psi Q^\dagger t^a Q$$

with $G = \alpha_s \log \frac{4\mu^2}{m_g^2} = \alpha_s \log \frac{4\pi}{\alpha_s} \ll 1$

- Ψ is light quark fields with 2Nf components of flavor and Nc colors. The 2 comes from spin d.o.f. in 4 dim.
- This is nothing but k-channel SU(N) Kondo model in 1+1 dim., where $k = 2N_f$, $N = N_c$.

Boundary CFT

Hamiltonian density of QCD Kondo effect

$$\mathcal{H} = i\Psi^\dagger \frac{\partial \Psi}{\partial x} + G\Psi^\dagger t^a \Psi C^a \delta(x)$$

where $Q^\dagger t^a Q \xrightarrow{M_Q \to \mathrm{large}} C^a \delta(x)$

Boundary CFT

Hamiltonian density of QCD Kondo effect

$$\mathcal{H} = i\Psi^\dagger \frac{\partial \Psi}{\partial x} + G\Psi^\dagger t^a \Psi C^a \delta(x)$$

where $Q^\dagger t^a Q \xrightarrow{M_Q \to \text{large}} C^a \delta(x)$

Currents

$J^a =: \Psi^\dagger t^a \Psi :$ color

$J^A =: \Psi^\dagger T^A \Psi :$ flavor $\quad : OO(x) := \lim_{\epsilon \to 0} \{OO(x+\epsilon) - \langle OO(x+\epsilon) \rangle\}$

$J =: \Psi^\dagger \Psi :$ charge

Boundary CFT

Hamiltonian density of QCD Kondo effect

$$\mathcal{H} = i\Psi^\dagger \frac{\partial \Psi}{\partial x} + G\Psi^\dagger t^a \Psi C^a \delta(x)$$

where $Q^\dagger t^a Q \xrightarrow{M_Q \to \text{large}} C^a \delta(x)$

Currents

$J^a =: \Psi^\dagger t^a \Psi :$ color

$J^A =: \Psi^\dagger T^A \Psi :$ flavor $\quad : OO(x) := \lim_{\epsilon \to 0} \{OO(x+\epsilon) - \langle OO(x+\epsilon) \rangle\}$

$J =: \Psi^\dagger \Psi :$ charge

The Sugawara form of the Hamiltonian density

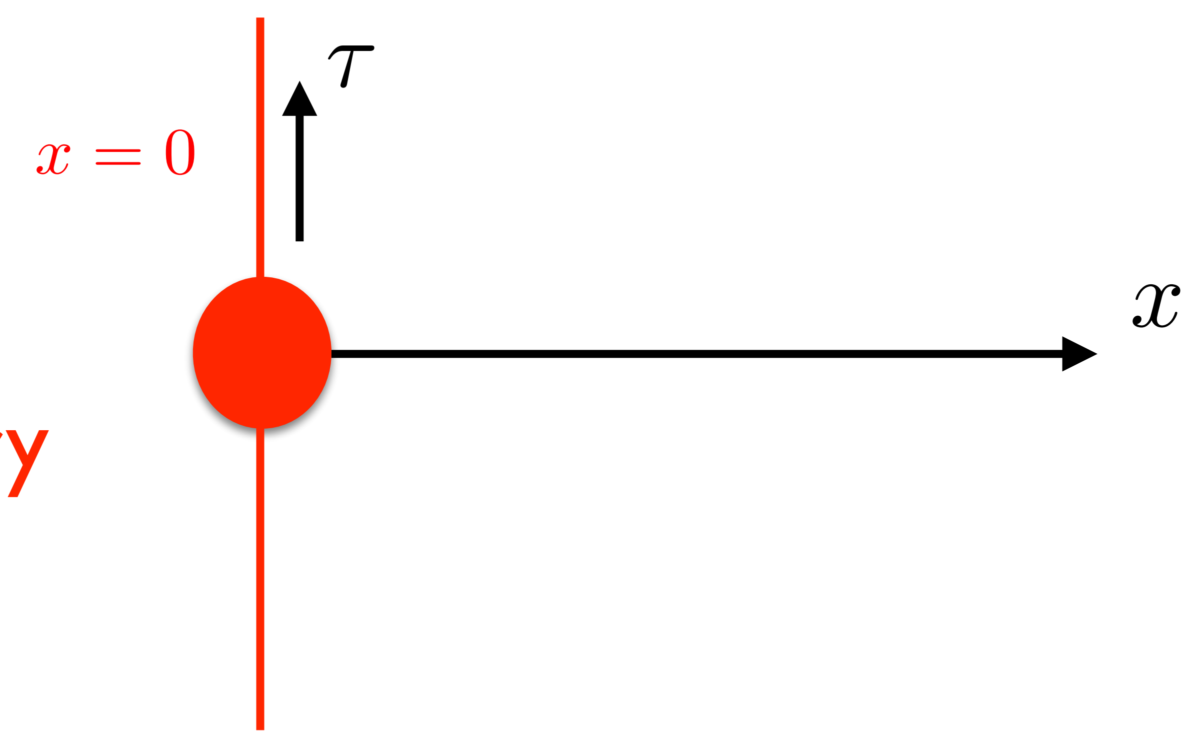
$$\mathcal{H} = \frac{1}{N_c + 2N_f} J^a J^a + \frac{1}{2N_f + N_c} J^A J^A + \frac{1}{4N_c N_f} J^2 + G J^a C^a \delta(x)$$

Boundary CFT

$$\mathcal{H}=\frac{1}{N_c+2N_f}J^aJ^a+\frac{1}{2N_f+N_c}J^AJ^A+\frac{1}{4N_cN_f}J^2+GJ^aC^a\delta(x)$$

$$\mathcal{J}^a=J^a+\frac{G}{2}(N_c+2N_f)C^a\delta(x)$$

$$\mathcal{H}=\frac{1}{N_c+2N_f}\mathcal{J}^a\mathcal{J}^a+\frac{1}{2N_f+N_c}J^AJ^A+\frac{1}{4N_cN_f}J^2$$



Impurity effect

⟶ Boundary of the theory

g-factor and Impurity entropy

Partition function

$$Z(L,\beta) \xrightarrow{L\to\infty} \underbrace{g_{R_{\rm imp}}}_{\text{boundary (impurity)}} \times \underbrace{e^{\frac{\pi c L}{6\beta}}}_{\text{bulk}},$$

universal quantities

$c = 2N_c N_f$: central charge

$g_{R_{\rm imp}} = \dfrac{S_{R_{\rm imp}0}}{S_{00}}$: g-factor

(S_{mn} : modular S-matrix)

Free energy

$$F = -\frac{1}{\beta}\log Z$$

Entropy

$$S(T) = -\frac{\partial F}{\partial T}$$

Impurity entropy at IR fixed point (T=0)

$$S_{\rm imp} = S(T) - S_{\rm bulk}(T)|_{T=0} = \log\left(g_{R_{\rm imp}}\right)$$

g-factor and Impurity entropy

g-factor provides the impurity entropy:

$$S_{\rm imp} = \log(g_{R_{\rm imp}})\,,$$

$R_{\rm imp}$: (anti-)fundamental representation

ex) SU(2), k=1 (standard Kondo effect)

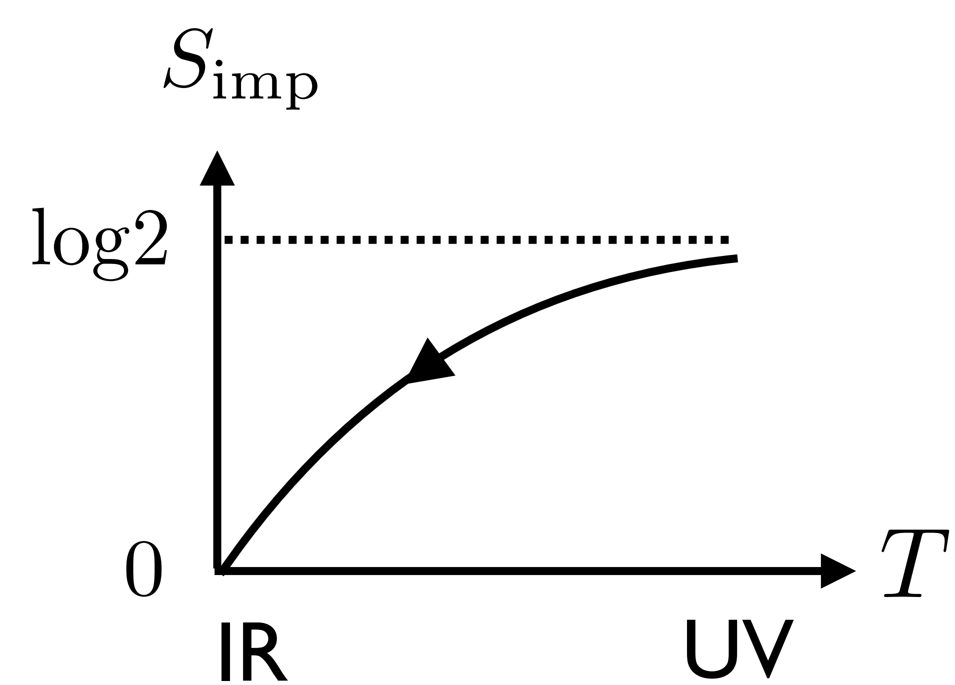
$$S_{\rm imp} = \log(2s+1)$$

In UV, $s = 1/2$

$$S_{\rm imp} \to \log 2$$

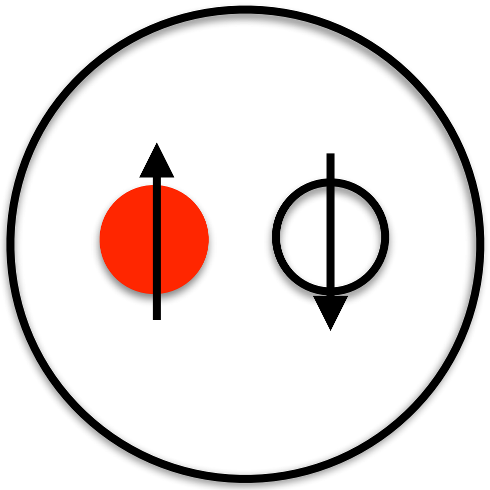
In IR, $s \to 0$ (Kondo singlet)

$$S_{\rm imp} \to 0$$



Overscreening Kondo effect in multi-channel SU(2) Kondo model

Standard Kondo effect

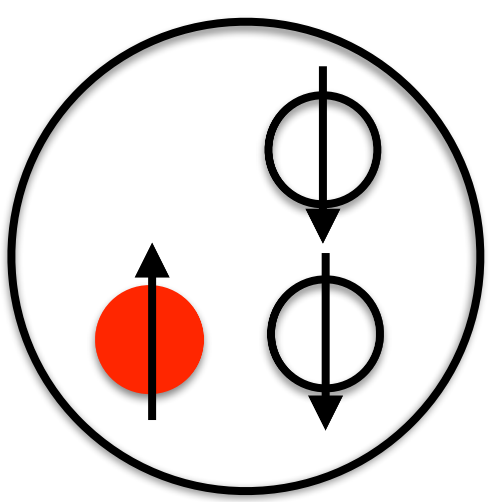


$k = 1$ (single channel)

$\longrightarrow$ Fermi liquid at IR fixed point

$\longrightarrow$ g : integer

Overscreening Kondo effect



$k = 2$ (two channel)

$\longrightarrow$ non-Fermi liquid at IR fixed point

$\longrightarrow$ g : non-integer

g-factor in QCD Kondo effect @ IR fixed point (zero temperature)

- $N_c = 3$ $\qquad k = 2N_f$

$$g = \frac{1+\sqrt{5}}{2} \qquad (N_f = 1)$$

$$g = 2.24598... \qquad (N_f = 2)$$

$$g = 2.53209... \qquad (N_f = 3)$$

- In general Nc and Nf, the g-factor is non-integer, and thus QCD Kondo effect has non-Fermi liquid IR fixed point.

- In large Nc limit: $N_c \to \infty, \quad N_f$: fixed

$g \to 1$ Fermi liquid at IR fixed point

1+1 dim. (boundary) CFT approach

Correlation functions are exactly determined by

- Conformal symmetry in 1+1 dim.
- Kac-Moody algebra

$$[\mathcal{J}^a(x), \mathcal{J}^b(y)] = i f^{abc} \mathcal{J}^c(x) \delta(x-y) + N_f \delta^{ab} \frac{\partial}{\partial x} \delta(x-y)$$

$$\langle O_1(x) O_2(y) \rangle = \frac{C_{1,2}}{|x-y|^{\Delta}}$$

Δ determined by conformal symmetry

$C_{1,2}$ determined by KM algebra

From the correlation functions, one can evaluate T-dep. of several observables of QCD Kondo effect in IR regions.

Specific heat, susceptibility and the Wilson ratio in QCD Kondo effect

- Bulk contributions to C & χ

$$C_{\text{bulk}} = \frac{\pi}{3} N k T$$

$$\chi_{\text{bulk}} = \frac{k}{2\pi}$$

These are well known properties of free Nk (bulk) fermions in 1+1 dim.

with $k = 2N_f\,,\ N = N_c$

Impurity contributions to C & χ

i) k=1 & arbitrary N [Affleck 1990]

Leading irrelevant operator

$$\delta\mathcal{H}_1 = \lambda_1 \mathcal{J}^a \mathcal{J}^a(x)\delta(x)$$

$$\lambda_1 \sim 1/T_{\mathrm{K}}$$

From the perturbation w.r.t. $\delta\mathcal{H}_1$

$$C_{\mathrm{imp}} = -\lambda_1 \frac{k(N^2-1)}{3}\pi^2 T$$

$$\chi_{\mathrm{imp}} = -\lambda_1 \frac{k(N+k)}{2}$$

with $k=1$

⟶ Typical Fermi liquid behaviors

ii) k > 1, Overscreening case (relevant for QCD Kondo effect)

Leading irrelevant operator

$$\delta\mathcal{H} = \lambda \mathcal{J}^a_{-1}\phi^a(x)\delta(x)$$

ϕ^a :adjoint operator, appearing when k = 2Nf >1
scaling dimension is $\Delta = N_c/(N_c + 2N_f)$

$\mathcal{J}^a_n$:Fourier mode of $\mathcal{J}^a(x)$

$\lambda \sim 1/T_{\mathrm{K}}^{\Delta}$ $\qquad \Delta < 1$

From the perturbation with respect to the leading irrelevant operator, we can evaluate $C_{\mathrm{imp}}, \chi_{\mathrm{imp}}$.

Observables

$$Z = \mathrm{e}^{-\beta F(T,\lambda,h)} \qquad\qquad L \to \infty$$

$$= \int \mathcal{D}\Psi\mathcal{D}\bar{\Psi}\ \mathrm{e}^{-\int d^2x\,\mathcal{H}} \exp\left\{+\int_{-\beta/2}^{\beta/2} d\tau \left[\lambda \mathcal{J}^a_{-1}\phi^a(\tau,0) + \frac{h}{2\pi}\int_{-L}^{L} dx\ \mathcal{J}^3(\tau,x)\right]\right\}$$

$$= Z_0 \left\langle \exp\left\{+\int_{-\beta/2}^{\beta/2} d\tau \left[\lambda \mathcal{J}^a_{-1}\phi^a(\tau,0) + \frac{h}{2\pi}\int_{-L}^{L} dx\ \mathcal{J}^3(\tau,x)\right]\right\}\right\rangle$$

Free energy can be divided in to bulk and impurity parts

$$F = L f_{\mathrm{bulk}} + f_{\mathrm{imp}}$$

which is expressed in terms of the correlation functions of the leading irrelevant operators.

$$\longrightarrow \quad C = -T\frac{\partial^2 F}{\partial T^2}\,, \quad \chi = -\left.\frac{\partial^2 F}{\partial h^2}\right|_{h=0}$$

Specific heat of QCD Kondo effect

$$C_{\mathrm{imp}} = \begin{cases} \frac{\lambda^2}{2}\pi^{1+2\Delta}(2\Delta)^2(N_c^2-1)(N_c+N_f)\left(\frac{1-2\Delta}{2}\right)\frac{\Gamma(1/2-\Delta)\Gamma(1/2)}{\Gamma(1-\Delta)}T^{2\Delta} & (2N_f > N_c) \\ \lambda^2\pi^{1+2\Delta}(N^2-1)(N_c+N_f)(2\Delta)^2\,T\log\left(\frac{T_{\mathrm{K}}}{T}\right) & (2N_f = N_c) \\ \underline{-\lambda_1\frac{2}{3}(N_c^2-1)\pi^2 T} + \underline{2\lambda^2\pi^2(N_c^2-1)(N_c+N_f)\frac{2\Delta}{1+2\Delta}\left(\frac{\beta_{\mathrm{K}}^{-2\Delta+1}}{2\Delta-1}\right)T} & (N_c > 2N_f) \end{cases}$$

$$\delta\mathcal{H}_1 = \lambda_1\mathcal{J}^a\mathcal{J}^a(x)\delta(x)$$

$$\delta\mathcal{H} = \lambda\mathcal{J}^a_{-1}\phi^a(x)\delta(x)$$

Specific heat of QCD Kondo effect

$$C_{\mathrm{imp}} = \begin{cases} \dfrac{\lambda^2}{2}\pi^{1+2\Delta}(2\Delta)^2(N_c^2-1)(N_c+N_f)\left(\dfrac{1-2\Delta}{2}\right)\dfrac{\Gamma(1/2-\Delta)\Gamma(1/2)}{\Gamma(1-\Delta)}\,\underline{T^{2\Delta}} & (2N_f > N_c) \\ \lambda^2\pi^{1+2\Delta}(N^2-1)(N_c+N_f)(2\Delta)^2\,\underline{T\log\left(\dfrac{T_{\mathrm{K}}}{T}\right)} & (2N_f = N_c) \\ -\lambda_1\dfrac{2}{3}(N_c^2-1)\pi^2\underline{T}+2\lambda^2\pi^2(N_c^2-1)(N_c+N_f)\dfrac{2\Delta}{1+2\Delta}\left(\dfrac{\beta_{\mathrm{K}}^{-2\Delta+1}}{2\Delta-1}\right)\underline{T} & (N_c > 2N_f) \end{cases}$$

Low T scaling

$$C_{\mathrm{imp}} \propto \begin{cases} T^{2\Delta} & (2N_f > N_c) & \text{Non-Fermi} \\ T\log(T_{\mathrm{K}}/T) & (2N_f = N_c) & \text{Non-Fermi} \\ T & (N_c > 2N_f) & \text{Fermi} \end{cases}$$

For $N_c > 2N_f$, although the g-factor (at IR fixed point) exhibits non-Fermi liquid signature, T-dep. of C_{imp} shows Fermi liquid behavior. ⟶ Fermi/non-Fermi mixing [T. Kimura and S. O, arXiv:1611.07284]

Susceptibility of QCD Kondo effect

$$\chi_{\text{imp}} = \begin{cases} \frac{\lambda^2}{2}\pi^{2\Delta-1}(N_c+N_f)^2(1-2\Delta)\frac{\Gamma(1/2-\Delta)\Gamma(1/2)}{\Gamma(1-\Delta)}T^{2\Delta-1} & (2N_f > N_c) \\ 2\lambda^2(N_c+N_f)^2\log\left(\frac{T_\text{K}}{T}\right) & (2N_f = N_c) \\ -\lambda_1 N_f(N_c+2N_f) + 2\lambda^2(N_c+N_f)^2\left(\frac{\beta_\text{K}^{-2\Delta+1}}{2\Delta-1}\right) & (N_c > 2N_f) \end{cases}$$

Low T scaling

$$\chi_{\text{imp}} = \begin{cases} T^{2\Delta-1} & (2N_f > N_c) \quad \text{Non-Fermi} \\ \log(T_\text{K}/T) & (2N_f = N_c) \quad \text{Non-Fermi} \\ \text{const.} & (N_c > 2N_f) \quad \text{Fermi} \end{cases}$$

The Wilson ratio of QCD Kondo effect

$$R_{\rm W} = \left(\frac{\chi_{\rm imp}}{C_{\rm imp}}\right) \Bigg/ \left(\frac{\chi_{\rm bulk}}{C_{\rm bulk}}\right)$$

$$= \frac{(N_c + N_F)(N_c + 2N_f)^2}{3N_c(N_c^2 - 1)} \qquad (2N_f \geq N_c)$$

Unknown parameters are canceled, and thus the Wilson ratio is universal.

$$R_{\rm W} = \frac{(N_c + N_f)(N_c + 2N_f/3)}{N_c^2 - 1} \frac{\gamma - \dfrac{2N_f(N_c + 2N_f)}{(N_c + N_f)^2}}{\gamma - \dfrac{2N_f(N_c + 2N_f/3)}{N_c(N_c + N_f)}} \qquad (N_c > 2N_f)$$

with $\gamma = 4\dfrac{\lambda^2}{\lambda_1} T_{\rm K}^{2\Delta - 1}$

For Nc >= 2Nf, the Wilson ratio is no longer universal, which depends on the detail of the system, such as $\lambda,\ T_{\rm K}$

IR behaviors of QCD Kondo effect (k-channel SU(N) Kondo effect)

	(k >= N)	(N > k >1)
	2Nf >= Nc	Nc > 2Nf
g-factor (IR fixed point)	non-Fermi	non-Fermi
Low T scaling	non-Fermi	Fermi
Wilson ratio	universal	non-universal

Fermi/non-Fermi mixing

T. Kimura and S. O, arXiv:1611.07284

Summary

- We propose QCD Kondo effect appearing at high density quark matter and in strong magnetic fields.

- We apply CFT approach to QCD Kondo effect and investigate its IR behaviors below the Kondo scale.

- In the vicinity of IR fixed point, QCD Kondo effect shows Fermi/non-Fermi mixing for Nc > 2Nf, while it shoes non-Fermi liquid behaviors for 2Nf >= Nc.