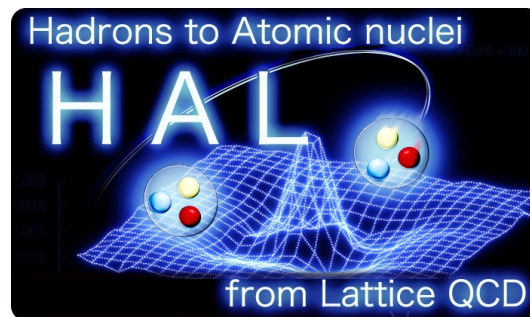


# *Lattice QCD studies on baryon interactions in the strangeness -2 sector*

Kenji Sasaki (*YITP, Kyoto University*)

for HAL QCD Collaboration



***HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration***

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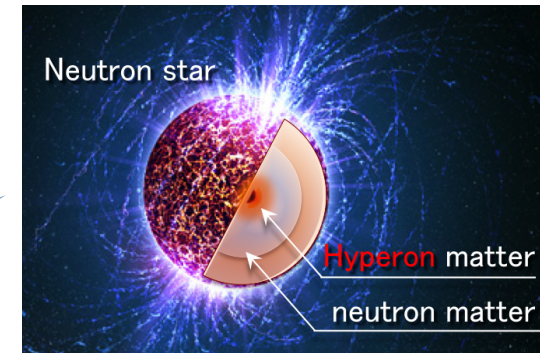
- Introduction
- HAL QCD method
- $S=-2$  BB potentials
  - H-dibaryon channel
- Summary

# *Introduction*

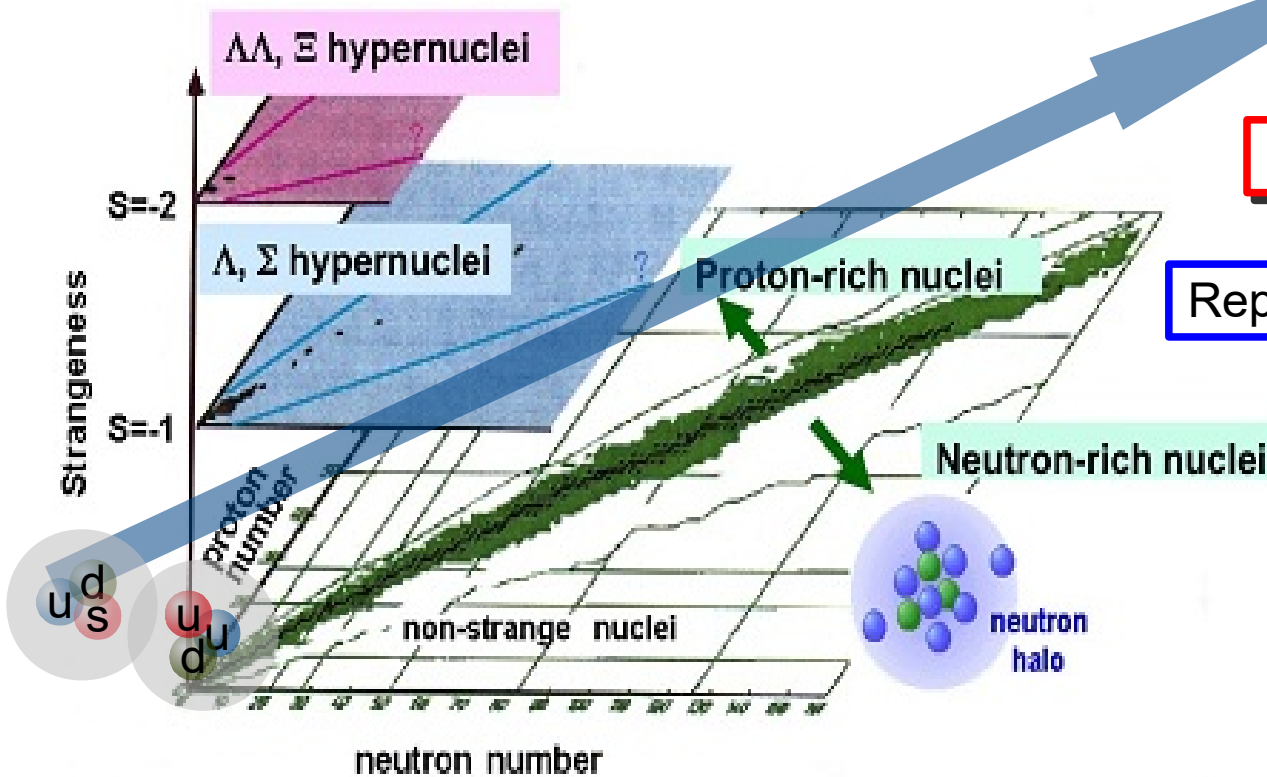
# Introduction

BB interactions are inputs for nuclear structure, astrophysical phenomena

Once we obtain a “reliable” nuclear potential,  
we apply them to the structure of (hyper-) nucleus  
and neutron star calculation

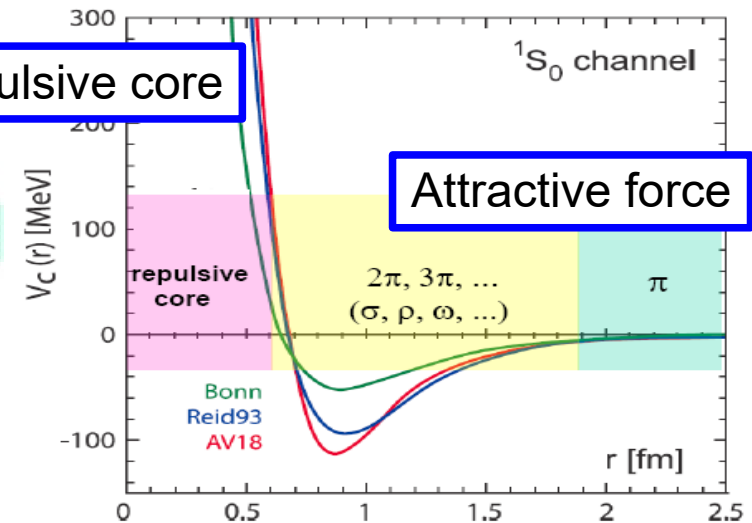


Stabilize the nucleus and the matter



Repulsive core

Attractive force

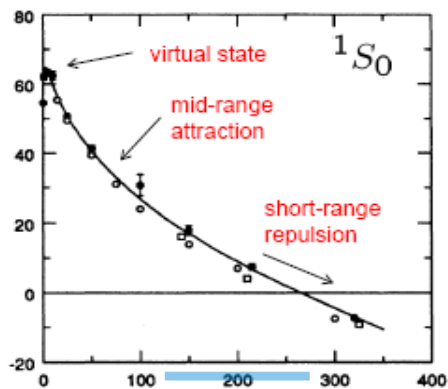


How do we obtain the nuclear force?

# Phenomenological descriptions

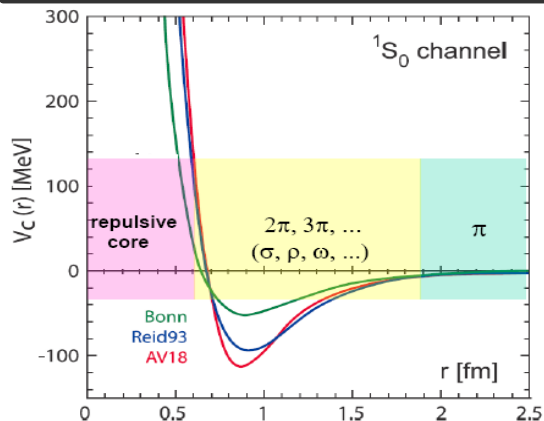
Traditional process to research the BB interaction / potential

## Scattering observables



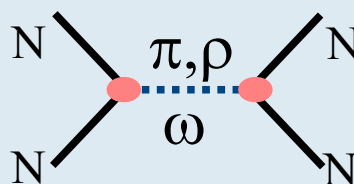
Model assumption

## BB interaction (potential)



## Meson exchange model

Described by hadron d.o.f.



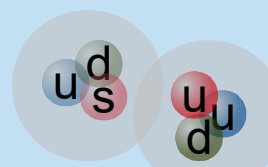
+ Phenomenological repul. core

H. Yukawa, PPS17(1935)48

Th.A.Rijken, PTPS185(2010)14

## Quark cluster model

Effective meson ex  
+ quark anti-symmetrization



Quark Pauli effects  
Color magnetic int.

M.Oka, PTPS137(2000)1

Y.Fujiwara, PPNP58(2007)439

## Effective Field theory

Systematic calc. respecting with

symmetry of QCD

LO



NLO



Short range interaction is  
parametrized by  
contact term

E. Epelbaum, RMP81(2009)1773

R. Machleidt, PRept.503(2011)1

The models would be highly ambiguous if experimental data are scarce!

# Introduction

Aim : Nuclear structures and exotic states from QCD

Lattice QCD simulation

- Advantageous for **more strange quarks**
- Signals getting worse as increasing the number of light quarks.
- **Complementary role to experiment.**

Main topics of  $S=-2$  multi-baryon system

▶ **H-dibaryon**

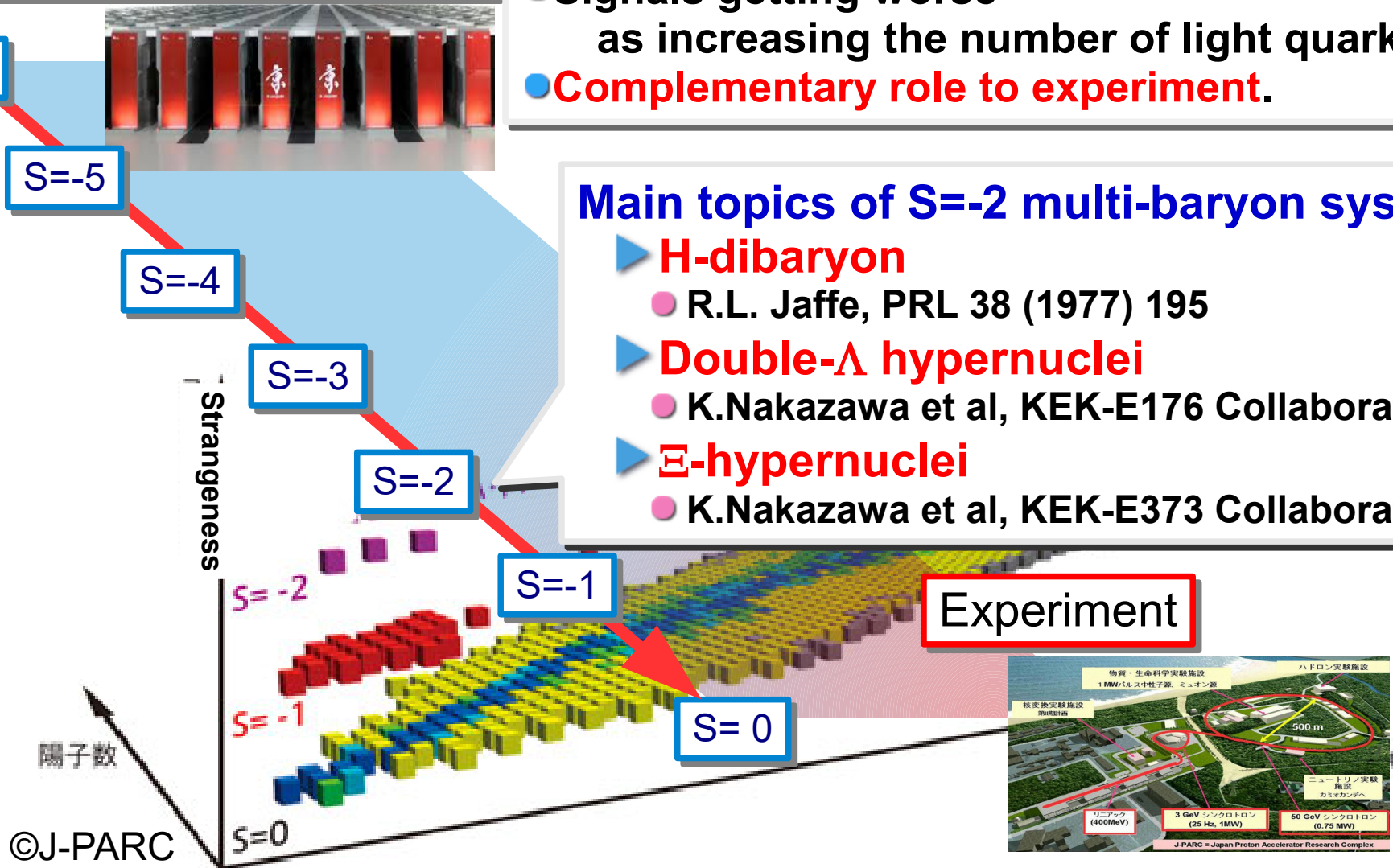
- R.L. Jaffe, PRL 38 (1977) 195

▶ **Double- $\Lambda$  hypernuclei**

- K.Nakazawa et al, KEK-E176 Collaboration

▶  **$\Xi$ -hypernuclei**

- K.Nakazawa et al, KEK-E373 Collaboration



# Interests of $S=-2$ multi-baryon system

## H-dibaryon

- The flavor singlet state with  $J=0$  predicted by R.L. Jaffe.
  - Strongly attractive color magnetic interaction.
  - No quark Pauli principle for flavor singlet state.

## Double- $\Lambda$ hypernucleus

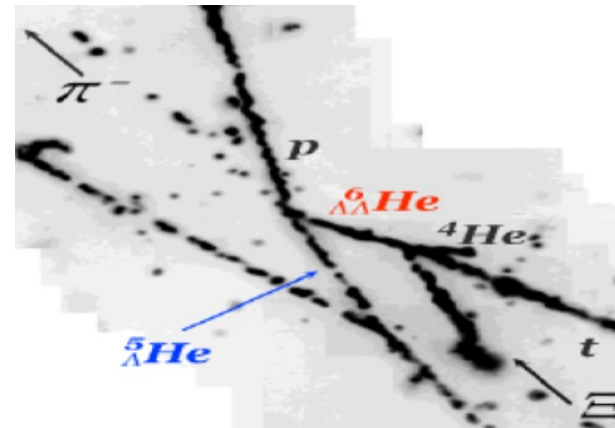
- Conclusions of the “NAGARA Event”

K.Nakazawa and KEK-E176 & E373 Collaborators

$\Lambda$ -N attraction

$\Lambda$ - $\Lambda$  weak attraction

$$m_H \geq 2m_\Lambda - 6.9\text{MeV}$$

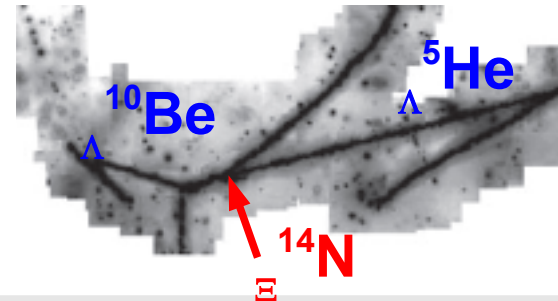


## $\Xi$ hypernucleus

- Conclusions of the “KISO Event”

K.Nakazawa and KEK-E373 Collaborators

$\Xi$ -N attraction



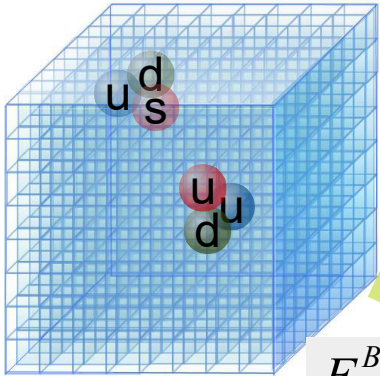
# *HAL QCD method*



# Derivation of hadronic interaction from QCD

Start with the fundamental theory, QCD

Lattice QCD simulation



Lüscher's finite volume method

M. Lüscher, NPB354(1991)531

1. Measure the discrete energy spectrum,  $E$
2. Put the  $E$  into the formula which connects  $E$  and  $\delta$

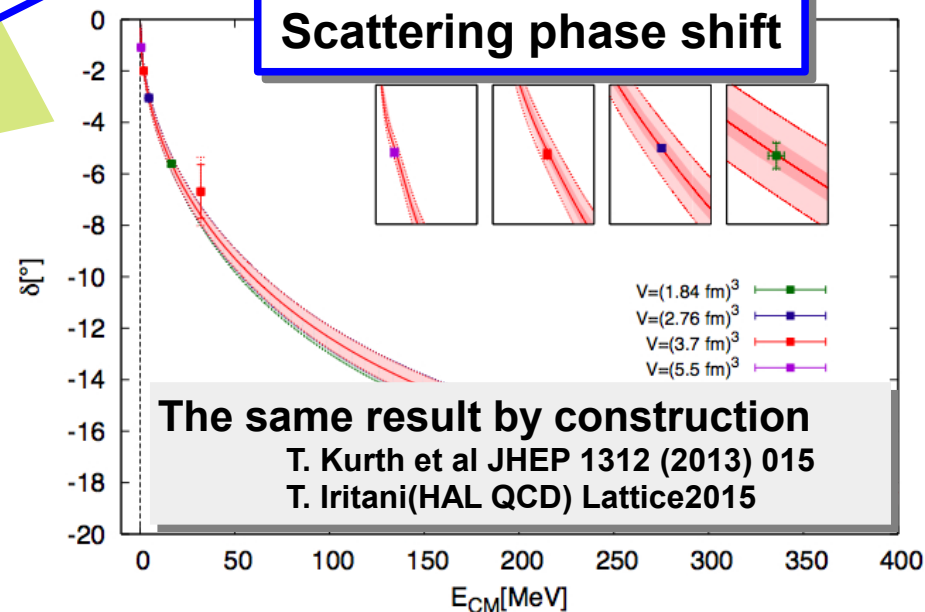
$$F^{B_1 B_2}_I(t, \vec{r}) = \langle 0 | B_1 B_2(t, \vec{r}) \bar{I}(t_0) | 0 \rangle = A_0 \Psi(\vec{r}, E_0) e^{-E_0(t-t_0)} + \dots$$

HAL QCD method

Ishii, Aoki, Hatsuda, PRL99 (2007) 022001

1. Measure the NBS wave function,  $\Psi$
2. Calculate potential,  $V$ , through Schrödinger eq.
3. Calculate observables by scattering theory

Scattering phase shift



# Nambu-Bethe-Salpeter wave function

**Definition : equal time NBS w.f.**

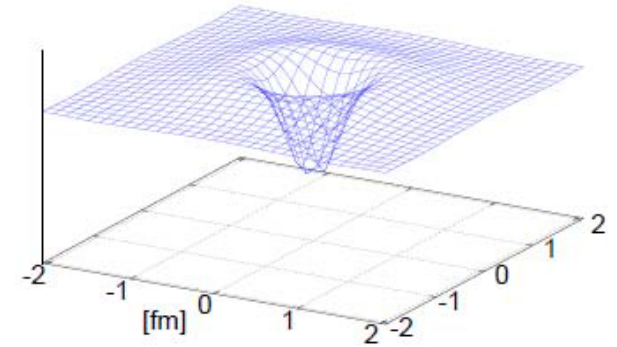
$$\Psi^\alpha(E, \vec{r}) e^{-Et} = \sum_{\vec{x}} \langle 0 | H_1^\alpha(t, \vec{x} + \vec{r}) H_2^\alpha(t, \vec{x}) | E \rangle$$

E : Total energy of the system

Local composite interpolating operators

$$B_\alpha = \epsilon^{abc} (q_a^T C \gamma_5 q_b) q_{c\alpha} \quad D_{\mu\alpha} = \epsilon^{abc} (q_a^T C \gamma_\mu q_b) q_{c\alpha}$$

$$M = (\bar{q}_a \gamma_5 q_a) \quad \text{Etc.....}$$

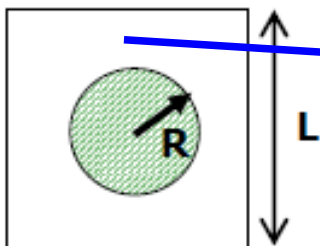


● It satisfies the Helmholtz eq. in asymptotic region :  $(p^2 + \nabla^2) \Psi(E, \vec{r}) = 0$

● Using the reduction formula,

C.-J.D.Lin et al., NPB619 (2001) 467.

$$\Psi^\alpha(E, \vec{r}) = \sqrt{Z_{H_1}} \sqrt{Z_{H_2}} \left( e^{i\vec{p}\cdot\vec{r}} + \int \frac{d^3q}{2E_q} \frac{T(q, p)}{4E_p(E_q - E_p - i\epsilon)} e^{i\vec{q}\cdot\vec{r}} \right)$$



$$\Psi(E, \vec{r}) \simeq A \frac{\sin(pr + \delta(E))}{pr}$$

Phase shift is defined as

$$S \equiv e^{i\delta}$$

**NBS wave function has a same asymptotic form with quantum mechanics.**  
(NBS wave function is characterized from phase shift)

# Time-dependent Schrödinger like equation

Start with the normalized four-point correlator.

$$R_I^{B_1 B_2}(t, \vec{r}) = F^{B_1 B_2}_I(t, \vec{r}) e^{(m_1 + m_2)t}$$

$$= A_0 \Psi(\vec{r}, E_0) e^{-(E_0 - m_1 - m_2)t} + A_1 \Psi(\vec{r}, E_1) e^{-(E_1 - m_1 - m_2)t} + \dots$$

$$\left( \frac{p_0^2}{2\mu} + \frac{\nabla^2}{2\mu} \right) \Psi(\vec{r}, E_0) = \int U(\vec{r}, \vec{r}') \Psi(\vec{r}', E_0) d^3 r'$$

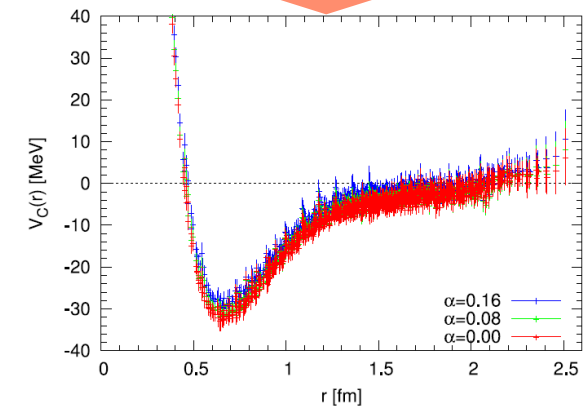
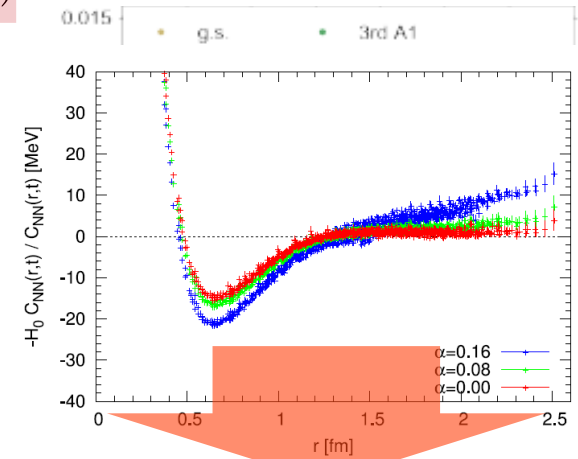
$$E_n - m_1 - m_2 \approx \frac{p_n^2}{2\mu} \quad \left( \frac{p_1^2}{2\mu} + \frac{\nabla^2}{2\mu} \right) \Psi(\vec{r}, E_1) = \int U(\vec{r}, \vec{r}') \Psi(\vec{r}', E_1) d^3 r'$$

**A single state saturation is not required!!**

$$\left( -\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu} \right) R_I^{B_1 B_2}(t, \vec{r}) = \int U(\vec{r}, \vec{r}') R_I^{B_1 B_2}(t, \vec{r}') d^3 r'$$

**Derivative (velocity) expansion of U**

$$U(\vec{r}, \vec{r}') = [V_C(r) + S_{12} V_T(r)] + [\vec{L} \cdot \vec{S}_s V_{LS}(r) + \vec{L} \cdot \vec{S}_a V_{ALS}(r)] + O(\nabla^2)$$



# HAL QCD method

## NBS wave function

$$\Psi(E, \vec{r}) e^{-E(t-t_0)} = \sum_{\vec{x}} \langle 0 | B_i(t, \vec{x} + \vec{r}) B_j(t, \vec{x}) | E, t_0 \rangle$$

E : Total energy of system

- In asymptotic region :  $(p^2 + \nabla^2) \Psi(E, \vec{r}) = 0$
- In interaction region :  $(p^2 + \nabla^2) \Psi(E, \vec{r}) = K(E, \vec{r})$

$$\Psi(E, \vec{r}) \simeq A \frac{\sin(pr + \delta(E))}{pr}$$

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

## Modified Schrödinger equation

$$\left( -\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu} \right) R_I^{B_1 B_2}(t, \vec{r}) = \int U(\vec{r}, \vec{r}') R_I^{B_1 B_2}(t, \vec{r}') d^3 r'$$

N. Ishii et al Phys. Lett. B712(2012)437

## Derivative expansion

$$U(\vec{r}, \vec{r}') = V_C(r) + S_{12} V_T(r) + \vec{L} \cdot \vec{S}_s V_{LS}(r) + \vec{L} \cdot \vec{S}_a V_{ALS}(r) + O(\nabla^2)$$

K. Murano et al Phys.Lett. B735 (2014) 19

## Potential

$$V(\vec{r}) = \left( -\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu} \right) R_I^{B_1 B_2}(t, \vec{r}) / R_I^{B_1 B_2}(t, \vec{r})$$

# HAL QCD method (coupled-channel)

## NBS wave function

$$\Psi^\alpha(E_i, \vec{r}) e^{-E_i t} = \langle 0 | (B_1 B_2)^\alpha(\vec{r}) | E_i \rangle \quad \int dr \tilde{\Psi}_\beta(E', \vec{r}) \Psi^\gamma(E, \vec{r}) = \delta(E' - E) \delta_\beta^\gamma$$

$$\Psi^\beta(E_i, \vec{r}) e^{-E_i t} = \langle 0 | (B_1 B_2)^\beta(\vec{r}) | E_i \rangle \quad R_E^{B_1 B_2}(t, \vec{r}) = \Psi_{B_1 B_2}(\vec{r}, E) e^{(-E + m_1 + m_2)t}$$

**Leading order of velocity expansion and time-derivative method.**

## Modified coupled-channel Schrödinger equation

$$\begin{pmatrix} \left(-\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_\alpha}\right) R_{E_0}^\alpha(t, \vec{r}) \\ \left(-\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_\beta}\right) R_{E_0}^\beta(t, \vec{r}) \end{pmatrix} = \begin{pmatrix} V_\alpha^\alpha(\vec{r}) & V_\beta^\alpha(\vec{r}) \Delta_\beta^\alpha(t) \\ V_\alpha^\beta(\vec{r}) \Delta_\alpha^\beta(t) & V_\beta^\beta(\vec{r}) \end{pmatrix} \begin{pmatrix} R_{E_0}^\alpha(t, \vec{r}) \\ R_{E_0}^\beta(t, \vec{r}) \end{pmatrix}$$

$$\begin{pmatrix} \left(-\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_\alpha}\right) R_{E_1}^\alpha(t, \vec{r}) \\ \left(-\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_\beta}\right) R_{E_1}^\beta(t, \vec{r}) \end{pmatrix} = \begin{pmatrix} V_\alpha^\alpha(\vec{r}) & V_\beta^\alpha(\vec{r}) \Delta_\beta^\alpha(t) \\ V_\alpha^\beta(\vec{r}) \Delta_\alpha^\beta(t) & V_\beta^\beta(\vec{r}) \end{pmatrix} \begin{pmatrix} R_{E_1}^\alpha(t, \vec{r}) \\ R_{E_1}^\beta(t, \vec{r}) \end{pmatrix}$$

$$\Delta_\beta^\alpha = \frac{\exp(-(m_{\alpha_1} + m_{\alpha_2})t)}{\exp(-(m_{\beta_1} + m_{\beta_2})t)}$$

S.Aoki et al [HAL QCD Collab.] Proc. Jpn. Acad., Ser. B, 87 509  
K.Sasaki et al [HAL QCD Collab.] PTEP no 11 (2015) 113B01

## Potential

**Considering two different energy eigen states**

$$\begin{pmatrix} V_\alpha^\alpha(\vec{r}) & V_\beta^\alpha(\vec{r}) \Delta_\beta^\alpha \\ V_\alpha^\beta(\vec{r}) \Delta_\alpha^\beta & V_\beta^\beta(\vec{r}) \end{pmatrix} = \begin{pmatrix} \left(\frac{\nabla^2}{2\mu_\alpha} - \frac{\partial}{\partial t}\right) R_{E_0}^\alpha(t, \vec{r}) & \left(\frac{\nabla^2}{2\mu_\beta} - \frac{\partial}{\partial t}\right) R_{E_1}^\alpha(t, \vec{r}) \\ \left(\frac{\nabla^2}{2\mu_\alpha} - \frac{\partial}{\partial t}\right) R_{E_0}^\beta(t, \vec{r}) & \left(\frac{\nabla^2}{2\mu_\beta} - \frac{\partial}{\partial t}\right) R_{E_1}^\beta(t, \vec{r}) \end{pmatrix} \begin{pmatrix} R_{E_0}^\alpha(t, \vec{r}) & R_{E_1}^\alpha(t, \vec{r}) \\ R_{E_0}^\beta(t, \vec{r}) & R_{E_1}^\beta(t, \vec{r}) \end{pmatrix}^{-1}$$

*S=-2 baryon-baryon potentials*

# Baryon-baryon system with $S=-2$

## Spin singlet states

Isospin	BB channels		
$I=0$	$\Lambda\Lambda$	$N\Xi$	$\Sigma\Sigma$
$I=1$	$N\Xi$	$\Lambda\Sigma$	---
$I=2$	$\Sigma\Sigma$	---	---

## Spin triplet states

Isospin	BB channels		
$I=0$	$N\Xi$	---	---
$I=1$	$N\Xi$	$\Lambda\Sigma$	$\Sigma\Sigma$

## Relations between BB channels and SU(3) irreducible representations

$$8 \times 8 = 27 + 8_s + 1 + 10 + 10 + 8_A$$

$J^P=0^+, I=0$

$$\begin{pmatrix} \Lambda\Lambda \\ N\Xi \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -\sqrt{5} & -\sqrt{8} & \sqrt{27} \\ \sqrt{20} & \sqrt{8} & \sqrt{12} \\ \sqrt{15} & -\sqrt{24} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 8 \\ 27 \end{pmatrix}$$

$J^P=1^+, I=0$

$$N\Xi \leftrightarrow 8$$

$J^P=0^+, I=1$

$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \end{pmatrix} = \frac{1}{5} \begin{pmatrix} \sqrt{2} & -\sqrt{3} \\ \sqrt{3} & \sqrt{2} \end{pmatrix} \begin{pmatrix} 27 \\ 8 \end{pmatrix}$$

$J^P=0^+, I=2$

$$\Sigma\Sigma \leftrightarrow 8$$

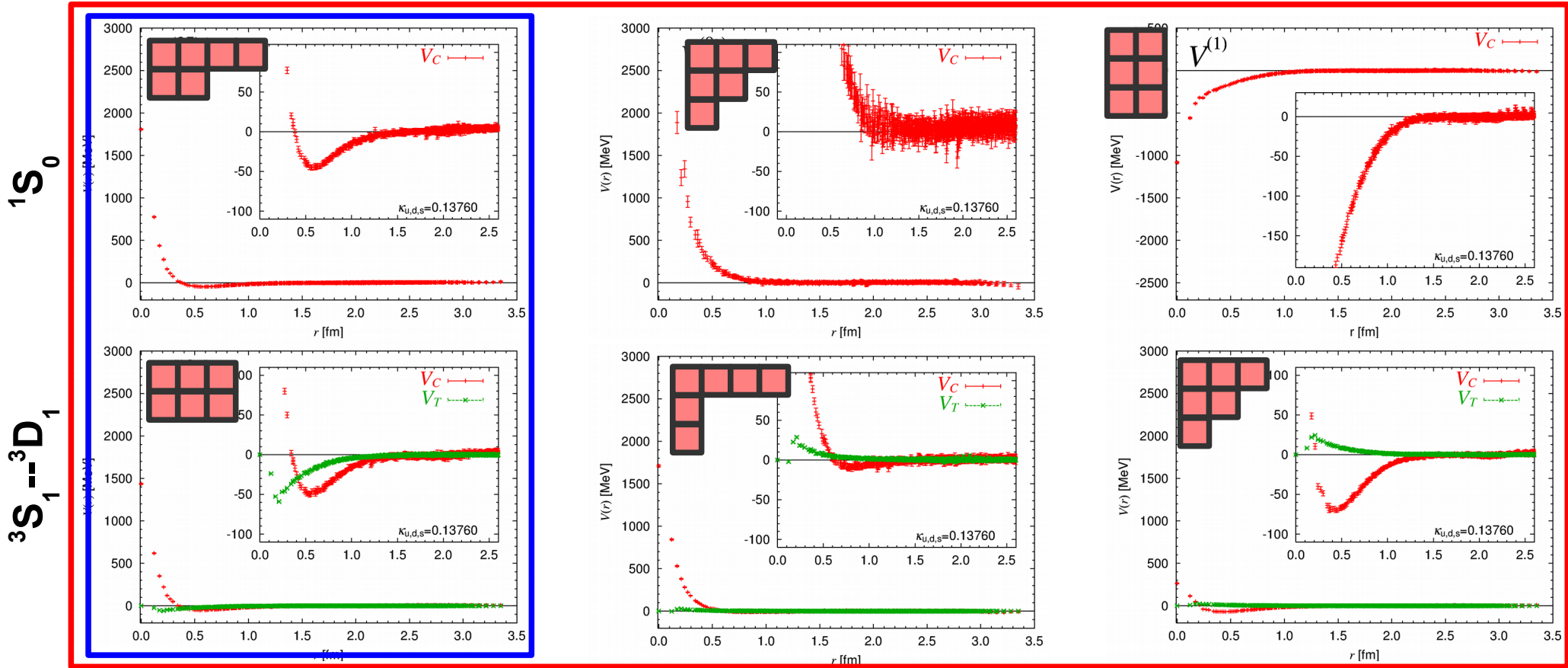
$J^P=1^+, I=1$

$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{2} & -\sqrt{2} & \sqrt{2} \\ \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & \sqrt{4} \end{pmatrix} \begin{pmatrix} 8 \\ 10 \\ 10 \end{pmatrix}$$

Features of flavor singlet interaction is integrated into the  $S=-2$   $J^P=0^+, I=0$  system.

# B-B potentials in SU(3) limit

$m_\pi = 469 \text{ MeV}$



Two-flavors

Three-flavors

- Quark Pauli principle can be seen at around short distances
  - No repulsive core in flavor singlet state
  - Strongest repulsion in flavor 8s state
- **Possibility of bound H-dibaryon in flavor singlet channel.**



# $SU(3)$ feature of BB interaction

In view of quark degrees of freedom

Oka, Shimizu and Yazaki NPA464 (1987)

- Short range repulsion in BB interaction could be a result of Pauli principle and color-magnetic interaction for the quarks.
- Strengths of repulsive core in YN and YY interaction are largely depend on their flavor structures.
- For the s-wave BB system, **no repulsive core** is predicted in **flavor singlet state** which is known as **H-dibaryon** channel.

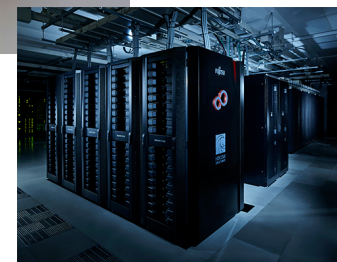
## ● Flavor symmetry aspect

	1	8s	27	8a	10	10*
Pauli	allowed	forbidden	---	allowed	forbidden	---
CMI	attractive	repulsive	repulsive	repulsive	repulsive	repulsive
Short range int.	attractive	repulsive	repulsive	repulsive	repulsive	repulsive

# Numerical setup

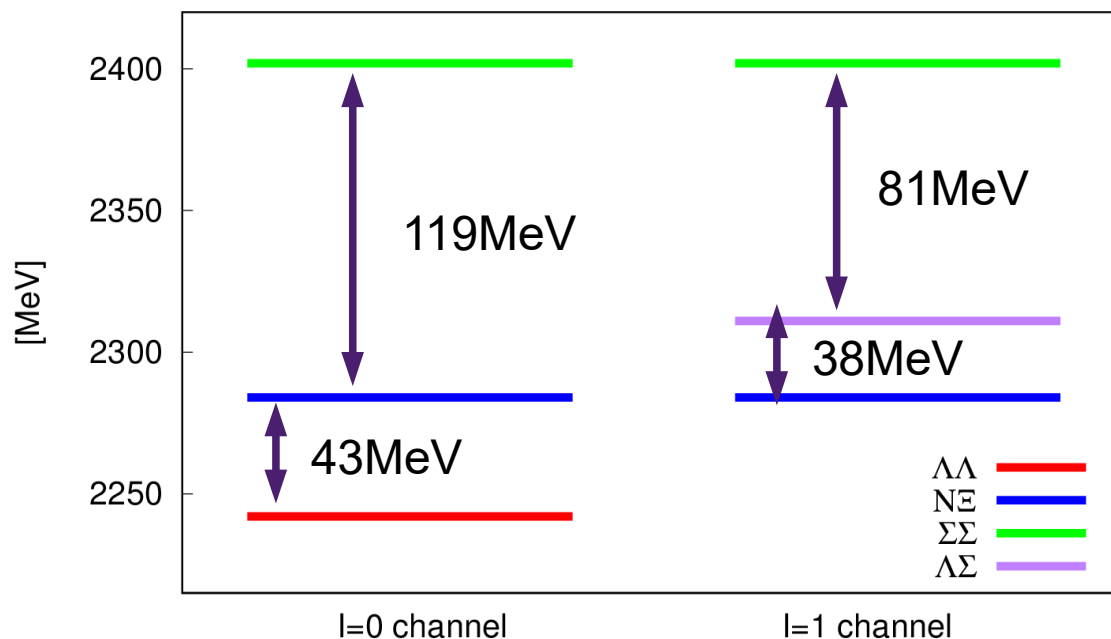
▶ **2+1 flavor** gauge configurations.

- Iwasaki gauge action &  $O(a)$  improved Wilson quark action
- $a = 0.085 [fm]$ ,  $a^{-1} = 2.333 \text{ GeV}$ .
- $96^3 \times 96$  lattice,  $L = 8.21 [fm]$ .
- 414 confs x 48 sources x 4 rotations.



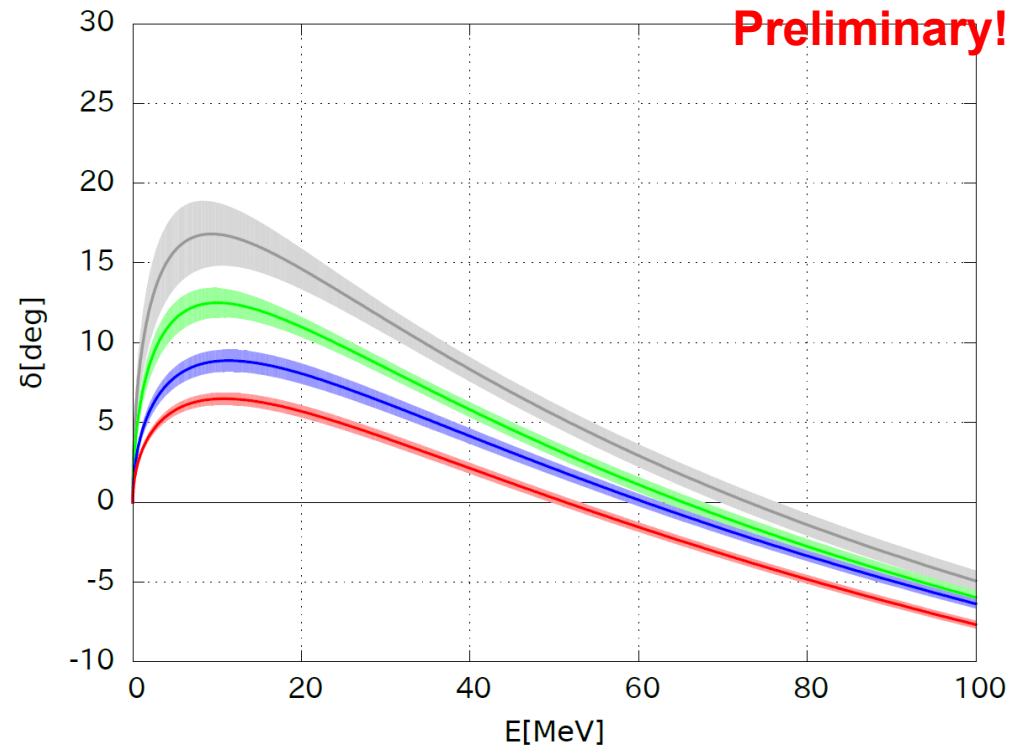
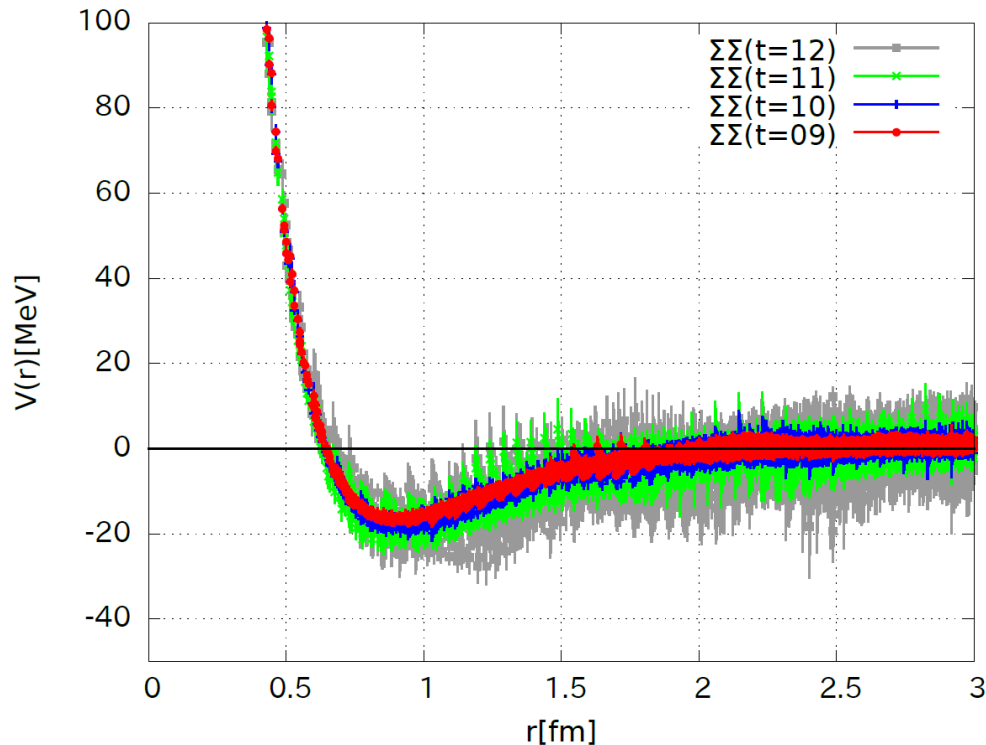
▶ **Wall source** is considered to produce S-wave B-B state.

	Mass [MeV]
$\pi$	146
$K$	525
$m_\pi / m_K$	0.28
$N$	$953 \pm 7$
$\Lambda$	$1123 \pm 3$
$\Sigma$	$1204 \pm 1$
$\Xi$	$1336 \pm 2$



# $\Sigma\Sigma (I=2) ^1S_0$ channel

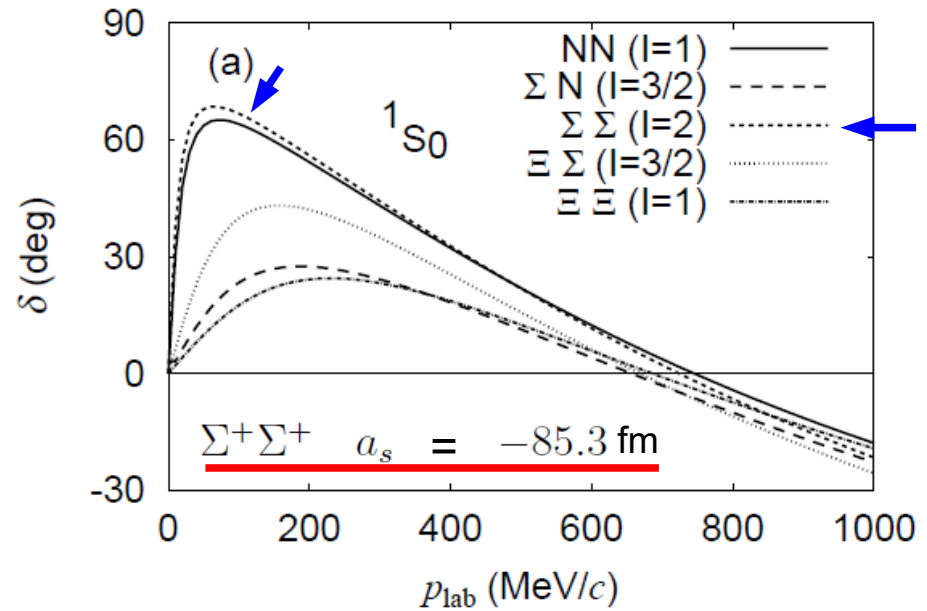
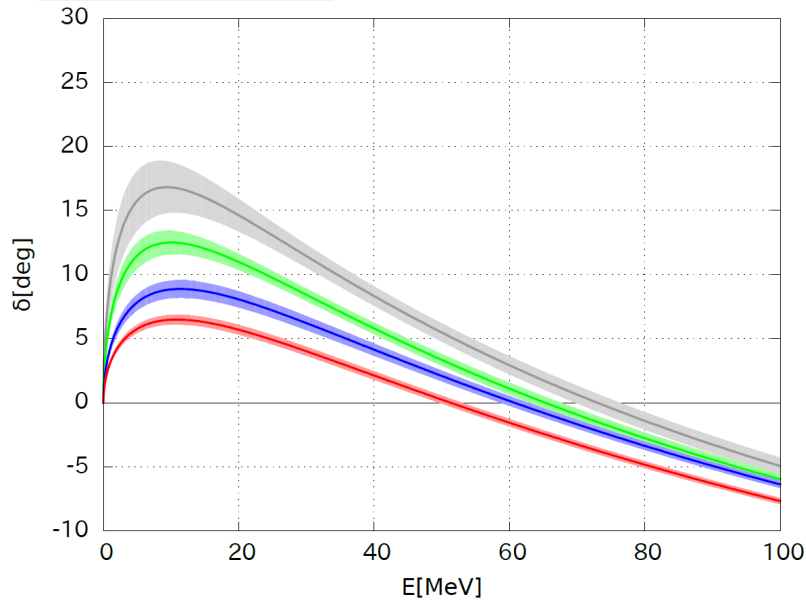
►  $N_f = 2+1$  full QCD  $m_\pi=146\text{MeV}$  with  $L = 8.1\text{fm}$



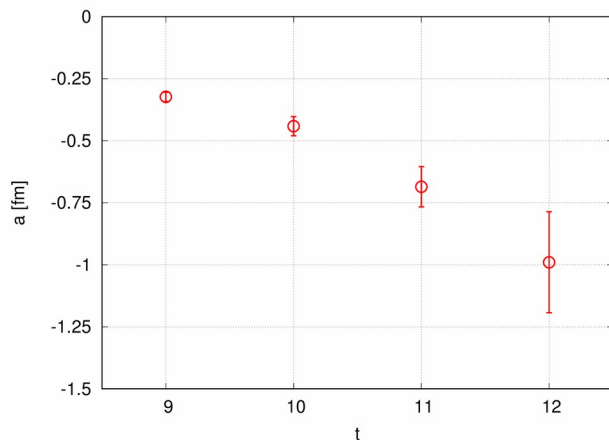
- $\Sigma\Sigma (I=2)$  potential belongs to the 27plet in flavor  $SU(3)$  limit.
- The potential has an attractive pocket and repulsive core.
- In this time range, potentials are qualitatively similar.
- Potential is saturated even  $t=12\dots$

# $\Sigma\Sigma (I=2) ^1S_0$ channel –comparison–

## Phase shift



## Scattering length



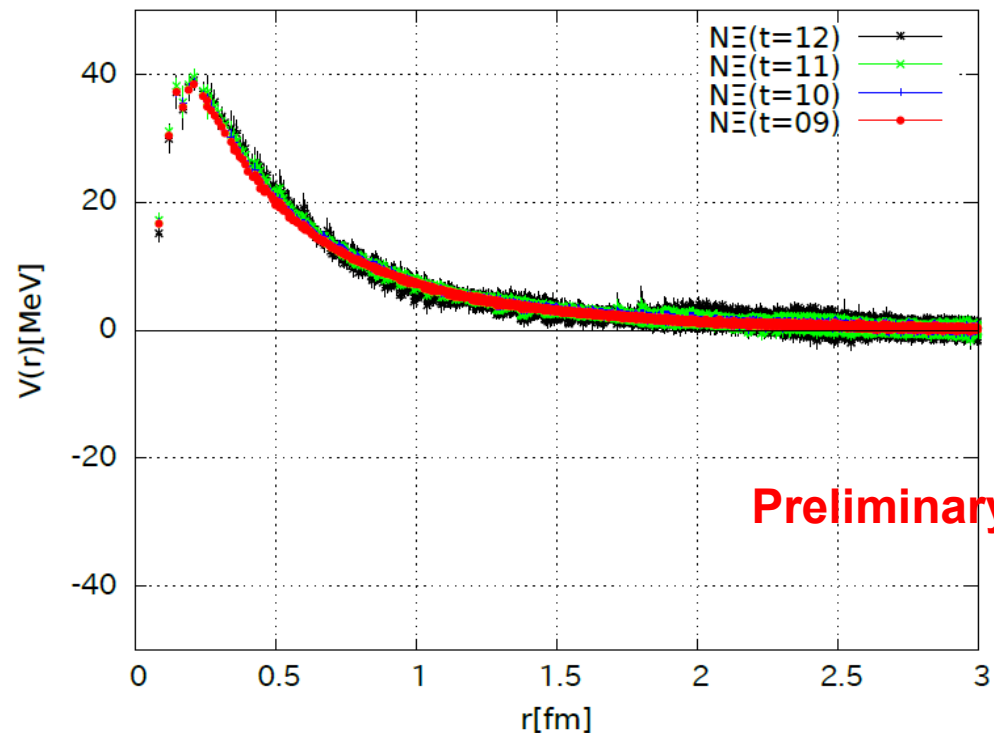
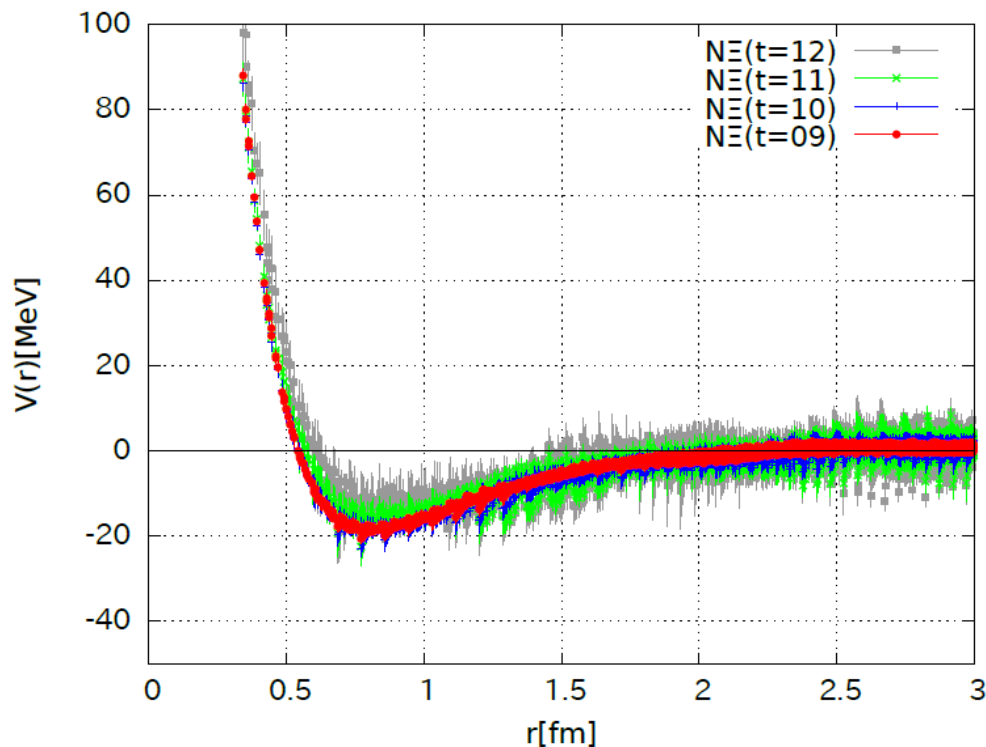
$\Lambda$		NLO				LO			
		500	550	600	650	550	600	650	700
$\Sigma^+\Sigma^+$	$a_{1S0}$	-2.19	-1.94	-1.83	-1.82	-6.23	-7.76	-9.42	-9.27

J. Haidenbauer et al, NPA954(2016)273

- Scattering length in  $\Sigma\Sigma (I=2)$  channel is not saturated.
- Attraction is strengthened as increasing  $t$ .
- Attraction is much weaker than the phenomenological one.

# $N\Xi (I=0) {}^3S_1 - {}^3D_1$ channel

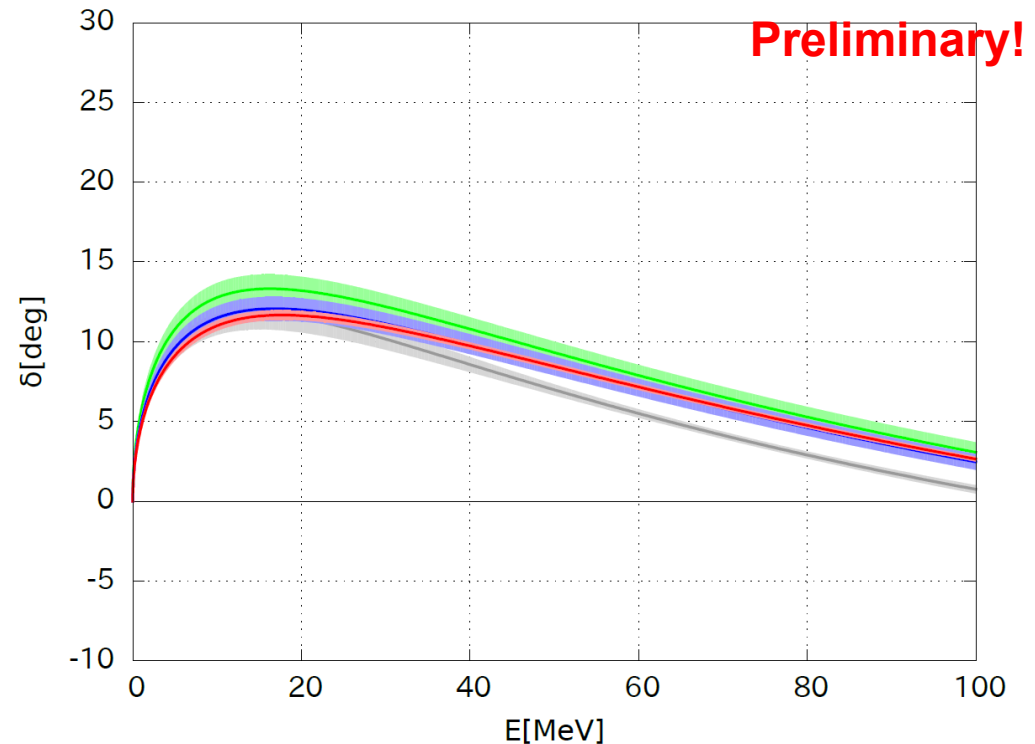
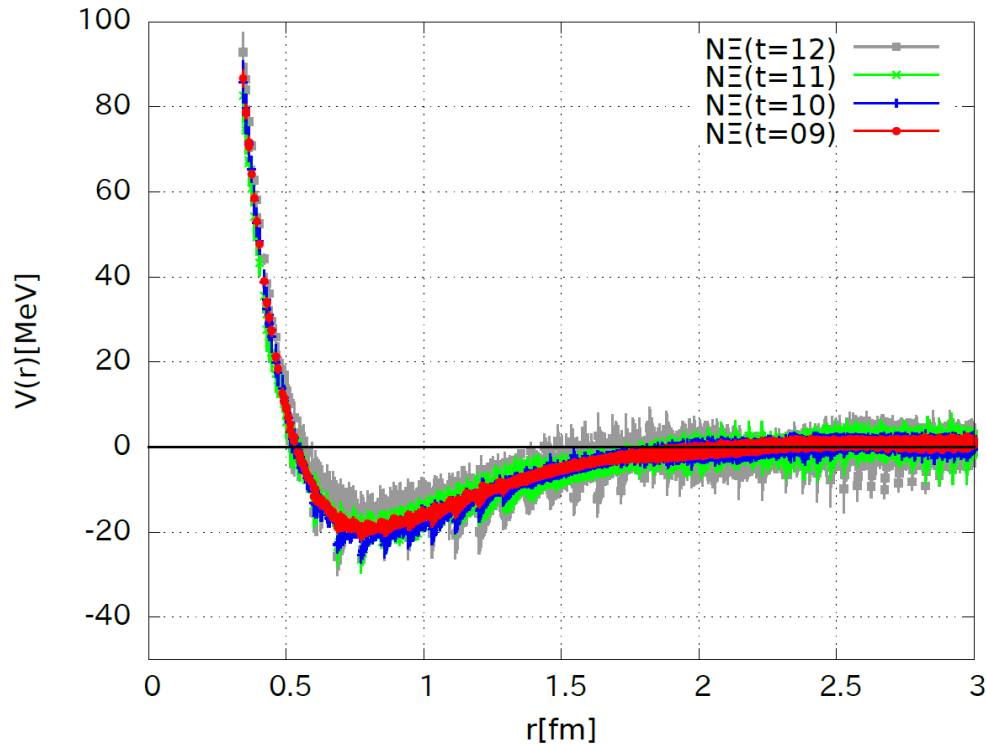
►  $N_f = 2+1$  full QCD  $m_\pi=146\text{MeV}$  with  $L = 8.1\text{fm}$



- $N\Xi (I=0, \text{spin triplet})$  potential belongs to the 8plet in flavor  $SU(3)$  limit.
- The potential has an attractive pocket and repulsive core.
- Tensor potential is weaker than the phenomenological NN tensor potential.

# $N\Xi (I=0) {}^3S_1$ channel (effective central potential)

►  $N_f = 2+1$  full QCD  $m_\pi=146\text{MeV}$  with  $L = 8.1\text{fm}$

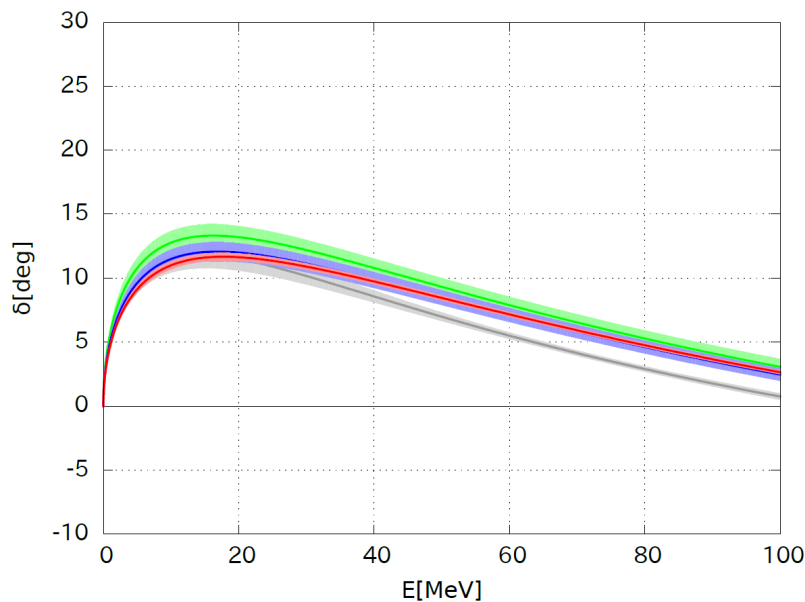


Preliminary!

- Effective  $N\Xi (I=0, \text{spin triplet})$  central potential is plotted.
- It belongs to the 8a-plet in the  $SU(3)$  limit.
- Phase shifts at low energies are same within the error bars.

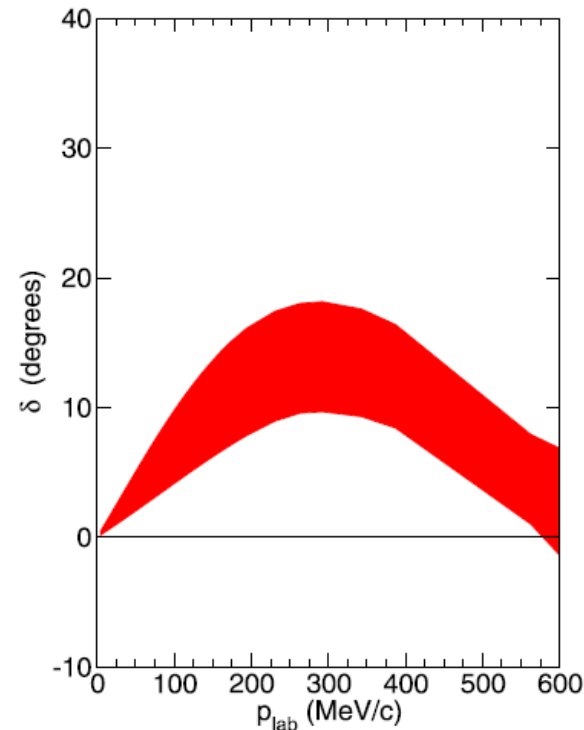
# $N\Xi (I=0) {}^3S_1$ channel –comparison–

## Phase shift

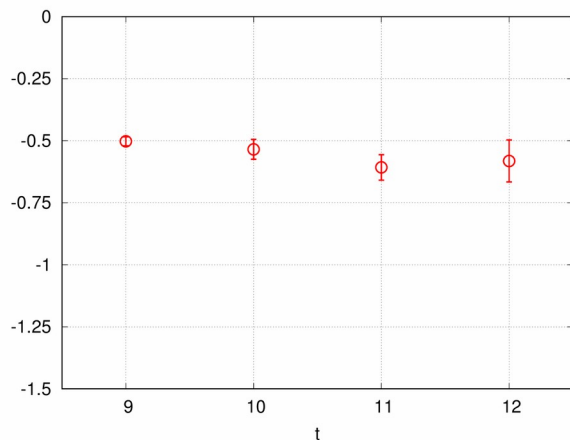


$N\Xi {}^3S_1 (I=0)$

Preliminary!



## Scattering length



$\Lambda$		NLO				LO			
		500	550	600	650	550	600	650	700
$I=0$	$a_{3S1}$	-0.33	-0.39	-0.62	-0.85	-0.85	-0.59	-0.43	-0.32
	$r_{3S1}$	-6.87	-1.77	1.00	1.42	-1.84	-3.93	-7.16	-12.56

J. Haidenbauer et al, NPA954(2016)273

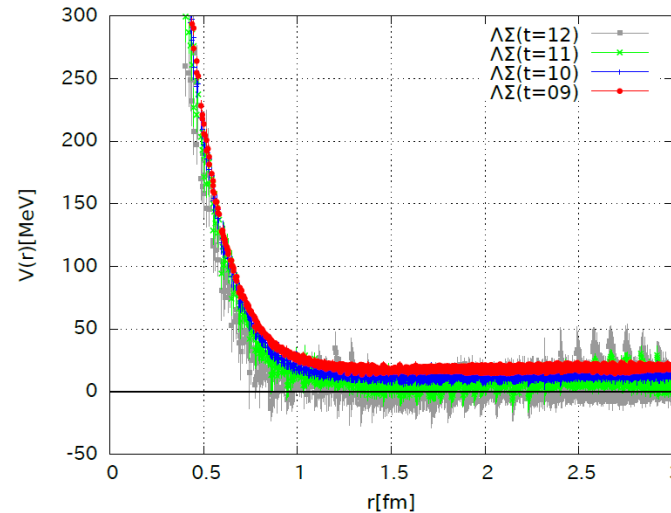
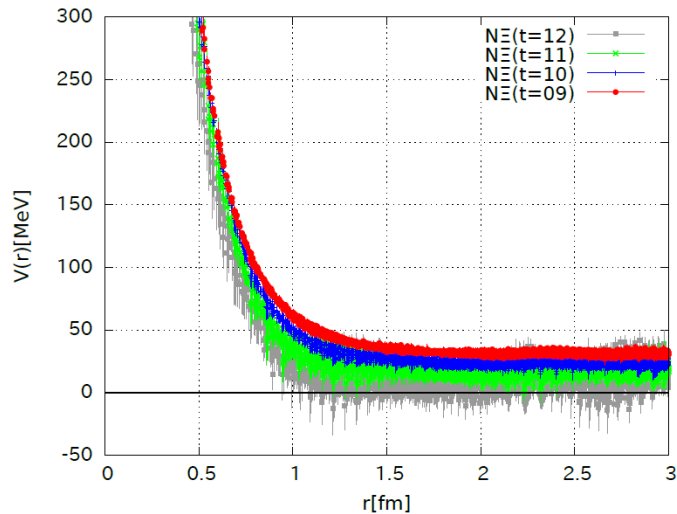
- Phase shift and scattering length are saturated (?).
- They are comparable with the results of NLO EFT calculation.

# $N\Xi, \Lambda\Sigma (l=1) {}^1S_0$ channel

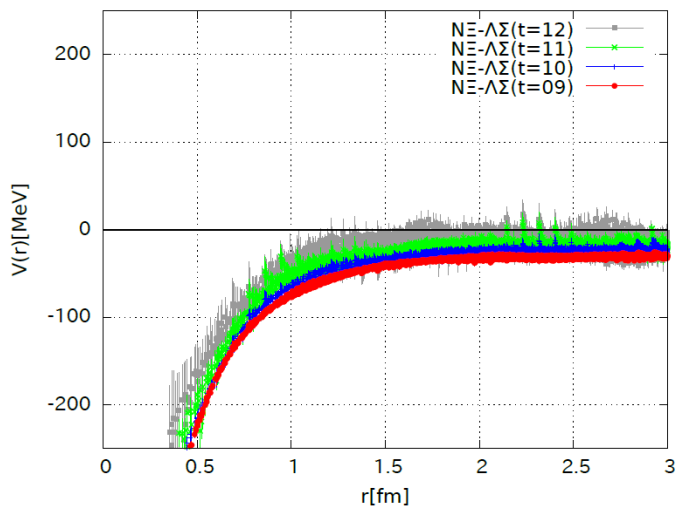
►  $N_f = 2+1$  full QCD  $m_\pi=146\text{MeV}$  with  $L = 8.1\text{fm}$

Preliminary!

## Diagonal elements



## Off-diagonal elements



$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \end{pmatrix} = \frac{1}{5} \begin{pmatrix} \sqrt{2} & -\sqrt{3} \\ \sqrt{3} & \sqrt{2} \end{pmatrix} \begin{pmatrix} 27 \\ 8 \end{pmatrix}$$

- Diagonal elements are repulsive in whole range.
- Diagonal  $N\Xi$  potential is strongly repulsive.
  - It means that the  $N\Xi$  potential is strongly depend on the channel.
- Potentials are not saturated in this time range.



# $N\Xi, \Lambda\Sigma, \Sigma\Sigma (l=1) {}^3S-D_1$ channel (central pot)

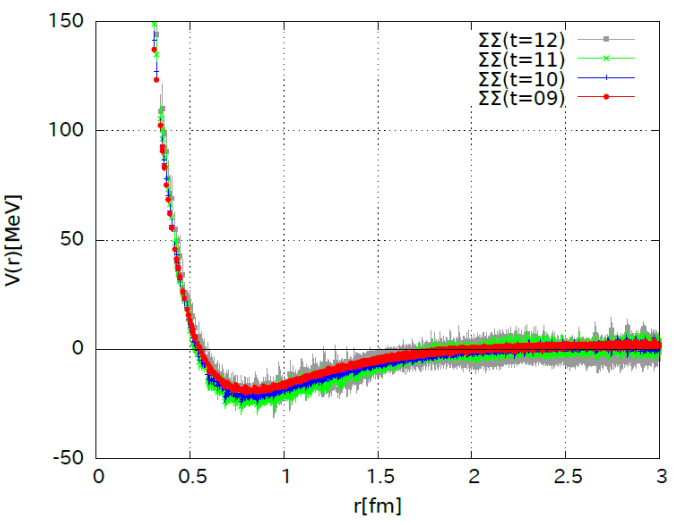
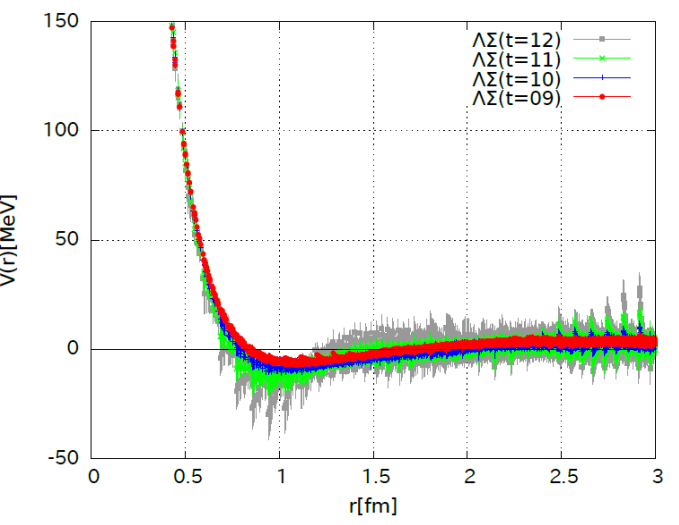
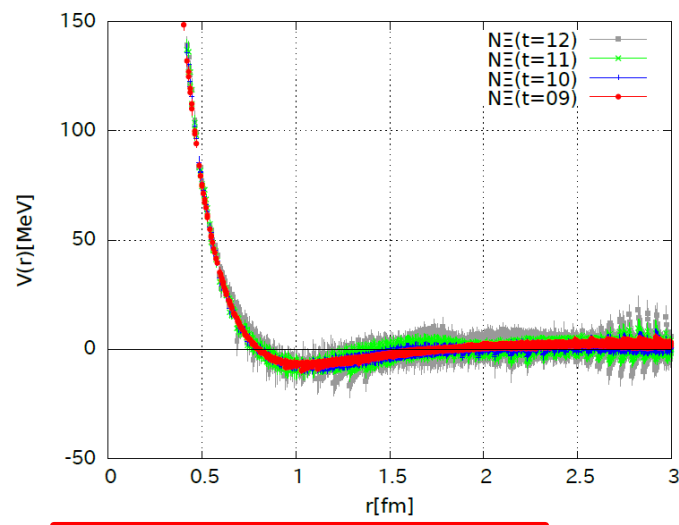
$t=09$   
 $t=10$   
 $t=11$   
 $t=12$

►  $N_f = 2+1$  full QCD  $m_\pi=146\text{MeV}$  with  $L = 8.1\text{fm}$

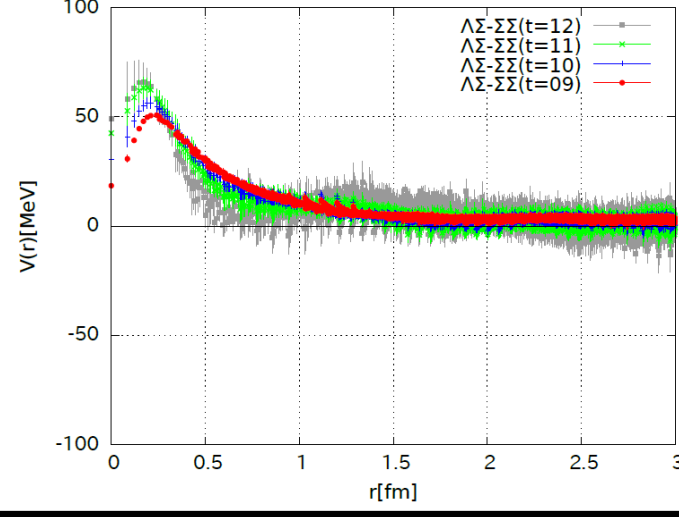
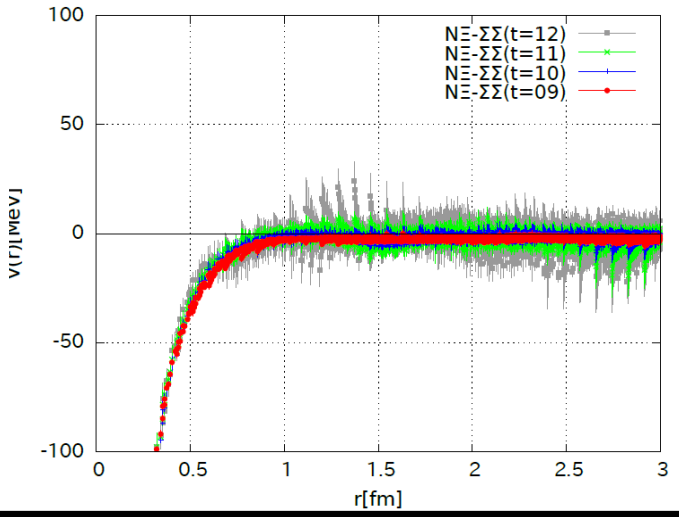
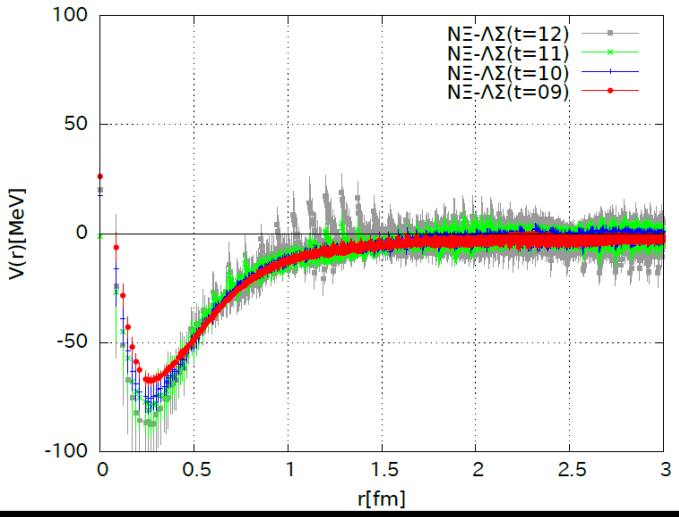
$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{2} & -\sqrt{2} & \sqrt{2} \\ \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & \sqrt{4} \end{pmatrix} \begin{pmatrix} 8 \\ 10 \\ 10 \end{pmatrix}$$

48src

## Diagonal elements



## Off-diagonal elements



# $N\Xi, \Lambda\Sigma, \Sigma\Sigma (I=1) {}^3S-D_1$ channel (tensor pot)

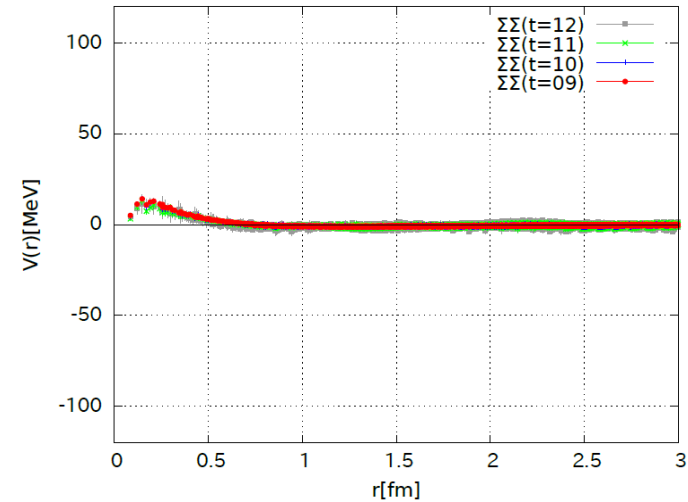
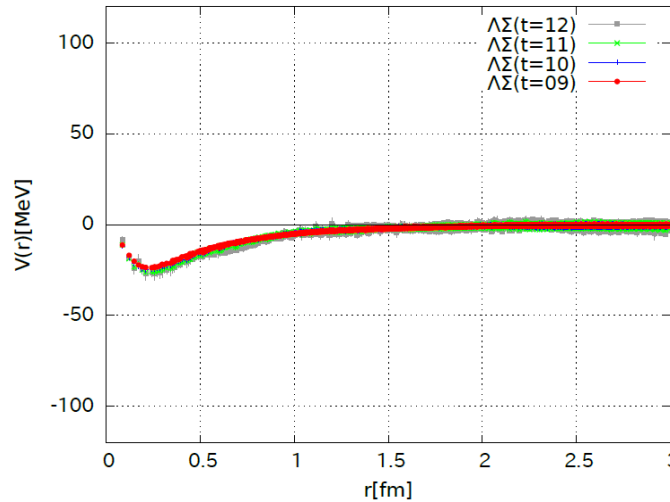
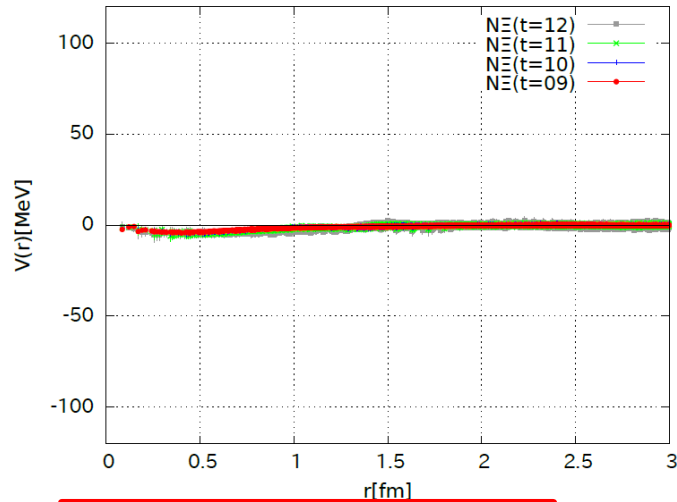
$t=09$   
 $t=10$   
 $t=11$   
 $t=12$

►  $N_f = 2+1$  full QCD  $m_\pi=146\text{MeV}$  with  $L = 8.1\text{fm}$

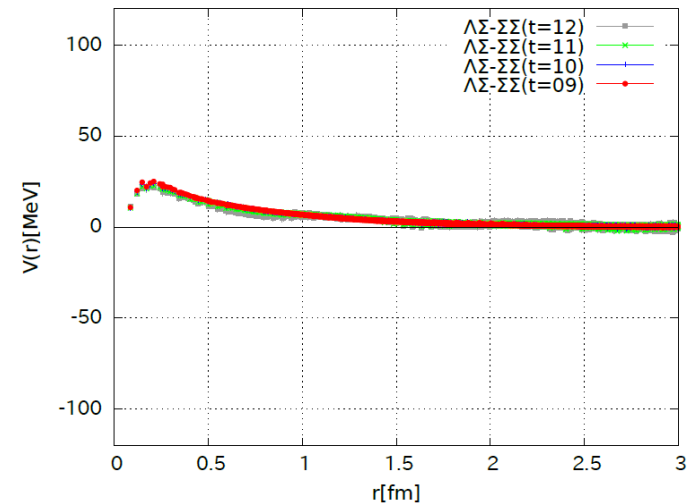
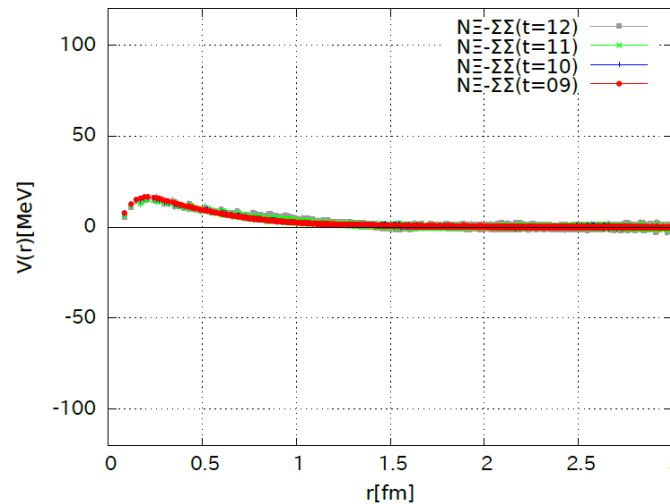
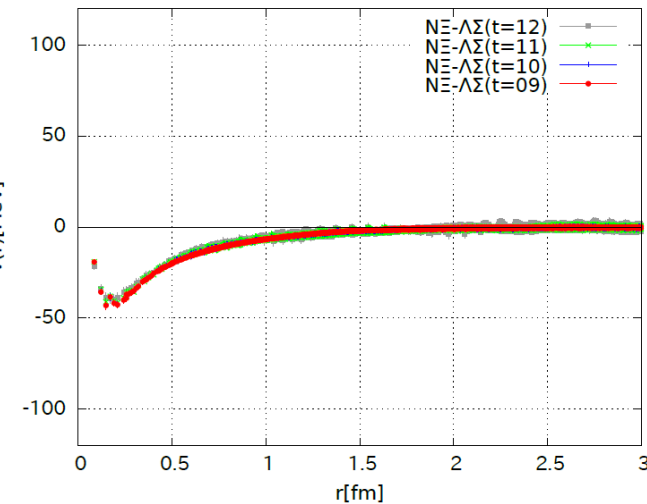
$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{2} & -\sqrt{2} & \sqrt{2} \\ \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & \sqrt{4} \end{pmatrix} \begin{pmatrix} 8 \\ 10 \\ 10 \end{pmatrix}$$

48src

## Diagonal elements



## Off-diagonal elements



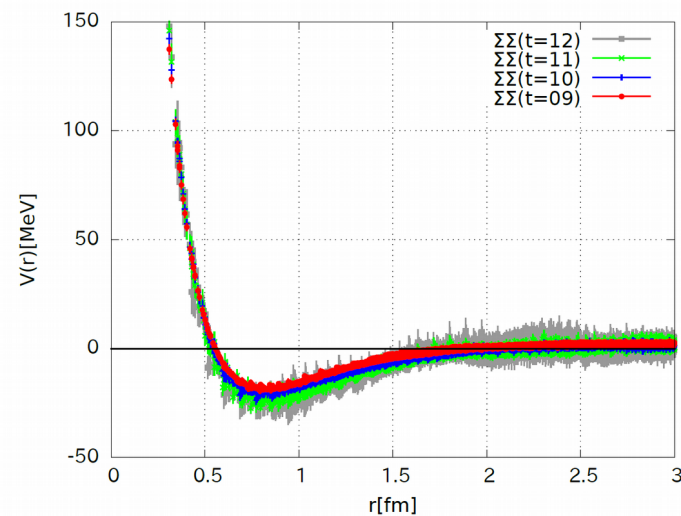
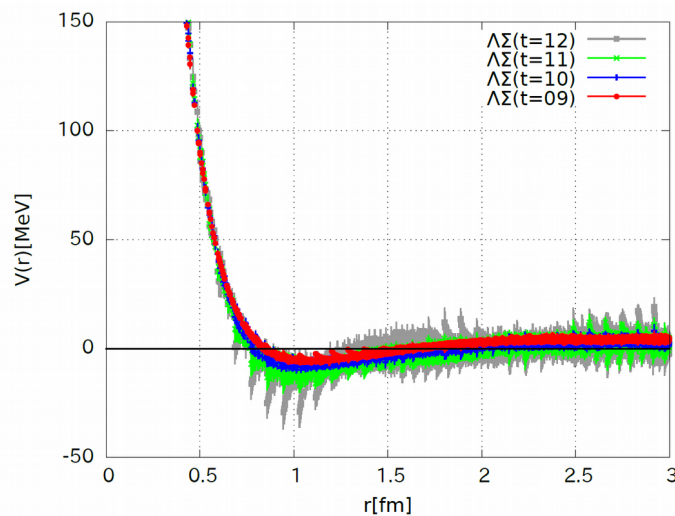
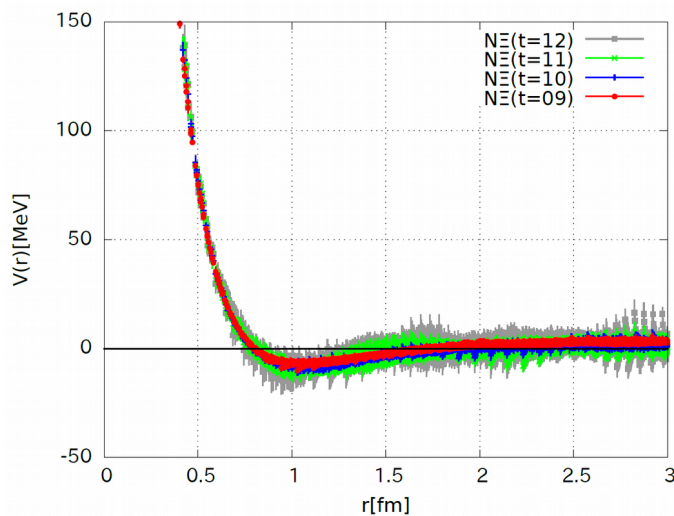
# $N\Xi, \Lambda\Sigma, \Sigma\Sigma (l=1) {}^3S_1$ channel (eff. pot.)

t=09  
t=10  
t=11  
t=12

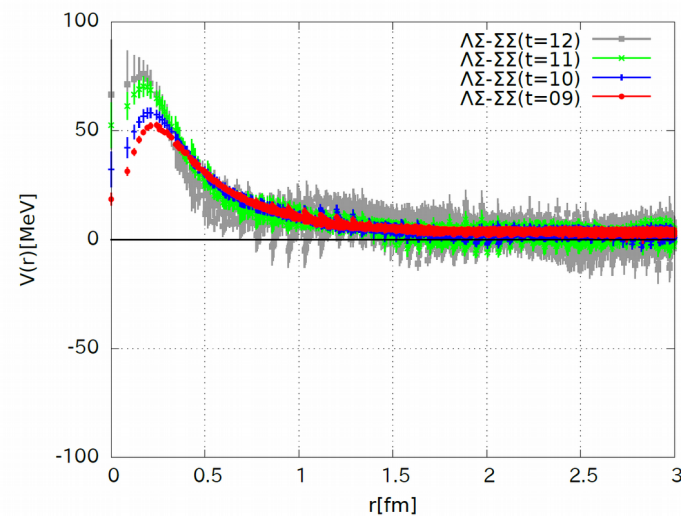
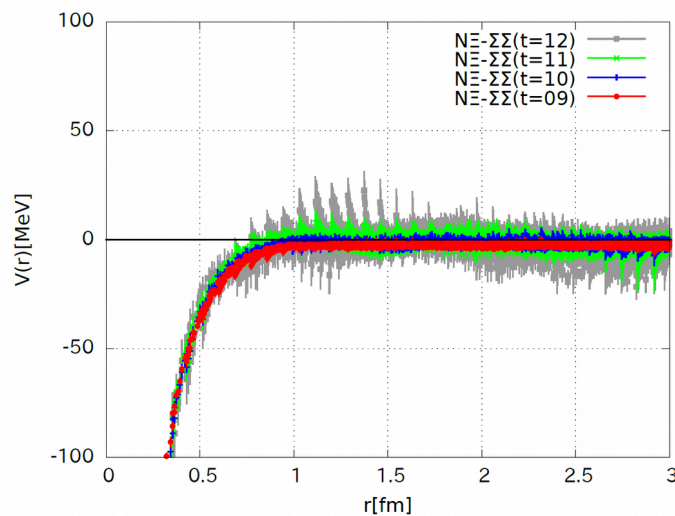
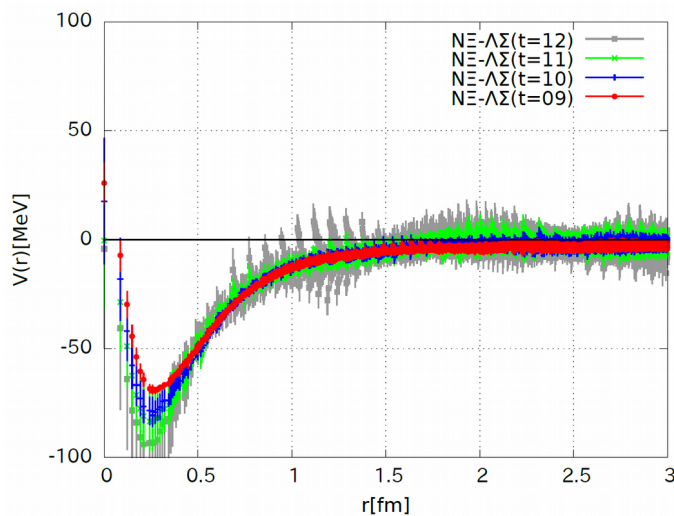
►  $N_f = 2+1$  full QCD  $m_\pi=146\text{MeV}$  with  $L = 8.1\text{fm}$

48src

## Diagonal elements



## Off-diagonal elements



# $\Lambda\Lambda, N\Xi, \Sigma\Sigma (I=0) ^1S_0$ channel

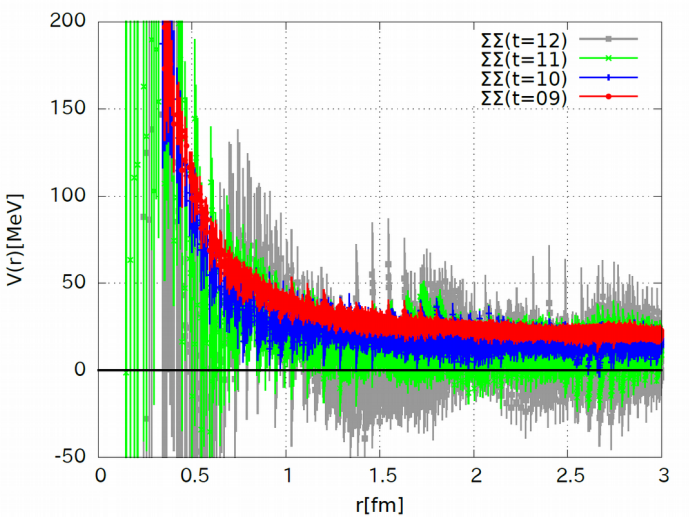
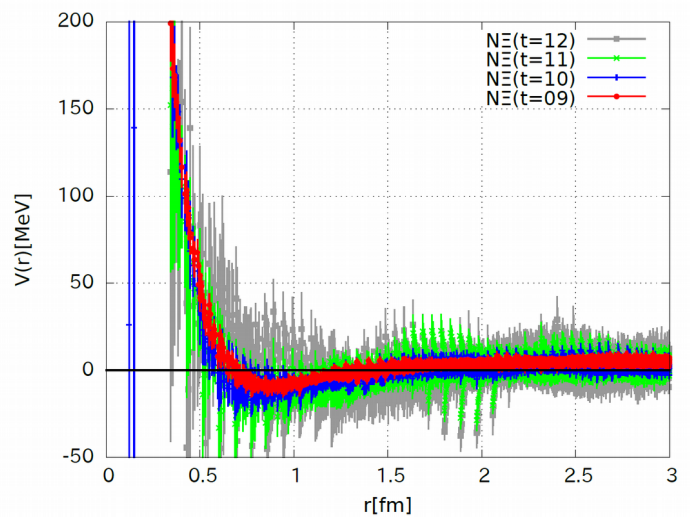
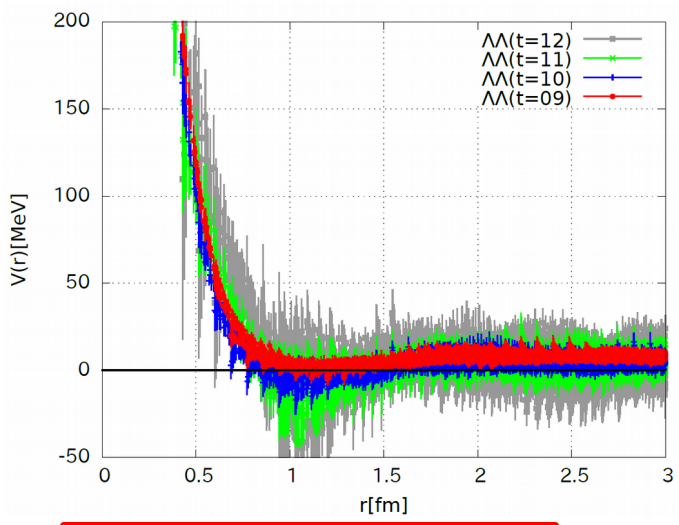
t=09  
t=10  
t=11  
t=12

► N<sub>f</sub> = 2+1 full QCD m<sub>π</sub>=146MeV with L = 8.1fm

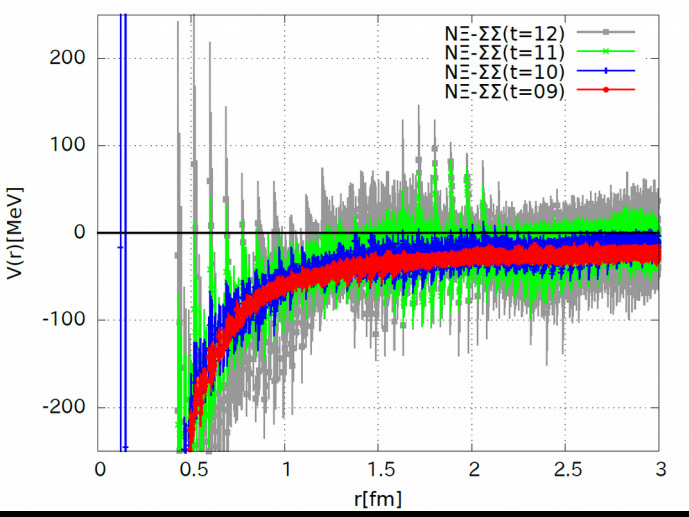
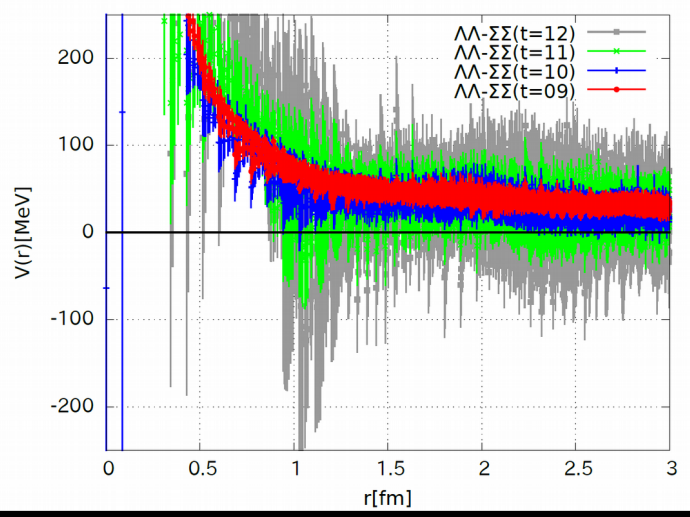
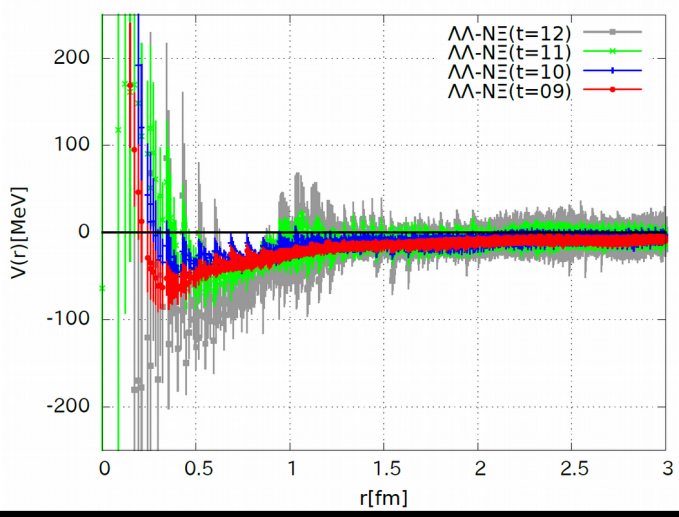
$$\begin{pmatrix} \Lambda\Lambda \\ N\Xi \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -\sqrt{5} & -\sqrt{8} & \sqrt{27} \\ \sqrt{20} & \sqrt{8} & \sqrt{12} \\ \sqrt{15} & -\sqrt{24} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 8 \\ 27 \end{pmatrix}$$

48src

## Diagonal elements



## Off-diagonal elements



# Comparison of potential matrices

Transformation of potentials

from the particle basis to the SU(3) irreducible representation (irrep) basis.

SU(3) Clebsh-Gordan coefficients

$$\begin{pmatrix} |1\rangle \\ |8\rangle \\ |27\rangle \end{pmatrix} = U \begin{pmatrix} |\Lambda\Lambda\rangle \\ |N\Xi\rangle \\ |\Sigma\Sigma\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{N\Xi} & V^{\Lambda\Lambda}_{\Sigma\Sigma} \\ V^{N\Xi}_{\Lambda\Lambda} & V^{N\Xi} & V^{N\Xi}_{\Sigma\Sigma} \\ V^{\Sigma\Sigma}_{\Lambda\Lambda} & V^{\Sigma\Sigma}_{N\Xi} & V^{\Sigma\Sigma} \end{pmatrix} U^t \rightarrow \begin{pmatrix} V_1 & & \\ & V_8 & \\ & & V_{27} \end{pmatrix}$$

$$\begin{pmatrix} \Lambda\Lambda \\ N\Xi \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -\sqrt{5} & -\sqrt{8} & \sqrt{27} \\ \sqrt{20} & \sqrt{8} & \sqrt{12} \\ \sqrt{15} & -\sqrt{24} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 8 \\ 27 \end{pmatrix}$$

In the SU(3) irreducible representation basis,

the potential matrix should be diagonal in the SU(3) symmetric configuration.



Off-diagonal part of the potential matrix in the SU(3) irrep basis would be an effective measure of the SU(3) breaking effect.

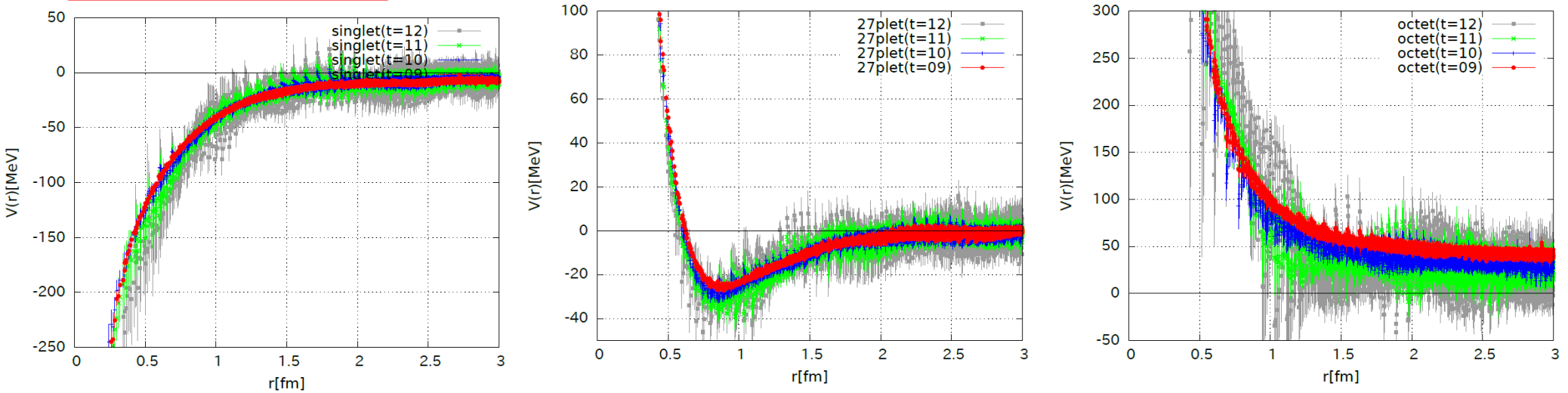
# 1, 8, 27plet ( $l=0$ ) $^1S_0$ channel

$t=09$   
 $t=10$   
 $t=11$   
 $t=12$

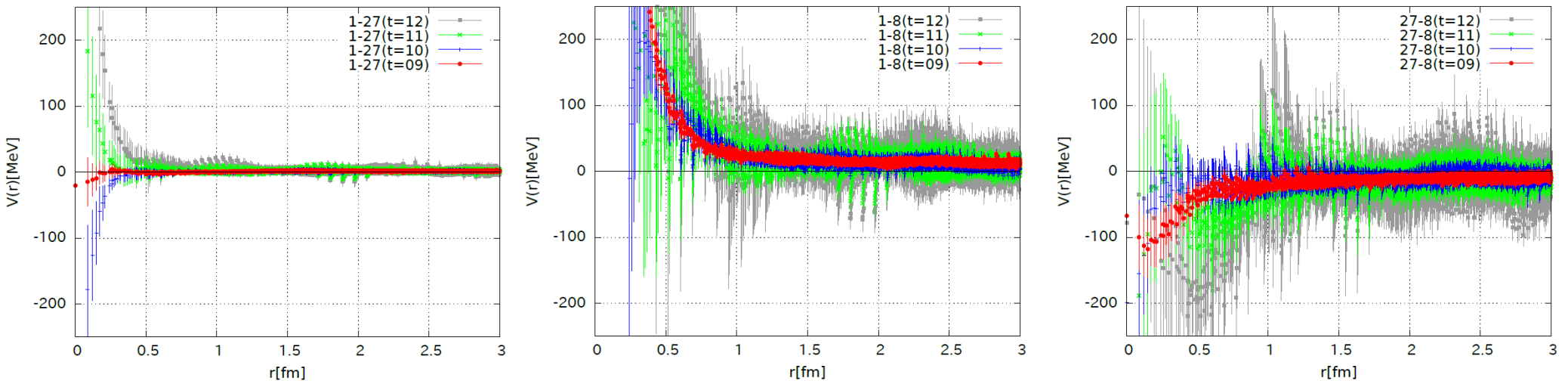
►  $N_f = 2+1$  full QCD  $m_\pi=146\text{MeV}$  with  $L = 8.1\text{fm}$

48src

## Diagonal elements



## Off-diagonal elements



*H-dibaryon channel*

# Keys to understand H-dibaryon state

A strongly bound state predicted by Jaffe in 1977 using MIT bag model.

H-dibaryon state is

- SU(3) flavor singlet [uuddss], strangeness S=-2.
- spin and isospin equals to zero, and  $J^P = 0^+$

► Strongly attractive interaction is expected in flavor singlet channel.

- Short range one-gluon exchange contributions

Strongly attractive **Color Magnetic Interaction**

- Symmetry of two-baryon system (**Pauli principle**)

Flavor singlet channel is free from Pauli blocking effect

	27	8	1	<u>10</u>	10	8
Pauli		forbidden	allowed		forbidden	
CMI	repulsive	repulsive	attractive	repulsive	repulsive	repulsive

$J^P=0^+, I=0$

$$\begin{pmatrix} \Lambda \Lambda \\ N \Xi \\ \Sigma \Sigma \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -\sqrt{5} & -\sqrt{8} & \sqrt{27} \\ \sqrt{20} & \sqrt{8} & \sqrt{12} \\ \sqrt{15} & -\sqrt{24} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 8 \\ 27 \end{pmatrix}$$

Oka, Shimizu and Yazaki NPA464 (1987)



# Works on H-dibaryon state

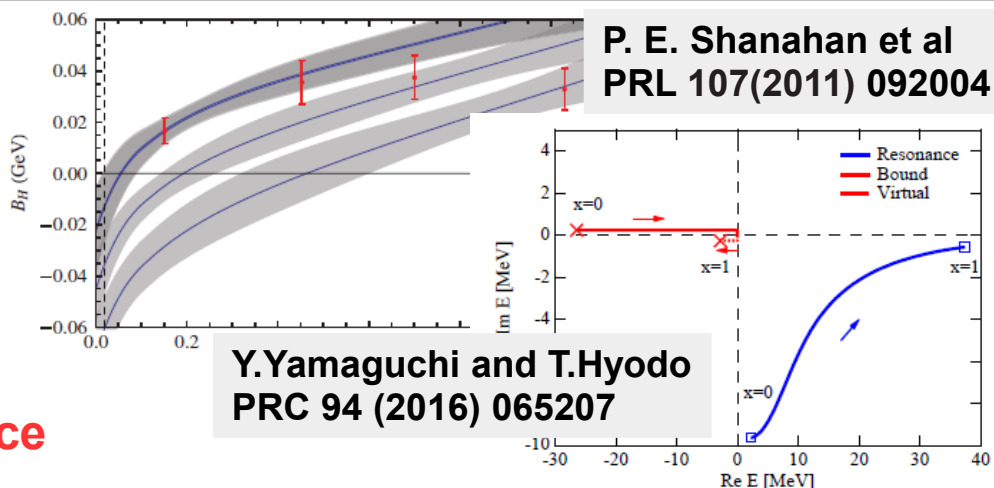
## Theoretical status

Several sort of calculations and results (bag models, NRQM, Quenched LQCD....)

There were no conclusive result.

Chiral extrapolations of recent LQCD data

Unbound or resonance

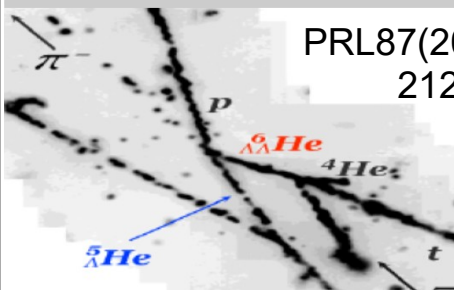


## Experimental status

### "NAGARA Event"

K. Nakazawa et al  
KEK-E176 & E373 Coll.

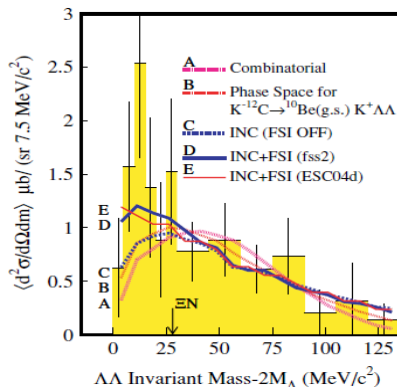
PRL87(2001)  
212502



Deeply bound dibaryon state is ruled out

### " $^{12}\text{C}(K^-, K^+ \Lambda\Lambda)$ reaction"

C.J. Yoon et al KEK-PS E522 Coll.

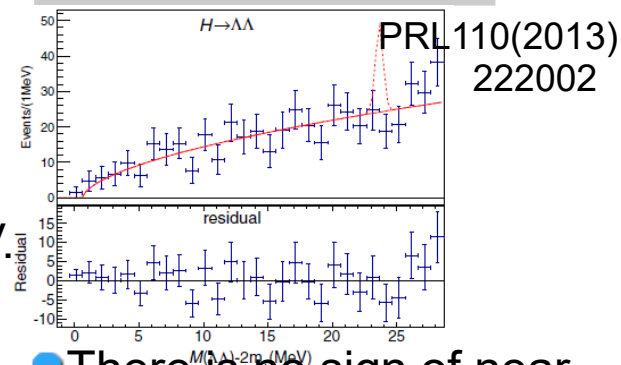


Significance of enhancements below 30 MeV.

Larger statistics  
J-PARC E42

### "Y(1S) and Y(2S) decays"

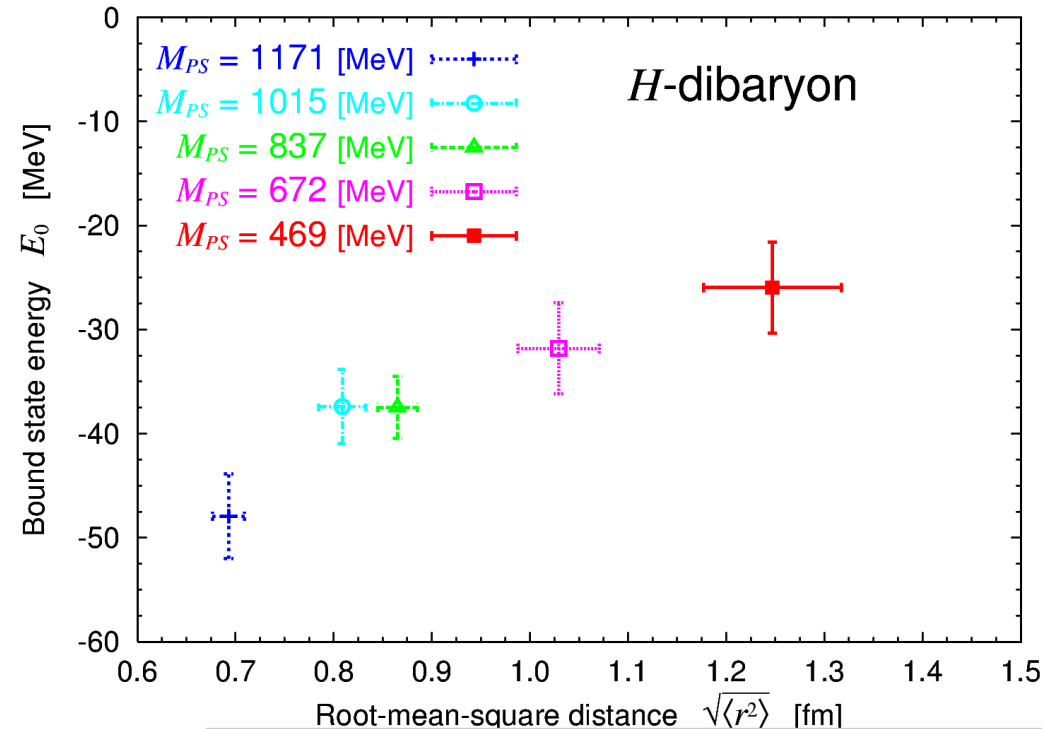
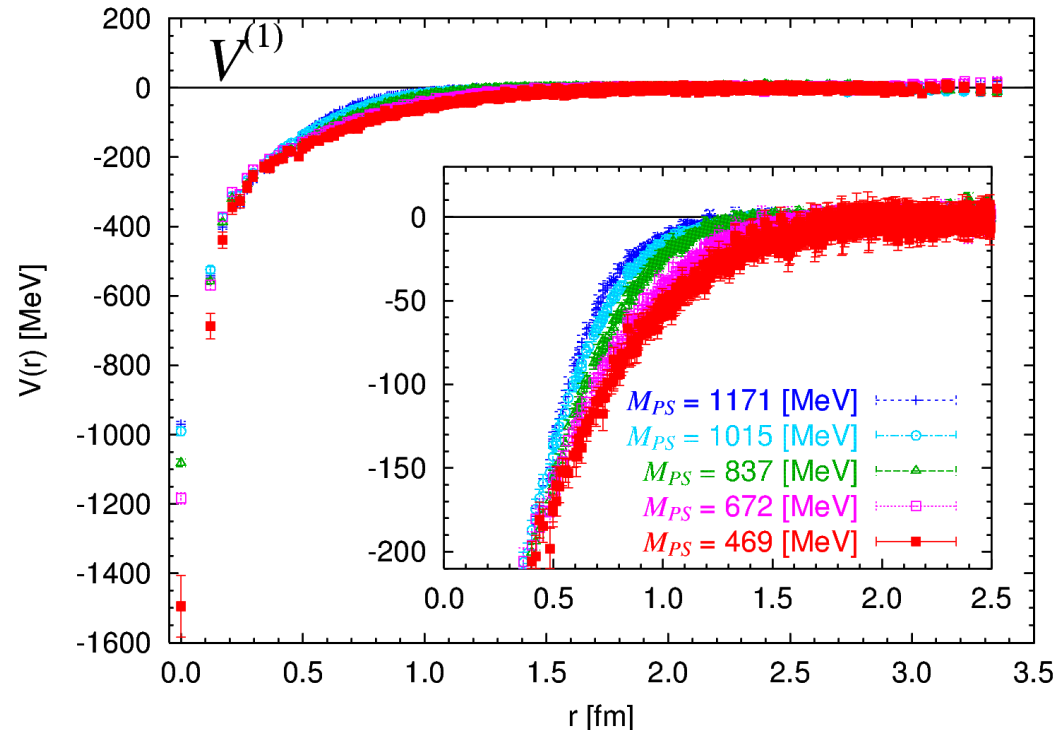
B.H. Kim et al Belle Coll.



There is no sign of near threshold enhancement.

# Hunting for H-dibaryon in SU(3) limit

Strongly attractive interaction is expected in flavor singlet channel.



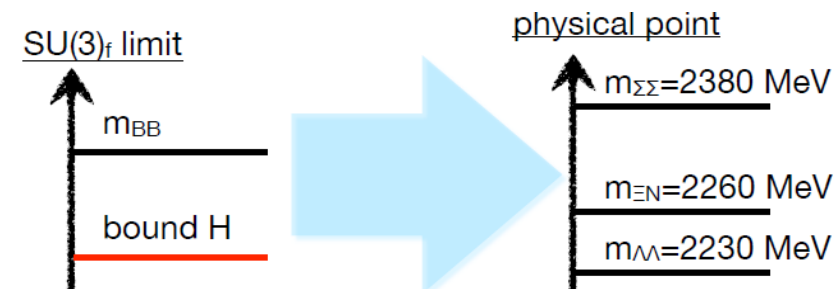
- Strongly attractive potential was found in the flavor singlet channel.
- Bound state was found in this mass range with SU(3) symmetry.

What happens at the physical point?

► SU(3) breaking effects

- Threshold separation
- Changes of interactions

T.Inoue et al[HAL QCD Coll.] NPA881(2012) 28



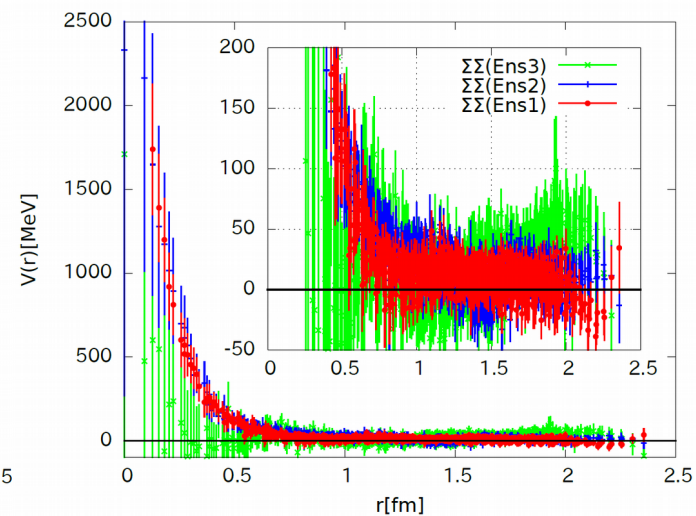
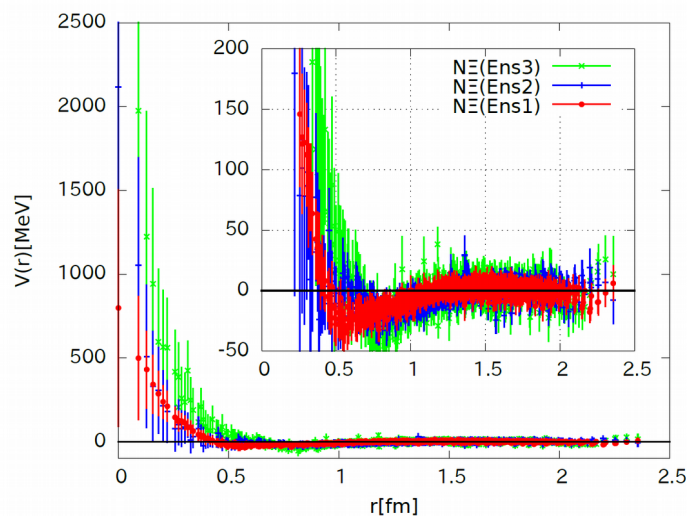
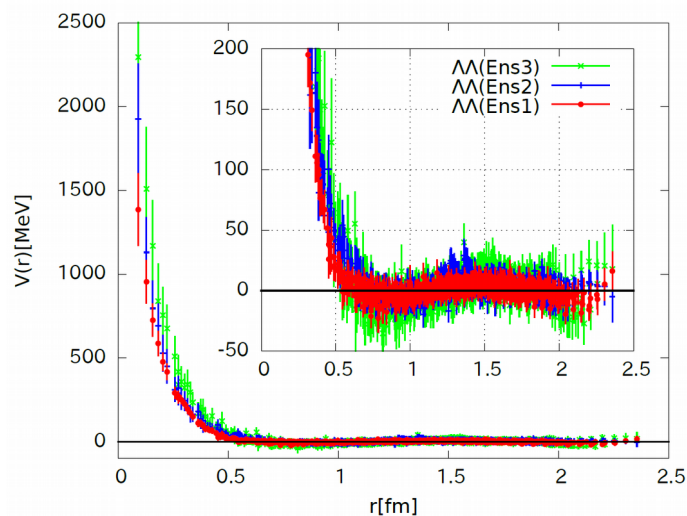
Non-trivial contributions

# $\Lambda\Lambda, N\Xi, \Sigma\Sigma$ ( $I=0$ ) $^1S_0$ channel

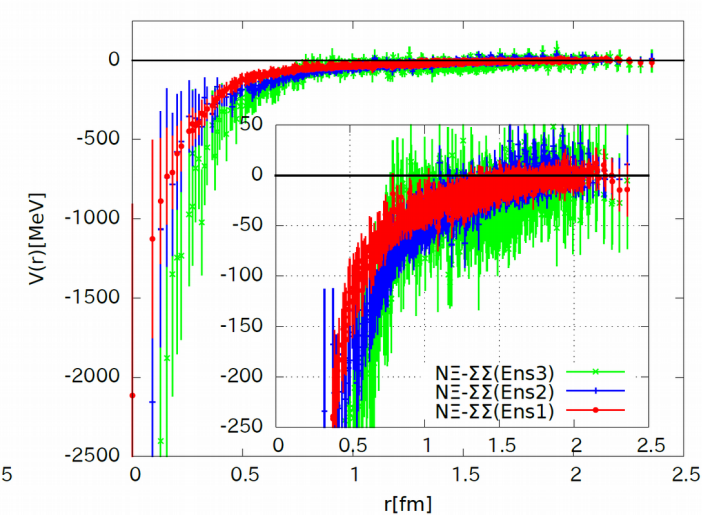
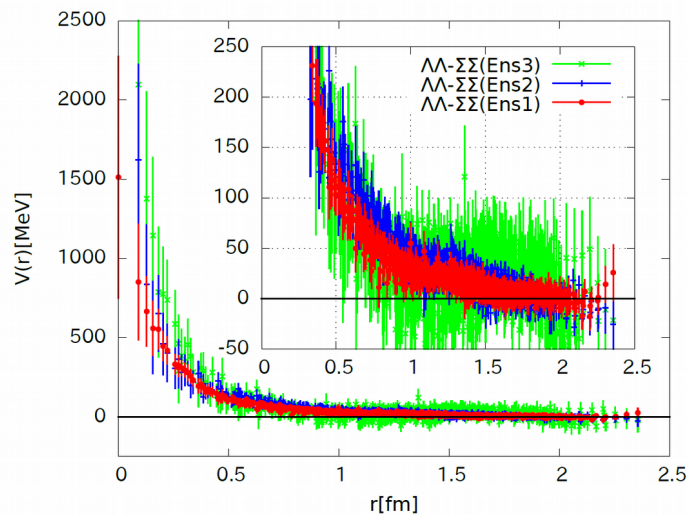
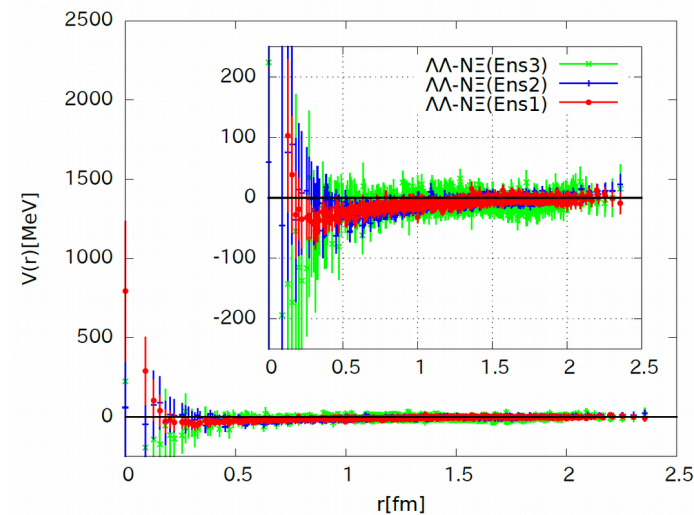
**Esb1** :  $m\pi = 701$  MeV  
**Esb2** :  $m\pi = 570$  MeV  
**Esb3** :  $m\pi = 411$  MeV

►  $N_f = 2+1$  full QCD with  $L = 2.9$  fm

## Diagonal elements



## Off-diagonal elements



# $\Lambda\Lambda$ and $N\Xi$ phase shifts

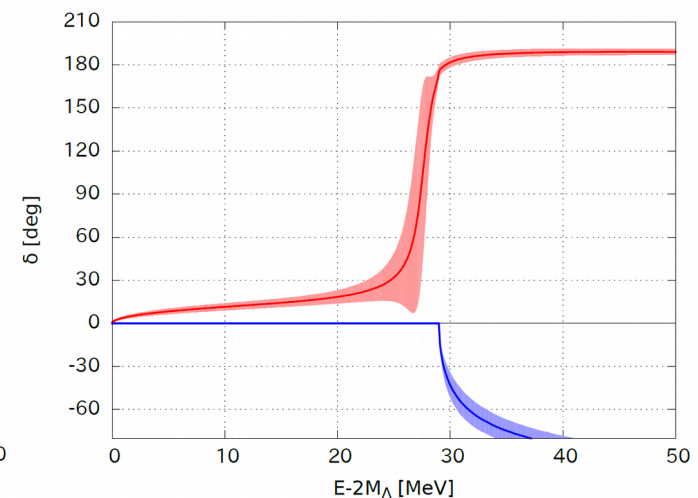
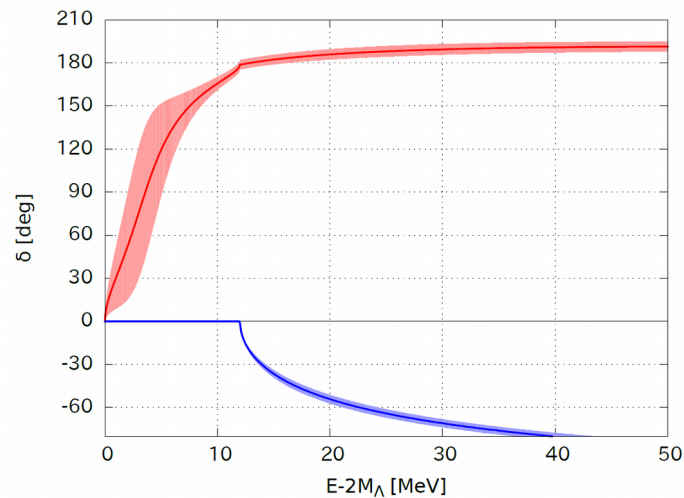
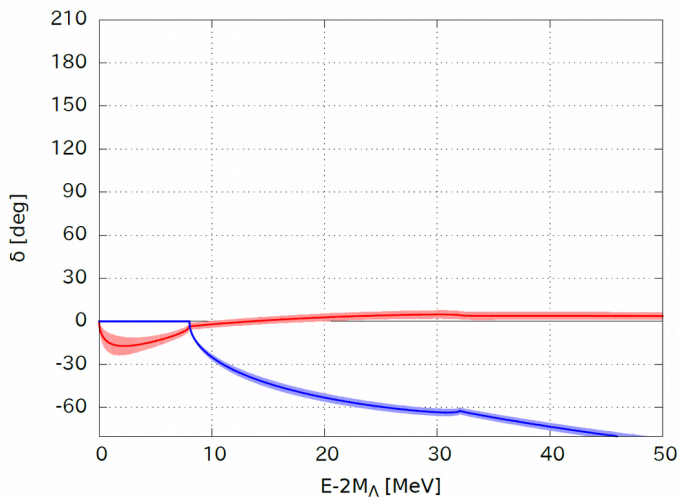
►  $N_f = 2+1$  full QCD with  $L = 2.9\text{fm}$

Preliminary!

$m_\pi = 700\text{ MeV}$

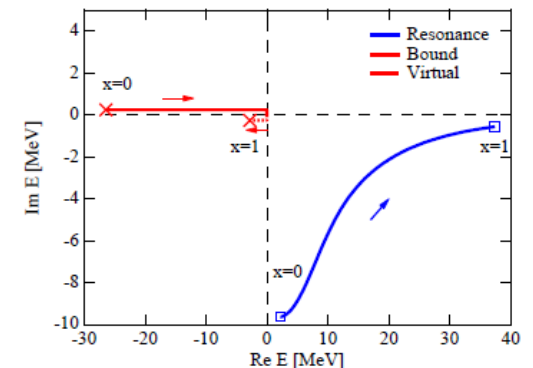
$m_\pi = 570\text{ MeV}$

$m_\pi = 410\text{ MeV}$



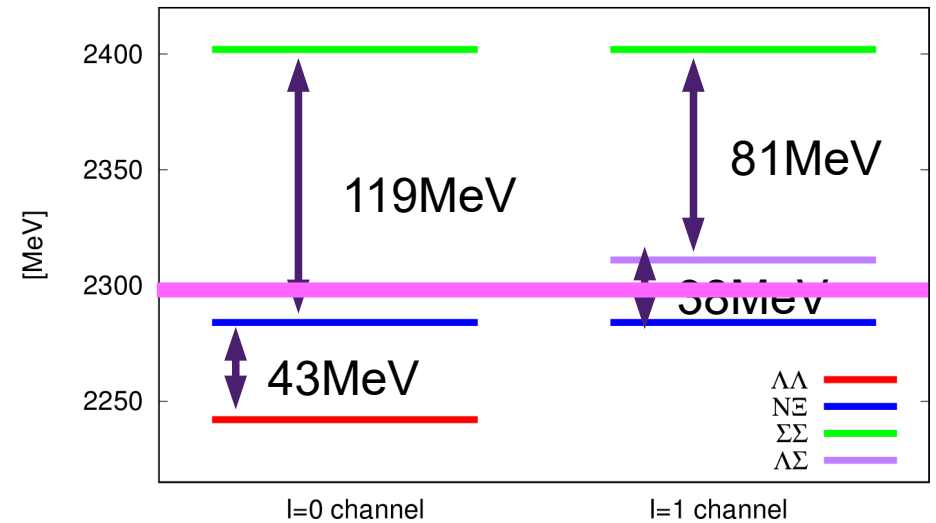
- $m_\pi = 700\text{ MeV}$  : bound state
- $m_\pi = 570\text{ MeV}$  : resonance near  $\Lambda\Lambda$  threshold
- $m_\pi = 410\text{ MeV}$  : resonance near  $N\Xi$  threshold..

Go to the physical point simulation!



Y.Yamaguchi and T.Hyodo  
PRC 94 (2016) 065207

# *Effective $\Lambda\Lambda$ and $N\Xi$ potential*



# Effective two channel potential

## ► Original coupled channel equation

$$\begin{pmatrix} (E^{\Lambda\Lambda} - H_0^{\Lambda\Lambda}) R^{\Lambda\Lambda}(\vec{r}, t) \\ (E^{\Xi N} - H_0^{\Xi N}) R^{\Xi N}(\vec{r}, t) \\ (E^{\Sigma\Sigma} - H_0^{\Sigma\Sigma}) R^{\Sigma\Sigma}(\vec{r}, t) \end{pmatrix} = \begin{pmatrix} V_{\Lambda\Lambda}^{\Lambda\Lambda}(\vec{r}) & V_{\Xi N}^{\Lambda\Lambda}(\vec{r}) & V_{\Sigma\Sigma}^{\Lambda\Lambda}(\vec{r}) \\ V_{\Lambda\Lambda}^{\Xi N}(\vec{r}) & V_{\Xi N}^{\Xi N}(\vec{r}) & V_{\Sigma\Sigma}^{\Xi N}(\vec{r}) \\ V_{\Lambda\Lambda}^{\Sigma\Sigma}(\vec{r}) & V_{\Xi N}^{\Sigma\Sigma}(\vec{r}) & V_{\Sigma\Sigma}^{\Sigma\Sigma}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\Lambda\Lambda}(\vec{r}, t) \\ R^{\Xi N}(\vec{r}, t) \\ R^{\Sigma\Sigma}(\vec{r}, t) \end{pmatrix}$$

Truncation of  $\Sigma\Sigma$  channel

## ► Reduced coupled channel equation

$$\begin{pmatrix} (E^{\Lambda\Lambda} - H_0^{\Lambda\Lambda}) R^{\Lambda\Lambda}(\vec{r}, t) \\ (E^{\Xi N} - H_0^{\Xi N}) R^{\Xi N}(\vec{r}, t) \end{pmatrix} = \begin{pmatrix} \overline{V_{\Lambda\Lambda}^{\Lambda\Lambda}}(\vec{r}) & \overline{V_{\Xi N}^{\Lambda\Lambda}}(\vec{r}) \\ \overline{V_{\Lambda\Lambda}^{\Xi N}}(\vec{r}) & \overline{V_{\Xi N}^{\Xi N}}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\Lambda\Lambda}(\vec{r}, t) \\ R^{\Xi N}(\vec{r}, t) \end{pmatrix}$$

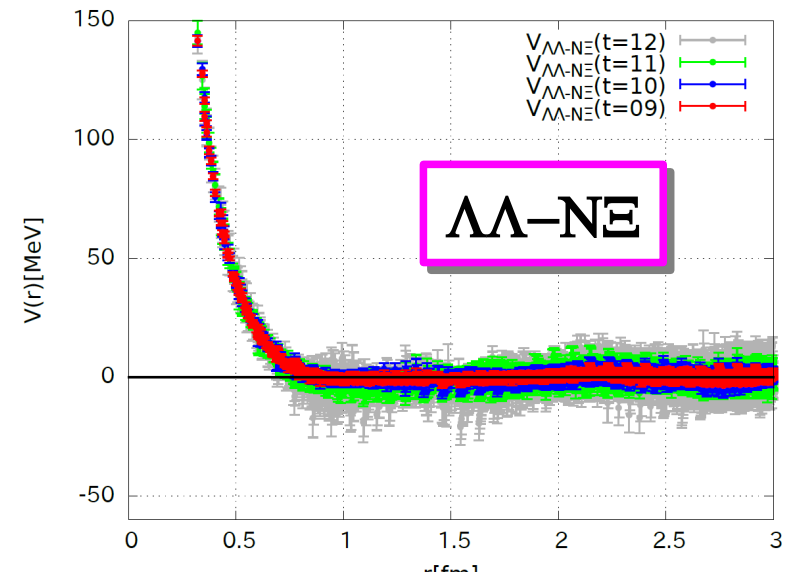
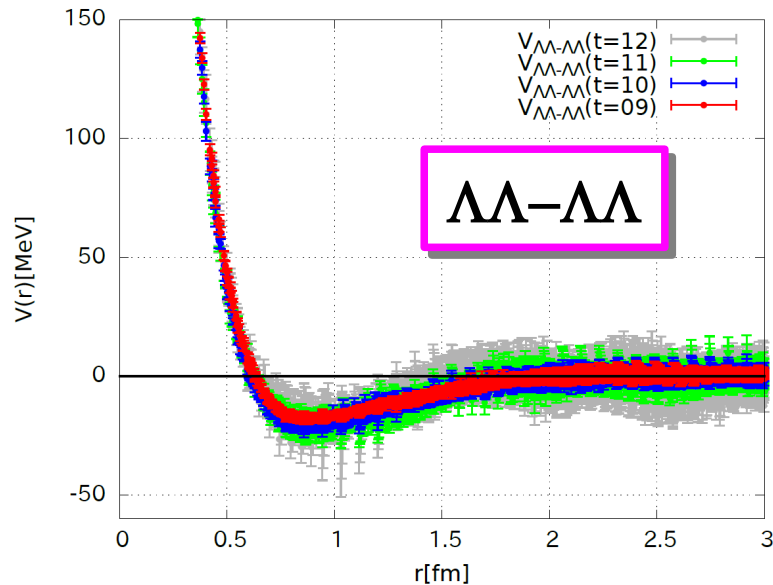
### Effective $\Lambda\Lambda$ - $N\Xi$ potential

- The same scattering phase shift would be expected in a low energy region.
- Non-locality (energy dependence, higher derivative contribution) of potential matrix could be enhanced.

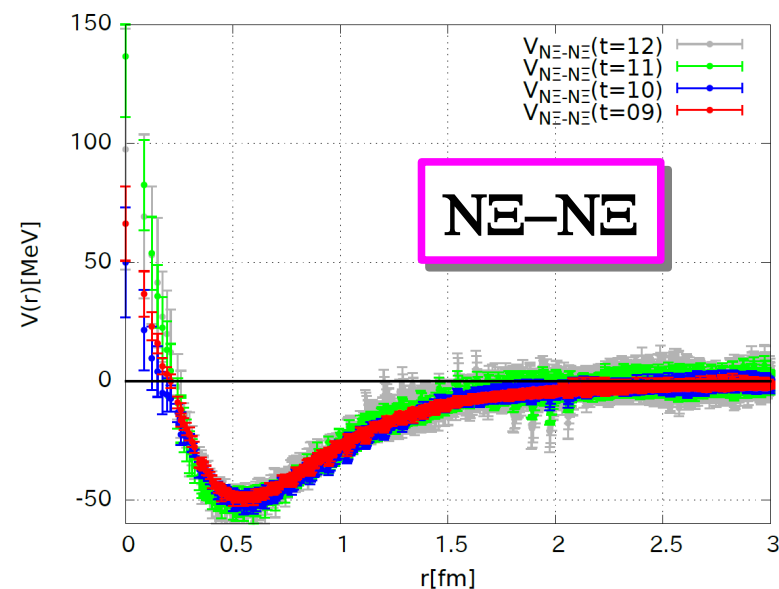
# $\Lambda\Lambda, N\Xi (I=0) ^1S_0$ potential (2ch calc.)

►  $N_f = 2+1$  full QCD with  $L = 8.1\text{fm}$ ,  $m_\pi = 146\text{ MeV}$

Preliminary!



- Potential calculated by only using  $\Lambda\Lambda$  and  $N\Xi$  channels.
- Long range part of potential is almost stable against the time slice.
- Short range part of  $N\Xi$  potential changes as time  $t$  goes.
- $\Lambda\Lambda-N\Xi$  transition potential is quite small in  $r > 0.7\text{fm}$  region

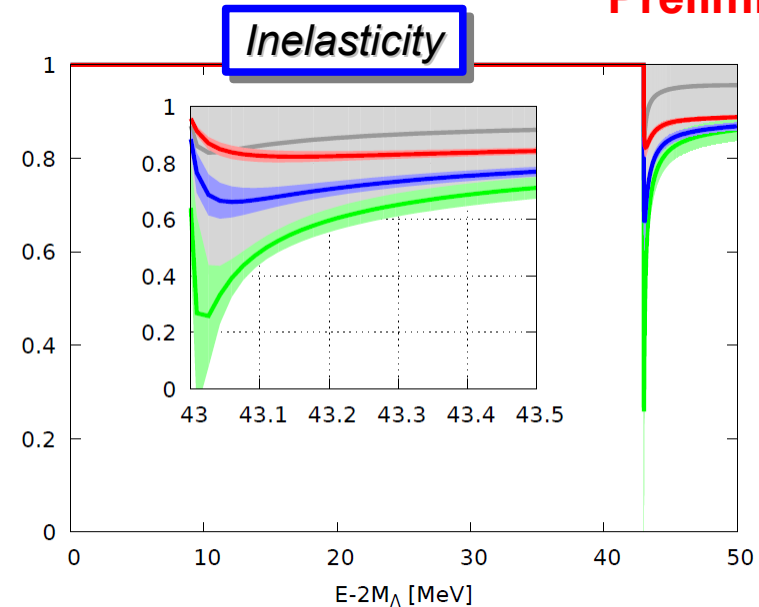
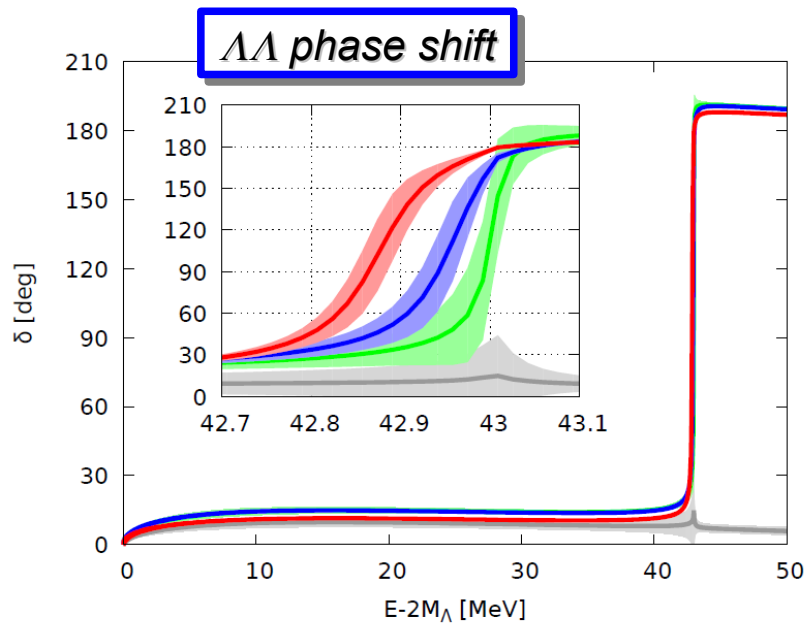


# $\Lambda\Lambda$ and $N\Xi$ phase shift and inelasticity

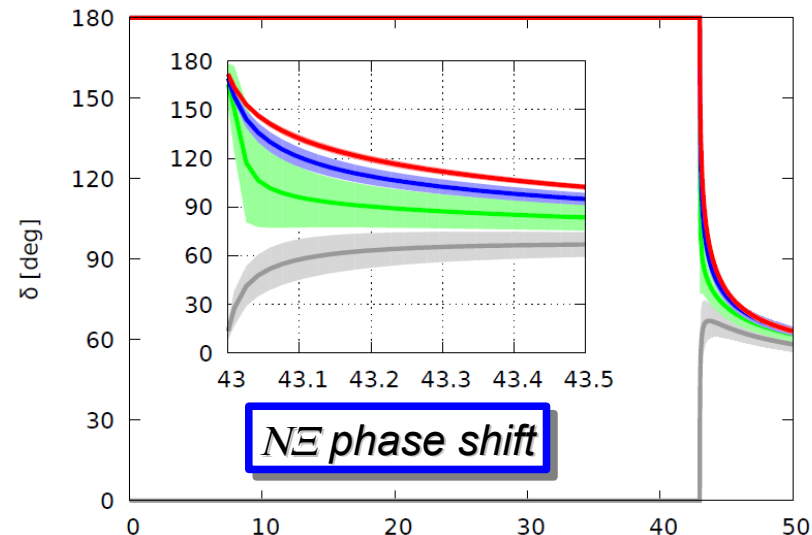
t=09  
t=10  
t=11  
t=12

►  $N_f = 2+1$  full QCD with  $L = 8.1\text{fm}$ ,  $m_\pi = 146\text{ MeV}$

Preliminary!



- $\Lambda\Lambda$  and  $N\Xi$  phase shift is calculated by using 2ch effective potential.
- A sharp resonance is found below the  $N\Xi$  threshold for  $t < 12$ .

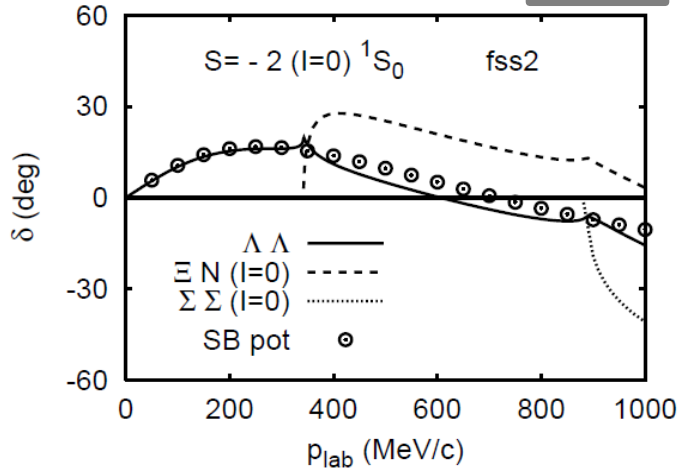
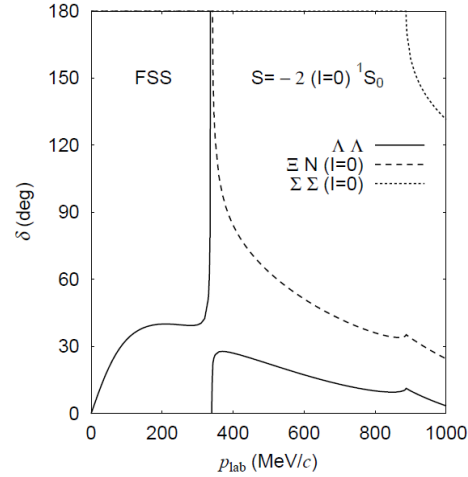
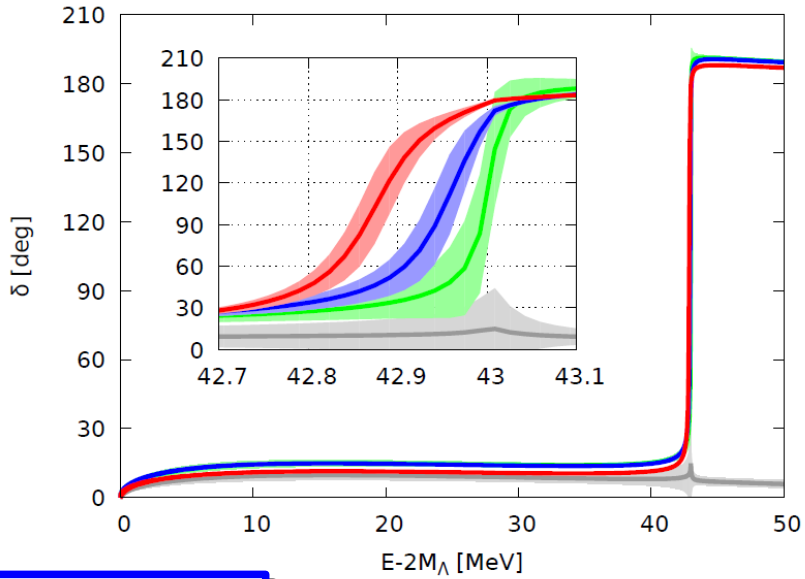




# $\Lambda\Lambda$ and $N\Xi$ phase shift –comparison–

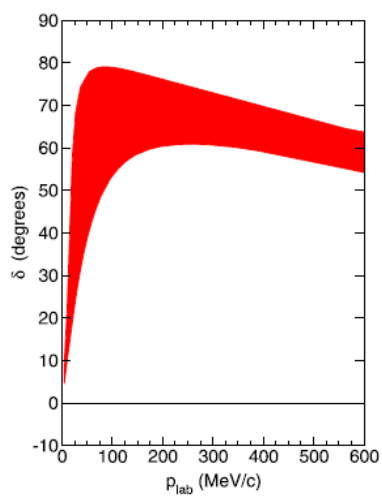
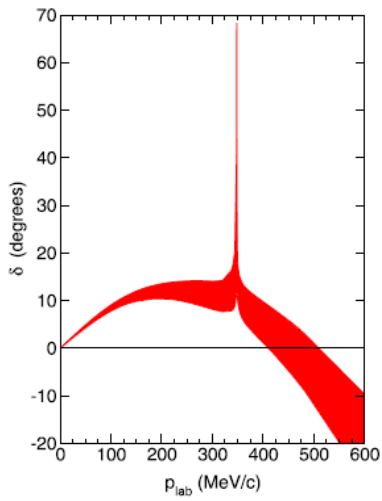
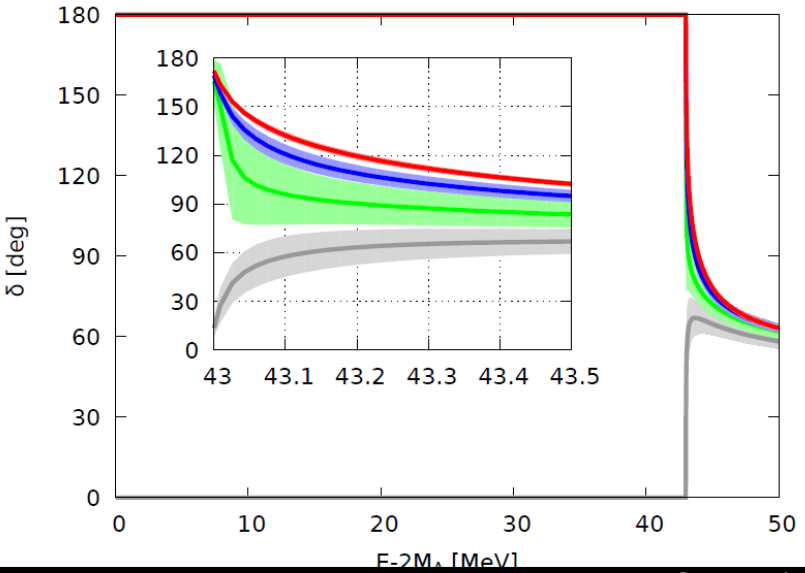
t=09  
t=10  
t=11  
t=12

## $\Lambda\Lambda$ phase shift



Y.Fujiwara et al, PPNP58(2007)439

## $N\Xi$ phase shift



J. Haidenbauer et al, NPA954(2016)273

● Our results are compatible with the phenomenological ones.

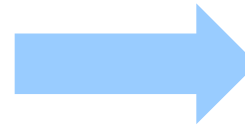
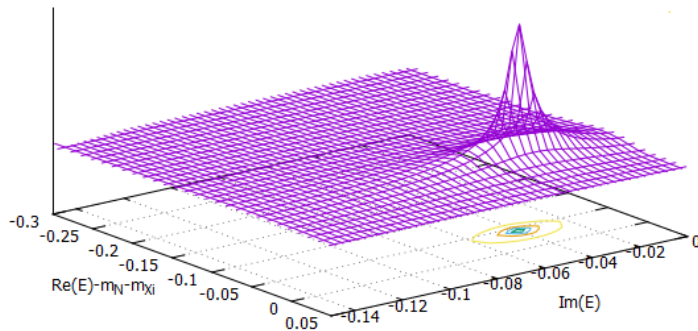
# Resonance energy and width

t=09  
t=10  
t=11

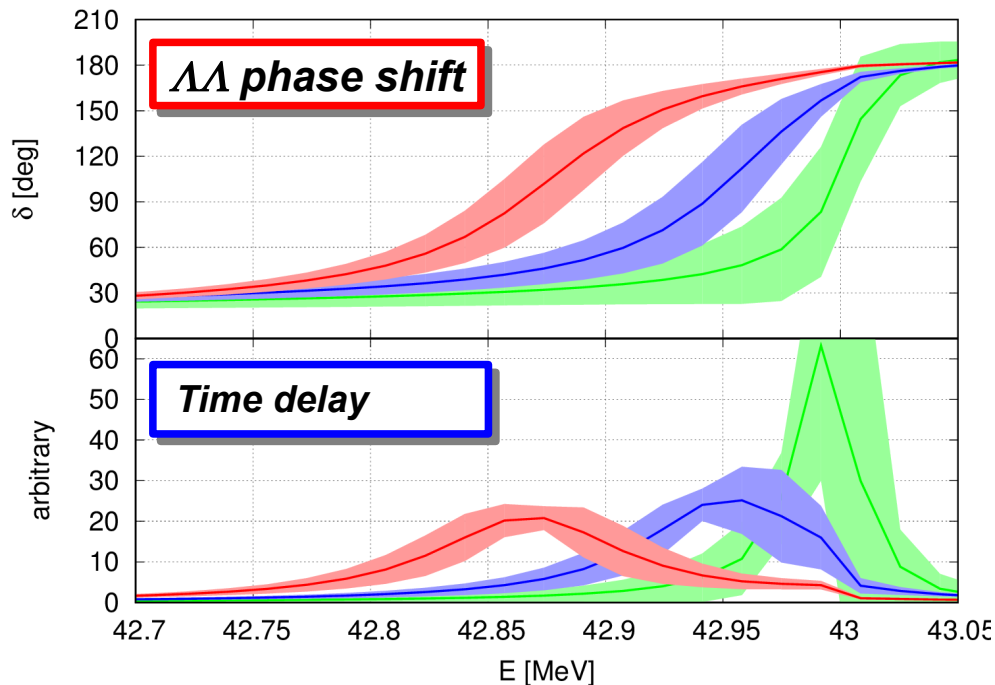
►  $N_f = 2+1$  full QCD with  $L = 8.1 \text{ fm}$ ,  $m_\pi = 146 \text{ MeV}$

Pole in t-matrix

Preliminary!



Energy and Width



Time delay

● In the vicinity of resonance point,

$$\delta(E) = \delta_B - \arctan\left(\frac{\Gamma/2}{E - E_r}\right)$$

thus

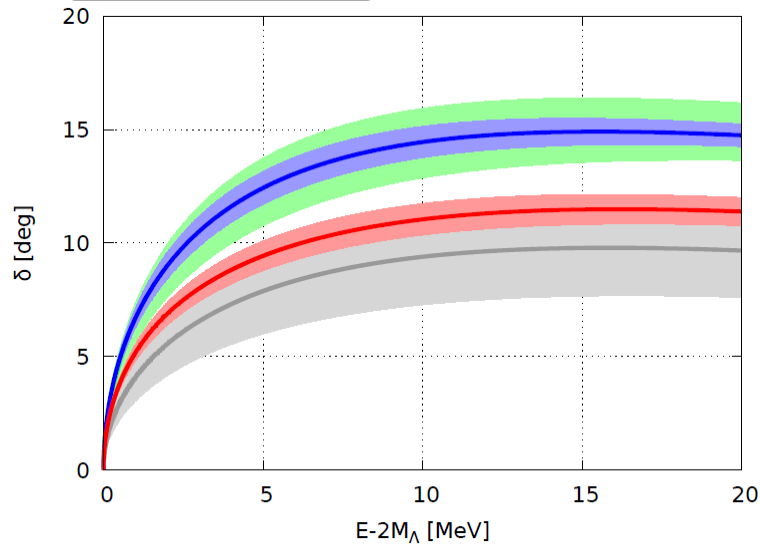
$$\frac{d\delta(E)}{dE} = \frac{\Gamma/2}{(E - E_r)^2 + (\Gamma/2)^2}$$

# $\Lambda\Lambda$ scattering length

$t=09$   
 $t=10$   
 $t=11$   
 $t=12$

## Phase shift

Preliminary!



$$-1.25 < a_{\Lambda\Lambda} < -0.56 \text{ fm}$$

$$a_{\Lambda\Lambda} = -0.821 \text{ fm}$$

$$a_{\Lambda\Lambda} = -0.97 \text{ fm}$$

K.Morita et al, PRC91 (2015)024916

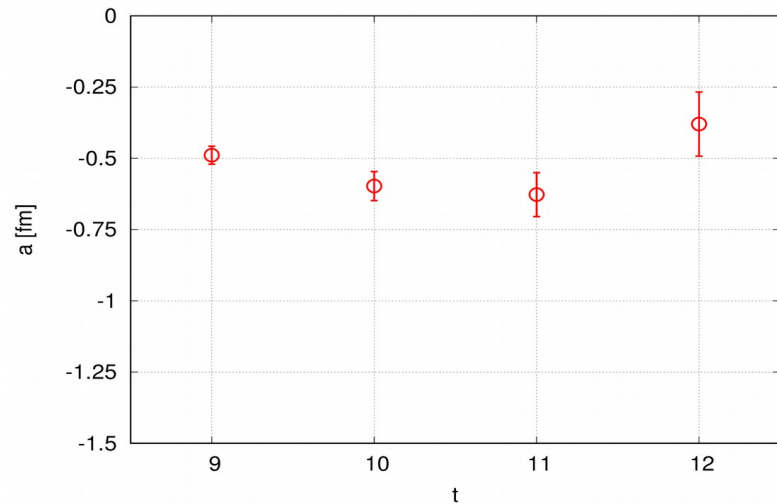
Y.Fujiwara et al, PPNP58(2007)439

Th.A.Rijken et al, Few-Body Syst 54(2013)801

$\Lambda$	550	600	650	700
$a_{1S0}^{\Lambda\Lambda}$	-1.52	-1.52	-1.54	-1.67
$r_{1S0}^{\Lambda\Lambda}$	0.82	0.59	0.31	0.34

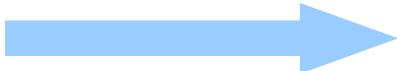
H. Polinder et al, PLB653(2007)29

## Scattering length



- Scattering length in  $\Lambda\Lambda$  ( $l=0$ ) channel is not saturated.
- Attraction is weaker than the phenomenological one.

# Summary

- ▶ We have investigated coupled channel baryonic interactions from lattice QCD.
  - ▶ We find that
    - Potential in  $\Lambda\Lambda$   $^1S_0$  channel is weakly attractive.
    - $\Lambda\Xi$  potential is largely depends on its channel.
    - Potential in flavor singlet  $^1S_0$  channel is strongly attractive.
  - ▶ We have studied dibaryon candidate states
    - H-dibaryon channel
      - We perform  $\Lambda\Lambda$ - $\Lambda\Xi$  coupled channel calculation.
      - Sharp resonance is found just below the  $\Lambda\Xi$  threshold in relatively small “t” region.  
(Time slice saturation is not achieved yet.)
-  ● Fate of H-dibaryon...
- We continue to study it by using higher statistical data.