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Transport coefficients of QGP in strong magnetic fields

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K. Hattori and **D. S.**,
Phys. Rev. D **94**, 114032 (2016).

K. Hattori, S. Li, **D. S.**, H. U. Yee,
Phys. Rev. D **95**, 076008 (2017).

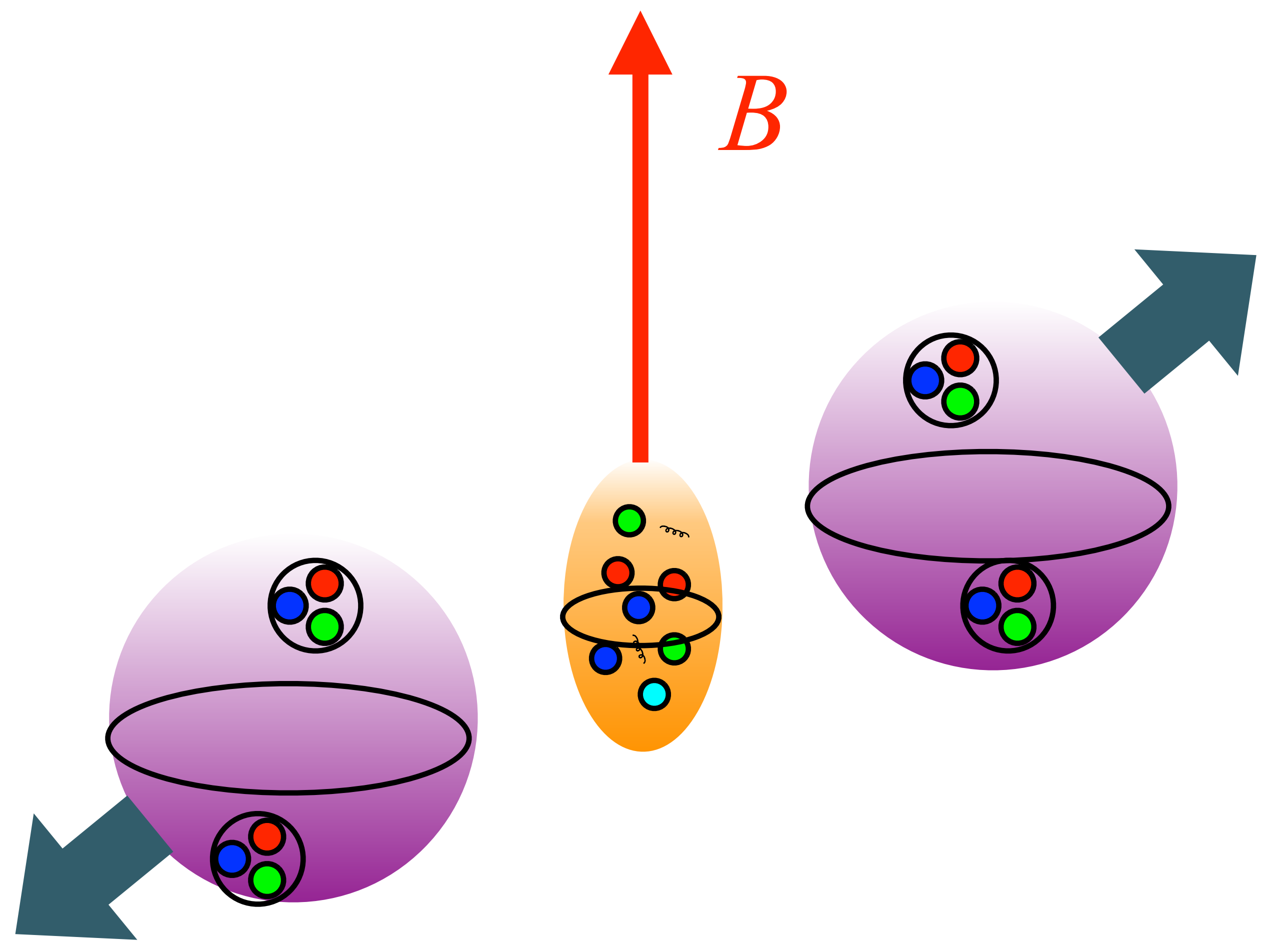



Outline

- (Long) Introduction
  —quarks and gluons in strong B
- Electrical Conductivity
- Bulk Viscosity
- Summary

Introduction

Strong magnetic field (*B*) may be generated in heavy ion collision due to Ampere's law.

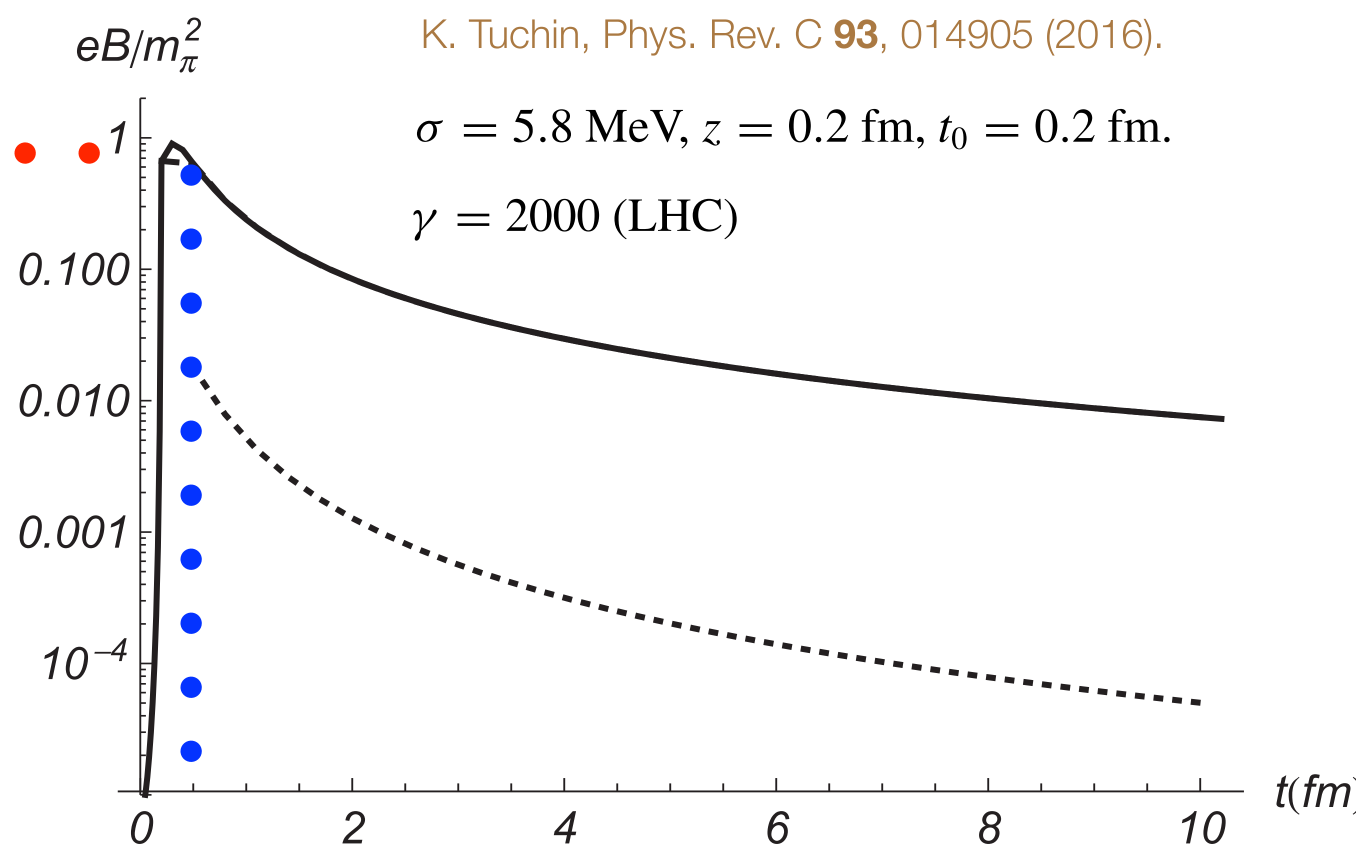


Introduction

strength:

$$\sqrt{eB} \sim 100\ [\mathrm{MeV}]$$

(Comparable with T)



lifetime: $\sim 0.3\,\mathrm{fm}$

At thermalization time (~0.5 fm), there still may be strong B.

Heavy ion collision may give a chance to investigate QCD matter at **finite temperature** in **strong magnetic field**.

Introduction

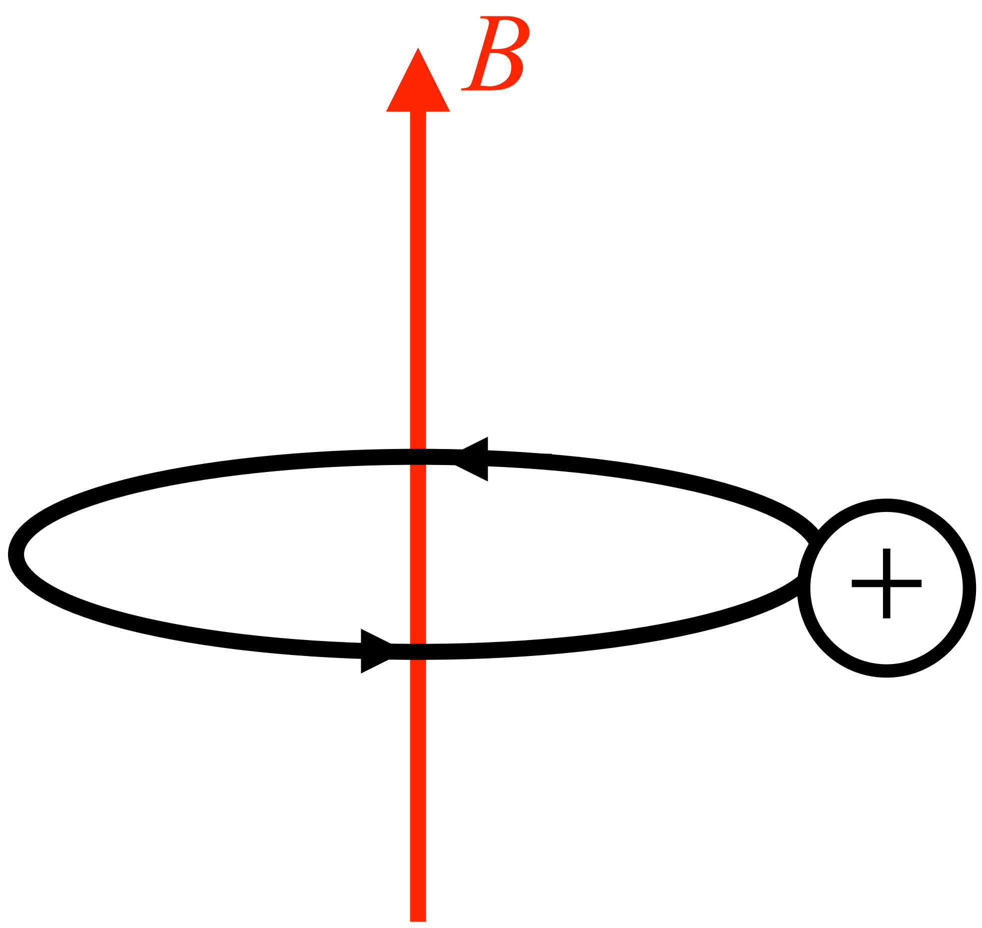
How the particles behaves when $\sqrt{eB}$ is much larger than the other energy scales of the system?

$$(\sqrt{eB} >> T, m, \Lambda_{\mathrm{QCD}}\ldots)$$

Quark in Strong B

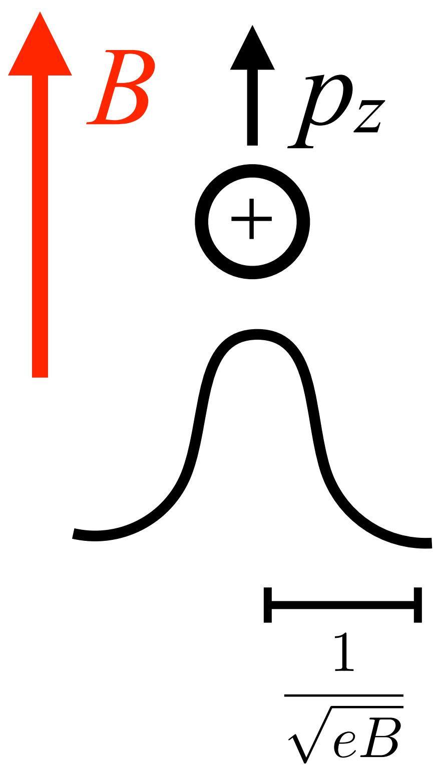
One-particle state of quark in magnetic field

Classical: Cyclotron motion due to Lorentz force



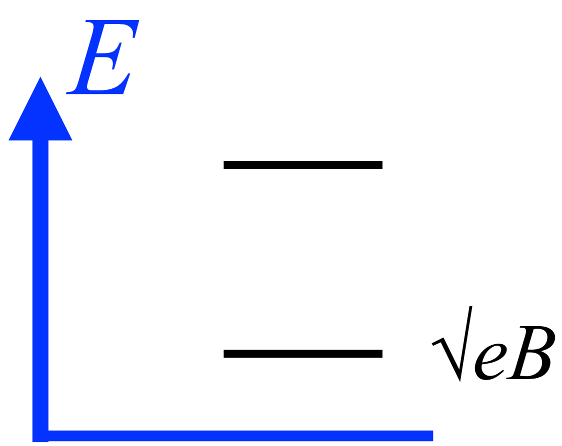
Quark in Strong B

Quantum: Landau Quantization



Longitudinal: Plane wave
Transverse: Gaussian

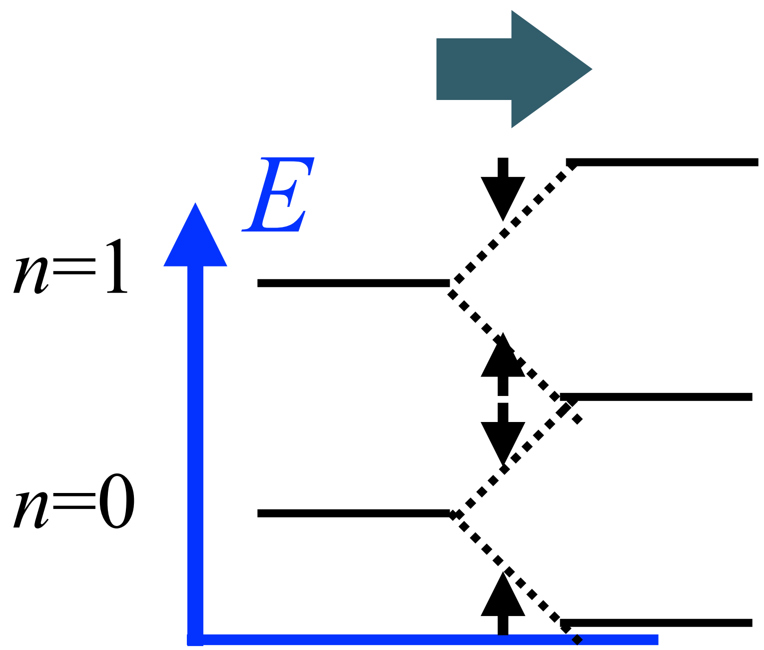
$$E_n = \sqrt{(p_z)^2 + m^2 + 2eB\left(n + \frac{1}{2}\right)}$$

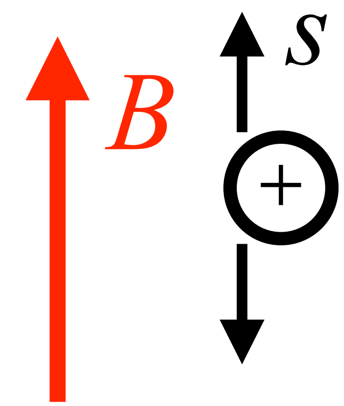


The gap ($\sim\sqrt{eB}$) is generated by zero-point oscillation.

Quark in Strong B

For spin-1/2 particle, we have Zeeman effect:





$$E_n = \sqrt{(p_z)^2 + m^2 + 2eB\left(n + \frac{1}{2} \mp \frac{1}{2}\right)}$$

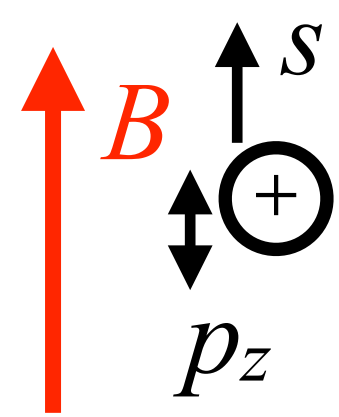
When n=0 (LLL), gap is small (m~1MeV).
When n>0, gap is large (~$\sqrt{eB}$~100MeV)

Lowest Landau Level (LLL) Approximation

When the typical energy of particle (T) is much smaller than gap ($\sqrt{eB}$), the higher LL does not contribute ($\sim\exp(-\sqrt{eB}/T)$), so **we can focus on the LLL**.

One-dimension dispersion, no spin degrees of freedom.

$$E_n = \sqrt{(p_z)^2 + m^2}$$

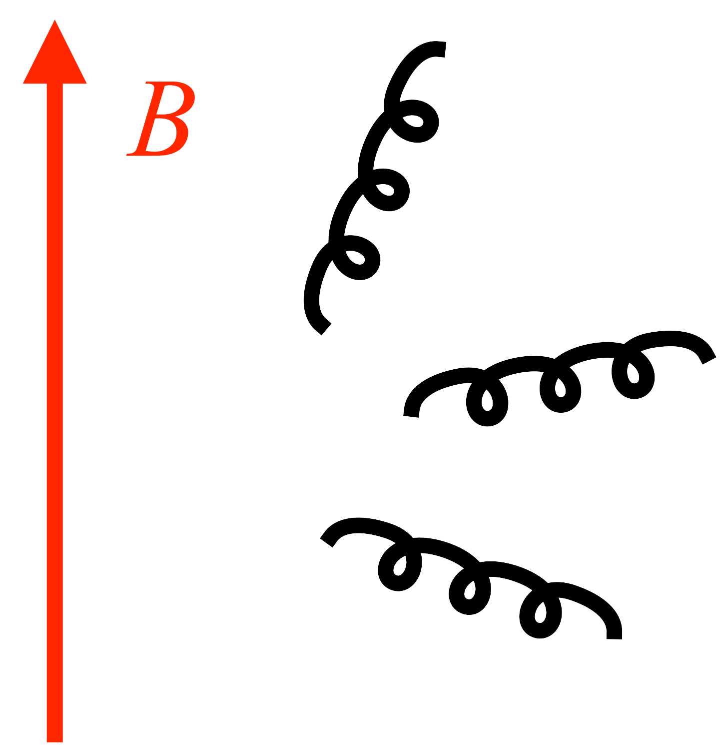


In heavy-ion collision, this condition can be marginally realized ($T\sim\sqrt{eB}\sim$100MeV).
But in Weyl semi-metal, it is already realized ($T\sim$1meV, $\sqrt{eB}\sim$10eV).

Q. Li, D. Kharzeev, et. al., nature physics **12**, 550 (2016)

Gluon in Strong B

Gluon does not have charge, so it does not feel B in the zeroth approximation.

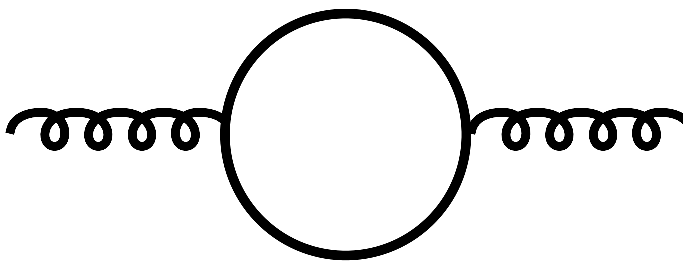


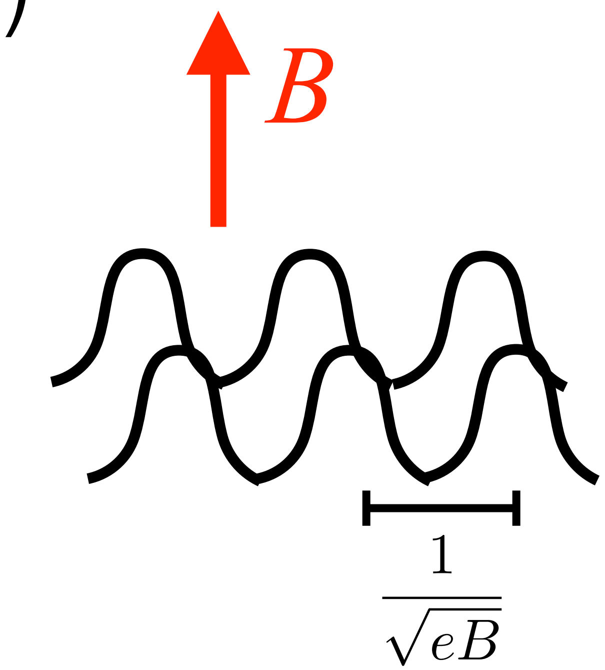
Massless boson in 3D

Gluon in Strong B

K. Fukushima, Phys. Rev. D **83**, 111501 (2011).

Coupling with (1+1)D quarks generates gluon mass.
(Schwinger mass generation)





(surface density)
~(average distance)$^{-2}$~eB

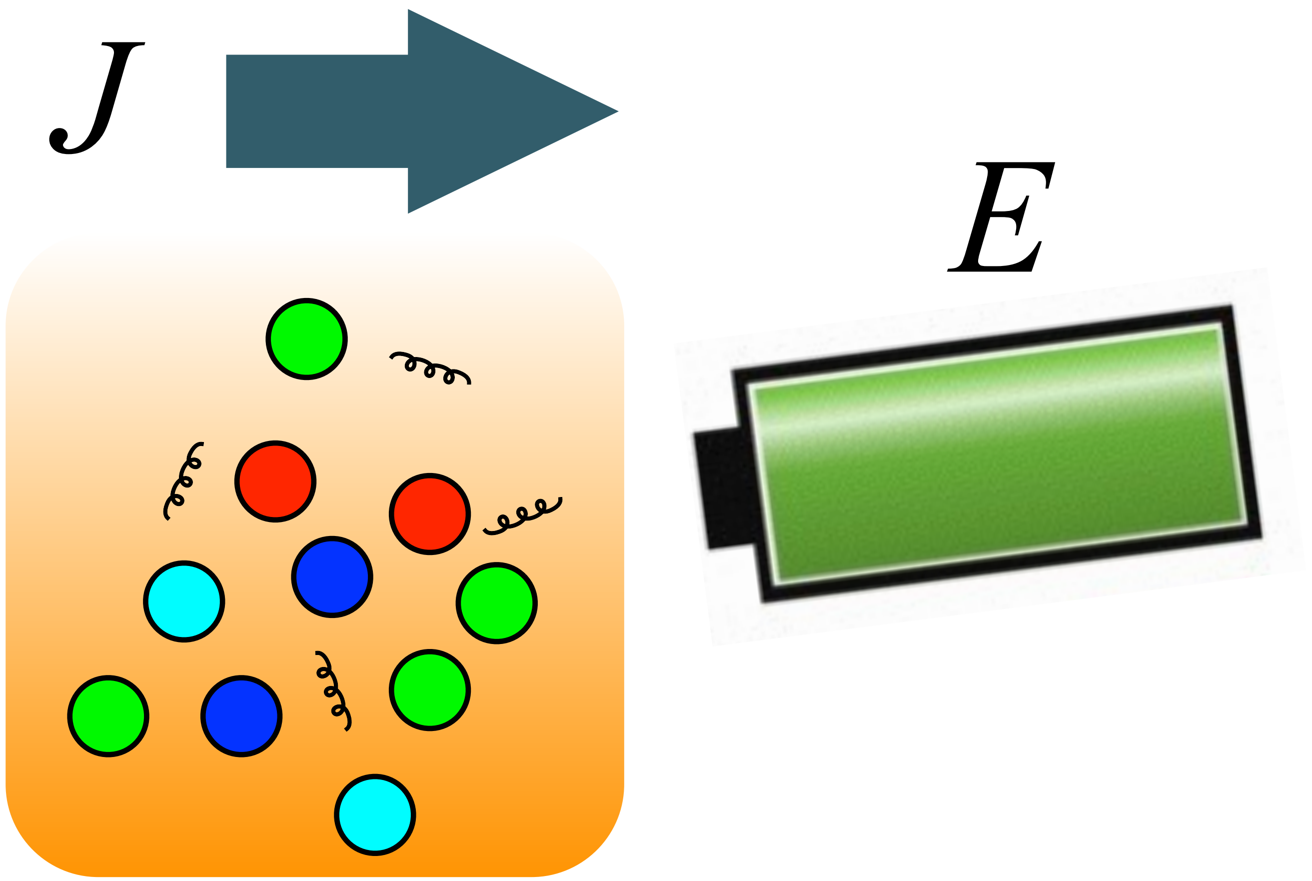
Color factor

$$M^2 \equiv \underbrace{\frac{1}{2}} \times \underbrace{\frac{g^2}{\pi}} \sum_f \underbrace{\frac{|eB|}{2\pi}} \sim g^2 eB$$

Schwinger mass **Landau degeneracy**

Electrical Conductivity

$$J = \sigma E$$



Motivation to Discuss Electrical Conductivity

Electrical conductivity is phenomenologically important because

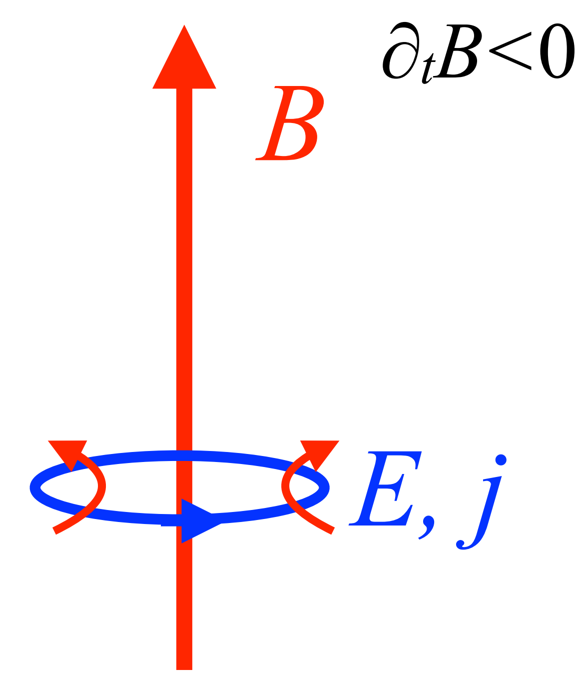
- Input parameter of magnetohydrodynamics (transport coefficient)

- May increase lifetime of $\boldsymbol{B}$ (Lenz's law)

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$$

$$\cancel{\partial_t \mathbf{E}} = \nabla \times \mathbf{B} - \mathbf{j}$$

When σ is large



Hierarchy of Energy Scale at LLL

For ordering of m and M, we consider the both cases.

(m<<M and m>>M) ($M \sim g\sqrt{eB}$)

E

$$\sqrt{eB} \gg T \gg M \gg m$$

LLL approximation

Thermally excited

E

$$\sqrt{eB} \gg T \gg m \gg M$$

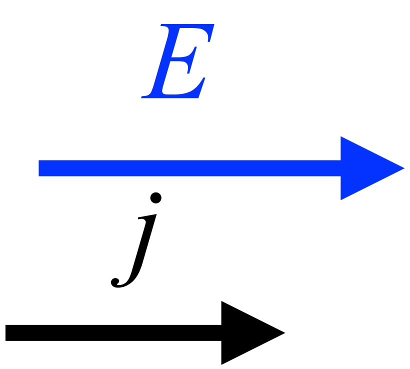
Electrical Conductivity

Conductivity at weak B ($\sqrt{eB} << T$)

$j^i = \sigma^{ij} E^j$

B=0

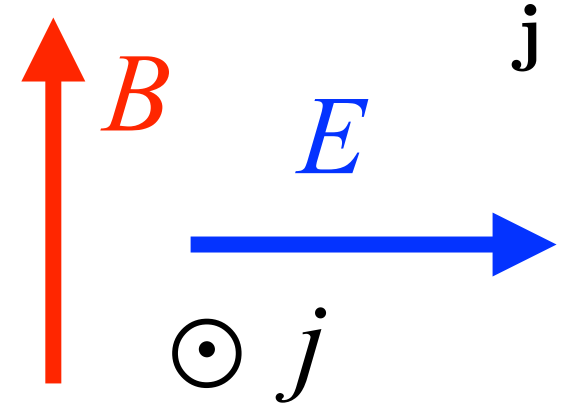
$$\mathbf{j} = \sigma_0 \mathbf{E}$$



$$\sigma^{ij} = \begin{bmatrix} \sigma_0 & 0 & 0 \\ 0 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{bmatrix}$$

weak B

$$\mathbf{j} = \sigma_1 \mathbf{E} \times \mathbf{B}$$

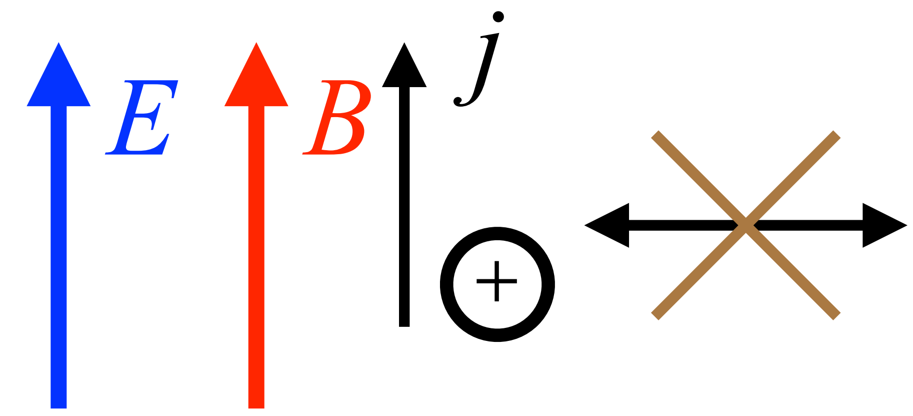


$$\sigma^{ij} = \begin{bmatrix} \sigma_0 & \sigma_1 & 0 \\ -\sigma_1 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{bmatrix}$$

Electrical Conductivity

Strong B (LLL)

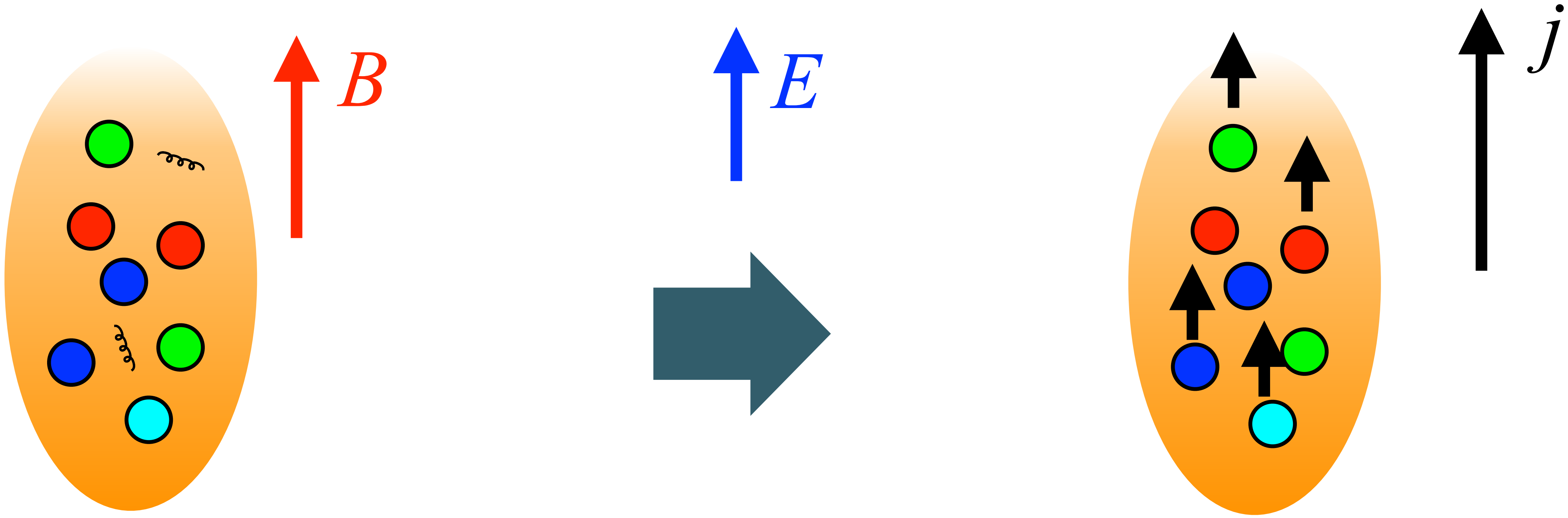
Quarks are confined in the direction of B,
so there is no current in other directions.



σ^{33} is finite, other components are zero.
(Very different from weak B case)

$$j^i = \sigma^{ij} E^j \qquad \sigma^{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}$$

Calculation of conductivity



Thermal equilibrium in strong B

Linear response against E

Slightly non-equilibrium, finite j

$$n^f(k^3, T, Z) = n_F(\epsilon_k^L)$$

$$\epsilon_k^L \equiv \sqrt{(k^3)^2 + m^2}$$

$$n^f(k^3, T, Z) = n_F(\epsilon_k^L) + \delta n^f(k^3, T, Z)$$

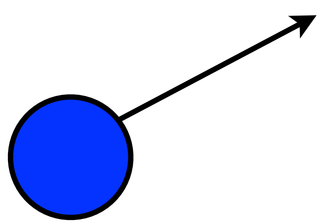
$$j^3(T, Z) = 2e \sum_f q_f N_c \underbrace{\frac{|eq_f B|}{2\pi}}_{\text{Landau degeneracy}} \int \frac{dk^3}{2\pi} v^3 \delta n^f(k^3, T, Z) = \sigma^{33} E^3$$

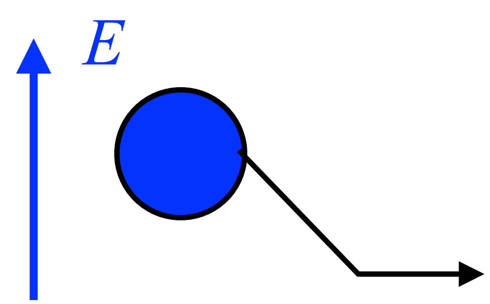
$$v^3 \equiv \partial \epsilon_k^L / (\partial k^3) = k^3 / \epsilon_k^L$$

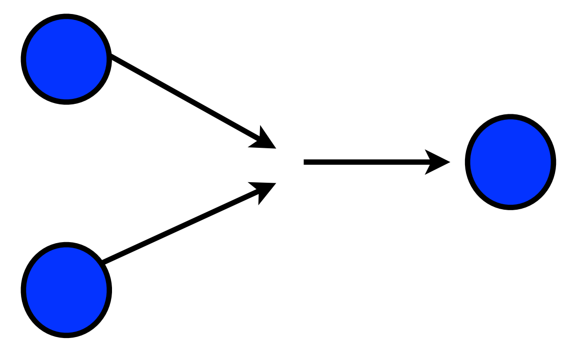
Evaluation of δn_F is necessary.

Calculation of conductivity

Evaluate n_F with **(1+1)D Boltzmann equation**







$$[\cancel{\partial_T + v^3\partial_Z} + eq_f E^3(T,Z)\partial_{k^3}]n^f(k^3,T,Z) = C[n]$$

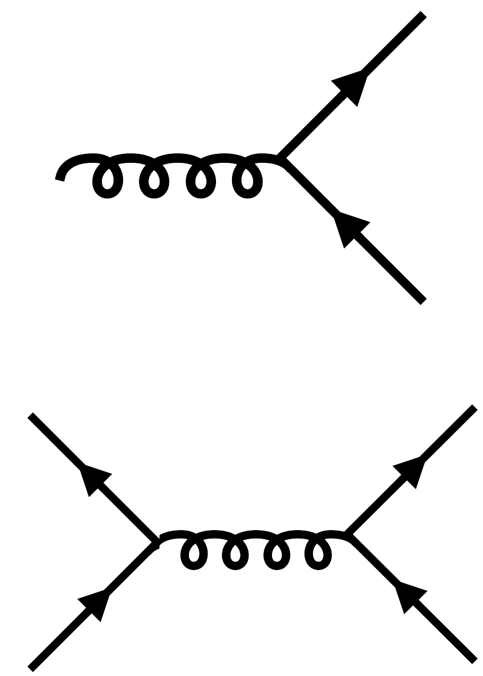
Constant and homogeneous E

Possible Scattering Process for Conductivity

The scattering process is very different from that in B=0…

B=0

1 to 2 scattering is kinematically forbidden; one massless particle can not decay to two massless particles.
2 to 2 is leading process.



strong B

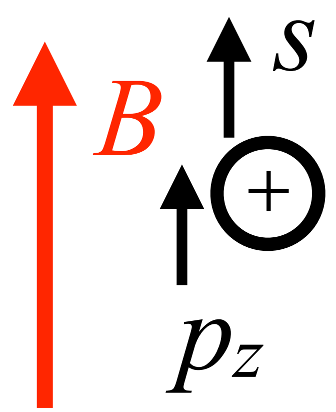
Gluon is effectively massive in (1+1)D

$$E = \sqrt{p_z^2 + p_\perp^2 + M^2}$$

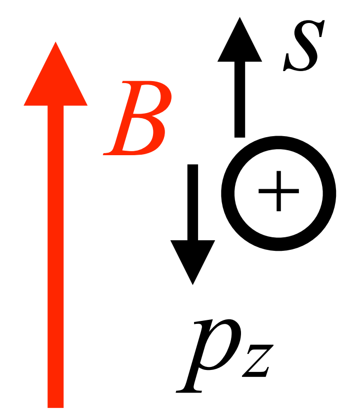
Decay of a gluon into quark and anti-quark becomes kinematically possible. (1 to 2)

Chirality in (1+1)D

Spin is always up.



χ=+1

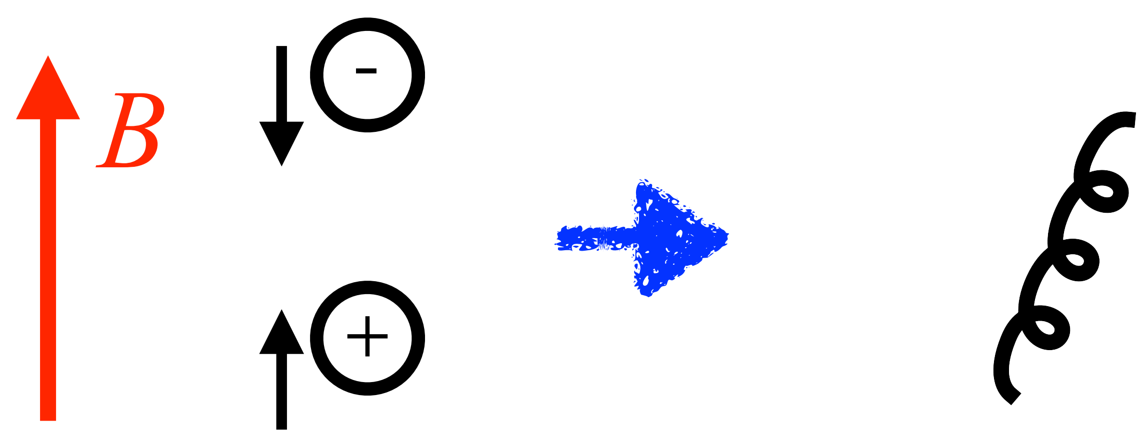


χ=-1

When m=0, the direction of p_z determines chirality.

Chirality in (1+1)D

Chirality is conserved at m=0:



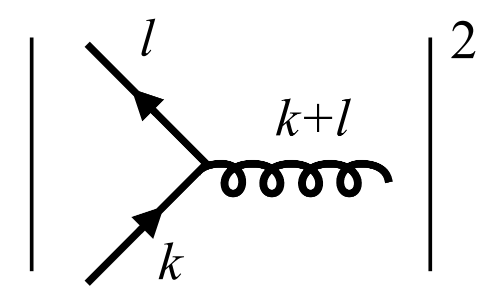
χ=+1+1=2 $\qquad\qquad$ χ=0

1 to 2 scattering is forbidden at m=0.

Possible Scattering Process for Conductivity

Collision term for 1 to 2:

$$C[n] = \frac{1}{2\epsilon_k^L}\int_l |M|^2[n_B^{k+l}(1-n_F^k)(1-n_F^l) - (1+n_B^{k+l})n_F^k n_F^l]$$

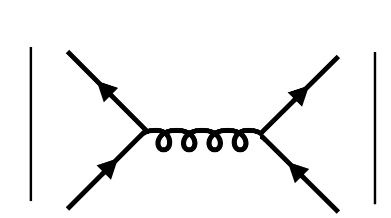


$$|M|^2 = 4g^2 C_f m^2$$

Vanishes at *m*=0!

(chirality conservation)

cf: 2 to 2



$\sim g^4$

Calculation of conductivity

$$eq_f E^3(T,Z)\partial_{k^3}\ n^f(k^3,T,Z) = C[n]$$

linearize $n^f(k^3,T,Z) = n_F(\epsilon_k^L) + \delta n^f(k^3,T,Z)$

$$\Rightarrow \quad eq_f E^3 \beta v^3 n_F(\epsilon_k^L)[1-n_F(\epsilon_k^L)] = C[\delta n^f(k^3,T,Z)]$$

(β=1/T)

$$C[n] = \frac{1}{2\epsilon_k^L}\int_l |M|^2[n_B^{k+l}(1-n_F^k)(1-n_F^l) - (1+n_B^{k+l})n_F^k n_F^l]$$

linearize

$$\Rightarrow \quad C[\delta n] = -\frac{1}{2\epsilon_k^L}\int_l |M|^2\left[\delta n_F^k\left(n_B^{k+l}+n_F^l\right) - \delta n_F^l\left(n_B^{k+l}+n_F^k\right)\right]$$

damping rate of quark (=-2$\xi_k \delta n^k{}_F$)

Calculation of conductivity

Solution for δn^F with damping rate ξ_k

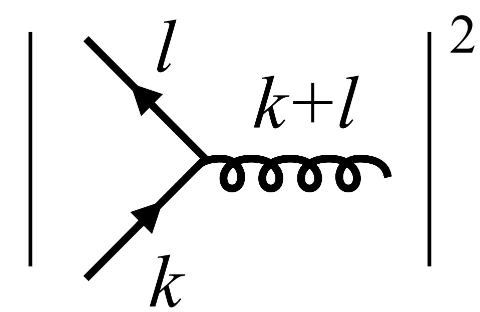
$$\delta n_F^k = -\frac{1}{2\xi_k} e q_f E^3 \beta v^3 n_F(\epsilon_k^L)[1 - n_F(\epsilon_k^L)]$$

$$j^3(T,Z) = 2e \sum_f q_f N_c \frac{|B_f|}{2\pi} \int \frac{dk^3}{2\pi} v^3 \delta n^f(k^3, T, Z)$$

$$\Rightarrow \quad j^3 = \underbrace{e^2 \sum_f q_f^2 N_c \frac{|e q_f B|}{2\pi} 4\beta \int \frac{dk^3}{2\pi} (v^3)^2 \frac{1}{2\xi_k} n_F(\epsilon_k^L)[1 - n_F(\epsilon_k^L)]}_{\sigma^{33}} E^3$$

Quark Damping Rate

$$\epsilon_k^L \xi_k = \frac{g^2 C_F m^2}{4\pi} \int_m^\infty dl^0 \frac{n_F(l^0) + n_B(l^0 + \epsilon_k^L)}{\sqrt{(l^0)^2 - m^2}}$$



leading-log approximation ($\ln[T/m] >> 1$)

$l^0 << T$ dominates

$$\epsilon_k^L \xi_k \simeq \frac{g^2 C_F m^2}{4\pi} \left[\frac{1}{2} + n_B(\epsilon_k^L)\right] \int_m^\infty dl^0 \frac{1}{\sqrt{(l^0)^2 - m^2}}$$

$$\simeq \frac{g^2 C_F m^2}{4\pi} \left[\frac{1}{2} + n_B(\epsilon_k^L)\right] \ln\left(\frac{T}{m}\right)$$

matrix element

soft fermion and hard boson

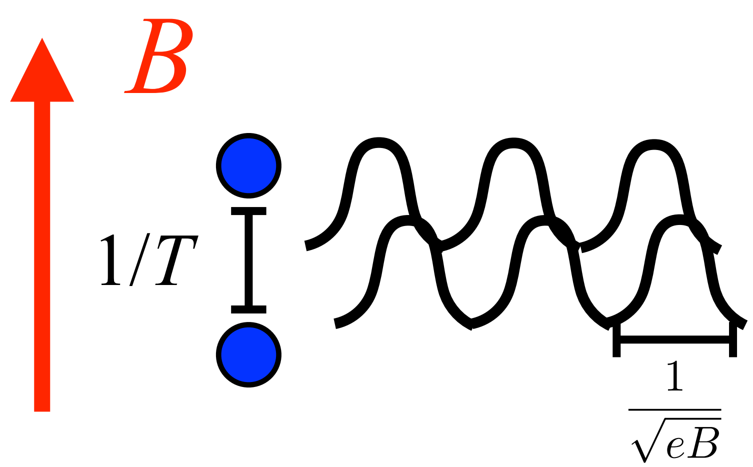
$n_F(1+n_B)+(1-n_F)n_B=n_F+n_B$

log divergence in phase space integral

UV cutoff: T

IR cutoff: m

Results



(average distance among quarks)~$1/T$
→(quark density in 1D)~T

Quark density in 1D

$$\sigma^{33} = e^2 \sum_f q_f^2 N_c \frac{|eq_f B|}{2\pi} \frac{4T}{g^2 C_F m^2 \ln(T/m)}$$

Landau degeneracy

Quark damping rate

Due to chirality conservation, collision is forbidden when m=0. Thus, σ~$1/m^2$.

When M>>m, $\ln(T/m)$→$\ln(T/M)$.

Other Term Does Not Contribute

$$C[\delta n] = -\frac{2g^2 C_F m^2}{\epsilon_k^L}\int_l [\delta n_F^k (n_B^{k+l} + n_F^l) - \delta n_F^l (n_B^{k+l} + n_F^k)]$$

Other Term

$\delta n_F^l = -\dfrac{eq_f}{2\xi_l} E^3 \partial_{l^3} n_F(\epsilon_l^L)$:odd in l^3

function of $(\varepsilon^{L}{}_{k}+\varepsilon^{L}{}_{l})$

(Other term)~ $\int_l \underbrace{(n_B^{k+l} + n_F^k)}_{\text{even in } l^3} \delta n_F^l \Rightarrow 0$

Our result (only retaining quark damping rate term) **is correct.**

Equivalent Diagrams

Our calculation is based on (unestablished) (1+1)D kinetic theory, but actually **we can reproduce the same result by field theory calculation**.

J. -S. Gagnon and S. Jeon, Phys. Rev. D **75**, 025014 (2007); **76**, 105019 (2007).

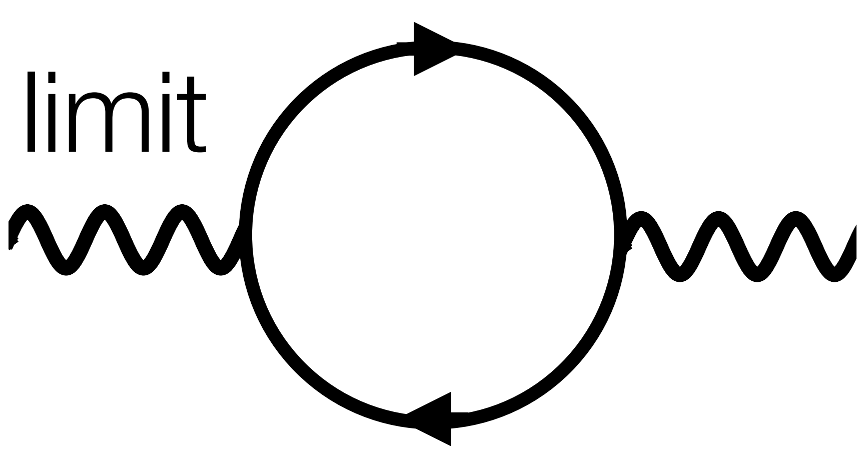
Kubo formula: $\sigma^{ij} \equiv \lim_{\omega \to 0} \frac{\Pi^{Rij}(\omega)}{i\omega}$

$j^{\mu} \equiv e \sum_f q_f \overline{\psi}_f \gamma^{\mu} \psi_f$

f: flavor index, q_{f}: electric charge

$\Pi^{R\mu\nu}(x) \equiv i\theta(x^0)\langle [j^{\mu}(x), j^{\nu}(0)] \rangle$

$p \to 0$ limit



Possible Phenomenological Implications

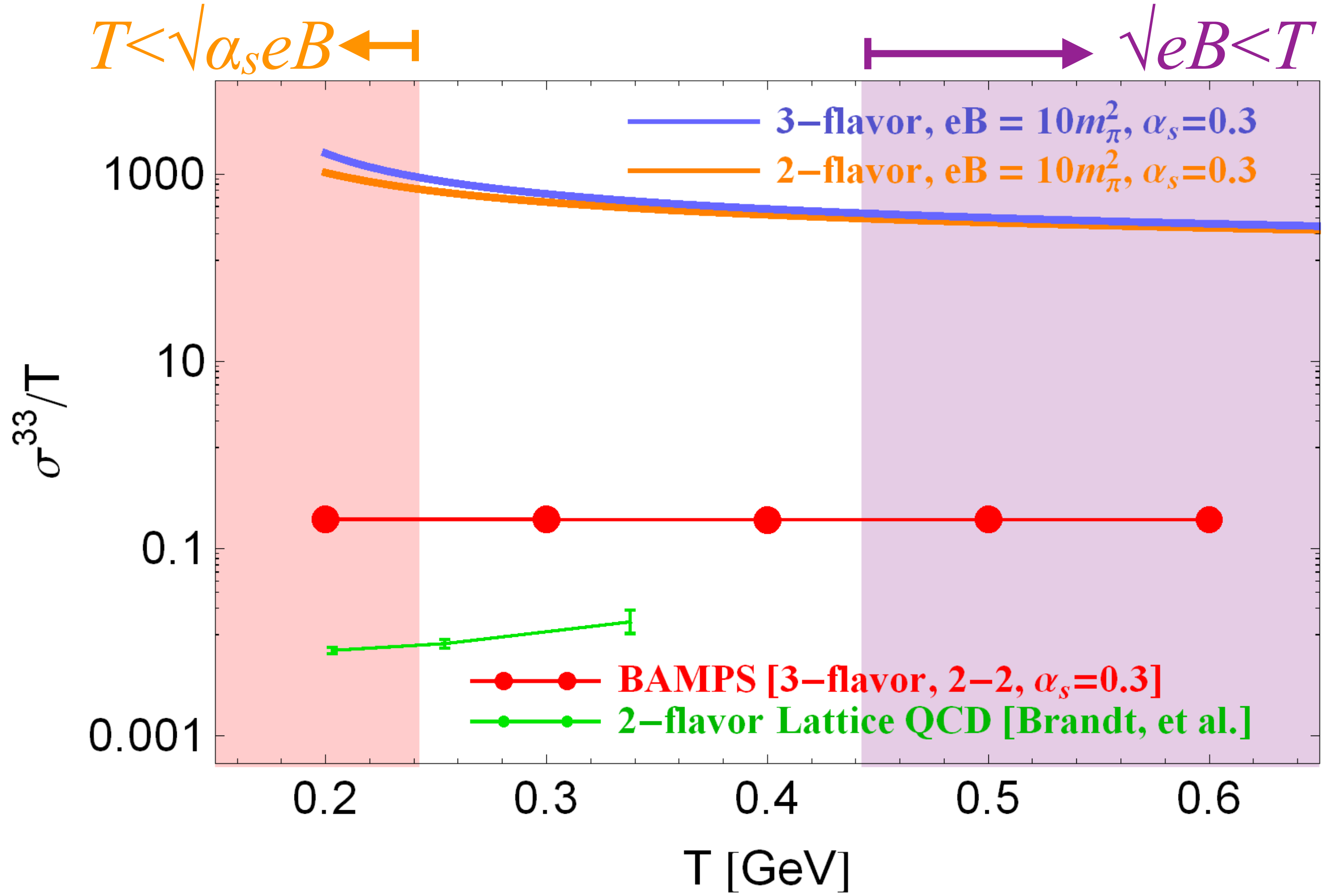
1. Order Estimate

$$\sigma^{33} = e^2 \sum_f q_f^2 N_c \frac{|eq_f B|}{2\pi} \frac{4T}{g^2 C_F m^2 \ln(T/M)}$$

Because of m^{-2} dependence, s contribution is very small.

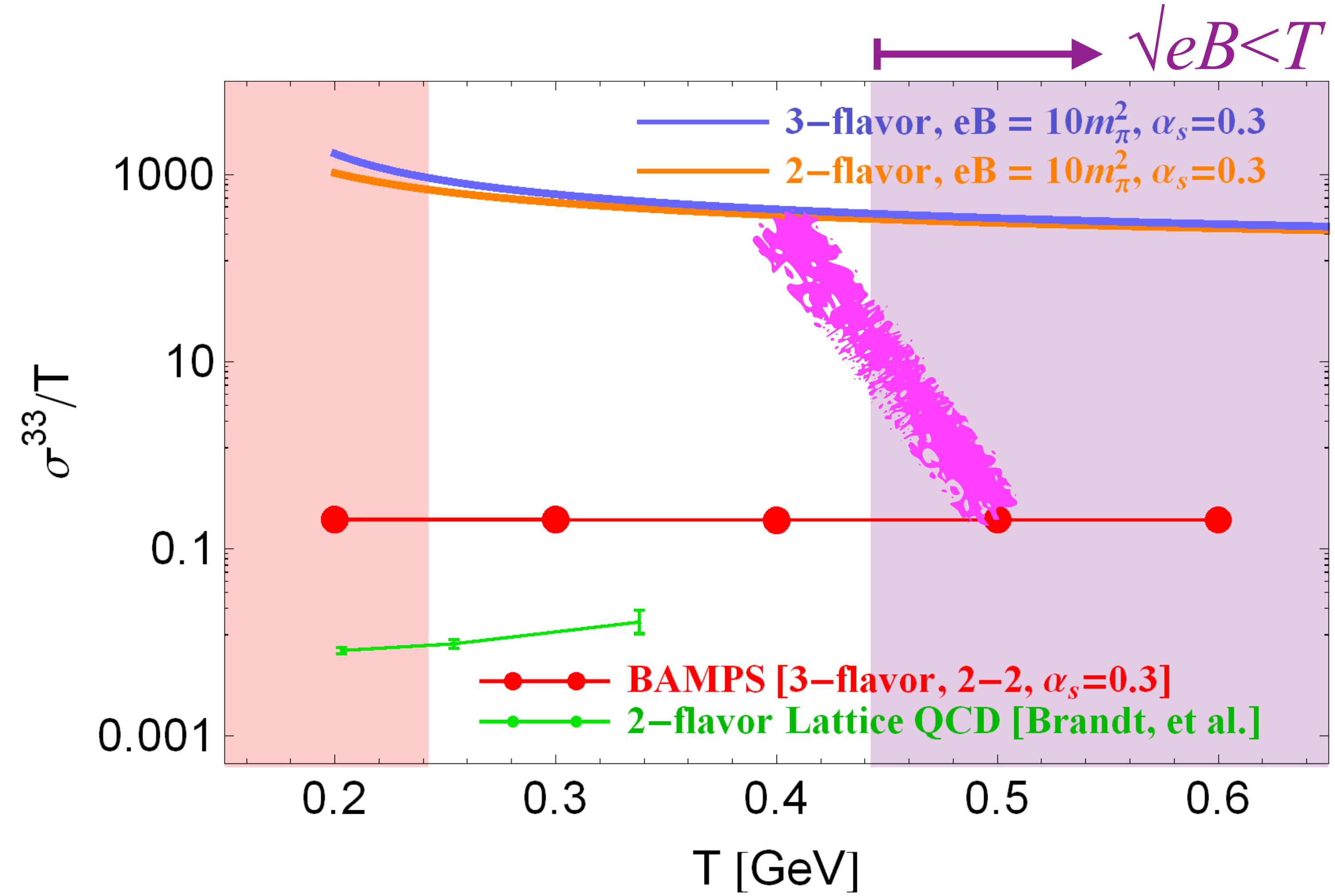
$$\alpha_s = \frac{g^2}{4\pi} = 0.3,$$
$$m = 3\mathrm{MeV}(u,d), 100\mathrm{MeV}(s),$$
$$eB = 10m_\pi^2 = (440\mathrm{MeV})^2.$$

M=160MeV>>m

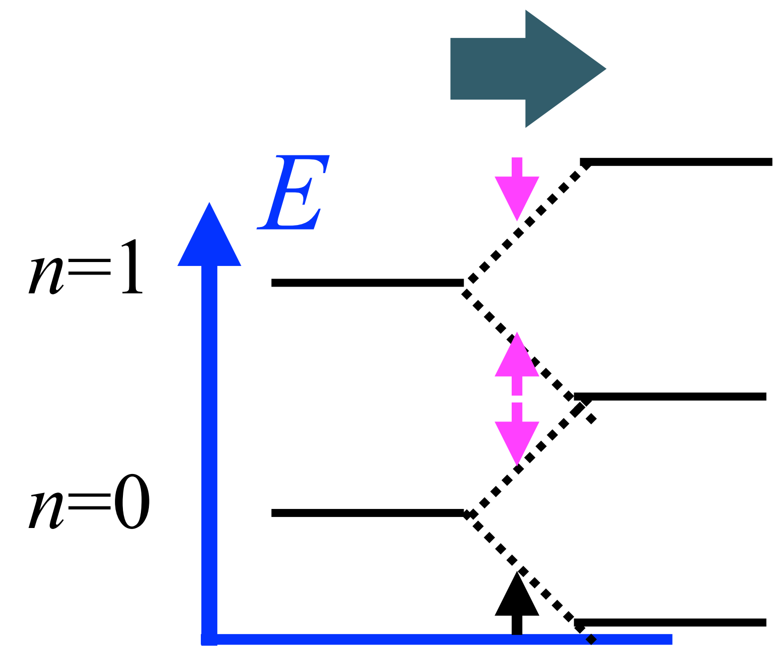


BAMPS: M. Greif, I. Bouras, C. Greiner and Z. Xu, Phys. Rev. D **90**, 094014 (2014).
Lattice: B. B. Brandt, A. Francis, B. Jaeger and H. B. Meyer, Phys. Rev. D **93**, 054510 (2016).

Possible Phenomenological Implications



$$\sigma^{33} = e^2 \sum_f q_f^2 N_c \frac{|eq_f B|}{2\pi} \frac{4T}{g^2 C_F \cancel{m^2} \ln(T/M)}$$

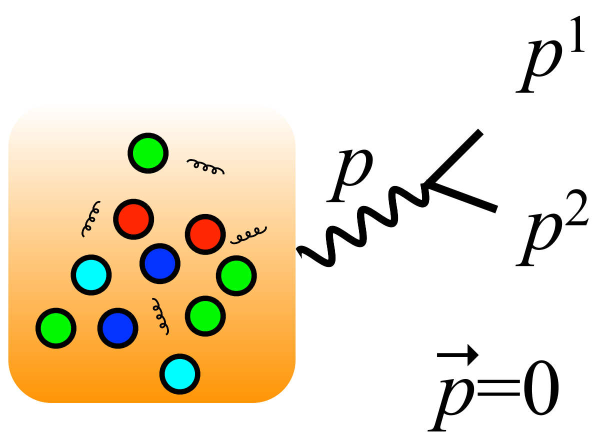


Beyond LLL approximation, there are also spin down particles, so the scattering is not suppressed by m^2.

σ^{33} is expected to be smaller at large T, so that it smoothly connects with B=0 result.

Possible Phenomenological Implications

2. Soft Dilepton Production



G. D. Moore and J. -M. Robert, hep-ph/0607172.

$$\frac{d\Gamma}{d^4p} = \frac{\alpha}{12\pi^4\omega^2} T\sigma^{33}$$

$\because$(virtual photon emission rate)$\sim n_B(\omega)\mathrm{Im}\Pi^{\mu}{}_{\mu} \sim T\sigma^{33}$

(photon interaction energy w leptons)

(quark mean free path)$^{-1}$

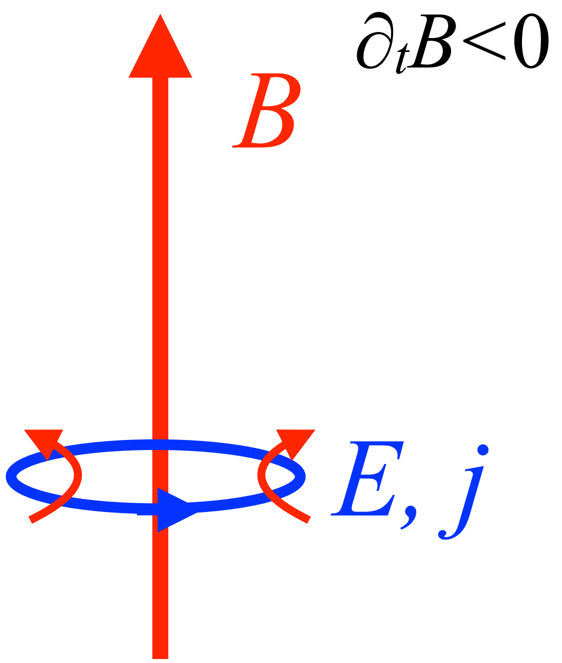
$$e\sqrt{eB} \ll \omega \ll \frac{g^2m^2}{T}\ln\left(\frac{T}{M}\right)$$

σ^{33} is large

Soft dilepton production is enhanced by B?

Possible Phenomenological Implications

3. Back Reaction to EM Fields



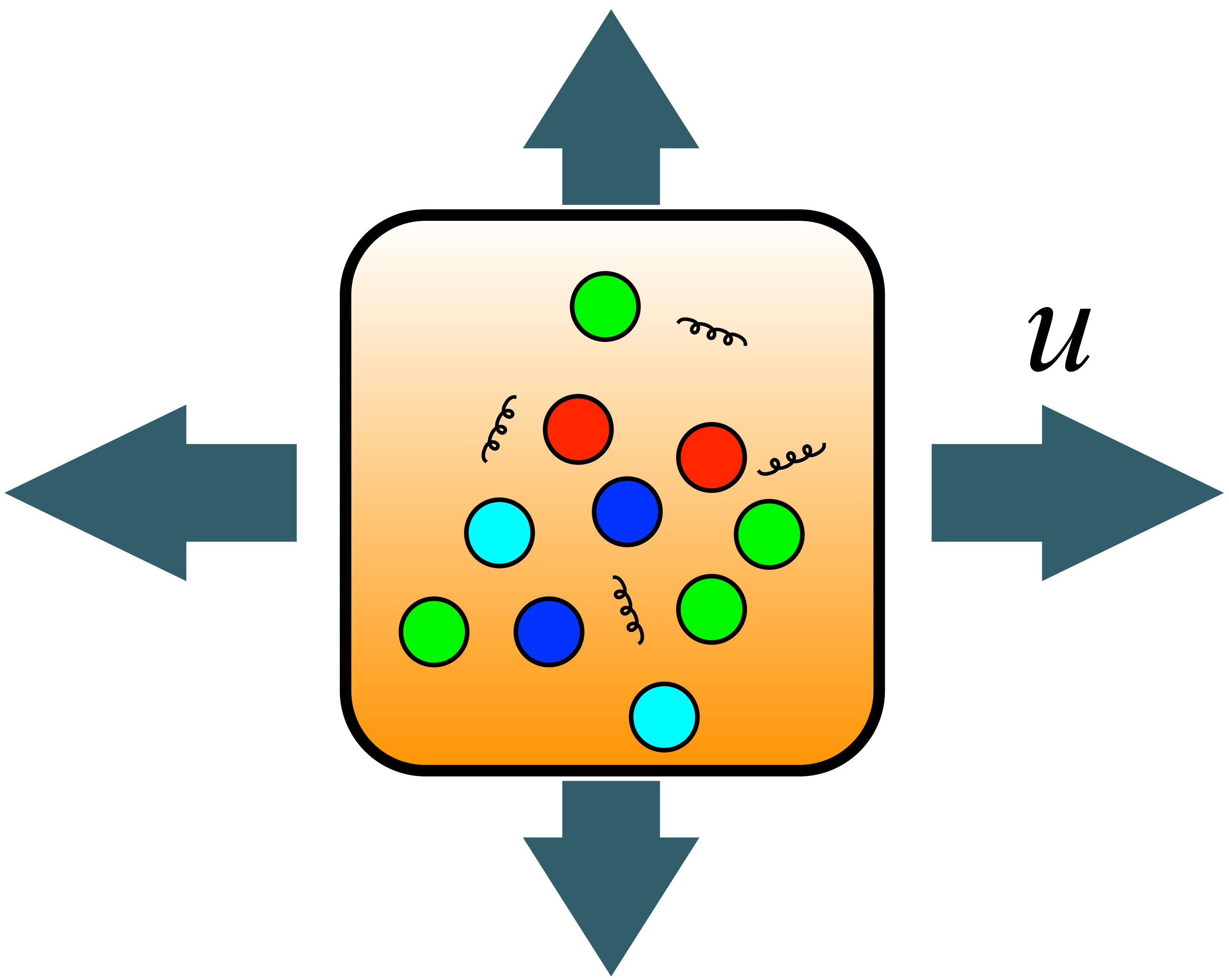
<u>Bad news</u>:

In LLL approximation, we have **no current in transverse plane**, so **Lenz's law does NOT work!**

The lifetime of B does not increase…

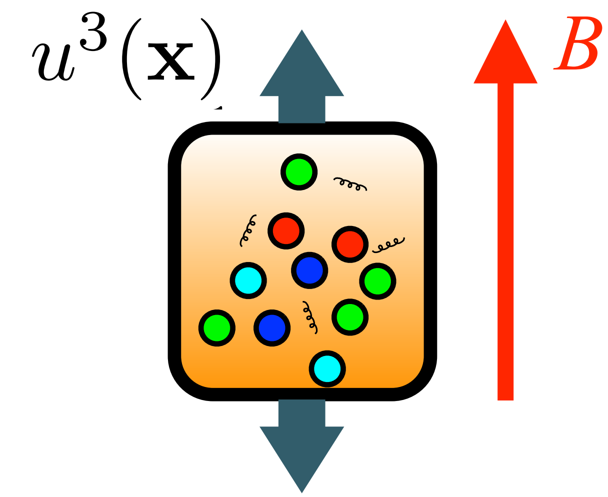
Bulk Viscosity

$$\Delta P = -\zeta \nabla \cdot u$$



Linearized Boltzmann equation

Boltzmann eq. without E $\quad \left(\partial_t + v^3 \partial_z\right) f(\mathbf{k}, \mathbf{x}, t) = C[f]$



Expansion in direction of B:

$$f_{\mathrm{eq}}(\mathbf{k}, \mathbf{x}, t) + \delta f(\mathbf{k}, \mathbf{x}, t) \qquad f_{\mathrm{eq}}(\mathbf{k}, \mathbf{x}, t) \equiv \left[\exp\{\beta(t)\gamma_u(\epsilon_k^L - k^3 u^3(\mathbf{x}))\} + 1\right]^{-1}$$

nonequilibrium deviation (responsible for viscosity)

linearize

$$-\beta n_F(\mathbf{k})[1 - n_F(\mathbf{k})] X(\mathbf{x}) \left[\Theta_\beta \epsilon_k^L - v^3 k^3\right] \simeq -\tau_k^{-1} \delta f(\mathbf{k}, \mathbf{x}, t)$$

expansion decreases T

$X(\mathbf{x}) \equiv \partial_z u^3(\mathbf{x})$: expansion (source term)

$\Theta_\beta \equiv \left(\dfrac{\partial P_\parallel}{\partial \varepsilon}\right)_{n,B}$: (speed of sound)^2

Linearized Boltzmann equation

solution: $\delta f(\mathbf{k}, \mathbf{x}, t) \simeq \beta \tau_k n_F(\mathbf{k})[1 - n_F(\mathbf{k})] X(\mathbf{x}) \left[\Theta_\beta \epsilon_k^L - v^3 k^3\right]$

Conformal case (m=0):

1 (not $1/3$, since the quarks live in one-dimension)

k^3

1

$\Rightarrow$ $\delta f = 0.$

In conformal case, the system is at equilibrium even after the expansion.
No nonequilibrium deviation, no bulk viscosity.

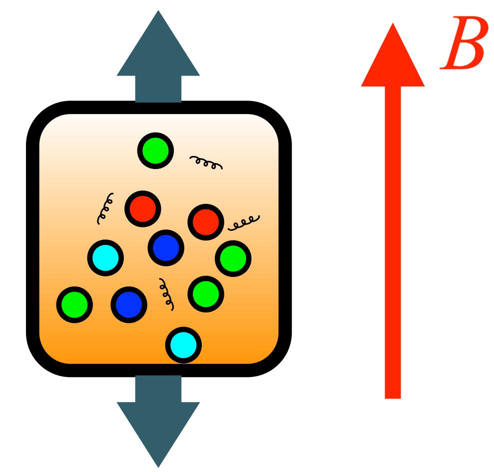
Linearized Boltzmann equation

solution: $\delta f(\mathbf{k},\mathbf{x},t) \simeq \beta\tau_k n_F(\mathbf{k})[1-n_F(\mathbf{k})]X(\mathbf{x})\,\underline{\left[\Theta_\beta \epsilon_k^L - v^3 k^3\right]}$

$-[(k^3)^2-\Theta_\beta(\varepsilon^L{}_k)^2]/\varepsilon^L{}_k$

$\Delta(P_\parallel-\Theta_\beta\varepsilon)=-3\zeta_\parallel X(x)$

X-G. Huang, A. Sedrakian, D. Rischke, Annals Phys. **326** 3075 (2011).



expansion decreases *T*.

So even in no-dissipative case, the pressure changes, and thus this contribution needs to be subtracted.

$$\delta[P_\parallel - \Theta_\beta \epsilon] = N_c \frac{|eq_f B|}{2\pi}\frac{1}{\pi}\int_{-\infty}^{\infty} dk^3\, \frac{(k^3)^2-\Theta_\beta(\epsilon_k^L)^2}{\epsilon_k^L}\,\delta f(\mathbf{k},\mathbf{x},t)$$

$[(k^3)^2-\Theta_\beta(\varepsilon^L{}_k)^2]$

Two conformal breaking factor $[(k^3)^2-\Theta_\beta(\varepsilon^L{}_k)^2]^2 \sim (m^2)^2$

Results

$$\zeta_{\parallel} = N_c \frac{|eq_f B|}{2\pi} \frac{9.6m^2}{g^2\pi^2 C_f T \ln\left(T/m\right)}$$

Conformal breaking

$$\frac{(m^2)^2}{m^2} = m^2$$

Chirality non-conservation

Same as the conductivity, except for the **extra m^4 dependence, due to the conformal breaking factor.**

s **quark contribution would dominates over *u*/*d* contribution, in contrast to the electrical conductivity.**

(Same as B=0 case) $\zeta \sim \frac{m^4}{g^4 T \ln(g^{-1})}$

P. Arnold, C. Dogan, G. Moore, Phys. Rev. D **74** 085021 (2006).

Possible Phenomenological Implications

1. Order Estimate

Contribution from s quark

$$\zeta_{\|} = N_c \frac{|eq_f B|}{2\pi} \frac{9.6 m_s^2}{g^2 \pi^2 C_f T \ln(T/m)}$$

$$\alpha_s = \frac{g^2}{4\pi} = 0.3,$$
$$m = 3\mathrm{MeV}(u,d), 100\mathrm{MeV}(s),$$
$$eB = 10 m_\pi^2 = (440\mathrm{MeV})^2.$$

M=160MeV>>m

(B=0)

$$\zeta_{\|} = \zeta_{\perp} \simeq 0.13 \frac{m_s^4}{T}$$

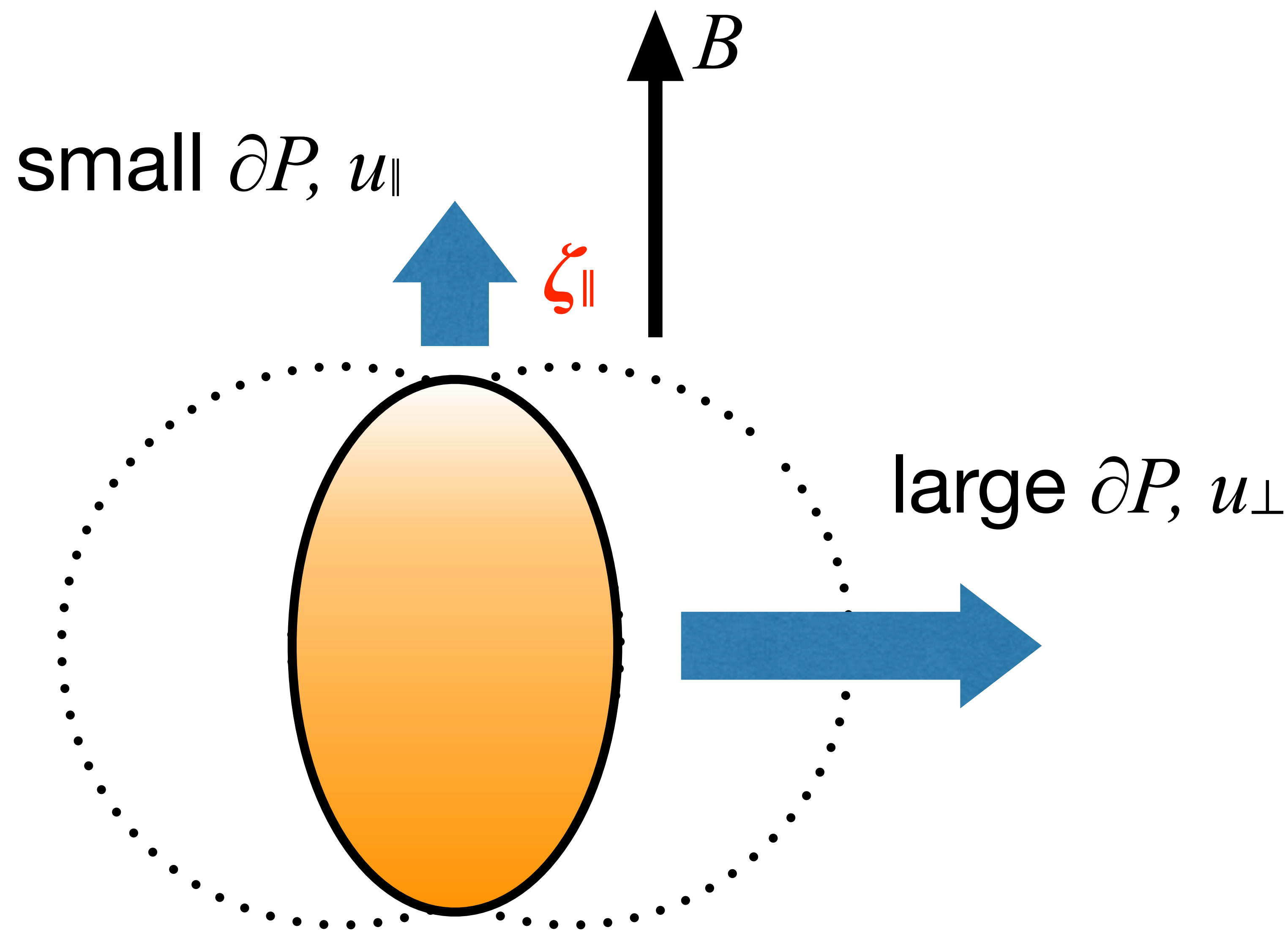
P. Arnold, C. Dogan, G. Moore, Phys. Rev. D **74** 085021 (2006).

$$\frac{\zeta_{\|}}{\zeta^s_{B=0}} \simeq 4.7 \frac{1}{\ln(T/M)}$$

***B* enhances $\zeta_{\|}$.**

Possible Phenomenological Implications

2. possible effect on flow



$\zeta_\parallel$ suppresses $u_\parallel$ $\Rightarrow$ **Enhances v_2?**

Summary (electrical conductivity)

Quark density in 1D

$$\sigma^{33} = e^2 \sum_f q_f^2 N_c \frac{|eq_f B|}{2\pi} \frac{4T}{g^2 C_F m^2 \ln(T/m)}$$

Landau degeneracy

Quark damping rate

When $M >> m$, $\ln(T/m) \rightarrow \ln(T/M)$.

The conductivity is enhanced by large *B*, and small *m*. The sensitivity to m was explained **in terms of chirality conservation.**

Summary (bulk viscosity)

$$\zeta_{\parallel} = N_c \frac{|eq_f B|}{2\pi} \frac{9.6 m^2}{g^2 \pi^2 C_f T \ln(T/m)}$$

When $M >> m$, $\ln(T/m) \rightarrow \ln(T/M)$.

Conformal breaking

$$\frac{(m^2)^2}{m^2} = m^2$$

Chirality non-conservation

The bulk viscosity is proportional to m^2, due to the conformal-breaking effect.

Future Perspective

- Go beyond LLL approximation... (more realistic B)
- Ask hydro guys to simulate MHD with our transport coefficients