Compositeness for the N^* and Δ^* resonances from the πN scattering amplitude

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1. Introduction
2. Two-body wave functions from scattering amplitudes
3. The N^* compositeness program
4. Summary

[1] T. S. , *Phys. Rev.* C95 (2017) 025206.

[2] T. S. , in preparation.

[3] T. S. , T. Hyodo and D. Jido, *PTEP* 2015 063D04.

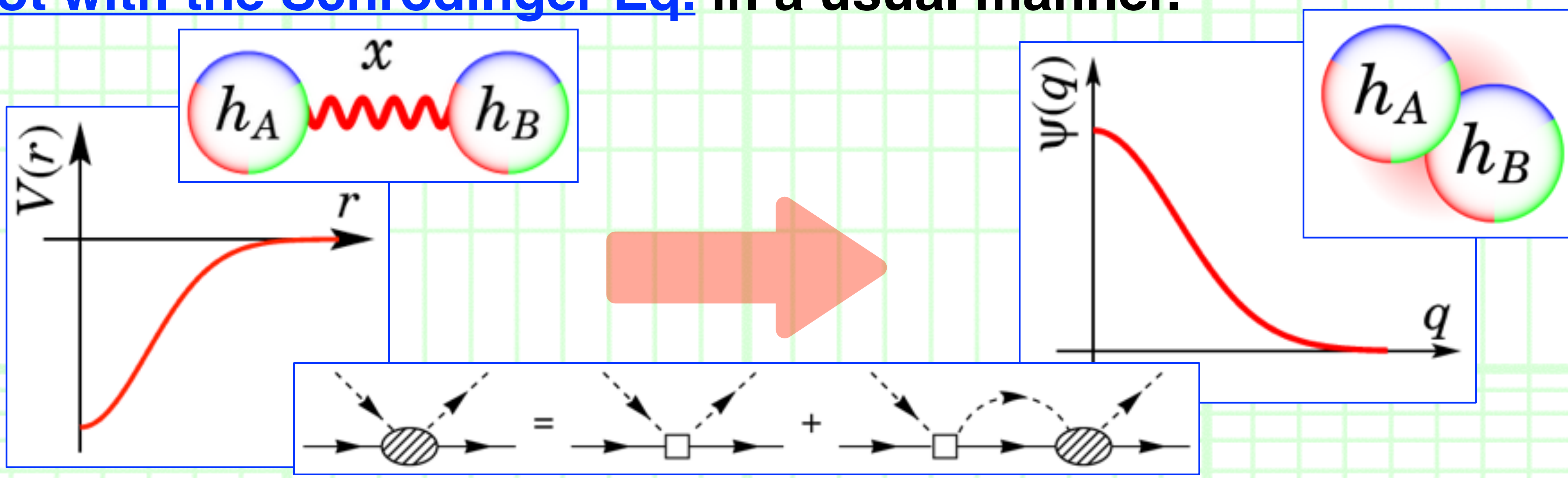
[4] T. S. , T. Arai, J. Yamagata-Sekihara and S. Yasui, *Phys. Rev.* C93 (2016) 035204.

1. Introduction

++ What we have done is ++

- **For a given interaction (potential) which generates a bound state, we can calculate the wave function of the bound state with the Lippmann-Schwinger Eq. (off-shell scattering amplitude for asymptotic two-body states).**

--- Not with the Schrödinger Eq. in a usual manner.



- **Furthermore, the wave function from the scattering amplitude is automatically scaled and shows the “correct” normalization.**

--- In contrast to the Schrödinger Eq. case,

we need not normalize the wave function by hand !

1. Introduction

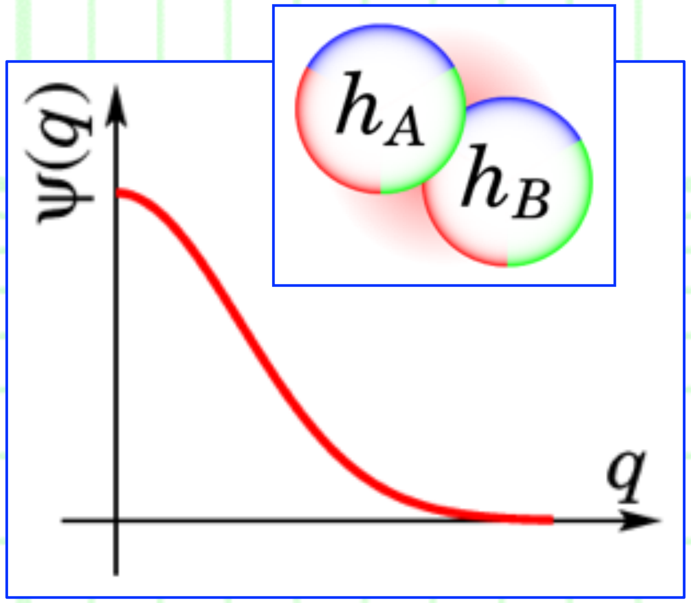
++ What we have done is ++

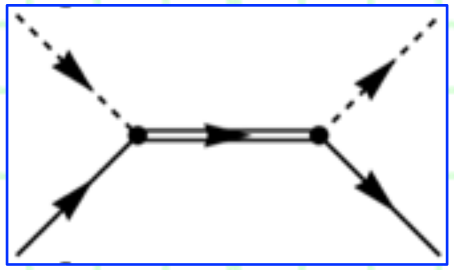
- "One can calculate the wave function for a given interaction."

--- Seems to be trivial ... ?

□ Energy dependent interaction.

--- Energy dependence of the interaction can be interpreted as a missing-channel contribution.



e.g. 

$$V_{\mathrm{miss}} = \frac{g_0^2}{E - M_0}$$

--> Then the norm of the bound state WF would deviate from unity.

□ Non-relativistic / semi-relativistic kinematics.

$$\hat{H}_0 |\mathbf{q}\rangle = \mathcal{E}(q) |\mathbf{q}\rangle \qquad \mathcal{E}(q) = M_{\mathrm{th}} + \frac{q^2}{2\mu} \quad \text{or} \quad \mathcal{E}(q) = \sqrt{m^2 + q^2} + \sqrt{M^2 + q^2}$$

□ Stable bound states / unstable resonances.

□ Coupled-channels effect. □ ...

- These points are clearly explained with the WF from the amplitude.

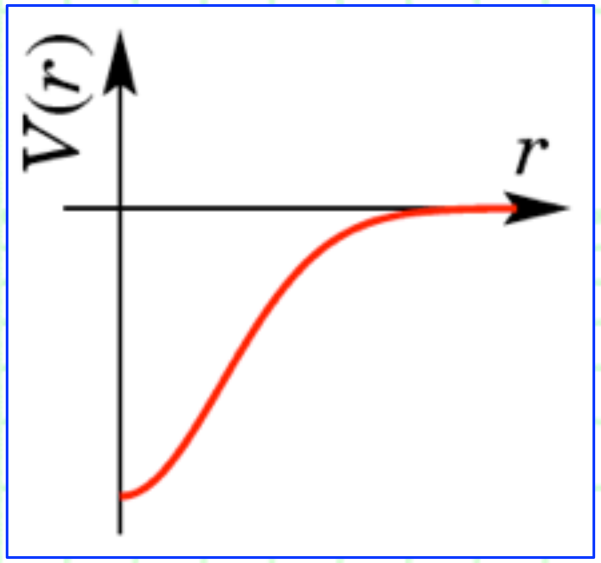
2. Wave functions from amplitudes

++ How to calculate the wave function ++

- There are **several approaches to calculate the wave function**.

  Ex.) A bound state in a NR single-channel problem.

  □ Usual approach: Solve the Schrödinger equation.



$$\hat{H}|\Psi\rangle = (\hat{H}_0 + \hat{V})|\Psi\rangle = E_{\text{pole}}|\Psi\rangle$$

--- Wave function in coordinate / momentum space:

$$\langle \mathbf{r}|\Psi\rangle = \psi(r) \qquad \langle \mathbf{q}|\Psi\rangle = \tilde{\psi}(q)$$

$$\left[M_{\text{th}} - \frac{\nabla^2}{2\mu} + V(r) \right] \psi(r) = E_{\text{pole}}\psi(r)$$

--- | q > is an eigenstate of free Hamiltonian H_0:

$$\hat{H}_0|\mathbf{q}\rangle = \mathcal{E}(q)|\mathbf{q}\rangle \qquad \mathcal{E}(q) = M_{\text{th}} + \frac{q^2}{2\mu}$$

--> After solving the Schrödinger equation, we have to **normalize the wave function by hand**.

$$\int d^3r\, [\psi(r)]^2 = 1 \quad \text{or} \quad \int \frac{d^3q}{(2\pi)^3} \left[\tilde{\psi}(q) \right]^2 = 1$$

<-- We require !

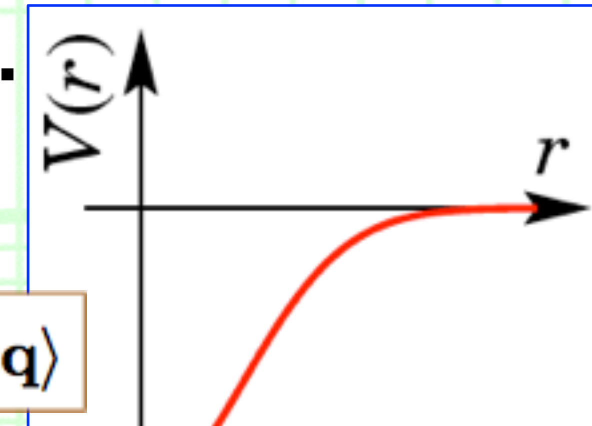
2. Wave functions from amplitudes

++ How to calculate the wave function ++

- There are several approaches to calculate the wave function.

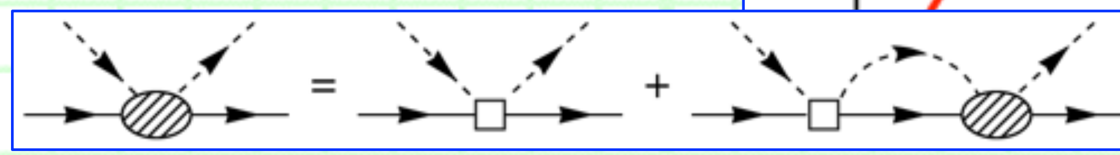
  Ex.) A bound state in a NR single-channel problem.

  - Our approach: Solve the Lippmann-Schwinger equation at the pole position of the bound state.



$$\hat{T}(E) = \hat{V} + \hat{V}\frac{1}{E - \hat{H}_0}\hat{T} = \hat{V} + \hat{V}\frac{1}{E - \hat{H}}\hat{V}$$

$$T(E;\, \mathbf{q}', \mathbf{q}) = \langle \mathbf{q}' | \hat{T}(E) | \mathbf{q} \rangle$$



--- Near the resonance pole position E_{pole}, amplitude is dominated by the pole term in the expansion by the eigenstates of H as

$$\langle \mathbf{q}' | \hat{T}(E) | \mathbf{q} \rangle \approx \langle \mathbf{q}' | \hat{V} | \Psi \rangle \frac{1}{E - E_{\text{pole}}} \langle \tilde{\Psi} | \hat{V} | \mathbf{q} \rangle$$

$|\Psi\rangle, \ |\mathbf{q}_{\text{full}}\rangle, \ldots$ $\quad \langle\tilde{\Psi}|, \ \langle \mathbf{q}_{\text{full}}|, \ldots$

$\Rightarrow$ $\mathbb{1} = |\Psi\rangle\langle\tilde{\Psi}| + \cdots$

--- The residue of the amplitude at the pole position has information on the wave function !

$$\langle \mathbf{q} | \hat{V} | \Psi \rangle = \langle \mathbf{q} | (\hat{H} - \hat{H}_0) | \Psi \rangle = [E_{\text{pole}} - \mathcal{E}(q)] \tilde{\psi}(q)$$

$$\langle \tilde{\Psi} | \hat{V} | \mathbf{q} \rangle = [E_{\text{pole}} - \mathcal{E}(q)] \tilde{\psi}(q)$$

$$\mathcal{E}(q) = M_{\text{th}} + \frac{q^2}{2\mu}$$

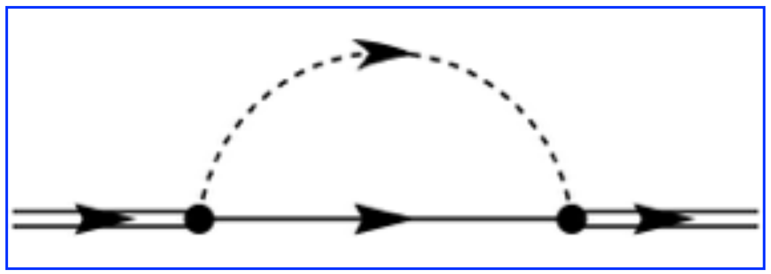
2. Wave functions from amplitudes

++ How to calculate the wave function ++

□ The idea of the renormalization for: $$\frac{1}{E-\hat{H}(E)} \approx |\Psi\rangle \frac{1}{E-E_{\rm pole}} \langle \tilde{\Psi}|$$

--- We "(re-)normalize" the total wave function as $\langle \tilde{\Psi}|\Psi\rangle = Z = 1$

cf.



$$\frac{1}{p^2-m_0^2-\Sigma(p^2)} \approx \frac{Z}{p^2-m_{\rm phys}^2}$$

by the pole term in the expansion by the eigenstates of H as

$$\langle \mathbf{q}'|\hat{T}(E)|\mathbf{q}\rangle \approx \langle \mathbf{q}'|\hat{V}|\Psi\rangle \frac{1}{E-E_{\rm pole}} \langle \tilde{\Psi}|\hat{V}|\mathbf{q}\rangle$$

$|\Psi\rangle$, $|\mathbf{q}_{\rm full}\rangle$, ... $\langle\tilde{\Psi}|$, $\langle \mathbf{q}_{\rm full}|$, ...

$$\mathbb{1} = |\Psi\rangle\langle\tilde{\Psi}| + \cdots$$

--- The residue of the amplitude at the pole position has information on the wave function !

$$\langle \mathbf{q}|\hat{V}|\Psi\rangle = \langle \mathbf{q}|(\hat{H}-\hat{H}_0)|\Psi\rangle = [E_{\rm pole}-\mathcal{E}(q)]\tilde{\psi}(q)$$

$$\langle \tilde{\Psi}|\hat{V}|\mathbf{q}\rangle = [E_{\rm pole}-\mathcal{E}(q)]\tilde{\psi}(q)$$

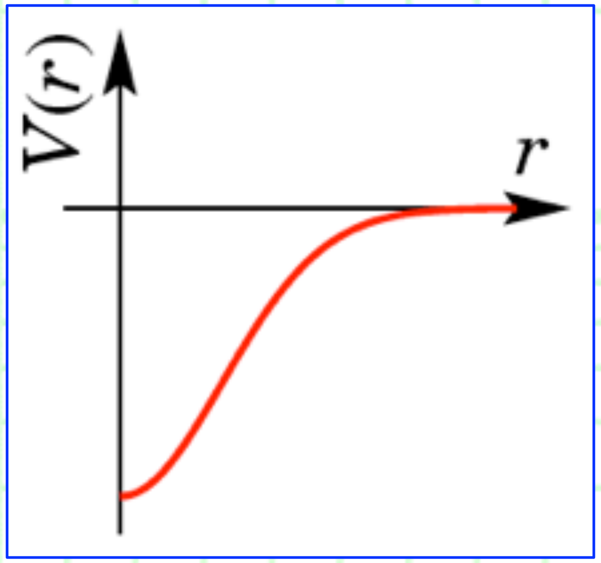
$$\mathcal{E}(q) = M_{\rm th} + \frac{q^2}{2\mu}$$

2. Wave functions from amplitudes

++ How to calculate the wave function ++

- There are **several approaches to calculate the wave function**.

  Ex.) A bound state in a NR single-channel problem.



  - Our approach: Solve the Lippmann-Schwinger equation at the pole position of the bound state.

--- The wave function can be extracted from the residue of the amplitude at the pole position:

$$T(E; \mathbf{q}', \mathbf{q}) = \langle \mathbf{q}' | \hat{T}(E) | \mathbf{q} \rangle \approx \frac{\gamma(q')\gamma(q)}{E - E_{\text{pole}}}$$

<-- Off-shell Amp. !

$$\gamma(q) \equiv \langle \mathbf{q} | \hat{V} | \Psi \rangle = [E_{\text{pole}} - \mathcal{E}(q)] \tilde{\psi}(q)$$

--> Because the scattering amplitude cannot be freely scaled (Lippmann-Schwinger Eq. is inhomogeneous !), the WF from the residue of the amplitude is automatically scaled as well !

If purely molecule -->

$$\int \frac{d^3 q}{(2\pi)^3} \left[\frac{\gamma(q)}{E_{\text{pole}} - \mathcal{E}(q)} \right]^2 = 1$$

<-- We obtain !

E. Hernandez and A. Mondragon, *Phys. Rev.* C29 (1984) 722.

2. Wave functions from amplitudes

++ Example 1: Stable bound state ++

- A Λ hyperon in $A \sim 40$ nucleus.

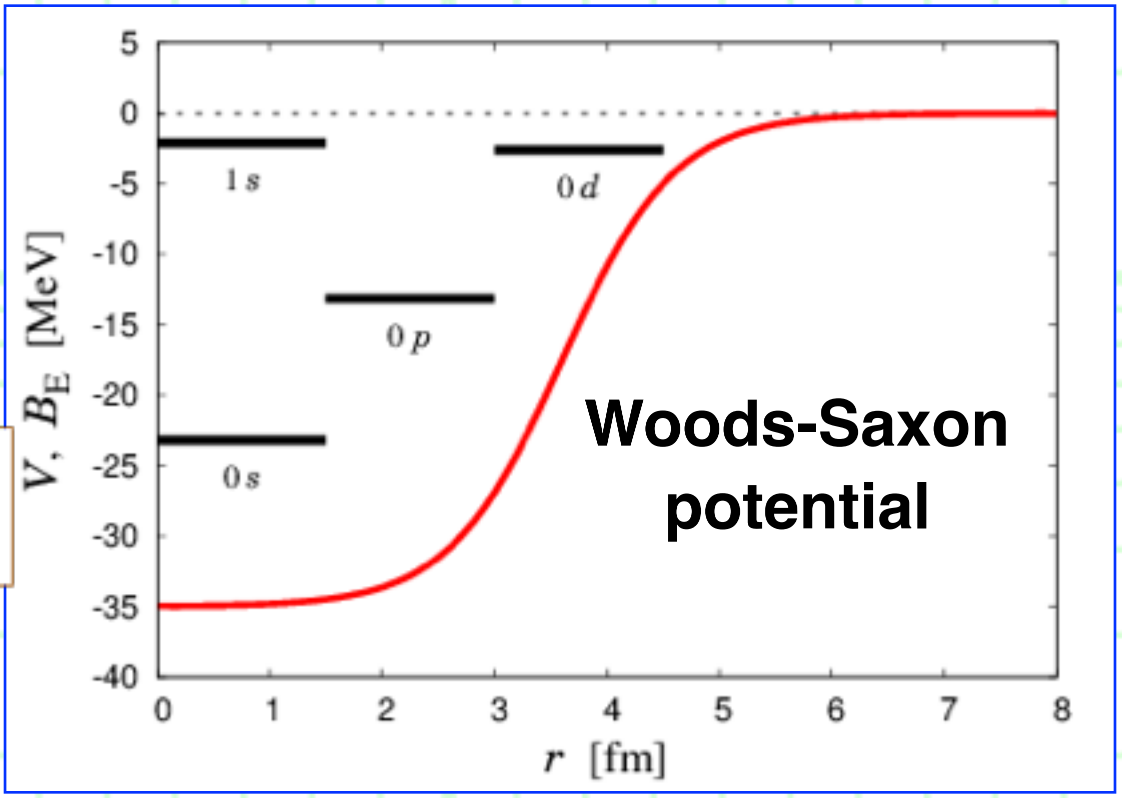
--> Calculate wave functions in 2 ways.

1. Solve Schrödinger equation:

$$\mathcal{E}(q)\tilde{\psi}(q) + \int \frac{d^3 q}{(2\pi)^3} \tilde{V}(\mathbf{q}, \mathbf{q}')\tilde{\psi}(q') = E_{\text{pole}}\tilde{\psi}(q)$$

--> Normalize ψ by hand !

$$\int \frac{d^3 q}{(2\pi)^3} \left[\tilde{\psi}(q)\right]^2 = 1$$



2. Solve Lippmann-Schwinger equation:

$$T(E; \mathbf{q}', \mathbf{q}) = V(\mathbf{q}', \mathbf{q}) + \int \frac{d^3 k}{(2\pi)^3} \frac{\tilde{V}(\mathbf{q}', \mathbf{k})T(E; \mathbf{k}, \mathbf{q})}{E - \mathcal{E}(k)}$$

--> Extract WF from the residue:

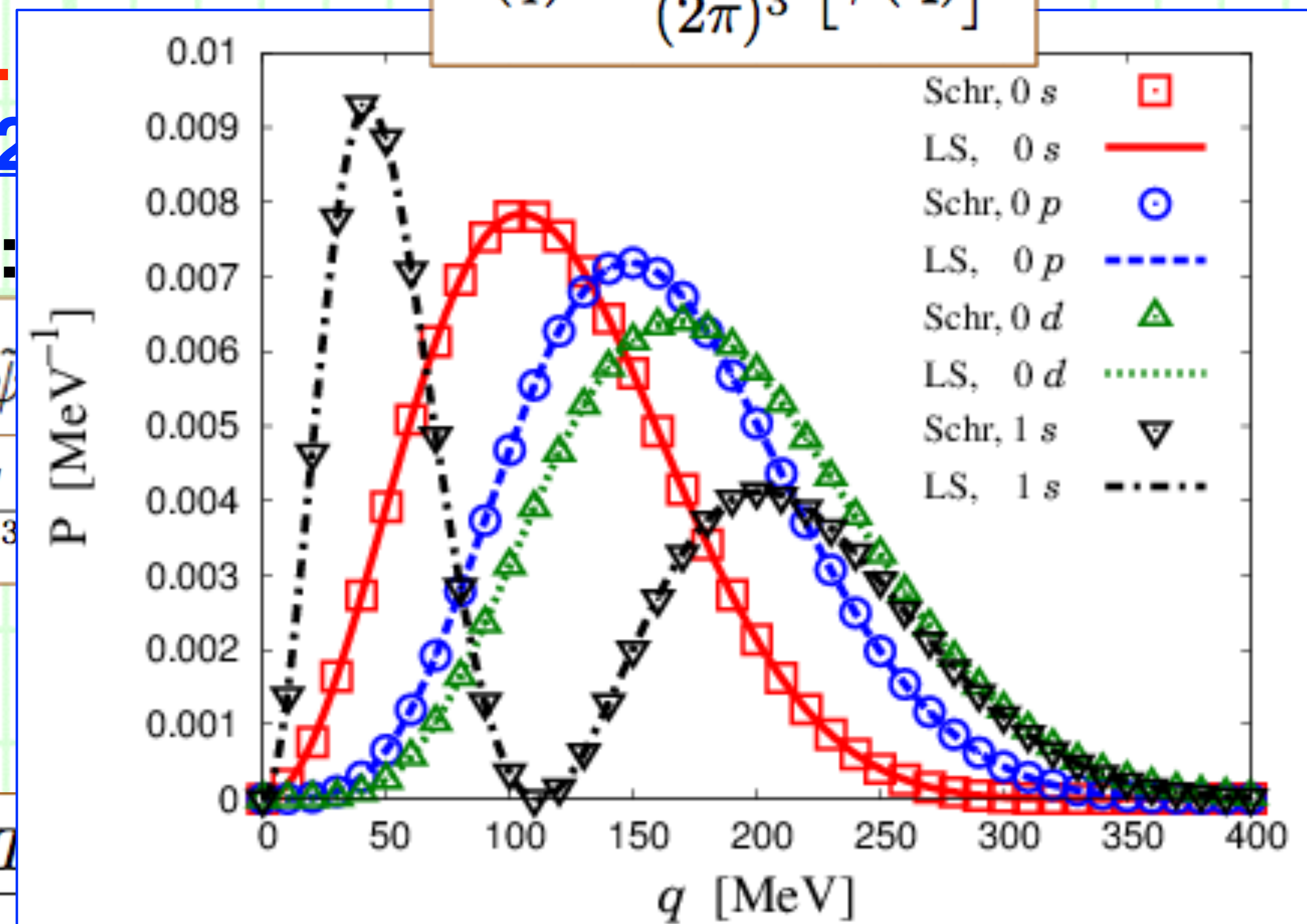
$$T(E; \mathbf{q}', \mathbf{q}) \approx \frac{\gamma(q')\gamma(q)}{E - E_{\text{pole}}} \quad \text{-->} \quad \tilde{\psi}(q) = \frac{\gamma(q)}{E_{\text{pole}} - \mathcal{E}(q)}$$

--- Without normalizing by hand !

2. Wave functions from amplitudes

++ Example 1: Stable bou

$$\mathrm{P}(q) = \frac{4\pi q^2}{(2\pi)^3} \left[\tilde{\psi}(\mathbf{q}) \right]^2$$



- A Λ hyperon in $A \sim 40$ nucleus.

--> Calculate wave functions in 2

1. Solve Schrödinger equation:

$$\mathcal{E}(q)\tilde{\psi}(q) + \int \frac{d^3 q}{(2\pi)^3} \tilde{V}(\mathbf{q}, \mathbf{q}')\tilde{\psi}(q') = E_{\mathrm{pole}}\tilde{\psi}$$

--> Normalize ψ by hand !

$$\int \frac{d^3 q}{(2\pi)^3}$$

2. Solve Lippmann-Schwinger equation:

$$T(E; \mathbf{q}', \mathbf{q}) = V(\mathbf{q}', \mathbf{q}) + \int \frac{d^3 k}{(2\pi)^3} \frac{\tilde{V}(\mathbf{q}', \mathbf{k})T}{E - \mathcal{E}(k)}$$

--> Extract WF from the residue:

$$T(E; \mathbf{q}', \mathbf{q}) \approx \frac{\gamma(q')\gamma(q)}{E - E_{\mathrm{pole}}} \quad \text{-->} \quad \tilde{\psi}(q) = \frac{\gamma(q)}{E_{\mathrm{pole}} - \mathcal{E}(q)}$$

--- Without normalizing by hand !

- In 1st way: Points.
  2nd way: Lines.
- Exact coincidence !

--- We obtain auto-matically normalized WF from the Amp. !

2. Wave functions from amplitudes

++ Example 1: Stable bound state ++

- We define **the compositeness X as the norm of the wave function**:

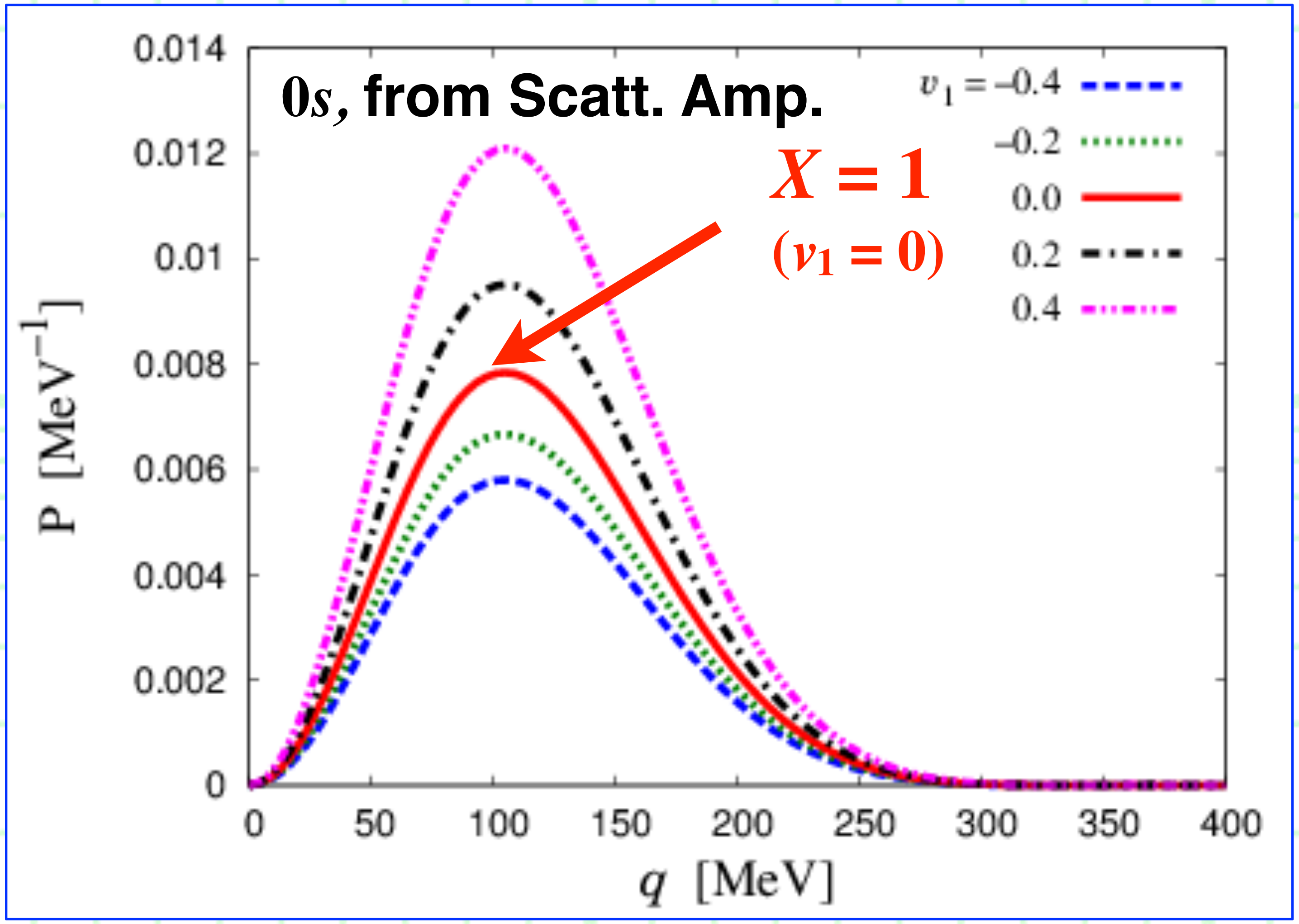
$$X \equiv \int \frac{d^3q}{(2\pi)^3} \langle \tilde{\Psi} | \mathbf{q} \rangle \langle \mathbf{q} | \Psi \rangle = \int_0^\infty dq \, \mathrm{P}(q)$$

$$\mathrm{P}(q) = \frac{4\pi q^2}{(2\pi)^3} \left[\frac{\gamma(q)}{E_{\mathrm{pole}} - \mathcal{E}(q)} \right]^2$$

--- In the following, we calculate X from the scattering amplitude.

□ The compositeness is **unity for energy independent interaction**.

Hernandez and Mondragon (1984).



□ However, if the interaction depends on the energy, the compositeness from the scattering amplitude **deviates from unity**.

$$V(r;\, E) \propto [v_0 + v_1(E - E_{\mathrm{pole}})]$$

2. Wave functions from amplitudes

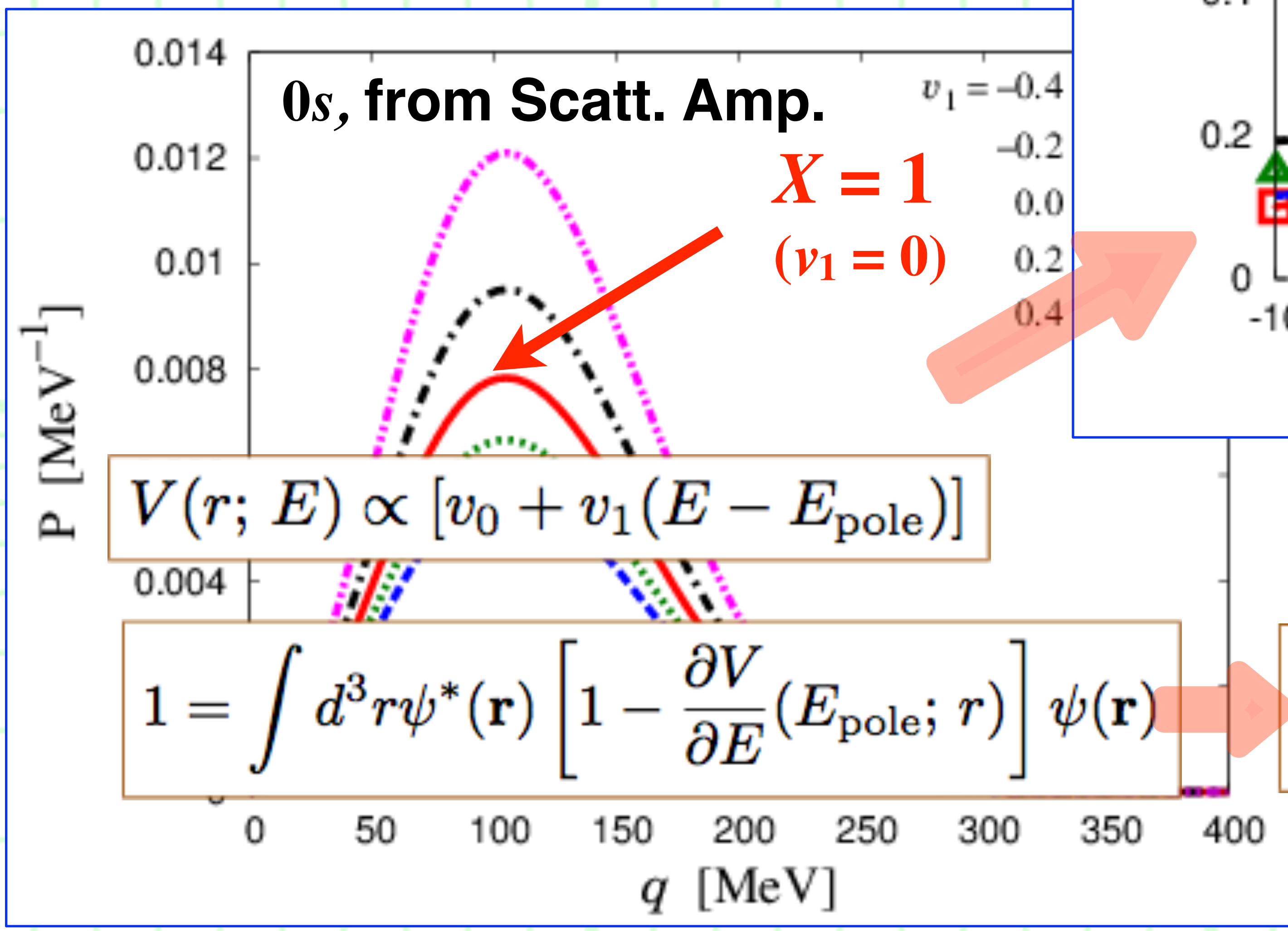
++ Example 1: Stable bound state ++

- We define **the compositenes**

$$X \equiv \int \frac{d^3q}{(2\pi)^3} \langle \tilde{\Psi} | \mathbf{q} \rangle \langle \mathbf{q} | \Psi \rangle =$$

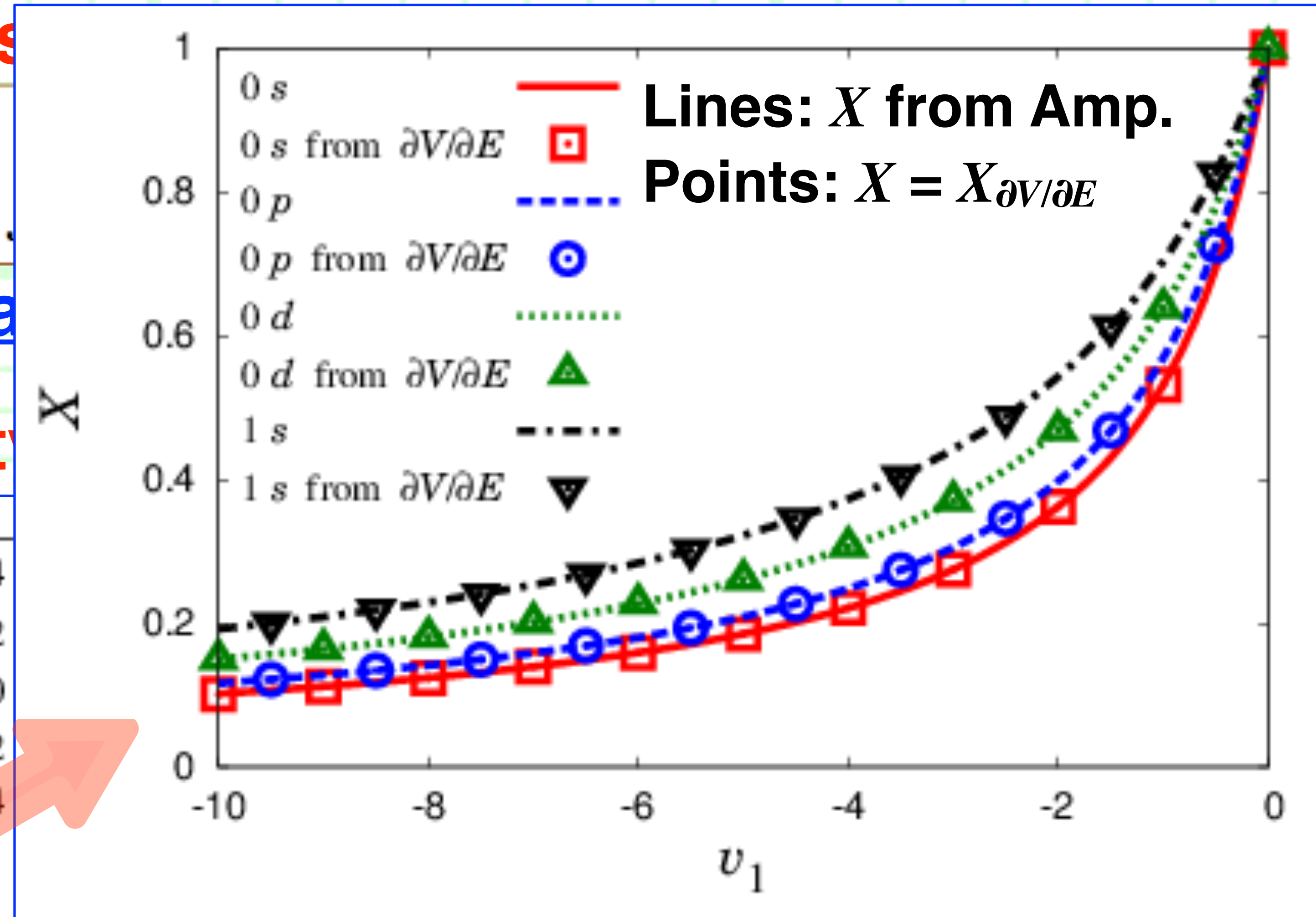
--- In the following, we calcula

- The compositeness is unit



$$V(r;\, E) \propto [v_0 + v_1(E - E_{\text{pole}})]$$

$$1 = \int d^3r \psi^*(\mathbf{r}) \left[1 - \frac{\partial V}{\partial E}(E_{\text{pole}};\, r) \right] \psi(\mathbf{r})$$



- **Consistent with the norm with energy-dep. interaction.**

$$X_{\partial V/\partial E} = 1 + \int d^3r \psi^*(\mathbf{r}) \frac{\partial V}{\partial E}(E_{\text{pole}};\, r) \psi(\mathbf{r})$$

Formanek, Lombard and Mares (2004);
Miyahara and Hyodo (2016).

2. Wave functions from amplitudes

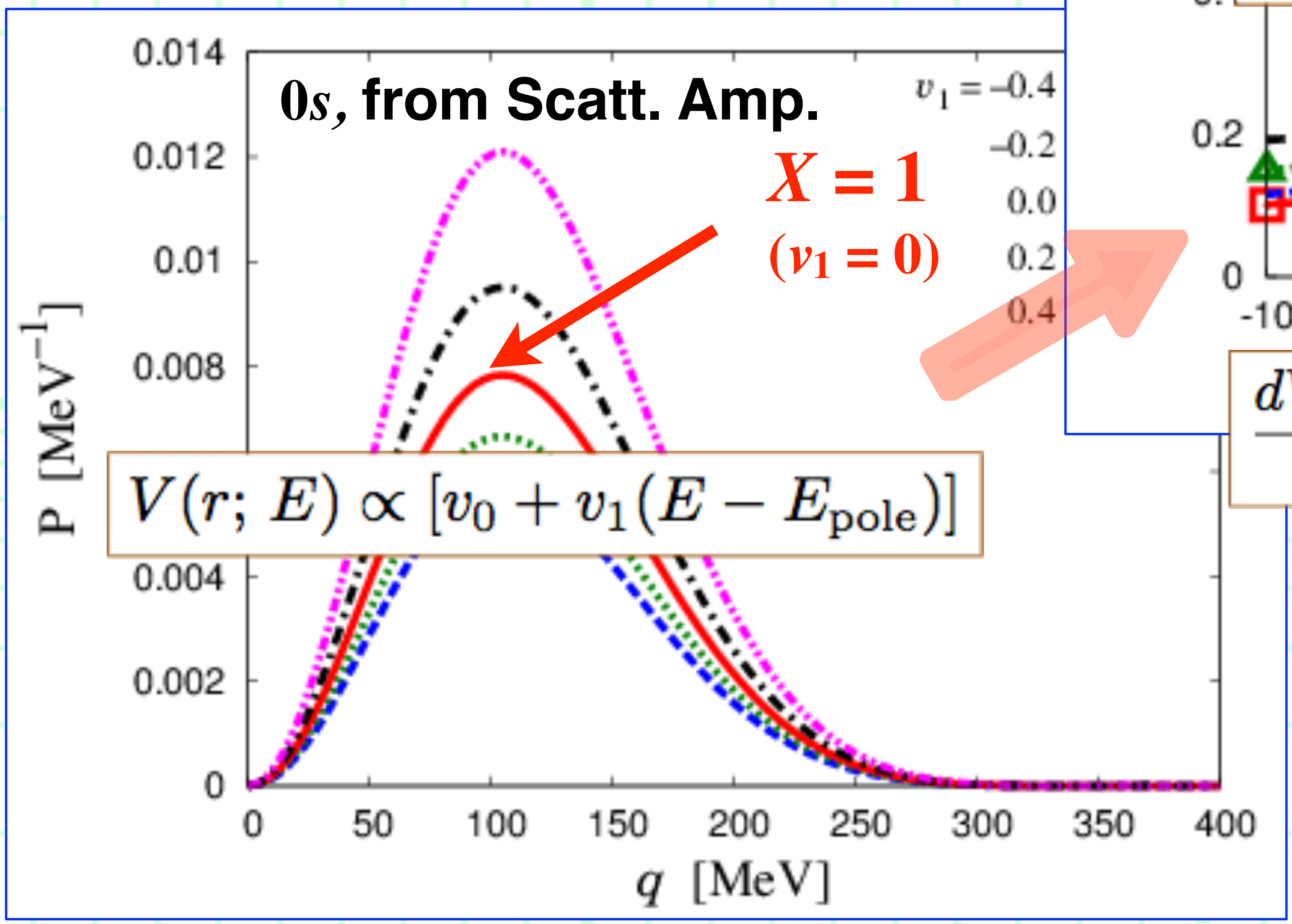
++ Example 1: Stable bound state ++

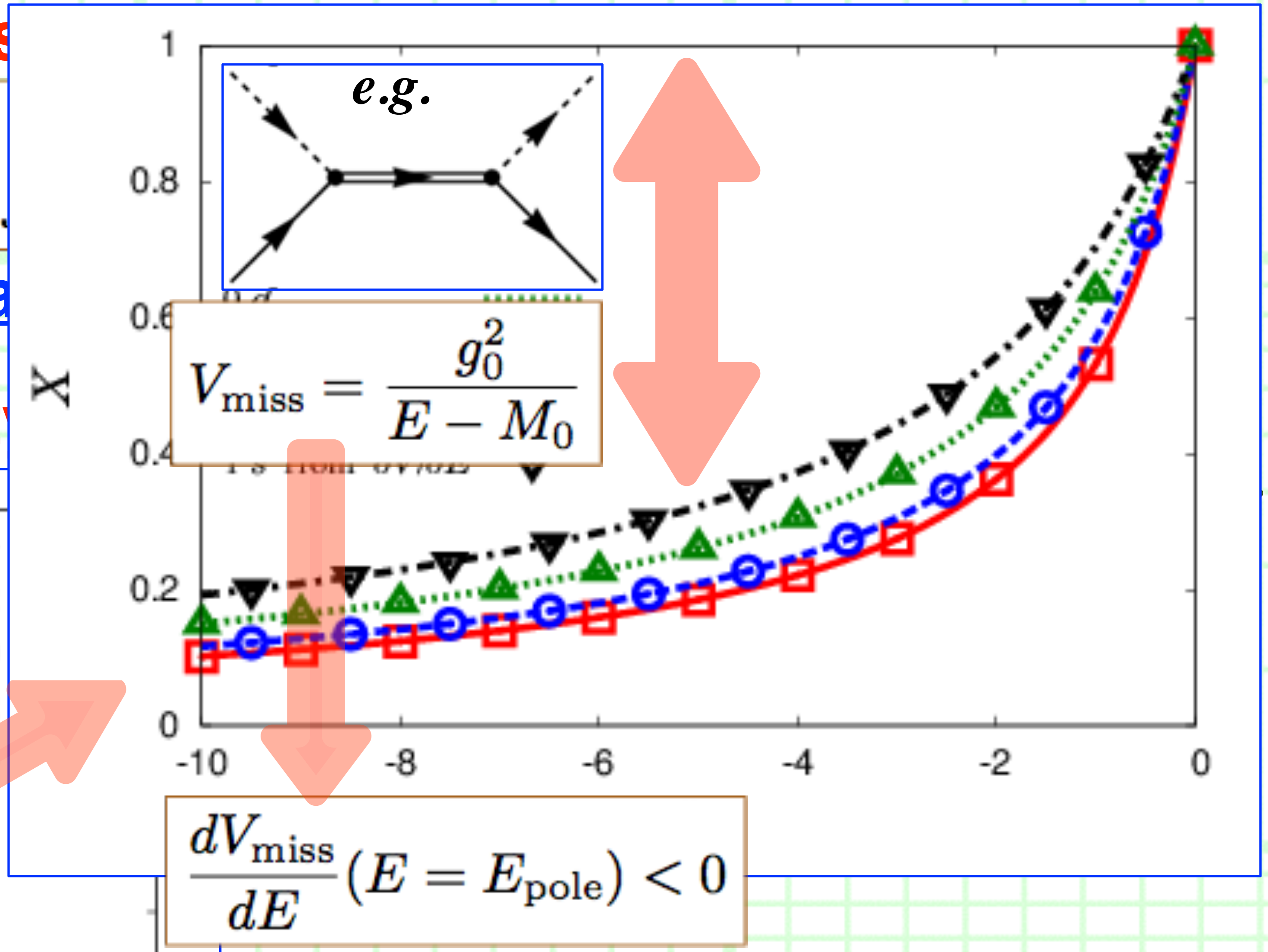
- We define **the compositenes**

$$X \equiv \int \frac{d^3q}{(2\pi)^3} \langle \tilde{\Psi} | \mathbf{q} \rangle \langle \mathbf{q} | \Psi \rangle =$$

--- In the following, we calcula

- The compositeness is unit





- **Deviation of compositeness from unity** can be interpreted **as a missing-channel part.**

T. S. , Hyodo and Jido, *PTEP* 2015 063D04.

2. Wave functions from amplitudes

++ Example 2: Unstable resonance state ++

- Unstable resonance in $\bar{K}N$-$\pi\Sigma$ system.

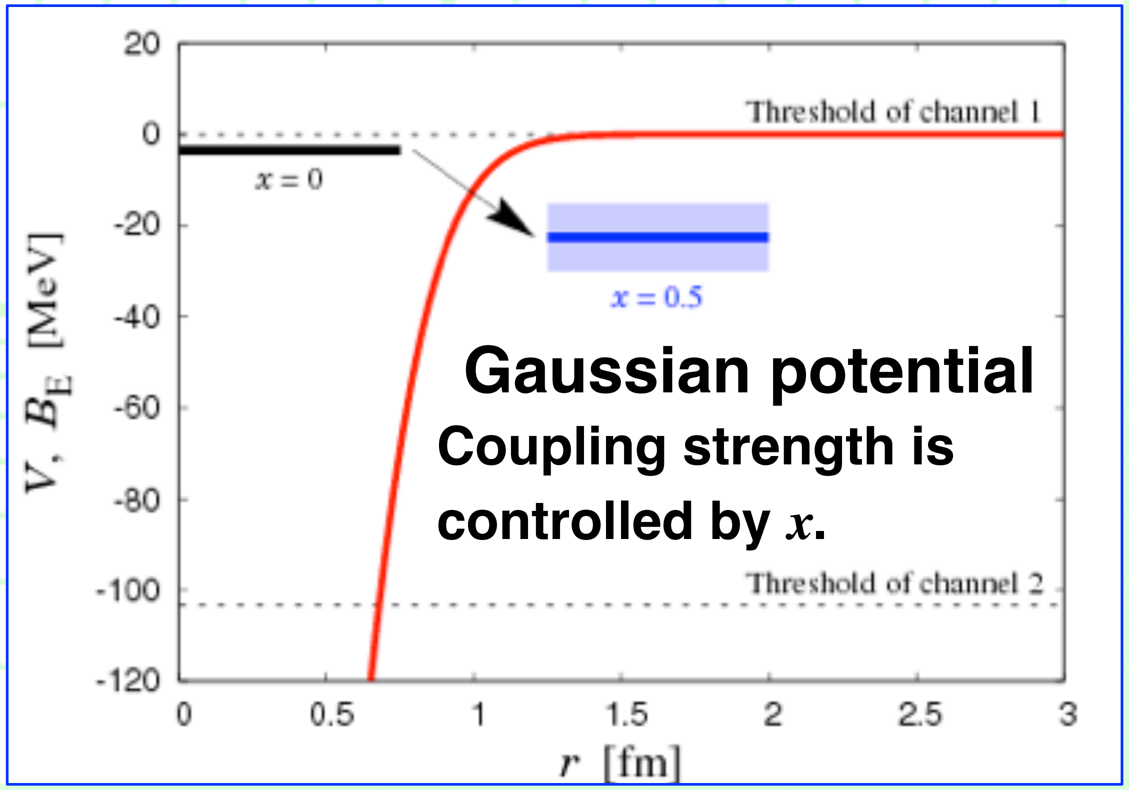
--> Calculate wave functions in 2 ways.

1. Solve Schrödinger equation:

$$E_j(q)\tilde{\psi}_j(\mathbf{q}) + \sum_k \int \frac{d^3q'}{(2\pi)^3} \tilde{V}_{jk}(\mathbf{q}, \mathbf{q}')\tilde{\psi}_k(\mathbf{q}') = E_{\text{pole}}\tilde{\psi}_j(\mathbf{q})$$

--> Normalize ψ_j by hand !

$$X_1 + X_2 = 1, \quad X_j \equiv \int \frac{d^3q}{(2\pi)^3} \left[\tilde{\psi}_j(\mathbf{q})\right]^2$$



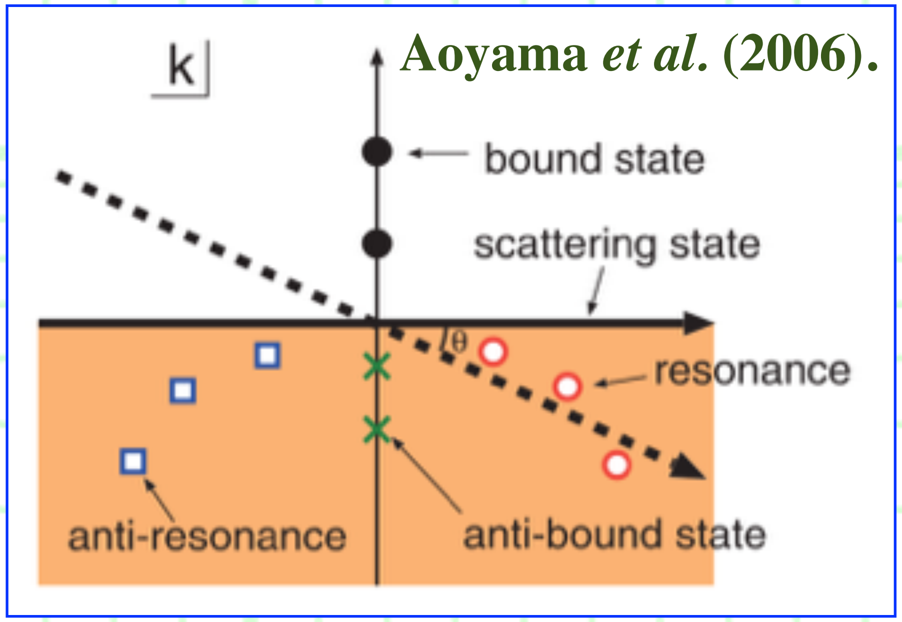
2. Solve Lippmann-Schwinger equation:

$$T_{jk}(\mathbf{q}', \mathbf{q}; E) = \tilde{V}_{jk}(\mathbf{q}', \mathbf{q}) + \sum_l \int \frac{d^3k}{(2\pi)^3} \frac{\tilde{V}_{jl}(\mathbf{q}', \mathbf{k})T_{lk}(\mathbf{k}, \mathbf{q}; E)}{E - E_l(k)}$$

--> Extract WF from the residue:

$$T_{jk}(\mathbf{q}', \mathbf{q}; E) \approx \frac{\gamma_j(\mathbf{q}')\gamma_k(\mathbf{q})}{E - E_{\text{pole}}} \quad \text{-->} \quad \tilde{\psi}_j(\mathbf{q}) = \frac{\gamma_j(\mathbf{q})}{E_{\text{pole}} - \mathcal{E}_j(q)}$$

--- Without normalizing by hand !



Complex scaling method.

2. Wave functions from amplitudes

++ Example 2: Unstable resona

$$\mathrm{P}_j(q) = \frac{4\pi q^2}{(2\pi)^3} \left[\tilde{\psi}_j(\mathbf{q}) \right]^2$$

- Unstable resonance in $\bar{K}N$-π

--> Calculate wave functions

1. Solve Schrödinger equati

$$E_j(q)\tilde{\psi}_j(\mathbf{q}) + \sum_k \int \frac{d^3 q'}{(2\pi)^3} \tilde{V}_{jk}(\mathbf{q},\, \mathbf{q}')\tilde{\psi}_k(\mathbf{q}') =$$

--> Normalize ψ_j by hand !

$$X_1 + X_2 = 1, \quad X_j \equiv \int \frac{d^3 q}{(2\pi)^3} \left[\tilde{\psi}_j(\mathbf{q}) \right]^2$$

2. Solve

equati

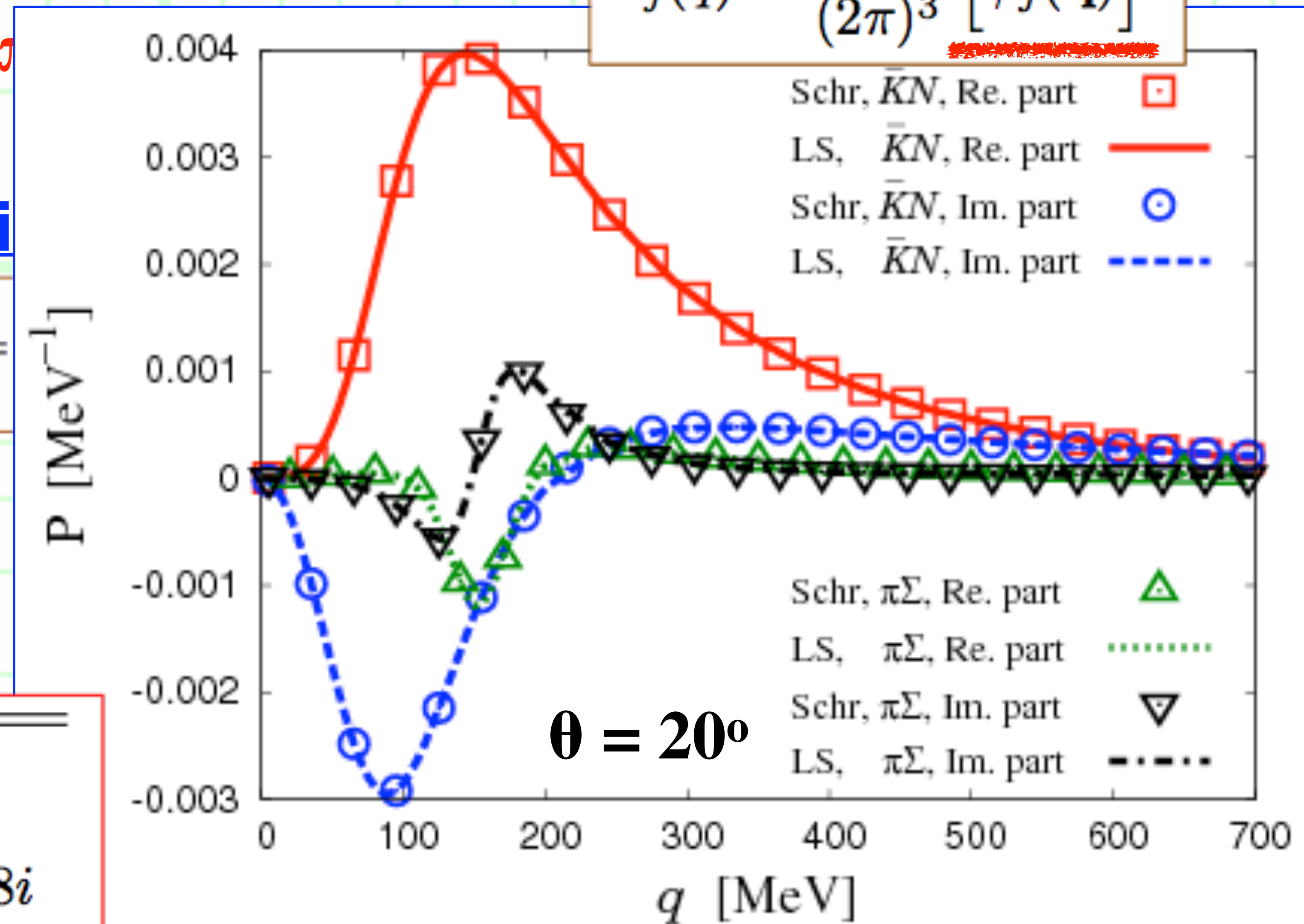
$$T_{jk}(\mathbf{q}',\, \mathbf{q};\, E \quad \quad E - E_l(k)$$

B_{E} [MeV]	22.6
Γ [MeV]	14.7
X_1	$0.99 - 0.08i$
X_2	$0.01 + 0.08i$
$X_1 + X_2$	$1.00 + 0.00i$

--> Extra e:

$$T_{jk}(\mathbf{q}',\, \mathbf{q};\, E) \approx \frac{\gamma_j(\mathbf{q}')\gamma_k(\mathbf{q})}{E - E_{\mathrm{pole}}} \quad \text{-->} \quad \tilde{\psi}_j(\mathbf{q}) = \frac{\gamma_j(\mathbf{q})}{E_{\mathrm{pole}} - \mathcal{E}_j(q)}$$

--- Without normalizing by hand !



- In 1st way: Points.
  2nd way: Lines.
- Coincidence again !

--- Our method is valid
even for resonances !

2. Wave functions from amplitudes

++ Example 2: Unstable resonance state ++

- We define **the compositeness X as the norm of the wave function**:

$$X \equiv \int \frac{d^3q}{(2\pi)^3} \langle \Psi^* | \mathbf{q} \rangle \langle \mathbf{q} | \Psi \rangle = \int_0^\infty dq \, \mathrm{P}(q)$$

$$\mathrm{P}(q) = \frac{4\pi q^2}{(2\pi)^3} \left[\tilde{\psi}(\mathbf{q}) \right]^2$$

--- **θ Indep. !**

--- In the following, we **calculate X from the scattering amplitude**.

<-- The compositeness is **unity for energy independent interaction**.

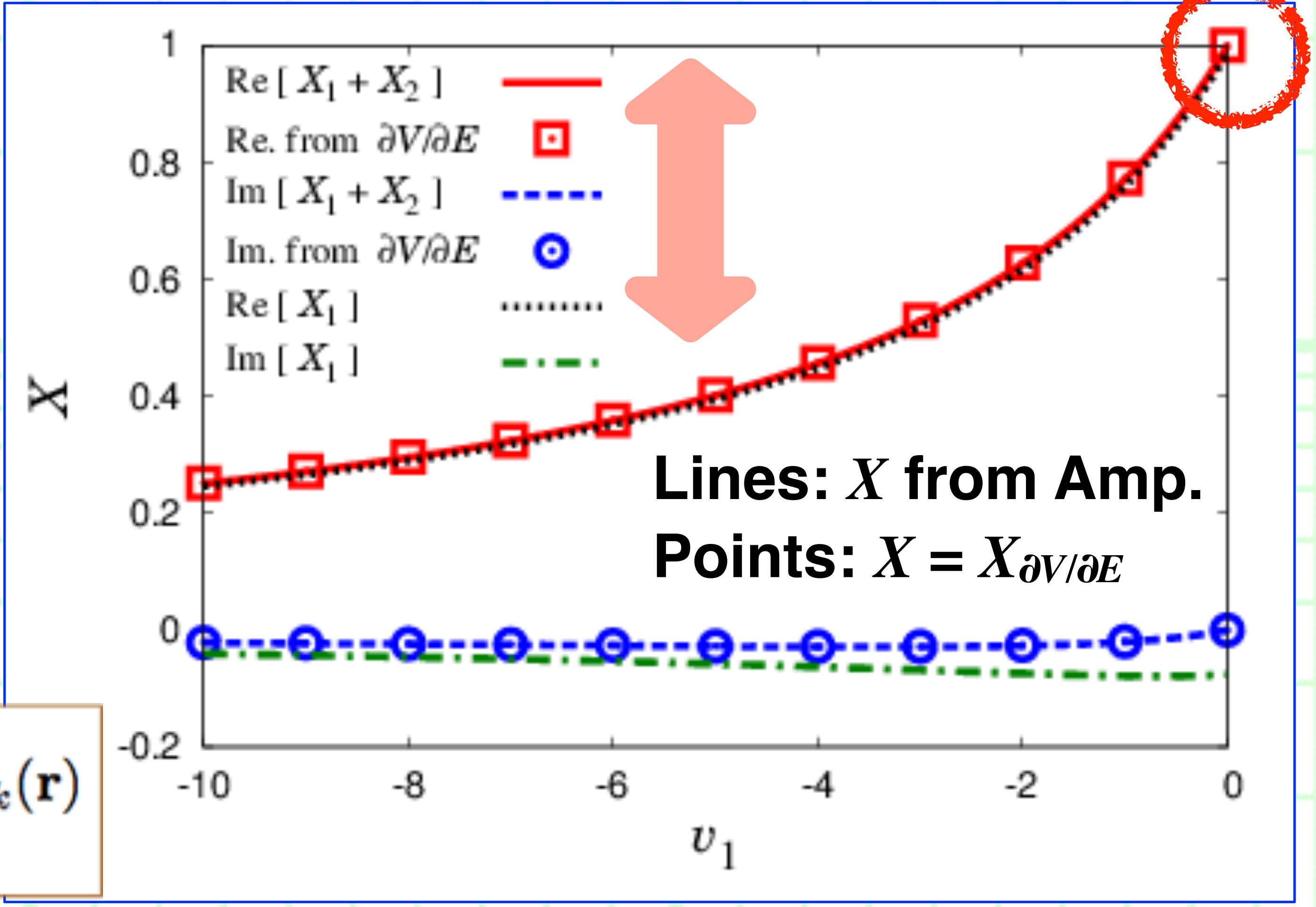
Hernandez and Mondragon (1984).

- When we consider **the energy dependence of the interaction**, **the compositeness from the scattering amplitude deviates from unity** because of **missing channel contribution**.

--- *e.g.*:

$$V_{\mathrm{miss}} = \frac{g_0^2}{E - M_0}$$

$$X_{\partial V / \partial E} = 1 + \sum_{j,k} \int d^3 r \psi_j(\mathbf{r}) \frac{\partial V_{jk}}{\partial E}(E_{\mathrm{pole}}; r) \psi_k(\mathbf{r})$$

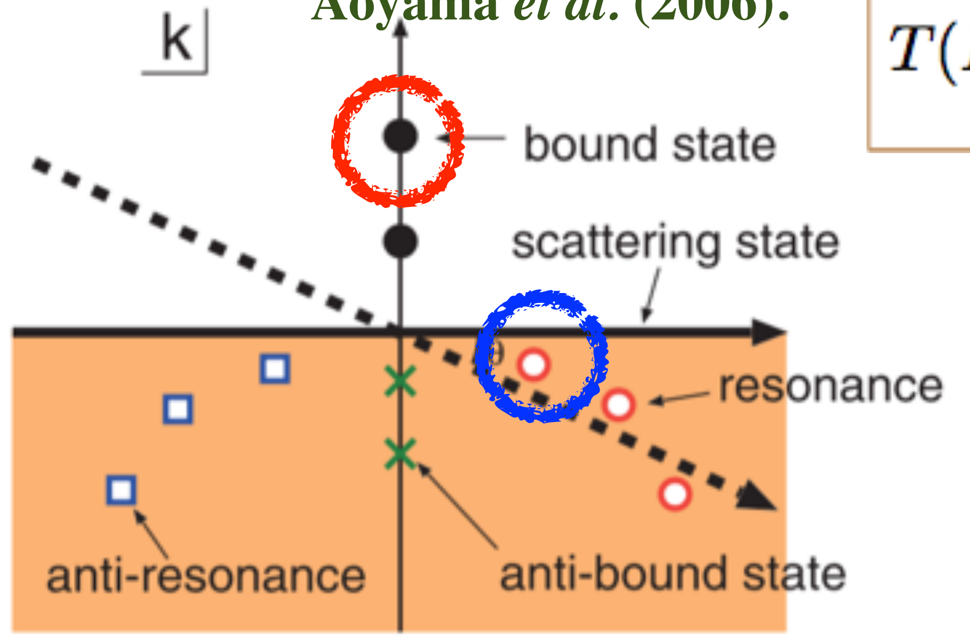


2. Wave functions from amplitudes

++ Lessons from schematic models ++

- For a given interaction, we can extract the two-body WF from the scattering amplitude at the pole position, both stable and unstable.

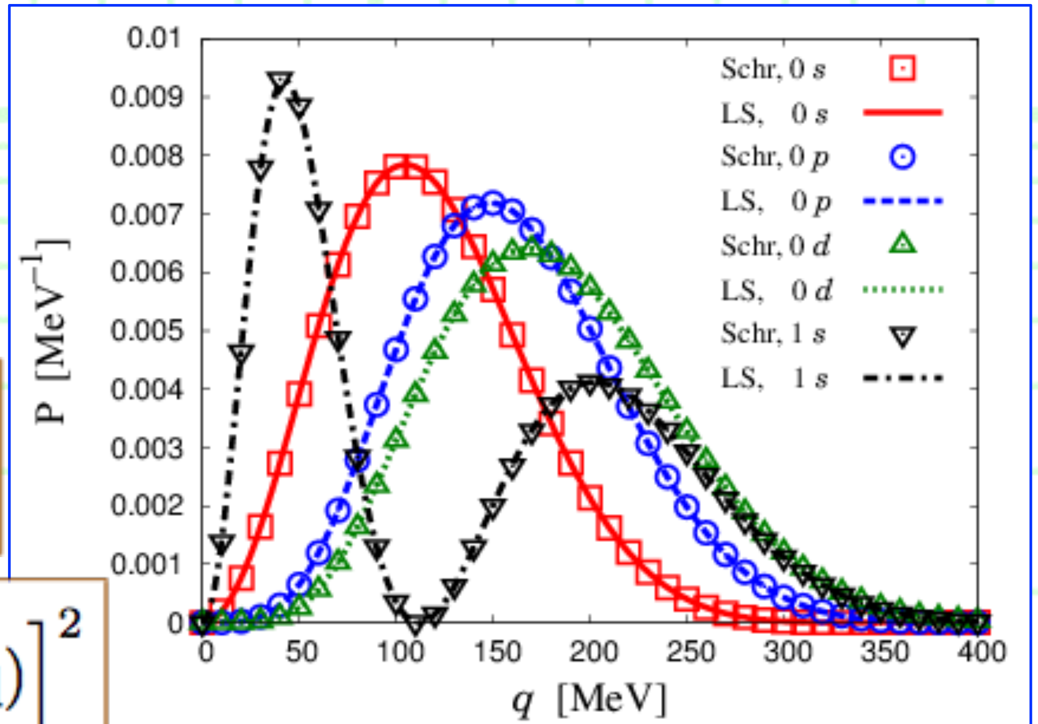
Aoyama *et al.* (2006).



$$T(E;\mathbf{q}',\mathbf{q}) \approx \frac{\gamma(q')\gamma(q)}{E - E_{\rm pole}}$$

$$\tilde{\psi}(q) = \frac{\gamma(q)}{E_{\rm pole} - \mathcal{E}(q)}$$

$$\mathrm{P}(q) = \frac{4\pi q^2}{(2\pi)^3}\left[\tilde{\psi}(\mathbf{q})\right]^2$$



T. S., *Phys. Rev.* C95 (2017) 025206.

- The WF from the scattering amplitude is automatically scaled.
  - The compositeness (= norm of the two-body WF) is unity for a bound state in an energy independent interaction.
  - For an energy dependent interaction, the compositeness deviates from unity, reflecting a missing channel contribution.

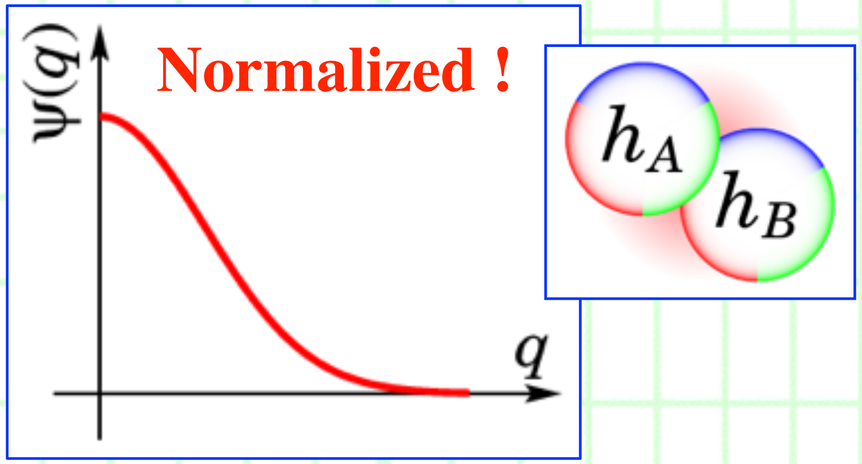
3. The N^* compositeness program

++ What I want to do is ++

- For a given interaction, we can calculate two-body wave functions from the scattering amplitude.

--- In particular, compositeness (= the norm of the wave function) is automatically normalized !

$$T(E; \mathbf{q}', \mathbf{q}) \approx \frac{\gamma(q')\gamma(q)}{E - E_{\rm pole}}$$



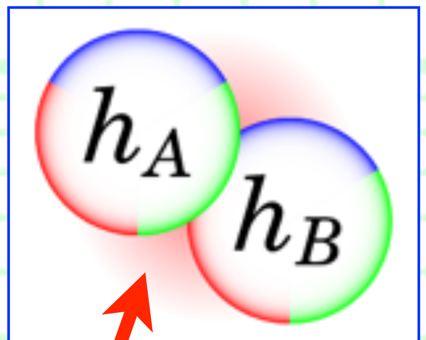
- Therefore, we can investigate:
  - Compositeness for "interesting" resonances from amplitudes.
  - Experimental information on the scattering amplitudes available.
  - Construction of detailed interactions possible.

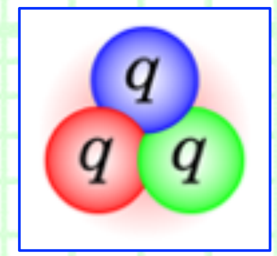
3. The N^* compositeness program

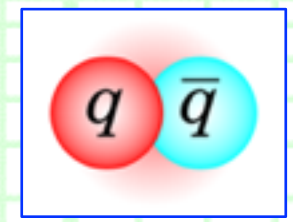
++ Wave functions for hadrons ++

- By using **the two-body wave function and compositeness** (norm), we can distinguish a certain configuration of hadrons in a model.

Hadronic molecules as a bound state of hadrons (*cf*. deuteron)



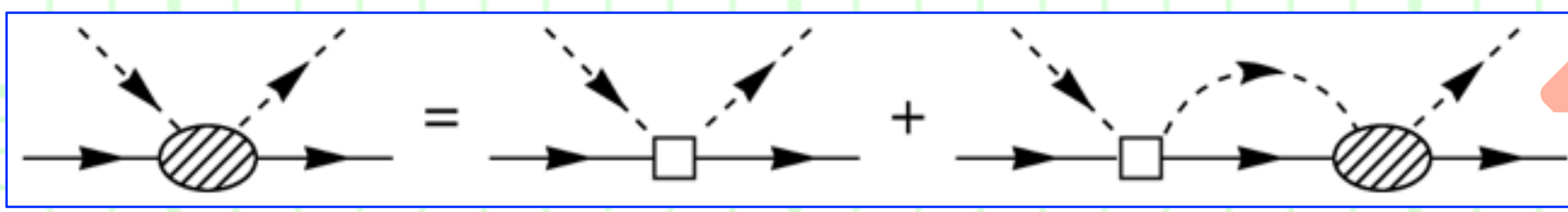




Ordinary hadrons

$$\langle \tilde{\Psi} | \Psi \rangle = X + Z = 1$$

$$X = \int \frac{d^3 q}{(2\pi)^3} \langle \tilde{\Psi} | \mathbf{q} \rangle \langle \mathbf{q} | \Psi \rangle = \int \frac{d^3 q}{(2\pi)^3} \left[\tilde{\psi}(\mathbf{q}) \right]^2$$



- In the previous studies, we have investigated:
  - □ Λ(1405). □ Ξ(1690). □ N(1535) & N(1650). □ ...

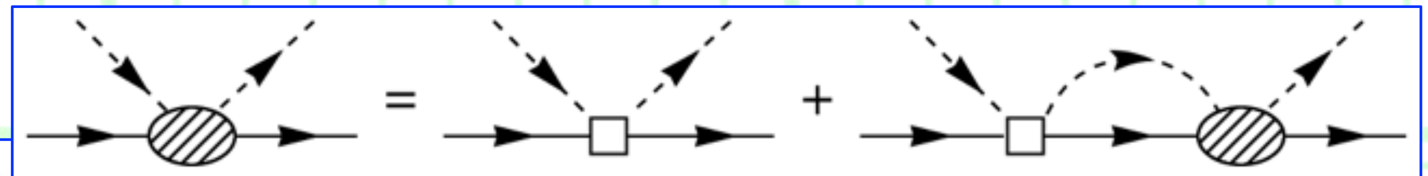
--- **Evaluated X for these "dynamically generated resonances".**

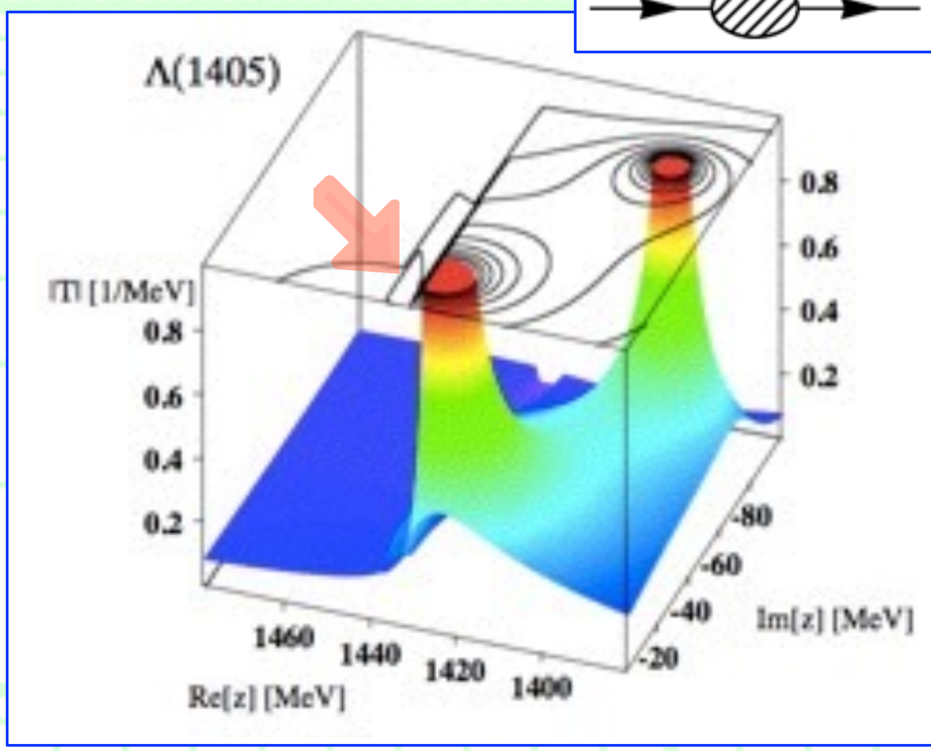
3. The *N** compositeness program

++ Example: compositeness for Λ(1405) ++

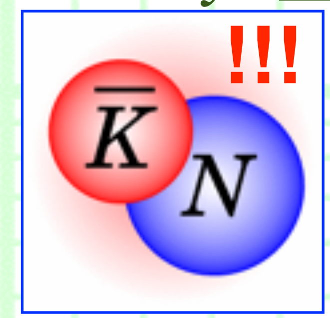
- **Compositeness *X* for Λ(1405) in the chiral unitary approach.**

Amplitude taken from: Ikeda, Hyodo and Weise, *Phys. Lett.* B706, (2011) 63; *Nucl. Phys.* A881 (2012) 98.





Hyodo and Jido ('12).



	Λ(1405), higher pole	Λ(1405), lower pole
$\sqrt{s_{\rm pole}}$	$1424 - 26i$ MeV	$1381 - 81i$ MeV
$X_{\bar{K}N}$	$1.14 + 0.01i$	$-0.39 - 0.07i$
$X_{\pi\Sigma}$	$-0.19 - 0.22i$	$0.66 + 0.52i$
$X_{\eta\Lambda}$	$0.13 + 0.02i$	$-0.04 + 0.01i$
$X_{K\Xi}$	$0.00 + 0.00i$	$-0.00 + 0.00i$
Z	$-0.08 + 0.19i$	$0.77 - 0.46i$

T. S. , Hyodo and Jido, *PTEP* 2015, 063D04.

--- **Large $\bar{K}N$ component for (higher pole) Λ(1405), since X_{KN} is almost unity with small imaginary parts.**

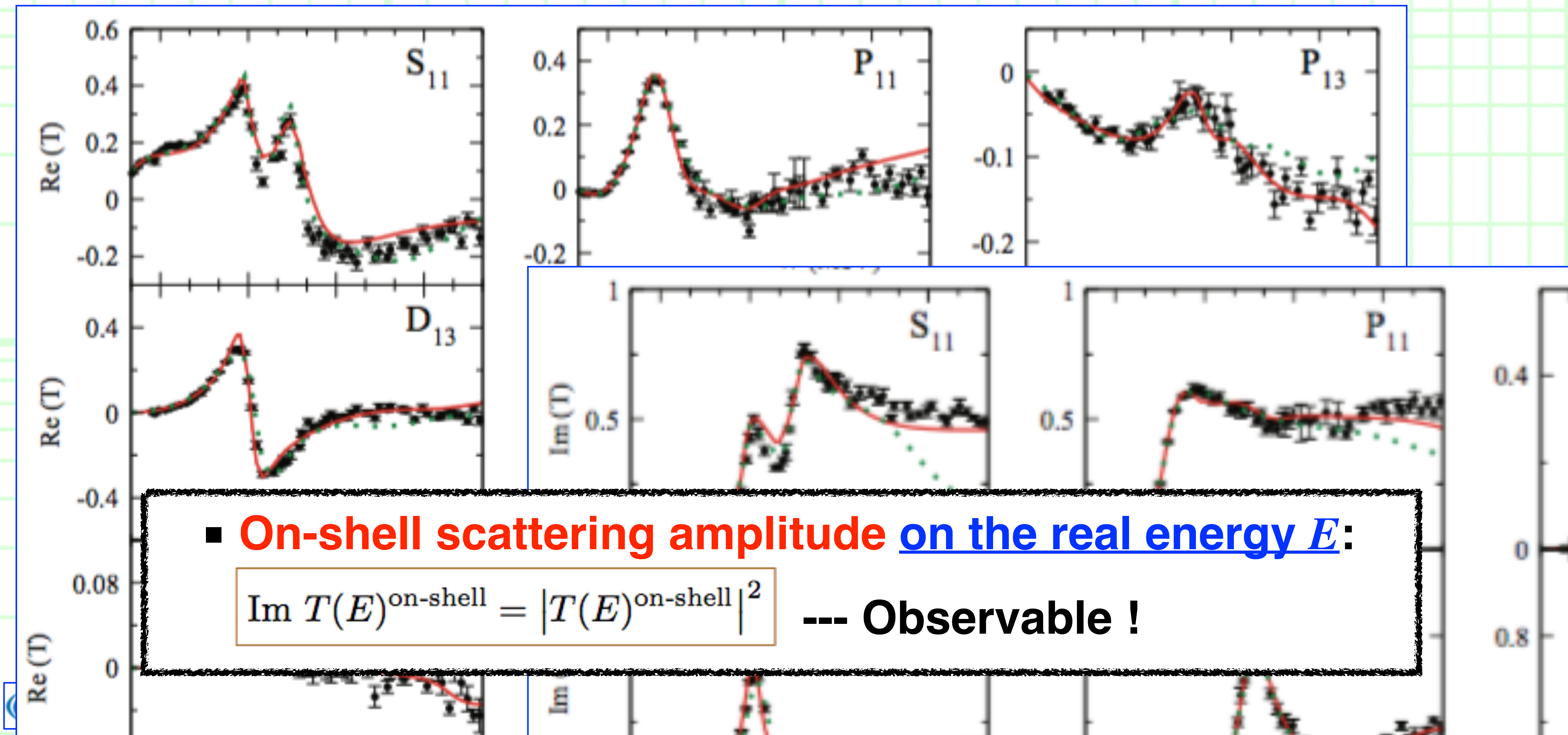
3. The N^* compositeness program

++ The N^* compositeness from πN amplitude ++

- Next target: **Comprehensive analysis** of the N^* and Δ^* resonances from the precise on-shell πN amplitude !

--- The precise on-shell πN scattering amplitude is available.

Kamano *et al.*, *Phys. Rev.* C88 (2014) 035209.



- On-shell scattering amplitude on the real energy E:

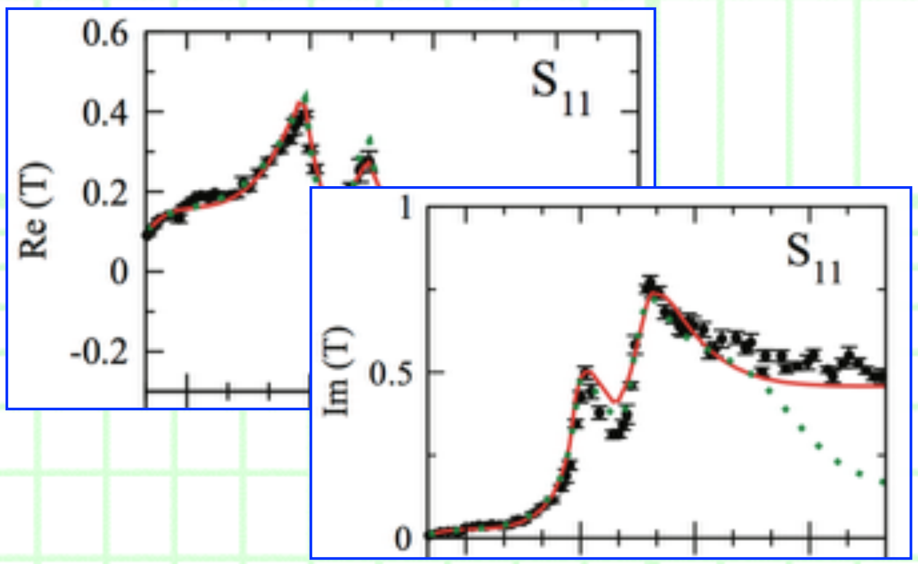
$$\mathrm{Im}\, T(E)^{\text{on-shell}} = \left| T(E)^{\text{on-shell}} \right|^2$$

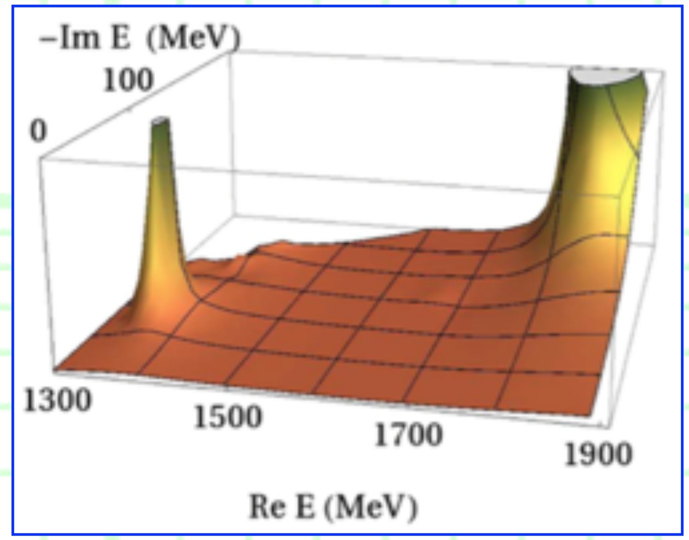
--- Observable !

3. The *N** compositeness program

++ Many *N** resonances ++

- **Many *N** and Δ* resonances** from the π*N* scattering amplitude.





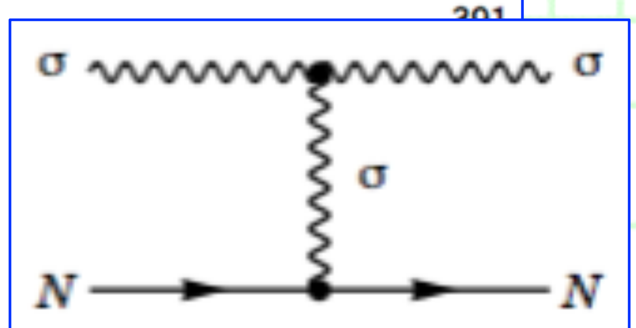
Suzuki *et al.*, *Phys. Rev. Lett.* 104 (2010) 042302.

- There are **several "interesting" *N** resonances**, such as:

N(1440) 1/2⁺ $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ PDG.

Breit-Wigner mass = 1410 to 1450 (≈ 1430) MeV
Breit-Wigner full width = 250 to 450 (≈ 350) MeV

N(1440) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\pi$	55–75 %	
$N\eta$	<1 %	
$N\pi\pi$	25–50 %	
$\Delta(1232)\pi$	20–30 %	
$\Delta(1232)\pi$, P-wave	13–27 %	
$N\sigma$	11–23 %	
$p\gamma$, helicity=1/2	0.035–0.048 %	407
$n\gamma$, helicity=1/2	0.02–0.04 %	406



- We can now investigate **their internal structure in terms of the meson-baryon component**.

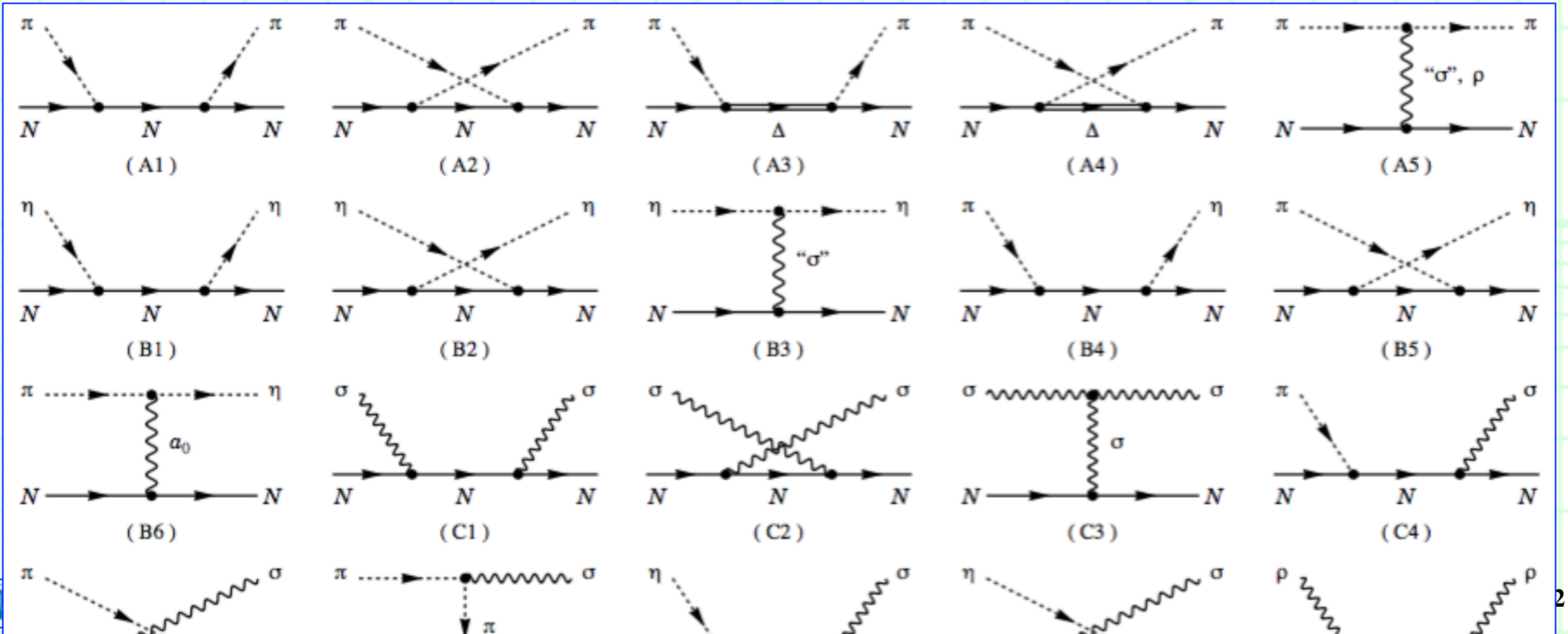
--- ***N*(1440) is a σ*N* bound state ?** *cf.* Jülich group.

Rönchen *et al.* (2013); ...

3. The N^* compositeness program

++ From on-shell to off-shell amplitude ++

- By using the on-shell πN amplitude (<-- observable), I construct the off-shell amplitude, where the N^* wave functions live.
- I take into account bare N^* states and appropriate diagrams for the meson-baryon interaction.
- How much the physical N^* are "dressed" ?



3. The N^* compositeness program

++ Numerical results ++

- Numerical results ...

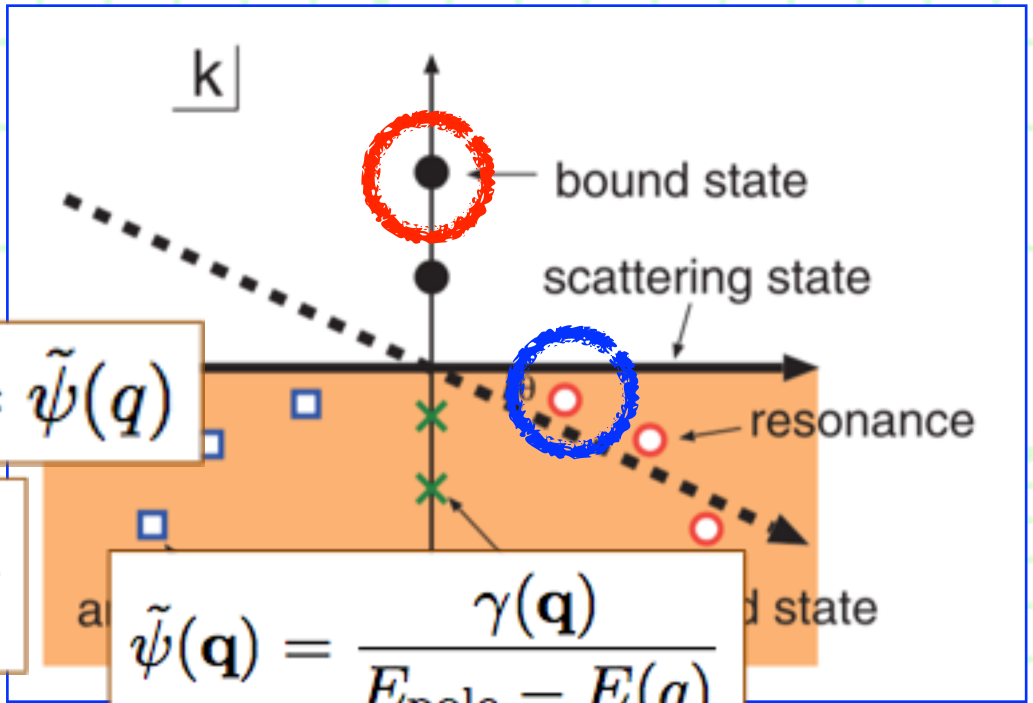
--- Sorry, but now on going !

- If you have your own πN amplitudes as solutions of the Lippmann-Schwinger Eq., you can calculate the N^* compositeness in the manner presented here.

--- Why don't you join me ?

4. Summary

- We can **extract the two-body WF** from **the residue of the scattering amplitude at the pole position**, both stable and unstable states.



$$\langle \mathbf{q} | \Psi \rangle = \tilde{\psi}(q)$$

Scattering amplitude:

$$T(E; \mathbf{q}', \mathbf{q}) = \langle \mathbf{q}' | \hat{T}(E) | \mathbf{q} \rangle \approx \frac{\gamma(q')\gamma(q)}{E - E_{\text{pole}}}$$

$$\tilde{\psi}(\mathbf{q}) = \frac{\gamma(\mathbf{q})}{E_{\text{pole}} - E(q)}$$

- **The WF from the scattering amplitude is automatically scaled.**
  - □ The compositeness (= norm of the two-body WF) is unity for a bound state in an energy independent interaction.
  - □ For an energy dependent interaction, the compositeness deviates from unity, reflecting a missing channel contribution.

- From the precise πN amplitude with appropriate models, we can evaluate the compositeness of the N^* and Δ^* resonances.
  - □ In particular, how is the structure of the $N(1440)$ resonance ?

Thank you very much for your kind attention !

Appendix

Appendix

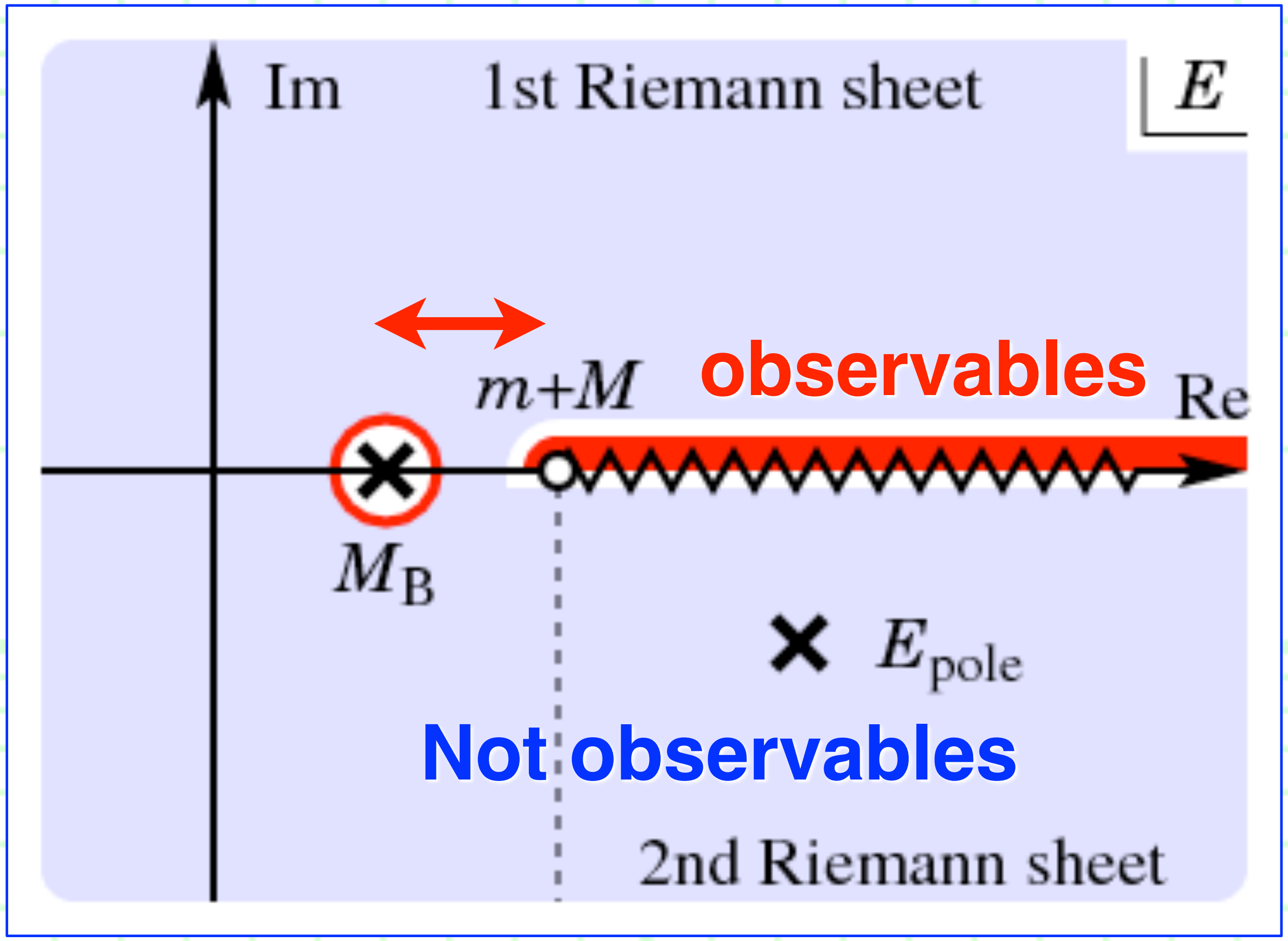
++ Compositeness and model (in-)dependence ++

- General case: Compositeness are model dependent quantity.

- Special case: Compositeness for near-threshold poles.

--- Compositeness can be expressed with threshold parameters such as scattering length and effective range.

- □ Deuteron.
  Weinberg ('65).
- □ $f_0(980)$ and $a_0(980)$.
  Baru *et al.* ('04),
  Kamiya-Hyodo, *Phys. Rev.* C93 (2016) 035203.
- □ Λ(1405).
  Kamiya-Hyodo, *Phys. Rev.* C93 (2016) 035203.
- □ ...



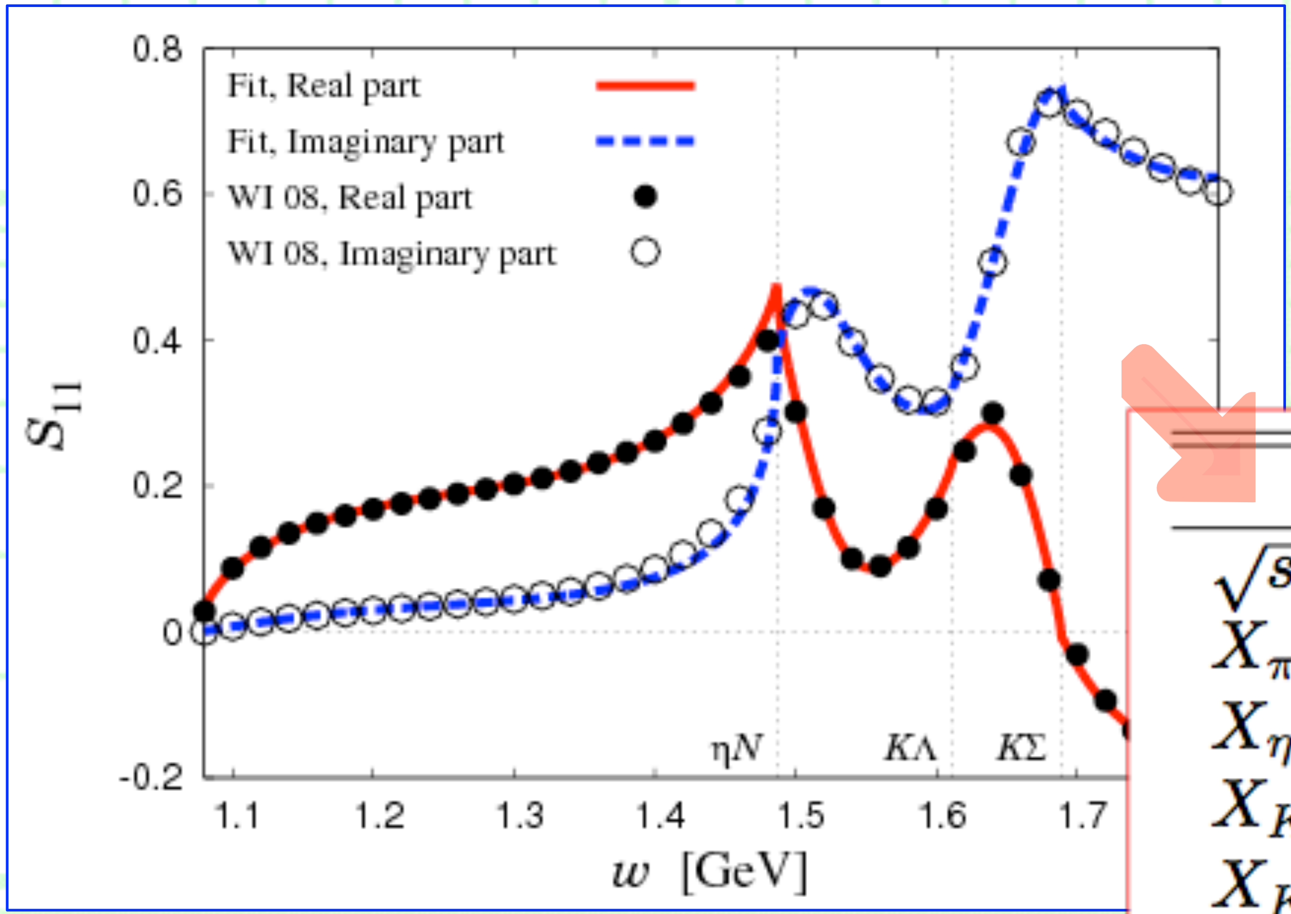
$$a = \frac{2(1-Z)}{2-Z} R + \mathcal{O}(m_\pi^{-1}), \quad r_e = -\frac{Z}{1-Z} R + \mathcal{O}(m_\pi^{-1}), \quad R \equiv \frac{1}{\sqrt{2\mu B}} = 4.318 \text{ fm}$$

Appendix

++ Compositeness for $N(1535)$ and $N(1650)$ ++

- **Compositeness X for $N(1535)$ & $N(1650)$ in chiral unitary approach.**

T. S. T. Arai, J. Yamagata-Sekihara and S. Yasui, *Phys. Rev.* C93 (2016) 035204.



	$N(1535)$	$N(1650)$
$\sqrt{s_{\rm pole}}$ [MeV]	$1496.4 - 58.7i$	$1660.7 - 70.0i$
$X_{\pi N}$	$-0.02 + 0.03i$	$0.00 + 0.04i$
$X_{\eta N}$	$0.04 + 0.37i$	$0.00 + 0.01i$
$X_{K\Lambda}$	$0.14 + 0.00i$	$0.08 + 0.05i$
$X_{K\Sigma}$	$0.01 - 0.02i$	$0.09 - 0.12i$
Z	$0.84 - 0.38i$	$0.84 + 0.01i$

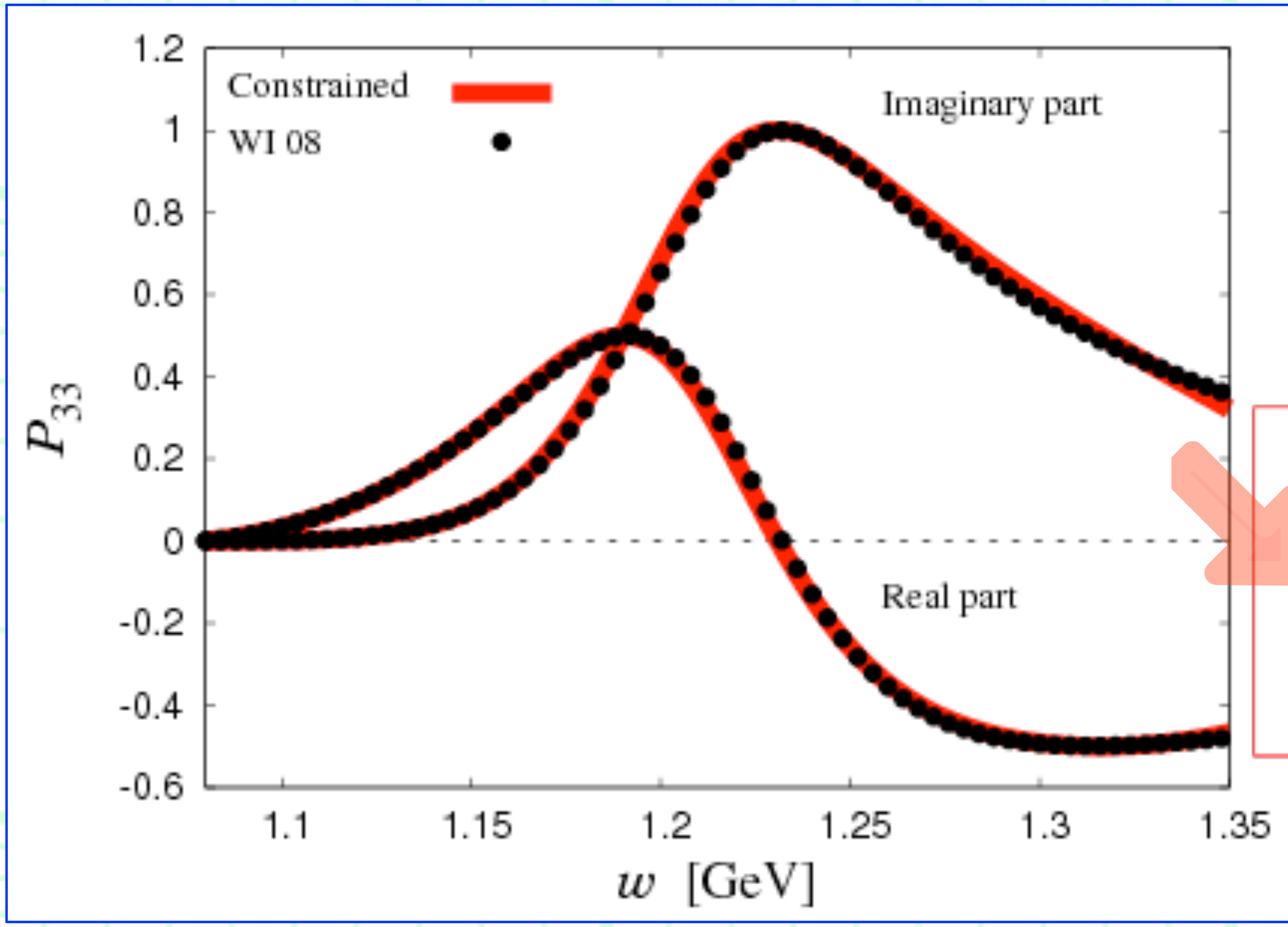
□ For both N^* resonances, the missing-channel part Z is dominant.

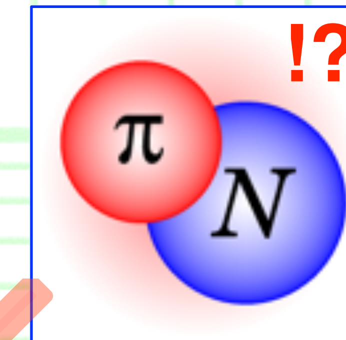
--> $N(1535)$ and $N(1650)$ have large components originating from contributions other than πN, ηN, $K\Lambda$, and $K\Sigma$.

Appendix

++ Compositeness for Δ(1232) ++

- **Compositeness X for Δ(1232) in chiral unitary approach.**





Constrained	Δ(1232)	$N(940)$
$\sqrt{s_{\text{pole}}}$ [MeV]	$1206.9 - 49.6i$	938.9
$X_{\pi N}$	$0.87 + 0.35i$	0.00
Z	$0.13 - 0.35i$	1.00

- □ **The πN compositeness $X_{\pi N}$ takes large real part !** **But non-negligible imaginary part as well.**

--> Large πN component in the Δ(1232) resonance !?