Tensor optimized antisymmetrized molecular dynamics (TOAMD) for nuclei using bare NN interaction

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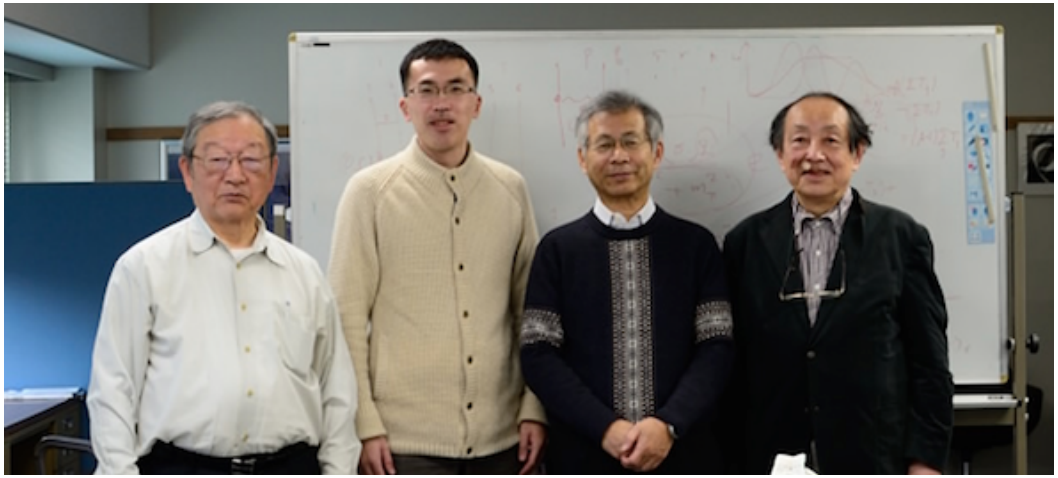


Tensor Optimized Antisymmetrized Molecular Dynamics (TOAMD)

Myo Toki Ikeda

Tensor optimized shell model (TOSM)

1. We include tensor interaction most effectively to shell model
2. Difficult to treat cluster structure

+

Horiuchi Enyo Kimura..

Antisymmetrized molecular dynamics (AMD)

1. Cluster+shell structure is handled on the same footing using effective interaction
2. Difficult to treat bare nucleon-nucleon interaction

↓

Study nuclear structure based on bare NN interaction

PTEP

Prog. Theor. Exp. Phys. **2015**, 073D02 (38 pages)
DOI: 10.1093/ptep/ptv087

Tensor-optimized antisymmetrized molecular dynamics in nuclear physics

Takayuki Myo[1,2,*], Hiroshi Toki[2], Kiyomi Ikeda[3], Hisashi Horiuchi[2], and Tadahiro Suhara[4]

Tensor-optimized antisymmetrized molecular dynamics as a successive variational method in nuclear many-body system

Takayuki Myo[a,b,*], Hiroshi Toki[b], Kiyomi Ikeda[c], Hisashi Horiuchi[b], Tadahiro Suhara[d]

Phys. Lett. B769 (2017) 213

PHYSICAL REVIEW C **95**, 044314 (2017)

Successive variational method of the tensor-optimized antisymmetrized molecular dynamics for central interaction in finite nuclei

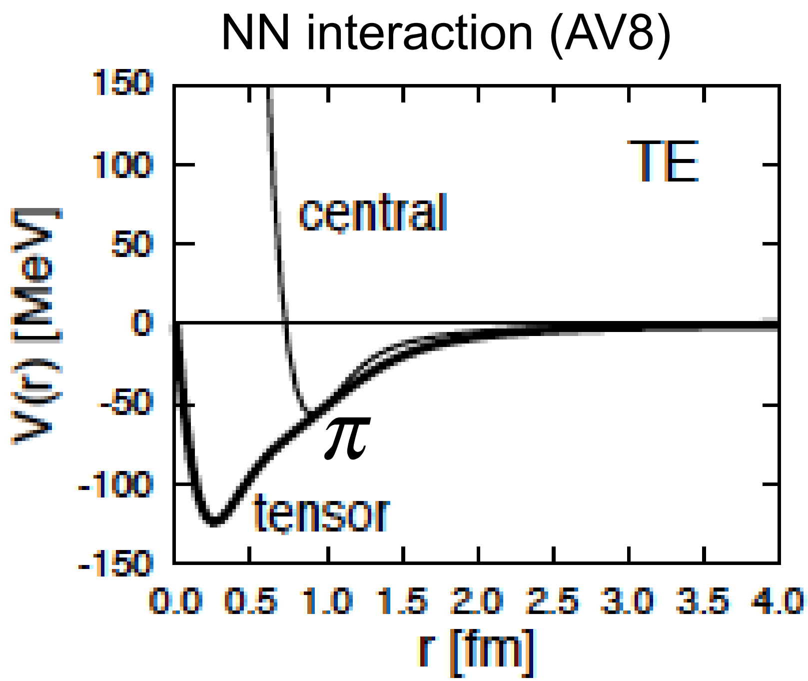
Takayuki Myo,[1,2,*] Hiroshi Toki,[2,†] Kiyomi Ikeda,[3,‡] Hisashi Horiuchi,[2,§] and Tadahiro Suhara[4,∥]

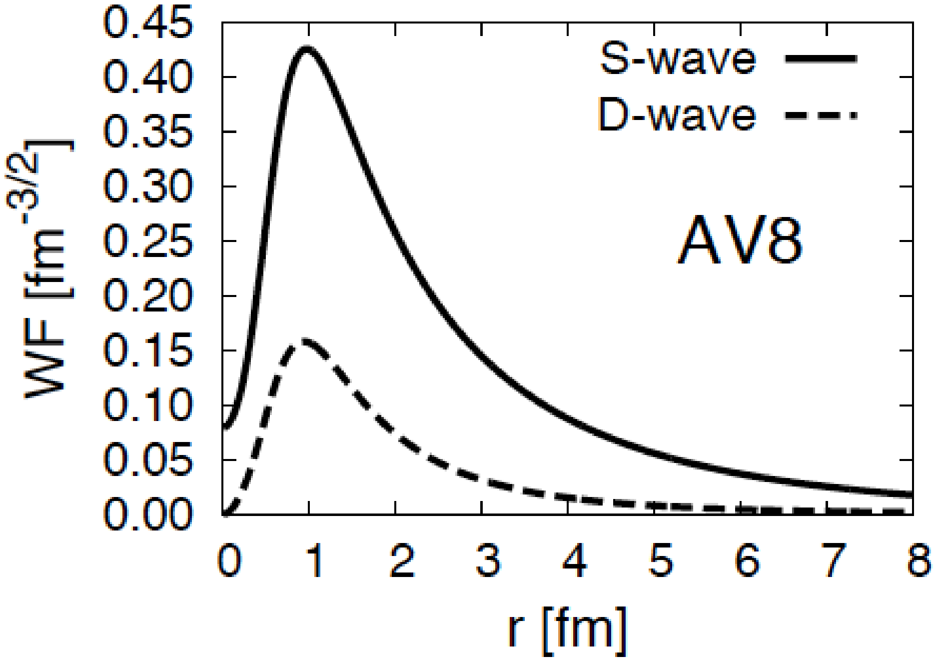
Two papers have been submitted

Deuteron (1^+)

S=1 and L=0 or 2

$$\Psi = \Phi_S + \Phi_D = (1 + F_D)\Phi_S$$





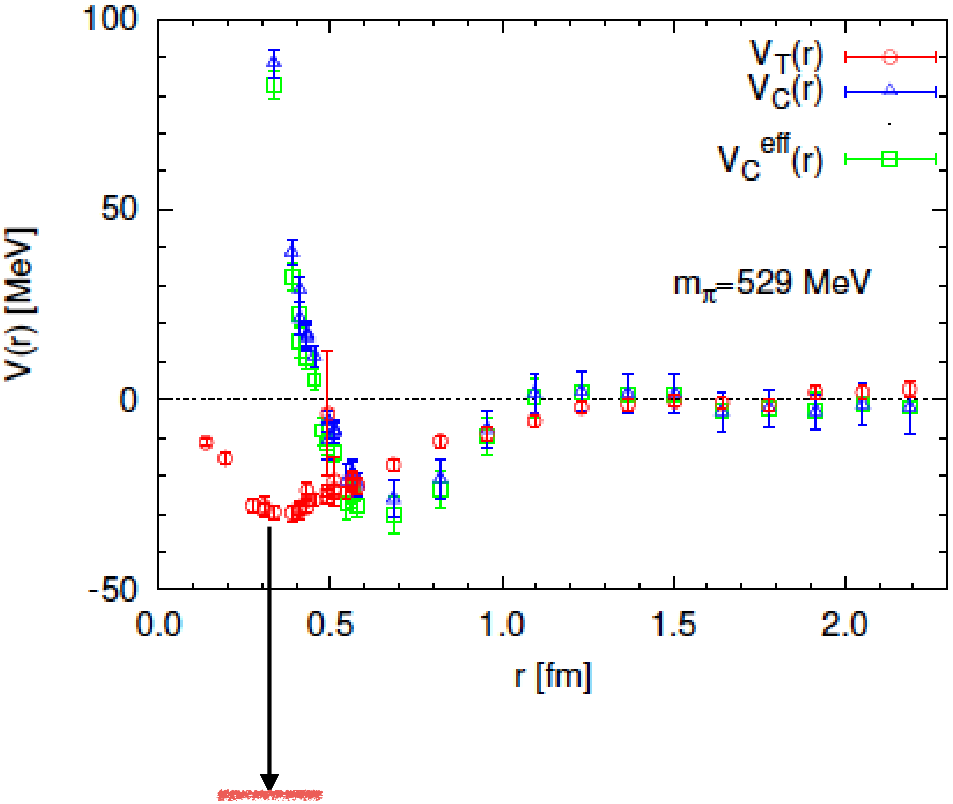
upion

Energy	−2.24 [MeV]
Kinetic	19.88
(SS)	11.31
(DD)	8.57
Central	−4.46
(SS)	−3.96
(DD)	−0.50
Tensorc	−16.64
(SD)	−18.93
(DD)	2.29
LS	−1.02
P(D)	5.78 [%]
Radius	1.96 [fm]
(SS)	2.00 [fm]
(DD)	1.22 [fm]

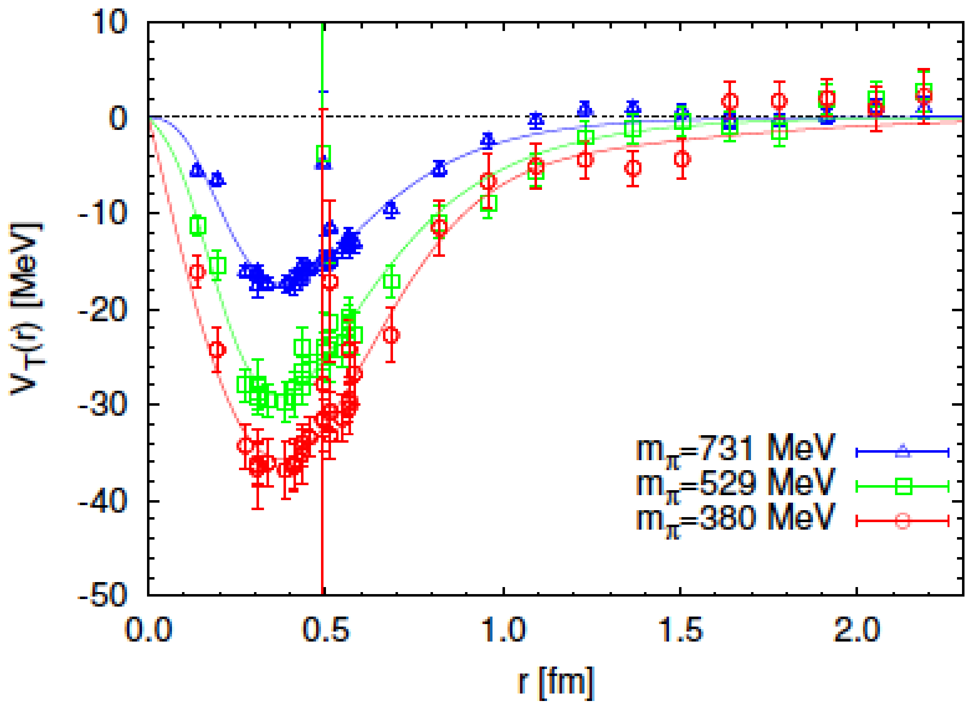
Progress of Theoretical Physics, Vol. 123, No. 1, January 2010

Theoretical Foundation of the Nuclear Force in QCD and Its Applications to Central and Tensor Forces in Quenched Lattice QCD Simulations

Sinya AOKI,[1] Tetsuo HATSUDA[2] and Noriyoshi ISHII[2]



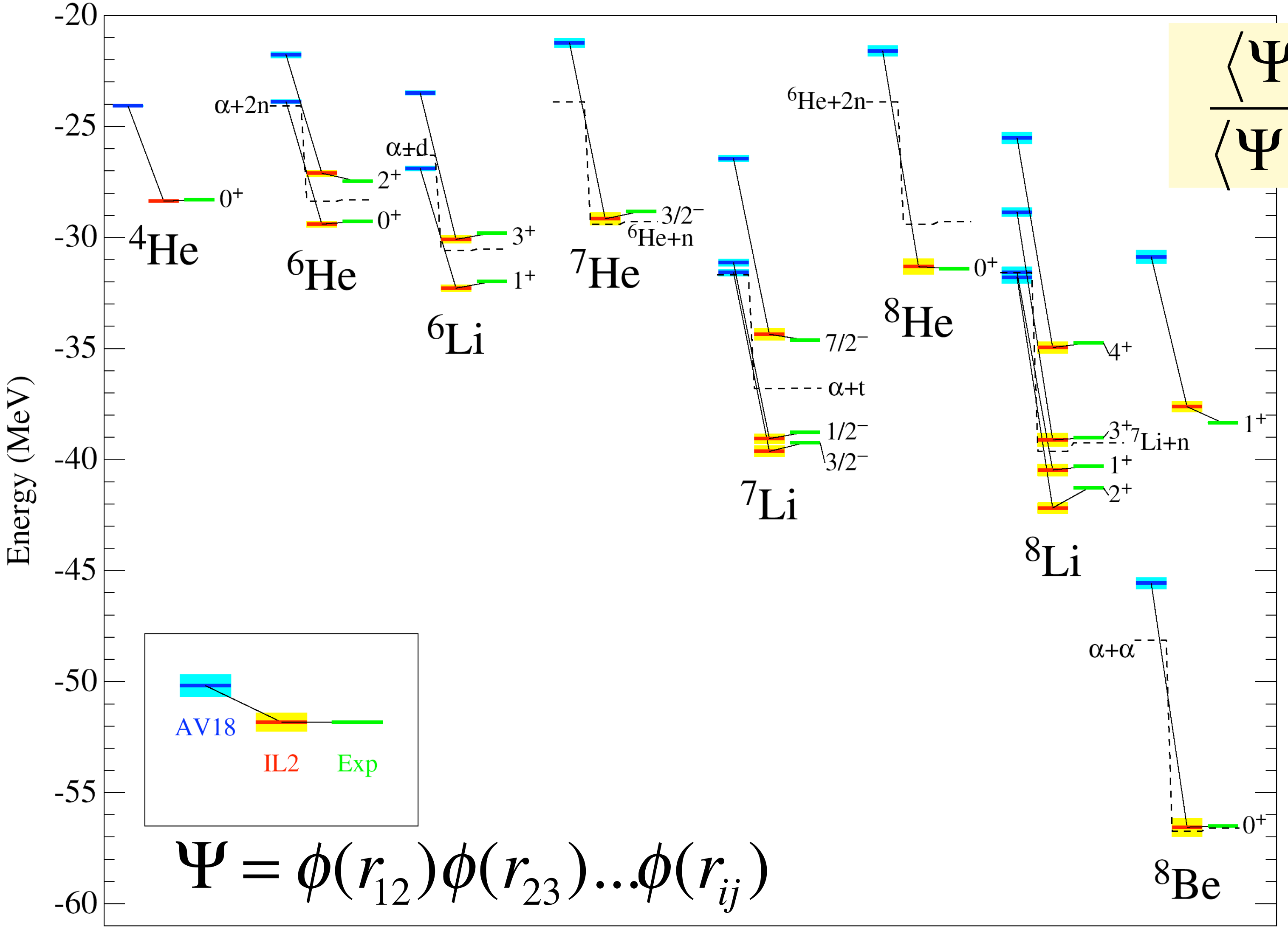
$m_\pi = 140 MeV$



Variational calculation of light nuclei with NN interaction

VMC+GFMC

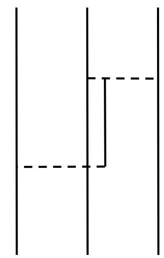
$$\frac{\langle \Psi | V_\pi | \Psi \rangle}{\langle \Psi | V_{NN} | \Psi \rangle} \sim 80\%$$



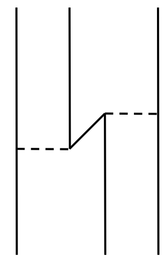
$$\Psi = \phi(r_{12})\phi(r_{23})\ldots\phi(r_{ij})$$

C. Pieper and R. B. Wiringa, Annu. Rev. Nucl. Part. Sci.51(2001)

V_{NNN}



Fujita-Miyazawa



Relativistic

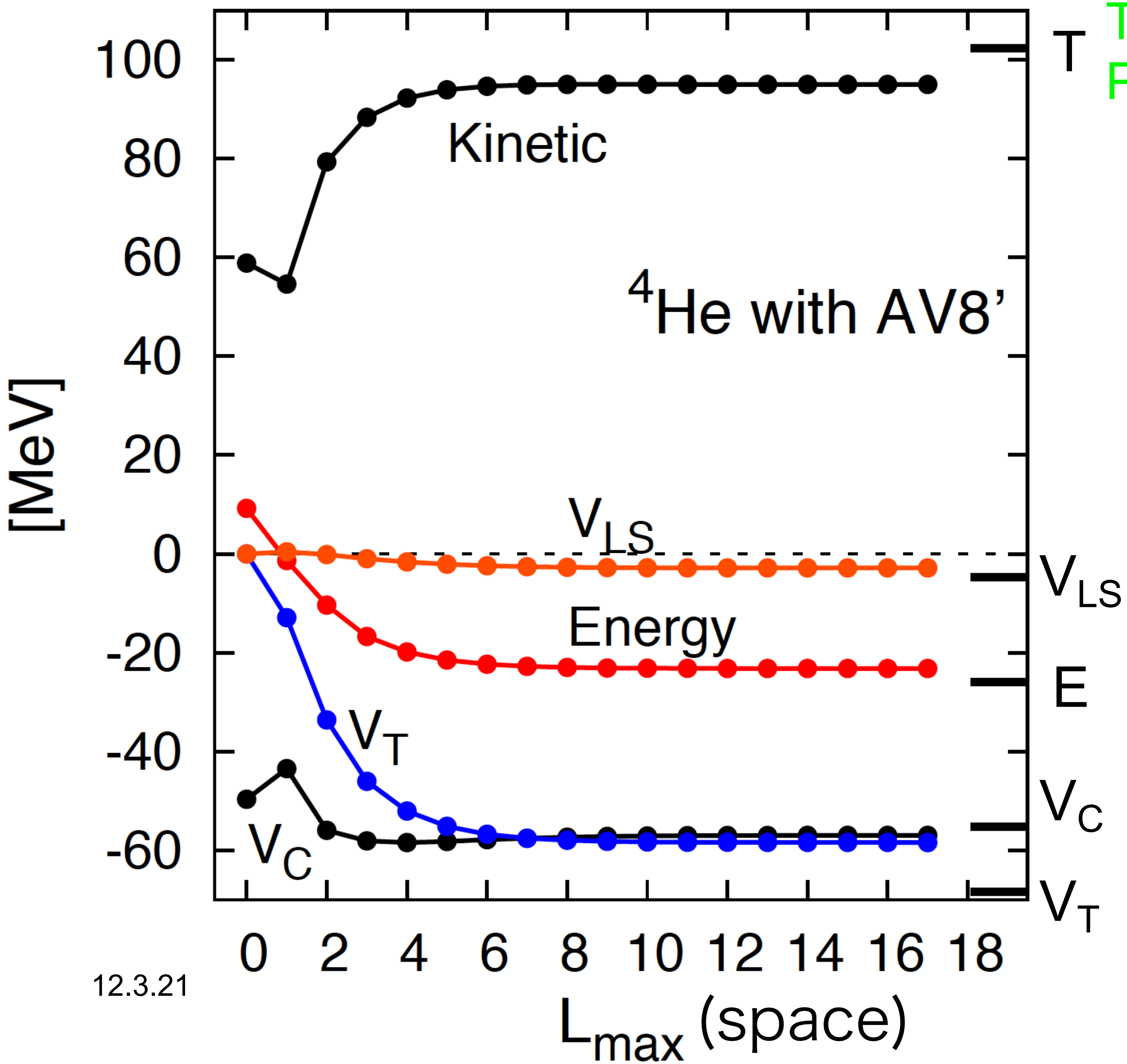
Heavy nuclei (Super model)

Pion is key

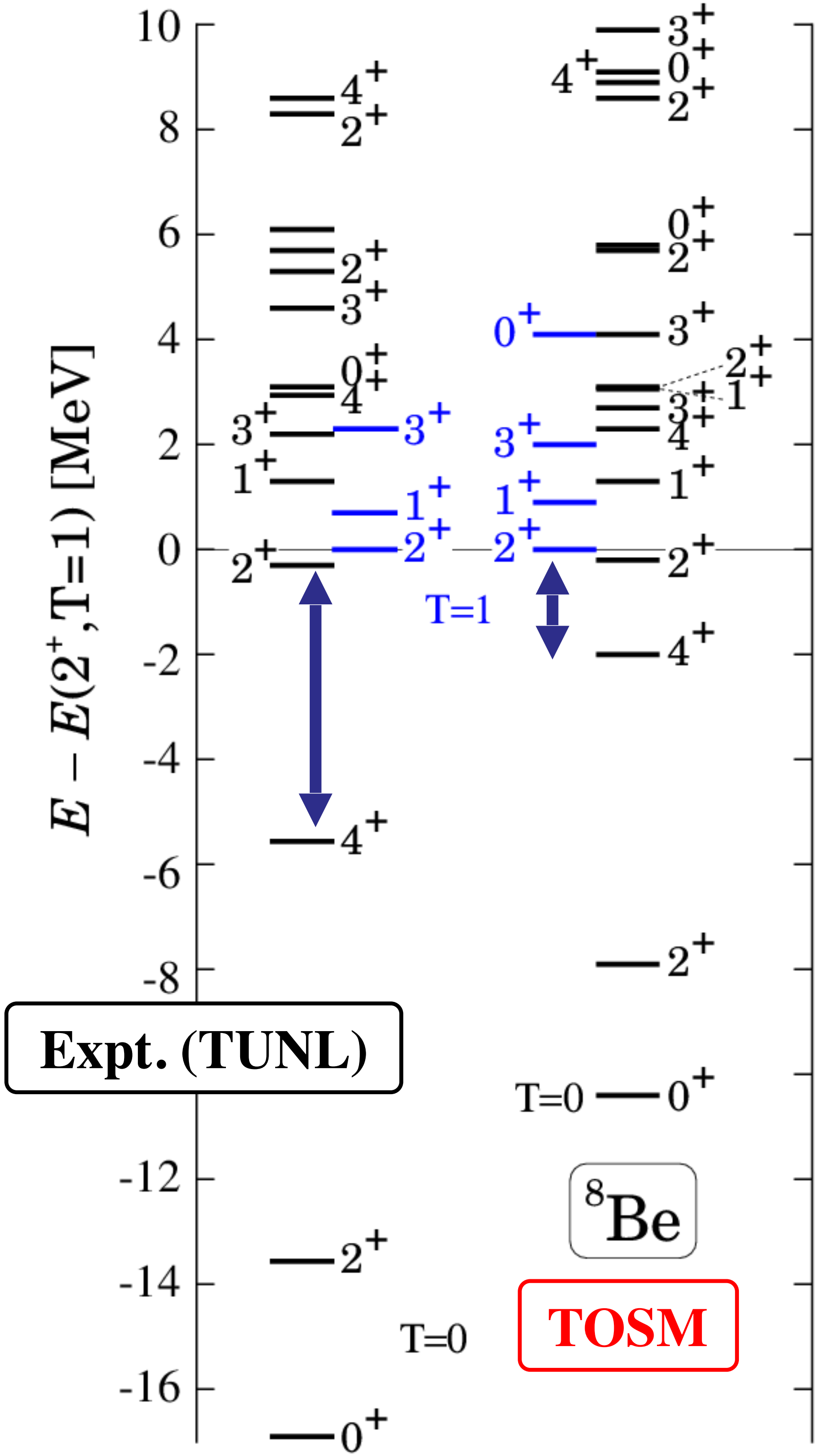
TOSM+UCOM with AV8'

$$\Psi = C_0|0\rangle + \sum_\alpha C_\alpha |2p2h:\alpha\rangle$$

T.Myo H.Toki K.Ikeda
PTP121(2009)511

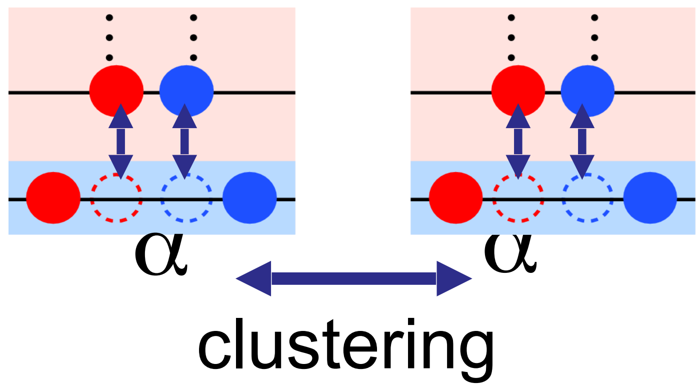


Few body Calculation (Kamda et al)



^{8}Be in TOSM – AV8' –

- correct level order (T=0,1)
- tensor contribution : T=0 > T=1
- α : 0p0h+2p2h with high-k
  - 2α needs 4p4h.
  - spatial asymptotic form of 2α



⇒ TOAMD

TOAMD

(Tensor optimized antisymmetrized molecular dynamics)

$$|\Psi\rangle = |AMD\rangle + F_D |AMD\rangle$$

$$\Psi = C_0|0\rangle + \sum_{\alpha} C_\alpha |2p2h:\alpha\rangle$$

(TOSM)

$$|AMD\rangle = A\prod_{i=1}^{A} \psi_{p_i}(\vec{r}_i)\chi_{p_i}(s_i)\xi_{p_i}(t_i)$$

$$\psi_{p_i}(\vec{r}_i) = \left(\frac{2\nu}{\pi}\right)^{3/4} e^{-\nu(\vec{r}_i - \vec{D}_{p_i})^2}$$

(shifted Gaussian)

$$\chi_{p_i}(s_i) = \beta_{p_i}|\uparrow\rangle + (1-\beta_{p_i})|\downarrow\rangle$$

$$\xi_{p_i}(t_i) = |proton\rangle \quad or \quad |neutron\rangle$$

$$F_D = \frac{1}{2}\sum_{i\neq j} f_D(r_{ij}) S_{12}(r_{ij}) \tau_i \cdot \tau_j$$

$$S_{12}(r_{ij}) = 3(\sigma_i \cdot \hat{r}_{ij})(\sigma_j \cdot \hat{r}_{ij}) - (\sigma_i \cdot \sigma_j)$$

$$= \sum_{xyx'y'} \hat{r}_{ijx}\hat{r}_{ijy}\sigma_{ix'}\sigma_{jy'}(3\delta_{xx'}\delta_{yy'} - \delta_{xy}\delta_{x'y'})$$

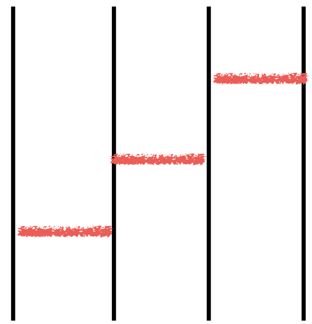
$$f_D(r_{ij}) = \sum_{\mu} C_\mu r_{ij}^2 e^{-a_\mu r_{ij}^2}$$

(Tensor correlation function)

Argonne wave function (Monte-Carlo method)

$$\left|\Psi\right\rangle = \prod_{i\neq j}\left(1+U_{ij}(r_{ij})\right)\left|\Psi_J(SM)\right\rangle$$

Jastrow(1955)

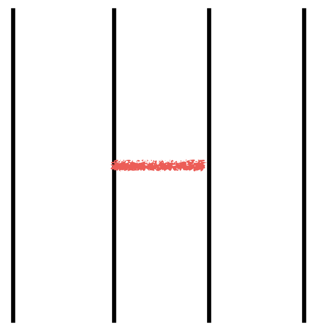


Multiple correlation functions are contained.

This is the reason why the calculations are time consuming. (very complicated) → VMC method

TOSM (Tensor optimized shell model)

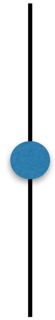
$$\left|\Psi\right\rangle = C_0\left|SM\right\rangle + \sum_{\alpha} C_\alpha \left|2p2h;\alpha\right\rangle$$



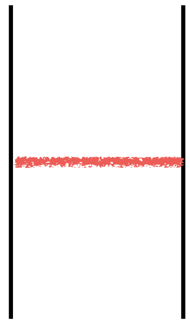
Relative correlation function is expressed by 2p2h states. (include 4p4h states are highly complicated)

Hamiltonian (AV18)

$$H = T + V + U$$

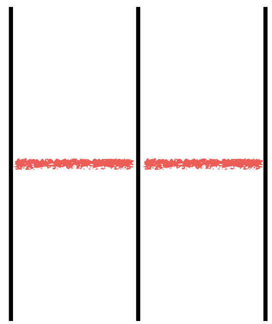


$$T = \sum_{i=1}^{A} \left(\frac{p_i^2}{2m} \right) - T_{CM}$$



$$V = \frac{1}{2} \sum_{p} \sum_{i \neq j} V^p(r_{ij}) O^p(ij)$$

$$V^p(r_{ij}) = \sum_{\mu} C_{\mu} e^{-a_{\mu}^p r_{ij}^2}$$



$$U = \frac{1}{2} \sum_{p} \sum_{i \neq j \neq k} U^p(ijk)$$

$$U^p(ijk) = V^{\pi}(ij) V^{\pi}(jk)$$

Energy (minimization)

$$E = \frac{\langle AMD | (1+F_D) H (1+F_D) | AMD \rangle}{\langle AMD | (1+F_D)(1+F_D) | AMD \rangle}$$

Difficulty

1. many body matrix elements (7 body)
2. antisymmetrization (exchange of particles)
3. tensor interaction (high momentum)

→ momentum space (separable)

$$e^{-ar_{ij}^2} = \left(\frac{\pi}{a}\right)^{3/2} \int_k e^{-k^2/4a} e^{ik(r_i - r_j)}$$

Advantage

1. Gaussian integral (analytical)
2. antisymmetrization (matrix technique)

Overlap integral

$$\langle AMD | AMD \rangle = \langle p_1 p_2 .. p_A | \det | q_1 q_2 .. q_A | \rangle$$

$$B = \begin{vmatrix} \langle p_1 | q_1 \rangle & \langle p_1 | q_2 \rangle & \cdots & \langle p_1 | q_A \rangle \\ \langle p_2 | q_1 \rangle & \langle p_2 | q_2 \rangle & \cdots & \langle p_2 | q_A \rangle \\ \cdots & \cdots & \cdots & \cdots \\ \langle p_A | q_1 \rangle & \langle p_A | q_2 \rangle & \cdots & \langle p_A | q_A \rangle \end{vmatrix}$$

$$\langle p | q \rangle = \langle \psi_p | \psi_q \rangle \langle \chi_p | \chi_q \rangle \langle \xi_p | \xi_q \rangle$$

$$\langle \psi_p | \psi_q \rangle = e^{-\frac{1}{2}\nu\left(\vec{D}_p - \vec{D}_q\right)^2}$$

$$\langle \chi_p | \chi_q \rangle = M^{pq} = \beta_p^* \beta_q + (1 - \beta_p^*)(1 - \beta_q)$$

$$\langle \xi_p | \xi_q \rangle = \bar{M}^{pq} = 1 \quad or \quad 0$$

One-body matrix element

$$\langle AMD | O_1 | AMD \rangle = \langle p_1 p_2 .. p_A | \sum_{i=1}^{A} O(i) \Big| \det | q_1 q_2 .. q_A | \rangle$$

$$M(O) = \sum_{r=1}^{A} \begin{vmatrix} \langle p_1 | q_1 \rangle & \langle p_1 | q_2 \rangle & \cdots & \langle p_1 | q_A \rangle \\ \langle p_r | O | q_1 \rangle & \langle p_r | O | q_2 \rangle & \cdots & \langle p_r | O | q_A \rangle \\ \cdots & \cdots & \cdots & \cdots \\ \langle p_A | q_1 \rangle & \langle p_A | q_2 \rangle & \cdots & \langle p_A | q_A \rangle \end{vmatrix}$$

$$M(O) = \sum_{r=1}^{A} \sum_{l=1}^{A} \langle p_r | O | q_l \rangle C(r:l)$$

$C(r:l)$ is a co-factor matrix of B.

Two-body matrix element

$$\left\langle AMD \middle| O_1 O_2 \middle| AMD \right\rangle = \left\langle p_1 p_2 .. p_A \middle| \sum_{i \neq j}^{A} O(i) O(j) \middle| \det \middle| q_1 q_2 .. q_A \right| \rangle$$

$$M(O_1 O_2) = \sum_{r_1 \neq r_2}^{A} \begin{vmatrix} \langle p_1 | q_1 \rangle & \langle p_1 | q_2 \rangle & \cdots & \langle p_1 | q_A \rangle \\ \langle p_{r_1} | O_1 | q_1 \rangle & \langle p_{r_1} | O_1 | q_2 \rangle & \cdots & \langle p_{r_1} | O_1 | q_A \rangle \\ \langle p_{r_2} | O_2 | q_1 \rangle & \langle p_{r_2} | O_2 | q_1 \rangle & \cdots & \langle p_{r_2} | O_2 | q_1 \rangle \\ \langle p_A | q_1 \rangle & \langle p_A | q_2 \rangle & \cdots & \langle p_A | q_A \rangle \end{vmatrix}$$

$$M(O_1 O_2) = \sum_{r_1 \neq r_2}^{A} \sum_{l_1 \neq l_2}^{A} \left\langle p_{r_1} \middle| O_1 \middle| q_{l_1} \right\rangle \left\langle p_{r_2} \middle| O_2 \middle| q_{l_2} \right\rangle C(r_1 r_2 : l_1 l_2)$$

$C(r_1 r_2 : l_1 l_2)$ is a co-factor matrix of B.

Three-body matrix element

$$\left\langle AMD \middle| O_1 O_2 O_3 \middle| AMD \right\rangle = \left\langle p_1 p_2 .. p_A \middle| \sum_{i \neq j \neq k}^{A} O(i) O(j) O(k) \middle| \det\left| q_1 q_2 .. q_A \right| \right\rangle$$

$$M(O_1 O_2 O_3) = \sum_{r_1 \neq r_2 \neq r_3}^{A} \sum_{l_1 \neq l_2 \neq l_3}^{A} \left\langle p_{r_1} \middle| O_1 \middle| q_{l_1} \right\rangle \left\langle p_{r_2} \middle| O_2 \middle| q_{l_2} \right\rangle \left\langle p_{r_3} \middle| O_3 \middle| q_{l_3} \right\rangle C(r_1 r_2 r_3 : l_1 l_2 l_3)$$

$C(r_1 r_2 r_3 : l_1 l_2 l_3)$ is a co-factor matrix of B.

$$C(r_1 r_2 .. r_n : l_1 l_2 .. l_n) = \begin{vmatrix} (B^{-1})_{l_1 r_1} & (B^{-1})_{l_1 r_2} & .. & (B^{-1})_{l_1 r_n} \\ (B^{-1})_{l_2 r_1} & (B^{-1})_{l_2 r_2} & .. & (B^{-1})_{l_2 r_n} \\ .. & .. & .. & .. \\ (B^{-1})_{l_n r_1} & (B^{-1})_{l_n r_2} & .. & (B^{-1})_{l_n r_n} \end{vmatrix} \det|B|$$

Central interaction

$$V^c = \frac{1}{2}\sum_{i\neq j}\sum_{\mu} C_\mu \left(\frac{\pi}{a_\mu}\right)^{3/2} \int_k e^{-k^2/4a_\mu} e^{ikr_i} e^{-ikr_j}$$

$$\langle AMD | V^c | AMD \rangle = \frac{1}{2}\sum_{p_1\neq p_2 : q_1\neq q_2}\sum_{\mu} \tilde{C}_\mu^{(0)} \int_k e^{-k^2/4a_\mu} \langle p_1 | e^{ikr_1} | q_1 \rangle \langle p_2 | e^{ikr_2} | q_2 \rangle C(p_1 p_2 : q_1 q_2)$$

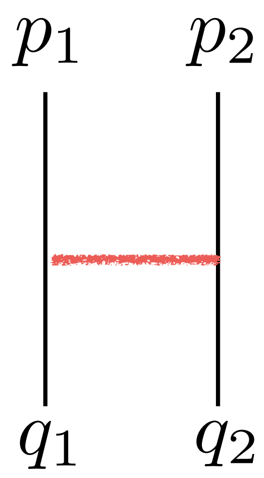
$$= \frac{1}{2}\sum_{p_1\neq p_2 : q_1\neq q_2}\sum_{\mu} \tilde{C}_\mu^{(0)} I^{(12)}(A,B,C) M^{p_1 q_1} M^{p_2 q_2} \bar{M}^{p_1 q_1} \bar{M}^{p_2 q_2} C(p_1 p_2 : q_1 q_2)$$

$$I^{(12)}(A,B,C) = \frac{1}{(2\pi)^3}\left(\frac{\pi}{A}\right)^{3/2} e^{-B^\dagger A^{-1} B/4 + C}$$

$$A = 1/4\nu + 1/4a_\mu$$

$$B = \frac{1}{2}\left(\vec{D}_{p_1} + \vec{D}_{q_1}\right) - \frac{1}{2}\left(\vec{D}_{p_2} + \vec{D}_{q_2}\right)$$

$$C = -\frac{1}{2}\nu\left[\left(D_{p_1} - D_{q_1}\right)^2 + \left(D_{p_2} - D_{q_2}\right)^2\right]$$



Tensor interaction

$$V^t = \frac{1}{2}\sum_{i\neq j}\sum_{\mu}\tilde{C}_\mu^{(2)}\sum_{xyx'y'}\int_k k_x k_y e^{-k^2/4a_\mu} e^{ikr_i} e^{-ikr_j}\sigma_{ix'}\sigma_{jy'}(3\delta_{xx'}\delta_{yy'}-\delta_{xy}\delta_{x'y'})$$

$$\langle AMD|V^t|AMD\rangle = \frac{1}{2}\sum_{p_1\neq p_2:q_1\neq q_2}\sum_{\mu}\tilde{C}_\mu^{(2)}\sum_{xyx'y'} I_{1x1y}^{(12)}(A,B,C) M_{x'}^{p_1q_1} M_{y'}^{p_2q_2} M^{p_1q_1} M^{p_2q_2}$$

$$(3\delta_{xx'}\delta_{yy'}-\delta_{xy}\delta_{x'y'})C(p_1p_2:q_1q_2)$$

$$\tilde{C}_\mu^{(m)} = C_\mu\left(\frac{\pi}{a_\mu}\right)^{3/2}\left(\frac{-i}{2a_\mu}\right)^m$$

$$I^{(ij:kl..)}(A,B,C:b) = \int_{k_1k_2..k_l} e^{-\vec{k}A\vec{k}+i\vec{B}\vec{k}+C}$$

$$I_{ixjykz..}^{(ij:kl..)}(A,B,C:b) = \int_{k_1k_2..k_l} k_{ix}k_{jy}k_{kz}\ldots e^{-\vec{k}A\vec{k}+i\vec{B}\vec{k}+C}$$

$$\vec{B} = \begin{pmatrix}\vec{B}_1\\ \vec{B}_2\\ \vec{B}_3\end{pmatrix} \rightarrow \begin{pmatrix}\vec{B}_1+\vec{b}_1\\ \vec{B}_2+\vec{b}_2\\ \vec{B}_3+\vec{b}_3\end{pmatrix}$$

$i\vec{b}_i\vec{k}_i$ is source term

Differentiation of Gaussian integral

$$I^{(ij:kl\,..)}_{ixjykz\,..}(A,B,C:b)=\int_{k_1k_2\,..k_l} k_{ix}k_{jy}k_{kz}\ldots e^{-\vec{k}A\vec{k}+i\vec{B}\vec{k}+C}$$

$$=\left(-i\frac{\partial}{\partial b_{ix}}\right)\cdots\left(-i\frac{\partial}{\partial b_{ix}}\right)I^{(ij:kl\,..)}(A,B,C:b)$$

$$I^{(ij:kl\,..)}(A,B,C:b)=\frac{1}{(2\pi)^{3n}}\left(\frac{\pi^n}{\det A}\right)^{3/2}e^{-B^\dagger A^{-1}B/4+C}$$

We can calculate differentiations systematically

Analytical expressions

Three-body central interaction

$$V^3=\sum_{i\neq j\neq k}\sum_{\mu_1\mu_2}\tilde{C}^{(0)}_{\mu_1}\tilde{C}^{(0)}_{\mu_2}\int_{k_1k_2}e^{-k_1^2/4a_{\mu_1}}e^{-k_2^2/4a_{\mu_2}}e^{ik_1r_i}e^{-ik_1r_j}e^{ik_2r_j}e^{-ik_2r_k}$$

$$\langle AMD|V^3|AMD\rangle=\sum_{p_1\neq p_2\neq p_3:q_1\neq q_2\neq q_3}\sum_{\mu_1\mu_2}\tilde{C}^{(0)}_{\mu_1}\tilde{C}^{(0)}_{\mu_2}\int_{k_1k_2}e^{-k_1^2/4a_{\mu_1}}e^{-k_2^2/4a_{\mu_2}}$$

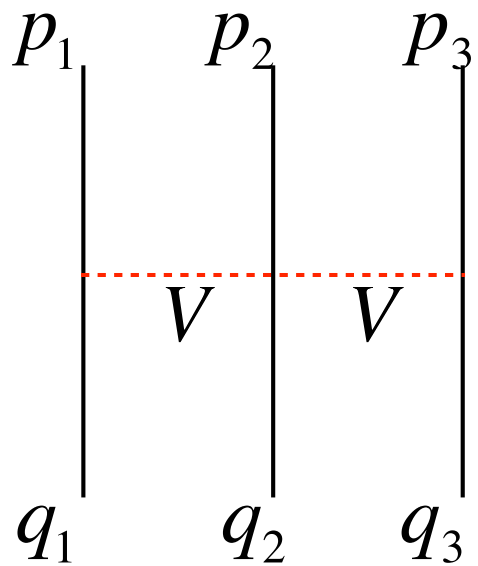
$$\langle p_1|e^{ik_1r_1}|q_1\rangle\langle p_2|e^{-ik_1r_2}e^{ik_2r_2}|q_2\rangle\langle p_3|e^{-ik_2r_3}|q_3\rangle C(p_1p_2p_3:q_1q_2q_3)$$

$$=\sum_{p_1\neq p_2\neq p_3:q_1\neq q_2\neq q_3}\sum_{\mu_1\mu_2}\tilde{C}^{(0)}_{\mu_1}\tilde{C}^{(0)}_{\mu_2}I^{(12:23)}(A,B,C)M^{p_1q_1}M^{p_2q_2}M^{p_3q_3}\bar{M}^{p_1q_1}\bar{M}^{p_2q_2}\bar{M}^{p_3q_3}C(p_1p_2p_3:q_1q_2q_3)$$

$$I^{(12:23)}(A,B,C)=\frac{1}{(2\pi)^{3n}}\left(\frac{\pi^n}{\det A}\right)^{3/2}e^{-B^\dagger A^{-1}B/4+C}$$

$$A=\begin{pmatrix}1/4\nu+1/4a_\mu & 1/4\nu\\ 1/4\nu & 1/4\nu+1/4a_\mu\end{pmatrix}\quad B=\begin{pmatrix}\frac{1}{2}\left(\vec{D}_{p_1}+\vec{D}_{q_1}\right)-\frac{1}{2}\left(\vec{D}_{p_2}+\vec{D}_{q_2}\right)\\ \frac{1}{2}\left(\vec{D}_{p_2}+\vec{D}_{q_2}\right)-\frac{1}{2}\left(\vec{D}_{p_3}+\vec{D}_{q_3}\right)\end{pmatrix}$$

$$C=-\frac{1}{2}\nu\left[\left(D_{p_1}-D_{q_1}\right)^2+\left(D_{p_2}-D_{q_2}\right)^2+\left(D_{p_3}-D_{q_3}\right)^2\right]$$



Many correlations

$$\left|\Psi\right\rangle = \left|AMD\right\rangle + F_D\left|AMD\right\rangle \rightarrow$$

$$\rightarrow \left|AMD\right\rangle + F_D\left|AMD\right\rangle + F_D F_D\left|AMD\right\rangle ..$$

$$\rightarrow (1+F_S)(1+F_D+F_D F_D+..)\left|AMD\right\rangle$$

$$F_D = \frac{1}{2}\sum_{i\neq j} C_\mu r_{ij}^2 e^{-a_\mu r_{ij}^2} S_{12}(r_{ij})$$ Tensor correlation

$$F_S = \frac{1}{2}\sum_{i\neq j} C_\mu e^{-a_\mu r_{ij}^2}$$ Short range correlation

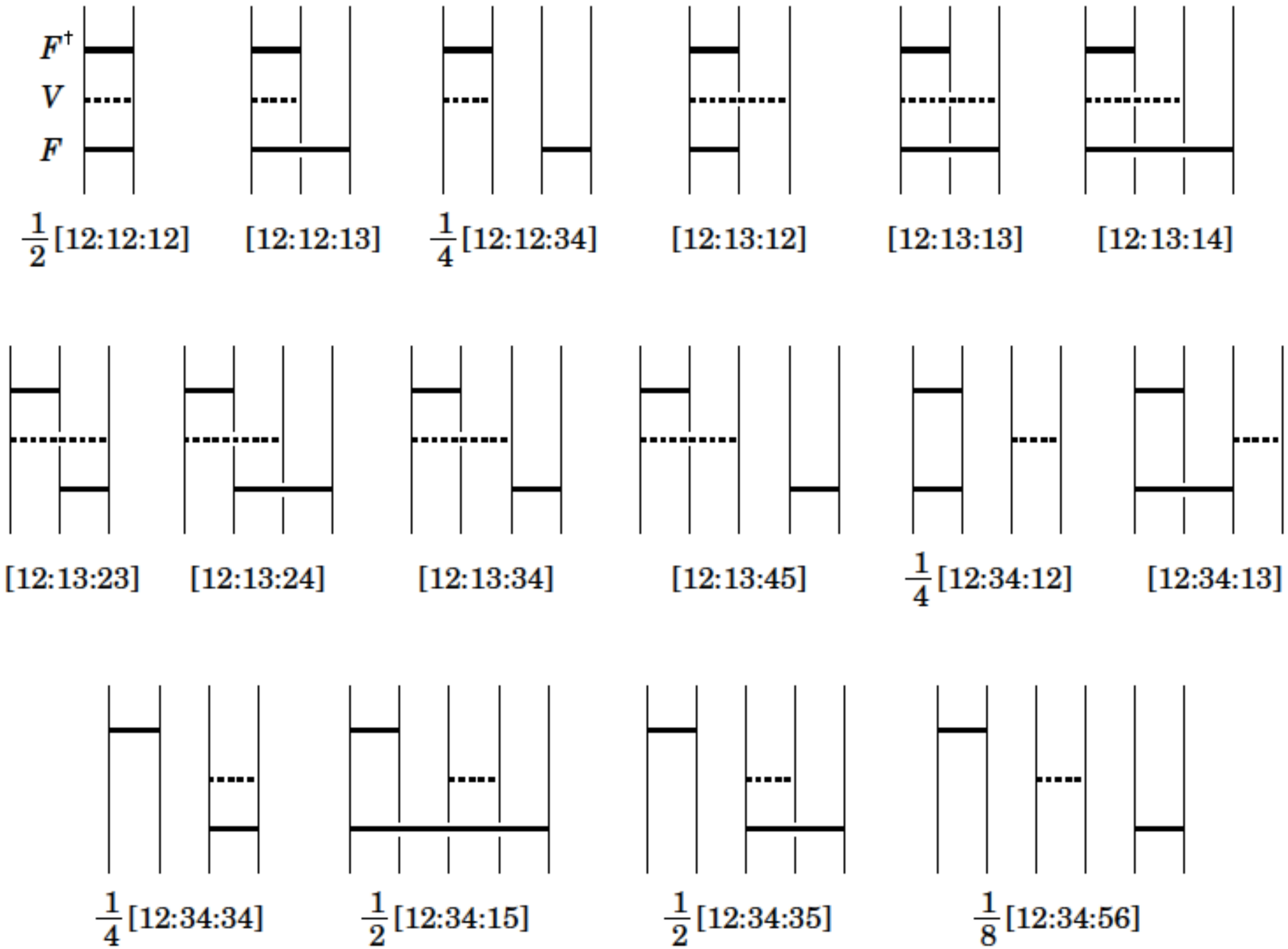


Table 1 Numbers of diagrams of many-body operators in the cluster expansion of $F^\dagger F^\dagger FF$ (norm), $F^\dagger F^\dagger TFF$ and $F^\dagger F^\dagger VFF$ appearing in the double TOAMD.

n-body	2	3	4	5	6	7	8	9	10
norm	1	13	46	47	25	6	1	–	–
T	1	40	183	259	163	55	10	1	–
V	1	40	295	587	516	235	65	10	1

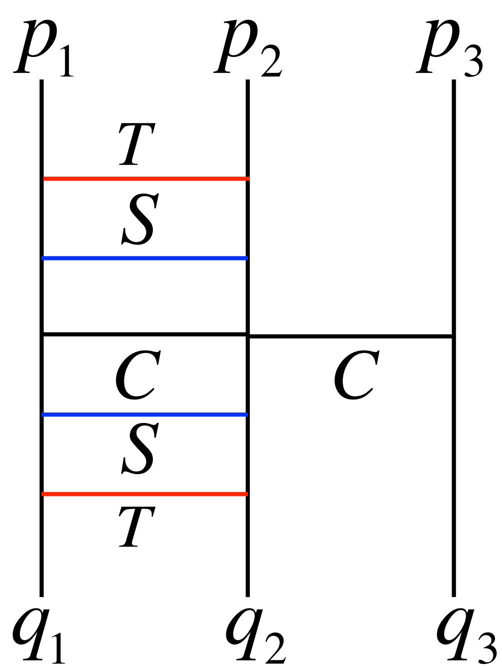
Some example of matrix element

$$\langle AMD | O | AMD \rangle = \sum_{p_1 \neq p_2 \neq p_3 : q_1 \neq q_2 \neq q_3} \sum_{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6} \tilde{C}^{(2)}_{\mu_1} \tilde{C}^{(0)}_{\mu_2} \tilde{C}^{(0)}_{\mu_3} \tilde{C}^{(0)}_{\mu_4} \tilde{C}^{(0)}_{\mu_5} \tilde{C}^{(2)}_{\mu_6}$$

$$\sum_{xyzux'y'z'u'} I^{((12)^3:23:(12)^2)}_{1x1y6z6u}(A,B,C) M^{p_1q_1}_{x'z'} M^{p_2q_2}_{y'u'} M^{p_3q_3} \bar{M}^{p_1q_1} \bar{M}^{p_2q_2} \bar{M}^{p_3q_3}$$

$$(3\delta_{xx'}\delta_{yy'} - \delta_{xy}\delta_{x'y'})(3\delta_{zz'}\delta_{uu'} - \delta_{zu}\delta_{z'u'}) C(p_1p_2p_3 : q_1q_2q_3)$$

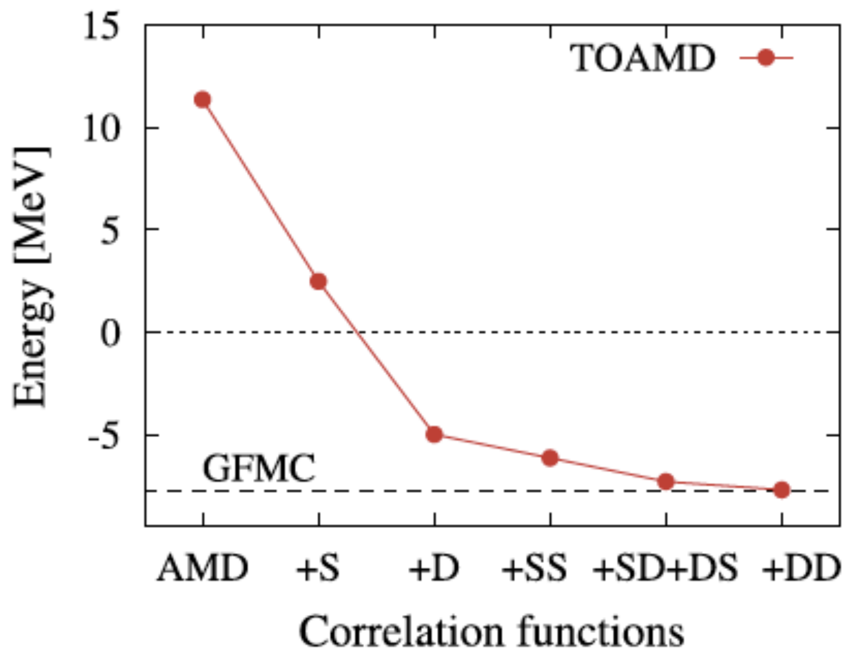
Any complicated matrix elements can be written in similar expressions

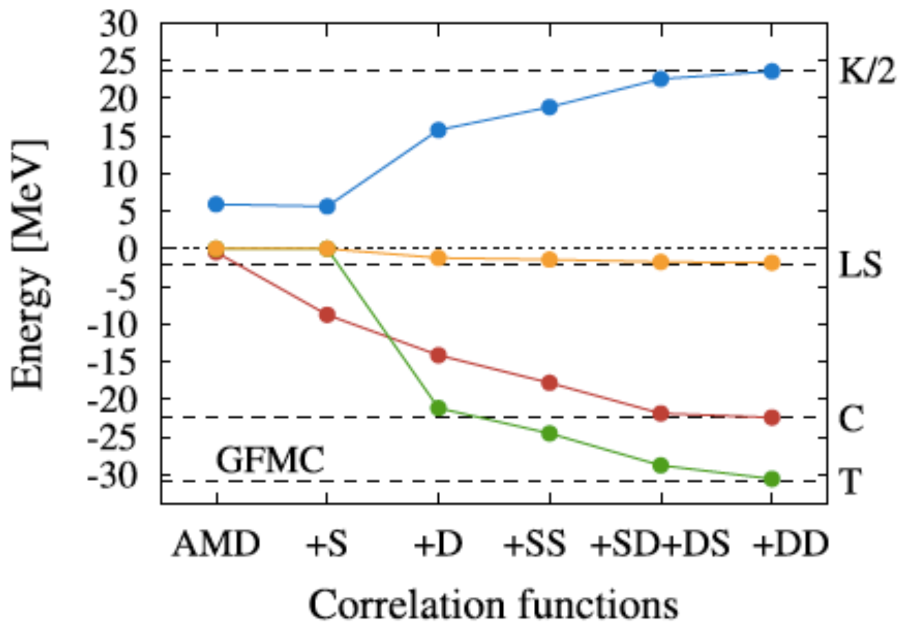


He(A=3)

Interaction is AV8'

TOAMD group: Phys. Lett. B769 (2017) 213





$$\Phi_{TOAMD} = (1 + F_S + F_D + F_S F_S + F_S F_D + F_D F_D)\Phi_{AMD}$$

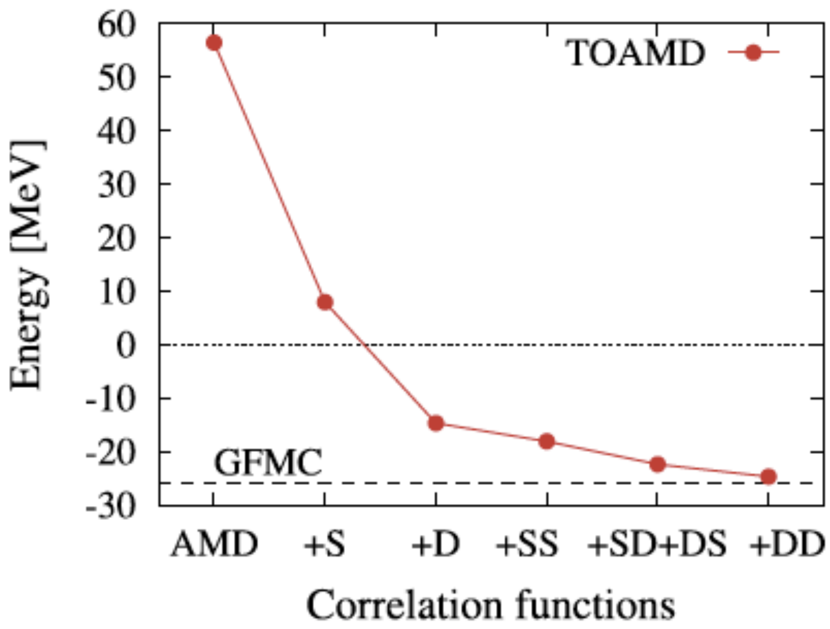
We achieve convergence successively.

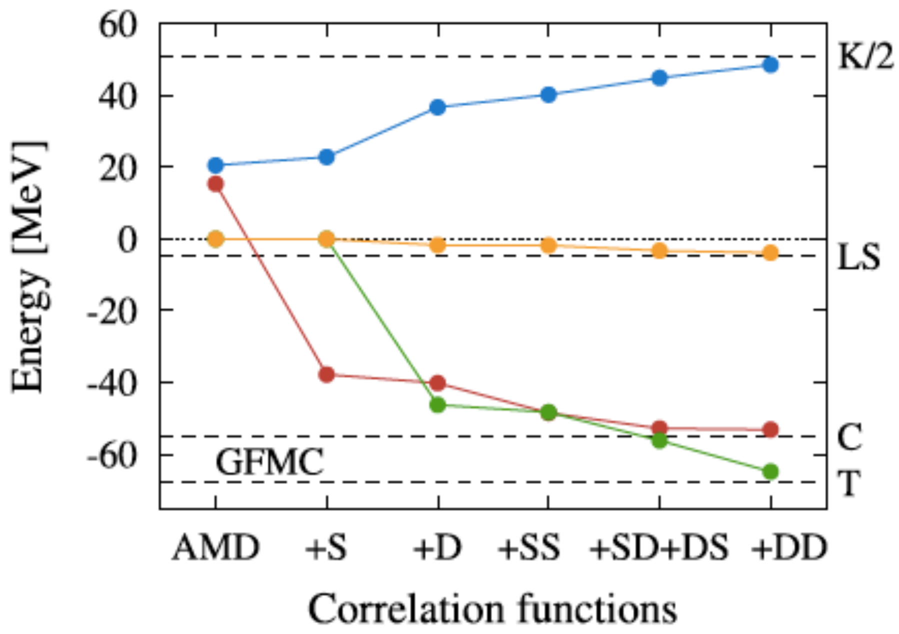
(Successive variational method)

He(A=4)

Interaction is AV8’

TOAMD group: Phys. Lett. B769 (2017) 213

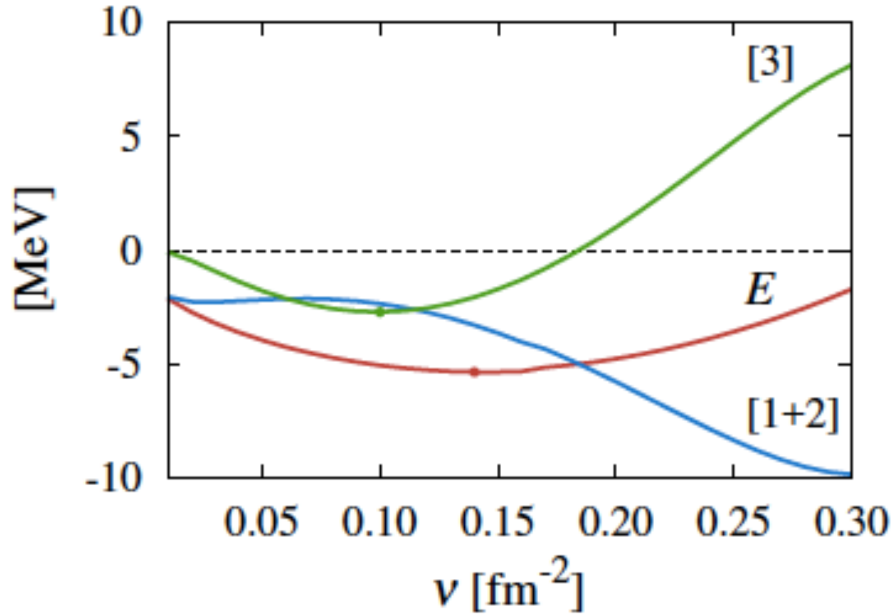




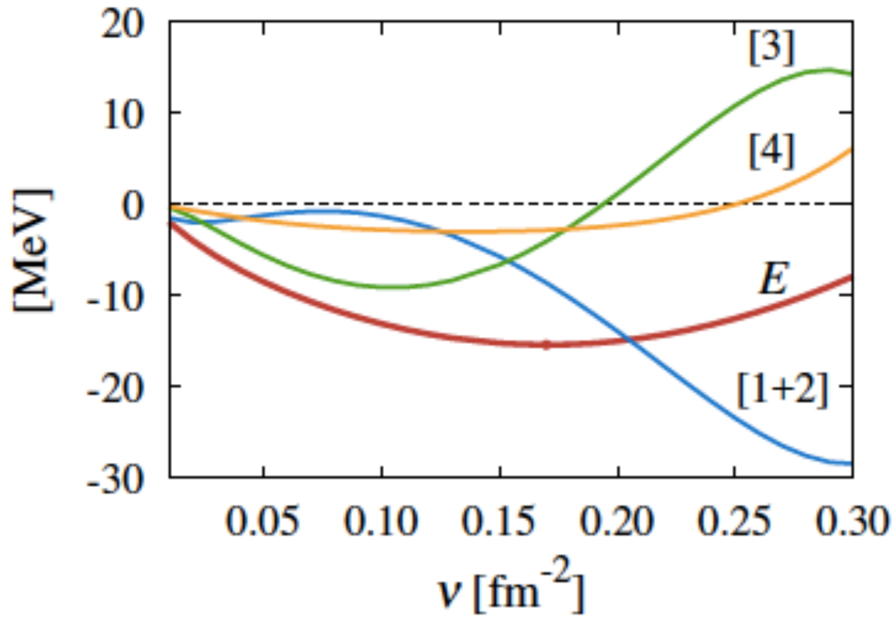
$$\Phi_{TOAMD} = (1 + F_S + F_D + F_S F_S + F_S F_D + F_D F_D)\Phi_{AMD}$$

TOAMD calculation

A=3



A=4

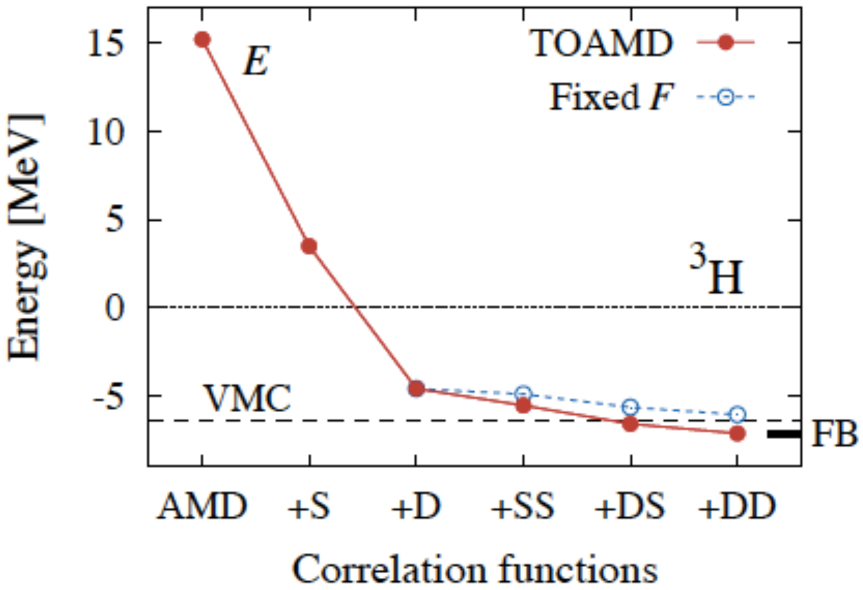


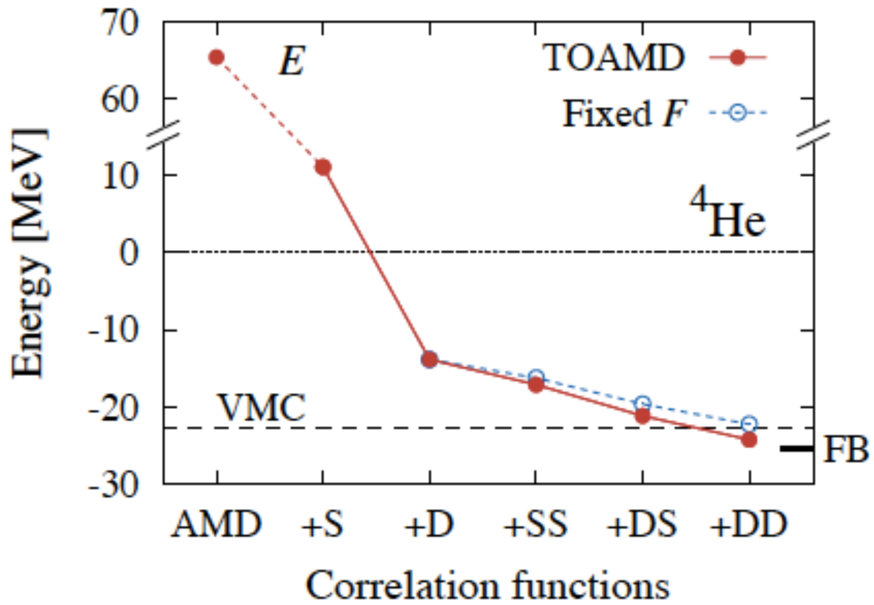
We have to calculate all the multi-body terms in order to get a variationally stable state

TOAMD vs Jastrow correlation (VMC)

TOAMD group: preliminary

AV8 interaction



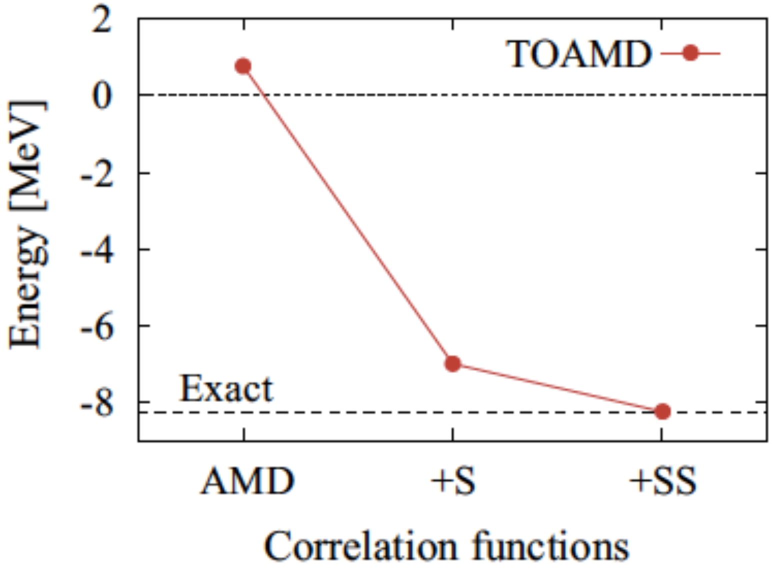


1. TOAMD is better than Jastrow correlation method
2. F(1)=/F(2) is significantly lower than F(1)=F(2)

Central interaction (MT-V potential)

TOAMD: Phys.Rev.C95(2017)044314

A=3



A=4

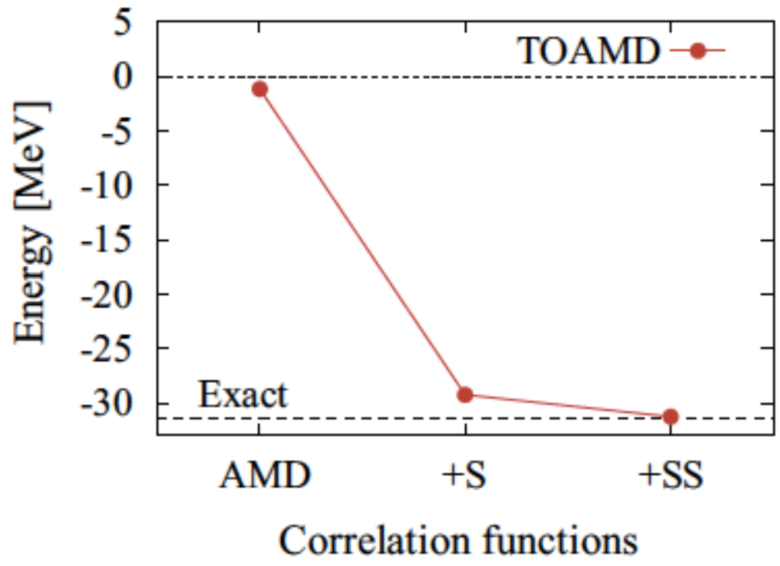


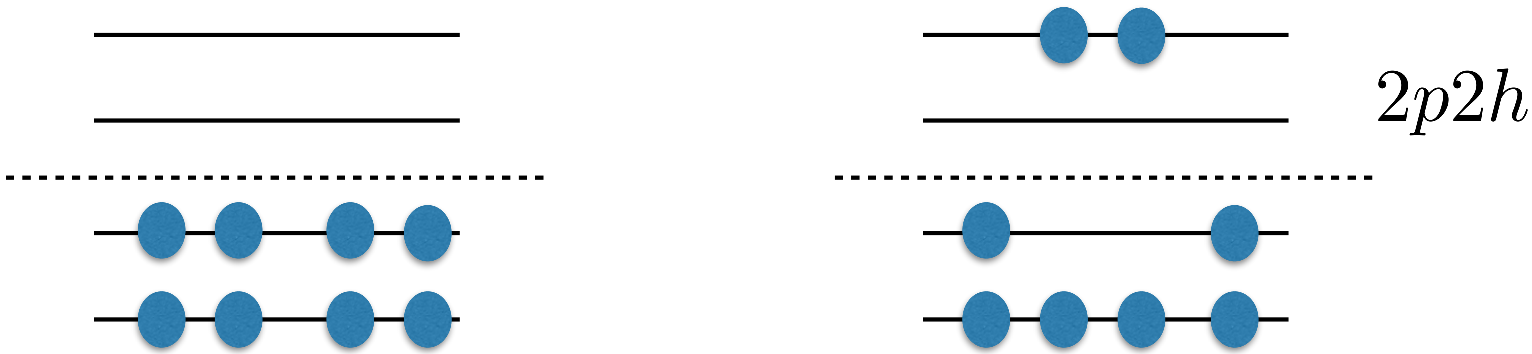
TABLE I: Energies of ^{3}H($\frac{1}{2}^+$) and ^{4}He (0^+) using MT-V potential in units of MeV in comparison with other theories.

	VMC [22]	Few-body [23]	TOAMD
^{3}H	−8.22(2)	−8.25	−8.24
^{4}He	−31.19(5)	−31.36	−31.28

Experiments

We have wave function of ground state

$$|A\rangle = (1+F)|AMD\rangle$$



$$\langle A|O|A\rangle \approx \langle \text{model:A}|O|\text{model:A}\rangle + \langle \text{model:A}|F_D O F_D|\text{model:A}\rangle$$

μ	Magnetic moment
$(S_p + S_n)^2$	Spin operators
e^{ikr}	Form factor

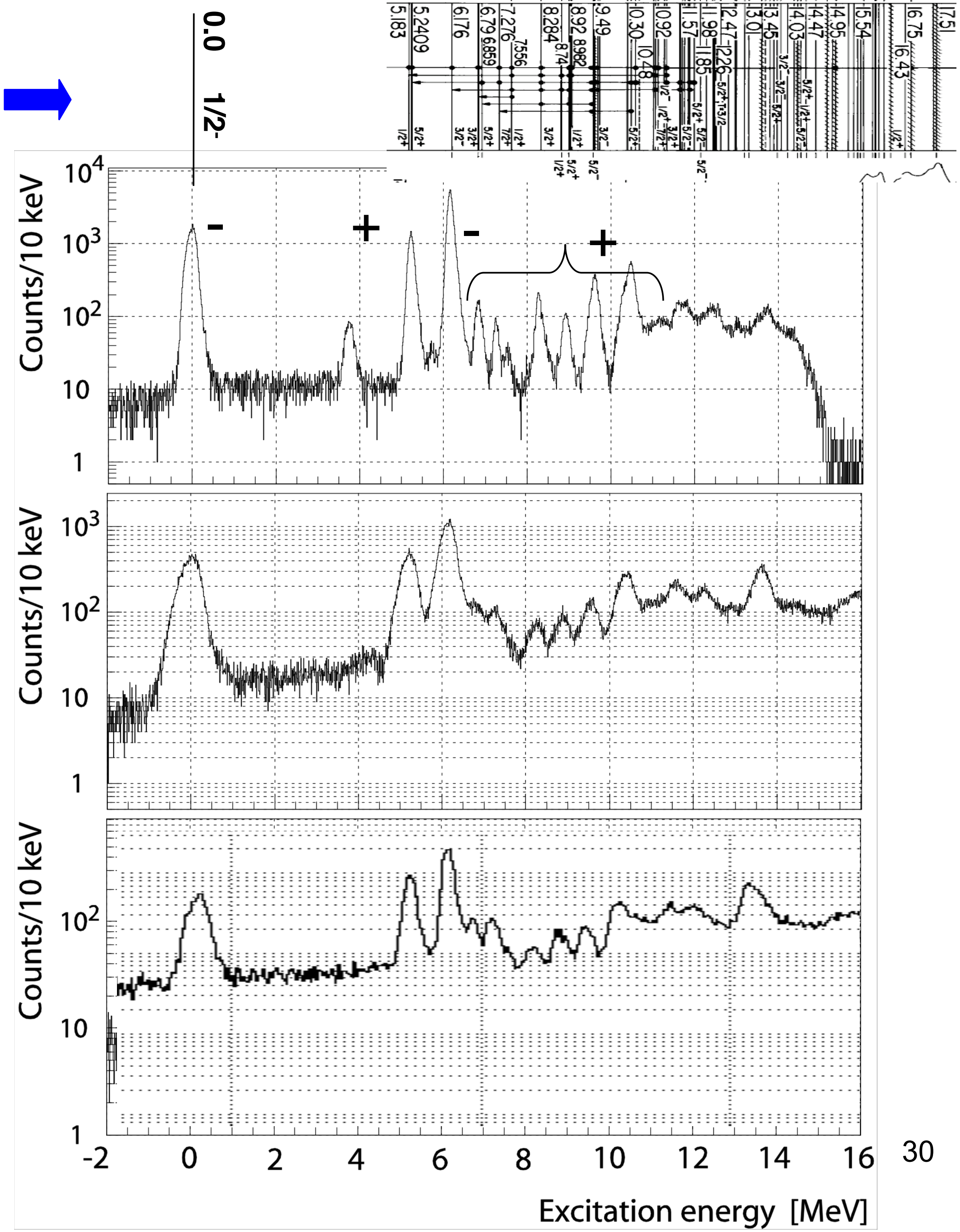
^{15}O Level scheme

Ong, Tanihata et al

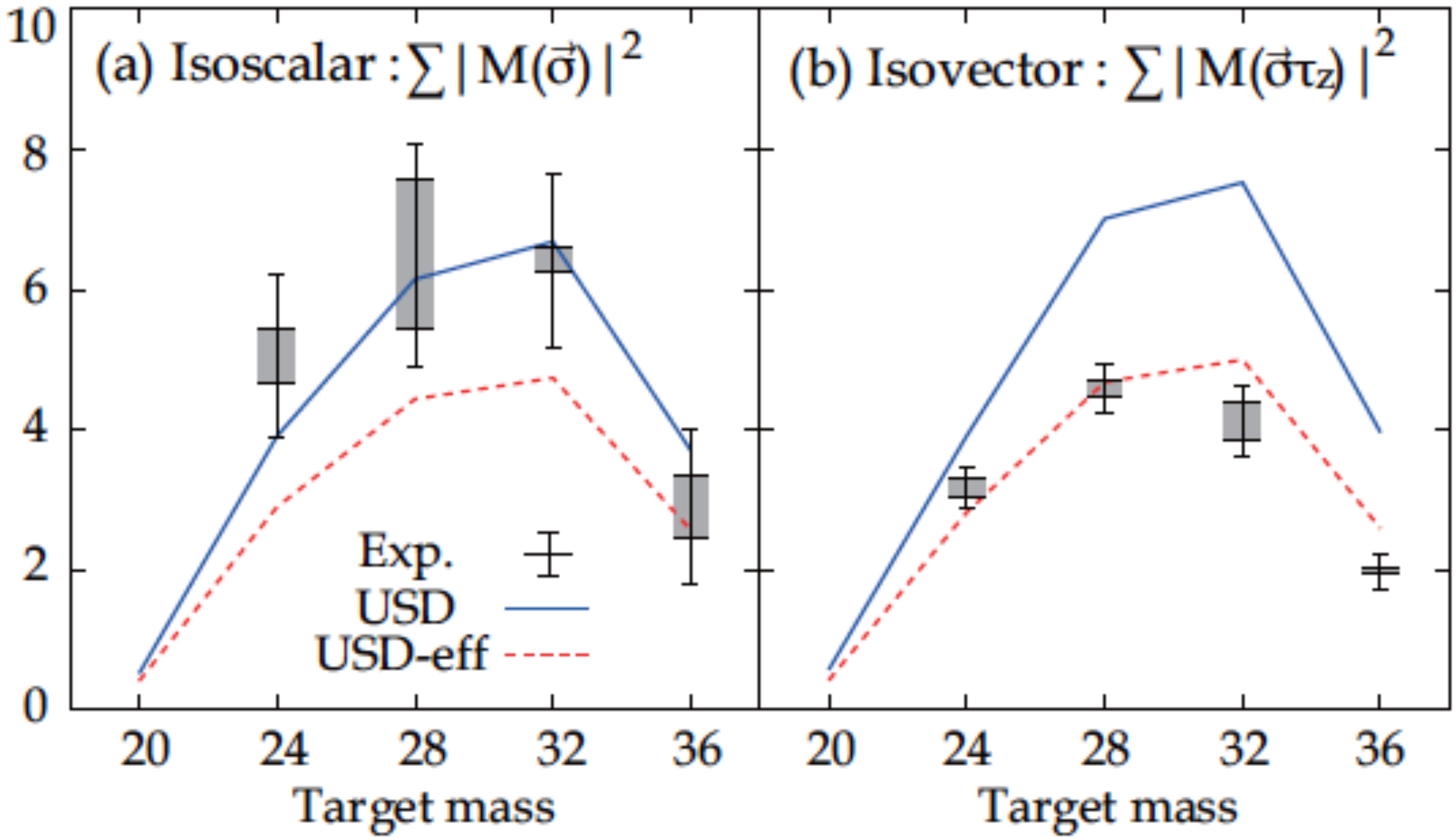
^{16}O (p,d)
$E_p = 198$ MeV
$\Theta_d = 10°$

^{16}O (p,d)
$E_p = 295$ MeV
$\Theta_d = 10°$

^{16}O (p,d)
$E_p = 392$ MeV
$\Theta_d = 10°$



Matsubara Tamii..PRL(2015)



$(S_p + S_n)^2$ $(S_p - S_n)^2$

Conclusion:

We formulated TOAMD (TOSM+AMD)

We calculated He3 and He4 using TOAMD

We achieved convergence successively

TOAMD is better than Jastrow correlation method

We will add delta excitation explicitly for 3-body int.

We will work p-shell nuclei using TOAMD