Strangeness and charm in hadrons and dense matter
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Early time production of the Husimi-Wehrl entropy in the Yang-Mills field from the McLerran-Venugopalan model initial condition

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Akira Ohnishi (YITP)
Toru T. Takahashi (Gumma Col.)

Outline

- Motivation
- Methods/Test in quantum mechanics

  H.T., H.Iida, T.Kunihiro, A.Ohnishi, and T.T.Takahashi, PTEP, 083A01 (2015).

- Entropy production in Yang-Mills field theory

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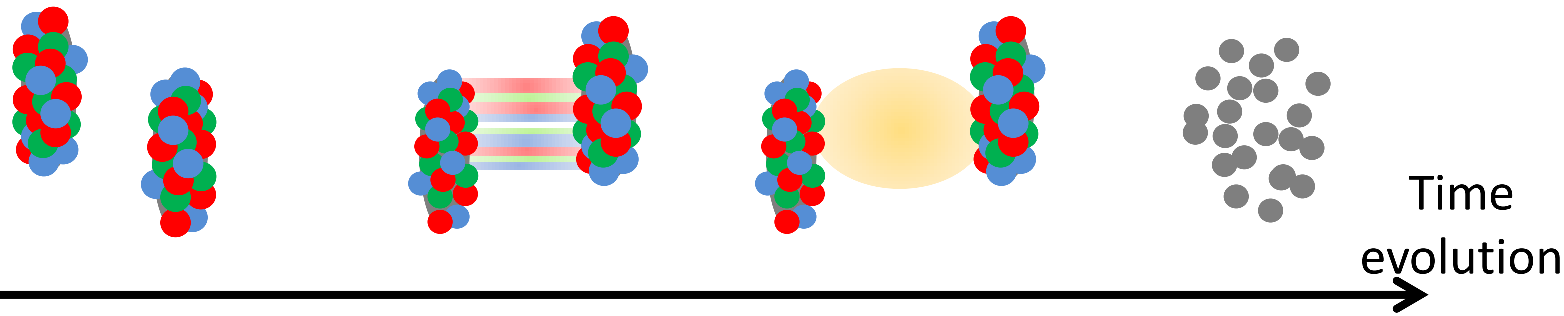
Motivation

Relativistic heavy ion collisions



Color Glass Condensate(CGC)

Classical approx. is valid.
[McLerran, Venugopalan(1993)]

Glasma

Classical approx.
[Lappi, McLerran(2006)]

Quark Gluon Plasma (QGP)

Hadron

Ideal liquid

τ_0= 0.6 - 1.0 fm/c

Perturbative QCD cannot explain this time scale.
[Baier, Alfred, Mueller, Schiff and Son(2001,2002)]

Early thermalization [Heinz(2002)]

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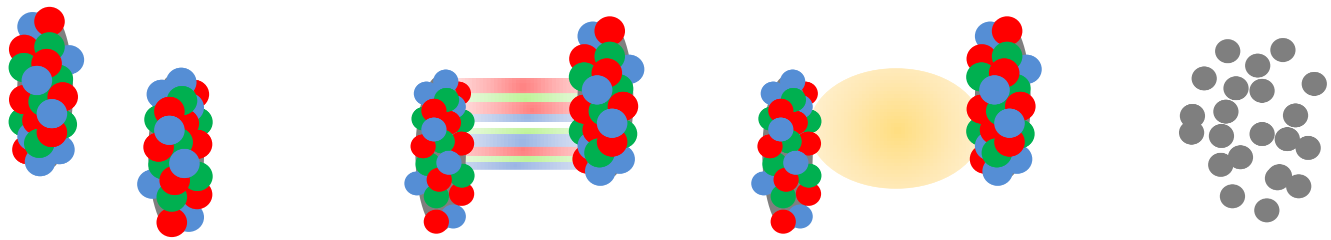

Time evolution

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Classical YM field
+
Quantum fluctuation
$\Rightarrow$ **Instability**

Early thermalization [Heinz(2002)]

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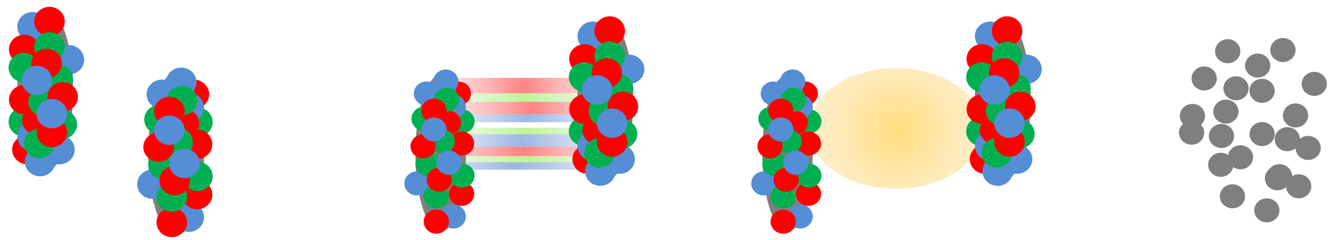
Relativistic heavy ion collisions



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Classical Statistical approximation

Early isotropization
[Epelbaum and Gelis(2013)]

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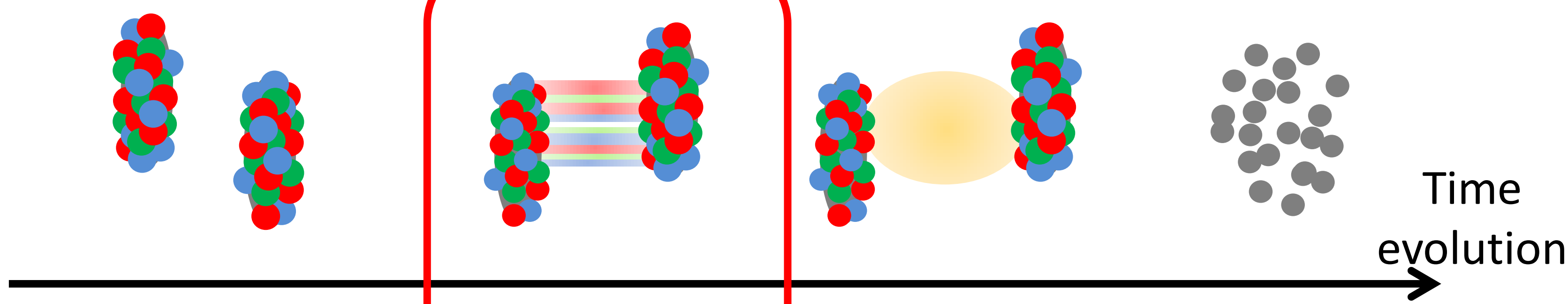

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Early thermalization [Heinz(2002)]

Large amount of entropy is produced in early stage of the collision.

We calculate entropy directly by using a semi-classical formalism with initial fluctuations.

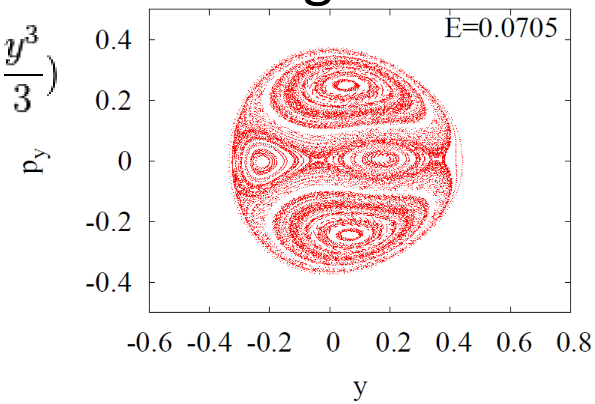
Thermalization scenario based on chaos

Ex.) Hénon-Heiles System

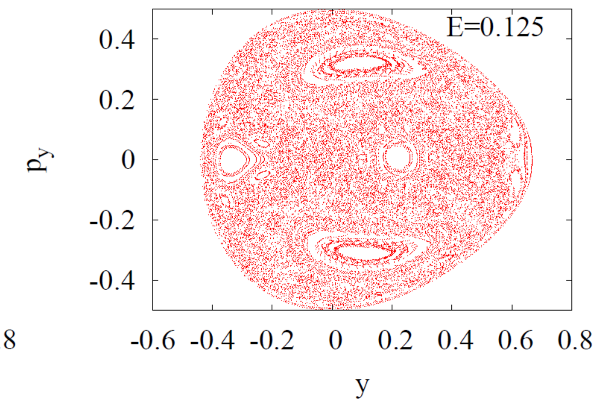
$$H = \frac{1}{2}(p_x^2 + p_y^2) + \frac{1}{2}(x^2 + y^2) + \lambda(x^2 y - \frac{y^3}{3})$$

Henon-Heiles system shows chaotic behavior when the energy is high enough.

Regular



Chaos

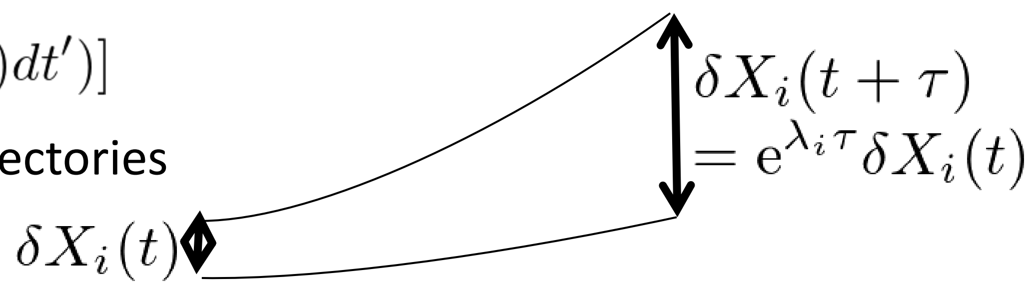


Chaotic systems have a sensitivity to initial value. This property is characterized by **Lyapunov exponents** λ_i, which is given from eigenvalue of a time evolution operator about distance $\delta\vec{X}$ in phase space;

$$U(t, t+\tau) = \mathcal{T}[\exp(\int_t^{t+\tau} \mathcal{H}(t')dt')]$$

$\delta\vec{X}$: distance between classical trajectories

$\mathcal{H}$: Hessian



The sum of positive Lyapunov exponents is positive in classical YM field.

[T.Kunihiro, B.Muller, A.Ohnishi, A.Schafer, T.T.Takahashi, A.Yamamoto, PRD **82**, 114015(2010)]
[H.Iida, T.Kunihiro, B.Muller, A.Ohnishi, A.Schafer, T.T.Takahashi, PRD **88**, 094006(2013)]

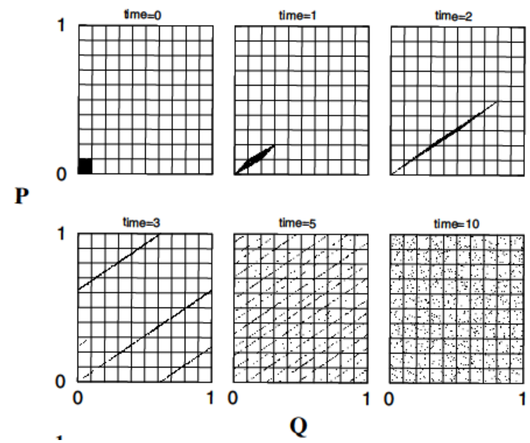
Thermalization scenario based on chaos

V. Latora and M.Baranger, PRL ('99);
M. Baranger, V. Latora and A.Rapisarda, Chaos, Soliton, Fractals (2002)

Generalized cat map(chaotic system)

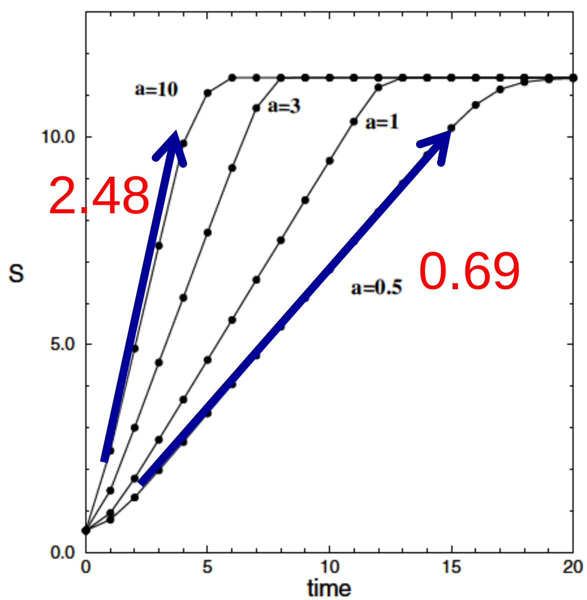
$$P = p + aq \quad (\text{mod } 1),$$

$$Q = p + (1+a)q \quad (\text{mod } 1)$$



Lyapunov exponent

$$\lambda = \log \frac{1}{2}(2 + a + \sqrt{a^2 + 4a})$$



Corse-grained Boltzmann Gibbs entropy

$$S(t) = -\sum_{i:\text{cell}} p_i(t) \log p_i(t)$$

$p_i(t)$:probability that the state of the system falls inside cell c_i of phase space at time t

The entropy production rate is consistent with Lyapunov exponent.

$$\lambda = 2.48, 1.57, 0.96, 0.69$$

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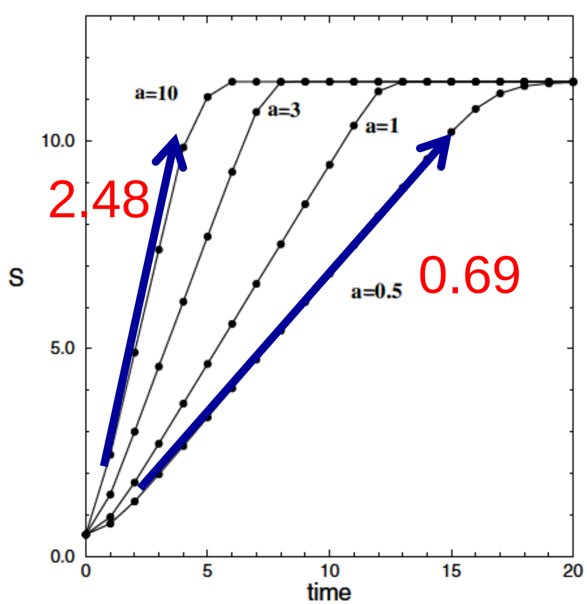
Chaos
↓
Entropy production
↓
Thermalization



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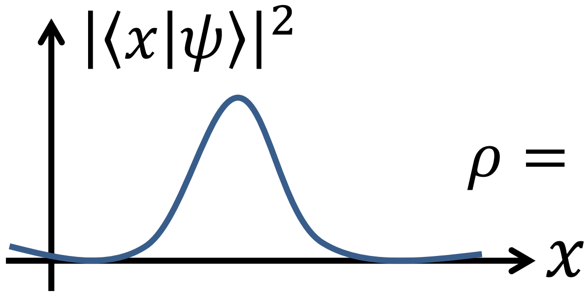
Semi-classical time evolution of Wigner func.

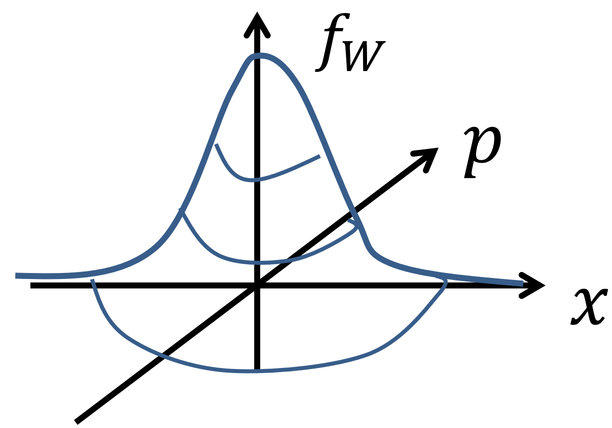
Wigner function [Wigner(1932)]

$$f_W(\vec{p},\vec{q};t) = \int d\vec{\eta}\exp(-i\vec{p}\cdot\vec{\eta}/\hbar)\langle\vec{q}+\vec{\eta}/2|\rho|\vec{q}-\vec{\eta}/2\rangle$$

Wigner function is the density matrix in Wigner representation.



$\rho = |\psi\rangle\langle\psi|$ ⇒ f_W



(Quasi-)distribution function

$$\langle\hat{A}\rangle = \int \frac{d\vec{p}d\vec{q}}{(2\pi\hbar)^n} f_W(\vec{p},\vec{q};t)A_W(\vec{p},\vec{q};t)$$

Wigner function has a problem in serving as a quantum distribution function.
It is **not positive definite**.

Semi-classical time evolution of Wigner func.

Wigner function [Wigner(1932)]

$$f_W(\vec{p},\vec{q};t) = \int d\vec{\eta}\exp(-i\vec{p}\cdot\vec{\eta}/\hbar)\langle \vec{q}+\vec{\eta}/2|\rho|\vec{q}-\vec{\eta}/2\rangle$$

In the case of $H = \frac{\vec{p}^2}{2m} + V(\vec{q})$,

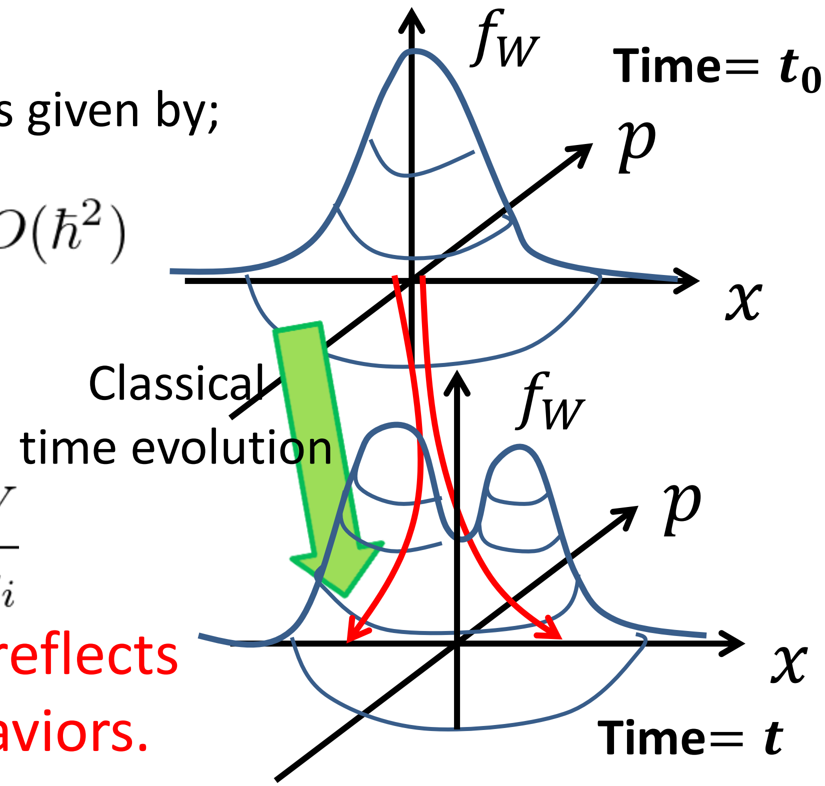
the **time evolution of Wigner function** is given by;

$$\frac{\partial}{\partial t} f_W = \sum_i^n \frac{\partial V}{\partial q_i}\frac{\partial f_W}{\partial p_i} - \sum_i^n \frac{p_i}{m}\frac{\partial f_W}{\partial q_i} + O(\hbar^2)$$

The semi-classical solution leads to

$$\frac{d}{dt} f_W(\vec{p},\vec{q};t) = 0$$

With classical EOM $\dot{q}_i = \frac{p_i}{m}, \dot{p}_i = -\frac{\partial V}{\partial q_i}$



The time evolution of Wigner function reflects the classical dynamics, the chaotic behaviors.

Husimi function

The figures are transferred from T.Kunihiro, B.Muller, A.Ohnishi, A.Schafer(2009).

Husimi function [Husimi(1940)]

$|\vec{\alpha}\rangle$; coherent state

$\rho = |\phi\rangle\langle\phi|$

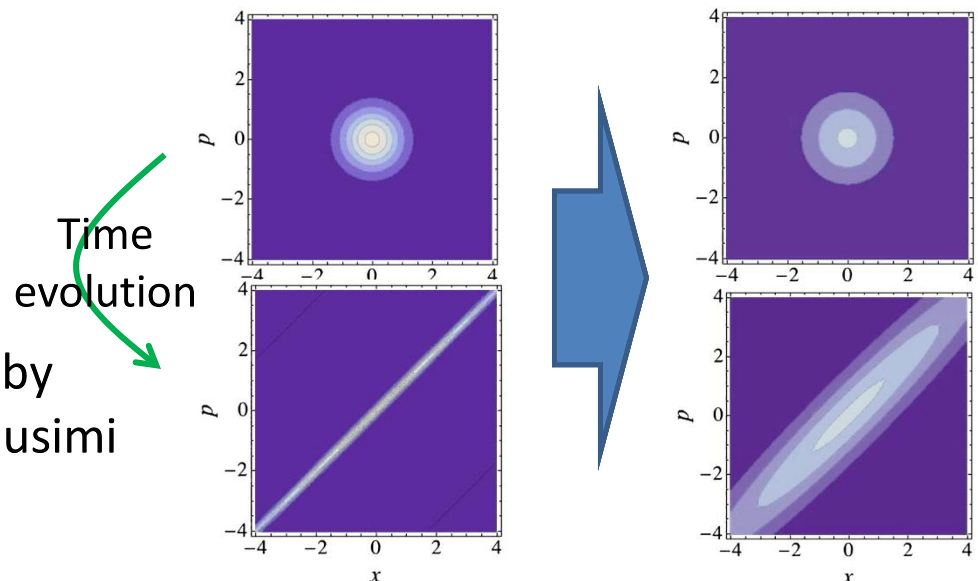
$$f_H(\Gamma;t) = \langle\vec{\alpha}|\hat{\rho}|\vec{\alpha}\rangle = |\langle\vec{\alpha}|\phi\rangle|^2 \geq 0$$

$$= \int \frac{d\Gamma'}{(\pi\hbar)^n} \exp\left(-\frac{1}{\hbar}(\Gamma - \Gamma')^2\right) f_W(\Gamma';t)$$

Where $\Gamma = (\vec{p}, \vec{q})$ is a point on the "phase space" in Wigner rep..

- Husimi function is semi-positive definite.

When Wigner function is lengthen by chaotic behaviors or instabilities, Husimi function spreads in "phase space".



Wigner func. Husimi func.

Husimi-Wehrl(HW) entropy

The figures are transferred from T.Kunihiro, B.Muller, A.Ohnishi, A.Schafer(2009).

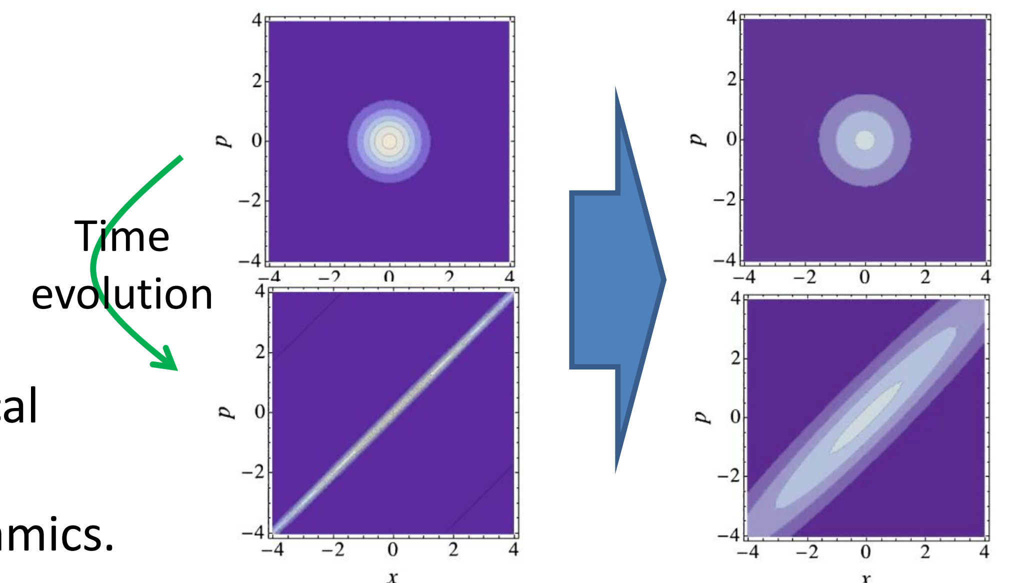
We can define entropy in terms of Husuimi function.

<u>Husimi-Wehrl entropy</u> [Wehrl(1978)]

$$S_{HW}(t) = -\int \frac{d\Gamma}{(2\pi\hbar)^n} f_H(\Gamma;t) \log f_H(\Gamma;t)$$

- Husimi function is semi-positive definite.
- Gauss smearing makes entropy production.

HW entropy is created when classical systems have chaos or instability.
The entropy evaluates chaotic dynamics.



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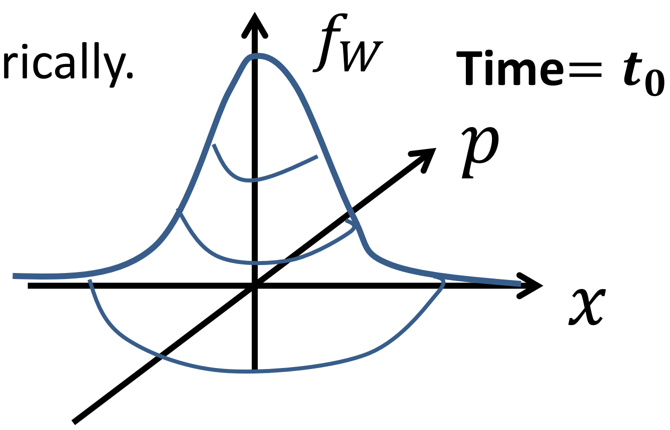
Husimi-Wehrl entropy in term of Wigner function

$$S_{HW}(t) = -\int \frac{d\vec{p}d\vec{q}}{(2\pi\hbar)^n} \exp(-\frac{1}{\Delta\hbar}\vec{p}^2 - \frac{\Delta}{\hbar}\vec{q}^2) \int \frac{d\vec{p'}d\vec{q'}}{(\pi\hbar)^n} f_W(\vec{p'}, \vec{q'}; t)$$
$$\times \log \int \frac{d\vec{p''}d\vec{q''}}{(\pi\hbar)^n} \exp(-\frac{1}{\Delta\hbar}(\vec{p}+\vec{p'}-\vec{p''})^2 - \frac{\Delta}{\hbar}(\vec{q}+\vec{q'}-\vec{q''})^2) f_W(\vec{p''}, \vec{q''}; t)$$

We would like to calculate these integrations numerically.



Test particle method

We assume that Wigner function is a sum of delta functions.

$$f_W(\vec{p}, \vec{q}; t) = \frac{(2\pi\hbar)^n}{N} \sum_i^N \delta^{(n)}(\vec{p} - \vec{\bar{p}}^i(t))\delta^{(n)}(\vec{q} - \vec{\bar{q}}^i(t)))$$

The test particles obey the classical equation of motion.

$$\dot{q}_i = \frac{p_i}{m}, \dot{p}_i = -\frac{\partial V}{\partial q_i}$$

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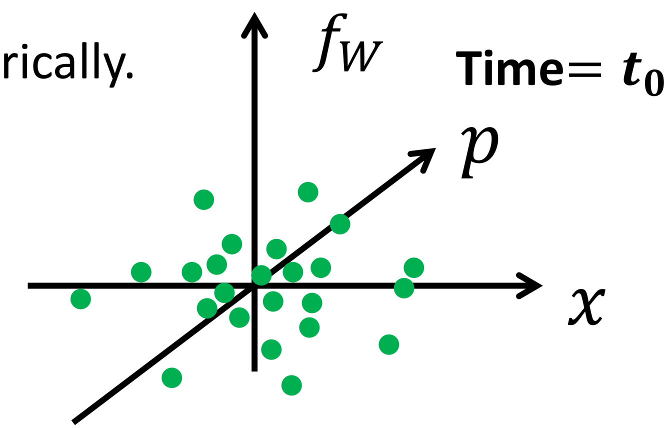

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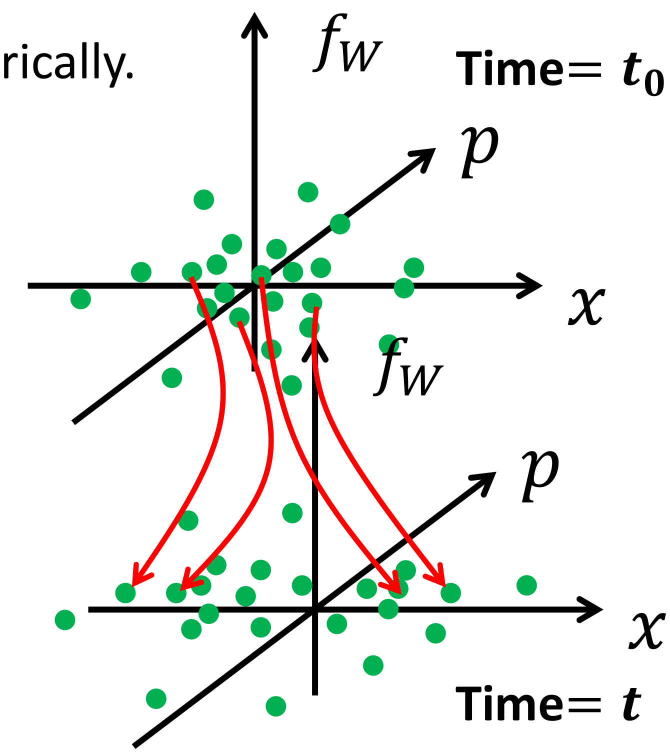

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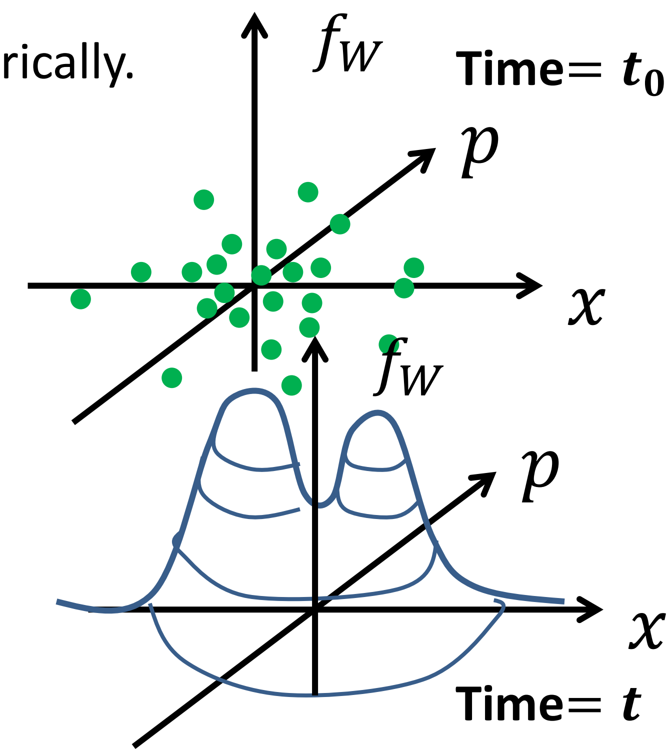

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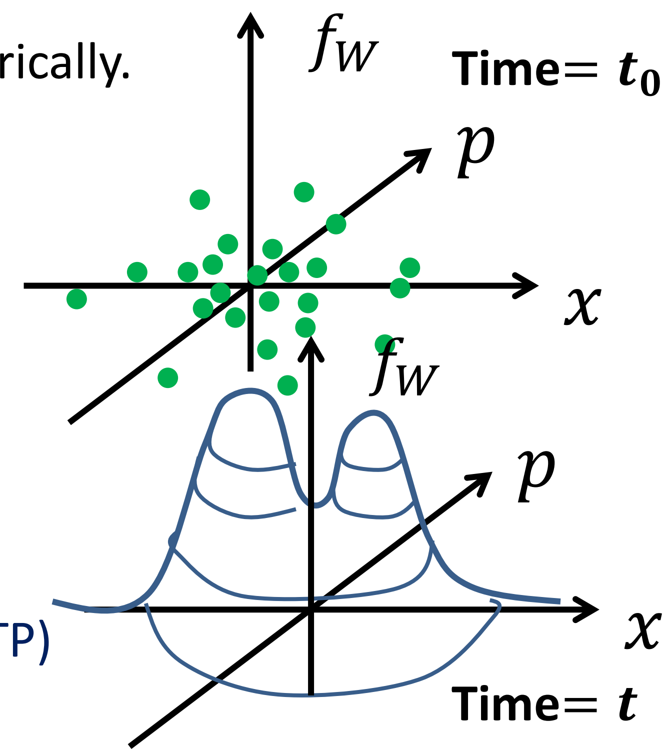

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Substitute
same test particle samples: test particle(TP) method
another test particle samples: parallel test particle(pTP) method

Examples in quantum mechanics

H.T., H.Iida, T.Kunihiro, A.Ohnishi, and T.T.Takahashi, PTEP, 083A01 (2015).
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Hamiltonian

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{1}{2}g^2 q_1^2 q_2^2 + \frac{\epsilon}{4} q_1^4 + \frac{\epsilon}{4} q_2^4$$

$$m = 1, g = 1, \epsilon = 0.1$$

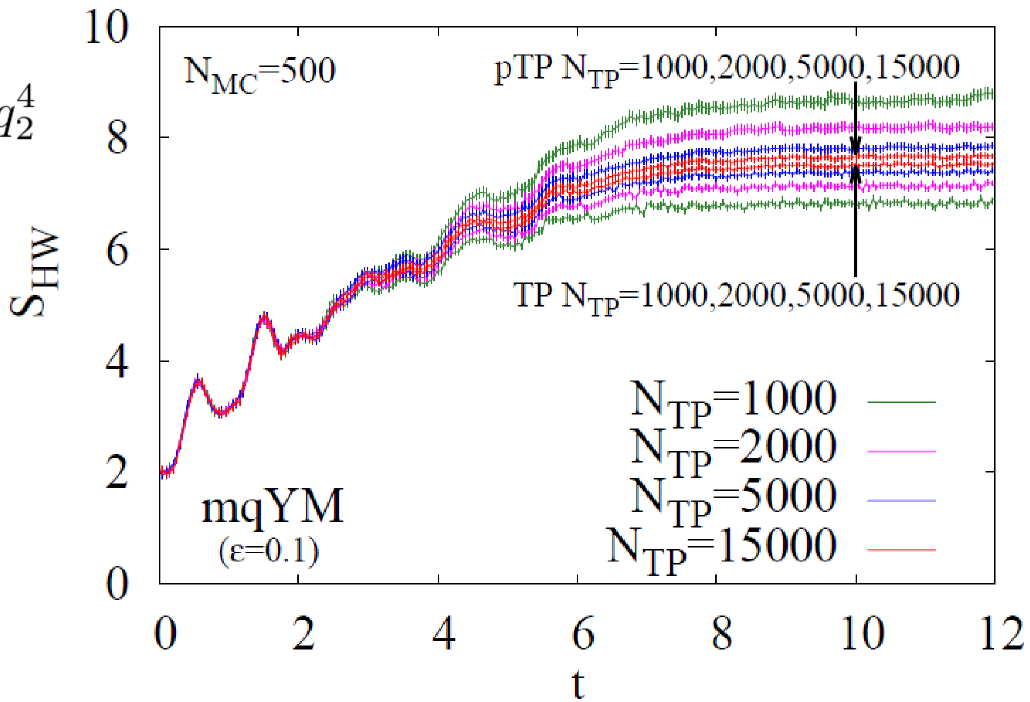
Initial condition: coherent state

$$f_W(\Gamma, t=0) = \exp[-\frac{1}{\hbar}\Gamma^2]$$

$$\Gamma = (p_1, p_2, q_1, q_2)$$

Our two numerical methods describe the entropy production.

Time evolution of Husimi-Wehrl entropy



The results in TP and pTP methods approach each other from below and above, respectively. We can guess the converged value between them.

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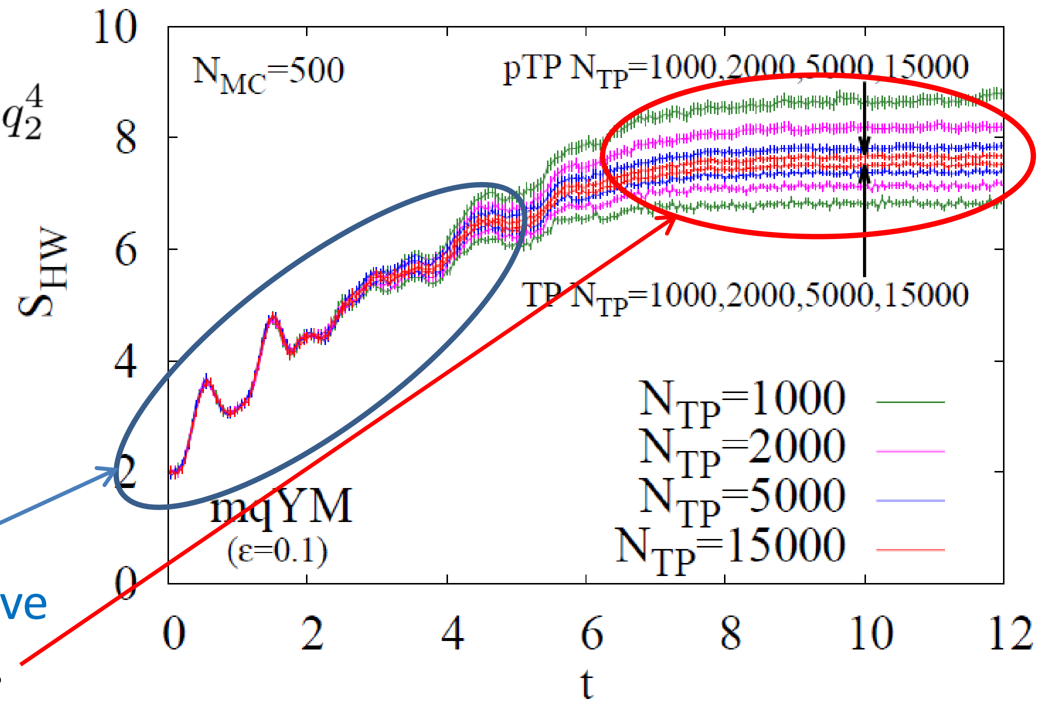
$$m = 1, g = 1, \epsilon = 0.1$$

Initial condition: coherent state

$$f_W(\Gamma, t=0) = \exp[-\frac{1}{\hbar}\Gamma^2]$$

$$\Gamma = (p_1, p_2, q_1, q_2)$$

Time evolution of Husimi-Wehrl entropy



Husimi function spreads with collective motion in early time and saturates.

The results in TP and pTP methods approach each other from below and above, respectively. We can guess the converged value between them.

Product ansatz

H.T., H.Iida, T.Kunihiro, A.Ohnishi, and T.T.Takahashi, PRD 94, 091502(R) (2016).

In higher dimension, we need a larger number of samples and test particles. We consider product ansatz to converge numerical results.

We assume that Husimi function is decomposed into the product of that of 1-dim degree of freedom.

$$f_H(q,p;t) = \prod_i^D h_i(q_i,p_i;t)$$

But we solve a equation of motion of full degrees of freedom unlike Hartree approximation.

Then Husimi-Wehrl entropy in product ansatz is written by

$$S_{HW}^{(PA)} = -\sum_i^D \int \frac{dq_i dp_i}{2\pi\hbar} h(q_i,p_i;t) \log h(q_i,p_i;t)$$

$$\geq S_{HW}$$ From subadditivity of entropy.

The Husimi-Wehrl entropy in product ansatz gives the upper bound of the entropy.

Check in the case of quantum mechanical systems

H.T., H.Iida, T.Kunihiro, A.Ohnishi, and T.T.Takahashi, PRD 94, 091502(R) (2016).

Hamiltonian

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{1}{2}g^2 q_1^2 q_2^2 + \frac{\epsilon}{4}q_1^4 + \frac{\epsilon}{4}q_2^4$$

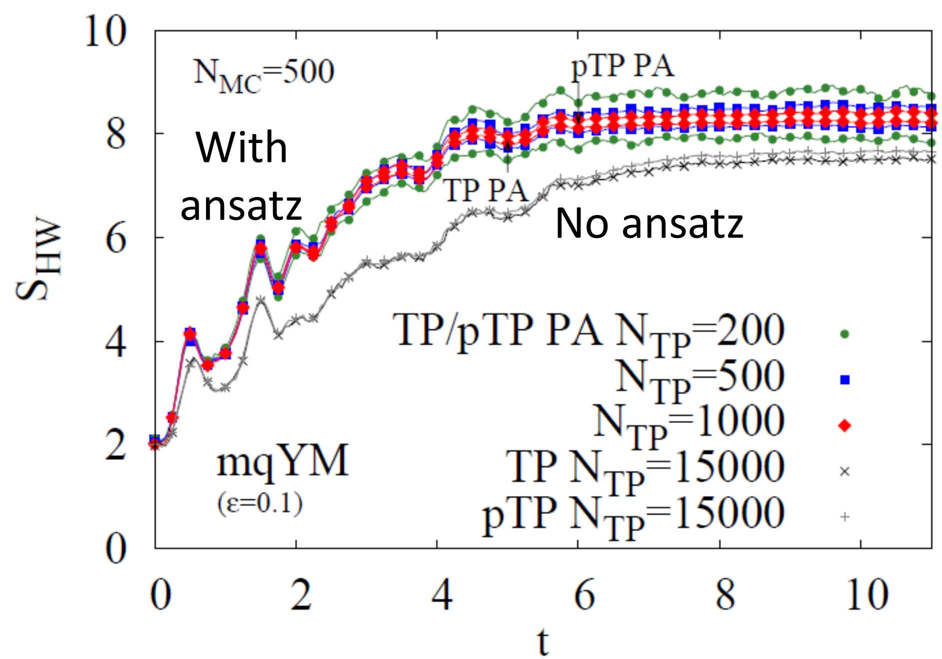
$$m = 1, g = 1, \epsilon = 0.1$$

Initial condition

$$f_W(\Gamma, t = 0) = \exp[-\frac{1}{\hbar}\Gamma^2]$$

$$\Gamma = (p_1, p_2, q_1, q_2)$$

Time evolution of Husimi-Wehrl entropy



Product ansatz gives the upper bound of entropy and consistent results within 10% error bar. The convergence with the number of the test particles is better.

Outline

- Motivation
- Methods/Test in quantum mechanics

H.T., H.Iida, T.Kunihiro, A.Ohnishi, and T.T.Takahashi, PTEP, 083A01 (2015).

- Entropy production in Yang-Mills field theory

H.T., H.Iida, T.Kunihiro, A.Ohnishi, and T.T.Takahashi, PRD 94, 091502(R) (2016).

- Results with phenomenological initial condition

H.T., T.Kunihiro, A.Ohnishi, and T.T.Takahashi, in preparation.

Classical Yang-Mills field

H.T., H.Iida, T.Kunihiro, A.Ohnishi, and T.T.Takahashi, PRD 94, 091502(R) (2016).
H.T., T.Kunihiro, A.Ohnishi, and T.T.Takahashi, in preparation.

We will work in temporal gauge $A_0^a = 0$
Then Hamiltonian in a non-compact formalism is given by

$$H = \frac{1}{2}\sum_{x,a,i} E_i^a(x)^2 + \frac{1}{4}\sum_{x,a,i,j} F_{ij}^a(x)^2$$

$$F_{ij}^a = \partial_i A_j^a(x) - \partial_j A_i^a(x) + \sum_{b,c} f^{abc} A_i^b(x) A_j^c(x)$$

Canonical variables are $(A_i^a(x), E_i^a(x))$

EOM is
$$\dot{A}_i^a(x) = E_i^a(x)$$
$$\dot{E}_i^a(x) = \sum_j \partial_j F_{ij}^a(x) + \sum_{b,c,j} f^{abc} A_j^b(x) F_{ji}^c(x)$$

For the extension, we consider

$$(q,p) \longrightarrow (A_i^a(x), E_i^a(x))$$

c.f. S. Mrowczynski, B. Muller(1994) (in a scalar field case)

Entropy production in SU(2) Yang-Mills field

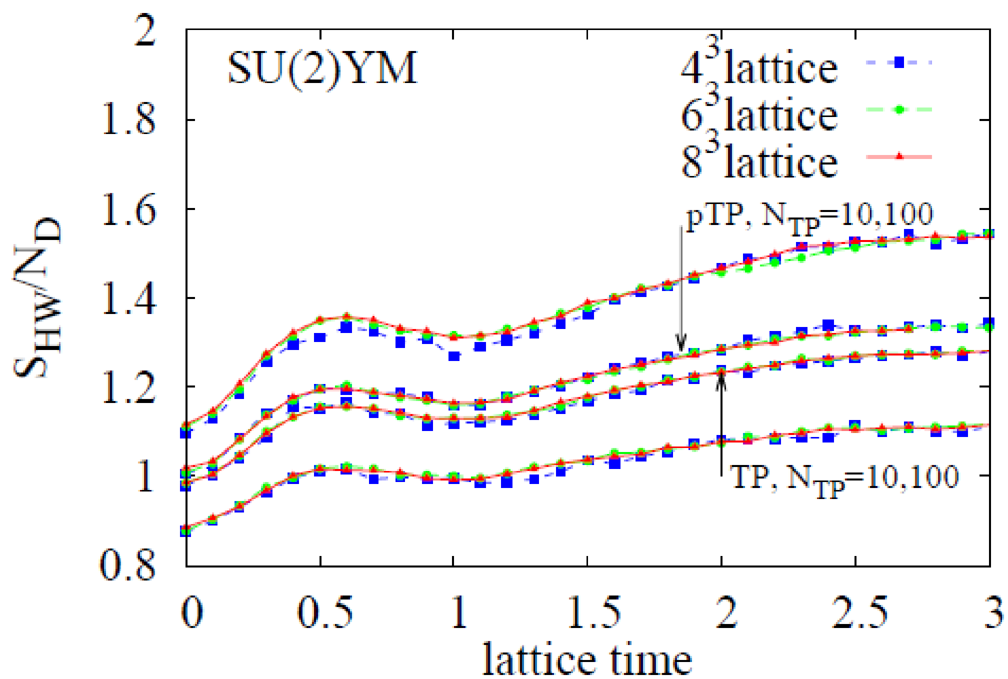
H.T., H.Iida, T.Kunihiro, A.Ohnishi, and T.T.Takahashi, PRD 94, 091502(R) (2016).

Husimi-Wehrl entropy is produced in YM field!

The results in TP and pTP approach each other from below and above.

The time evolution of the entropy on each lattice size agrees with each other.

Time evolution of Husimi-Wehrl entropy per one degrees of freedom



We see that the entropy as given by Husimi-Wehrl entropy is created in Yang-Mills theory though in the product ansatz.

Entropy production in SU(2) Yang-Mills field

H.T., H.Iida, T.Kunihiro, A.Ohnishi, and T.T.Takahashi, PRD 94, 091502(R) (2016).

The growth rate is consistent with the sum of positive Lyapunov exponents in [Kunihiro, et. al.(2010)]

$$U(t, t+\tau) = \mathcal{T}[\exp(\int_t^{t+\tau} \mathcal{H}(t')dt')]$$

Lyapunov exponents are given from eigenvalue of a time evolution operator.

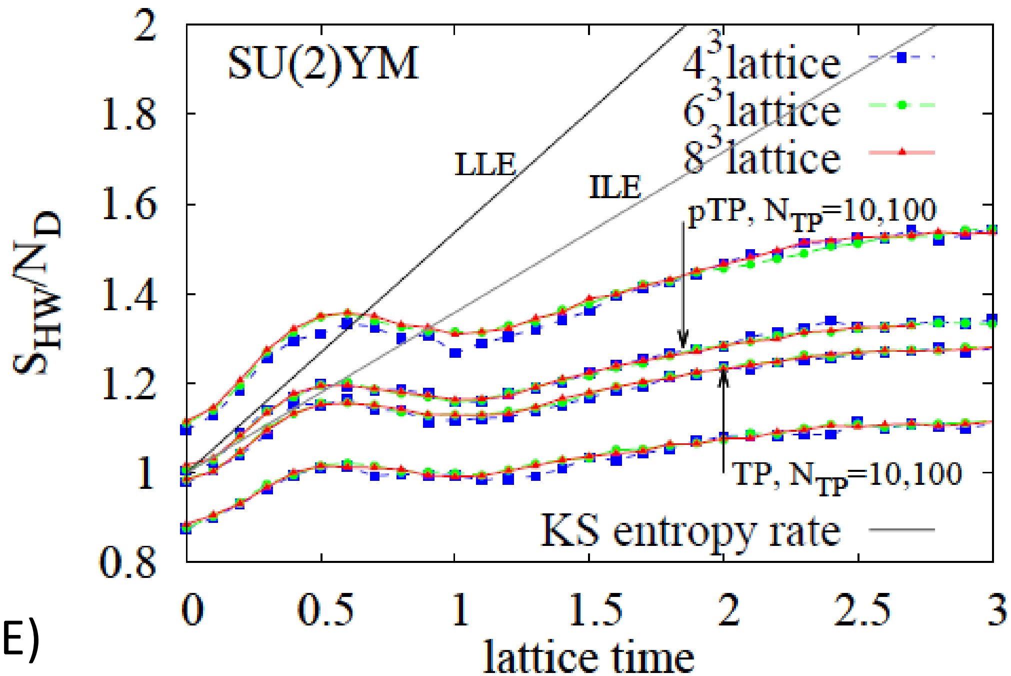
When τ is infinitesimal;

local Lyapounov exponent(LLE)

When τ is intermediate time scale;

intermediate Lyapounov exponent(ILE)

Time evolution of Husimi-Wehrl entropy per one degrees of freedom



The production of Husimi-Wehrl entropy is caused by the chaotic behavior of Yang-Mills field.

Outline

- Motivation
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- Entropy production in Yang-Mills field theory

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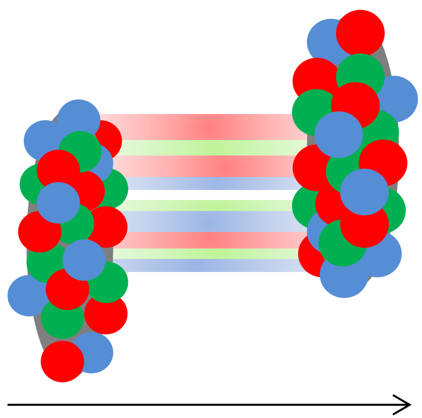
- Results with phenomenological initial condition

  H.T., T.Kunihiro, A.Ohnishi, and T.T.Takahashi, in preparation.

McLarran-Venugopalan(MV) model

McLerran and Venugopalan (1994).
H.T., T.Kunihiro, A.Ohnishi, and T.T.Takahashi, in preparation.



(Non expanding geometry)

Physical scale

$$\alpha_s = 0.15$$

$$aL = \sqrt{\pi}R_A = 7\sqrt{\pi}[\mathrm{fm/c}]$$

$$\mu = Q_s = 2\mathrm{GeV}$$

$$\Leftrightarrow g^2\mu a L = 120$$

a: lattice spacing
L: lattice size

Initial condition

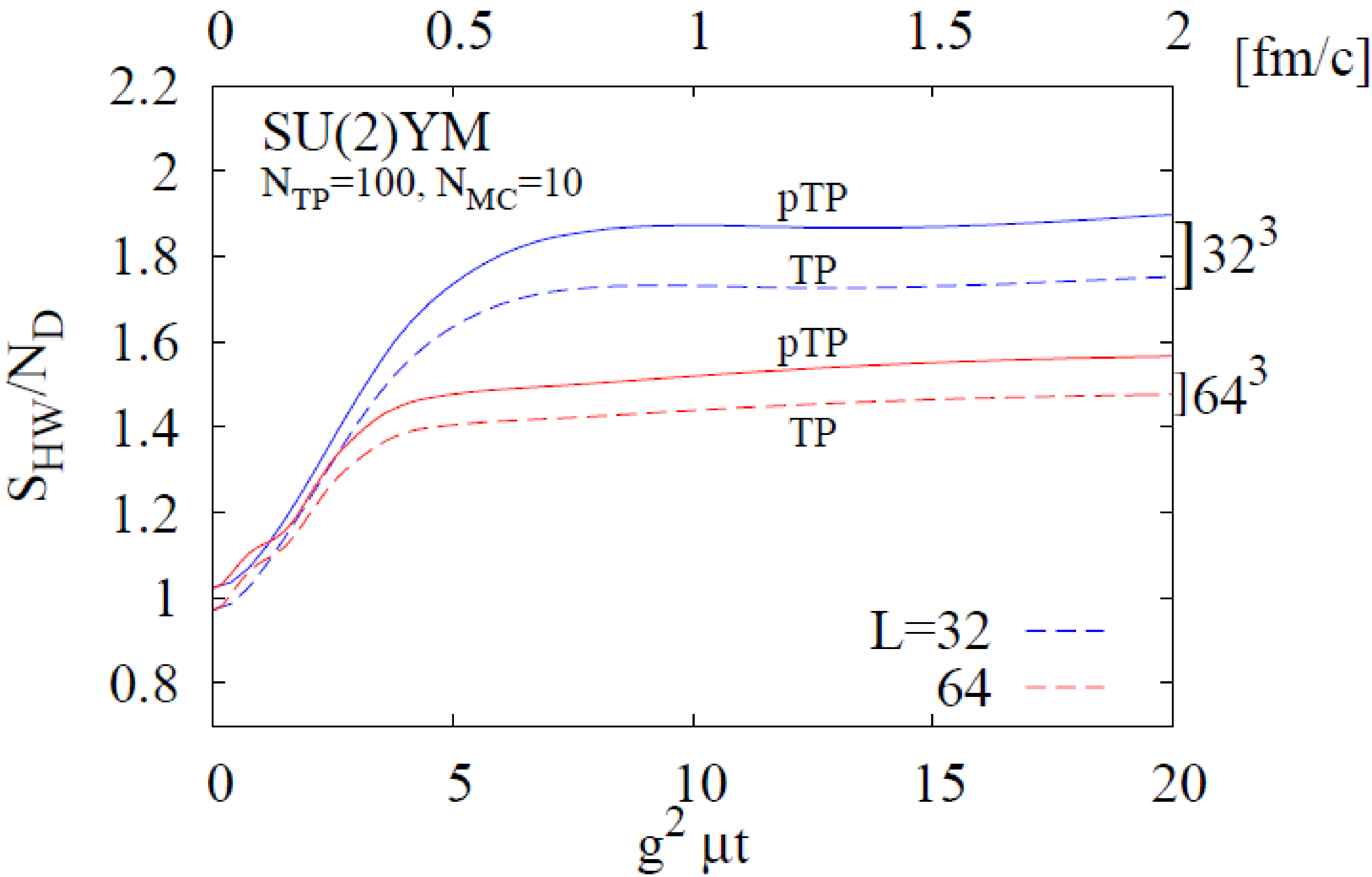
One event + Quantum fluctuation

$$f_W(\Gamma, t=0) = 2^{N_D}\exp[-\frac{\omega_L(A-A_{\mathrm{MV}})^2}{\hbar g^2} - \frac{(E-E_{\mathrm{MV}})^2}{\hbar g^2\omega_L}]$$

The ration of width of Gaussian $\omega_L = a\omega$ depends on lattice size.
We set $\omega = Q_s$.

HW entropy production

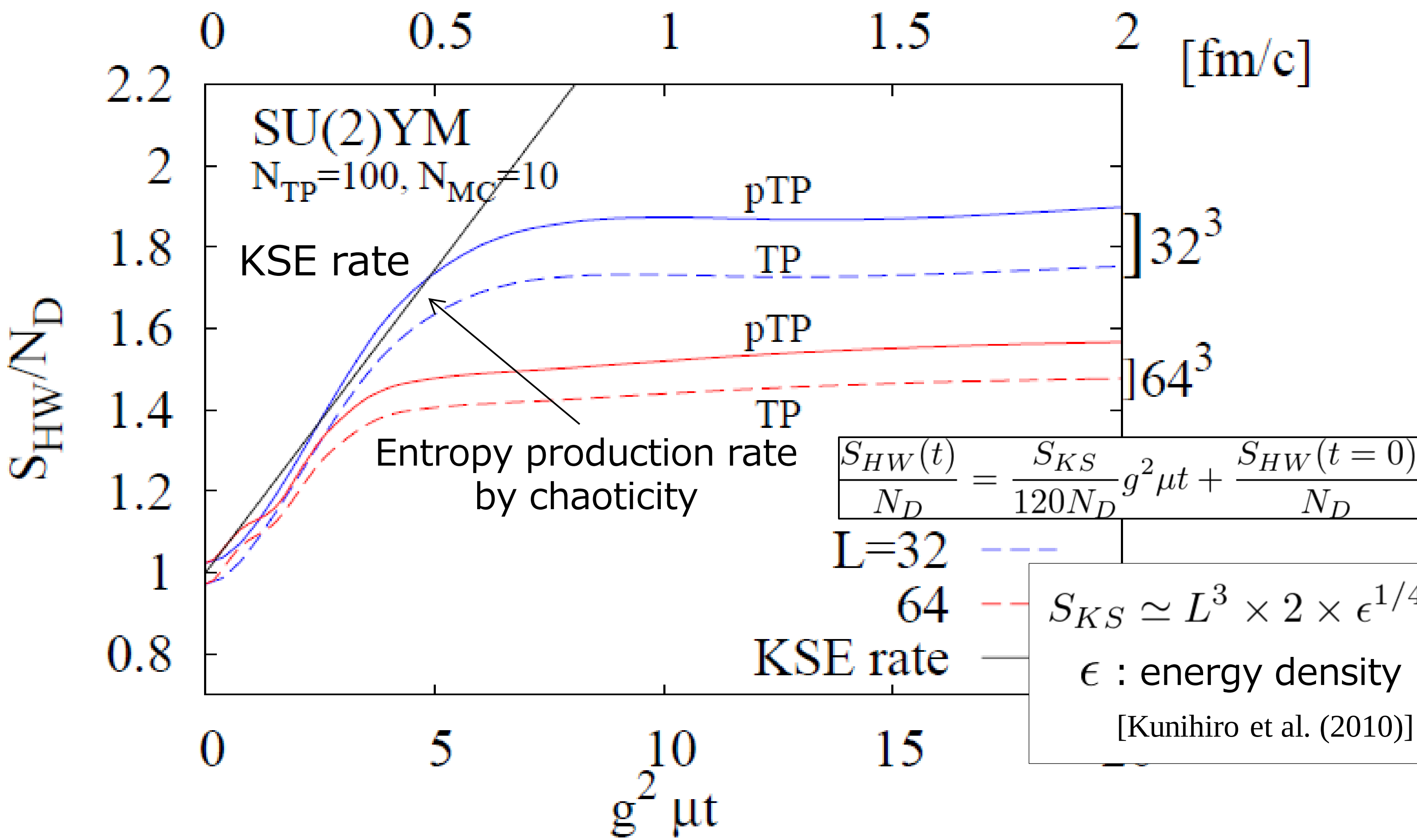
H.T., T.Kunihiro, A.Ohnishi, and T.T.Takahashi, in preparation.



The HW entropy is produced within 1[fm/c].

HW entropy production

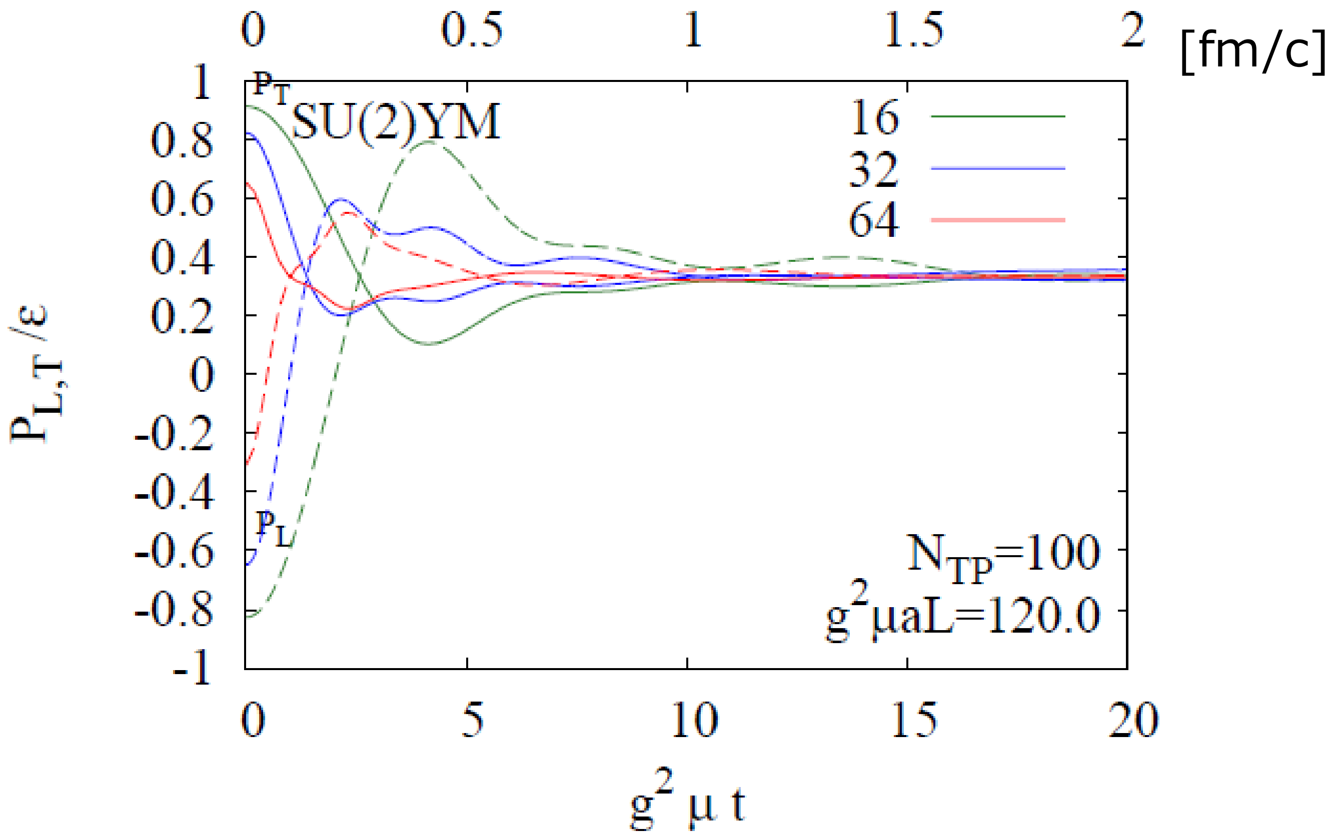
H.T., T.Kunihiro, A.Ohnishi, and T.T.Takahashi, in preparation.



The production rate of the HW entropy is characterized by Kolmogorov-Sinai entropy(KSE), which suggests the chaoticity plays an important role in the tharmalization.

Isotropization of pressure

H.T., T.Kunihiro, A.Ohnishi, and T.T.Takahashi, in preparation.



The isotropization occurs within 1 [fm/c] in $L = 32{,}64$.
This time scale is the almost same as that of the HW entropy production.

Plaquette energy distribution

H.T., T.Kunihiro, A.Ohnishi, and T.T.Takahashi, in preparation.

Plaquette energy

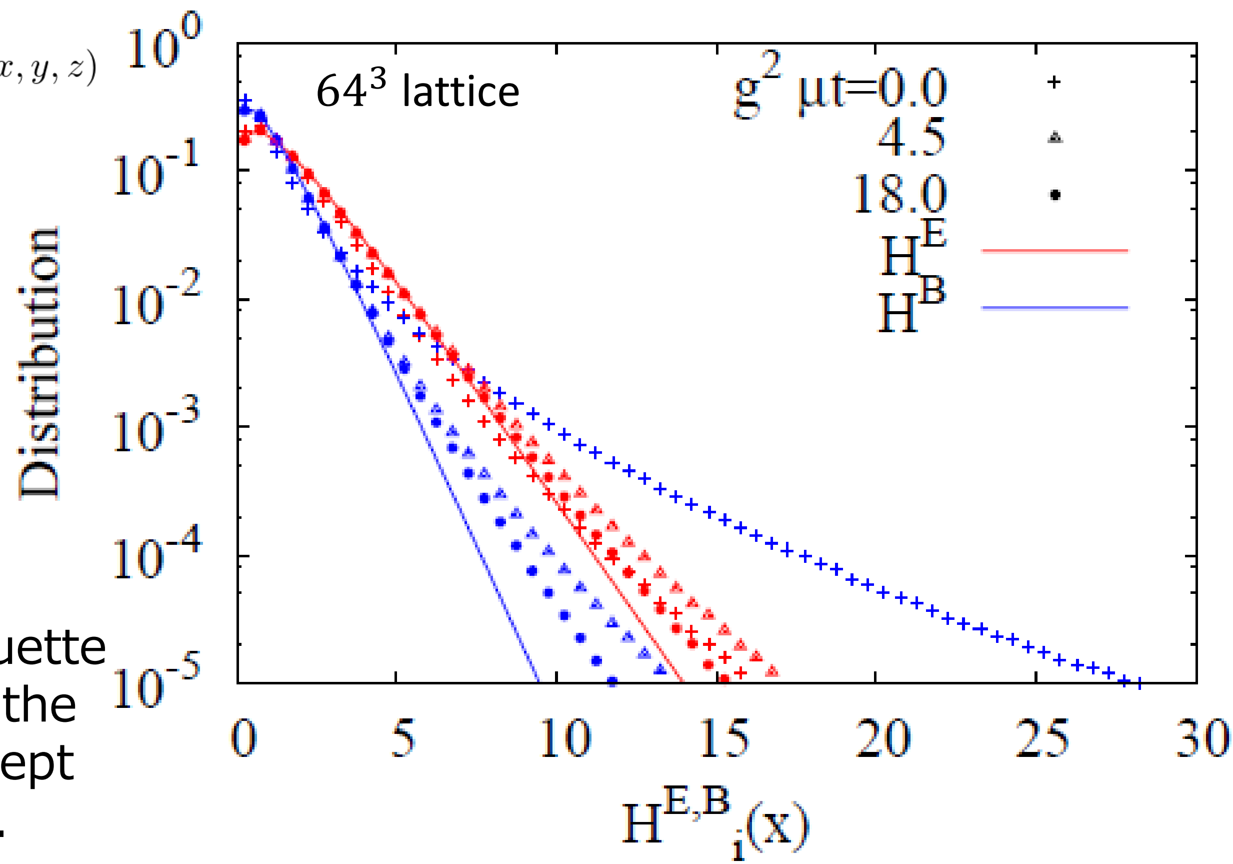
$$H_i^E(\mathbf{x}) = \frac{1}{2}\sum_{a=1}^{N_c^2-1}(E_i^a(\mathbf{x}))^2 \quad (i = x, y, z)$$

$$H_i^B(\mathbf{x}) = \frac{1}{2}\sum_{a=1}^{N_c^2-1}(B_i^a(\mathbf{x}))^2$$

Boltzmann distribution

(Solid lines)

$$\sqrt{H^{E,B}}\exp(-H^{E,B}/T)$$



Electro and magnetic plaquette energy distributions reach the Boltzmann distribution except for high momentum mode.

The electric and magnetic distribution have different temperatures, which suggests that the saturation of the HW entropy is related to the quasi-stationary state.

Summary

- We calculate Husimi-Wehrl (HW) entropy in Yang-Mills field with random initial condition and phenomenological initial condition given by McLerran-Venugopalan model.
- In the case of random initial condition, the production rate of the HW entropy agrees with the Kolmogorov-Sinai entropy.
- In the case of phenomenological initial condition, we show that the HW entropy is produced within 1 [fm/c], which suggests the early thermalization of the gluon fields.
- When the HW entropy saturates, the plaquette energy distribution reach the Boltzmann distribution.