Side-Jumps and Collisions in Chiral Kinetic Theory from Quantum Field Theories

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Phys.Rev. D95 (2017) no.9, 091901

arXiv:17XX…ongoing

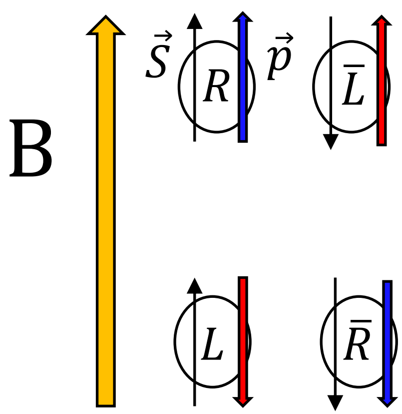
Anomalous Currents

- Normal currents : $\mathbf{J} = \sigma_e \mathbf{E}$
- Anomalous currents : $\mathbf{J} = \sigma_B \mathbf{B}?$ chiral magnetic effect (CME)

A simple picture : D.E. Kharzeev, L.D. McLerran, H.J. Warringa, NPA 803, 227

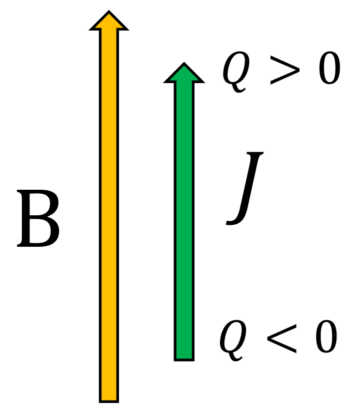
- Landau levels under strong B fields : $E_s = \pm\sqrt{p_z^2 \mp 2eB(n + 1/2 \mp s)}$

LLL : $E_s = \pm|p_z|$ (no degeneracy of spins)



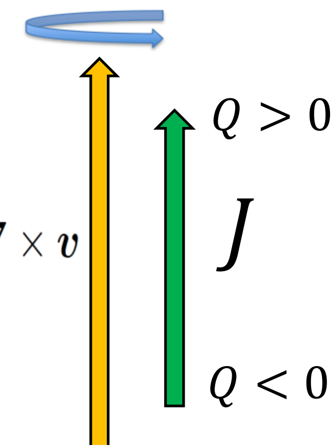
massless particles :
helicity $(\hat{p}\cdot\vec{S})$ is conserved

Chiral imbalance :
$n_R - n_L = n_5 > 0$



vorticity :

$\boldsymbol{\omega} = \frac{1}{2}\boldsymbol{\nabla}\times\boldsymbol{v}$



CME : $\mathbf{J} = \frac{e^2}{2\pi^2}\mu_5\mathbf{B}$

CVE : $\mathbf{J} = \frac{1}{\pi^2}\mu_5\mu\,\boldsymbol{\omega}$

$\mu = \frac{\mu_R + \mu_L}{2}, \quad \mu_5 = \frac{\mu_R - \mu_L}{2}$

Chiral anomaly & CME

- A simple argument for the relation to anomaly : K. Fukushima et.al.('08)

$$\mathbf{J} = \sigma_B \mathbf{B} \xrightarrow[\text{weak E}]{} \text{power : } \mu_5 \frac{dN_5}{dt} = \int_{\mathbf{x}} \mathbf{J}\cdot\mathbf{E} = \int_{\mathbf{x}} \sigma_B \mathbf{B}\cdot\mathbf{E}$$

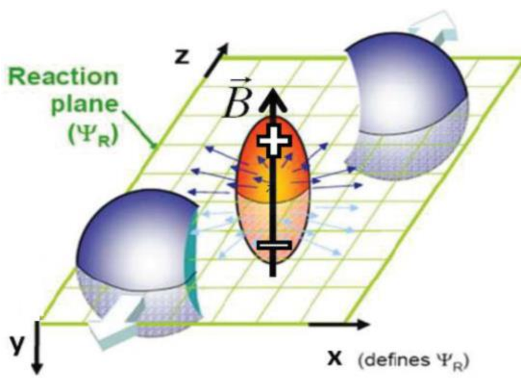
chiral anomaly : $\frac{dN_5}{dt} = \frac{e^2}{2\pi^2}\int_x \mathbf{B}\cdot\mathbf{E} \longrightarrow$ CME conductivity : $\sigma_B = \frac{e^2\mu_5}{2\pi^2}$

- CME(in equilibrium) was found from different approaches :
  - Kubo formula (perturbative calculations) in thermal equilibrium: for both strong & weak B
    K. Fukushima, D. E. Kharzeev, H. J. Warringa, PRD78, 074033
    D. E. Kharzeev and H. J. Warringa, Phys. Rev. D80, 034028 (2009)
  - Hydrodynamics : based on the 2nd law of thermodynamics
    D. T. Son and P. Surowka, Phys. Rev. Lett. 103, 191601 (2009)
    A. V. Sadofyev and M. V. Isachenkov, Phys. Lett. B697, 404 (2011)
  - AdS/CFT(QCD)
    J. Erdmenger, M. Haack, M. Kaminski, and A. Yarom, JHEP 01, 055 (2009)
    M. Torabian and H.-U. Yee, JHEP 08, 020 (2009)
- CME conductivity ($\sigma_B = e^2\mu_5/(2\pi^2)$) is independent of the coupling(associated with the chiral anomaly).

Observations of CME

- Observations of CME in the real world?

Heavy ion collisions ($m_q \ll T$):

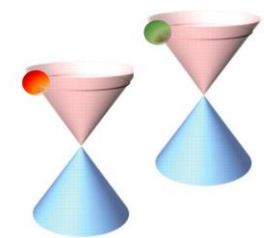


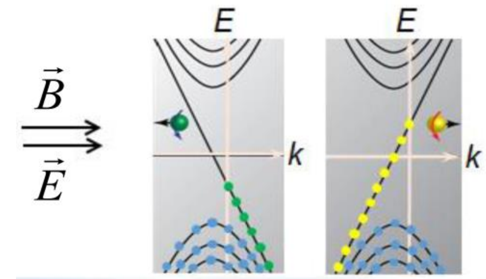
- strong B fields from collisions
- local n_5 from topological excitations :

$$Q = \frac{g^2}{32\pi^2}\int d^4x\, F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$$

- CME signal “might be measurable“ from 3-pt correlations
- strong background : under debate

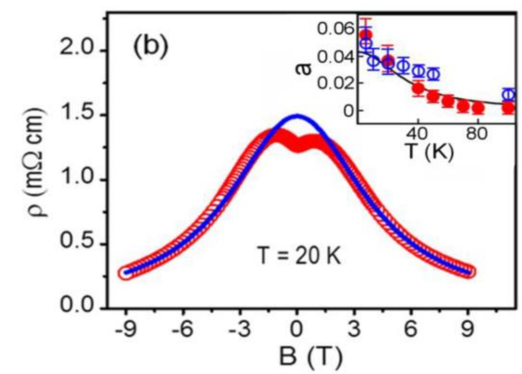
Weyl semimetal :





charge pumping via parallel E & B : generate μ_5

$\rho \sim 1/\sigma$



“negative magnetoresistance” (the signal of CME)

Qiang Li, et.al., Nature Phys. 12 (2016) 550-554

Kinetic theory with chiral anomaly

- The chiral kinetic theory (CKT) : to investigate anomalous transport in and "out of" equilibrium and to manifest the microscopic dynamics.
- Validity : rare collisions

D. T. Son and N. Yamamoto, Phys. Rev. Lett. 109,181602 (2012)
M. Stephanov and Y. Yin, Phys. Rev. Lett. 109, 162001 (2012)

- The semi-classical approach :

➢ Change of momentum triggers the rotation of spin and results in the Berry phase.

Berry phase $\mathcal{O}(\hbar)$

a_p: Berry connection (a fictitious gauge field)

$$\mathcal{I} = \int \left[(\mathbf{p} + \mathbf{A}) \cdot d\mathbf{x} - (\epsilon_{\mathbf{p}} + A_0) dt - \boxed{\mathbf{a_p} \cdot d\mathbf{p}} \right]$$

$$a_+^\mu = i c_+^\dagger \frac{\partial}{\partial p_\mu} c_+$$

energy shift : $\epsilon_{\mathbf{p}} = |\mathbf{p}| - \boxed{\frac{\mathbf{B} \cdot \mathbf{p}}{2|\mathbf{p}|^3}}$

Magnetic moment coupling

Berry curvature : $\boxed{\mathbf{\Omega_p} = \nabla_{\mathbf{p}} \times \mathbf{a_p} = \frac{\mathbf{p}}{2|\mathbf{p}|^3}}$

Monopole as source/sink of the particle number current.

($|p| \leq \sqrt{B}$: quantum region)

➢ EOM :

$$\begin{aligned} \sqrt{\omega}\dot{\mathbf{x}} &= \hat{\mathbf{p}} + (\mathbf{E} \times \mathbf{\Omega_p}) + \mathbf{B}(\hat{\mathbf{p}} \cdot \mathbf{\Omega_p}), \quad \sqrt{\omega} = 1 + \mathbf{B} \cdot \mathbf{\Omega_p}, \\ \sqrt{\omega}\dot{\mathbf{p}} &= \mathbf{E} + (\hat{\mathbf{p}} \times \mathbf{B}) + \mathbf{\Omega_p}(\mathbf{E} \cdot \mathbf{B}). \end{aligned}$$

➢ Change of phase space : $\mathrm{d}\Gamma = \sqrt{\omega}\, \mathrm{d}\xi = (1 + \mathbf{B} \cdot \mathbf{\Omega_p}) \frac{\mathrm{d}\mathbf{p}\, \mathrm{d}\mathbf{x}}{(2\pi)^3}$

Kinetic theory with chiral anomaly

- Modified distribution functions : $\rho = \sqrt{\omega} f$
- CKT : $\dot{\rho} + \partial_a(\dot{\xi}^a \rho) = 0$

$$\Rightarrow \left[(1 + \hbar \mathbf{B} \cdot \mathbf{\Omega_q})\, \partial_t + (\tilde{\mathbf{v}} + \hbar \mathbf{E} \times \mathbf{\Omega_q} + \hbar (\tilde{\mathbf{v}} \cdot \mathbf{\Omega_q}) \mathbf{B}) \cdot \mathbf{\nabla} + \left(\tilde{\mathbf{E}} + \tilde{\mathbf{v}} \times \mathbf{B} + \hbar (\tilde{\mathbf{E}} \cdot \mathbf{B}) \mathbf{\Omega_q} \right) \cdot \frac{\partial}{\partial \mathbf{q}} \right] f = 0$$

$$\tilde{\mathbf{v}} = \partial \epsilon_{\mathbf{p}} / \partial \mathbf{p}$$

D. T. Son and N. Yamamoto, Phys. Rev. D87, 085016 (2013)

$$\tilde{\mathbf{E}} = \mathbf{E} - \partial \epsilon_{\mathbf{p}} / \partial \mathbf{x}$$

- Number density and current : $J^0 = \int \frac{d^3 q}{(2\pi)^3} (1 + \hbar \mathbf{B} \cdot \mathbf{\Omega_q})\, f$

$$\mathbf{J} = \int \frac{d^3 q}{(2\pi)^3} \Big(\tilde{\mathbf{v}} + \underbrace{\boxed{\hbar \mathbf{E} \times \mathbf{\Omega_q}}}_{\text{AHE}} - \underbrace{\boxed{\hbar \epsilon_{\mathbf{q}} \mathbf{B}\, \mathbf{\Omega_q} \cdot \frac{\partial}{\partial \mathbf{q}}}}_{\text{CME}} - \underbrace{\boxed{\hbar \epsilon_{\mathbf{q}} \mathbf{\Omega_q} \times \nabla}}_{\text{magnetization current}} \Big) f$$

- The magnetization current is associated with CVE.

J.-Y. Chen, et.al. Phys. Rev. Lett. 113, 182302 (2014)

- Derivation from QFT is desired : systematic inclusion of collisions
- Previous studies from QFT (Wigner-function approach) are subject to either near equilibrium or predominant chemical potentials.

J.-W. Chen, et.al. Phys. Rev. Lett. 110, 262301 (2013) D. T. Son and N. Yamamoto, Phys. Rev. D87, 085016 (2013)

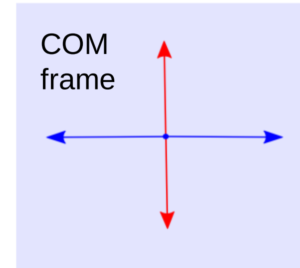
Puzzles

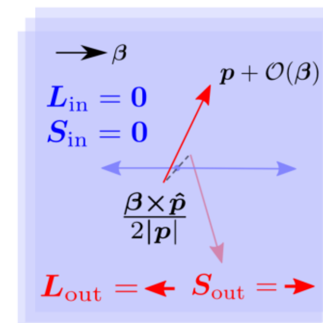
- Lorentz invariance (L.I.) : J.-Y. Chen, et.al. Phys. Rev. Lett. 113, 182302 (2014)

LT of the action : $\delta_{\boldsymbol{\beta}}\mathcal{I} = \int \left[\frac{\boldsymbol{\beta}\times\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|}(\dot{\boldsymbol{p}} - \boldsymbol{E} - \hat{\boldsymbol{p}}\times\boldsymbol{B}) + \frac{\boldsymbol{B}\cdot\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|}\boldsymbol{\beta}\cdot(\dot{\boldsymbol{x}} - \hat{\boldsymbol{p}})\right] dt$

⇨ Modified L.T. (side-jumps) :

$$\delta'_{\boldsymbol{\beta}}\boldsymbol{x} = \boldsymbol{\beta}t + \frac{\boldsymbol{\beta}\times\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|};\quad \delta'_{\boldsymbol{\beta}}\boldsymbol{p} = \boldsymbol{\beta}\mathcal{E} + \frac{\boldsymbol{\beta}\times\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|}\times\boldsymbol{B};\qquad \delta t = \boldsymbol{\beta}\cdot\boldsymbol{x}.$$





- Finite Lorentz transformation : frame transformation

J.-Y. Chen, D. T. Son, and M. A. Stephanov, Phys. Rev. Lett. 115, 021601 (2015)

$$\begin{aligned} X'^{\mu} &= \Lambda^{\mu}{}_{\nu}X^{\nu} \\ p'^{\mu} &= \Lambda^{\mu}{}_{\nu}p^{\nu} \end{aligned} \longleftrightarrow n'^{\mu} = (\Lambda^{-1})^{\mu}{}_{\nu}\, n^{\nu}$$

- Collisions "without" background fields : (cons. of total L)

$$f' - f = -\Delta\cdot\partial f + \int_{BCD} C_{ABCD}\frac{\Delta\cdot\bar{n}}{p\cdot\bar{n}}.$$

$$j^{\mu} = \underbrace{p^{\mu}f}_{\text{normal current}} + \underbrace{S^{\mu\nu}\partial_{\nu}f}_{\text{magnetization current}} + \underbrace{\int_{BCD} C_{ABCD}\,\bar{\Delta}^{\mu}}_{\text{jump current}}$$

side-jumps for frame transf. $\Delta^{\mu}_{nn'} = -\frac{S^{\mu\nu}_{n'}n_{\nu}}{p\cdot n} = \lambda\frac{\epsilon^{\mu\alpha\beta\gamma}p_{\alpha}n_{\beta}n'_{\gamma}}{(p\cdot n)(p\cdot n')}$

"The distribution function is not a scalar (frame-dependent) and the current is modified."

- Previous studies in field theories are unable to address the issues about Lorentz symmetry of non-equilibrium CKT with collisions.

What we want to resolve

- To derive the CKT for more general conditions from quantum field theories.
- To realize the side-jumps and the modified Lorentz transformation from the field-theory point of view.
- To systematically incorporate collisions from field theories and investigate the influence on side-jumps.

Theoretical setup

- We consider only “right-handed Weyl fermions” under U(1) background fields.
- Wigner functions : less (greater) propagators under Wigner transformation.

$$S^>(x,y) = \langle \psi(x) \mathcal{P}\mathcal{U}^\dagger(A_\mu, x, y) \psi^\dagger(y) \rangle$$
$$S^<(x,y) = \langle \psi^\dagger(y) \boxed{\mathcal{P}\mathcal{U}(A_\mu, x, y)} \psi(x) \rangle$$

gauge link

$$\Longrightarrow \quad \grave{S}^{<(>)}(q,X) = \int d^4Y e^{\frac{iq\cdot Y}{\hbar}} S^{<(>)}\left(X + \frac{Y}{2}, X - \frac{Y}{2}\right)$$

$X = \frac{x+y}{2}$, $Y = x - y$ $\quad$ q is canonical momentum

H. T. Elze, M. Gyulassy, and D. Vasak, Nucl. Phys. B276, 706 (1986)

- Without $\mathcal{O}(\hbar)$ corrections :

fermions & anti-fermions

$$\grave{S}^>(q,X) = 2\pi\left(\theta(q^0) - \theta(-q^0)\right)\left(1 - f(q,X)\right) q^\mu \bar{\sigma}_\mu \delta(q^2)$$
$$\grave{S}^<(q,X) = 2\pi\left(\theta(q^0) - \theta(-q^0)\right) \boxed{f(q,X)} \boxed{q^\mu \bar{\sigma}_\mu \delta(q^2)}$$

distribution function $\quad$ spectral density

- Wigner functions are always covariant :

four-current : $J^\mu = \int \frac{d^4q}{(2\pi)^4} \mathrm{tr}\left(\sigma^\mu \grave{S}^<\right)$

- Dirac equations (collisionless):

$$\sigma^\mu\left(\hbar\Delta_\mu - 2iq_\mu\right)\grave{S}^< = 0, \quad \left(\hbar\Delta_\mu + 2iq_\mu\right)\grave{S}^<\sigma^\mu = 0,$$

$$\Delta_\mu = \partial_\mu + F_{\nu\mu}\partial/\partial q_\nu$$
$$\partial_\mu = \partial/\partial X^\mu$$

- Solving Dirac equations perturbatively up to $\mathcal{O}(\hbar)$

(equivalent to gradient expansion) $\quad (\partial_X/q \sim \hbar \ll 1)$

- Caveat : the perturbative solution is subject to weak fields or large momenta.

(strong fields : solve Dirac eq. non-perturbatively. e.g. Landau levels with large B)

Perturbative solutions

- Trace and traceless parts of Dirac equations (collisionless) : $\grave{S}^{<} = \bar{\sigma}^{\mu}\grave{S}^{<}_{\mu}$,

$$\Delta_\mu \grave{S}^{\mu<} = 0, \quad q^\mu \grave{S}^{<}_{\mu} = 0,$$

kinetic theory

dispersion relation

$$\Delta_\mu = \partial_\mu + F_{\rho\mu}\partial^\rho_q$$

$$\boxed{\hbar\Delta_{[i}\grave{S}^{<}_{0]} - 2\epsilon^{ijk}q_j\grave{S}^{<}_{k} = 0, \quad \hbar\epsilon^{ijk}\Delta_j\grave{S}^{<}_{k} + 2q_{[i}\grave{S}^{<}_{0]} = 0.}$$

The traceless part (linear to σ^i) leads to side-jumps

- Perturbative solutions up to $\mathcal{O}(\hbar)$

$$\grave{S}^{<\mu}(q) = 2\pi\left[\boxed{\delta(q^2)q^\mu f} + \hbar\delta(q^2)\delta^{\mu i}\epsilon^{ijk}\Delta_k\frac{q_j}{2q_0}f + \boxed{\hbar\epsilon^{\mu\nu\alpha\beta}q_\nu F_{\alpha\beta}\frac{\partial\delta(q^2)}{2\partial q^2}f}\right]$$

L.O. sol.

Modification from E & B :
e.g. energy shift by B fields

- the side-jump term: $\boxed{\delta\grave{S}^{f<}_{\mu} = 2\pi\delta_{\mu i}\epsilon_{ijk}\frac{\delta(q^2)q_j}{2q_0}\Delta_k f}$

(implies side-jumps in f)
Not cov. (frame dep.)

- This solution is not unique : we can add arbitrary corrections $\delta\grave{S}^{<}_{\mu} \propto q_\mu\delta(q^2)$.

(related to side-jumps and non-scalar f)

(Re-)Derivation of CKT

- Deriving CKT from $\Delta_\mu \grave{S}^{\mu <} = 0$:

$$\delta\left(q^2 + \hbar\frac{\mathbf{B}\cdot\mathbf{q}}{q_0}\right)\left[q^\mu \Delta_\mu - \frac{\hbar\partial_k(\mathbf{B}\cdot\mathbf{q})}{2q_0}\frac{\partial}{\partial q_k} + \frac{\hbar\epsilon^{ijk}E_i q_j}{2q_0^2}\Delta_k\right] f = 0$$

the shift of energy : $q_0 = \epsilon_{\mathbf{q}} = |\mathbf{q}|\left(1 - \frac{\hbar\mathbf{B}\cdot\mathbf{q}}{2|\mathbf{q}|^3}\right)$ for positive energy

  - The q derivatives of the side-jump term also give $d\delta(q^2)/dq^2$ terms.
  - We employ the mathematical trick $\frac{q^2 d\delta(q^2)}{dq^2} = -\delta(q^2)$.

- The full CKT is reproduced :

$$\left[(1+\hbar\mathbf{B}\cdot\boldsymbol{\Omega}_{\mathbf{q}})\,\partial_t + (\tilde{\mathbf{v}} + \hbar\mathbf{E}\times\boldsymbol{\Omega}_{\mathbf{q}} + \hbar(\tilde{\mathbf{v}}\cdot\boldsymbol{\Omega}_{\mathbf{q}})\mathbf{B})\cdot\boldsymbol{\nabla} + \left(\tilde{\mathbf{E}} + \tilde{\mathbf{v}}\times\mathbf{B} + \hbar(\tilde{\mathbf{E}}\cdot\mathbf{B})\boldsymbol{\Omega}_{\mathbf{q}}\right)\cdot\frac{\partial}{\partial\mathbf{q}}\right] f = 0$$

- Number density and current : $J^0 = \int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}\left(\grave{S}^<\right) = \int\frac{d^3q}{(2\pi)^3}\left(1+\hbar\mathbf{B}\cdot\boldsymbol{\Omega}_{\mathbf{q}}\right) f.$

$$\mathbf{J} = \int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}\left(\sigma\grave{S}^<\right) = \int\frac{d^3q}{(2\pi)^3}\left(\tilde{\mathbf{v}} + \hbar\mathbf{E}\times\boldsymbol{\Omega}_{\mathbf{q}} - \hbar\epsilon_{\mathbf{q}}\mathbf{B}\,\boldsymbol{\Omega}_{\mathbf{q}}\cdot\frac{\partial}{\partial\mathbf{q}} - \hbar\epsilon_{\mathbf{q}}\boldsymbol{\Omega}_{\mathbf{q}}\times\nabla\right) f$$

(consistent with the results in D. T. Son and N. Yamamoto, Phys. Rev. D87, 085016 (2013))

CME with a dynamical B field

- Reproduce the results in D. Kharzeev, M. Stephanov, and H.-U. Yee, Phys. Rev. D 95, 051901 (2017) D. Satow and H. U. Yee, Phys. Rev. D 90, 014027 (2014).
- CKT from field theories :

R-handed fermions :

$$\Big\{q^0\partial_0+\Big[\mathbf{q}-\frac{\lambda\hbar}{2q_0^2}(\mathbf{q}\times\mathbf{E})\Big]\cdot\nabla+\lambda\Big[(\mathbf{q}\times\mathbf{B})+q_0\mathbf{E}-\frac{\hbar}{2q_0^2}\left((\mathbf{q}\cdot B)\mathbf{E}-(\mathbf{E}\cdot\mathbf{B})\mathbf{q}\right)\Big]\cdot\nabla_{\mathbf{q}}$$
$$+\frac{\lambda\hbar}{2q_0}(\mathbf{q}\cdot\nabla\mathbf{B})\cdot\nabla_{\mathbf{q}}+\lambda\Big[\mathbf{q}\cdot\mathbf{E}+\hbar\left(\frac{\mathbf{q}\cdot(\nabla\times\mathbf{E})}{2q_0}\right)\Big]\partial_{q0}\Big\}\grave{f}_q=-(\grave{f}_q-f_0)\frac{q^0}{\tau_R}.$$

time-dep. B field :

$$k^\mu=(\omega,\mathbf{k})$$
$$\omega\mathbf{B}=\mathbf{k}\times\mathbf{E}$$

- The perturbative solution : $\grave{f}_q=\grave{f}_0(q_0)+\boxed{\lambda\delta\grave{f}(q,X)}+\boxed{\lambda\hbar\delta^2\grave{f}(q,X)}$

(neglect λ^2 terms) weak fields quantum corrections

$$f_0(q_0)=\frac{1}{e^{\beta(q_0-\mu)}+1}$$

$$\Rightarrow\quad \grave{f}_q=f_0(q_0)+\lambda\left(q\cdot k+i\frac{q_0}{\tau_R}\right)^{-1}\left(-i\mathbf{q}\cdot\mathbf{E}+\hbar\left(\frac{\mathbf{q}\cdot(\mathbf{k}\times\mathbf{E})}{2q_0}\right)\right)\partial_{q0}f_0$$

- The current from particles: $J^i_+=\int\frac{d^4q}{(2\pi)^4}2\grave{S}^{<i}_+(q,X)$

magnetization current

$$J^i_+=\int\frac{d^4q}{(2\pi)^3}2\delta(q^2)\theta(q_0)\Big\{q^i\left(f_0+\lambda\delta\grave{f}+\lambda\hbar\delta^2\grave{f}\right)+\frac{\lambda\hbar}{2q_0}\Big[\left((\mathbf{q}\times\mathbf{E})^i-q_0B^i\right)\partial_{q0}f_0\boxed{-(\mathbf{q}\times\nabla)^i\delta\grave{f}}\Big]\Big\}$$

- The full CME current :

$$J^i_{\rm CME}(\omega)=\delta\left(J^i_++J^i_-\right)=-\frac{\lambda\hbar}{2}\int\frac{d^3\mathbf{q}}{(2\pi)^3|\mathbf{q}|}\left[1-\frac{2\omega}{3(\omega+i\tau_R^{-1})}\right]B^i\left(\partial_{|\mathbf{q}|}f_+(|\mathbf{q}|)-\partial_{|\mathbf{q}|}f_-(|\mathbf{q}|)\right)$$

$$\Rightarrow\quad\begin{cases}J^i_{\rm CME}(\omega)\to J^i_{\rm sCME} & \text{for } \tau_R\omega\to 0\\ J^i_{\rm CME}(\omega)\to \frac{1}{3}J^i_{\rm sCME} & \text{for } \tau_R\omega\to\infty\end{cases}$$

Equilibrium CME :

$$J^i_{\rm sCME}=\frac{\lambda\hbar}{4\pi^2}\mu B^i$$

Covariant currents and side-jumps

- Revisiting the Wigner functions :

$$\grave{S}^{<\mu} = 2\pi\left[\delta(q^2)\left(q^\mu + \boxed{\hbar\delta^{\mu i}\epsilon^{ijk}\frac{q_j}{2q_0}\Delta_k}\right) + \hbar\epsilon^{\mu\nu\alpha\beta}q_\nu F_{\alpha\beta}\frac{\partial\delta(q^2)}{2\partial q^2}\right]\boxed{f(q,X)}$$

frame dep.

- Introducing a frame u^μ: (the original expression is in $u^\mu = (1,\mathbf{0})$)

$$\delta\grave{S}^{f<}_\mu = 2\pi\delta(q^2)\epsilon_{\mu\nu\alpha\beta}\frac{q^\alpha u^\beta}{2q\cdot u}\Delta^\nu f^{(u)}$$

$$\begin{aligned} X'^\mu &= \Lambda^\mu{}_\nu X^\nu \\ q'^\mu &= \Lambda^\mu{}_\nu q^\nu \end{aligned} \longleftrightarrow u'^\mu = (\Lambda^{-1})^\mu{}_\nu u^\nu$$

- Wigner functions (currents) are covariant : $(\Lambda^{-1})_\mu{}^\nu \grave{S}'^<_\nu - \grave{S}^<_\mu = 0$

taking $f'^{(u)}(q',X') = f^{(u)}(q,X) + \hbar\delta f^{(u)}(q,X)$

$$(\Lambda^{-1})_\mu{}^\nu \grave{S}'^<_\nu(q',X') - \grave{S}^<_\mu(q,X) = \hbar 2\pi\delta(q^2)\left(q_\mu\delta f^{(u)} + \epsilon_{\mu\nu\alpha\beta}\left(\frac{q^\alpha u'^\beta}{2q\cdot u'} - \frac{q^\alpha u^\beta}{2q\cdot u}\right)\Delta^\nu f^{(u)}\right)$$

- Modified Lorentz transformation :

$$f'^{(u)}(q',X') = f^{(u)}(q,X) + \hbar N^\mu_{uu'}\Delta_\mu f^{(u)}(q,X)\,, \quad N^\nu_{uu'} = -\frac{\epsilon^{\mu\nu\alpha\beta}q_\alpha u'_\beta u_\mu}{2(q\cdot u')(u\cdot q)}.$$

- Side-jumps : $X^\mu \to X^\mu + \hbar N^\mu_{uu'}$ $\quad q_\mu \to q_\mu + \hbar N^\nu_{uu'}F_{\mu\nu}$

(consistent with the infinitesimal L.T. in J.-Y. Chen, et.al. Phys. Rev. Lett. 113, 182302 (2014))

Origin of side-jumps

- Considering the free case (no background fields).
- Nontrivial phase for massless particles with helicity: $|p,\lambda\rangle \rightarrow e^{-i\Phi(p,\Lambda)}|\Lambda p,\lambda\rangle$

S. Weinberg, The Quantum Theory of Fields, Volume I

- The wave function of a particle with positive energy :

L. T. : $v_+(\Lambda p) = e^{i\Phi(p,\Lambda)}U(\Lambda)v_+(p), \quad v_+(p) = \begin{pmatrix} \sqrt{|\mathbf{p}|+p^3} \\ \frac{p^1+ip^2}{\sqrt{(|\mathbf{p}|+p^3)}} \end{pmatrix}$

- Second quantization : $\psi(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2|\mathbf{p}|}} e^{-ip\cdot x} v_+(p) a_{\mathbf{p}}$ (neglect anti-fermions)

$$\grave{S}^{<}(x,y) = \langle \psi^\dagger(y)\psi(x)\rangle = \int \frac{d^3p}{(2\pi)^3\sqrt{2|\mathbf{p}|}} \int \frac{d^3p'}{(2\pi)^3\sqrt{2|\mathbf{p}'|}} v_+(p)v_+^\dagger(p')\langle a^\dagger_{\mathbf{p}'}a_{\mathbf{p}}\rangle e^{i(p'-p)\cdot X - \frac{i}{2}(p'+p)\cdot Y}$$

nontrivial phase

- Under the L.T. : $N(p',p) \equiv \langle a^\dagger_{\mathbf{p}'}a_{\mathbf{p}}\rangle \rightarrow e^{-i\left(\boxed{\Phi(\Lambda,p)-\Phi(\Lambda,p')}\right)}\langle a^\dagger_{\mathbf{p}'}a_{\mathbf{p}}\rangle$
- Could we define a scalar distribution function?
- Introduce a phase field : $\phi(p) \rightarrow \phi'(\Lambda p) = \phi(p) - \Phi(p,\Lambda)$
- Reparametrize the wave function and annihilation operator :

$$v_+(p) \rightarrow e^{i\phi(p)}v_+(p)$$
$$a_{\mathbf{p}} \rightarrow e^{-i\phi(p)}a_{\mathbf{p}}$$

Manifestation of Lorentz symmetry

- From $a_{\mathbf{p}} \to e^{-i\phi(p)} a_{\mathbf{p}}$, we may define a scalar distribution function :

scalar non-scalar

$$\boxed{\check{N}(p', p)} \equiv e^{-i\left(\phi(p) - \phi(p')\right)} \boxed{N(p', p)} \quad \Rightarrow \quad \check{f}(q, X) \equiv \int \frac{d^3\bar{p}}{(2\pi)^3} \check{N}\left(q - \frac{\bar{p}}{2}, q + \frac{\bar{p}}{2}\right) e^{-i\bar{p}\cdot X}$$

$$N(p', p) \equiv \langle a^\dagger_{\mathbf{p}'} a_{\mathbf{p}} \rangle$$

- From $v_+(p) \to e^{i\phi(p)} v_+(p)$, the spectral density should be covariant :

$$\grave{S}^{<}_{\mu}(q, X) = (2\pi)\theta(q^0)\delta(q^2) \boxed{\left(q_\mu \left(1 - \hbar(\partial^\nu_q \phi - a^\nu)\partial_\nu\right) + \hbar\delta_{\mu i}\epsilon_{ijk} \frac{q_j}{2|\mathbf{q}|}\partial_k\right)} \check{f}(q, X) \,,$$

covariant

- Compare to the previous expression :

$$\grave{S}^{<\mu} = 2\pi\theta(q^0)\delta(q^2) \left(q^\mu + \hbar\delta^{\mu i}\epsilon^{ijk} \frac{q_j}{2|\mathbf{q}|}\partial_k\right) f(q, X)$$

"the origin of side-jumps"

$$\Rightarrow f(q, X) = \check{f}\left(q_\mu, X^\mu \boxed{- \hbar\partial^\mu_q \phi(q) + \hbar a^\mu}\right)$$

non-scalar

- Choices of phase field corresponds to the gauge degrees of freedom for the Berry connection.
- The perturbative solution could be uniquely determined by Lorentz symmetry.

Collisions

- Collisions in terms of self-energy :

J.-P. Blaizot and E. Iancu, Phys. Rept. 359, 355 (2002)

$$\left(i\not{D}_x - \Sigma^\delta(x)\right) S^<(x,y)$$
$$= \int_{-\infty}^{\infty} d^4z \left(\Sigma_R(x,z)S^<(z,y) - \Sigma^<(x,z)S_A(z,y)\right)$$
$$S^<(x,y)\left(-i\not{D}_y^\dagger - \Sigma^\delta(y)\right)$$
$$= \int_{-\infty}^{\infty} d^4z \left(S^<(x,z)\Sigma_A(z,y) - S_R(x,z)\Sigma^<(z,y)\right)$$

- Trace and traceless parts of Dirac equations:

Assuming $\Sigma^\delta = \mathrm{Re}[\Sigma^{R/A}] = 0$

$$\Delta_\mu \grave{S}^{<\mu} = \Sigma^<_\mu \grave{S}^{>\mu} - \Sigma^>_\mu \grave{S}^{<\mu}, \quad q_\mu \grave{S}^{<\mu} = 0,$$
$$\hbar\Delta_{[i}\grave{S}^<_{0]} - 2\epsilon^{ijk} q_j \grave{S}^<_k = \hbar\left(\Sigma^<_{[i}\grave{S}^>_{0]} - \Sigma^>_{[i}\grave{S}^<_{0]}\right),$$
$$\hbar\epsilon^{ijk}\Delta_j \grave{S}^<_k + 2q_{[i}\grave{S}^<_{0]} = \hbar\epsilon^{ijk}\left(\Sigma^<_j\grave{S}^>_k - \Sigma^>_j\grave{S}^<_k\right).$$

- The perturbative solution : only the side-jump term is modified

$$\delta\grave{S}^{f<}_\mu = (2\pi)\delta_{\mu i}\epsilon_{ijk}\delta(q^2)\frac{q_j}{2q_0}\left(\Delta_k f - C_k\right), \qquad C_\beta[f] = \Sigma^<_\beta \bar{f} - \Sigma^>_\beta f$$

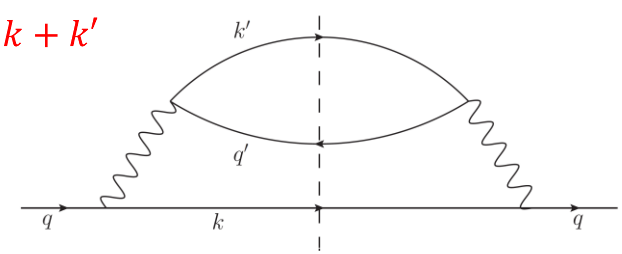
the jump current

Side-jumps with collisions

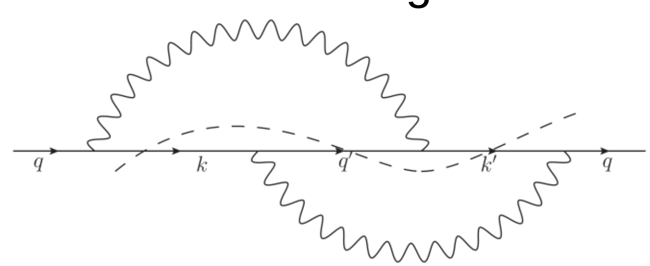
- Introducing a frame : $\delta\grave{S}^{f<}_{\mu}=2\pi\delta(q^2)\epsilon_{\mu\alpha\beta\nu}\frac{q^{\alpha}u^{\beta}}{2q\cdot u}\left(\Delta^{\nu}f^{(u)}-C^{\nu}[f^{(u)}]\right)$
- Side-jumps : $f'^{(u)}=f^{(u)}+\hbar N^{\mu}_{uu'}\left(\Delta_{\mu}f^{(u)}-C_{\mu}[f^{(u)}]\right)$
- Full CKT : $$\mathrm{CKT}_0-(1+\hbar\mathbf{B}\cdot\mathbf{\Omega_q})\,C_0$$ $$+(\tilde{\mathbf{v}}+\hbar\mathbf{E}\times\mathbf{\Omega_q}+\hbar(\tilde{\mathbf{v}}\cdot\mathbf{\Omega_q})\mathbf{B})\cdot\mathbf{C}$$ $$-\hbar\epsilon_{\mathbf{q}}\mathbf{\Omega_q}\cdot\left(\bar{f}(\mathbf{\Delta}^{>}\times\mathbf{\Sigma}^{<})-f(\mathbf{\Delta}^{<}\times\mathbf{\Sigma}^{>})\right)=0,\quad \mathbf{\Delta}^{<(>)}=\mathbf{\Delta}+\mathbf{\Sigma}^{<(>)}$$
- Further approximations for Σ are needed in practice.

- A simple example : the leading-order 2-2 Coulomb scattering between right-handed fermions with positive energy in the absence of background fields.

$q+q'\to k+k'$



t & *u* channels



interference

crossing symmetry : $(q,k,q',k')\to(q,-q',-k,k')$ for *s* & *u* channels

The no-jump frame in 2-2 scattering

- Introducing a frame : $\tilde{S}^{<}_{\mu} = 2\pi\delta(q^2)\Big[q_\mu f - \frac{\hbar}{2q\cdot u}\epsilon_{\mu\nu\alpha\beta}u^\nu q^\alpha\Big(\partial^\beta f + \Sigma^{>\beta} f - \Sigma^{<\beta}\bar{f}\Big)\Big]$

- Conservation of the angular momentum : COM frame = no-jump frame

  J.-Y. Chen, D. T. Son, and M. A. Stephanov, Phys. Rev. Lett. 115, 021601 (2015)

- Choosing the COM frame : $\boxed{u^\mu_c = (q+q')^\mu/\sqrt{s}}$

$$\mathcal{P}(q',k,k') = 4e^4\left(\frac{1}{(q-k)^2} + \frac{1}{(q-k')^2}\right)^2,$$

$$\Sigma^{<}_{\mu} = \int_{q',k,k'} \mathcal{P}(q',k,k')\tilde{S}^{>}_{\mu}(q')\big(\tilde{S}^{<}(k)\cdot\tilde{S}^{<}(k')\big),$$

$$\int_{\mathbf{q}',\mathbf{k},\mathbf{k}'} = \int \frac{d^3\mathbf{q}'d^3\mathbf{k}d^3\mathbf{k}'}{(2\pi)^5}\frac{\delta^{(4)}(q+q'-k-k')}{8E_{q'}E_kE_{k'}}.$$

$$\Rightarrow \tilde{S}^{>\mu}\Sigma^{<}_{\mu} = 2\pi\delta(q^2)\int_{\mathbf{q}',\mathbf{k},\mathbf{k}'} \mathcal{P}(q',k,k')(k\cdot k')(q\cdot q') \times \bar{f}^{(u_c)}(q)\bar{f}^{(u_c)}(q')f^{(u_c)}(k)f^{(u_c)}(k'),$$

- No "explicit" $\mathcal{O}(\hbar)$ corrections in C_μ : $\partial_\mu \tilde{S}^{\mu<} = 2\pi\delta(q^2)q^\mu C_\mu\big[f^{(u_c)}\big],$

$$q^\mu C_\mu\big[f^{(u_c)}\big] = \frac{1}{4}\int_{\mathbf{q}',\mathbf{k},\mathbf{k}'} \boxed{|\mathcal{M}|^2 \times \Big[\bar{f}^{(u_c)}(q)\bar{f}^{(u_c)}(q')f^{(u_c)}(k)f^{(u_c)}(k') - f^{(u_c)}(q)f^{(u_c)}(q')\bar{f}^{(u_c)}(k)\bar{f}^{(u_c)}(k')\Big]}$$

no side-jumps

- Practical form :

$$q\cdot\partial f^{(u_O)}_q = \int_{\mathbf{q}',\mathbf{k},\mathbf{k}'}(k\cdot k')\mathcal{P}(q',k,k')\Big\{(q\cdot q')\Big[\bar{f}^{(u_O)}_q\bar{f}^{(u_c)}_{q'}f^{(u_c)}_k f^{(u_c)}_{k'} - f^{(u_O)}_q f^{(u_c)}_{q'}\bar{f}^{(u_c)}_k\bar{f}^{(u_c)}_{k'}\Big]$$
$$+\frac{\hbar\epsilon^{ijk}q_jq'_k}{2q_0}\Big[\bar{f}^{(u_O)}_q\partial_i\Big(\bar{f}^{(u_c)}_{q'}f^{(u_c)}_k f^{(u_c)}_{k'}\Big) - f^{(u_O)}_q\partial_i\Big(f^{(u_c)}_{q'}\bar{f}^{(u_c)}_k\bar{f}^{(u_c)}_{k'}\Big)\Big]\Big\},\quad u^\mu_O = (1,\mathbf{0}).$$

Conclusions & outlook

- The relativistic CKT with collisions can be derived from quantum field theories.
- Side-jumps stem from parametrization of the Wigner functions and they are associated with the spin structure of massless particles.
- It is also found in field theories that the current is "explicitly" modified by collisions via side-jumps.

- Generalization to QCD.
- Do quantum corrections in collisions lead to new anomalous effects?
- The 1-2 scattering under background fields is allowed due to the modified dispersion relation.