Non-Equilibrium, Dissipation, and Evolution in Astrophysics

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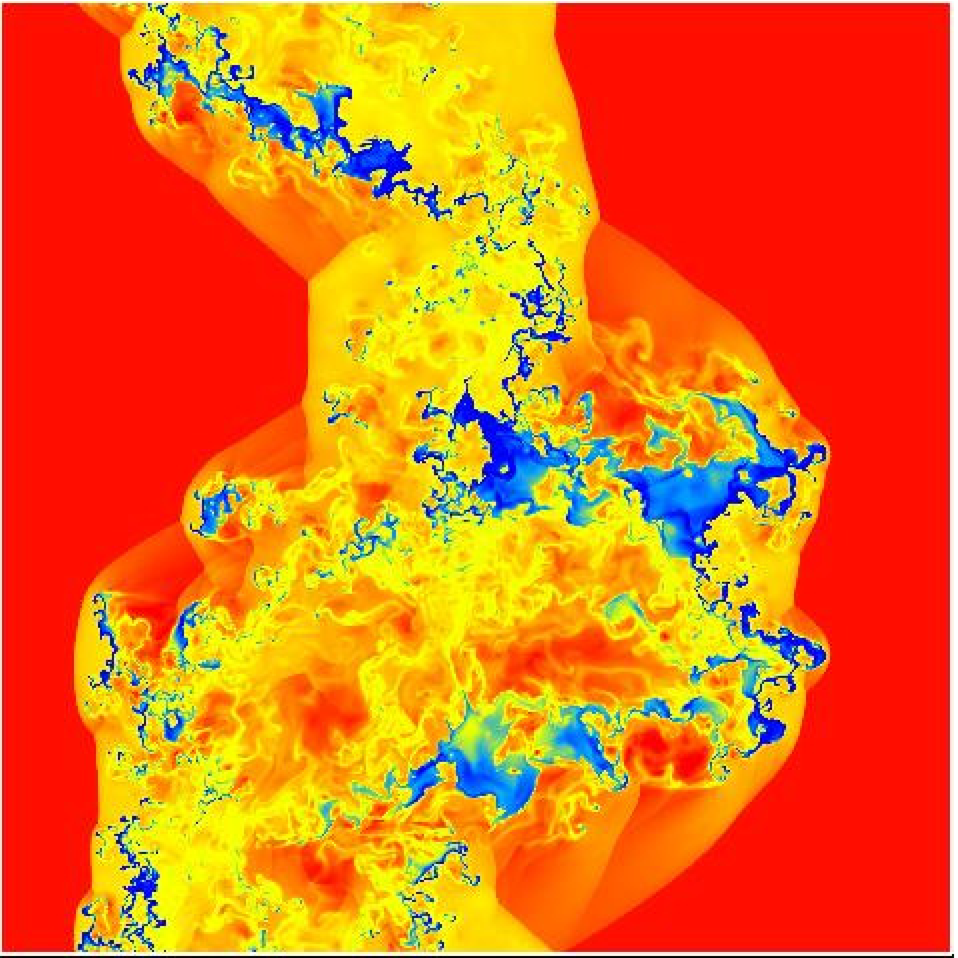
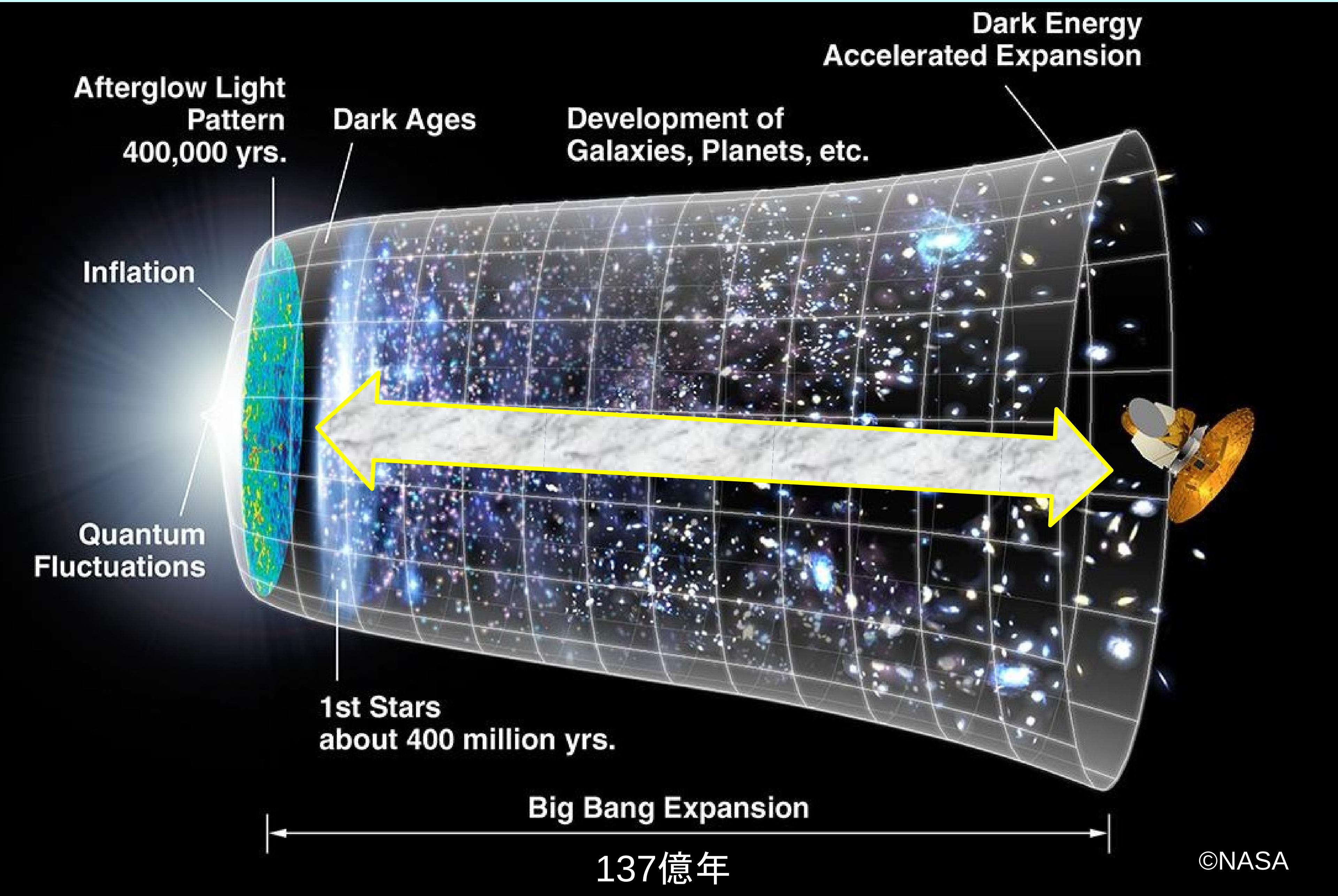


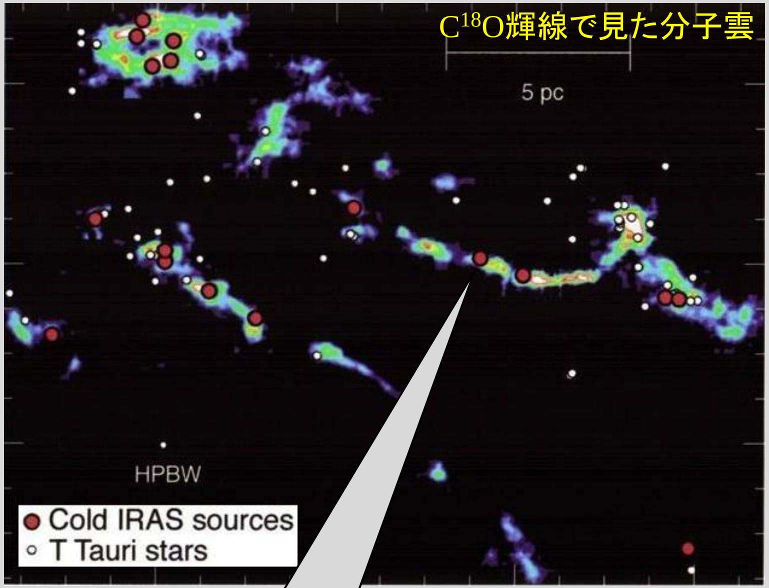
1. Structure Formation
2. Phase Transition Dynamics in Interstellar Medium
3. Dissipation in Relativity
4. Summary

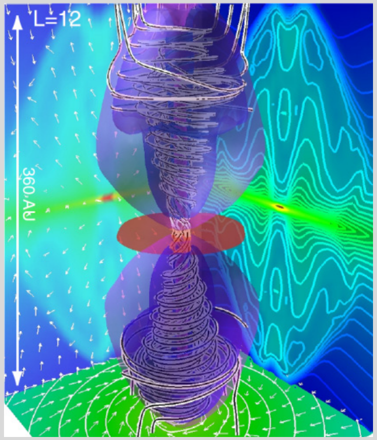
Feb. 18, 2009@Kyoto GCOE

宇宙の進化：熱平衡状態から秩序化



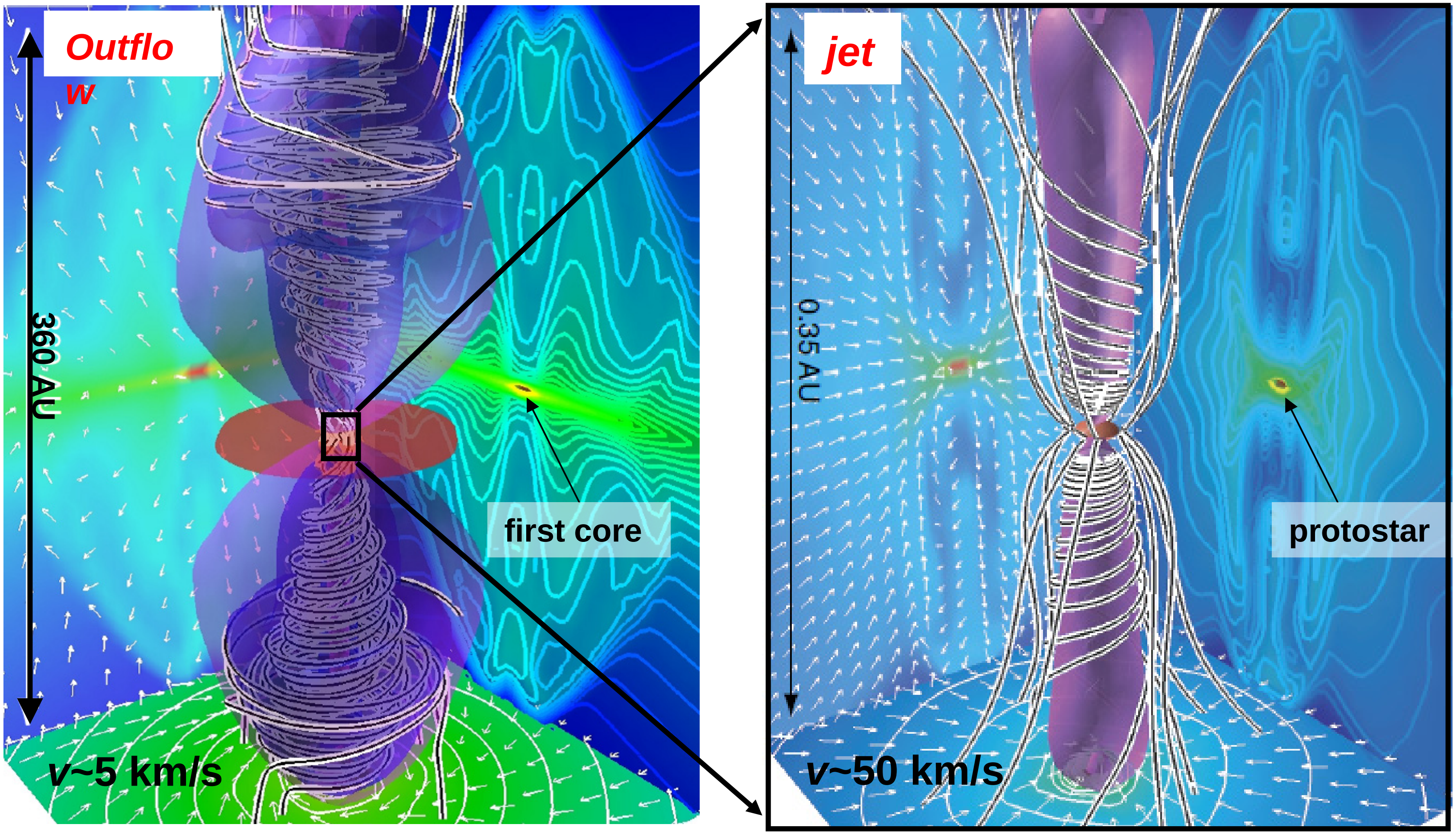
分子雲コアの重力収縮と星形成





磁束問題と
角運動量
問題

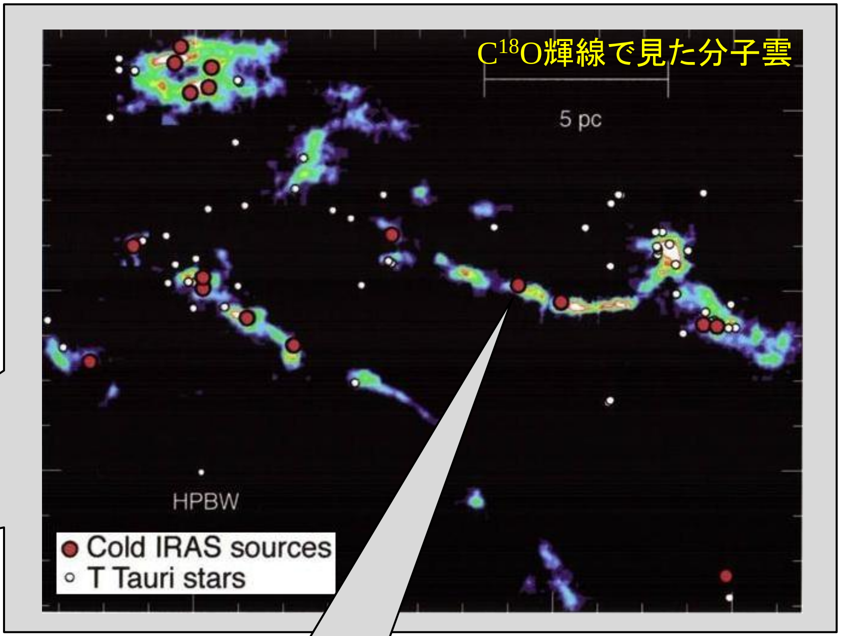
解明された原始星形成期

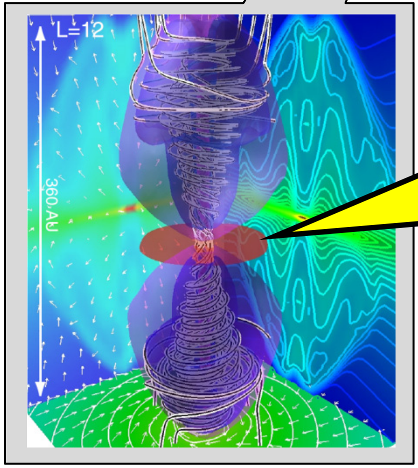


Machida, SI, & Matsumoto (2006,2007,2008)

３Ｄで解明されたのは，$M = 0.01\ M_{\odot}$になる頃まで！

惑星形成の舞台としての星周円盤





原始惑星系
円盤へ進化

Planet Formation

1. 塵粒子が沈殿できない！

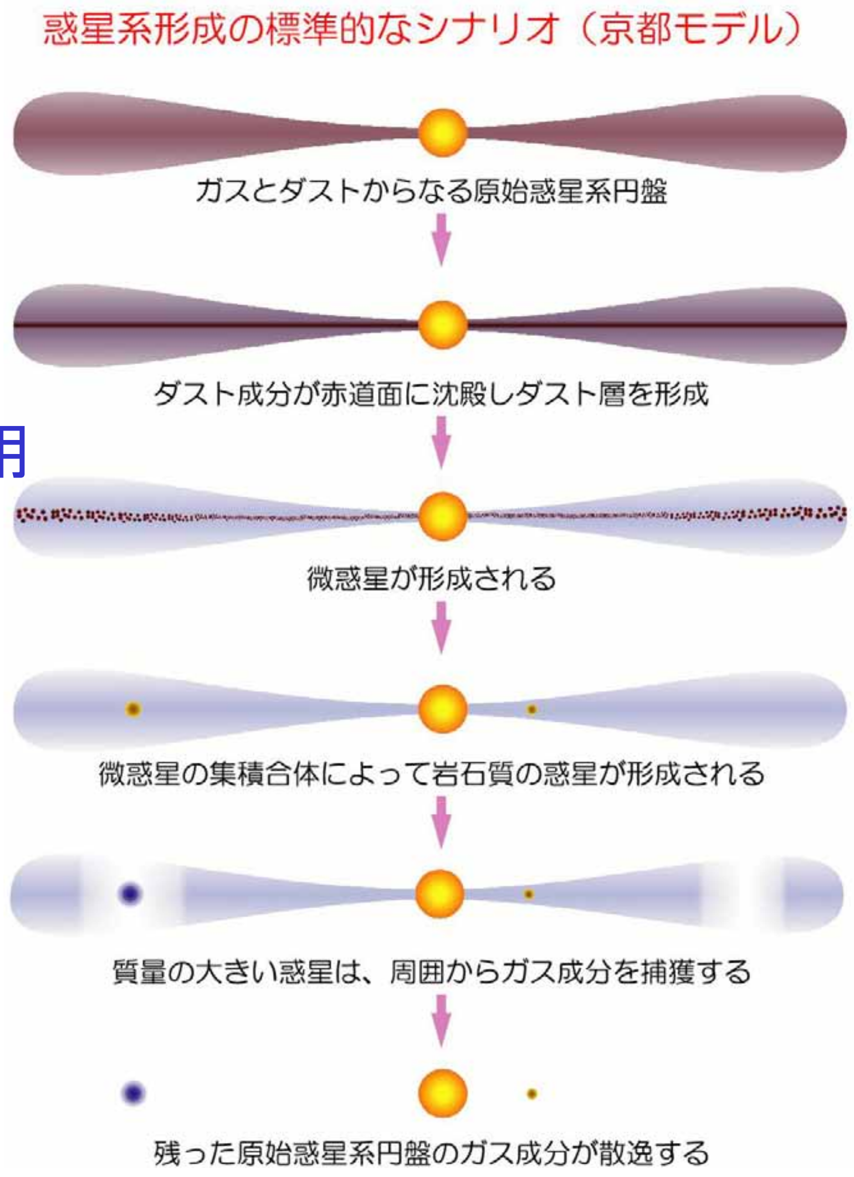
   沈殿するとKH不安定となり乱流化

2. できた惑星も中心星に落下する！

   質量の大きなガス円盤との相互作用

   – Type I Migration… Gap無し
   – Type II Migration… Gap有り

極めて困難



Scaling in Spherical Collapse

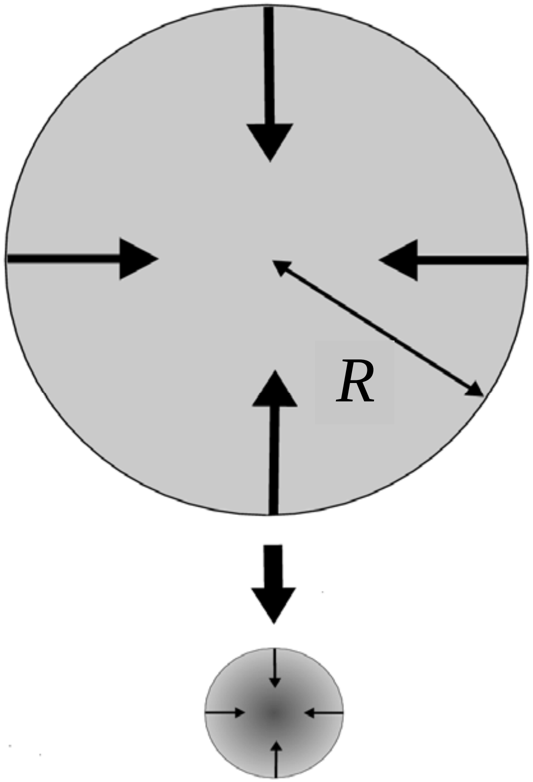
Homologous Collapse

$P \propto \rho^{\gamma}, \quad C_S^2 \propto \rho^{\gamma-1}$

$\rho \propto 1/\mathrm{R}^3, \quad M = \text{const.}$

$F_P \equiv (1/\rho)dP/dR \propto C_S^2/R$

$F_G \equiv GM/R^2 \propto 1/\mathrm{R}^2$



$F_P \,/\, F_G \propto R^{-(3\gamma-4)}$

if $\gamma > 4/3$ ➔ stable

$\gamma_{\mathrm{crit}} = 4/3$ ➔ Cooling is essential.

自己重力系は比熱が負

Virial Theorem for Self-Gravitating System :

U: Internal Energy, W: Grav. Energy

$$3(\gamma-1)U + W = 0$$

Total Energy: $E = U + W$

$$= (3\gamma-4) / [3(\gamma-1)]W = -(3\gamma-4)U$$

In the case of $\gamma = 5/3$, emission of energy ➔

$dE < 0 \Rightarrow dW = 2\, dE < 0,$

$$dU = -dE = -dW/2 > 0 \;\; ➔ \;\; T\uparrow$$

Effective specific heat is negative!

非効率な星形成過程

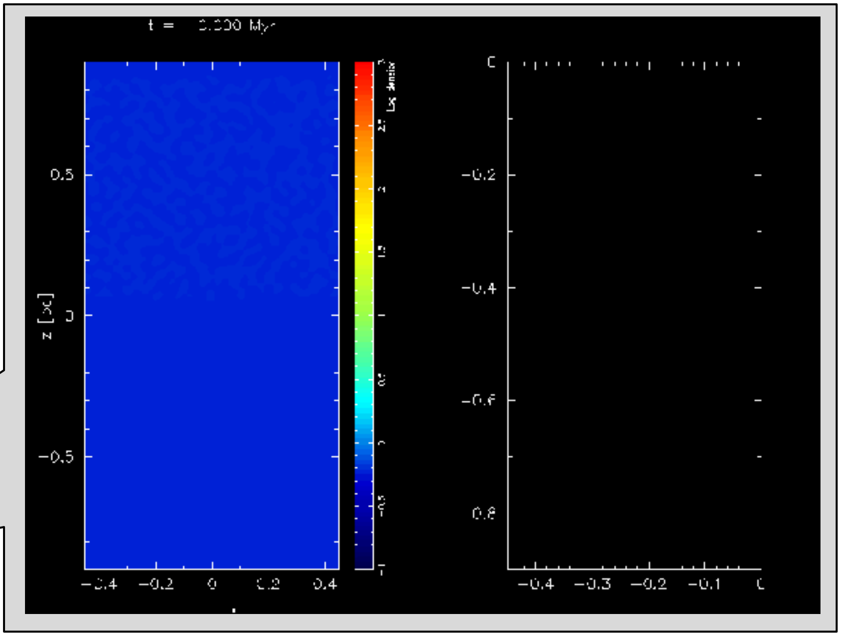
- 銀河系内の分子雲(~10K)の総量 $M_{MC} \sim 10^9 M_\odot$
- COで観測される密度 $n_{CO} \sim 10^2 /\mathrm{cm}^3$
- その自由落下時間 $\sim 10^6$ yr
- もし全部が動的に星になるとすると

星形成率 $= 10^9 M_\odot / 10^6 \mathrm{yr} = 10^3 M_\odot /\mathrm{yr}$

$>>$ 観測される星形成率 $\sim 10^0 M_\odot /\mathrm{yr}$

つまり，効率は1%以下(~10^{-3})のはずである！

Turbulent Interstellar Medium

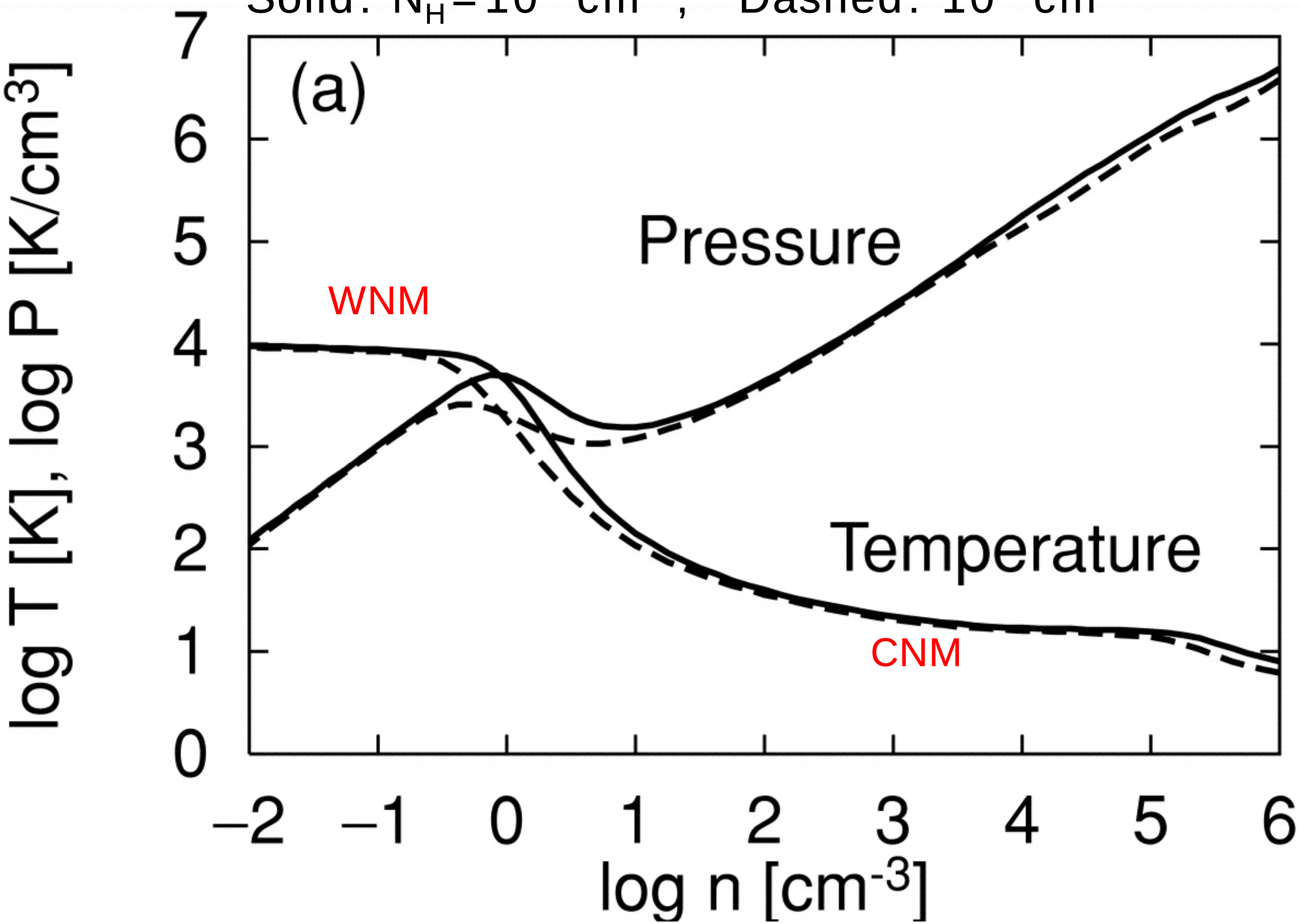


Koyama & SI (2006) ApJ **652**, 1131

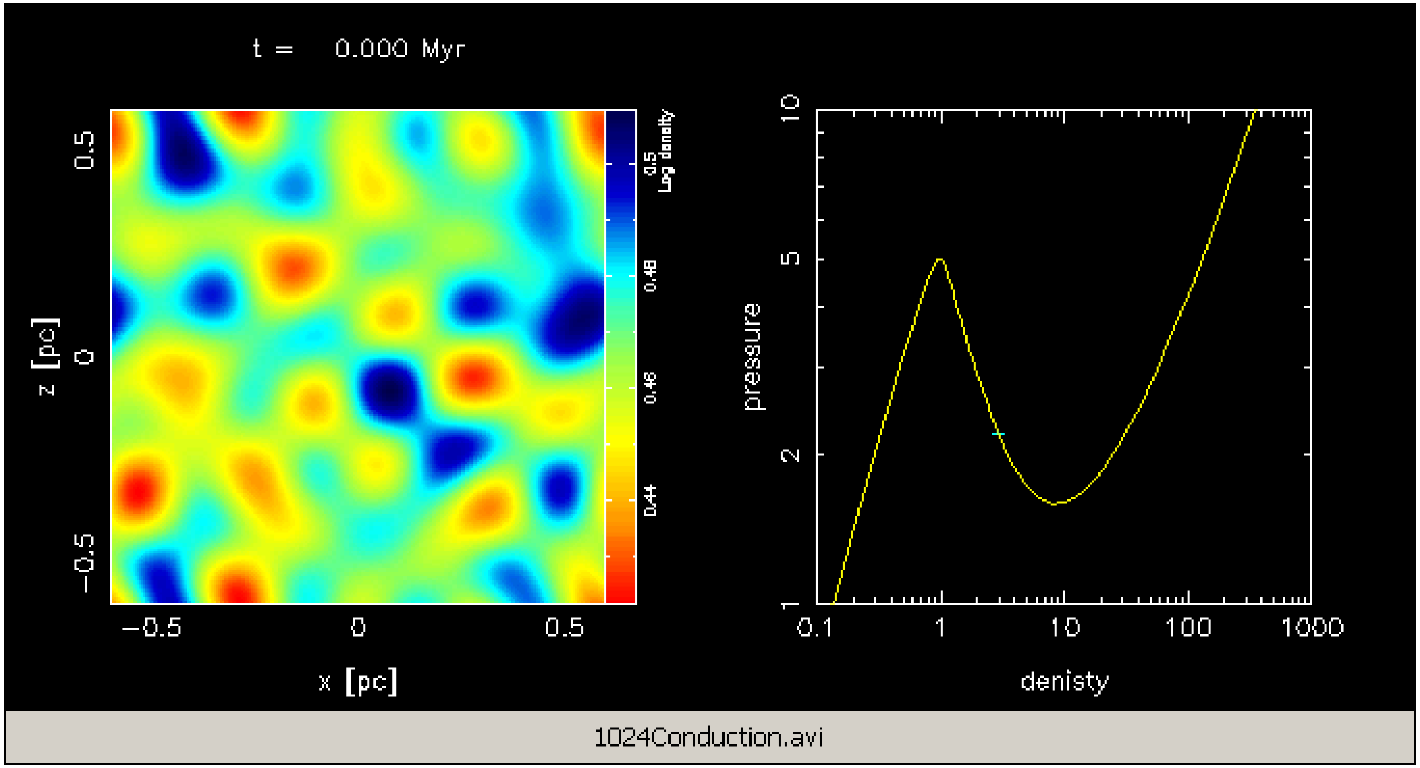
多相ガスにおける乱流駆動メカニズムの解明、及び相転移面の不安定性の発見

Radiative Equilibrium

Solid: $N_H = 10^{19} cm^{-2}$,   Dashed: $10^{20} cm^{-2}$



2D Evolution from Unstable Equilibrium



Periodic Box Evolution <u>without Shock Driving</u>

With Cooling/Heating and Thermal Conduction

Without Physical Viscosity $\rightarrow$ $\mathrm{Pr} = 0$

2相平衡状態

Van der Waals gas の

状態方程式

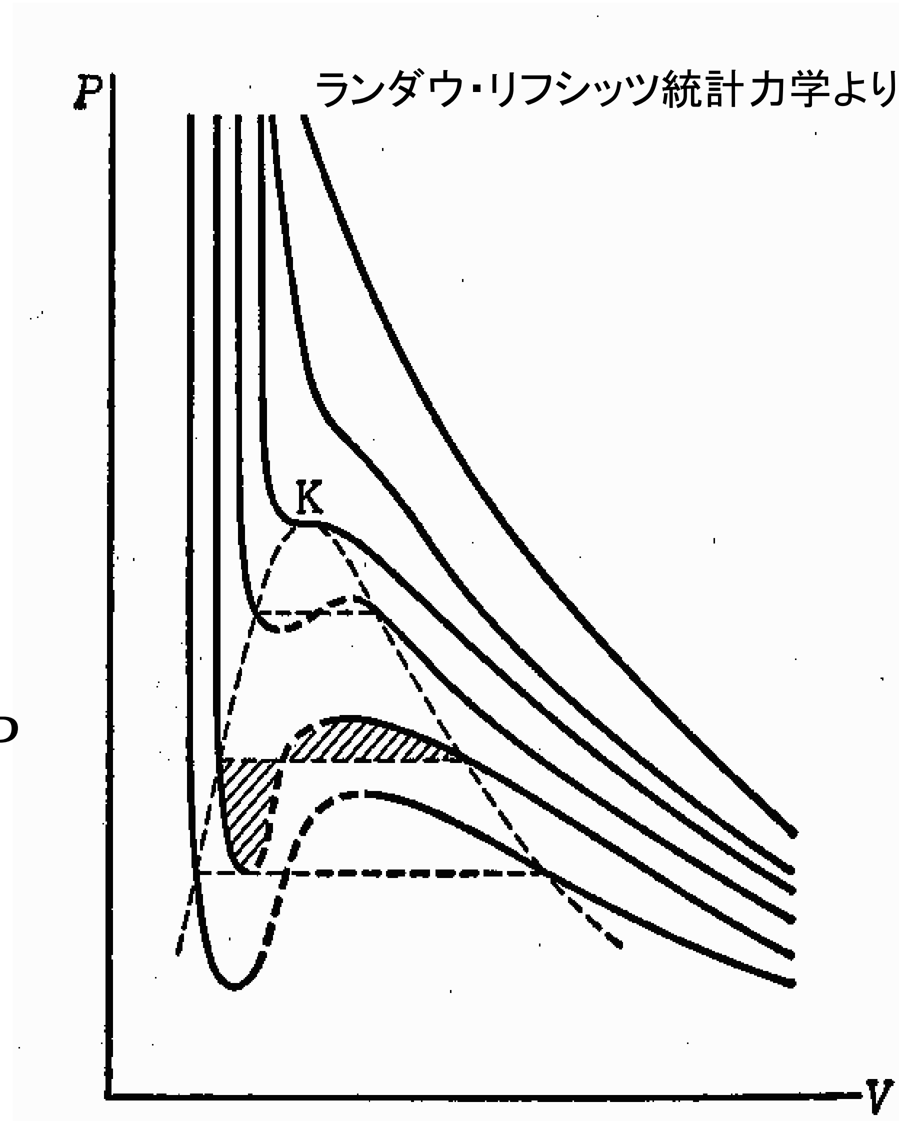
$$P = \frac{NT}{V - nb} - \frac{N^2 a}{V^2}$$

$$\mu_1 = \mu_2 \Leftrightarrow 0 = \int_1^2 d\mu$$

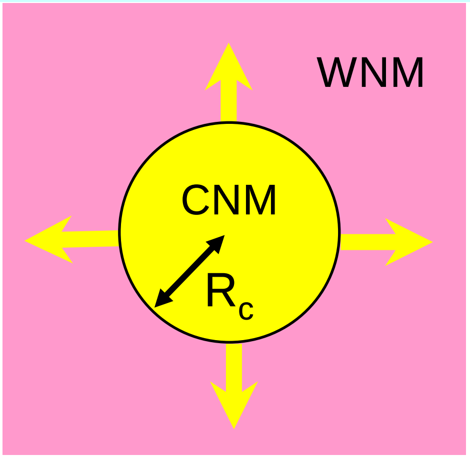
$$= \int_1^2 V(P, T = \text{一定}) dP$$

斜線部分の面積が等しい

（Maxwellの規則）

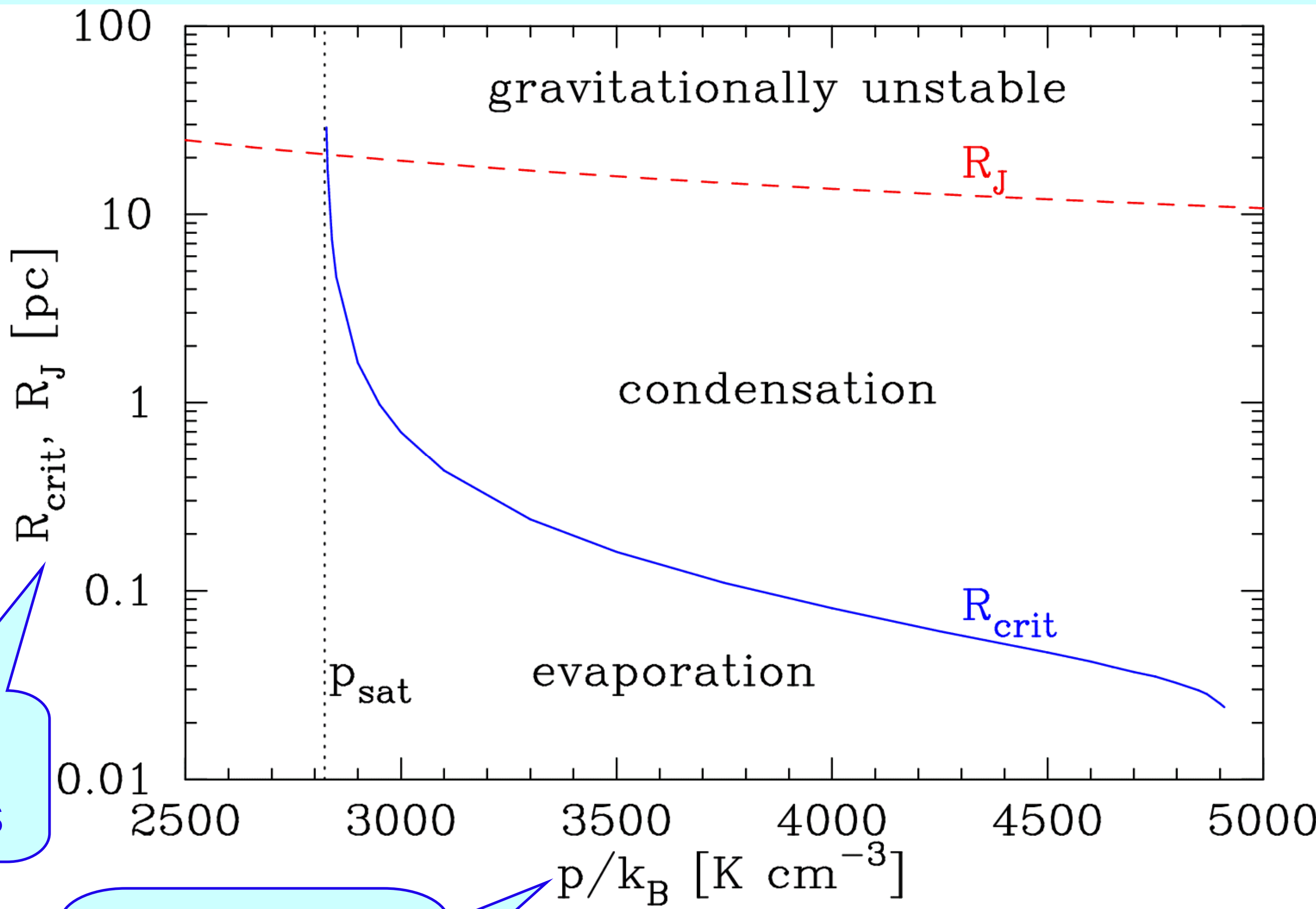


Evaporation of Spherical CNM in WNM



pressure is larger, the critical size of the stable cloud is smaller.

Critical Radius



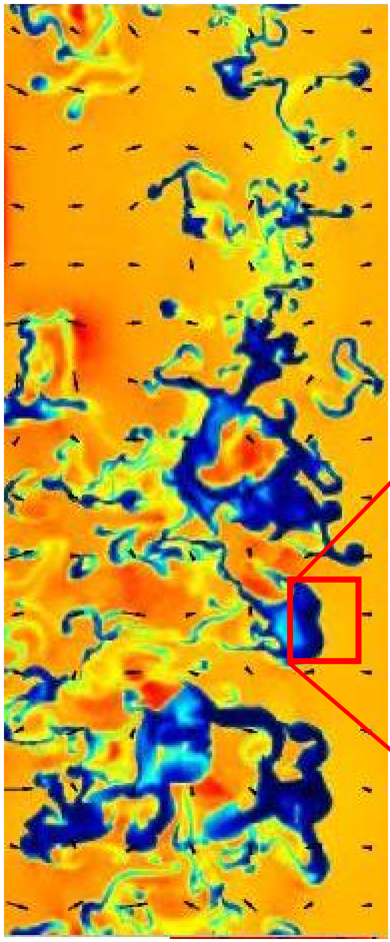
Ambient Pressure

Bigger clouds grow and smaller clouds evaporate.

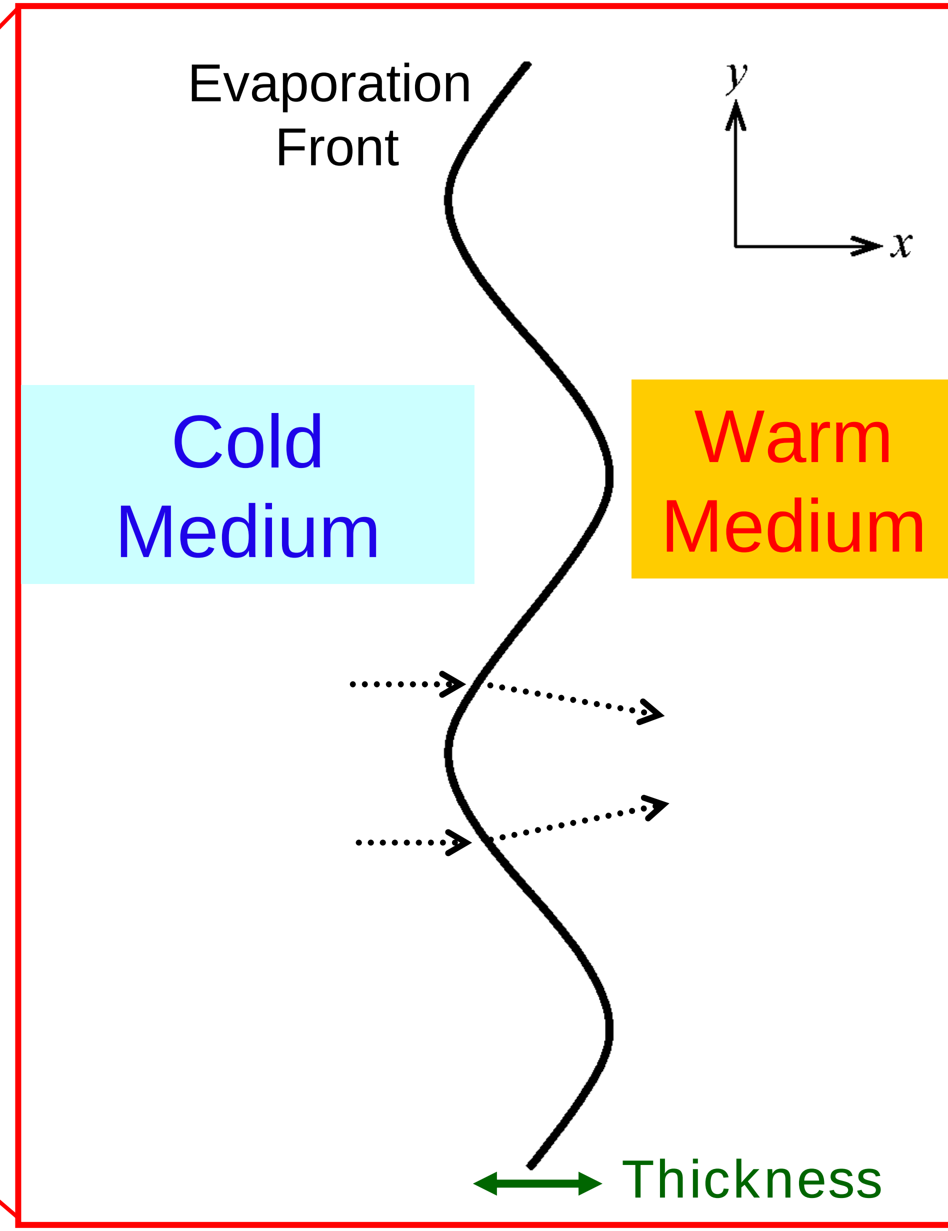
Nagashima, SI, & Koyama 2006, ApJL **652**, L41

Yatou & Toh 2009, PRE submitted (arXiv:0901.2395)

Instability of Transition Layer



importance in maintaining "turbulence"



Inoue, SI, & Koyama 2006, ApJ **652**, 1131

Instability of Transition Layer

Similar Mechanisms…

1) Darrieus-Landau (DL) Instability

Flame-Front Instability

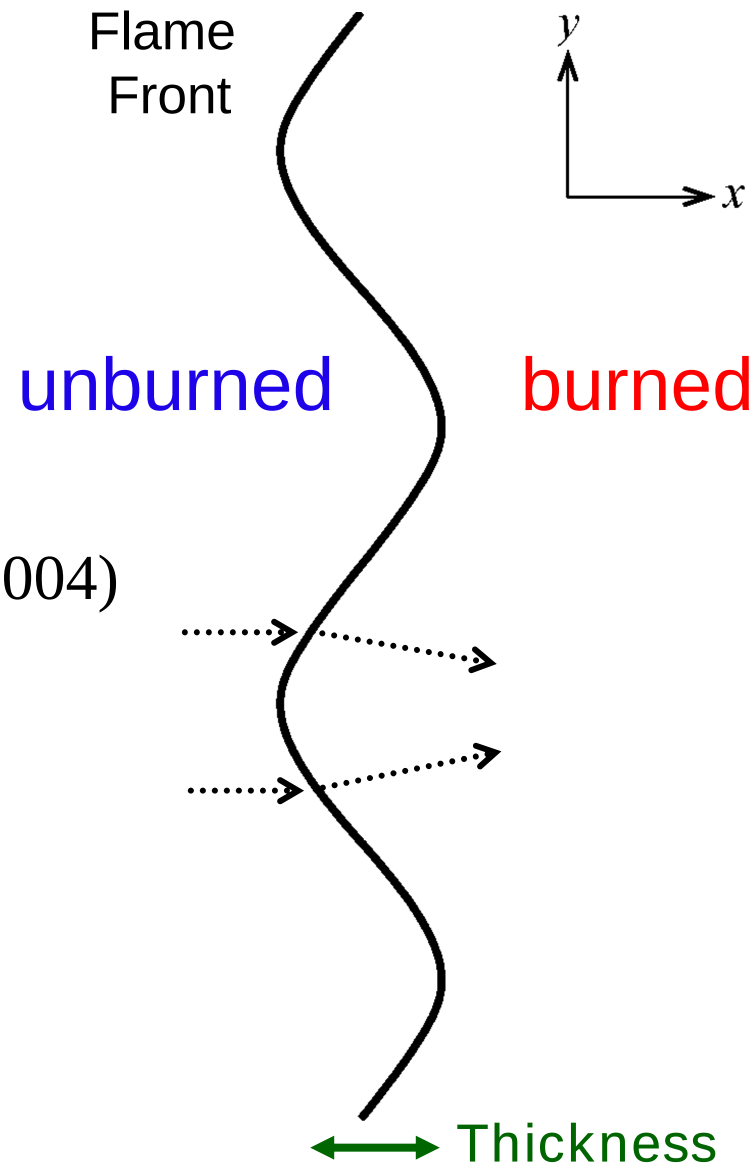
Important in SNe Ia

Effect of Magnetic Field (Dursi 2004)

2) Corrugation Instability in MHD Slow Shock

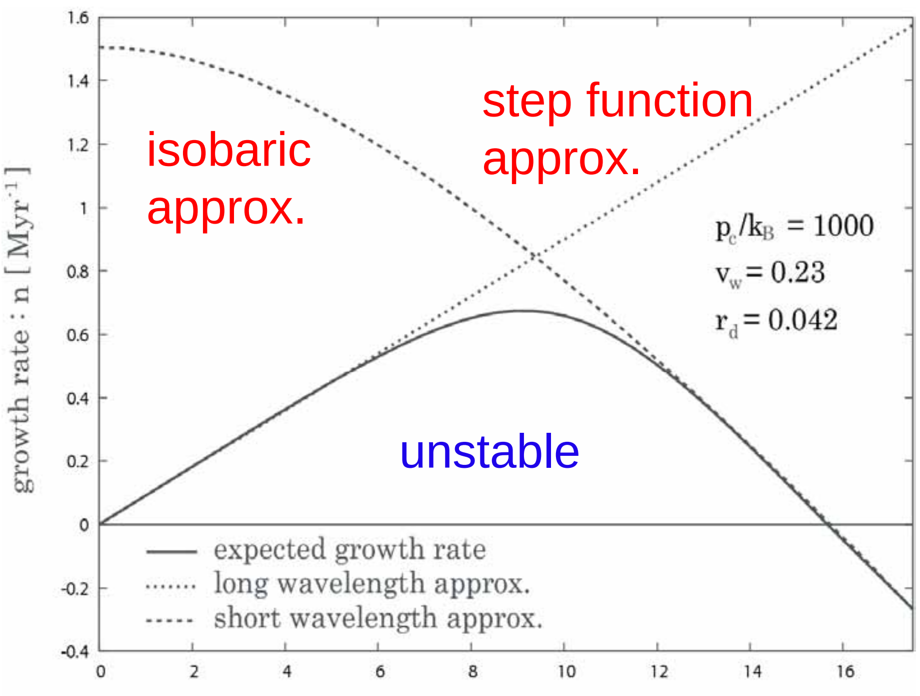
– Edelman 1990

– Stone & Edelman 1995



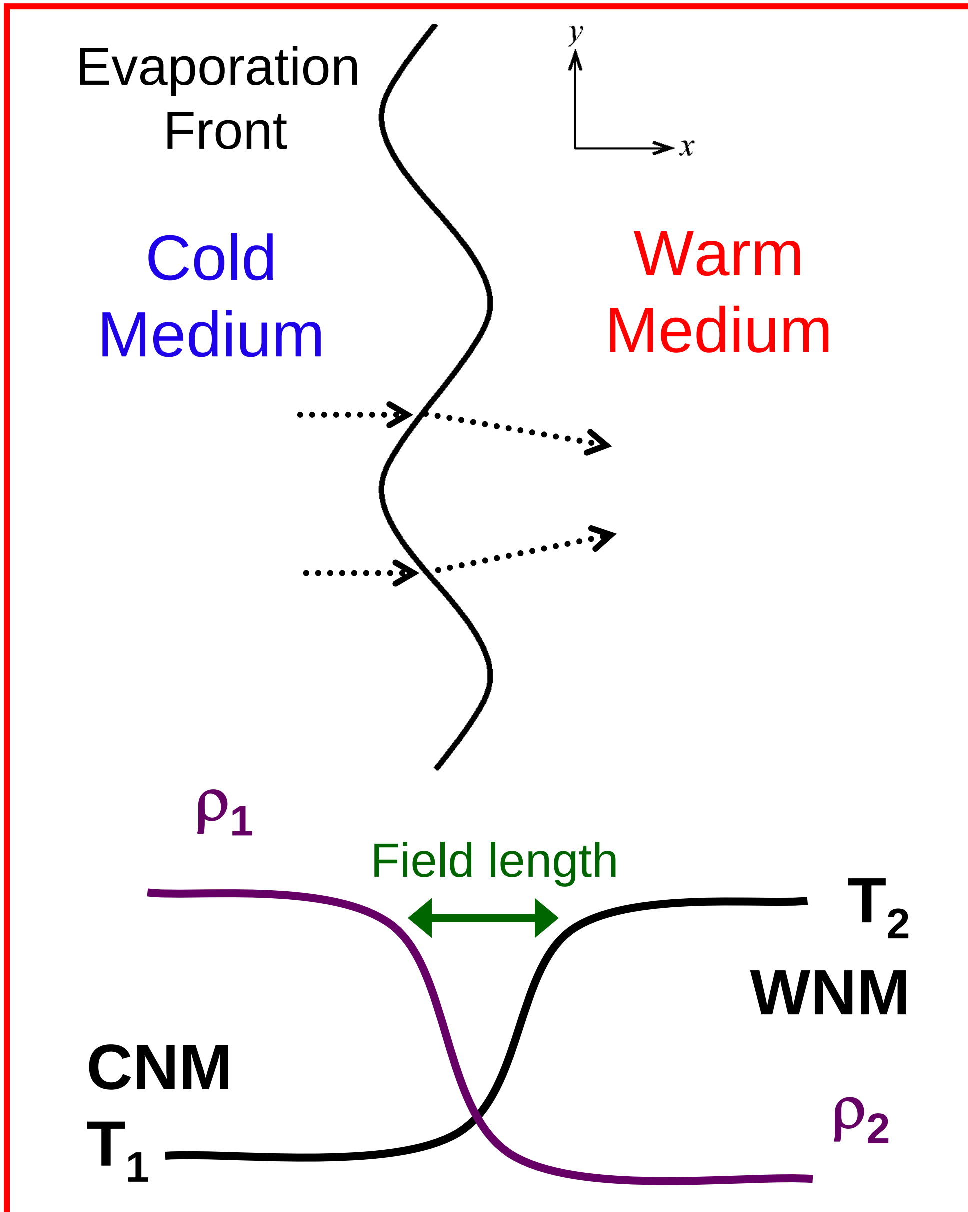
Linear Analysis of New Instability

Growth Rate (Myr^{-1})



wavenumber $k_y/2\pi$ [pc^{-1}]

Inoue, SI, & Koyama 2006, ApJ **652**, 1131



Summary of TI-driven Turbulence

- 2D/3D Calculations of The Propagation of Shock Wave into WNM

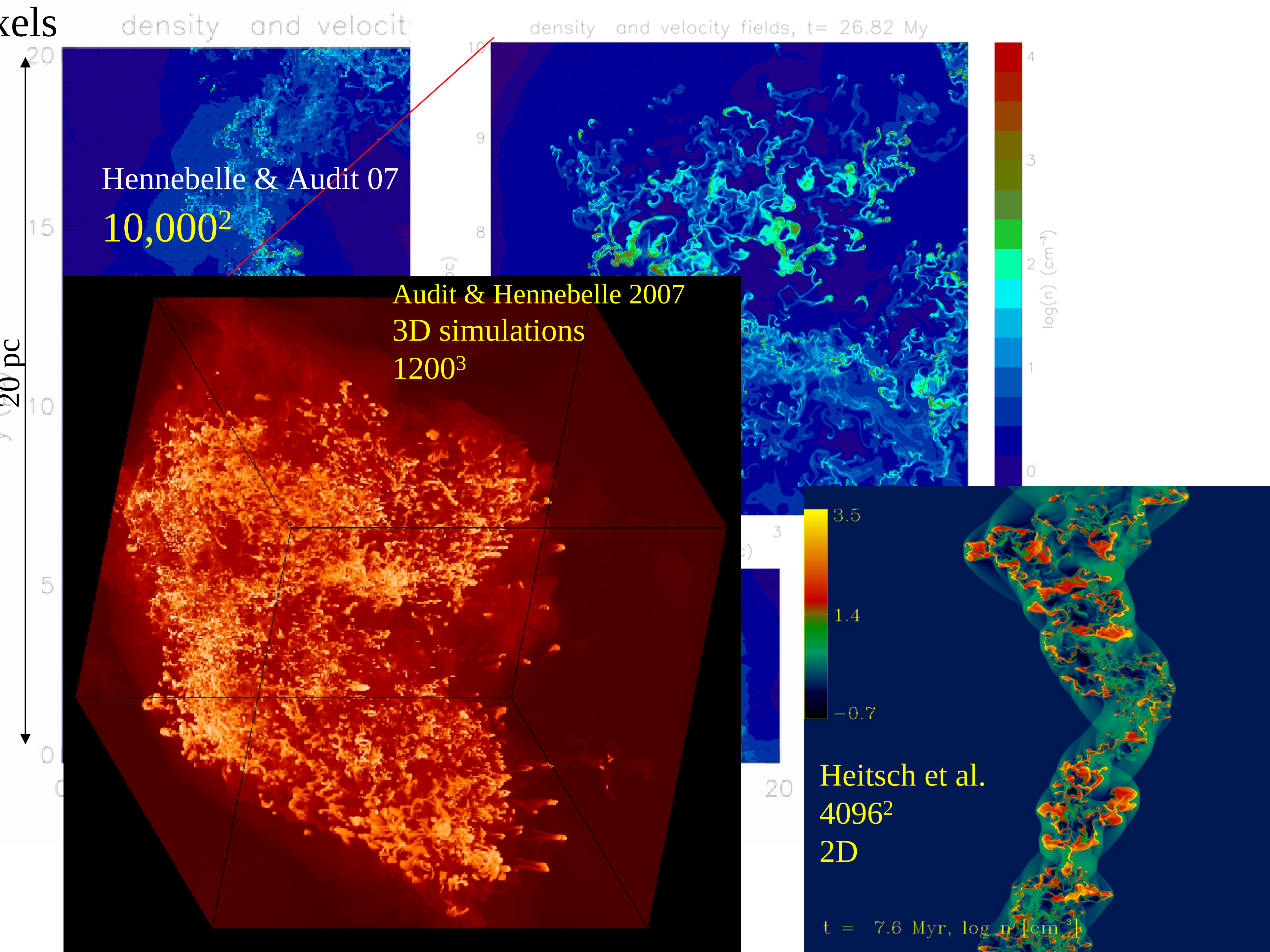
via Thermal Instability

➔ fragmentation of the cold layer into cold clumps with long-sustained supersonic velocity dispersion (~ km/s)

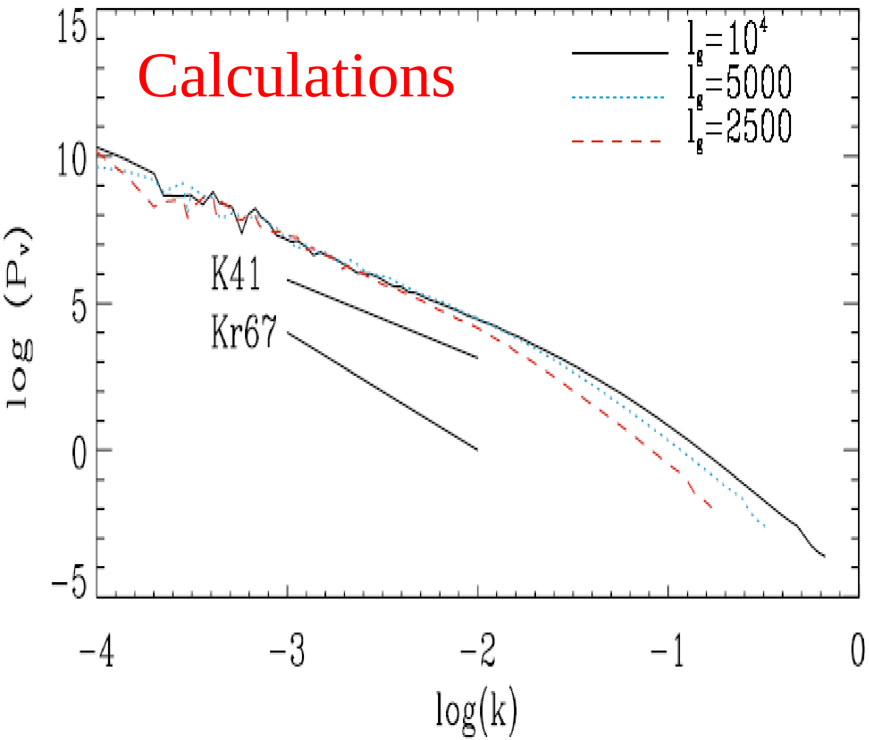
1D: Shock $\Rightarrow E_{th} \Rightarrow E_{rad}$

2D&3D: Shock $\Rightarrow E_{th} \Rightarrow E_{rad} + E_{kin}$

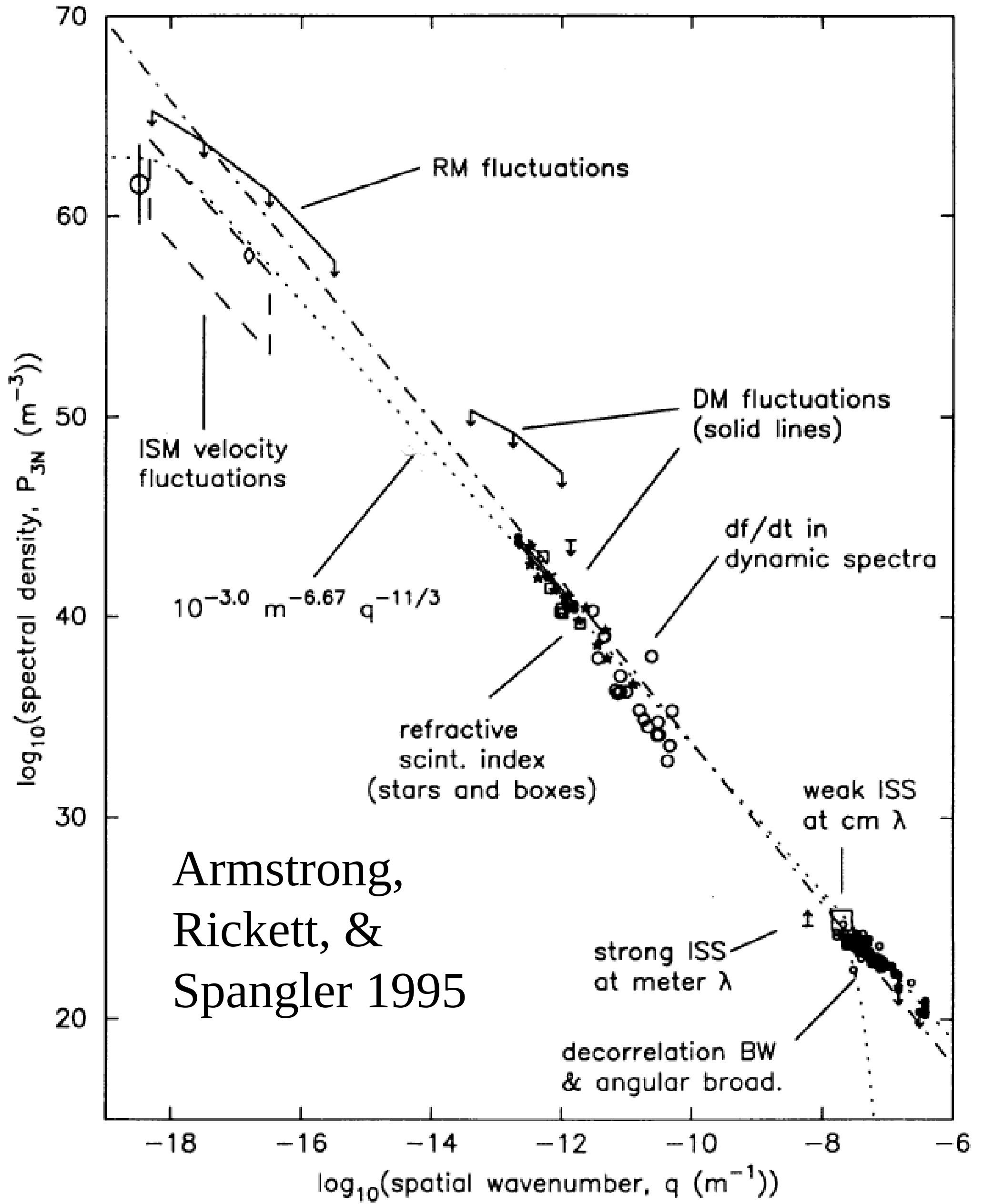
$$\delta v \sim \text{a few km/s} < C_{S,WNM}$$


xels
density and velocit
density and velocity fields, t= 26.82 My
20 pc
Hennebelle & Audit 07
10,000²
Audit & Hennebelle 2007
3D simulations
1200³
log(n) (cm⁻³)
Heitsch et al.
4096²
2D
3.5
1.4
−0.7
t = 7.6 Myr, log n [cm⁻³]

Property of "Turbulence"



Observation



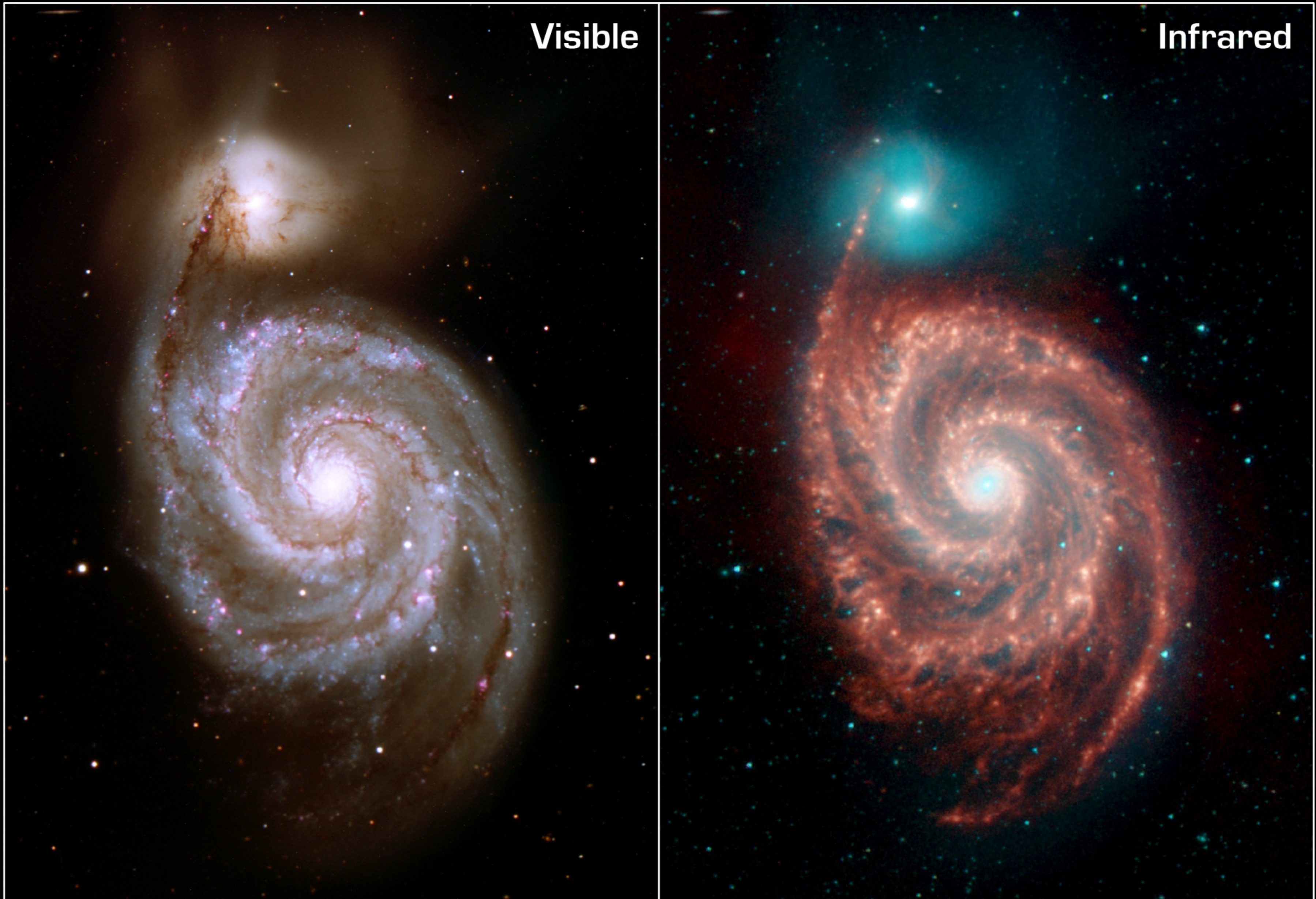
$\delta v < C_{S,WNM}$

➔ Kolmogorov-like Spectrum

Main Components in Galaxies

- Stars
- Gas
- Photons

What else?

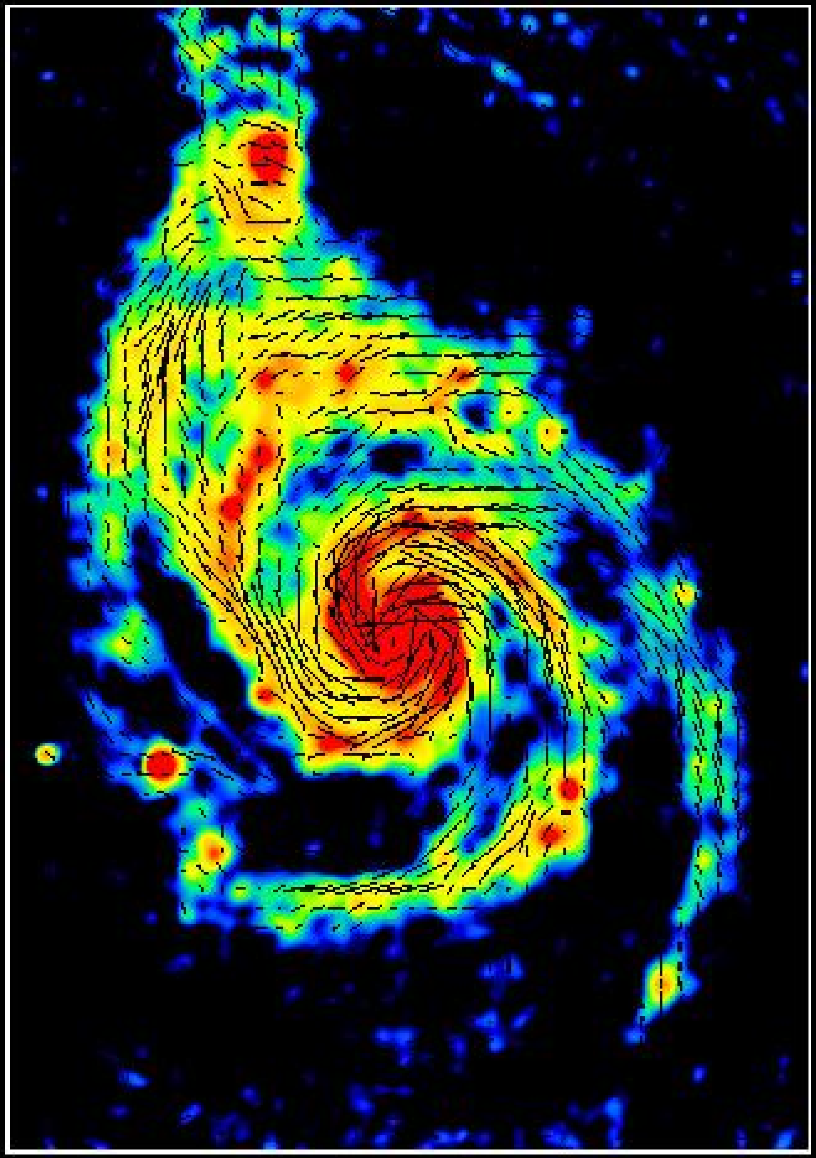


Spiral Galaxy M51 ("Whirlpool Galaxy") Spitzer Space Telescope • IRAC

NASA / JPL-Caltech / R. Kennicutt (Univ. of Arizona) ssc2004-19a

M51 Synchrotron

M51 6cm Tot.Int.+D-Vectors (VLA+Effelsberg)



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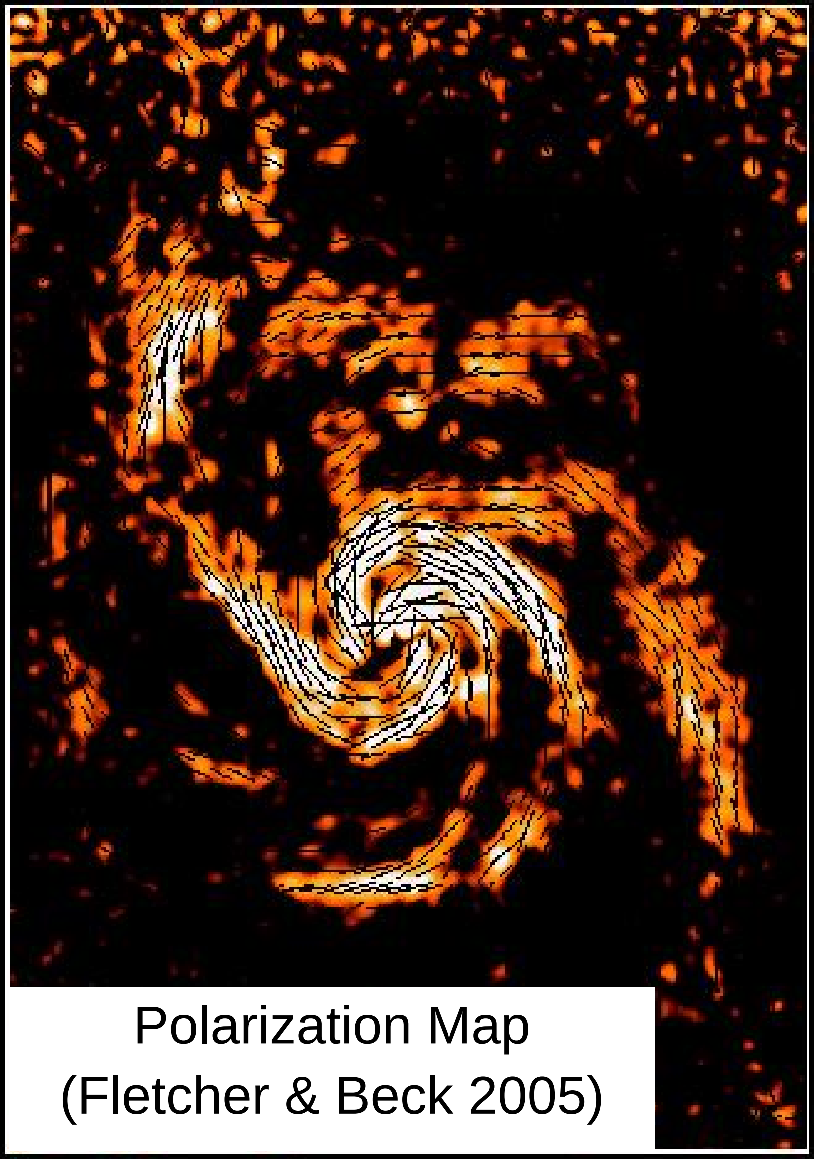
M51 6cm Pol.Int.+D-Vectors (VLA+Effelsberg)



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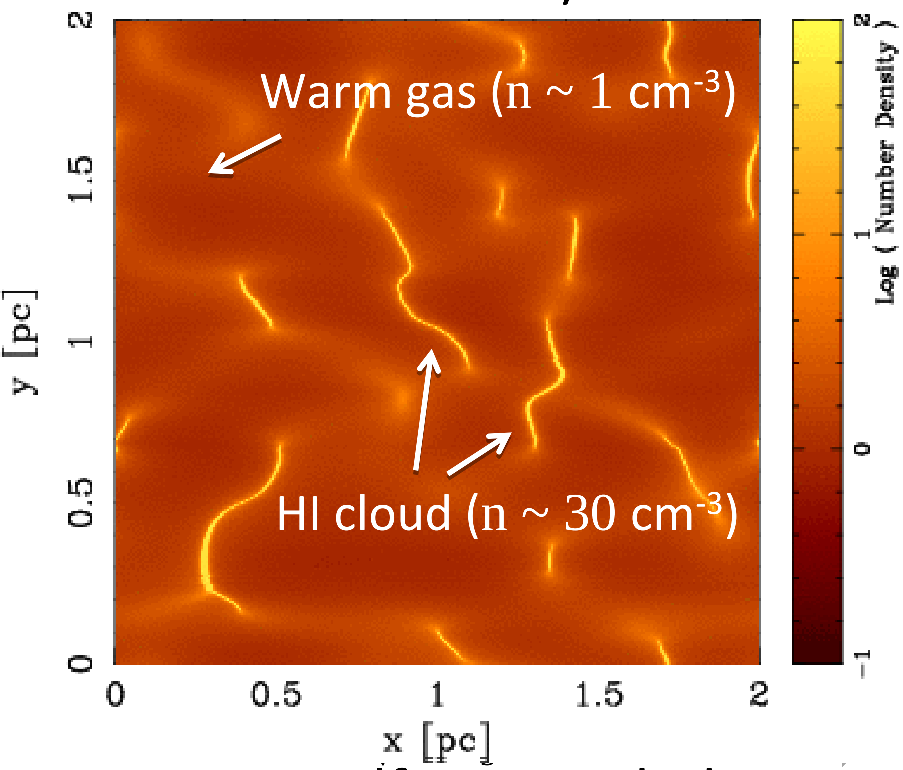
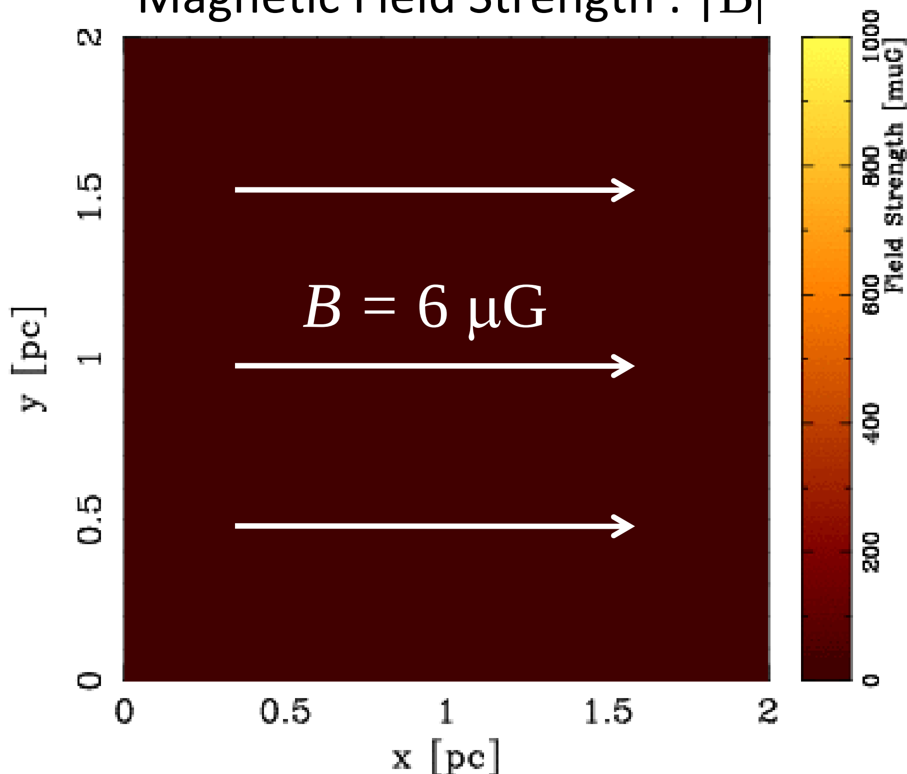
Supernova Shock in 2-Phase ISM

■ Inoue, Yamazaki, & SI (2009) ApJ in press (arXiv:0901.0486)

Number Density : n



Magnetic Field Strength : |B|

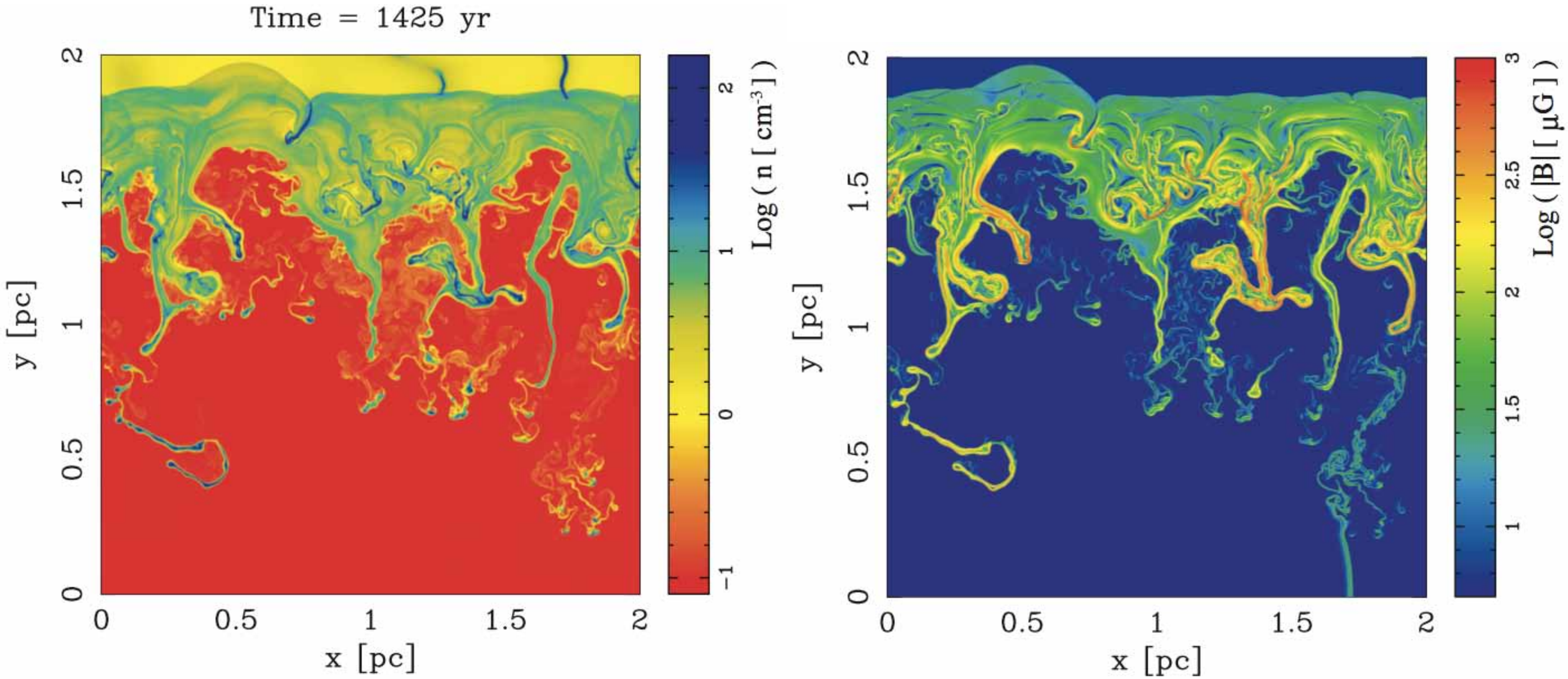


Super-Alfvenic turbulence

磁場増幅: B_{max} ~ 1 mG (β~1), <B> ~ 100 μG

● Supernova Age ~ 1000 yr ...(Bamba+2002, Uchiyama 2008, etc.)

2-pahse ISM を掃く超新星衝撃波



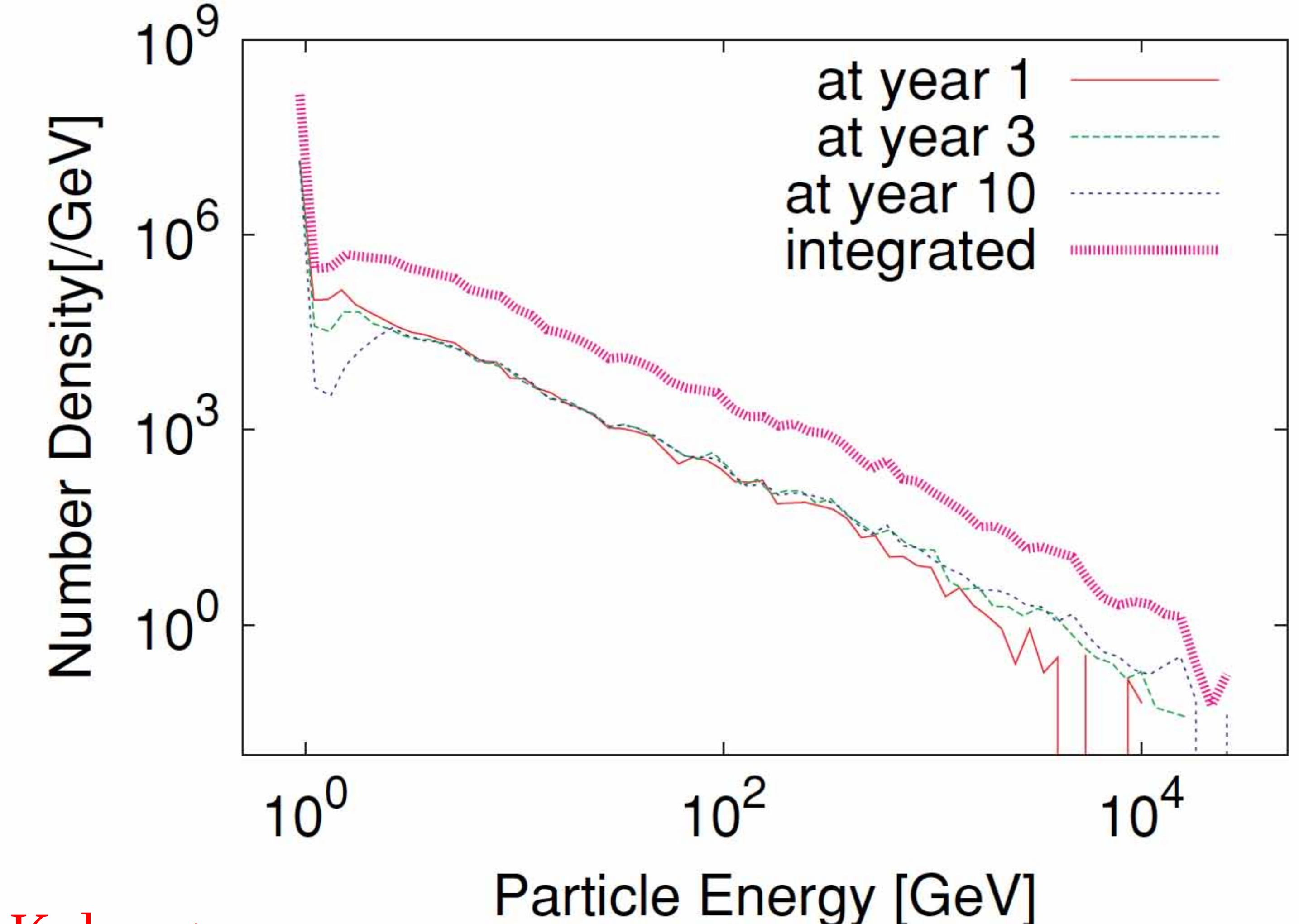
- $\nabla\rho \times \nabla p \neq 0 \rightarrow$ shock 面で渦生成 ($\delta v \sim c_s$)
- Turbulent Dynamo で磁場増幅 : Bmax ~ 1mG (post shock の $\beta \sim 1$)

■ パラメータは異なるが分子雲の乱流駆動へのヒント？

分子雲は超音速乱流 → 密度構造は clumpy

乱流減衰 ~ 1 Myr ⟺ SN shock に掃かれる ~ 1 回/Myr

1st Order Fermi Acceleration



Kolmogorov
Background Turbulence

Muranushi & SI (2009) ApJL **691**, L24

Mystery: Energy Equipartition?

銀河系の中のエネルギー分布

- 銀河系内の（単位体積当り）星起源の輻射場のエネルギーは，
  $E_{\gamma,\mathrm{stellar}} \sim 10^0$ eV/cc

$$E_{\gamma,\,星} \sim E_{\mathrm{th,\,gas}} \sim E_{乱流} \sim E_{宇宙線} \sim E_{磁場} \gtrsim E_{\mathrm{CMB}}$$

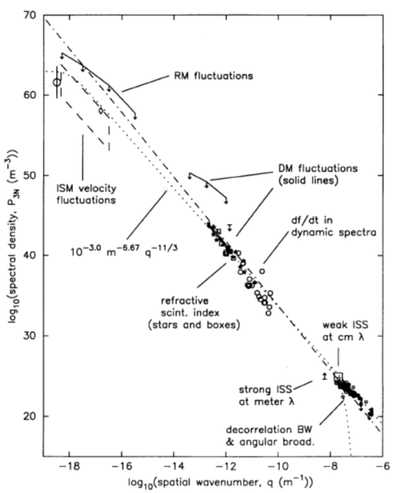
「～」の意味は±１桁程度の精度で… 理由は不明？

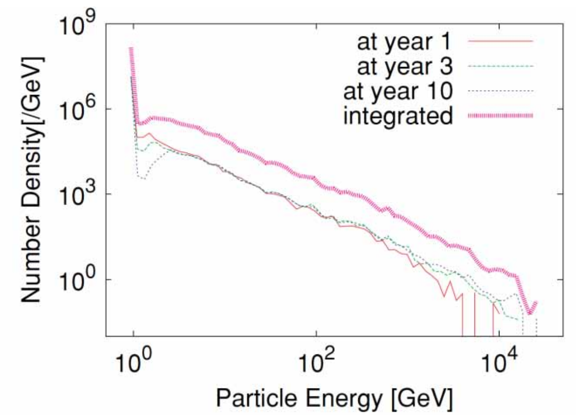
Overall Equilibrium???

Spectrum of Various Components

Every component has energy density ~ 10^0 eV/cc .

$$E_{\gamma,\,\mathrm{stellar}} \sim E_{\mathrm{th,\,gas}} \sim E_{\mathrm{turb}} \sim E_{\mathrm{CR}} \sim E_{\mathrm{mag}} \gtrsim E_{\mathrm{CMB}}$$





A New Type of Quasi-Equilibrium?

第２部：散逸の相対論

Property of Conduction Equation

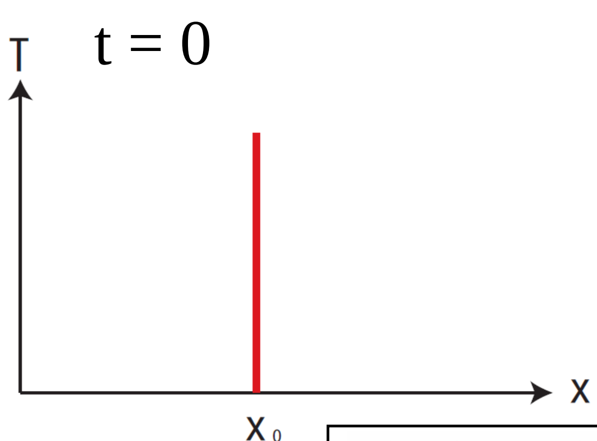
ex） The energy equation （if considering the **first-order** theory）

$$n c_V \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q} = \nabla \cdot (\kappa \nabla T)$$

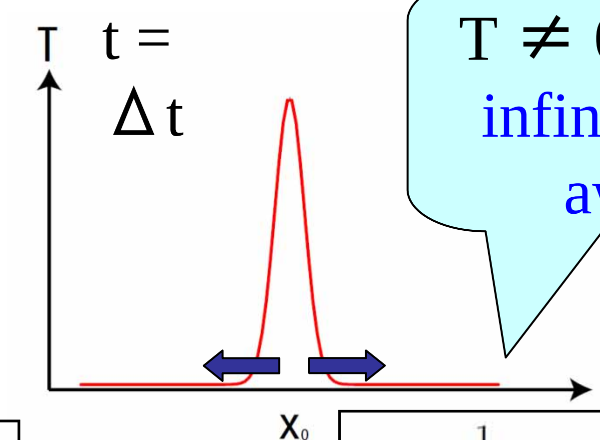
: Parabolic equation

Heat propagates instantaneously in this model!

Unphysical Instability in Covariant Analogue



$$T = T_0 \; \delta_D(x - x_0)$$



$$T = \frac{1}{2\sqrt{\pi \chi t}} \exp\left[- \frac{(x - x_0)^2}{4 \chi t} \right]$$

T ≠ 0 even at infinitely far away!

Classical Answer: Israel-Stewart Theory

$$\nabla_b T^{ab} = 0$$

$$\nabla_a N^a = 0$$

$$N^a = n u^a$$

$$T^{ab} = \rho u^a u^b + (P + \tau)\Delta^{ab} + q^a u^b + q^b u^a + \tau^{ab}$$

$$\tau = -\zeta_V (\nabla_a u^a + \boxed{\beta_0 \dot{\tau}} - \alpha_0 \nabla_a q^a)$$

$$q^a = -\kappa T \Delta^{ab} \left(\frac{1}{T} \nabla_b T + \dot{u}_b + \boxed{\beta_1 \dot{q}_b} - \alpha_0 \nabla_b \tau - \alpha_1 \nabla_c \tau_b^c \right)$$

$$\tau^{ab} = -2\zeta_S \left\langle \nabla^a u^b + \boxed{\beta_2 \dot{\tau}^{ab}} - \alpha_1 \nabla^a q^b \right\rangle$$

features:
- hyperbolic equations and stable
- propagation speed is less than speed of light (causal)
- many parameters on the order of collision timescale (impractical ?)

How to Solve Israel-Stewart Eqs.

2.1 静的な場合 (共動系で)

Israel-Stuart 理論の熱伝導部分は以下の系に相当している．

$$\tau \dot{\mathbf{q}} = -\kappa \nabla T - \mathbf{q}, \quad \rho C_{\mathrm{v}} \dot{T} = -\nabla \cdot \mathbf{q}, \tag{14}$$

変数を一つに消去すると，以下の電信方程式になる．

$$\partial_t^2 T = c_\tau^2 \partial_x^2 T - \tau^{-1} \partial_t T, \quad c_\tau^2 \equiv \kappa / \rho C_{\mathrm{v}} \tau. \tag{15}$$

時間尺度 τ は衝突時間程度であり，実際の数値計算で $\mathbf{q}$ の時間発展は追いたくはない．そのため，(14) の第 1 式を積分して，

$$\mathbf{q}(t) = -\int_{t-\Delta t}^{t} \frac{\kappa}{\tau} \nabla T(s) \exp \frac{(s-t)}{\tau} ds + \mathbf{q}(t-\Delta t) \exp\left(-\frac{\Delta t}{\tau}\right), \tag{16}$$

但し，ここで，$T(t)$ は電信方程式の Green 関数を用いて時刻 $t-\Delta t$ での $T, \partial_t T$ で表す． ⇒ 利点：この方法では，Δt を大きくできる！

ラボ系での電信方程式

流体共同系 (x,t) で電信方程式

$$c_\tau^{-2}\partial_t^2 T - \partial_x^2 T = -\alpha^{-1}\partial_t T, \tag{17}$$

はラボ系 (x',t') では、以下のような形となる。

$$\gamma_c\, c_\tau^{-2}(\partial_{t'} + c_1\partial_{x'})(\partial_{t'} - c_2\partial_{x'})T = -\alpha^{-1}(\partial_{t'} + v\partial_{x'})T, \tag{18}$$

ここで、

$$\gamma_c \equiv \gamma(1 - \frac{c_\tau v}{c^2})(1 + \frac{c_\tau v}{c^2}). \tag{19}$$

$$c_1 \equiv \frac{c_\tau + v}{1 + c_\tau v/c^2}, \quad c_2 \equiv \frac{c_\tau - v}{1 - c_\tau v/c^2}. \tag{20}$$

この式の解を $t' = 0$ 面での初期データ $T(x)$, $\partial T/\partial t(x)$ で表したい。
でも、Green 関数は？

動いている場合：共動系の解➔ラボ系

共同系でGreen関数を使って解いて、それをラボ系で使うことにする。但し、共同系(x,t)での初期データは“$t=$一定”面では与えられていない！しかし、頑張ると、以下のように$T(x,t)$が解ける。

$$
\begin{aligned}
T(x,t)\exp\left(\frac{c_\tau^2}{2\alpha}t\right) &= \frac{1}{2}\left[T(x'-c_1t')+T(x'+c_2t')\right] \\
&+ \frac{\gamma^2}{2}\frac{v}{c_\tau}\left(1-\frac{c_\tau^2}{c^2}\right)\int_{x'-c_1t'}^{x'+c_2t'} I_0(w)\partial_s T(s)ds \\
&+ \frac{\gamma^2}{2}\frac{1}{c_\tau}\left(1-\frac{c_\tau^2v^2}{c^4}\right)\int_{x'-c_1t'}^{x'+c_2t'} I_0(w)\partial_t T(s)ds \\
&+ \frac{c_1+c_2}{2}t\int_{x'-c_1t'}^{x'+c_2t'} I_1(w)T(s)\frac{c_\tau/(2\alpha)}{\sqrt{(u-\xi)(\eta-u)}}ds \qquad (21)
\end{aligned}
$$

ここで、I_0, I_1は変形Bessel関数で、引数wは

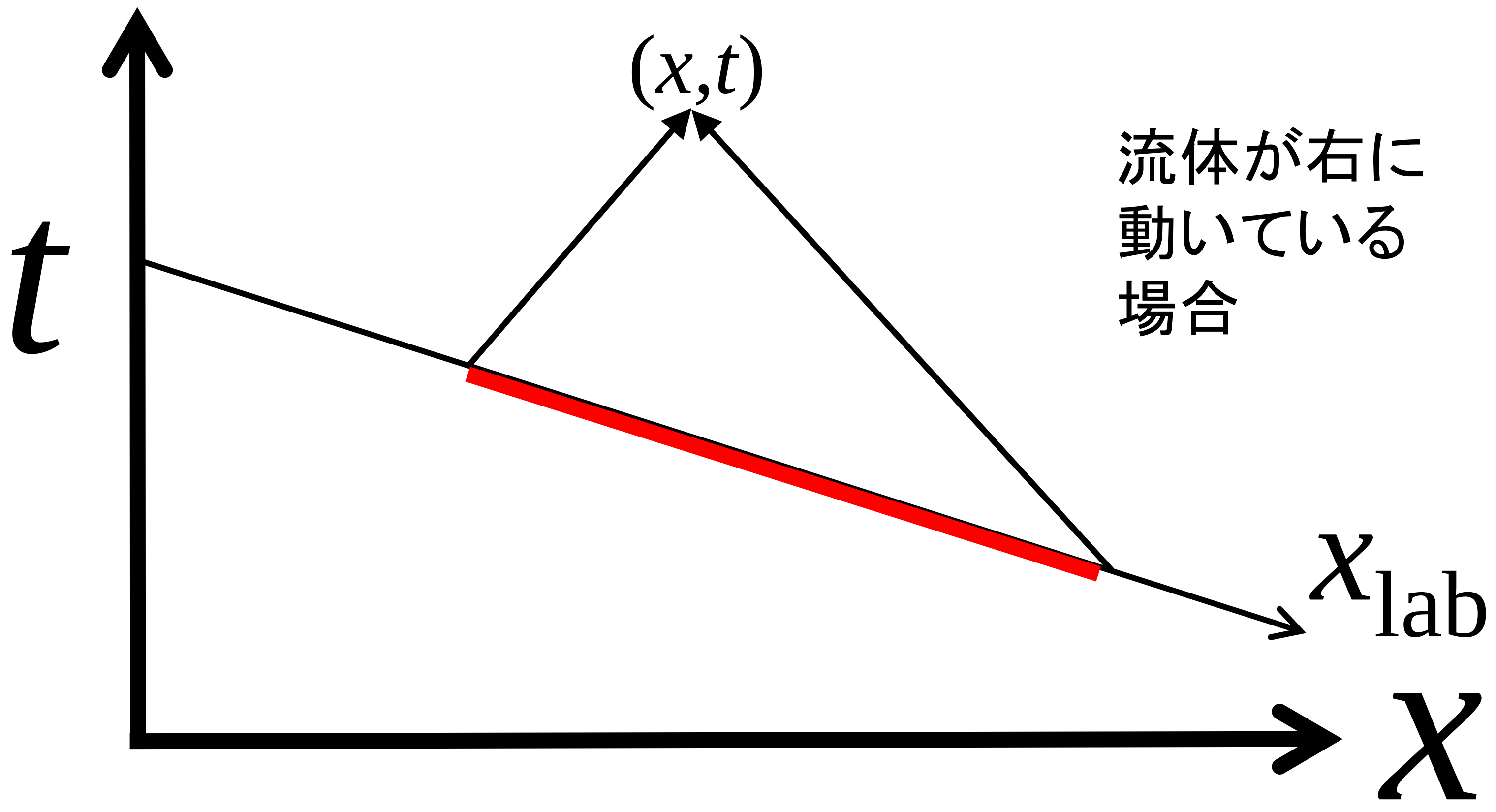
$$
w=\frac{c_\tau}{2\alpha}\sqrt{(u-x'-c_1t')(x'+c_2t'-u)}, \qquad (22)
$$

$$
u=\gamma s\left(1+\frac{c_\tau v}{c^2}\right). \qquad (23)
$$

流体共同系での概念図

ラボ系の “t = const.” スライスは傾いている

傾いた床面で積分が定義されたグリーン関数を作れば良い！



疑問

Israel-Stewart モデルは正しいのか？

Israel-Stewart の式をちゃんと解く価値があるのか？

➔ 運動論的な記述と比較すれば判るはず！

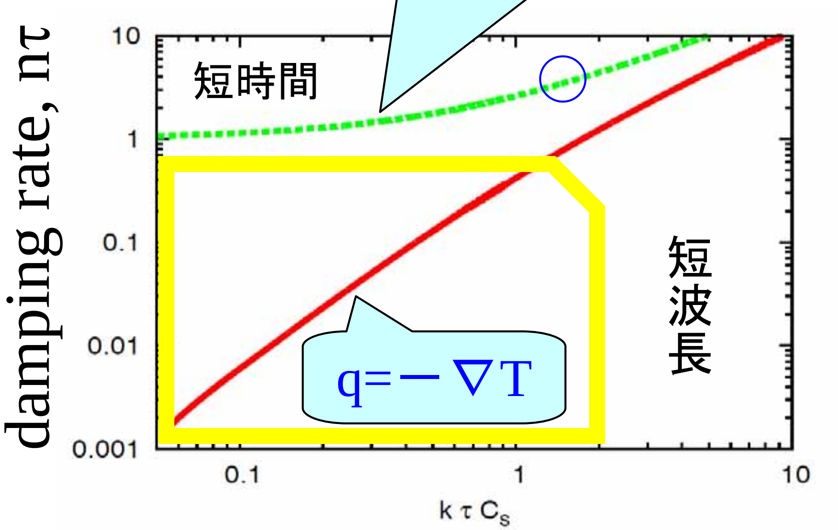
Dispersion Relation for Heat Flow

(low temperature case)

$$\delta \sim \exp(\, i\,k\,x - nt\,)$$

relaxation mode

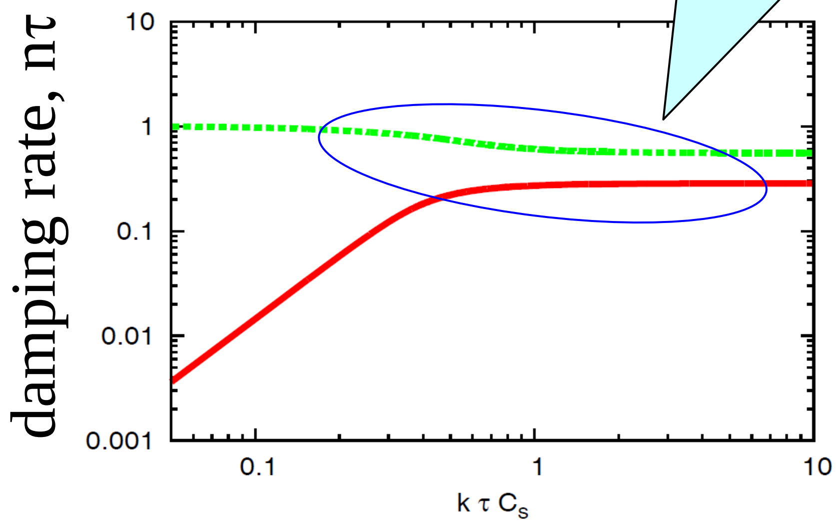
qualitatively inaccurate



wave number, $kc_s\tau$

Kinetic Theory

高本君の修士論文



wave number, $kc_s\tau$

Israel-Stuart Theory
（流体近似）

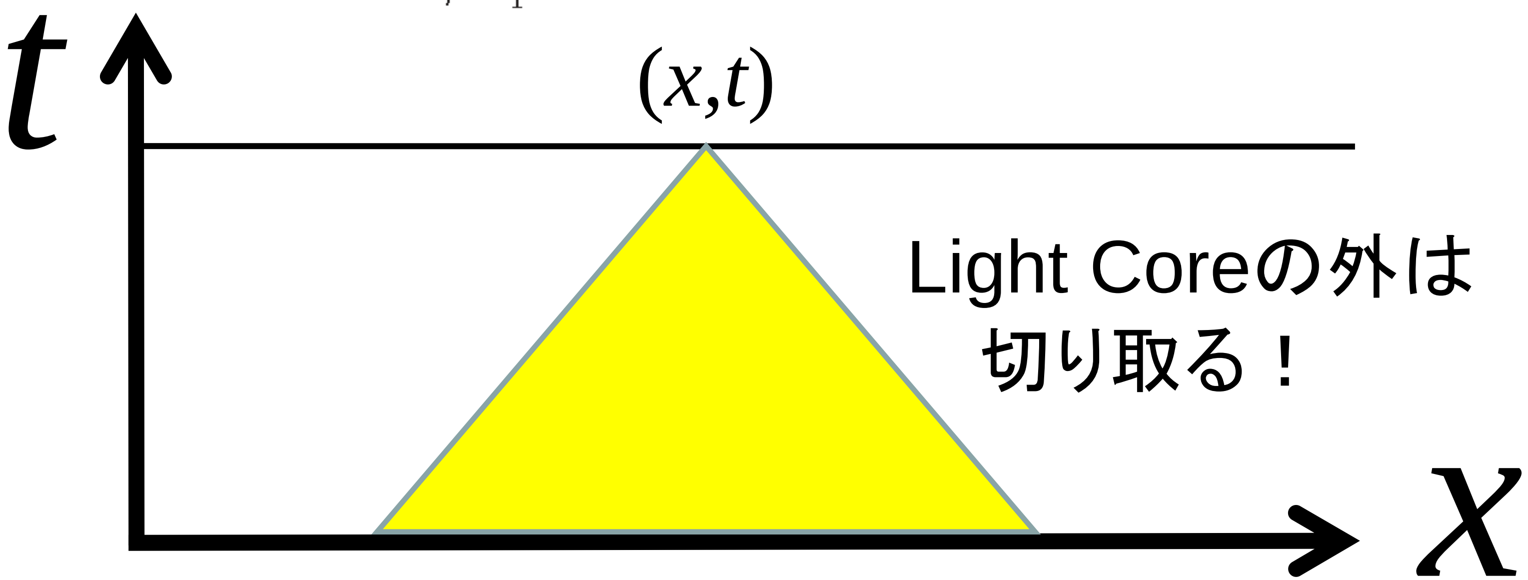
もっと簡単な究極(?)の方法

Israel-Stuart 理論が安定なのは因果律を満たしているからのはず．通常の拡散方程式の Green 関数 $G(x,t) = (2\sqrt{t\pi})^{-1}e^{-x^2/4t}$ を用いると，流体静止の場合 (簡潔にするため，$\kappa = \rho C_{\rm v} = 1$ の unit で)

$$T(x,t) = \int_{-\infty}^{\infty} G(x-\xi,t)T(\xi,t-\Delta t)d\xi, \tag{24}$$

である。そこで、因果律を満たすように，形式解を変形し、

$$T(x,t) = \int_{x-c_\tau\Delta t_1}^{x+c_\tau\Delta t_2} G(x-\xi,\Delta t(\xi))T(\xi,t-\Delta t(\xi))d\xi, \tag{25}$$



Summary

- 熱平衡状態からの種々の構造形成と秩序化
  - 宇宙膨張による冷却➔不安定性➔“散逸構造”
    - 重力不安定性、熱的不安定性, etc.
  - 自然は急激な変化を嫌う！

    … 暗黒の極未来への抵抗？

- 散逸を含む相対論的（磁気）流体力学理論
  - 数値計算手法の構築とその応用へ
  - 天体物理以外への応用
    - 例：クオーク-グルーオン・プラズマ系への応用