Quantum criticality and the search for novel phases

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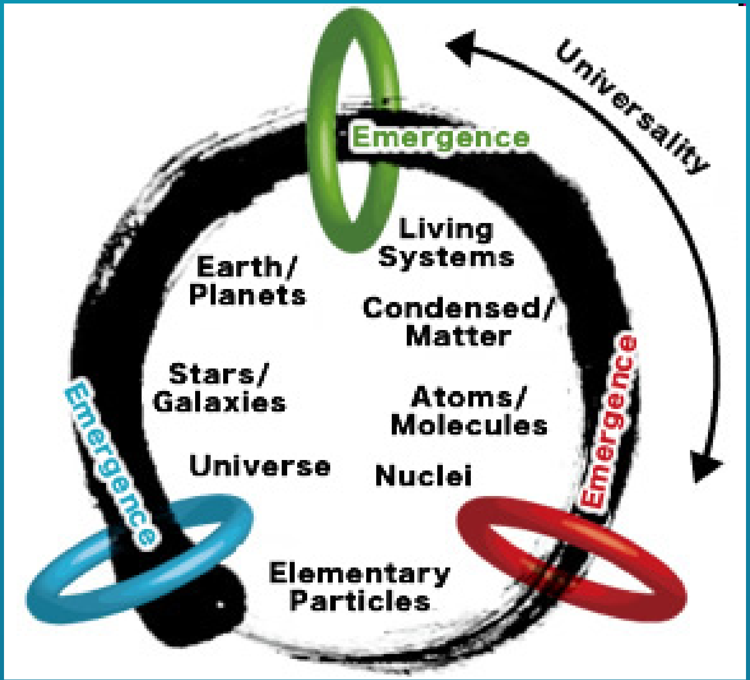
Global Centre of Excellence Symposium:

Kyoto Feb 2010



"Spun from universality and emergence"

Kyoto Feb 2010

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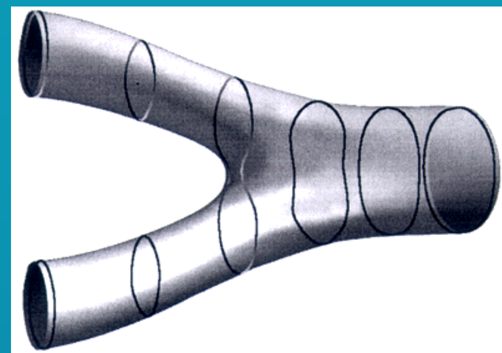
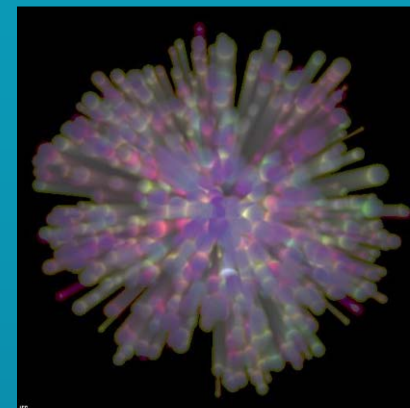
Outline

- The emergence physics of the metallic state
  - Fermionic excitations: The Fermi liquid.
  - Bosonic excitations: Critical fluctuations at a phase transition.
- Quantum criticality: demolishing those foundations
  - What is quantum criticality?
  - A theory in crisis?
    - Destroying the Fermionic quasiparticle at a quantum critical point.
    - Destroying the Landau universality at a quantum critical point.
  - New forms of emergent behaviour

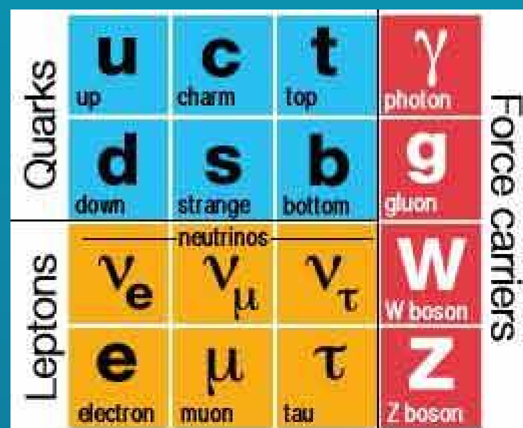
High energy challenge: unravelling emergence

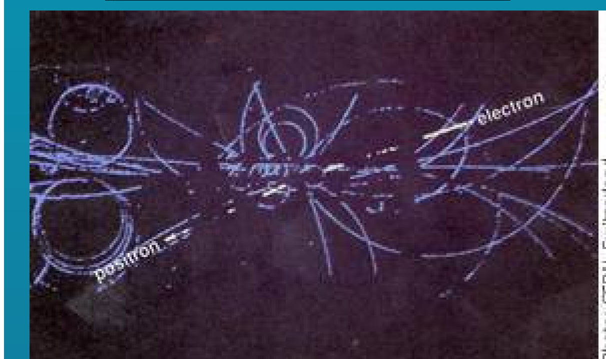
Unification?



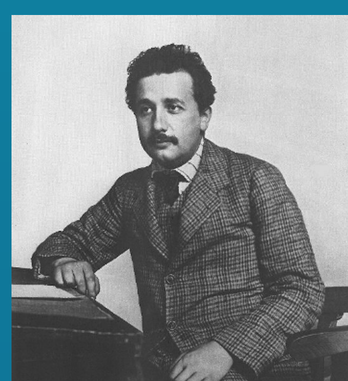


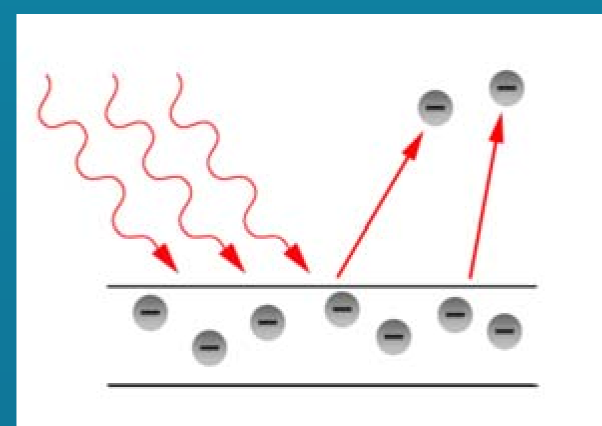
"Standard Model"
+ Einstein's GR





Quantum
theory





Big bang

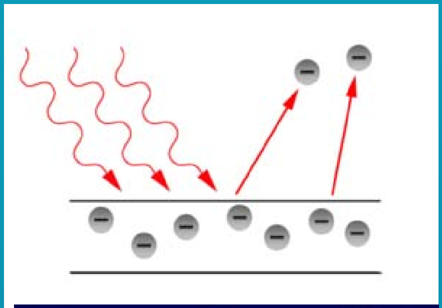
1TeV

10eV

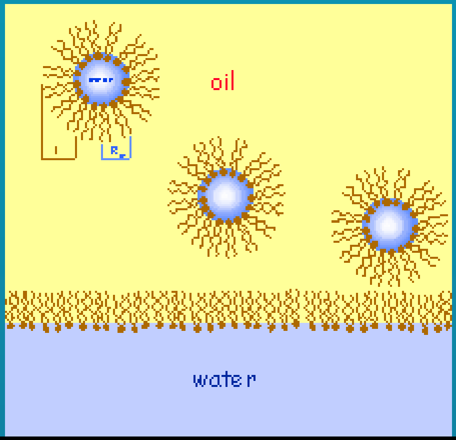
Condensed matter challenge: understanding emergence

"More is different", PW Anderson

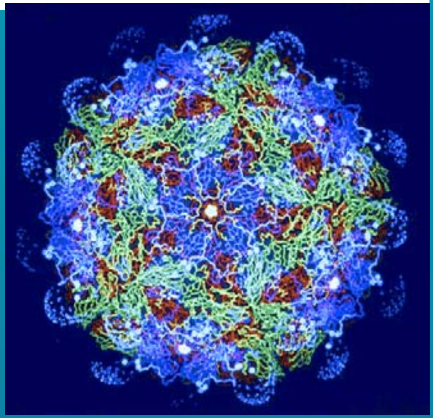
Schrödinger equation



$10\,\mathrm{eV} \sim 10^5\,\mathrm{K}$

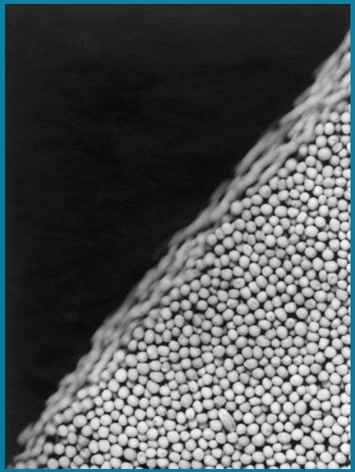


Soft matter
k_B, ($\hbar \sim 0$)

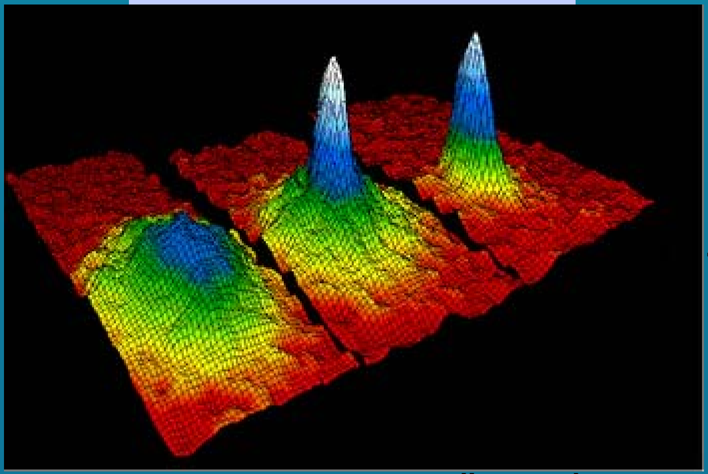


Biological matter

10^2 K

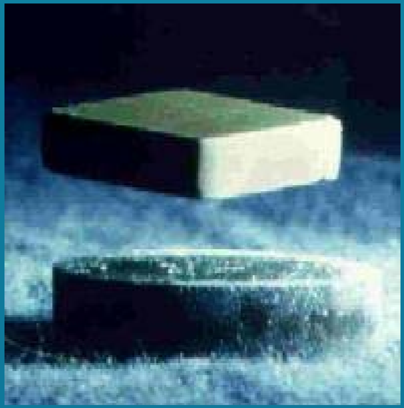


Non-equilibrium



Atom traps (k_B, $\hbar$)

10^{-9} K



1 K

"hard" condensed matter (k_B, $\hbar$)

Two key pillars of our understanding of emergence

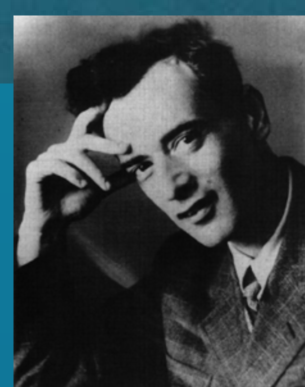
Metals can be understood from electrons

Universality at continuous phase transitions

The electron quasiparticle
Fermi liquid theory

Critical fluctuations
(Quantum criticality)

Fermionic excitations



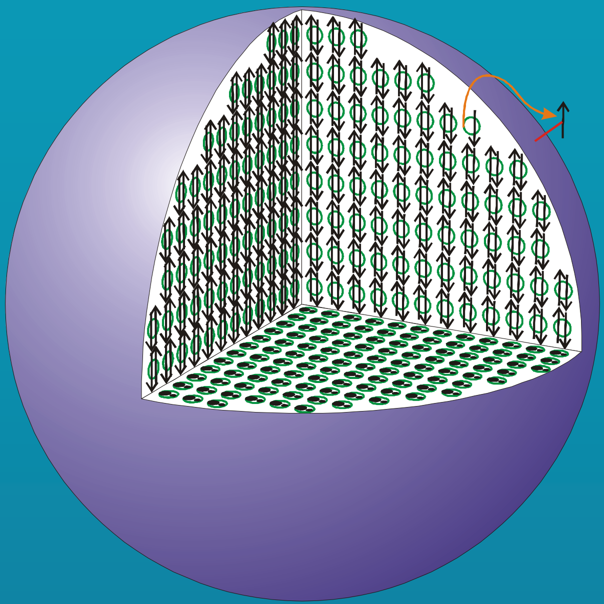
Landau

Bosonic excitations

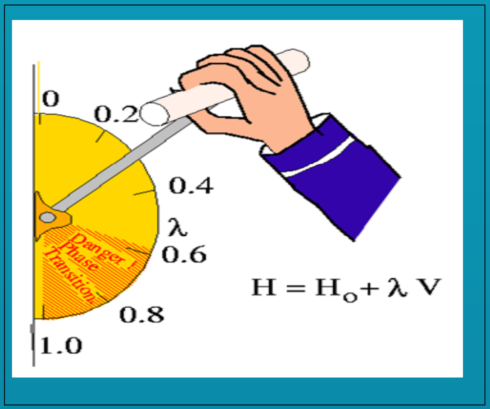
Slide credit: Ashvin Vishwanath

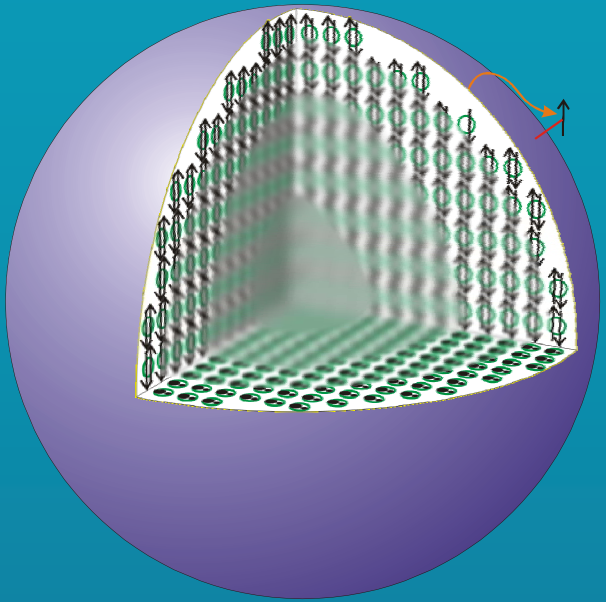
Fermi liquid theory: why electrons “exist” in a metal
Landau: 1956

Free electron gas



Fermi liquid of quasiparticles





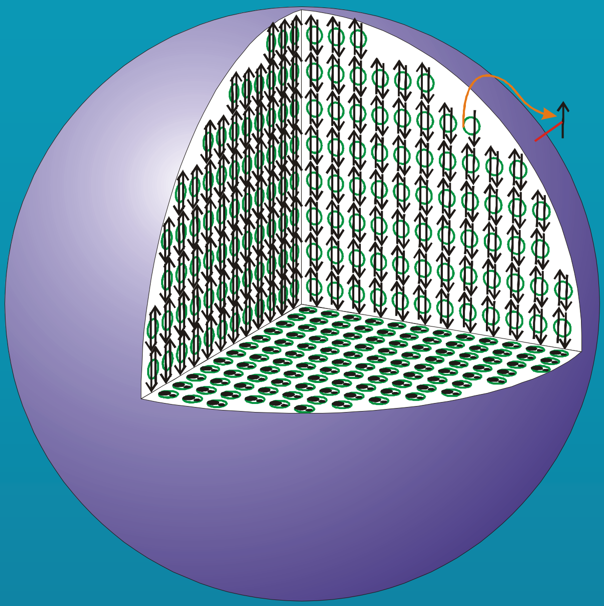
Fermi surface still controls the properties:

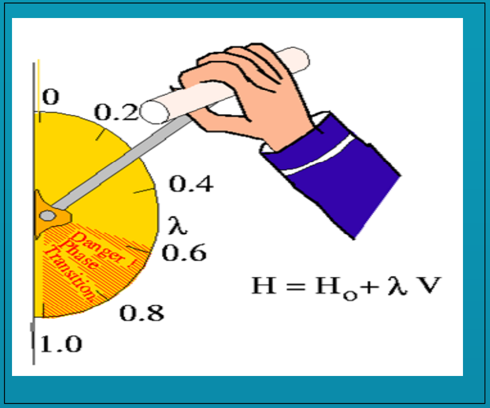
- Interactions preserve number: → Fermi volume unchanged (Luttinger)
- Specific heat: $C_v \sim m\,T$: but mass is renormalized $m \rightarrow m^*$
- Spin susceptibility: $\chi \sim m^*$: but modified by interactions

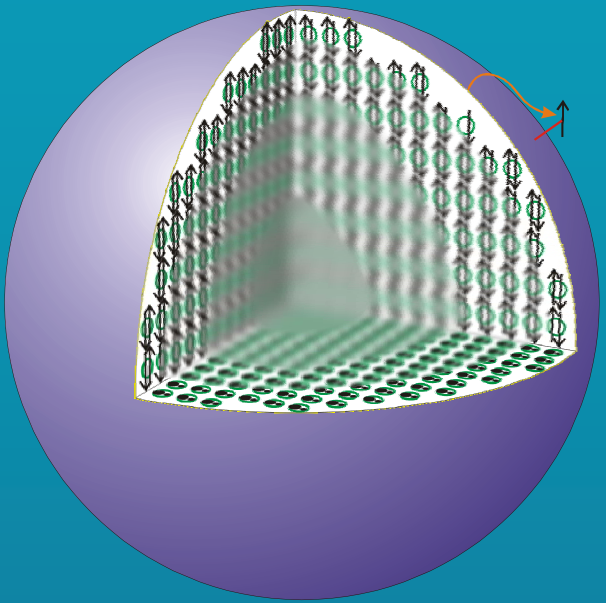
Fermi liquid theory: why electrons "exist" in a metal
Landau: 1956

Free electron gas

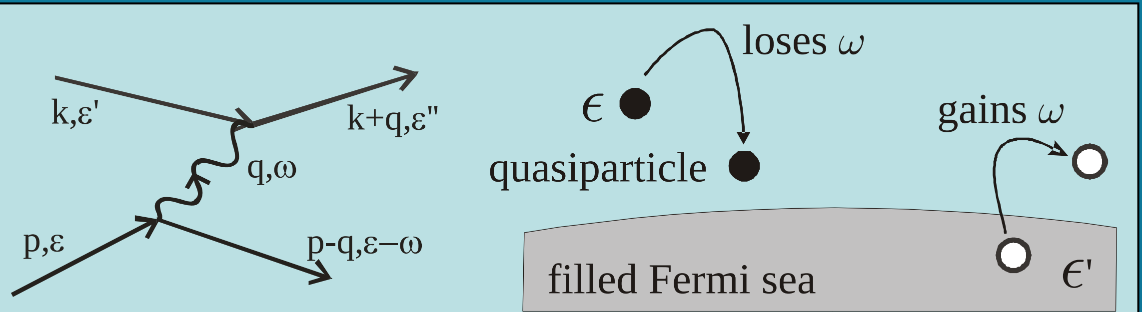
Fermi liquid of quasiparticles





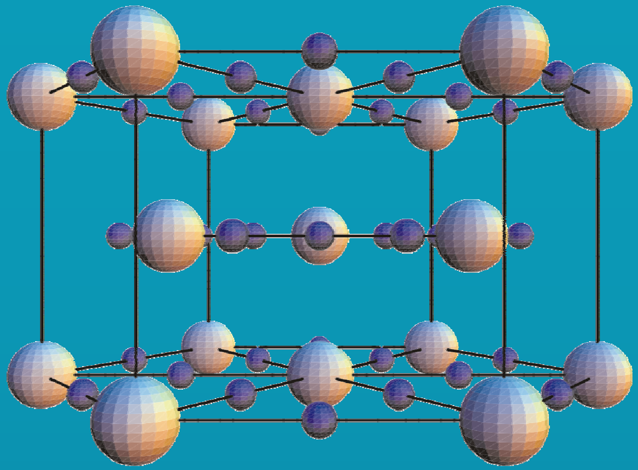


Scattering severely restricted by the Fermi surface – makes theory self-consistent



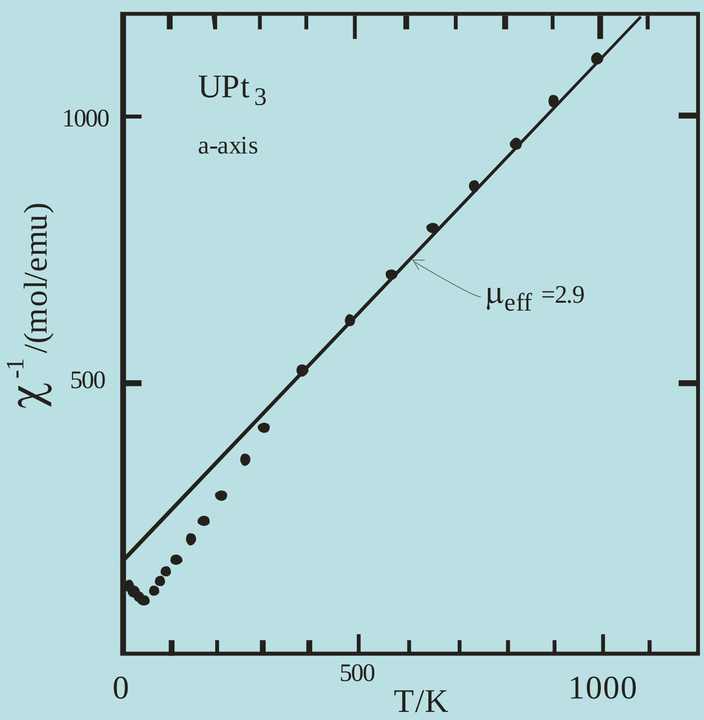
$$\rho \sim T^2$$

Experimental example: UPt_3



A heavy Fermi liquid: U ~ $5f^3$

High temp: f electrons bound to form
a local moment via Hund's rule (~ 1eV):



Frings *et al.*,
JMMM 31-34, 240 (1983).

Experimental example: UPt_3

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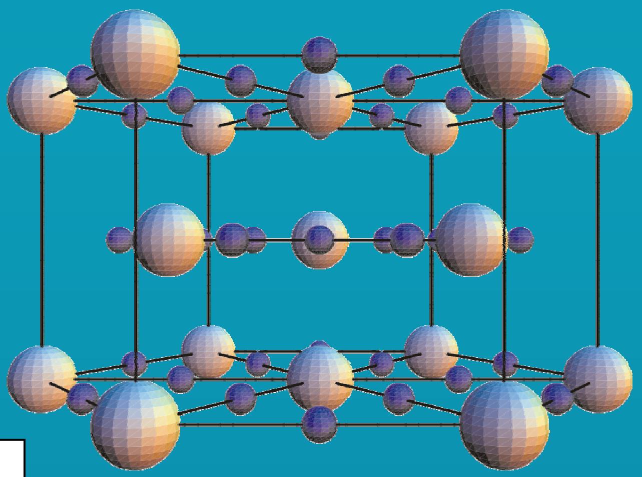
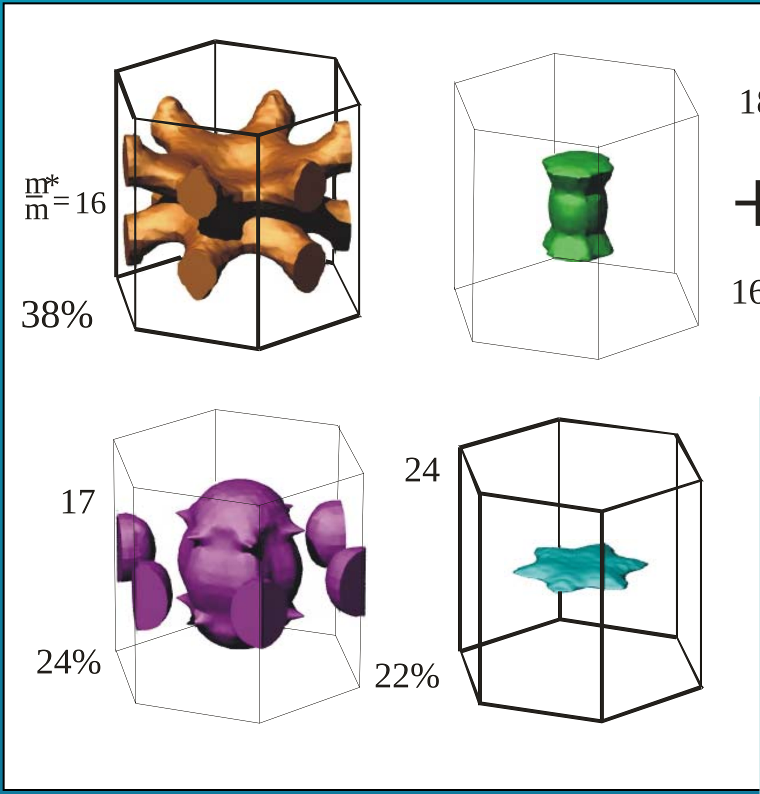
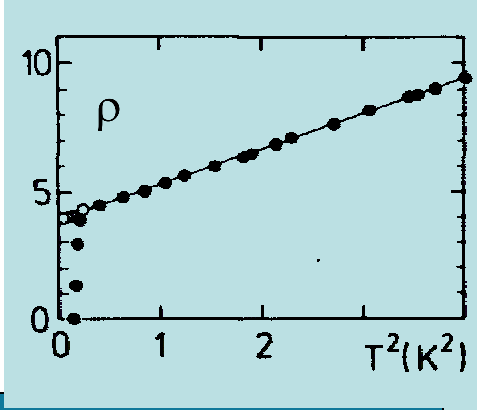
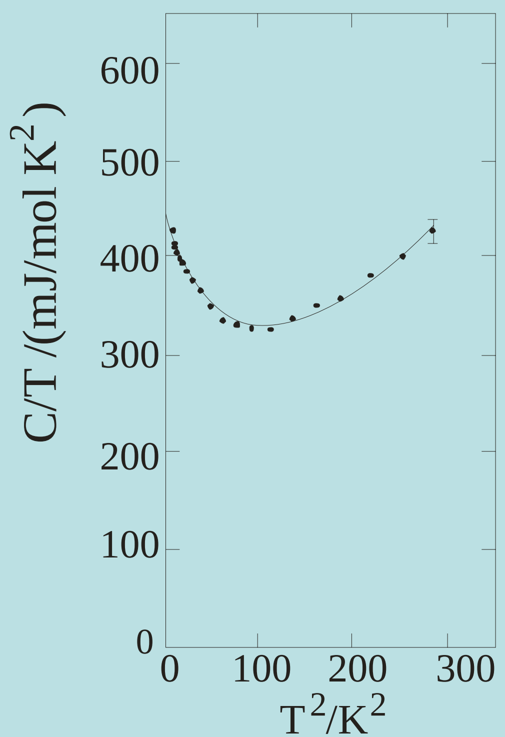


S.R. Julian and G. McMullan, unpublished (1998)





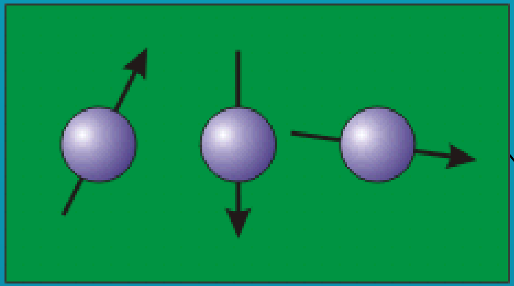


G.R. Stewart *et al.*, PRL, 52, 679 (1984)

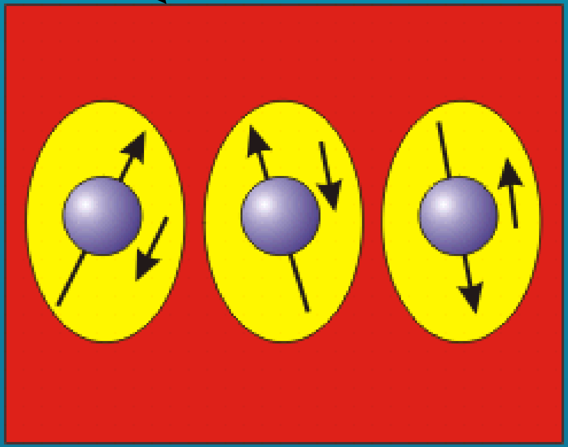
Low temp: f electrons delocalize to make up a heavy Fermi liquid

The Kondo origin of heavy fermion behaviour

High temperature
free magnetic moments
+ conduction electrons
$N_e = N_c$



Kondo
screening



Low temperatures
No free spins but very heavy electrons ($m \sim 10^3\, m_e$) and
$N_e = N_c + N_f$ (large Fermi volume)

“Asymptotic freedom in a cryostat” – Piers Coleman

Two key pillars of our understanding of emergence

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Universality at continuous phase transitions

The electron quasiparticle
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Critical fluctuations
(Quantum criticality)

Fermionic excitations

Landau

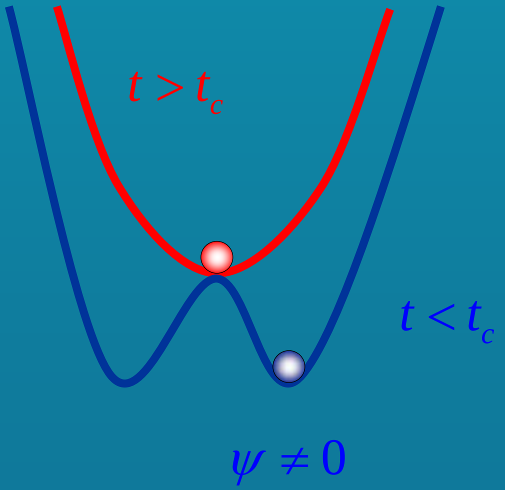
Bosonic excitations

Universality at phase transitions

Landau 1937

$$F = (t - t_c)|\psi|^2 + b|\psi|^4$$

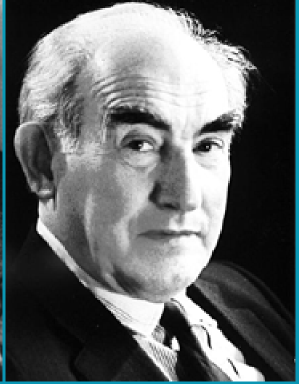
An order parameter: ψ .



Slide credit: Piers Coleman

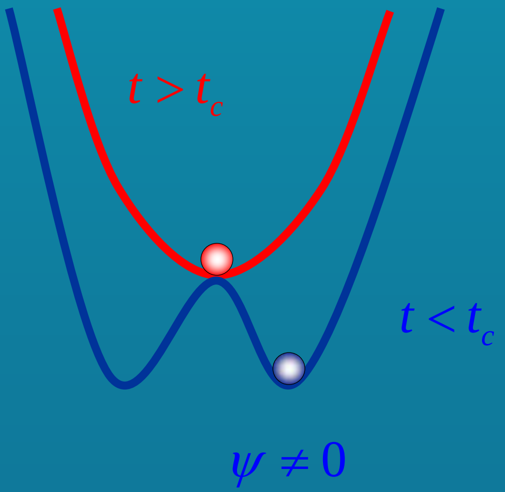
Universality at phase transitions

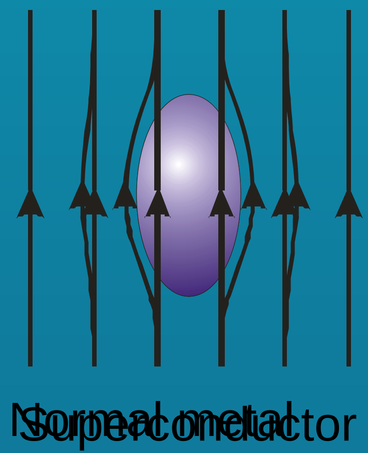
Landau 1937



Ginzburg 1951

$$F = \int d^3x \left[(t - t_c)|\psi|^2 + b|\psi|^4 + |(-i\vec{\nabla} - e^* \vec{A})\psi|^2 \right]$$

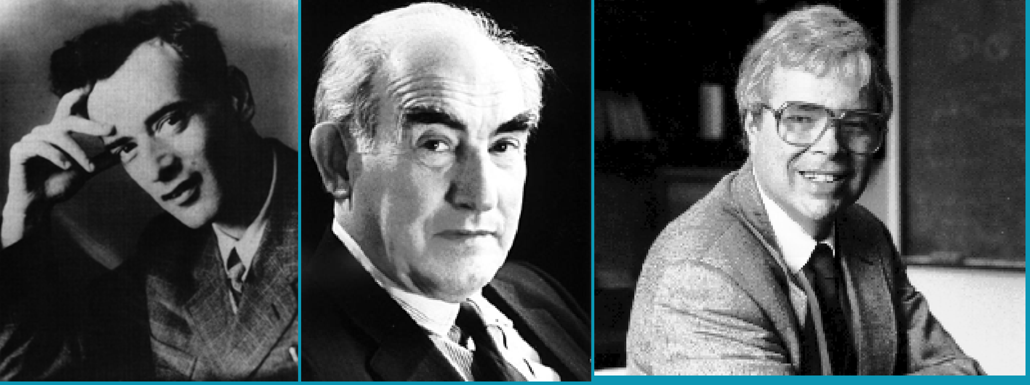




Diverging correlation length: $\xi \sim \dfrac{\xi_0}{(t - t_c)^{1/2}}$

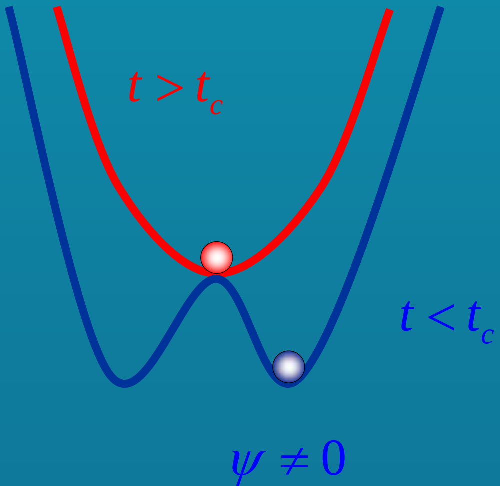
Slide credit: Piers Coleman

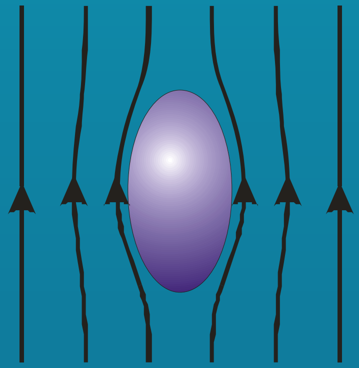
Universality at phase transitions



Landau 1937 Ginzburg 1951 Wilson, Fisher, Kadanoff…'70s

$$F = \int d^3x \left[(t - t_c)|\psi|^2 + b|\psi|^4 + |(-i\vec{\nabla} - e^* \vec{A})\psi|^2 \right]$$





Superconductor

Role of (interacting) fluctuations.

$d<d_u$ (upper critical dimension)

- Dominate…scaling, modified exponents…

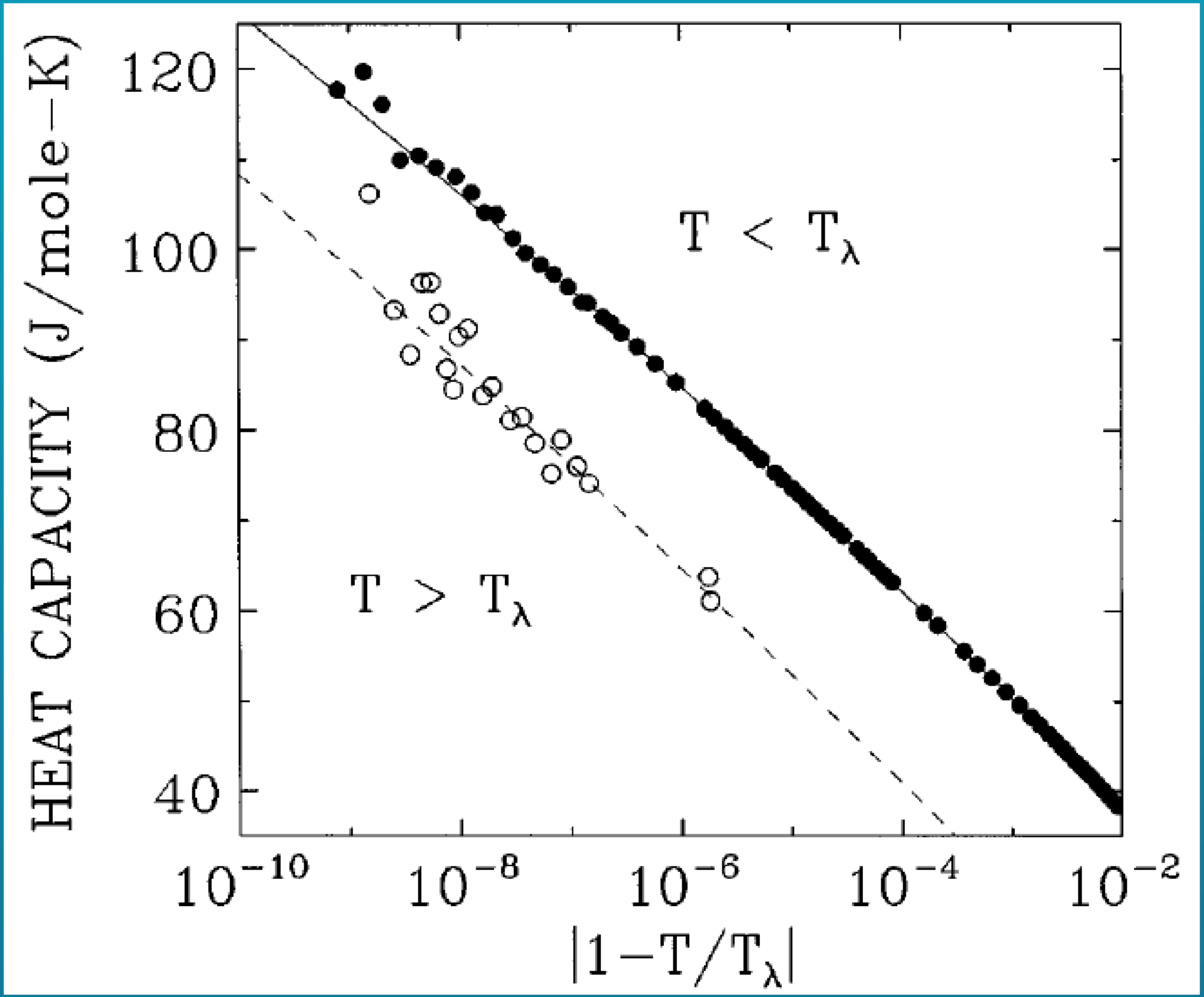
$d=d_u$, $d>d_u$

- Negligible…Gaussian results okay.

Diverging correlation length: $\xi \sim \dfrac{\xi_0}{(t - t_c)^{\nu}}$

Slide credit: Piers Coleman

Theory of classical phase transitions remarkably successful



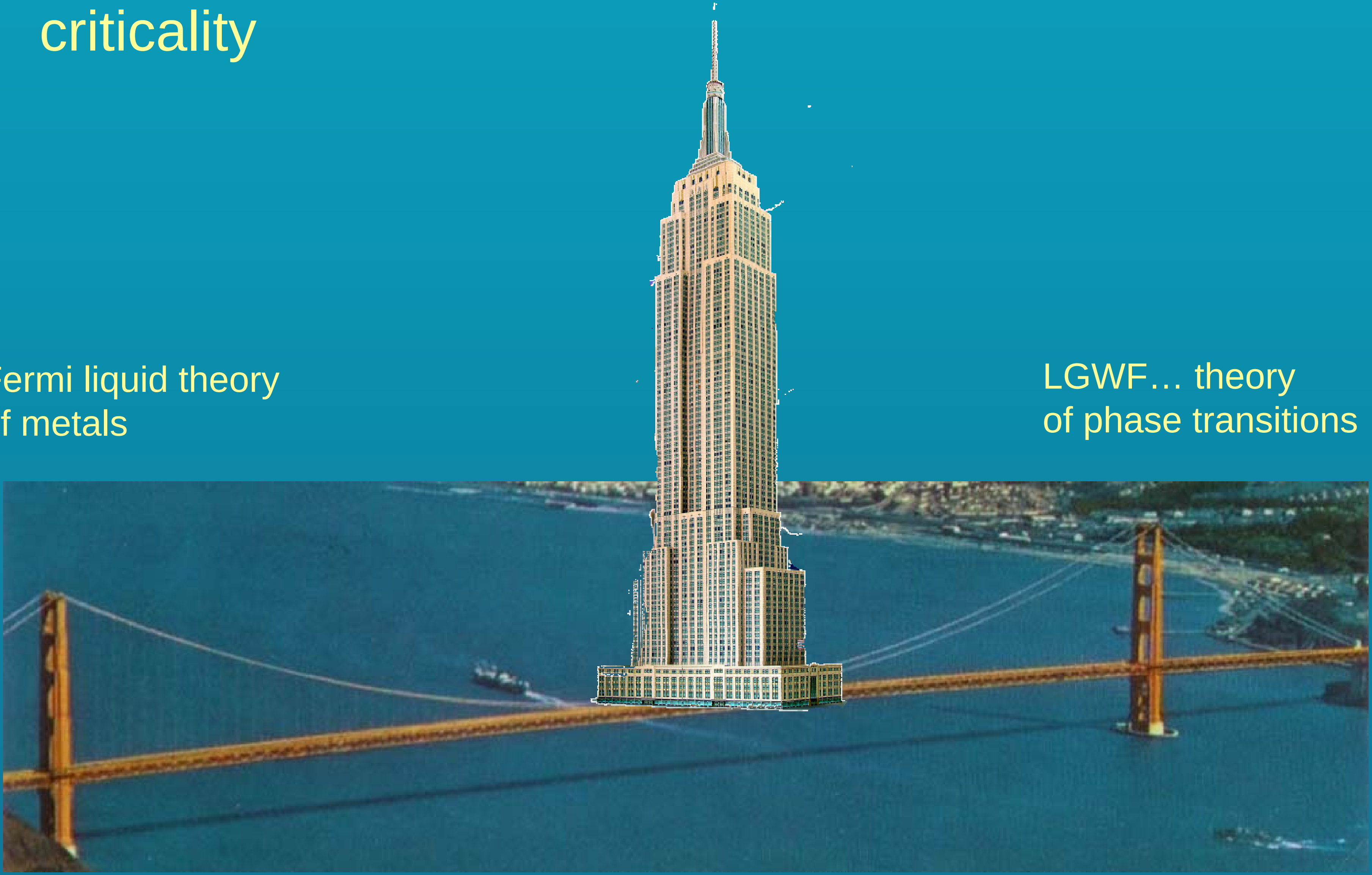
Specific heat of Helium at the lambda transition: $\alpha = 0.01285 \pm 0.00038$

Lipa et al, Phys Rev Lett **76**, 944 (1996).

Pushing these two pillars to the limit: Quantum criticality

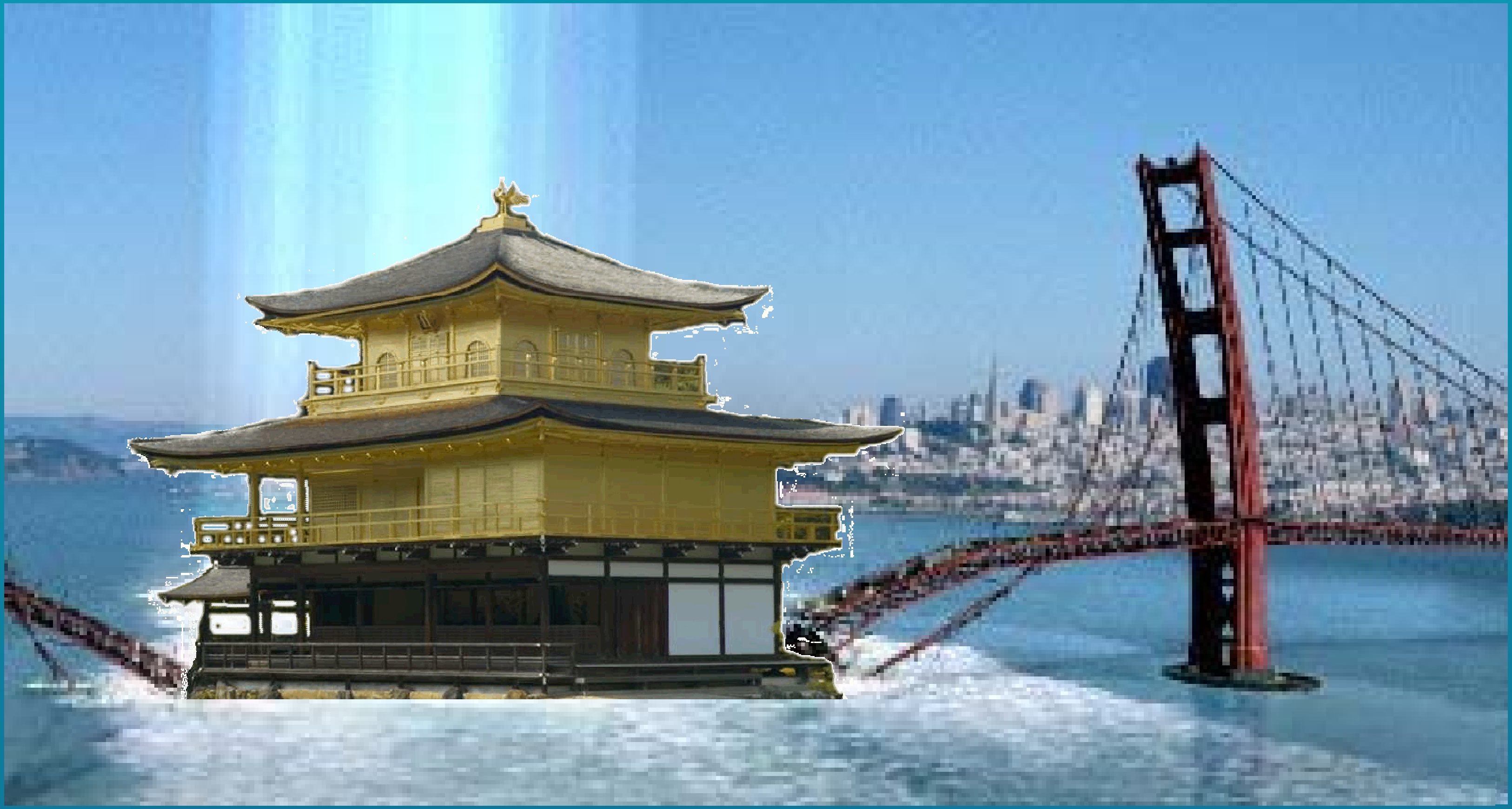
Fermi liquid theory
of metals

LGWF… theory
of phase transitions



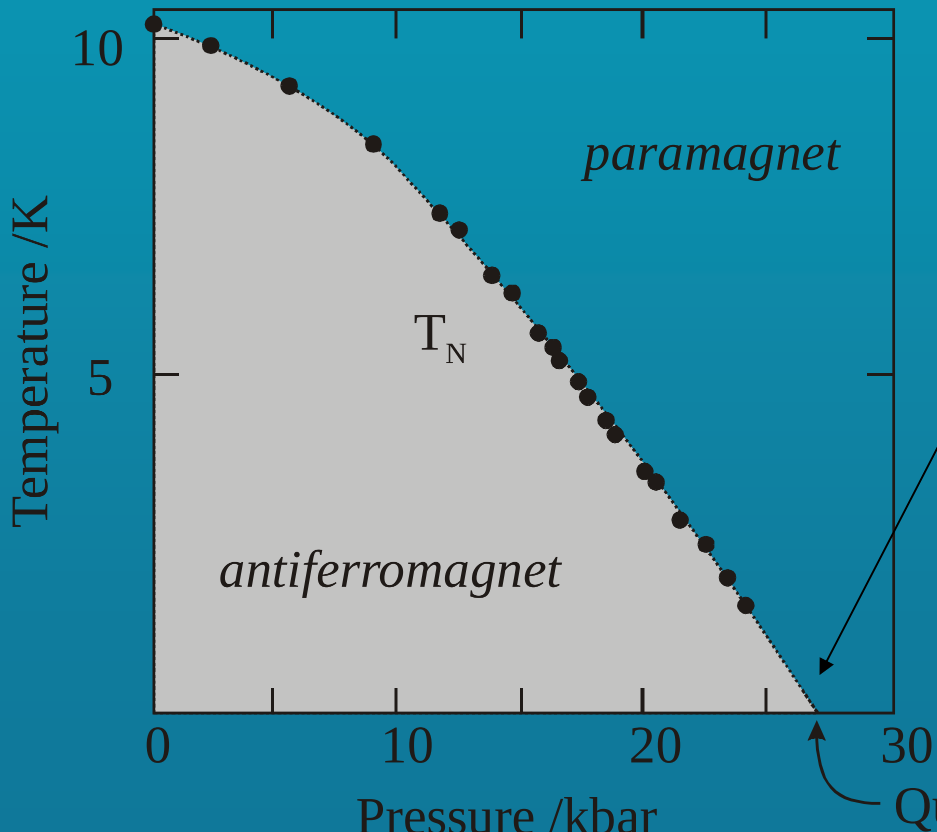
Pushing these two pillars to the limit: Quantum criticality

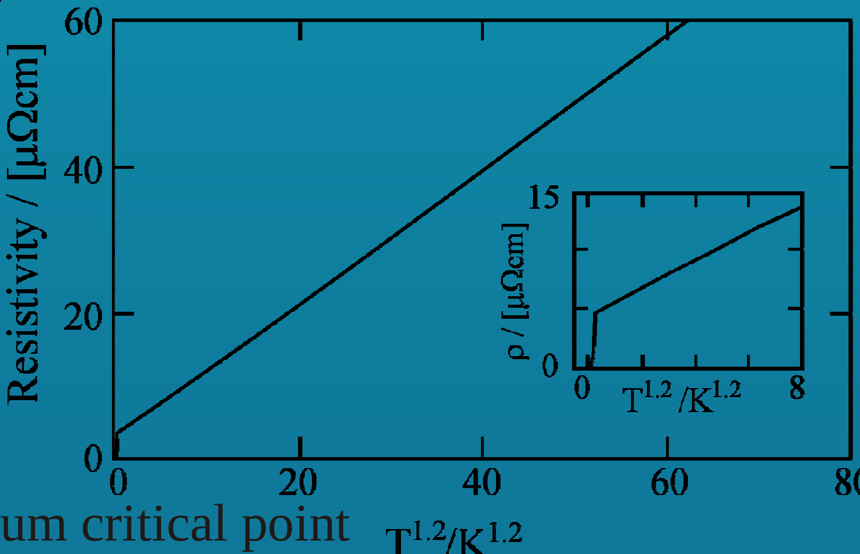
Out of quantum criticality something new can emerge: novel phases



Quantum criticality – a route to a non Fermi liquid

e.g. $CePd_2Si_2$ under pressure



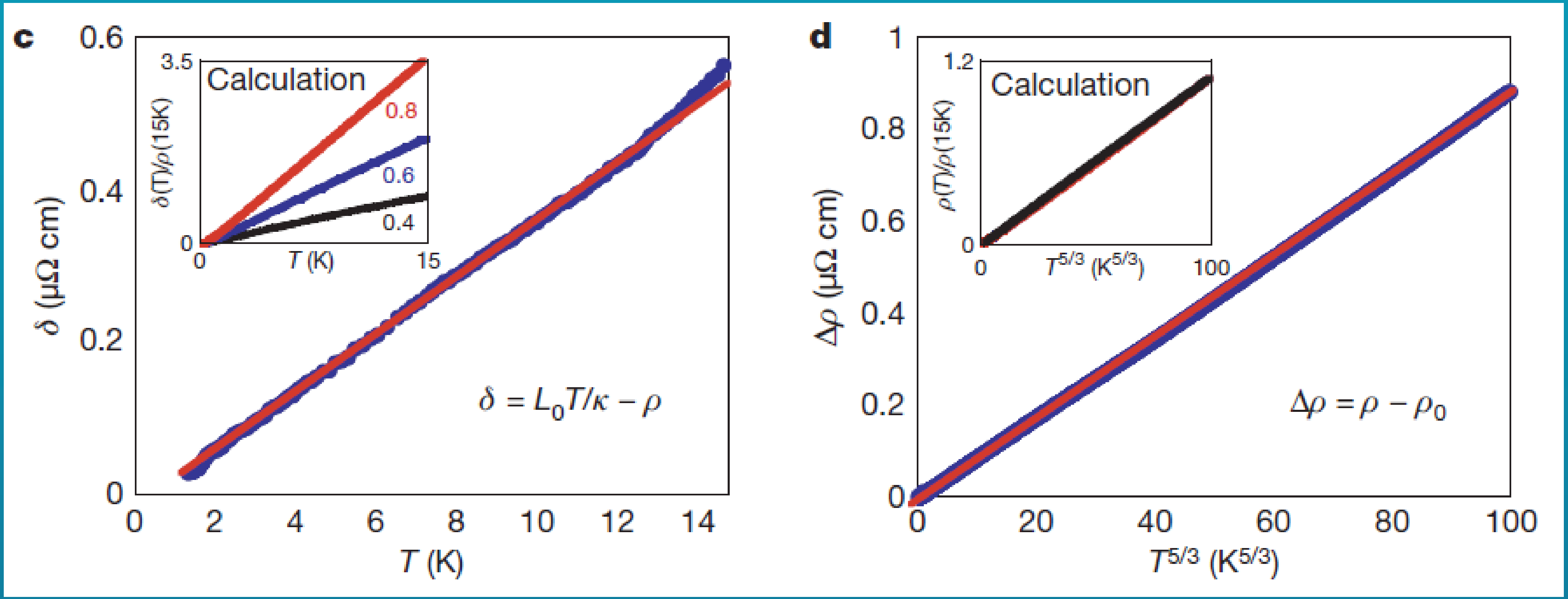


S. R. Julian *et al.* J. Phys: Condens. Matt., **8**, 9675 (1996)

The fermionic excitations - A Fermi liquid pushed to the limit: The marginal Fermi liquid of $ZrZn_2$

Energy scattering rate $\sim T$

Momentum scattering rate $\sim T^{5/3}$

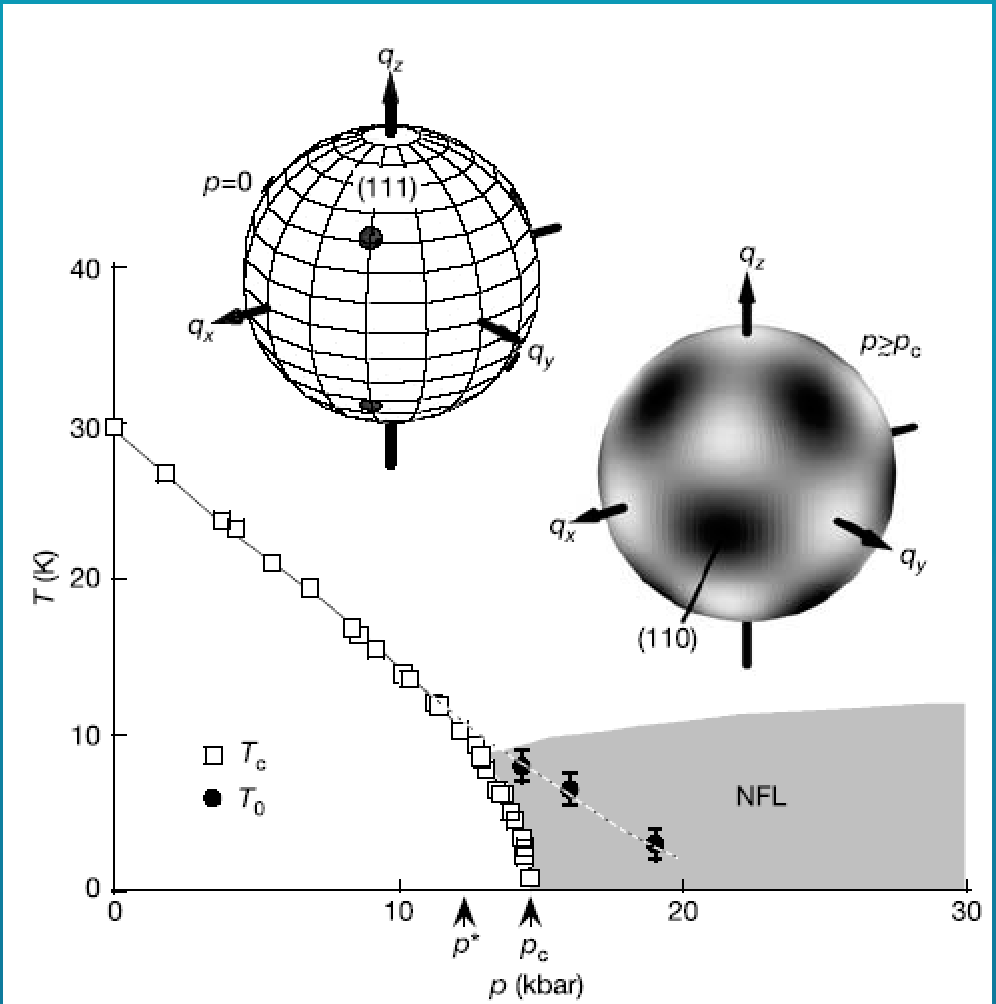


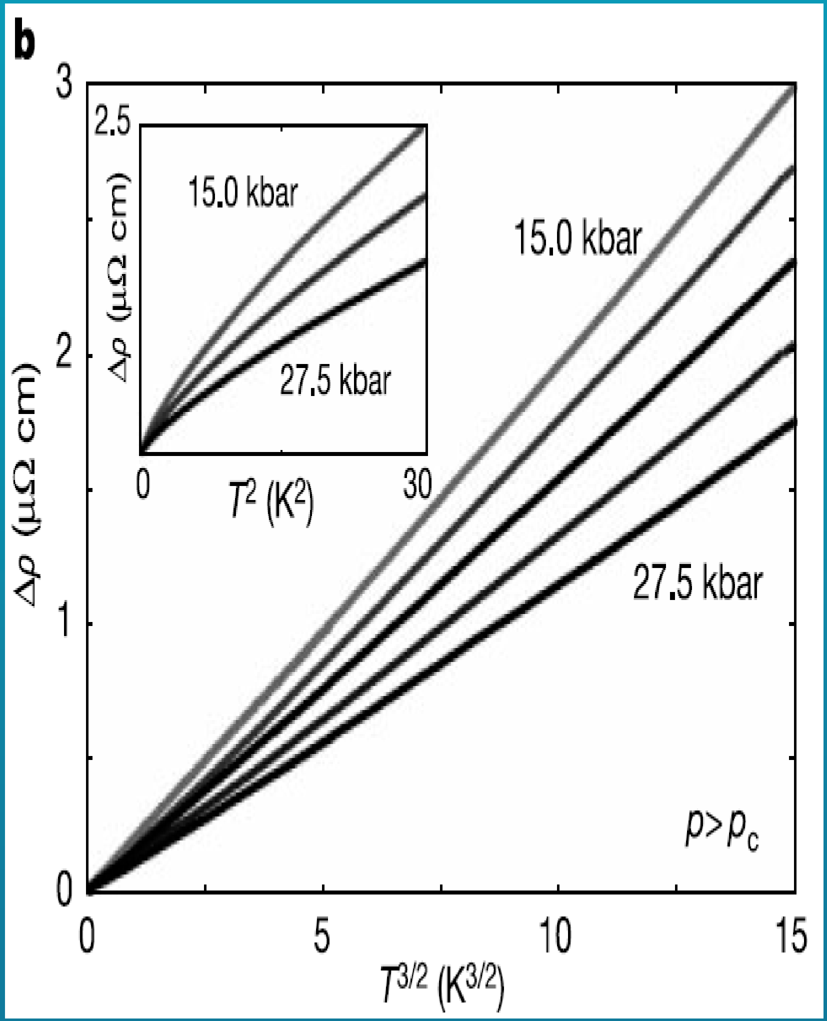
R. P. Smith, M. Sutherland, G. G. Lonzarich, S. S. Saxena, N. Kimura, S. Takashima, M. Nohara and H. Takagi
Nature **455**, *#7217*, 1220-1223 (2008).

Exposing physics normally too weak to see: e.g. Michael Reizer (1989)
Not the physics of a tuned quantum critical point…but of a phase!

See: **Non-Fermi liquids** *A. J. Schofield* Contemp. Phys. **40**, *#2*, 95-115 (1999).

Fermionic quasiparticles destroyed: A quantum critical phase in MnSi?





C. Pfleiderer, D. Reznik, L. Pintschovius, H. v. Lohneysen, M. Garst and A. Rosch, Nature **427**, 227 (2004).

N. Doiron-Leyraud, I. R. Walker, L. Taillefer, M. J. Steiner, S. R. Julian & G. G. Lonzarich, Nature **425**, 595 (2004).

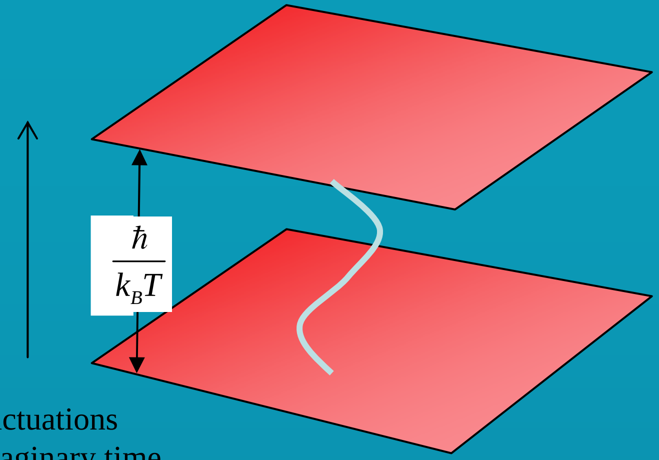
Hertz '76

Quantum criticality

$$Z = e^{-\beta F} = \sum_i e^{-\beta \epsilon_i}$$

$$Z = \sum_i \langle \psi_i | e^{-\beta \hat{H}} | \psi_i \rangle$$

$[\tau]=[L]^z$



Quantum fluctuations
evolve in imaginary time

$$S = \int_0^\beta d\tau \int d^D x \left[(x - x_c)|\psi|^2 + b|\psi|^4 + |\vec{\nabla}\psi|^2 + \text{dissip.} \right]$$

Slide credit: Piers Coleman

Hertz '76

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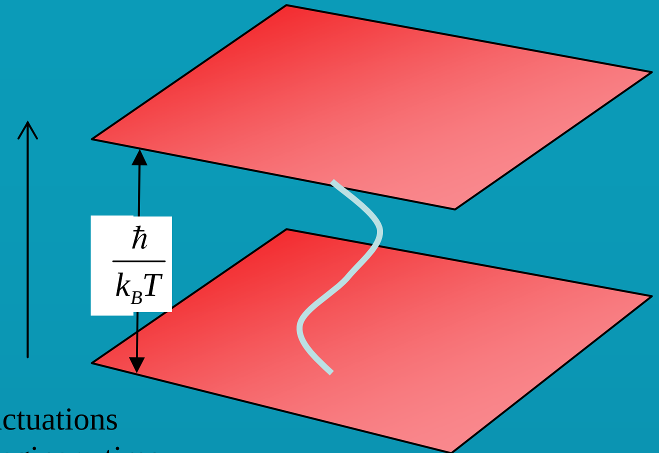

Quantum fluctuations evolve in imaginary time

$$S = \int \sum_\omega d^D q \left[\left((x - x_c) + q^2 + \frac{i\omega}{q^{z-2}} \right) |\psi|^2 + O(b|\psi|^4) \right]$$

$D + z \leq d_u$

$$\tau^{-1} \sim \frac{k_B T}{\hbar}$$

$$\chi''(E) = \frac{1}{E^{1-\alpha}} G\left(\frac{E}{T}\right)$$

α>0 E/T Scaling.

One energy scale- the temperature.

Sachdev and Ye, PRL 69, 2411 (92),
Sachdev, QPT, pp234 (Cambridge, 99)

$D + z > d_u$

$$\tau^{-1} \sim b T^{D/2}, \ (z = 2)$$

$$\alpha = 0$$

"Gaussian fixed point" α=0.

T is not the only energy scale.

Typically, z=2 (antiferromagnet) or 3 (ferromagnet) and d_u=4 so experimental examples should be Gaussian – no scaling..

Hertz '76

Quantum criticality

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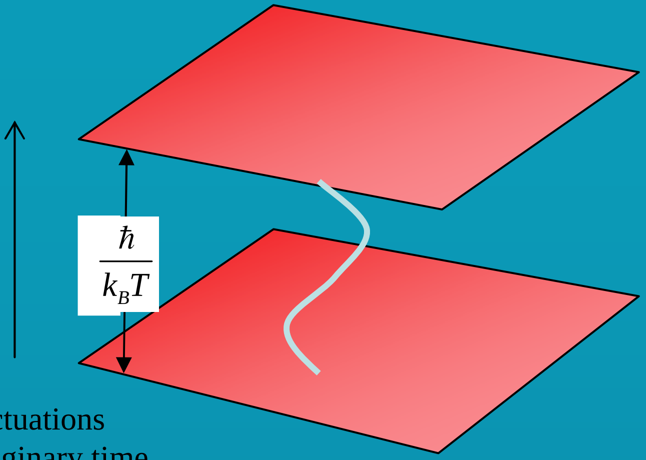

Quantum fluctuations
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$$S = \int \sum_\omega d^D q \left[\left((x - x_c) + q^2 + \frac{i\omega}{q^{z-2}} \right) |\psi|^2 + O(b|\psi|^4) \right]$$

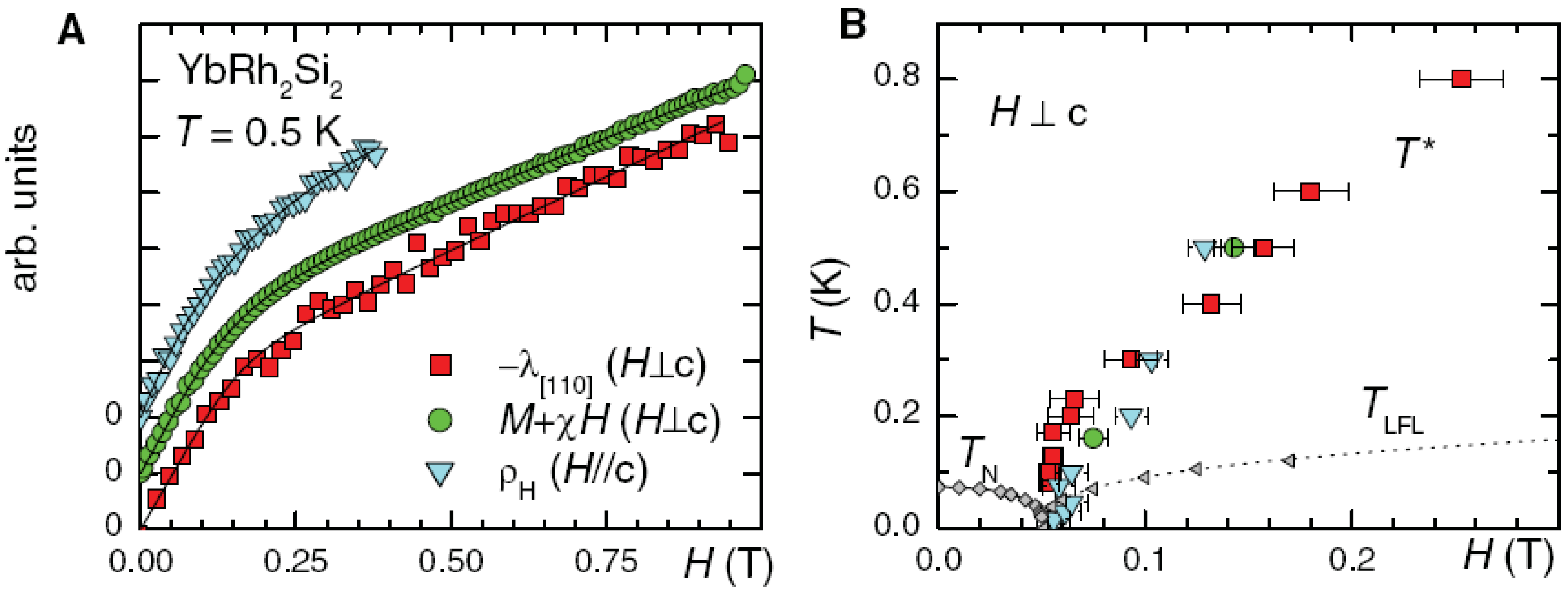
Summary of LGWH theory of quantum criticality

- A single massless mode at the quantum critical point
- All $T \neq 0$ transitions are ultimately classical.
- All quantum critical points should be above their upper critical dimension:
  - Critical exponents can be calculated
  - but no simple E/T scaling

Slide credit: Piers Coleman

Yet - evidence for new physics at a heavy fermion quantum critical point

P. Gegenwart, T. Westerkamp, C. Krellner, Y. Tokiwa, S. Paschen, C. Geibel, F. Steglich, E. Abrahams, Q. Si Science **315**, *#5814*, 969-971 (2007).



An additional energy scale appears to converge on the quantum phase transition

Summary – quantum criticality
The "Standard Model" in crisis

- Initial experiments seemed to agree with theory:
  - e.g. $Pd_{1-x}Ni_x$ (a disordered ferromagnetic qcp)
- But now there are now clear examples of failure:
  - Clean ferromagnets always first order (e.g. $ZrZn_2$)
  - A non-Fermi liquid phase in MnSi
  - E/T scaling and local criticality in $CeCu_{6-x}Au_x$
  - Multiple energy scales in $YbRh_2Si_2$
- Novel theoretical ideas emerging:
  - Fluctuations driving feromagnets first order
  - Kondo competing with RKKY
    - Local criticality, spin-charge separation, supersymmetry…
  - New excitations: deconfined criticality

Yet from the crisis – new forms of emergent behaviour at a quantum critical point

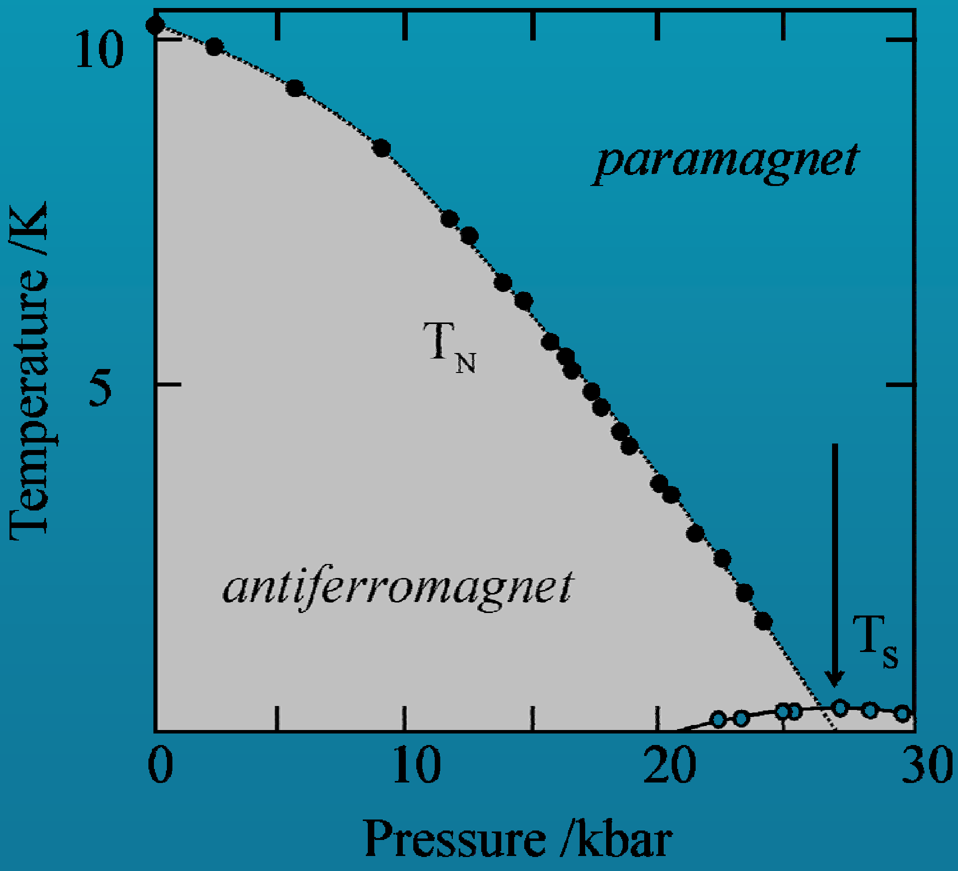
The singularity at the QCP…

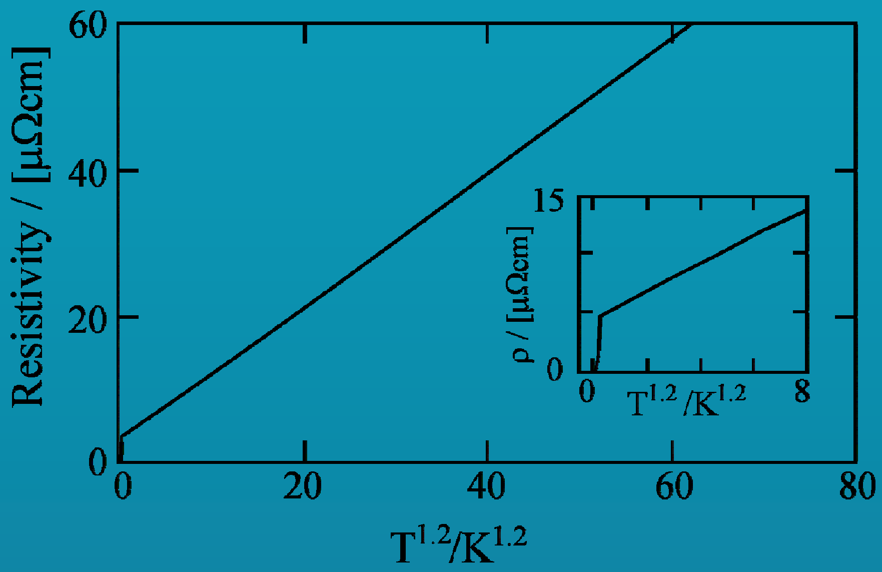
…might be to your advantage

Right image idea: Malte Grosche

Quantum criticality – a route to superconductivity

$CePd_2Si_2$

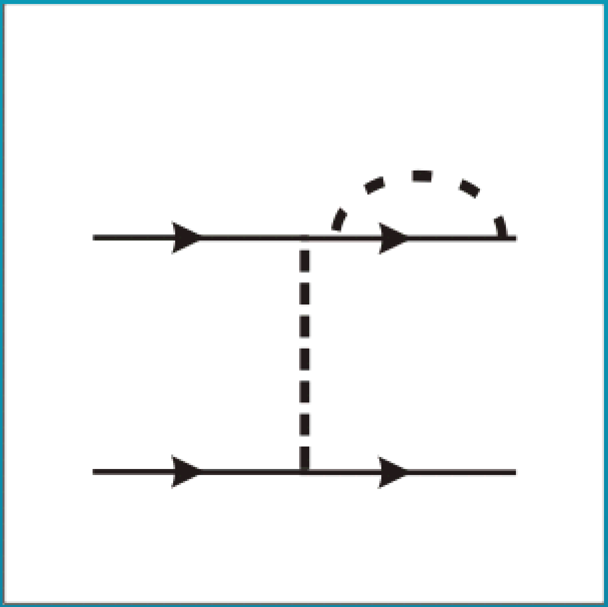




$$\rho \sim T^{1+\epsilon}$$

Quantum criticality drives superconductivity

"Lonzarich's Rules" for finding spin-mediated superconductors



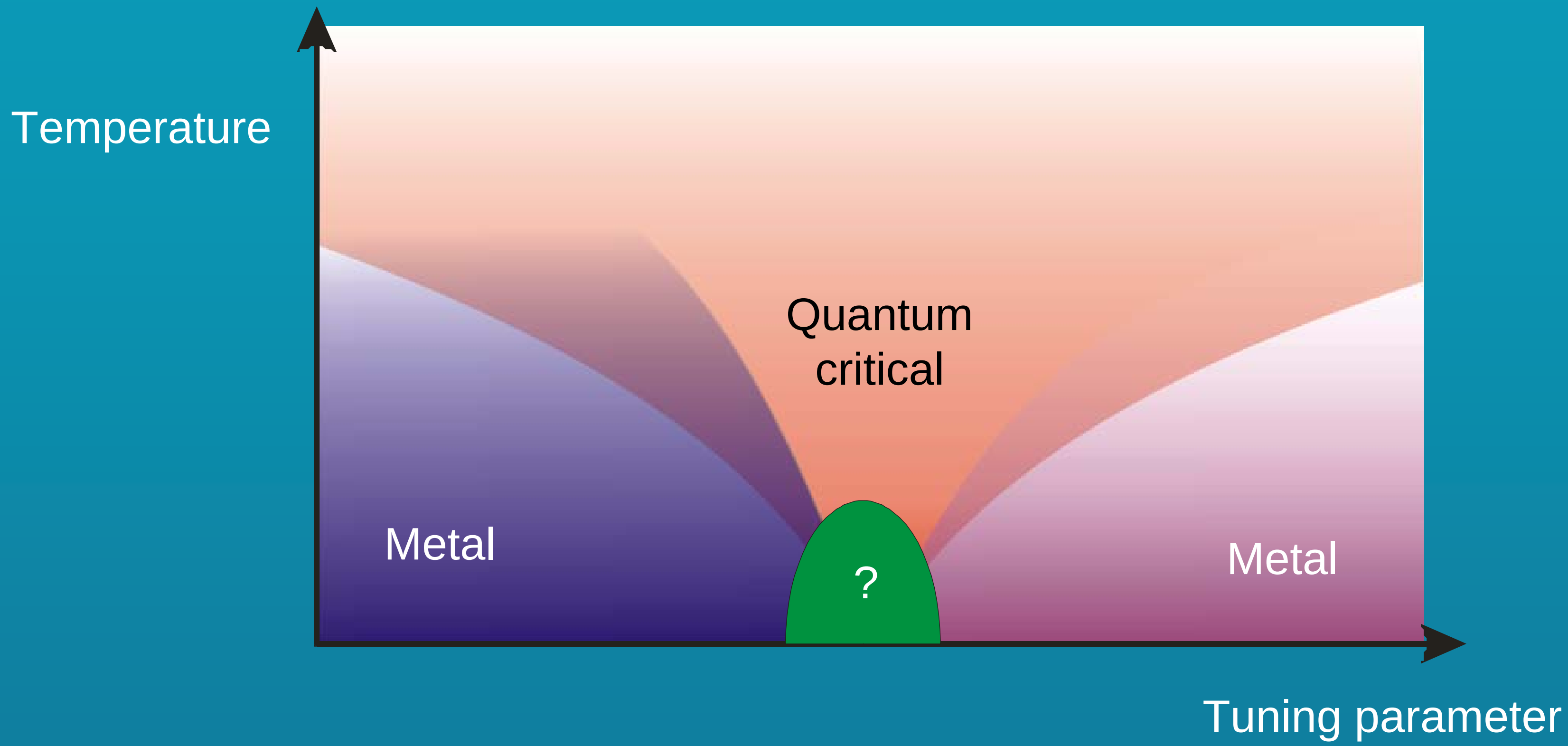
$$T_c \sim T_{\mathrm{sf}} e^{-\frac{1+\lambda_z}{\lambda_\Delta}}$$

- Look on the border of magnetism (i.e. at a QCP)
- AFM preferred to FM
- If FM, then uniaxial
- 2D better than 3D
- Want a large T_{sf}
- Single band or nested multiband is good.
- Avoid joint AFM and FM fluctuations

P. Monthoux, D. Pines and G. G. Lonzarich Nature **450**, #*7173*, 1177-1183 (2007)

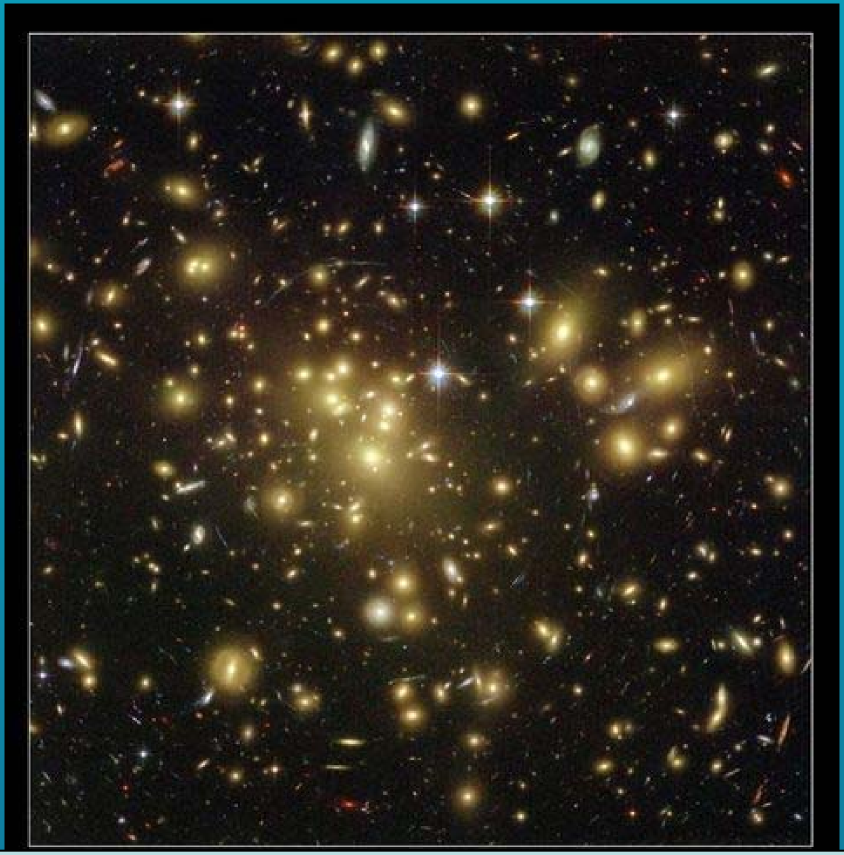
A. J. Schofield: arXiv:1001.4279v1.

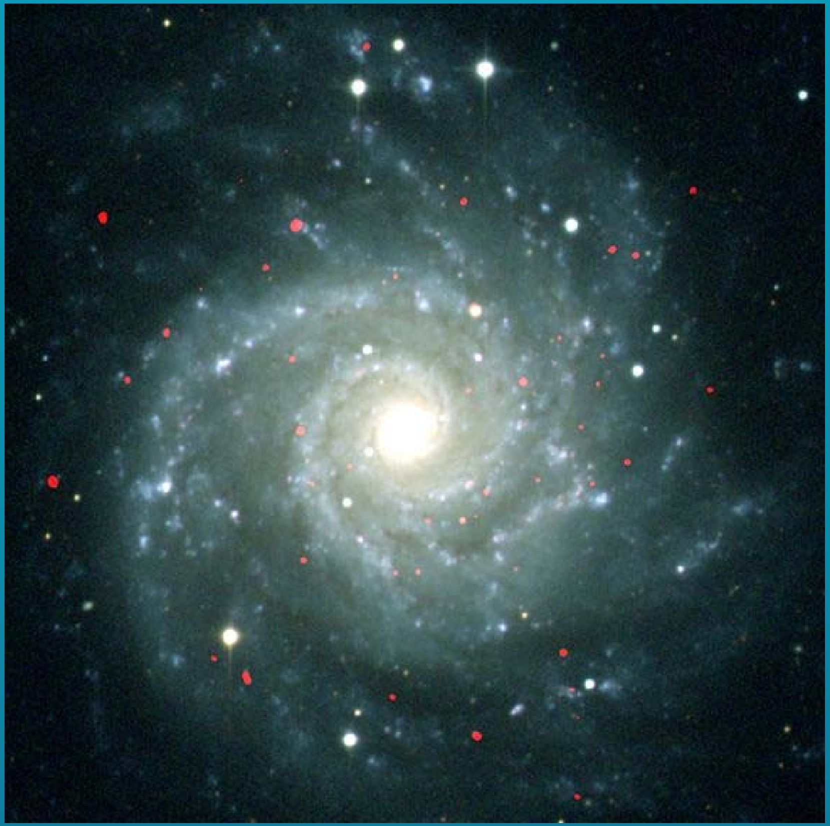
New forms of emergence beneath a quantum critical region



Experimentally, there are a growing number of “dark order” states …

The dark matter problem

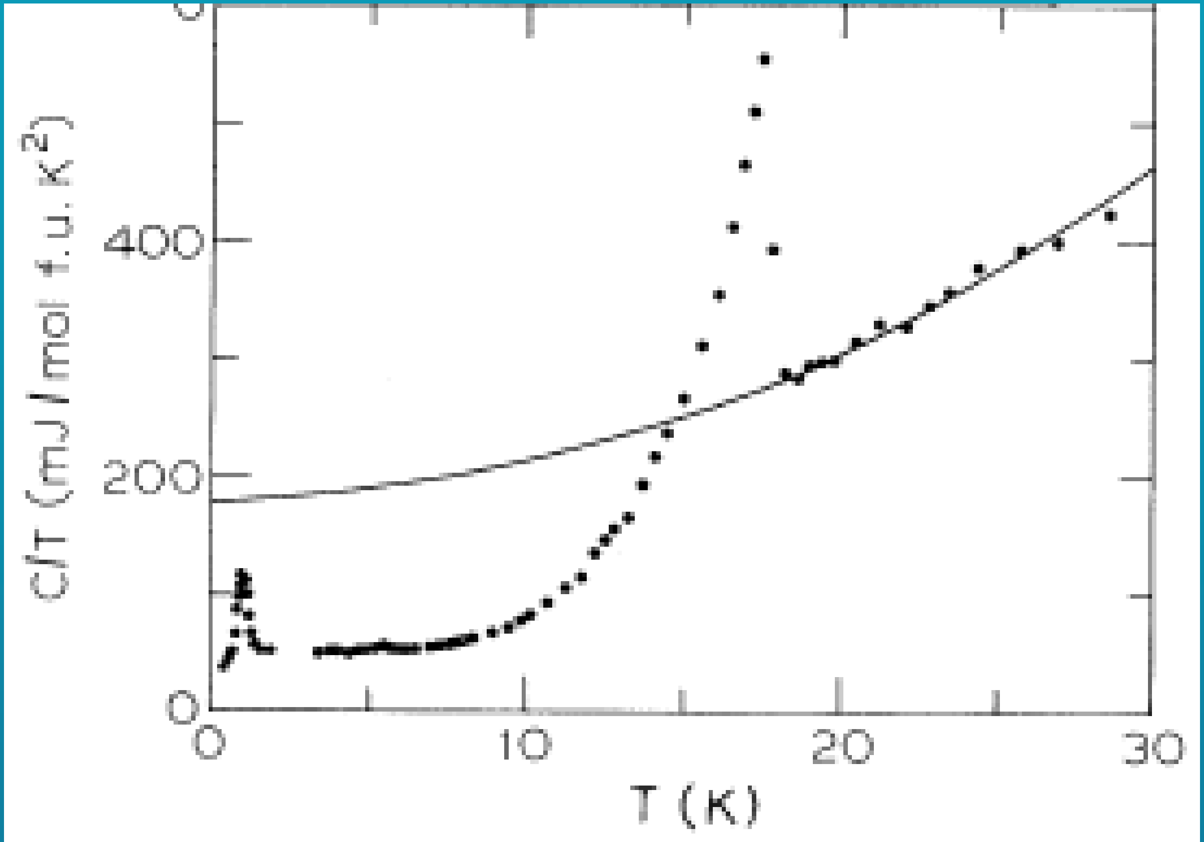




- Zwicky (1933) Viral theorem in clusters,
- Rubin & Ford (1965) galactic rotation curves, …

Dark Matter: has a gravitational effect but is transparent to the current observational probes… unless you know what to look for.

The dark order problem



URu_2Si_2: T. T. M. Palstra, A. A. Menovsky, J. van den Berg, A. J. Dirkmaat, P. H. Kes, G. J. Nieuwenhuys and J. A. Mydosh Physical Review Letters **55**, 2727 (1985)

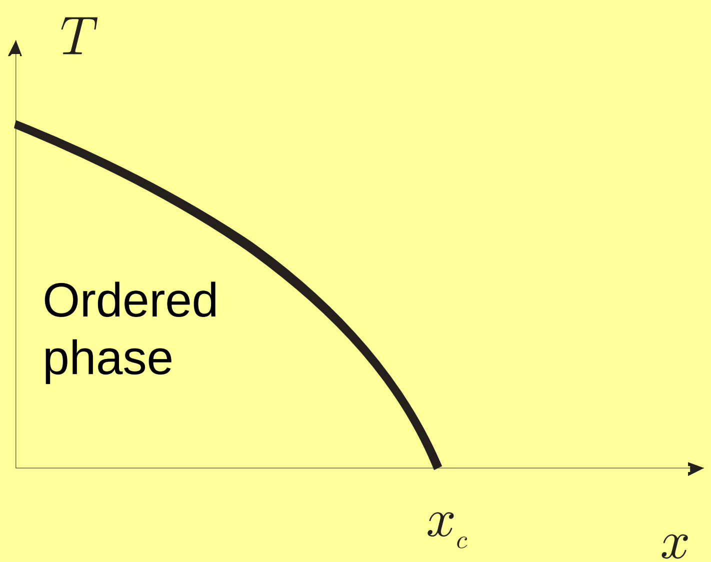
- URu_2Si_2 UCu_5, $Sr_3Ru_2O_7$, cuprates(?) …

Dark order: ordered states that have a thermodynamic effect but whose order parameter is transparent to current probes.

Dark order at the quantum critical end-point

Conventional view: an itinerant picture of density wave order developing in a metal.
J. A. Hertz Phys. Rev. B **14**, 1165 (1976).

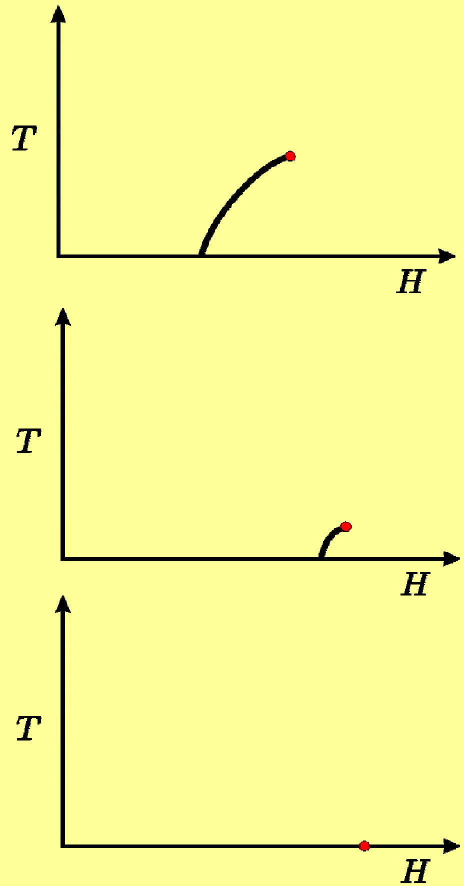
Continuous phase transition driven to T=0



Example: $\mathbf{CePd_2Si_2}$
Antiferromagnetism tuned by pressure.

[S. R. Julian *et. al.* J. Phys. C. (1996)]

Critical end-point driven to T=0

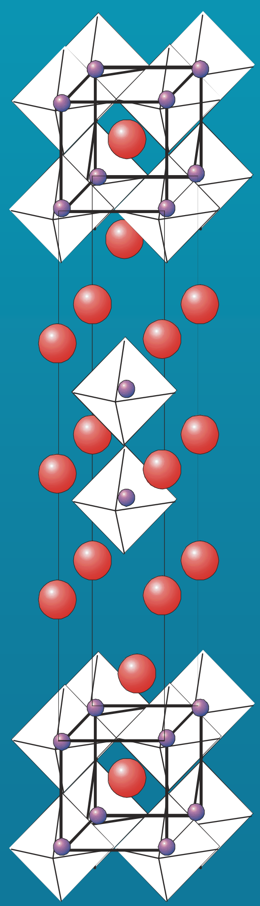


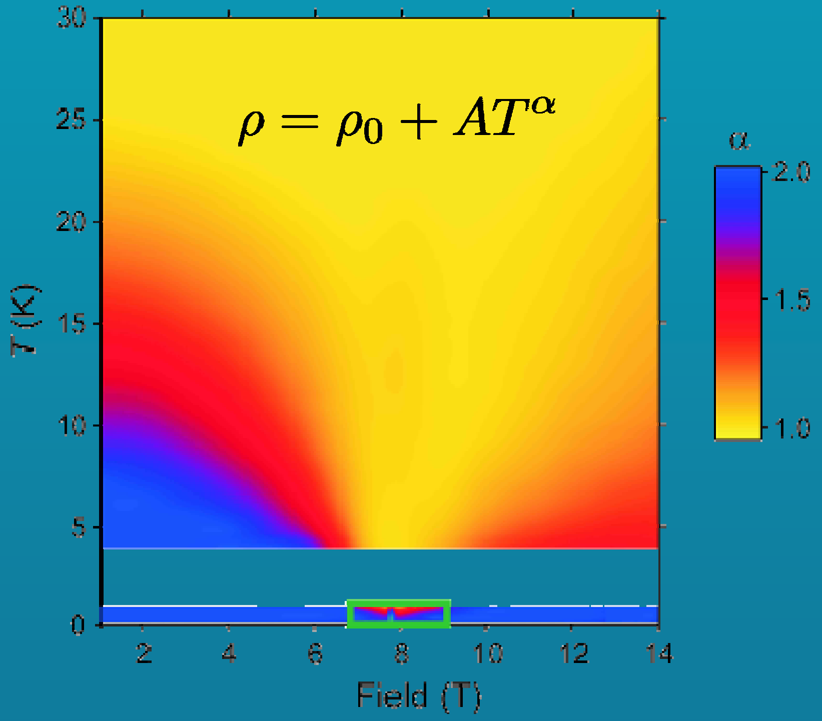
Example: $\mathbf{Sr_3Ru_2O_7}$ Metamagnetism tuned by magnetic field angle

[S.A. Grigera *et al.*, Science (2001)]

The metamagnetic quantum critical end-point

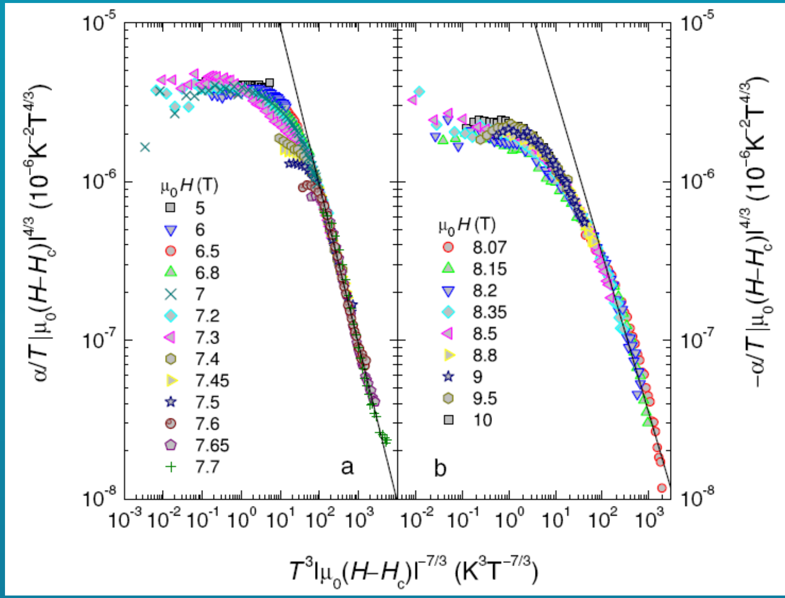
Theory of the metamagnetic quantum critical endpoint:
A.J. Millis, A. J. Schofield, G.G. Lonzarich and S.A. Grigera, Phys. Rev. Lett. **88**, 217204 (2002)





S.A.Grigera, R.S.Perry, A.J.Schofield, M.Chiao, S.R.Julian, G.G.Lonzarich, S.I.Ikeda, Y.Maeno, A.J.Millis, A.P.Mackenzie,

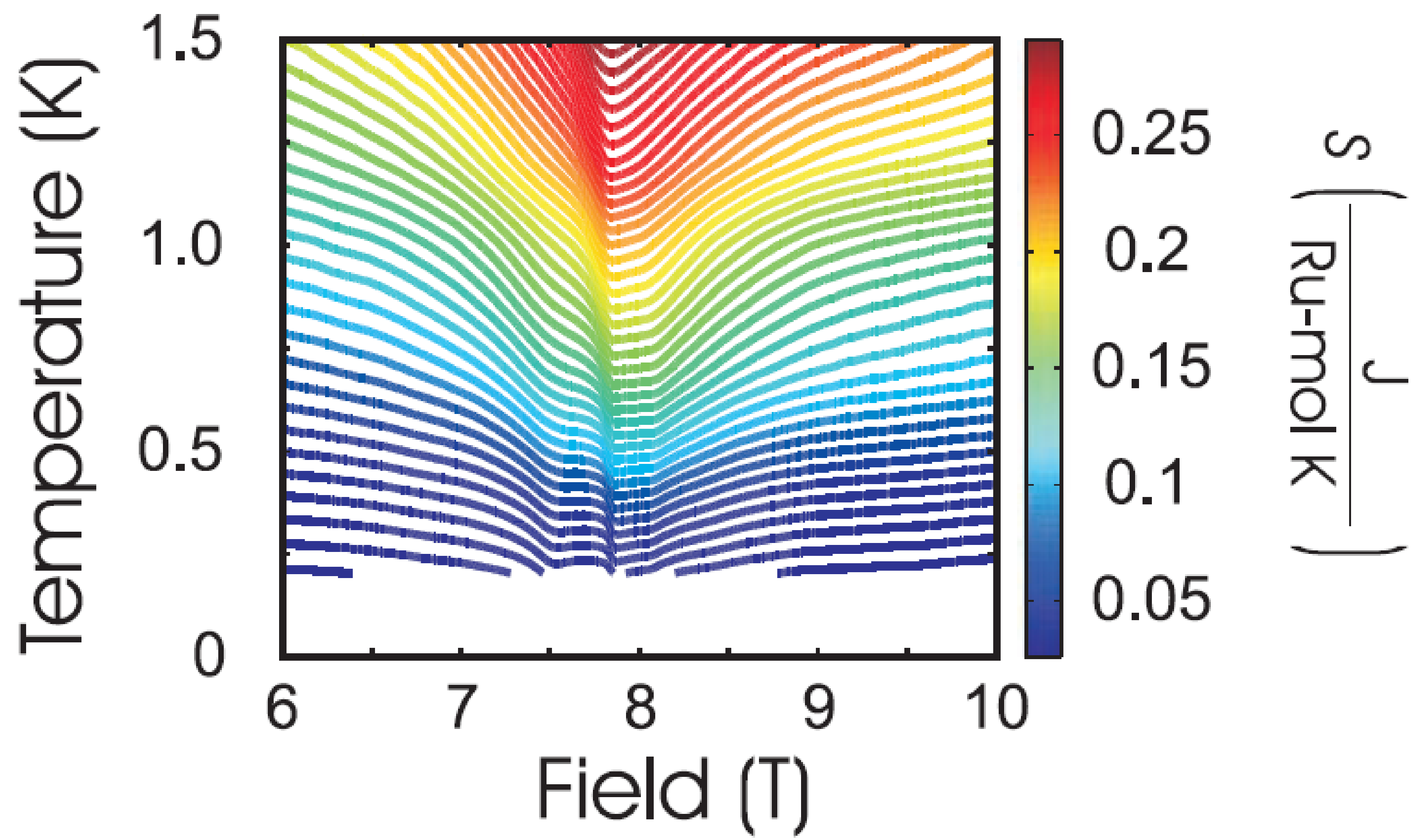
Science, **294**, 329 (2001).



P. Gegenwart, F. Weickert, M. Garst, R.S. Perry and Y. Maeno,

Phys. Rev. Lett. **96**, 136402 (2006).

Quantum criticality concentrates the entropy



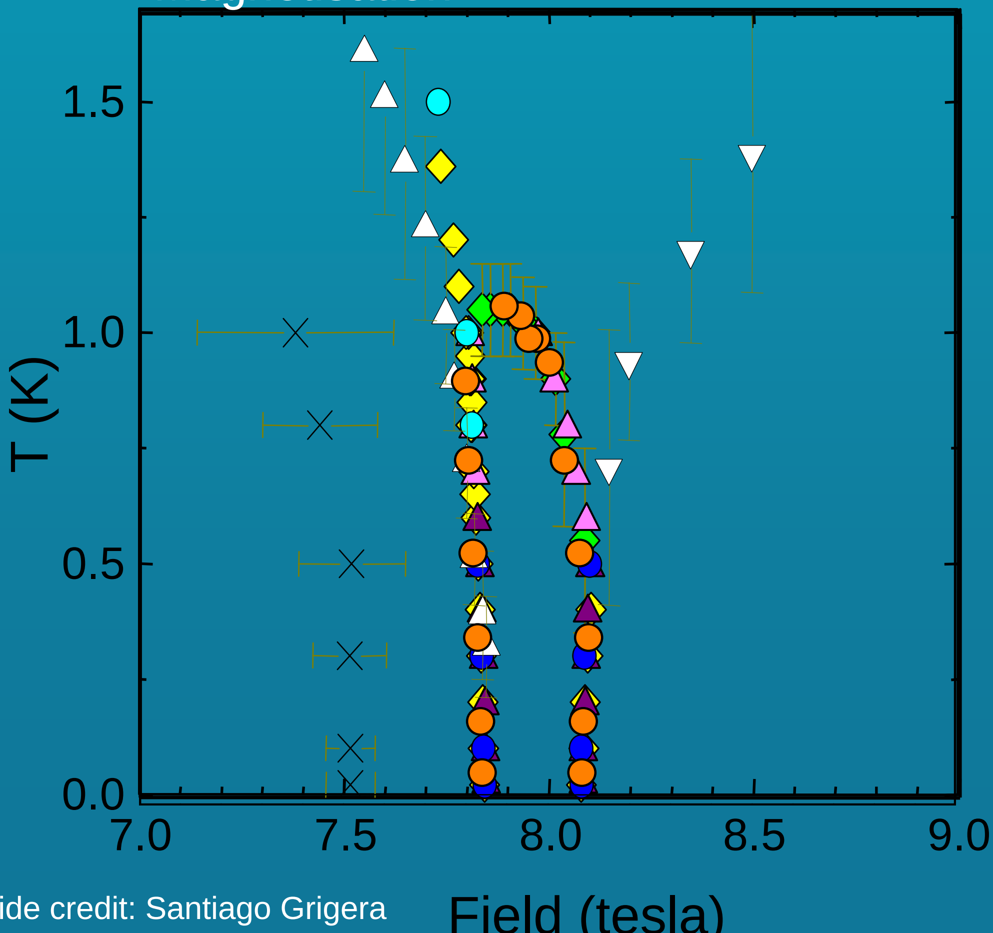
A. W. Rost, R. S. Perry, J.-F. Mercure, A. P. Mackenzie and S. A. Grigera
Science **325**, *#5946*, 1360-1363 (2009).

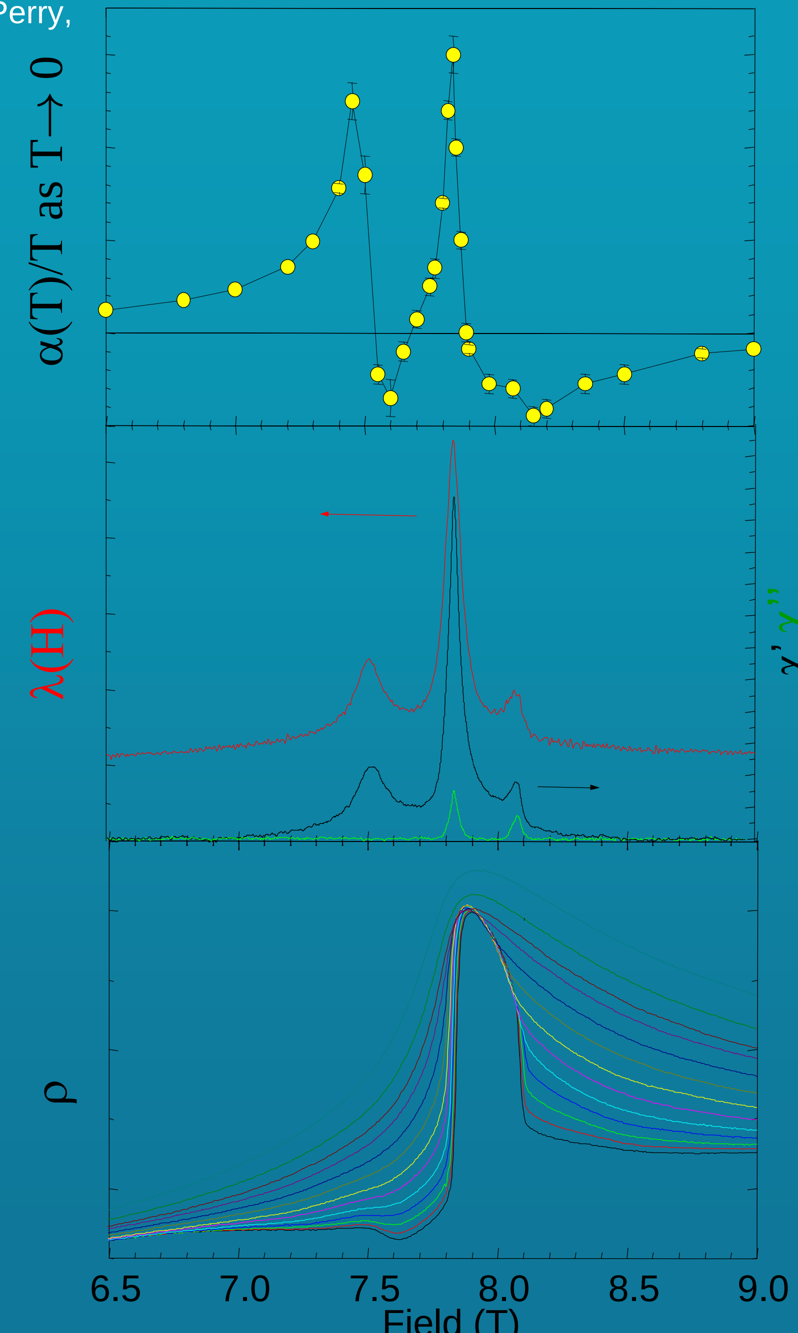
Near the "QCP"

S. A. Grigera, P. Gegenwart, R. A. Borzi, F. Weickert, A. J. Schofield, R. S. Perry, T. Tayama, T. Sakakibara, Y. Maeno, A. G. Green and A. P. Mackenzie

Science **306**,1154 (2004)

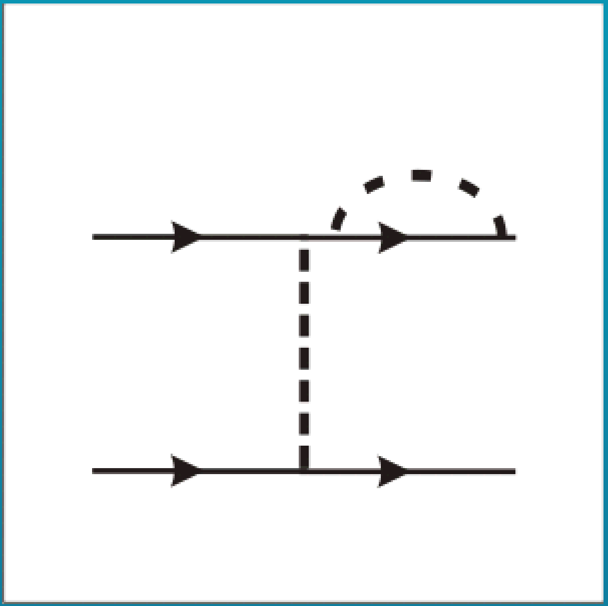
- Resistivity: $\partial\rho/\partial T$ and $\partial^2\rho/\partial T^2$
- Susceptibility: χ' and χ''
- Magnetostriction: $\lambda(H)$
- Thermal expansion: $\alpha(T)$
- Magnetisation





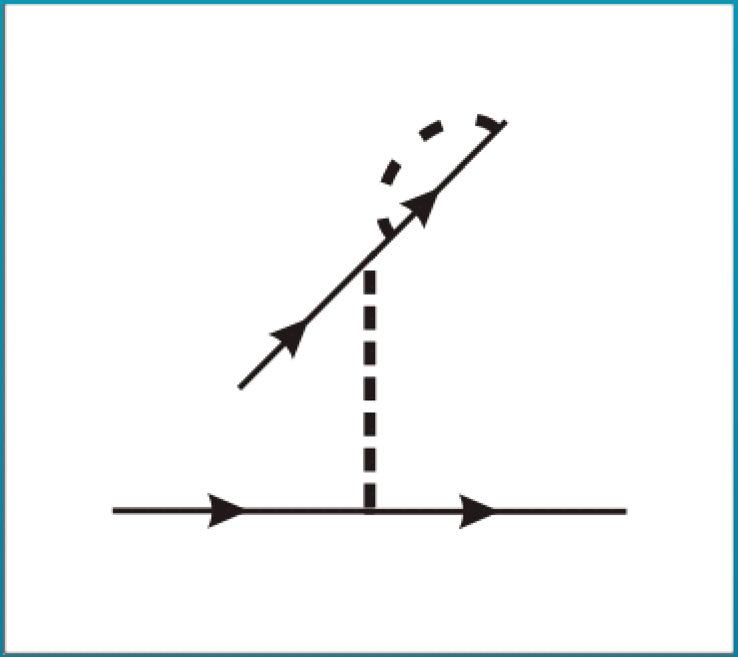
Slide credit: Santiago Grigera

From particle-particle pairing to particle-hole pairing



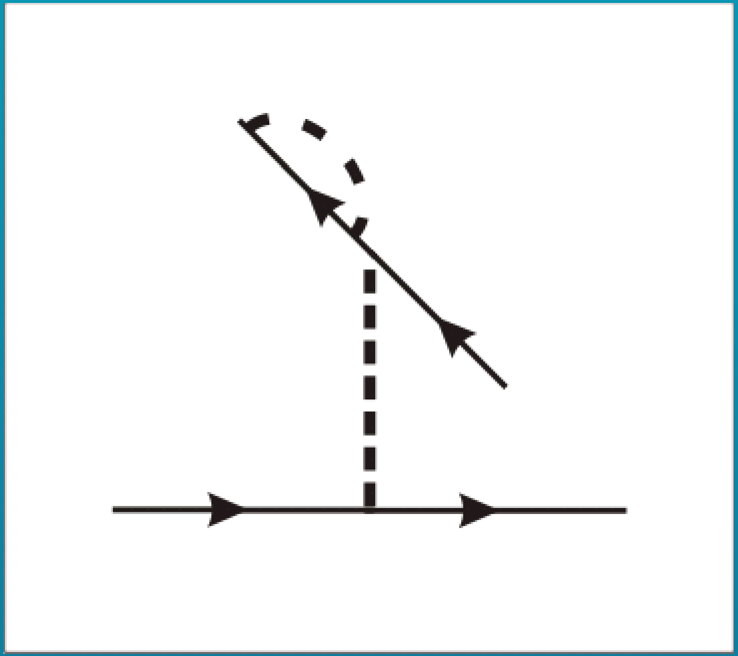
Discussions: Gil Lonzarich

From particle-particle pairing to particle-hole pairing



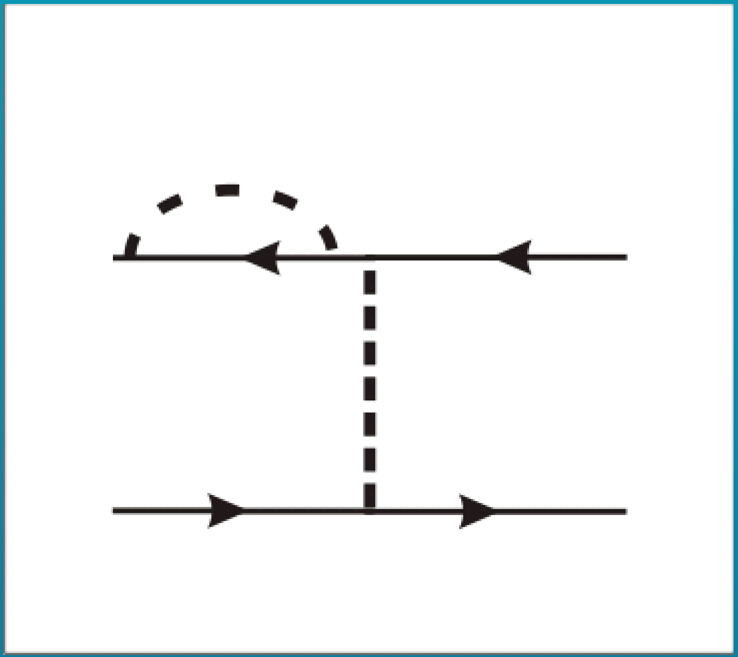
Discussions: Gil Lonzarich

From particle-particle pairing to particle-hole pairing



Discussions: Gil Lonzarich

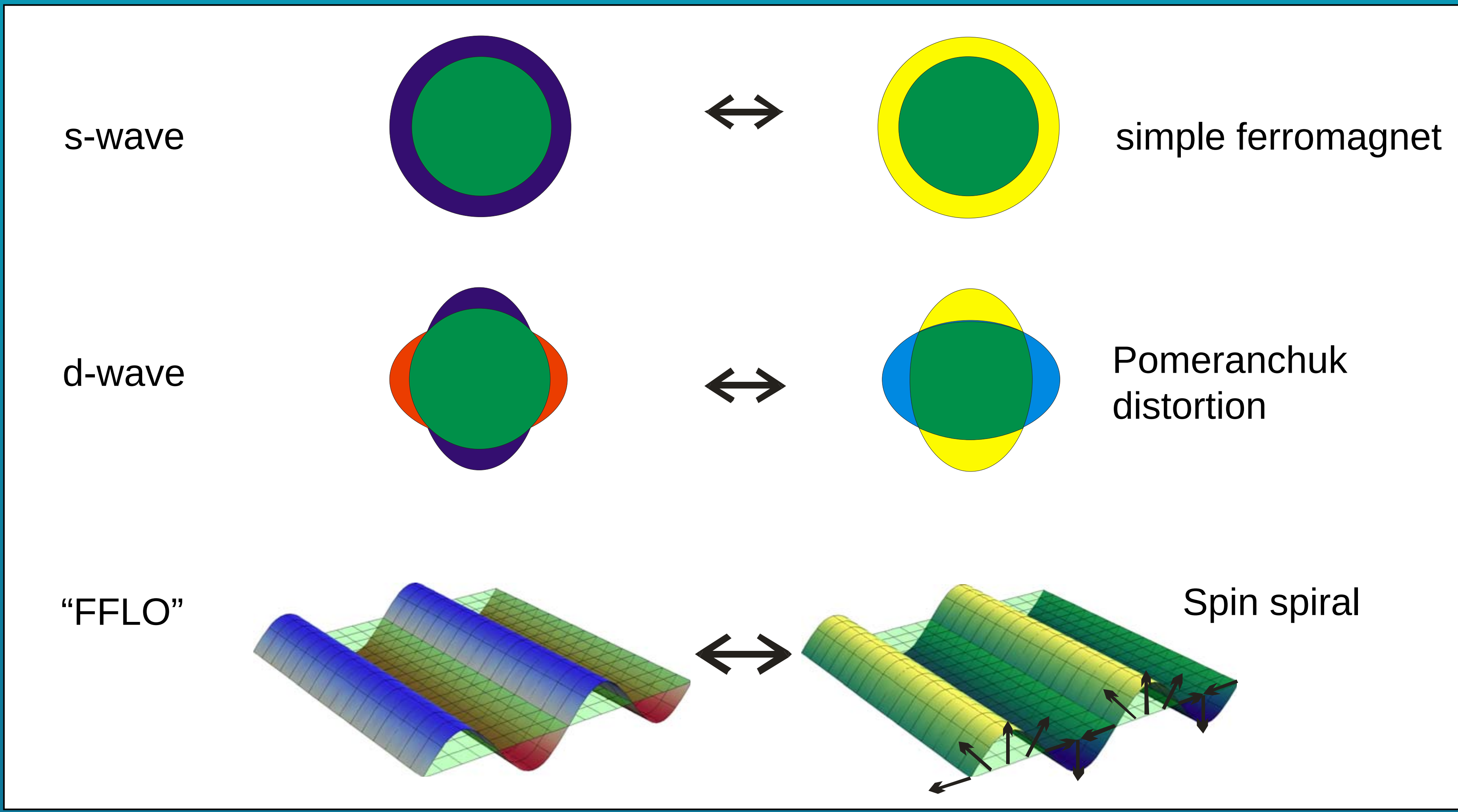
From particle-particle pairing to particle-hole pairing



Discussions: Gil Lonzarich

Dark order states as magnetic analogues of unconventional superconductors [see A. J. Schofield, Physics **2**, 93 (2009)]

Superconductors: particle-particle   Magnets: particle-hole



Dark order states as magnetic analogues of unconventional superconductors: A. J. Schofield arXiv:1001.4279v1

Superconductors: part-part

Conventional: *s*-wave

$$\Delta = \sum_{k,k',\sigma} V_{k,k'} \langle c^\dagger_{k'\sigma} c^\dagger_{-k'\bar{\sigma}} \rangle$$

Unconventional: *p*-wave, *d*-wave,…

$$\Delta(k) = \sum_{k,k',\sigma} V_{k,k'} \langle c^\dagger_{k'\sigma} c^\dagger_{-k'\bar{\sigma}} \rangle$$

Inhomogeneous: FFLO

$$\Delta(q) = \sum_{k,k',\sigma} V_{k,k'} \langle c^\dagger_{k'+q/2,\sigma} c^\dagger_{-k'+q/2,\bar{\sigma}} \rangle$$

Magnets: part-hole

Conventional: Stoner ferromagnetism

$$M = \sum_{k,\sigma,\sigma'} g_{\sigma,\sigma'} \langle c^\dagger_{k\sigma} c_{k\sigma'} \rangle$$

“Pomeranchuk”: *p*-wave, *d*-wave,…

$$M(k) = \sum_{k,\sigma,\sigma'} g_{k,k';\sigma,\sigma'} \langle c^\dagger_{k'\sigma} c_{k'\sigma'} \rangle$$

Inhomogeneous: “spirals”, density waves

$$M(q) = \sum_{k,\sigma} g_{k,k';\sigma,\sigma'} \langle c^\dagger_{k'+q/2,\sigma} c_{k'-q/2,\sigma'} \rangle$$

Dark order states as magnetic analogues of unconventional superconductors

Superconductors: part-part

Magnets: part-hole

Conventional: *s*-wave

$$\Delta = \sum_{k,k',\sigma} V_{k,k'} \langle c^\dagger_{k'\sigma} c^\dagger_{-k'\bar{\sigma}} \rangle$$

Conventional: Stoner ferromagnetism

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"Pomeranchuk": *p*-wave, *d*-wave,…

$$M(k) = \sum_{k,\sigma,\sigma'} g_{k,k';\sigma,\sigma'} \langle c^\dagger_{k'\sigma} c_{k'\sigma'} \rangle$$

Inhomogeneous: FFLO

$$\Delta(q) = \sum_{k,k',\sigma} V_{k,k'} \langle c^\dagger_{k'+q/2,\sigma} c^\dagger_{-k'+q/2,\bar{\sigma}} \rangle$$

Inhomogeneous: "spirals", density waves

$$M(q) = \sum_{k,\sigma} g_{k,k';\sigma,\sigma'} \langle c^\dagger_{k'+q/2,\sigma} c_{k'-q/2,\sigma'} \rangle$$

"Unconventional Magnets"
Pomeranchuk (1958) instabilities

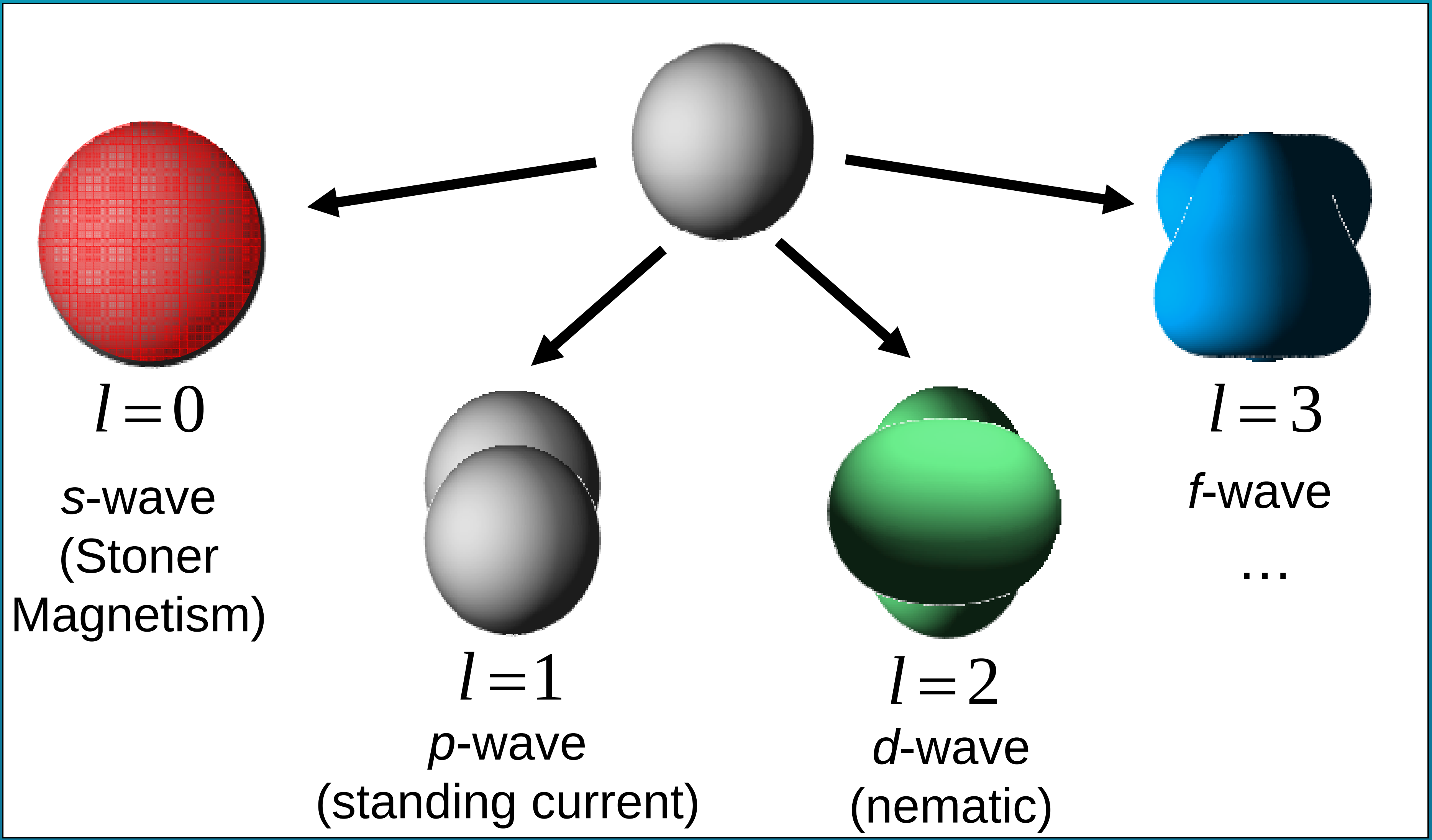
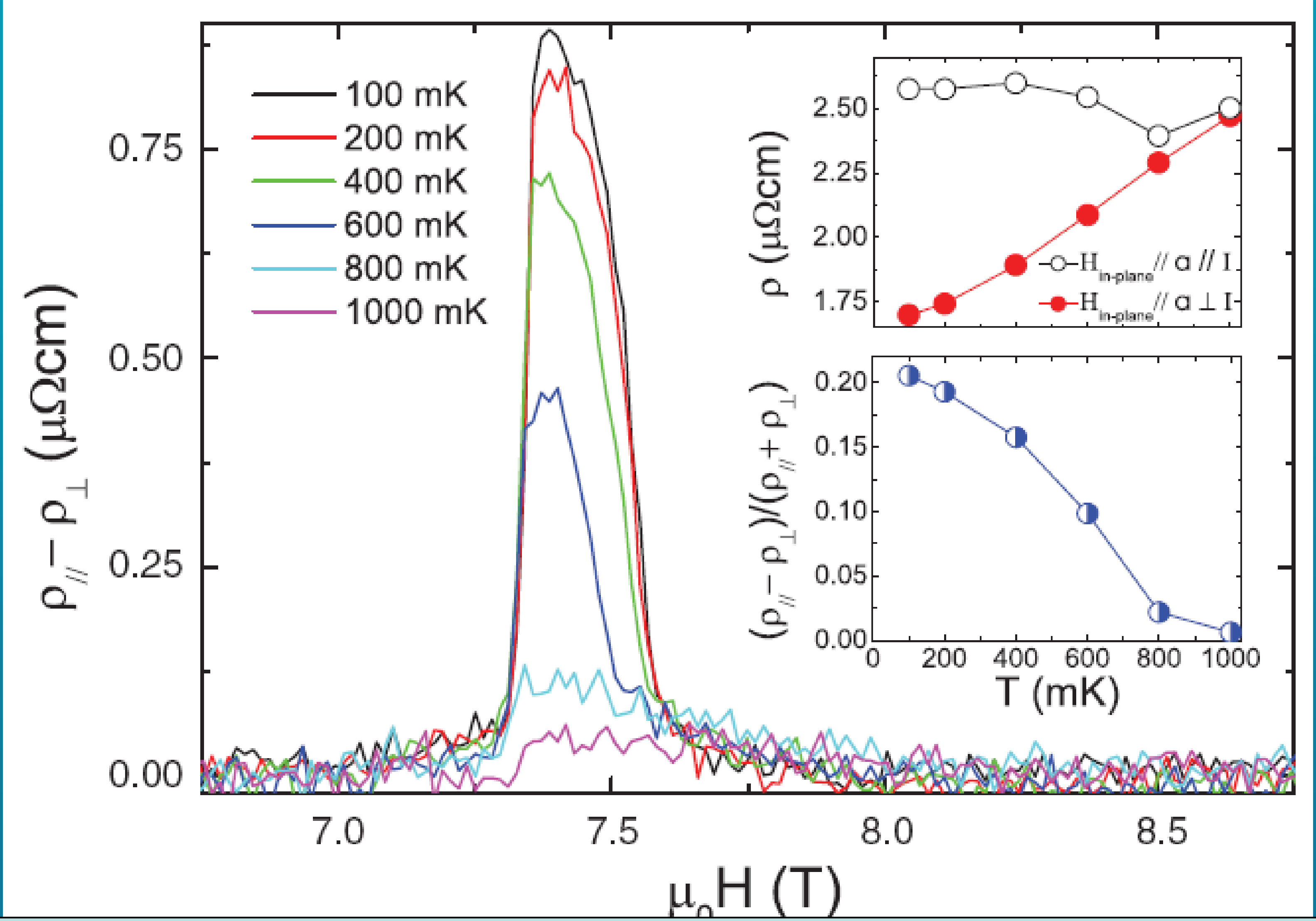


Image credit: Jorge Quintanilla

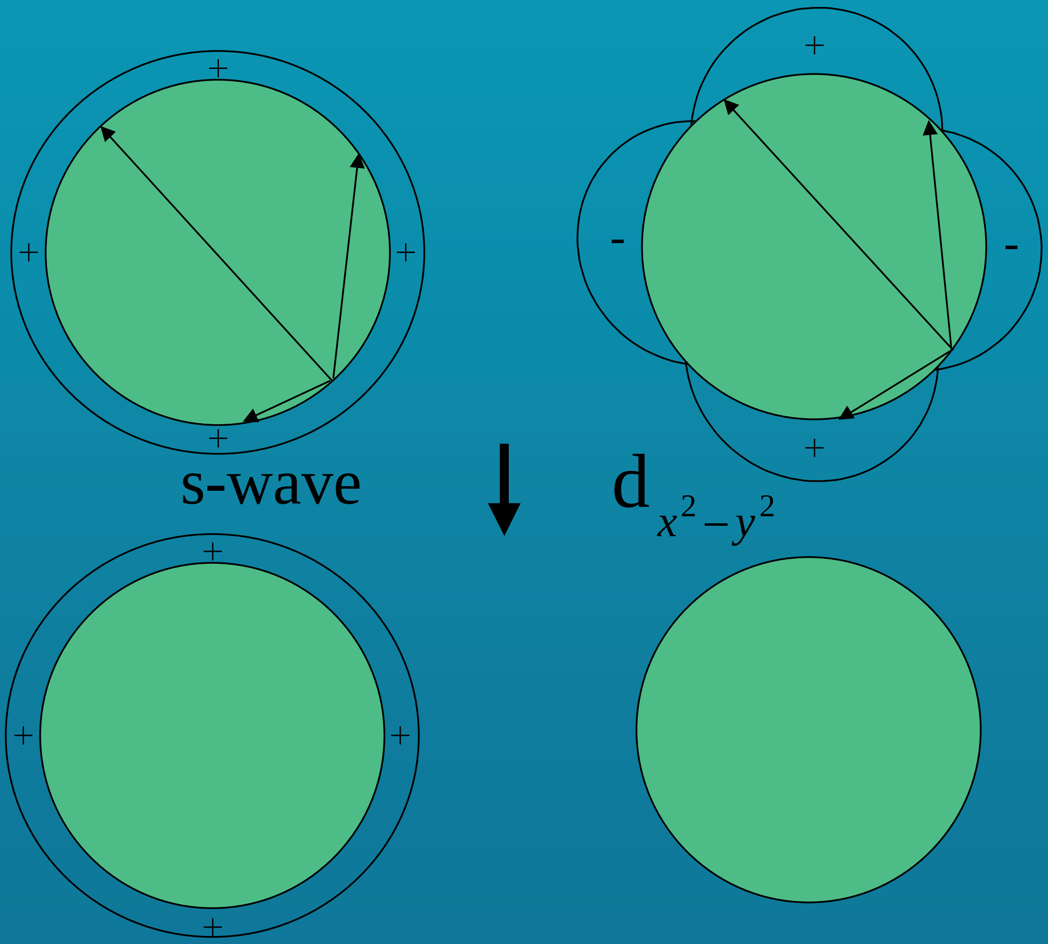
Evidence for this in $Sr_3Ru_2O_7$: resistivity anisotropy as if magnetic field aligns domains



R.A. Borzi, S. A. Grigera, J. Farell, R.S.Parry, S. J. S. Lister, S. L. Lee, D. A. Tennant, Y. Maeno, A. P. Mackenzie, Science, **315**, 214 (2007).

Can we use disorder dependence to identify a Fermi surface shape change transition?

A. F. Ho and A. J. Schofield EPL **84**, #2, 27007 (2008).



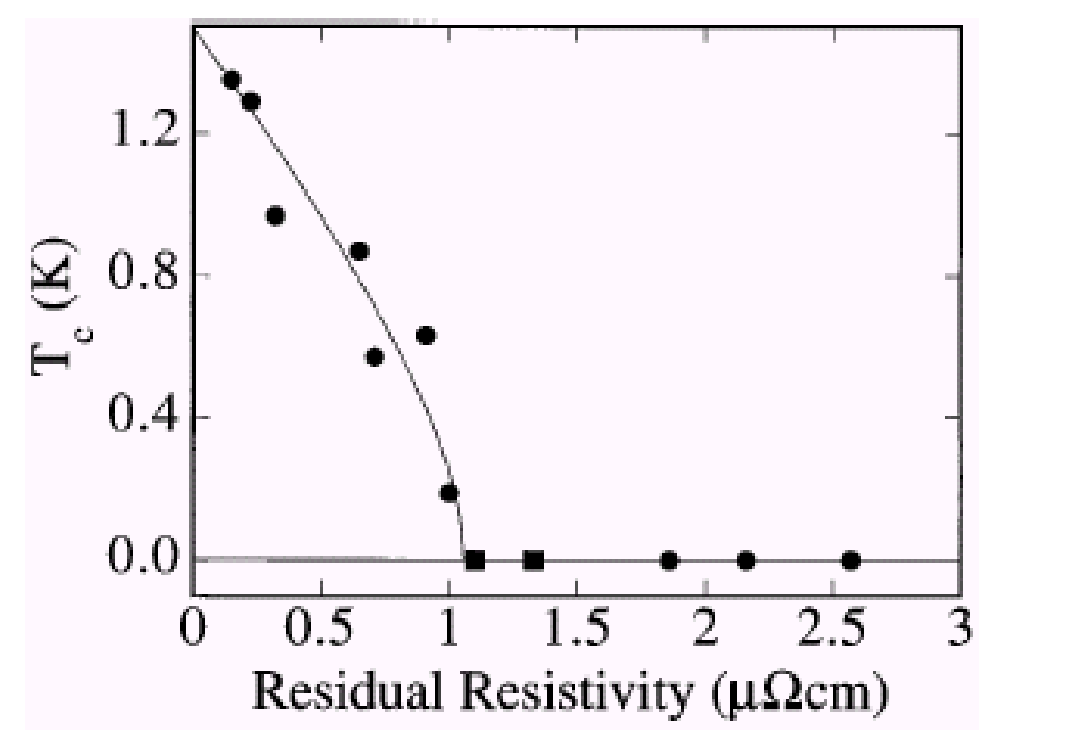
Analogy with superconductivity

s-wave superconductor is robust against disorder (Anderson's theorem)

non s-wave superconductivity is not protected by Anderson's theorem: Larkin

This has become the de-facto standard method for identifying unconventional superconductors

Example: $Sr_2 RuO_4$

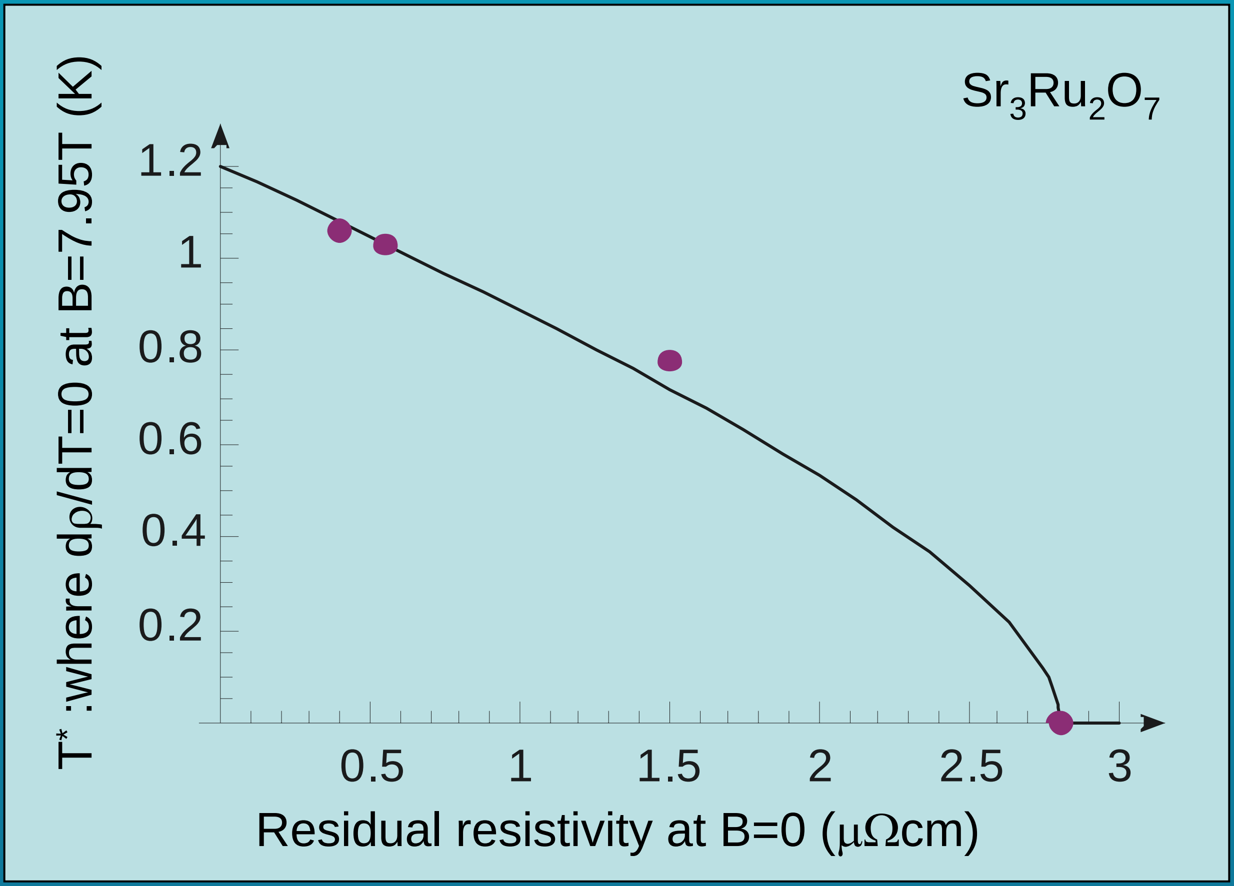


In Sr_2RuO_4, l must be greater than 1000Å for superconductivity to be observed at all, and an order of magnitude higher still for there to be negligible impurity pair-breaking.

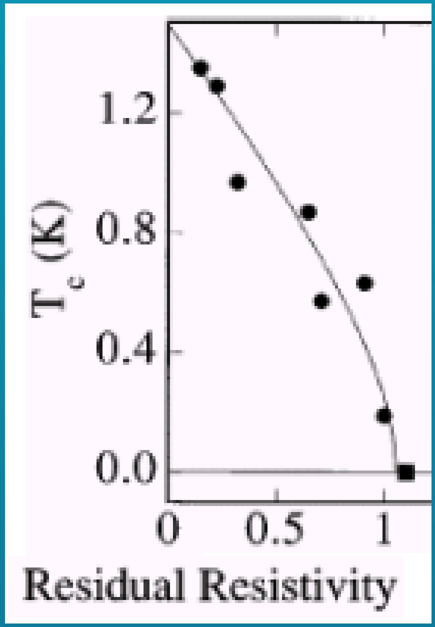
Mackenzie, A.P., R.K.W. Haselwimmer, A.W. Tyler, G.G. Lonzarich, Y. Mori, S. NishiZaki and Y. Maeno, Phys. Rev. Lett. **80**, 161 (1998)

Comparison with "Dark Order" state in $Sr_3Ru_2O_7$

- No systematic studies. Existing data from S. A. Grigera



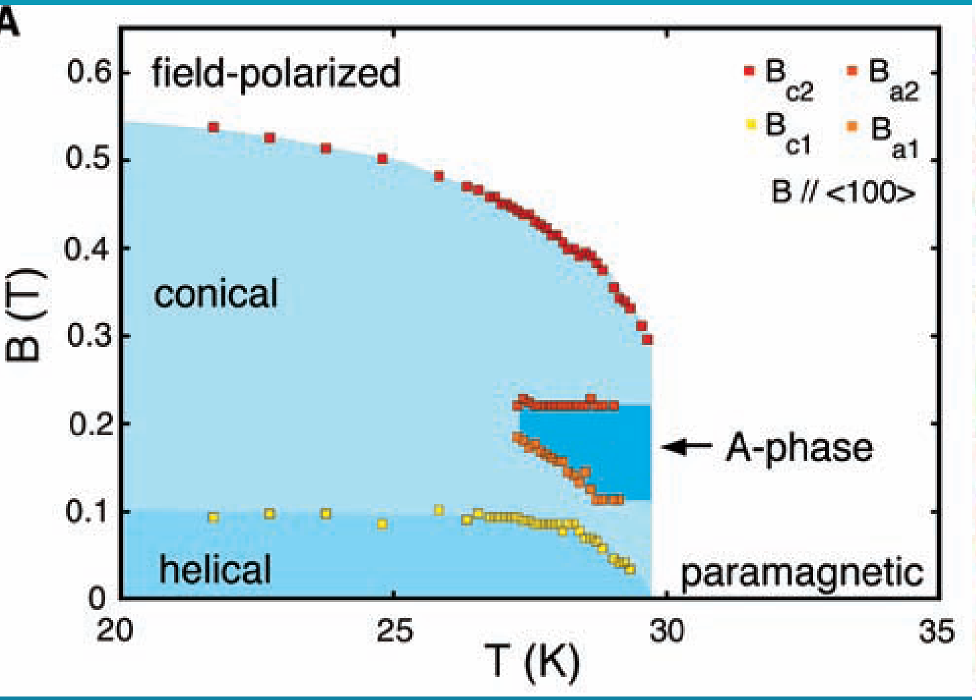
c.f. Sr_2RuO_4

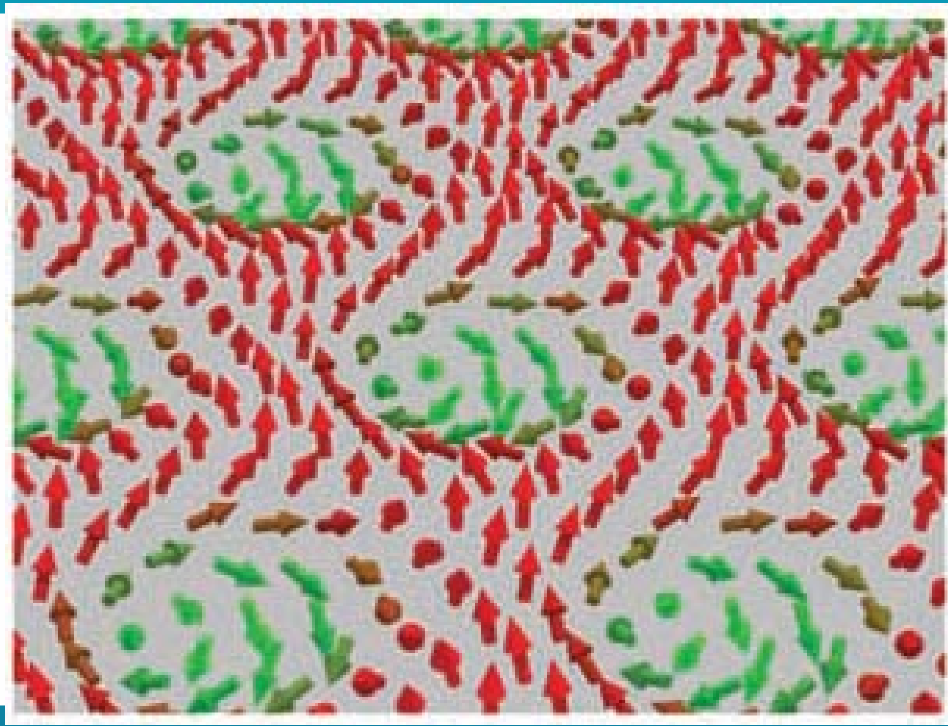


A. P. Mackenzie *et al.* Phys. Rev. Lett. **80**, 161 (1998)

A. F. Ho and A. J. Schofield EPL **84**, #2, 27007 (2008).

Yet more magnetic analogues of superconducting states





MnSi in a magnetic field: a skyrmion lattice – the magnetic analogue of an Abrikosov flux lattice

*S. Muhlbauer, B. Binz, F. Jonietz, C. Pfleiderer, A. Rosch, A. Neubauer, R. Georgii and P. Boni. Science **323**, #5916, 915-919 (2009).*

Summary

- Emergence: a guiding principle in condensed matter
  - Fermi liquid theory
  - Theory of phase transitions
- Quantum criticality: pushing these theories to the limit
  - Evidence for a breakdown in the conventional models
  - non-Fermi liquid phases, multiple energy scales…
- “Dark order” – puzzling phases near quantum criticality
  - magnetic analogues of superconductors
  - $Sr_3Ru_2O_7$: a candidate for Pomeranchuk type metallic state.
- Disorder dependence as a possible “smoking gun”
  - Disorder on Pomeranchuk is like its role on an unconventional superconductor.
- The hope for new theory…

Synergy: condensed matter and high energy theory

40/50's	**Many Body Physics** (imaginary time)	←	**QED** (Field Theory, Feynman diagrams)
50/60's	Broken Symmetry, **Superconductivity**	→	Revival of Field Theory **Higgs**, Standard Model
60/70's	**Critical phenomena** (stat mech/expt)	↔	**Renormalization**, Field Theory
	Nucleation	→	Inflation
70-90's	Kondo, Heavy Electrons, 1D conductors, Edge states	←	QCD, Gross Neveu, Large N expansion, conformal field theory

21st Century

SPECULATION

Supersymmetric quantum critical point ↔ Supersymmetric field theories

Slide credit: Piers Coleman