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Cosmological Structure from Quantum Fluctuations

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outline of talk

- **Cosmic structure**
  - *mapping our Universe*
- **Primordial sound waves**
  - *classical waves an expanding Universe*
  - *problem of initial conditions*
- **Inflation in the very early Universe**
  - *quantum fluctuations*

Hubble's law

Edwin Hubble made the most remarkable scientific discovery of the 20th century

Distant galaxies are moving away from us

and their *speed* is proportional to their *distance*

=> our universe is expanding

Sloan Digital Sky Survey

today we use Hubble's law to build 3-dimensional maps of galaxies in our universe

the largest galaxy survey to date is the SDSS

over 200 scientists from 14 institutions
(including Japan Participation Group)

it has mapped one million galaxies

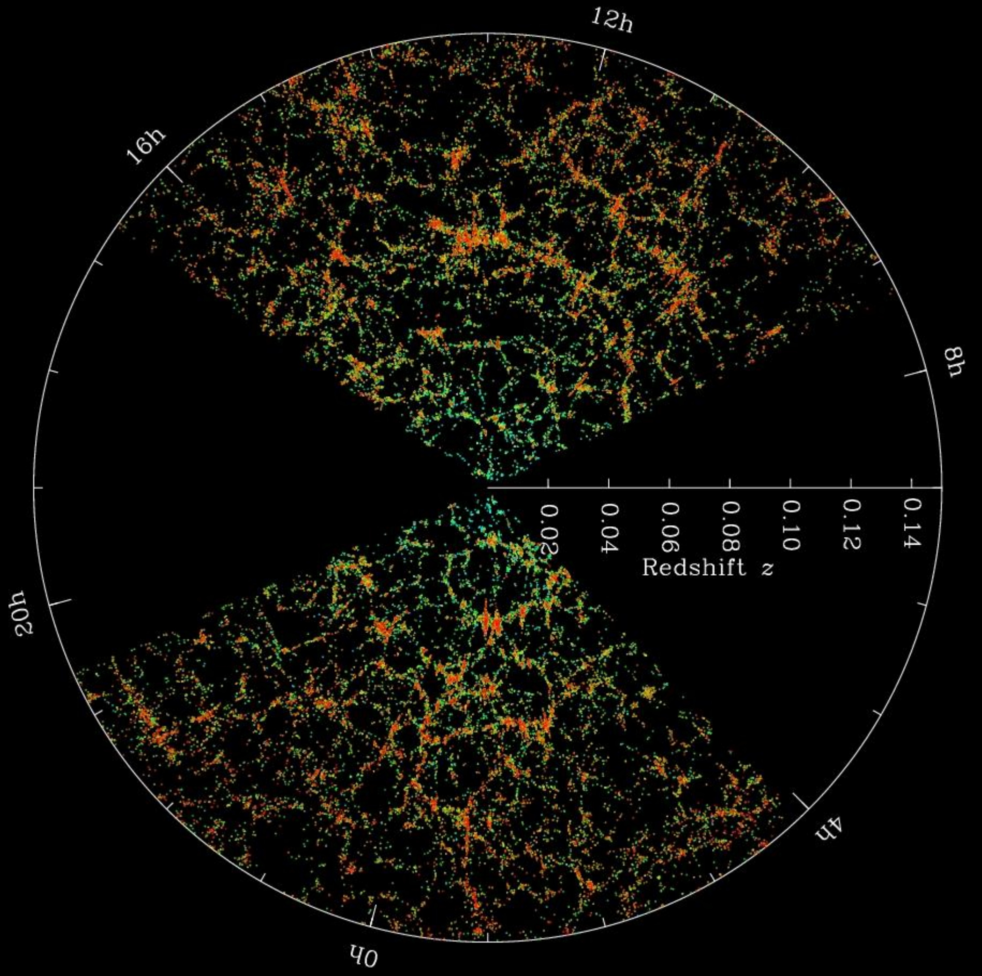
see *www.galaxyzoo.org*

movies from COSMUS University of Chicago

SDSS: a map of one million galaxies

movie from COSMUS University of Chicago

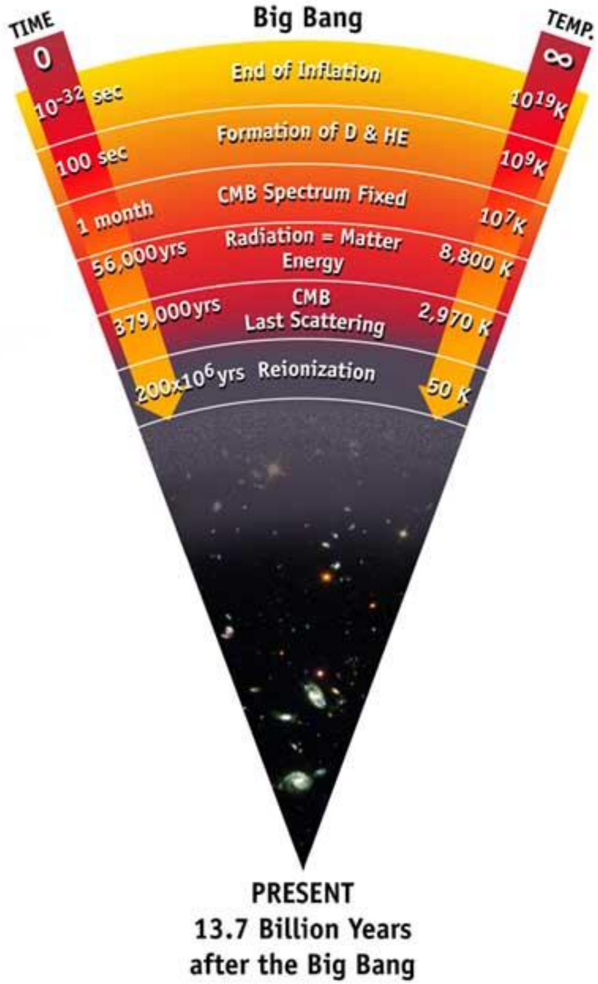
astrophysics gets complicated...



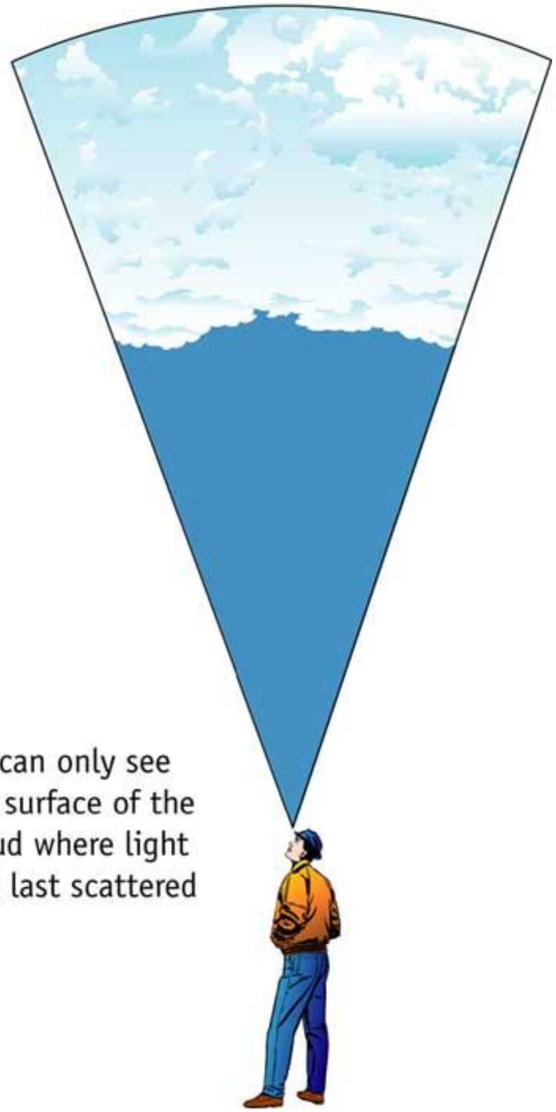
– atoms
– stars
– galaxies
– people...

– cosmologists look back to a simpler past...

how far back can we look?



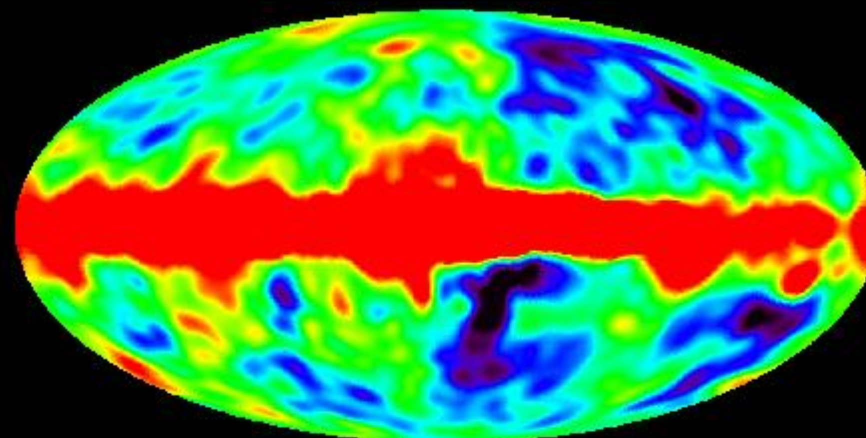
The cosmic microwave background Radiation's "surface of last scatter" is analogous to the light coming through the clouds to our eye on a cloudy day.



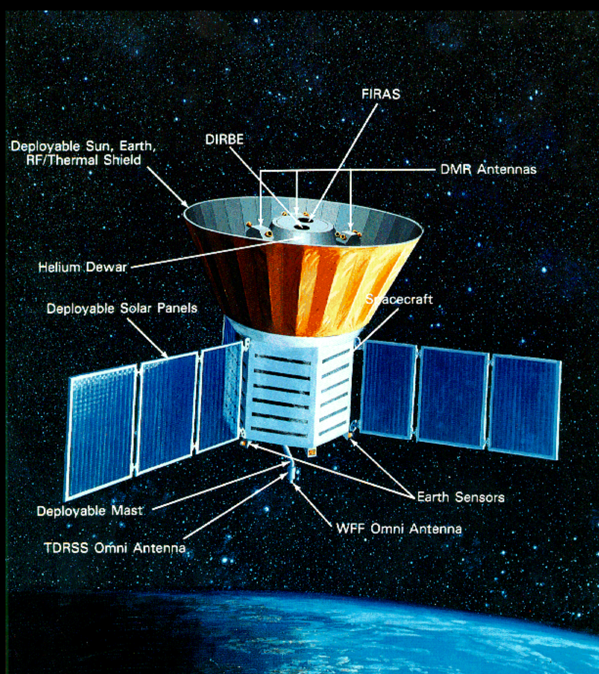
We can only see the surface of the cloud where light was last scattered

Cosmic microwave background radiation

- discovered by Penzias and Wilson 1965
- relic thermal radiation from the hot big bang



COBE satellite



2.7 K everywhere

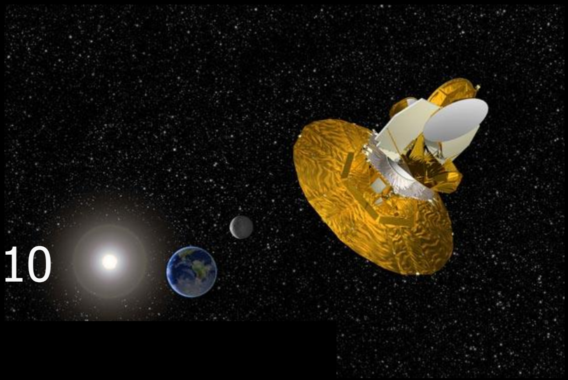
+/- 3.3 mK redshift due local motion
(at 1 million miles per hour)

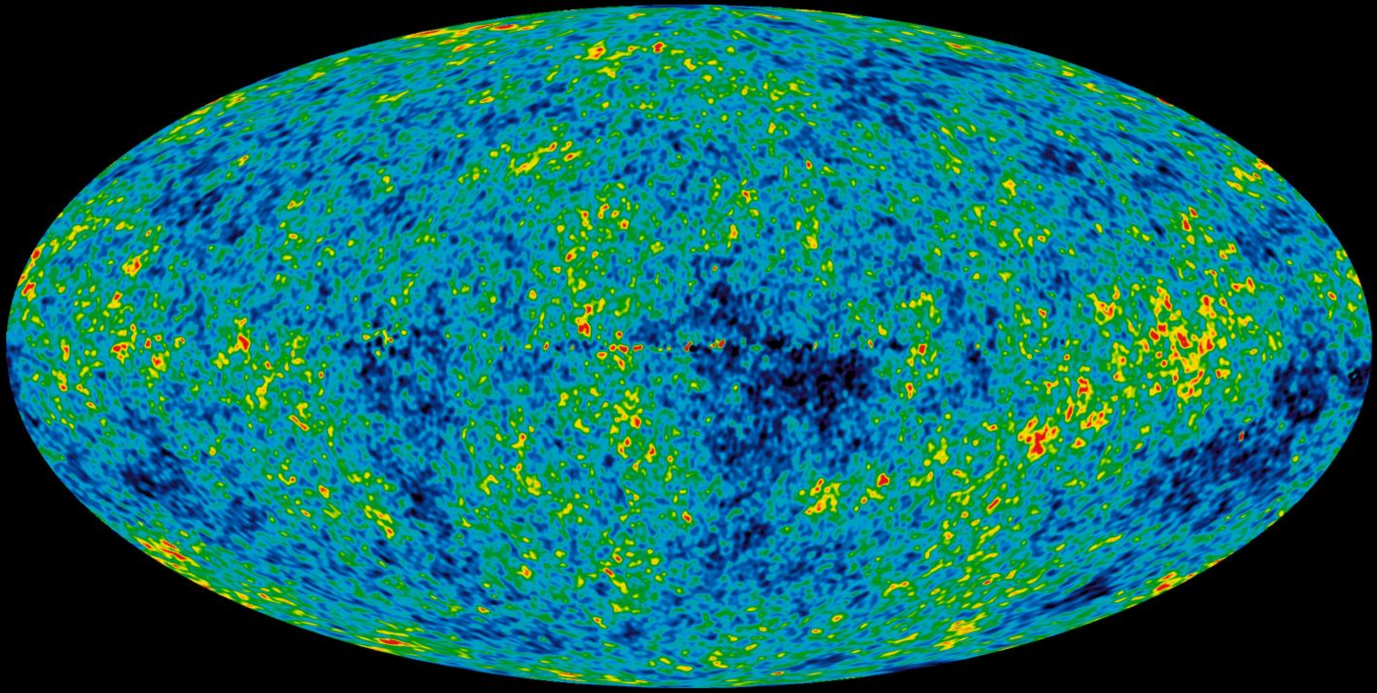
+/- 18 μK intrinsic anisotropies

COBE data release 1994

WMAP first data released February 2003

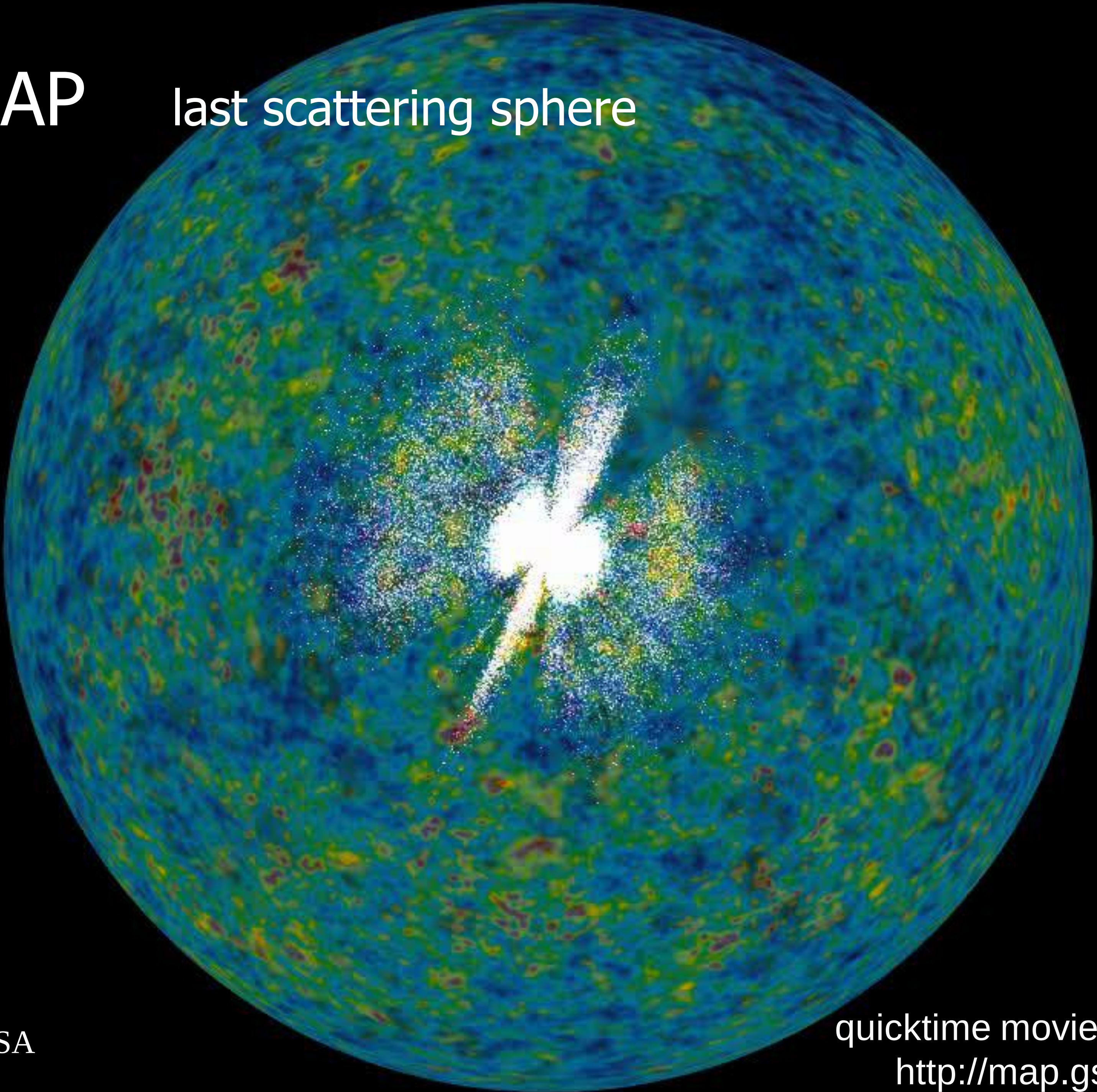
latest data released February 2010





the surface of last scattering

WMAP last scattering sphere



quicktime movie from WMA
http://map.gsfc.nasa.gov

cosmic fluid equations:

energy density ρ

pressure P

momentum density $\vec{q} = (\rho + P)\vec{v}$

$$\dot{\rho} = -\vec{\nabla}.\vec{q} - 3H\rho$$

$$\dot{\vec{q}} = -\vec{\nabla}p - 3H\vec{q}$$

Hubble expansion, H, dilutes density and momentum

eliminate pressure $\vec{\nabla}P = c_s^2\vec{\nabla}\rho$

where c_s^2 = adiabatic sound speed = 1/3 in hot big bang

to obtain second-order wave equation

$$\ddot{\rho} + 3H\dot{\rho} = c_s^2\nabla^2\rho$$

cosmic sound waves:

wave equation in an expanding spacetime:

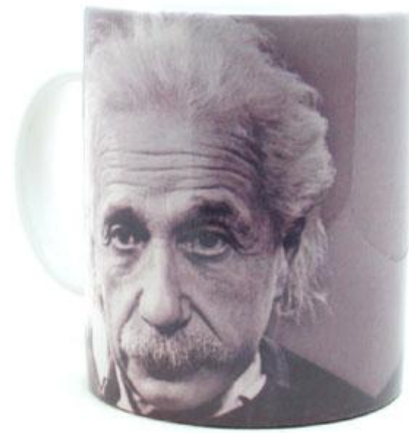
$$\frac{d^2}{dt^2}\rho + 3H\frac{d}{dt}\rho = c_s^2\frac{d^2}{dx^2}\rho$$

Characteristic timescales for wavelength λ

- *oscillation period/wavelength* λ / c_s
- *Hubble damping time-scale* H^{-1}

- *small-scales* $\lambda < c_s H^{-1}$ *under-damped oscillator*
- *large-scales* $\lambda > c_s H^{-1}$ *over-damped ("frozen-in")*

cosmo-seismology:



in general relativity should consider both

- effect of curved spacetime on matter
- effect of matter on curved spacetime

...leading to non-linear coupled equations

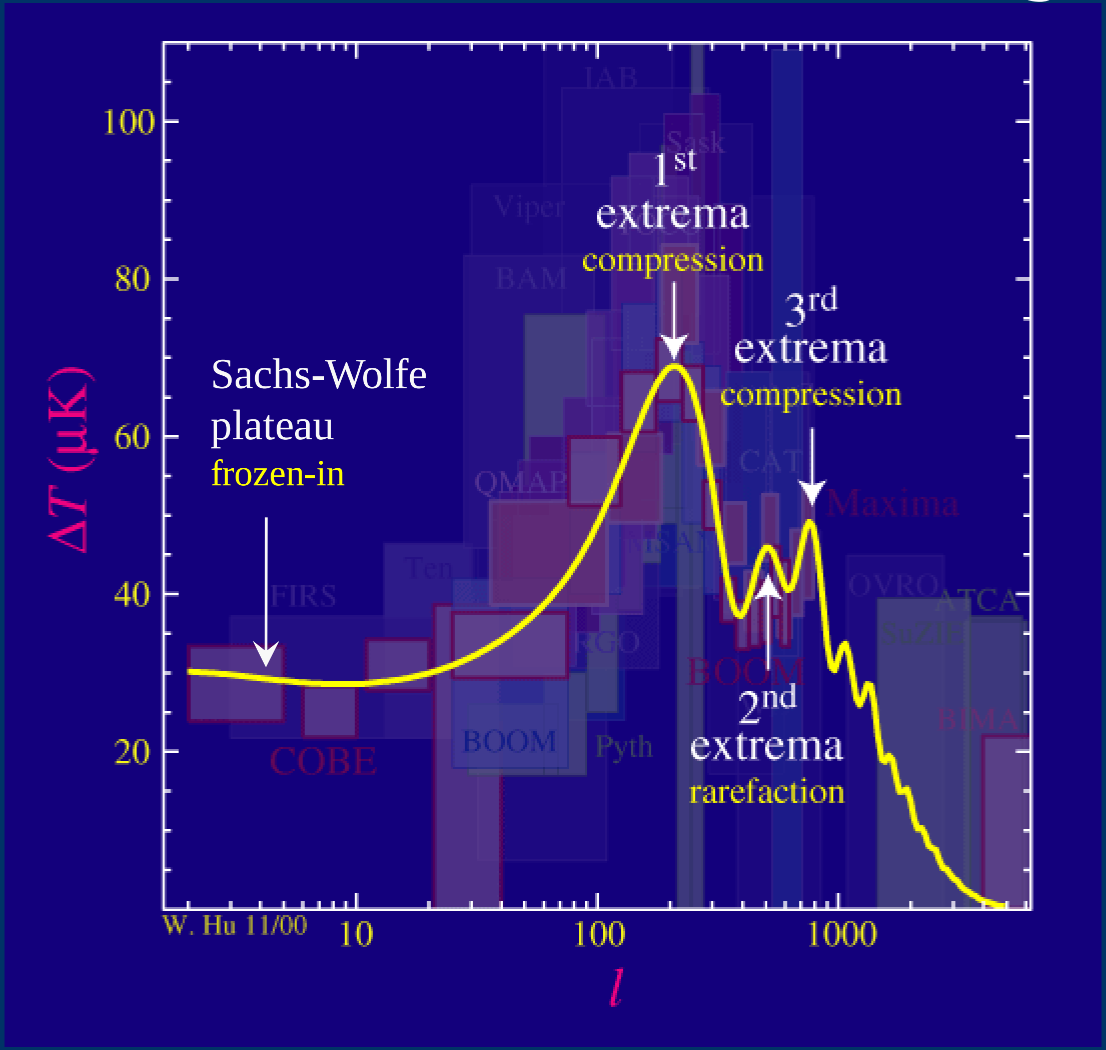
but often the simple picture works for linearised perturbations

two independent modes:

i. density waves (sound waves)
   = scalar metric perturbations
ii. gravitational waves (spacetime waves)
   = tensor metric perturbations

study the detailed response of the system (our Universe) to inhomogeneous perturbations

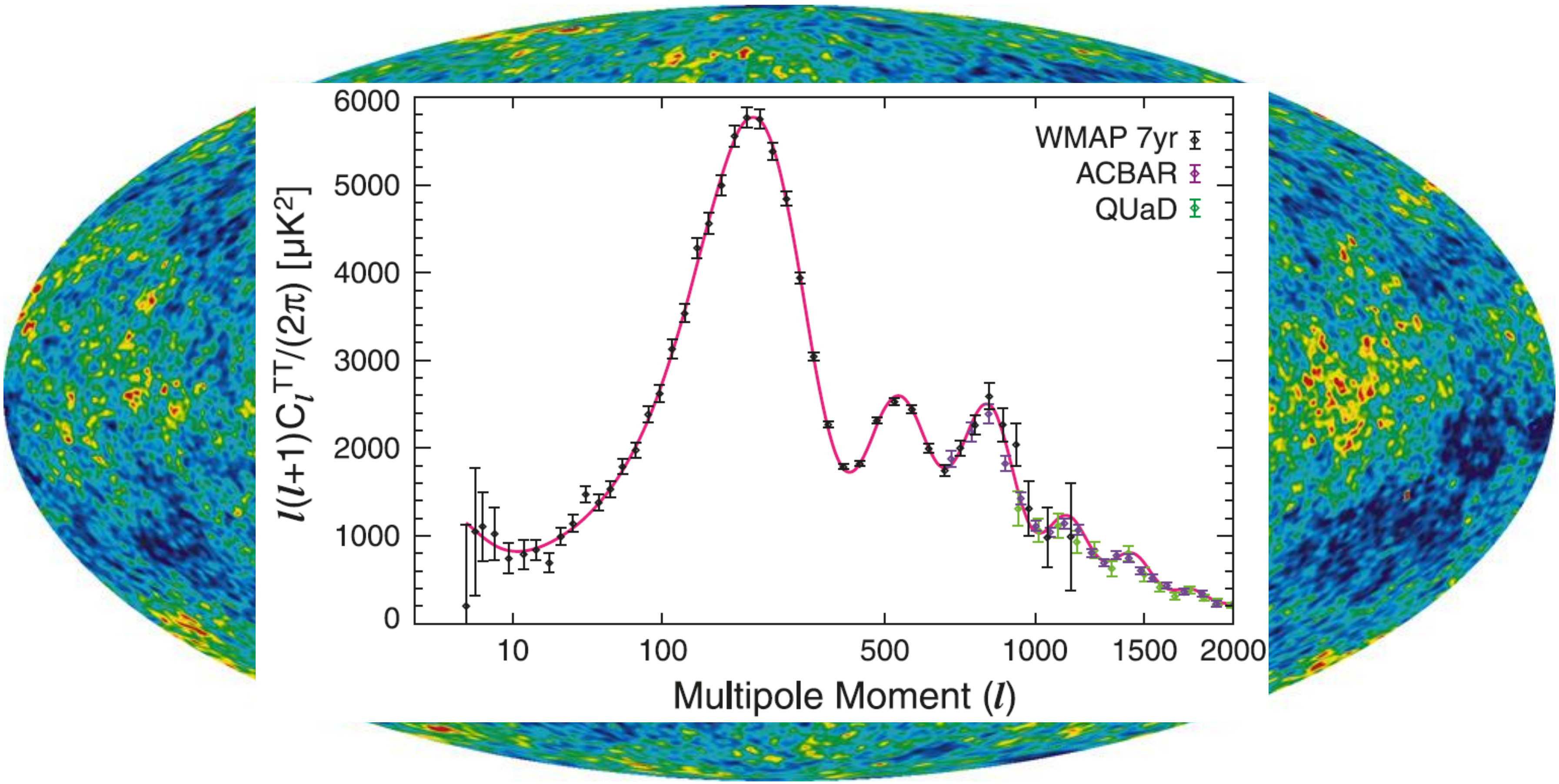
Characteristic scales on last-scattering surface



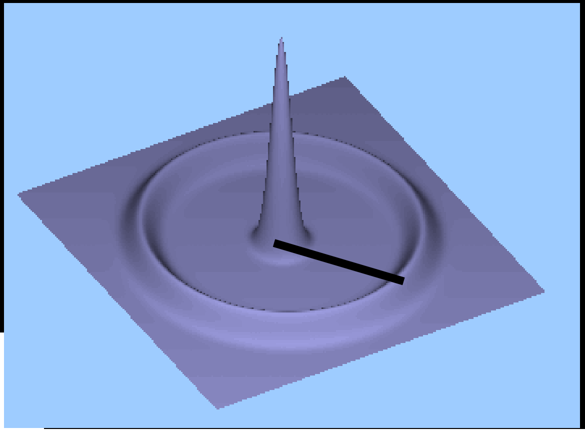
coherent oscillations of a relativistic plasma (photons + electrons + protons)

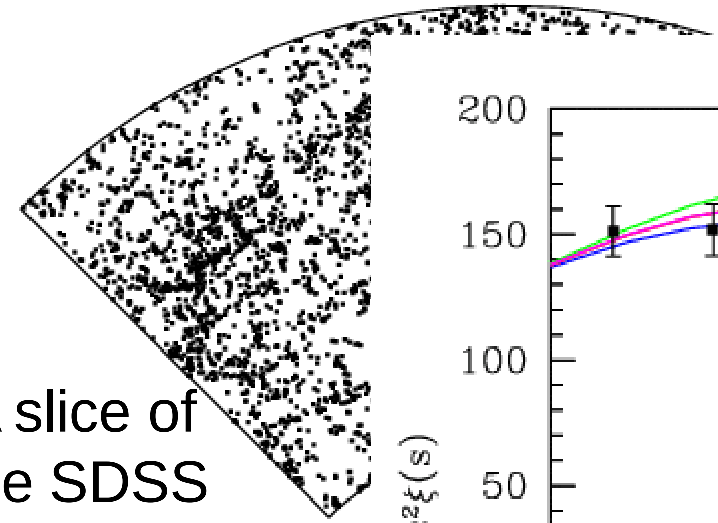
much more at background.uchicago.edu/~whu

WMAP angular power spectrum

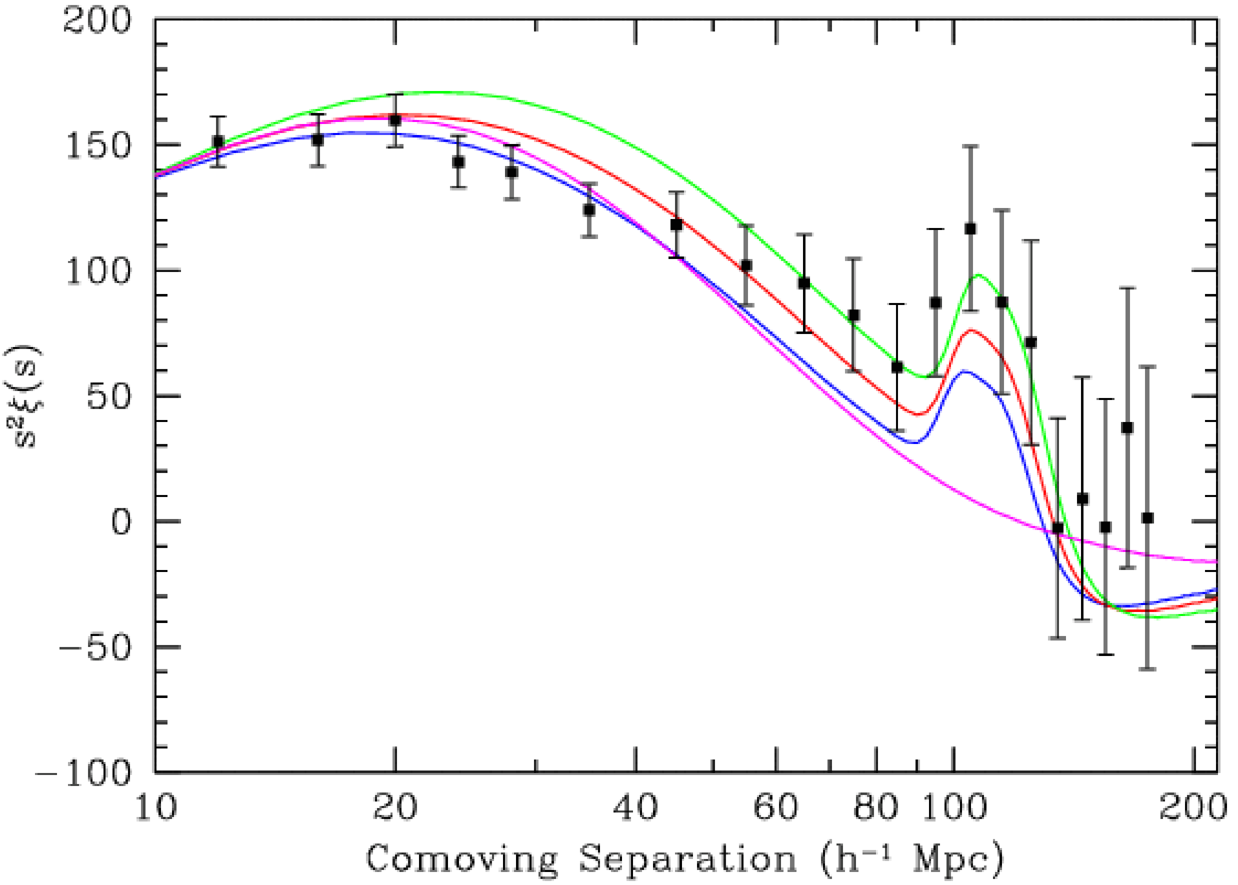


Baryon Acoustic Oscillations also seen in galaxy distribution



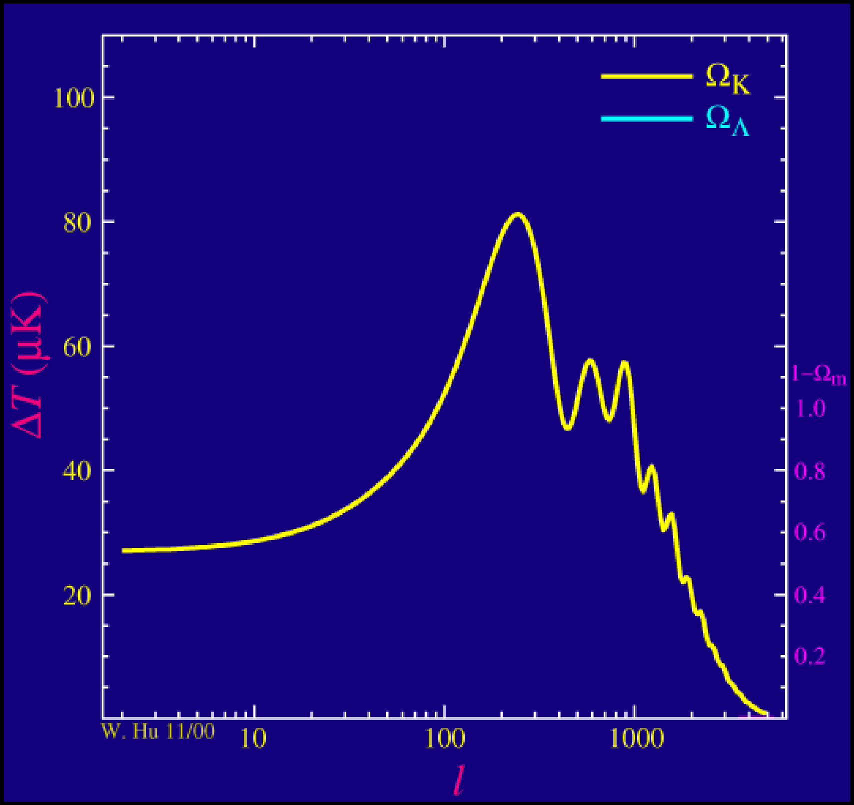


A slice of the SDSS



Precision cosmology

e.g., angular scale of first peak indicates flat space geometry,
but also depends on nature of energy density in the universe



c/o Wayne Hu, Chicago

background.uchicago.edu/~whu

compare many models against data (MCMC) to determine

– cosmological parameters
– *primordial perturbations*

one question leads to another:

where do the primordial perturbations come from?

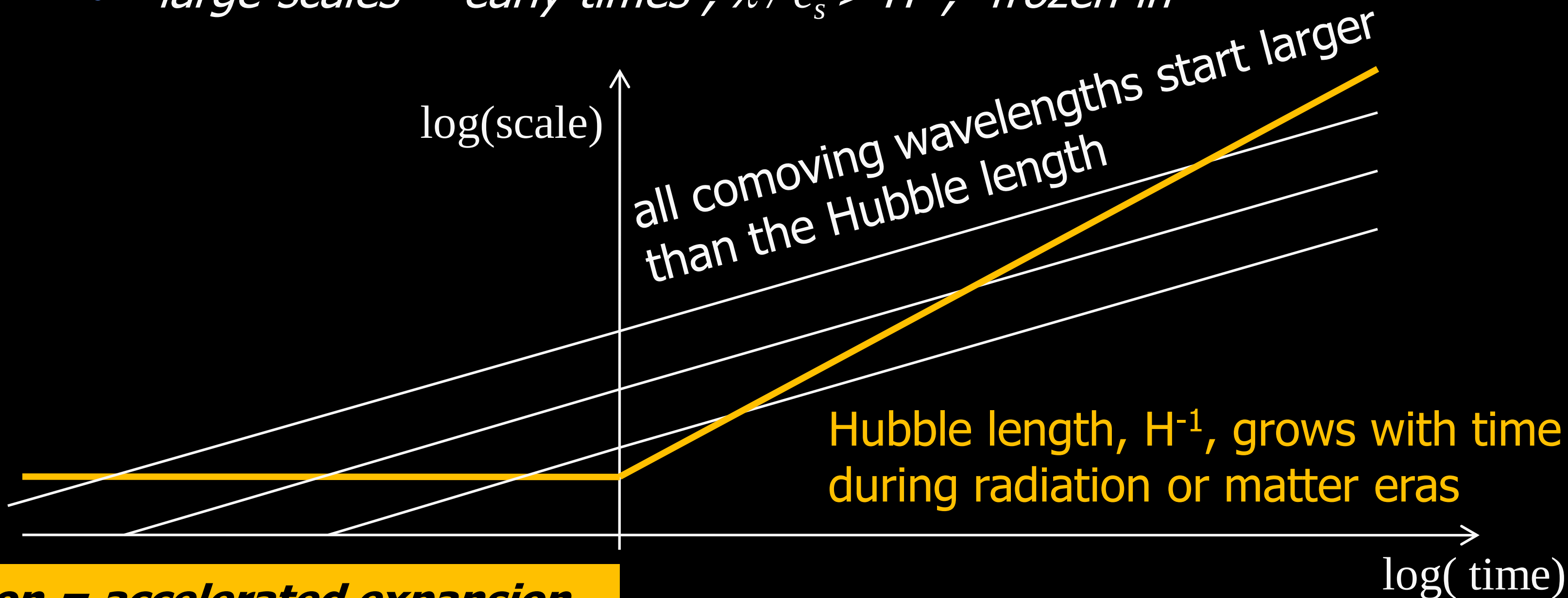
...quantum fluctuations during a period of inflation in the very early universe

problem of initial conditions:

$$\delta\ddot{\rho} + 3H\delta\dot{\rho} + (c_s / \lambda)^2 \delta\rho = 0$$

Characteristic timescales for waves, fixed comoving wavelength

- *small-scales = late times,* $\lambda / c_s < H^{-1}$ *, under-damped oscillator*
    - *large-scales = early times ,* $\lambda / c_s > H^{-1}$*, "frozen-in"*



inflation = accelerated expansion with almost constant Hubble rate

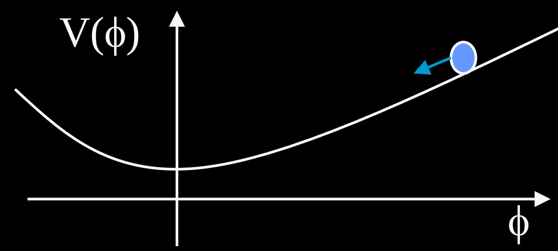
Cosmological inflation:

Starobinsky (1980)

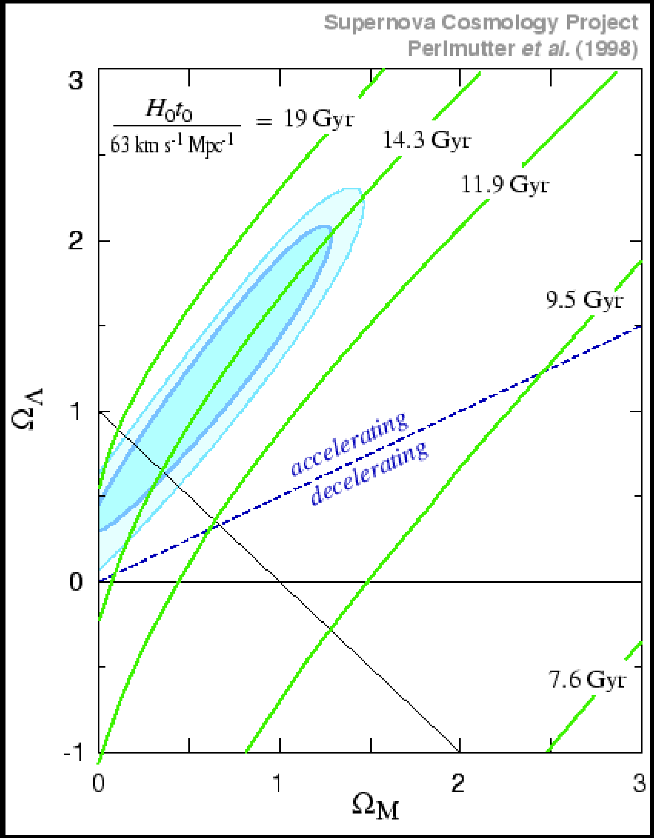
Guth (1981)

- period of accelerated expansion in the very early universe
- driven by vacuum energy

  e.g. self-interacting scalar field

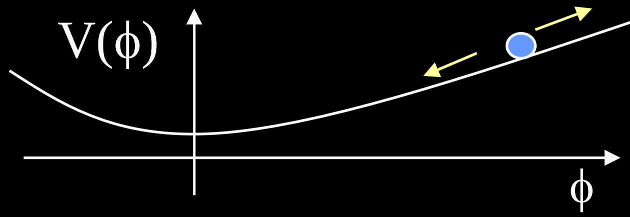


- speculative and uncertain physics
- just the kind of peculiar cosmological behaviour we observe today



Vacuum fluctuations

Hawking '82, Starobinsky '82, Guth & Pi '82



- *small-scale/underdamped zero-point fluctuations* $\delta\phi_k \approx \frac{e^{-ik\eta}}{\sqrt{2k}}$
- *large-scale/overdamped perturbations in growing mode*

  linear evolution $\Rightarrow$ *Gaussian random field*

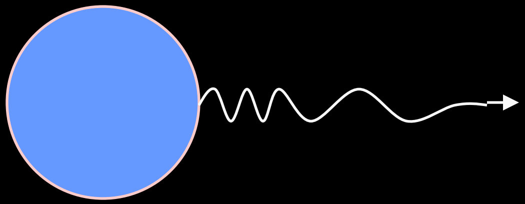
$$\left\langle \delta\phi^2 \right\rangle_{k=aH} \approx \frac{4\pi\, k^3 \left|\delta\phi_k\right|^2}{(2\pi)^3} = \left(\frac{H}{2\pi}\right)^2$$

fluctuations of any light fields (m<3H/2) 'frozen-in' on large scales

cosmological Hawking radiation

semi-classical gravity - quantum fields in curved spacetime

from a black-hole event horizon



Hawking '73

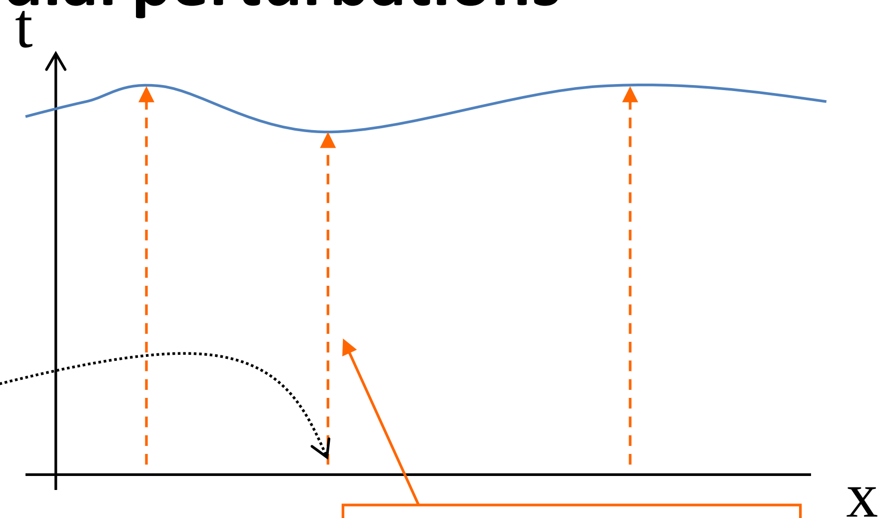
or from a cosmological event horizon during a period of inflation in the very early universe

Hawking '82
Starobinsky '82
Guth and Pi '82

scalar metric/density perturbations from inflation

the δN formalism for primordial perturbations

in radiation-dominated era
curvature perturbation ζ on
uniform-density hypersurface



during inflation
field perturbations $\phi(x,t_i)$ on
initial spatially-flat hypersurface

$$N = \int_{initial}^{final} H\, dt$$

on large scales, neglect spatial gradients, treat as "separate universes"

$$\zeta = N(\phi_{initial}) - \overline{N} \approx \sum_I \frac{\partial N}{\partial \phi_I} \delta\phi_I$$

Starobinsky `85; Sasaki & Stewart `96
Lyth & Rodriguez '05 – works to any order

density perturbations from inflaton field

- *quantum field fluctuations on unperturbed (flat) hypersurfaces during inflation leads to scalar metric perturbation*

$$\zeta = \frac{dN}{d\phi}\delta\phi = \left(-\frac{H}{\dot\phi}\delta\phi\right)_{k=aH}$$

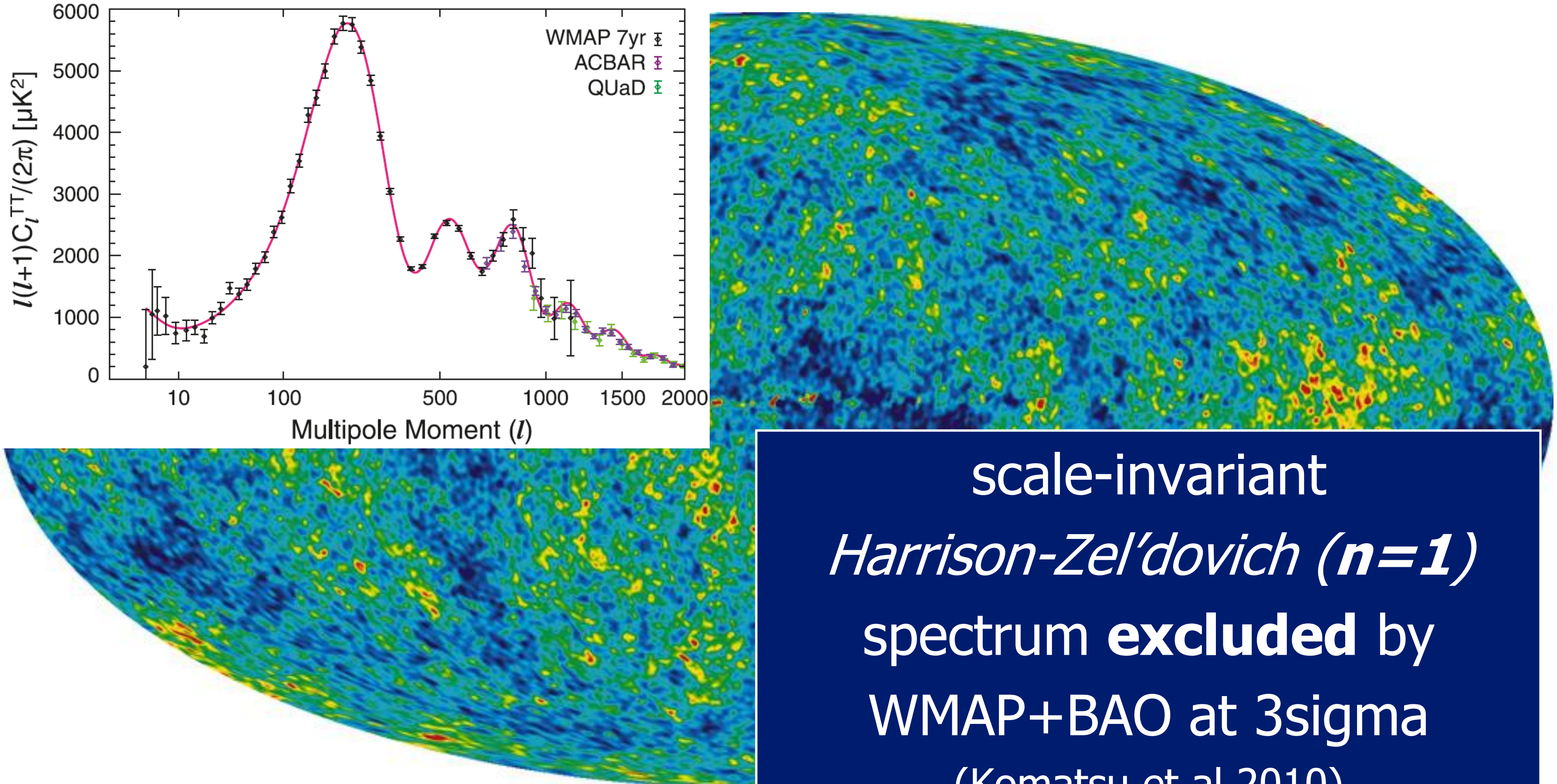
- *produce primordial density perturbations in radiation-dominated era*

$$\Rightarrow \quad \left\langle \frac{\delta T^2}{T^2} \right\rangle_{SW} \approx \frac{1}{25}\left\langle \zeta^2 \right\rangle \approx \frac{1}{25}\left(\frac{H^2}{2\pi\dot\phi}\right)^2_{k=aH}$$

slow time-dependence during inflation

$\Rightarrow$ *weak scale-dependence*

WMAP 7 year data February 2010



scale-invariant

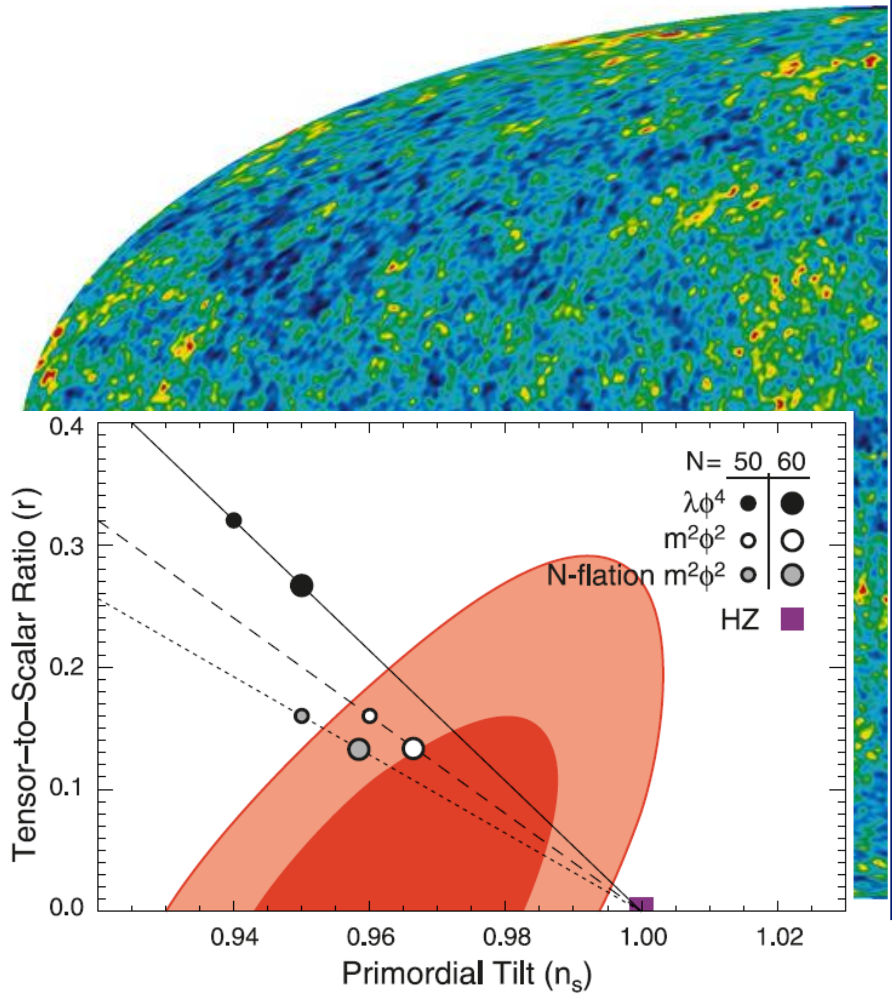
*Harrison-Zel'dovich (**n=1**)*

spectrum **excluded** by

WMAP+BAO at 3sigma

(Komatsu et al 2010)

what next?

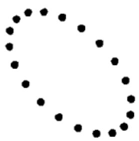


WMAP 7-year data (Komatsu et al 2010)

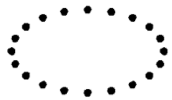
- running spectral index
  $-0.086 < d\,n / d \ln k < 0.018$
  higher-order in slow-roll

- gravitational waves
  tensor-scalar ratio
  $r < 0.36$

- non-Gaussianity
  $-10 < f_{NL} < 74$



gravitational waves



- *transverse, traceless "tensor" metric perturbations*

$$\delta g_{ij}(t,x) \approx \int d^3k \; h_k(t) \, e_{ij}^{(+,\times)}(x)$$

- *amplitude,* $h(t)$, *obeys same wave equation for massless field*
    - *early time quantum fluctuation also frozen-in at late times*

- *decoupled from matter perturbations*

$$\Rightarrow \quad \left\langle T^2 \right\rangle_{today} \quad \approx \quad \left(\frac{64\pi}{M_{Pl}^{\;2}} \right) \left(\frac{H}{2\pi} \right)^2_{k=aH}$$

constrain the Hubble rate during inflation relative to Planck scale

ESA Planck satellite launched! ***14th May 2009***

next all-sky survey

50 million pixels
improved polarisation sensitivity

data release 2012

Gravitational waves
$r \approx 0.1$?

Non-Gaussianity
$f_{NL} = 8$?

summary:

- ***large-scale structure of our Universe*** *can* come from small scale ***quantum fluctuations***

- "golden era" for cosmological discovery, and ambitious plans
  - cosmic microwave background (CMBPol, B-pol, LiteBIRD)
  - large-scale structure in galaxy surveys (dark energy survey...)
  - radio surveys (Square Kilometre Array)
  - gravitational wave detectors (LIGO, LISA...)

- can go beyond linear theory and study non-linear interactions in early universe
  - non-Gaussianity of density perturbations
  - gravitational waves from early universe