Symmetry Breaking in Hot and Dense QCD

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Symmetry-related Phenomena in QCD

Chiral Symmetry (Nambu, Jona-Lasinio, Goldstone, Hatsuda, Kunihiro...)

- Symmetry of QCD in the chiral limit ($m_q = 0$)
- Chiral phase transition at high T and/or μ

Center Symmetry (Polyakov, Susskind, Svetitsky, Yaffe...)

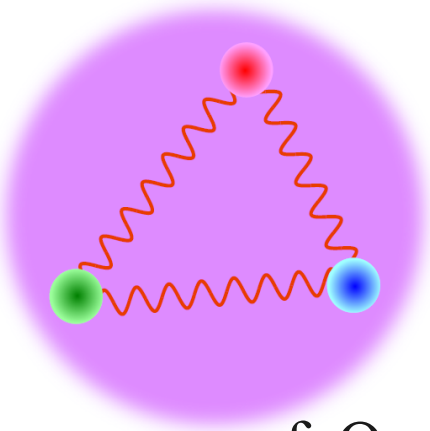
- Symmetry of QCD in the quench limit ($1/m_q = 0$)
- Deconfinement phase transition at high T and/or μ

Parity ($\mathcal{P}$) and Charge-Parity ($\mathcal{CP}$) Symmetry

- Explicit breaking of $\mathcal{CP}$ – "strong $\mathcal{CP}$ problem"
- Topological excitations at high T and/or μ (Khazeev, Pisarski, McLerran...)

Energy Scales

- **Typical QCD energy scale** $\Lambda_{QCD} \sim 200\text{MeV}$
- **Energy scales in the strong interactions**
  - Temperature – $T \sim \Lambda_{\mathrm{QCD}} \sim 10^{12}\,\mathrm{K}$
  - Baryon density – $\rho_{\mathrm{B}} \sim \Lambda^3 \sim \mathrm{fm}^{-3}$
  - Nucleon mass – $M_N \sim N_c \Lambda_{\mathrm{QCD}} \sim \mathrm{GeV}$
  - Magnetic field – $eB \sim \Lambda_{\mathrm{QCD}}^2 \; (B \sim 10^{18}\,\mathrm{Gauss})$

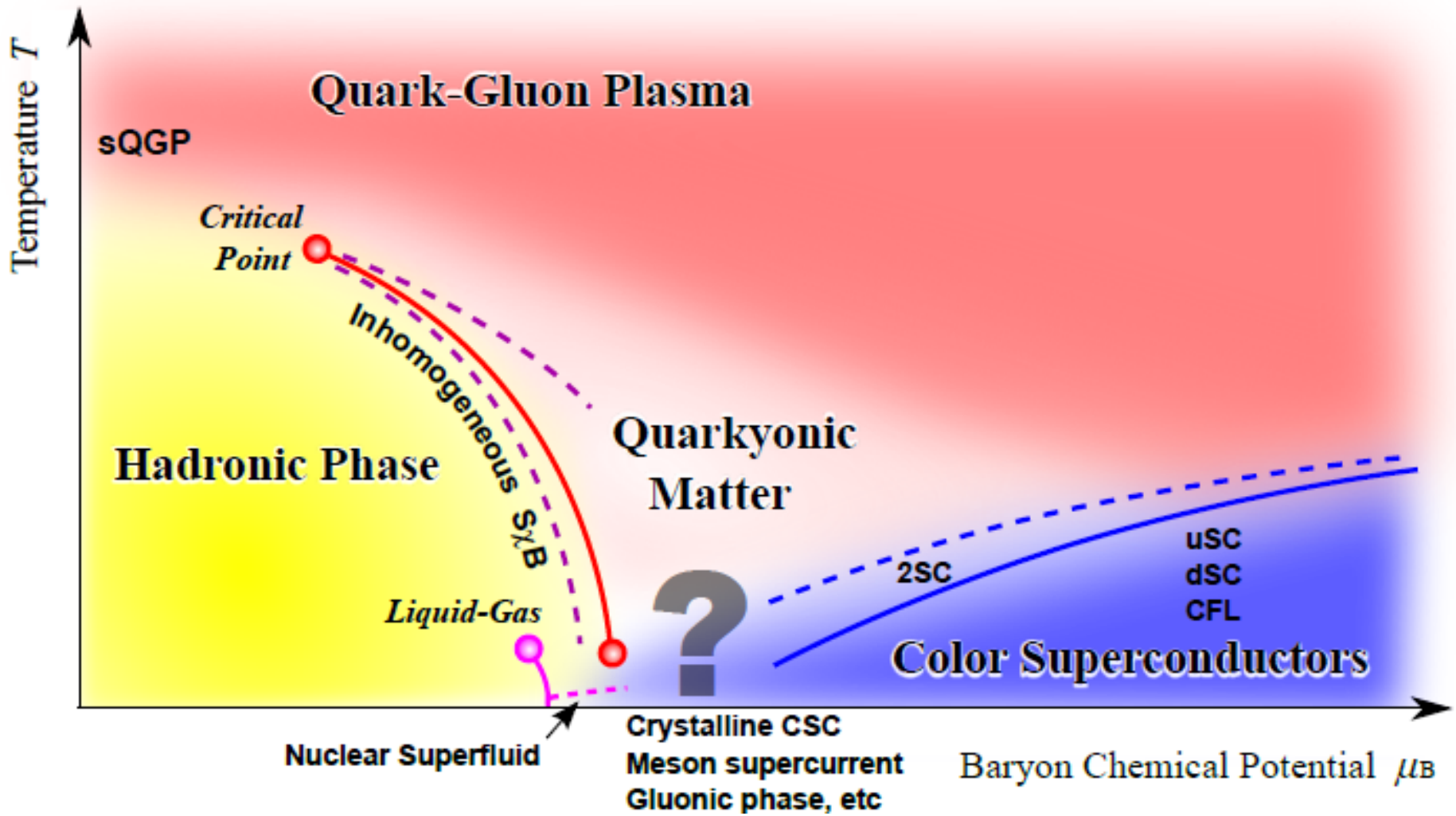


QCD Physics

Confinement
Chiral Symmetry Breaking
Quantum Anomaly

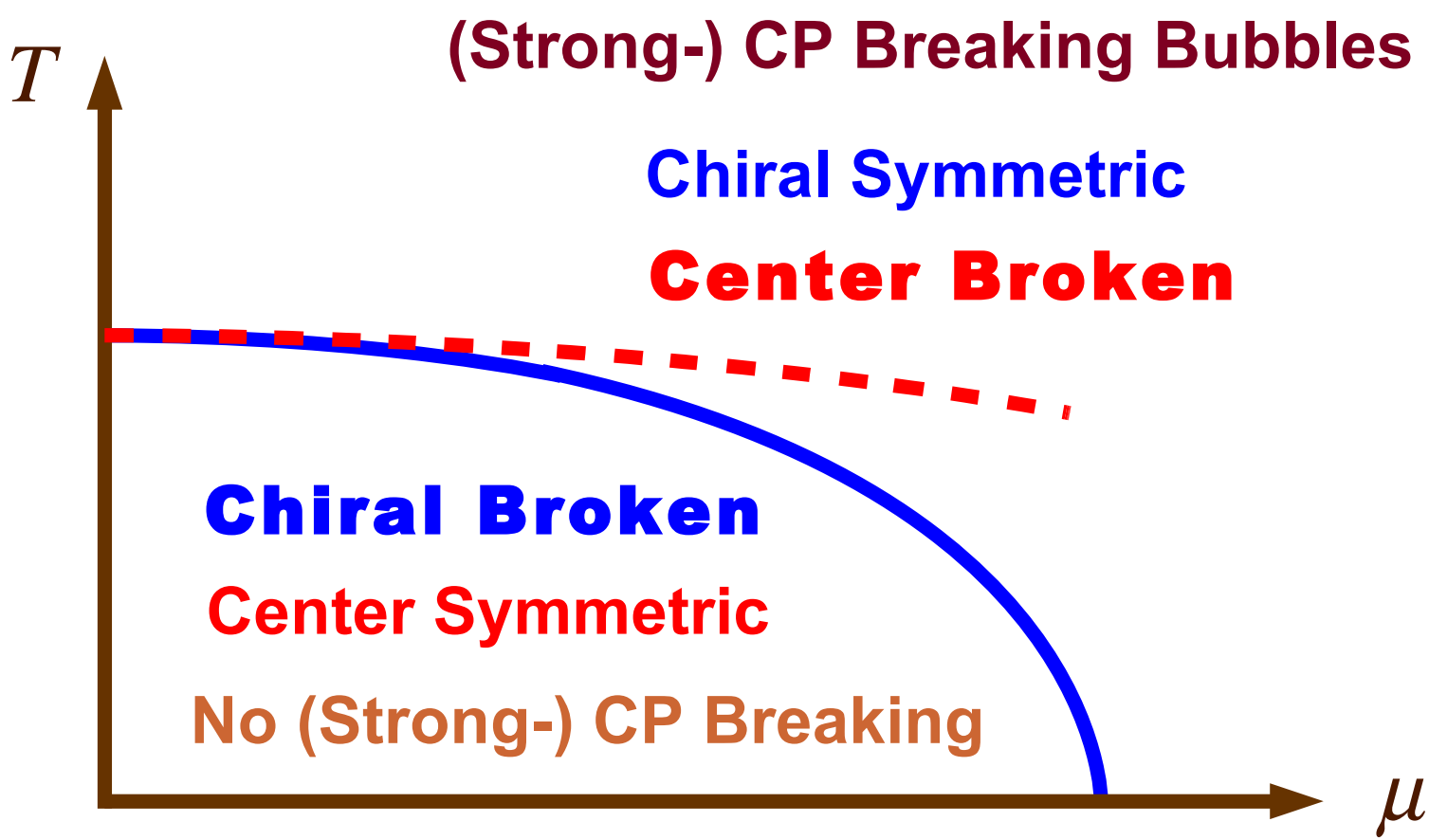
c.f. Quark mass – $m_{\mathrm{ud}} < 5\,\mathrm{MeV}$

Phase Diagram of QCD



From a review by Fukushima-Hatsuda

Minimal Phase Diagram

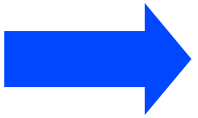


Chiral Phase Transition (m_q~0)

Constituent Quarks

$M_q \sim \Lambda_{QCD}$

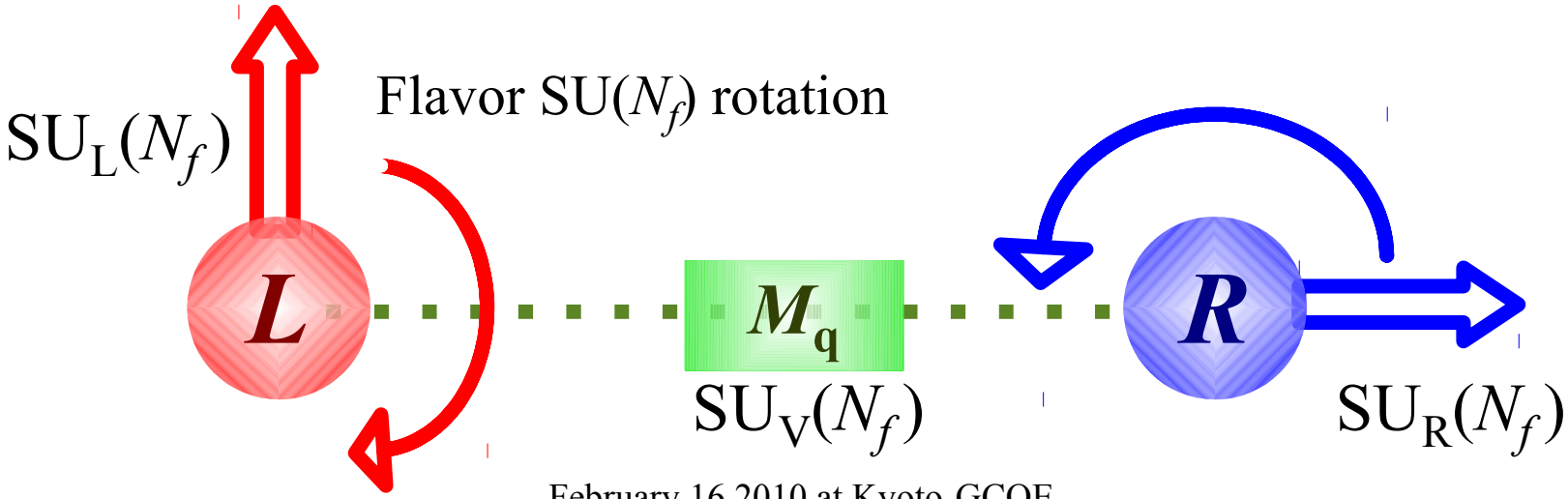
Chiral Condensate $\langle \bar{\psi}\psi \rangle$



$m_q \sim 0$

Chiral Symmetry Breaking

$$\langle \bar{\psi}\psi \rangle = \langle \bar{u}u + \bar{d}d + \cdots \rangle = \langle u_R^\dagger u_L + u_L^\dagger u_R + d_R^\dagger d_L + d_L^\dagger d_R + \cdots \rangle$$



Deconfinement Transition ($1/m_q \sim 0$)

Inter- (heavy) Quark Potential

$$\langle \mathrm{tr}\, L^\dagger(r)\, \mathrm{tr}\, L(0) \rangle \sim \exp[-V(r)/T]$$

Confined Phase	$V(r) \sim \sigma r$	decays exponentially
Deconfined Phase	$V(r) \sim \left(e^{-mr}/r\right)^2$	goes to a finite number

$$\langle \mathrm{tr}\, L^\dagger(r)\, \mathrm{tr}\, L(0) \rangle \to |\langle \mathrm{tr}\, L \rangle|^2$$

Analogy to Spin Systems

Spin System

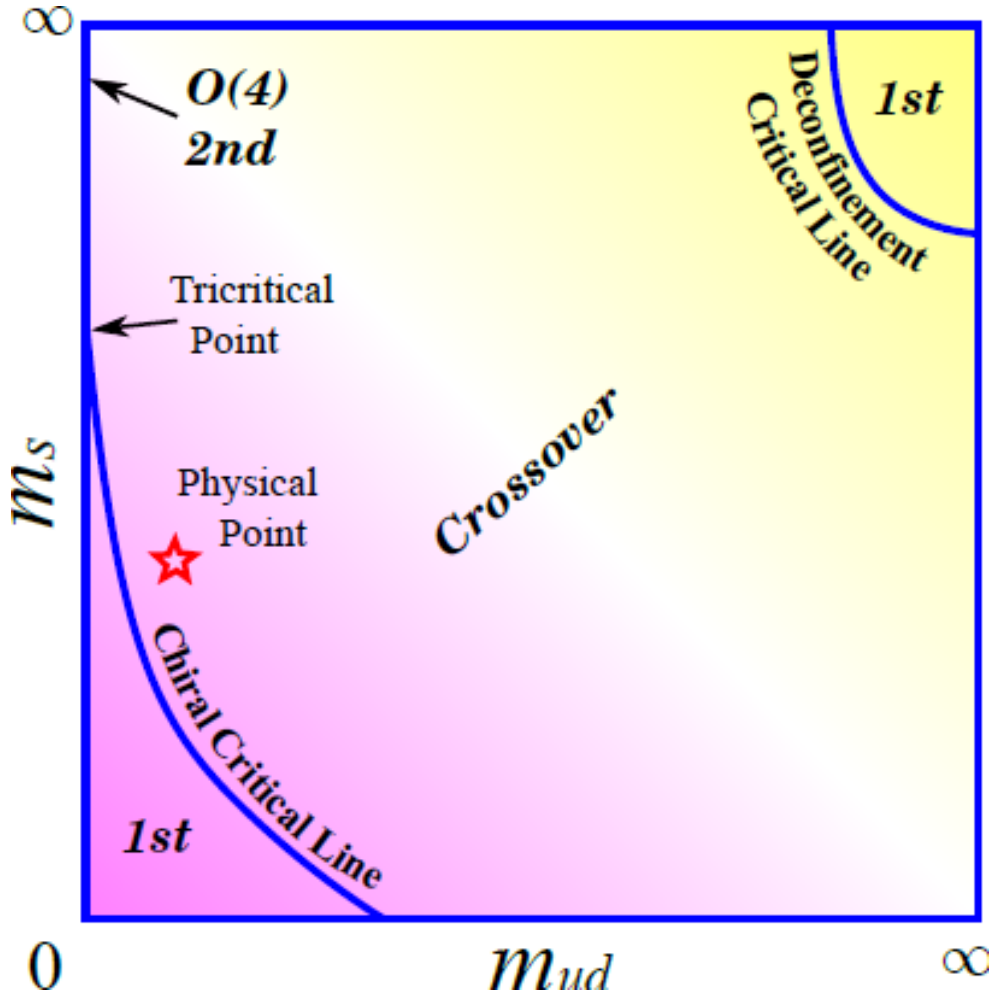
	Ordered	Disordered
$\langle s \rangle$	$\neq 0$	$= 0$
$\langle s(r)\, s(0) \rangle$	$\to \lvert\langle s \rangle\rvert^2$	$\to \exp[-r/\xi]$

Polyakov Loop

	Disordered	Ordered
$\langle \mathrm{tr}\, L \rangle$	$\neq 0$	$= 0$
$\langle \mathrm{tr}\, L^\dagger(r)\, \mathrm{tr}\, L(0) \rangle$	$\to \lvert\langle \mathrm{tr}\, L \rangle\rvert^2$	$\to \exp[-r/\xi]$

Underlying Symmetry = Center of the Gauge Group
= Z_{N_c} Symmetry (Z_3 in QCD)

Two Transitions in terms of M_q



Deconfinement Phase Transition

Chiral Phase Transition

Columbia Plot
Lattice QCD group at Columbia U.
de Forcrand, Philipsen

Two Transitions in terms of T

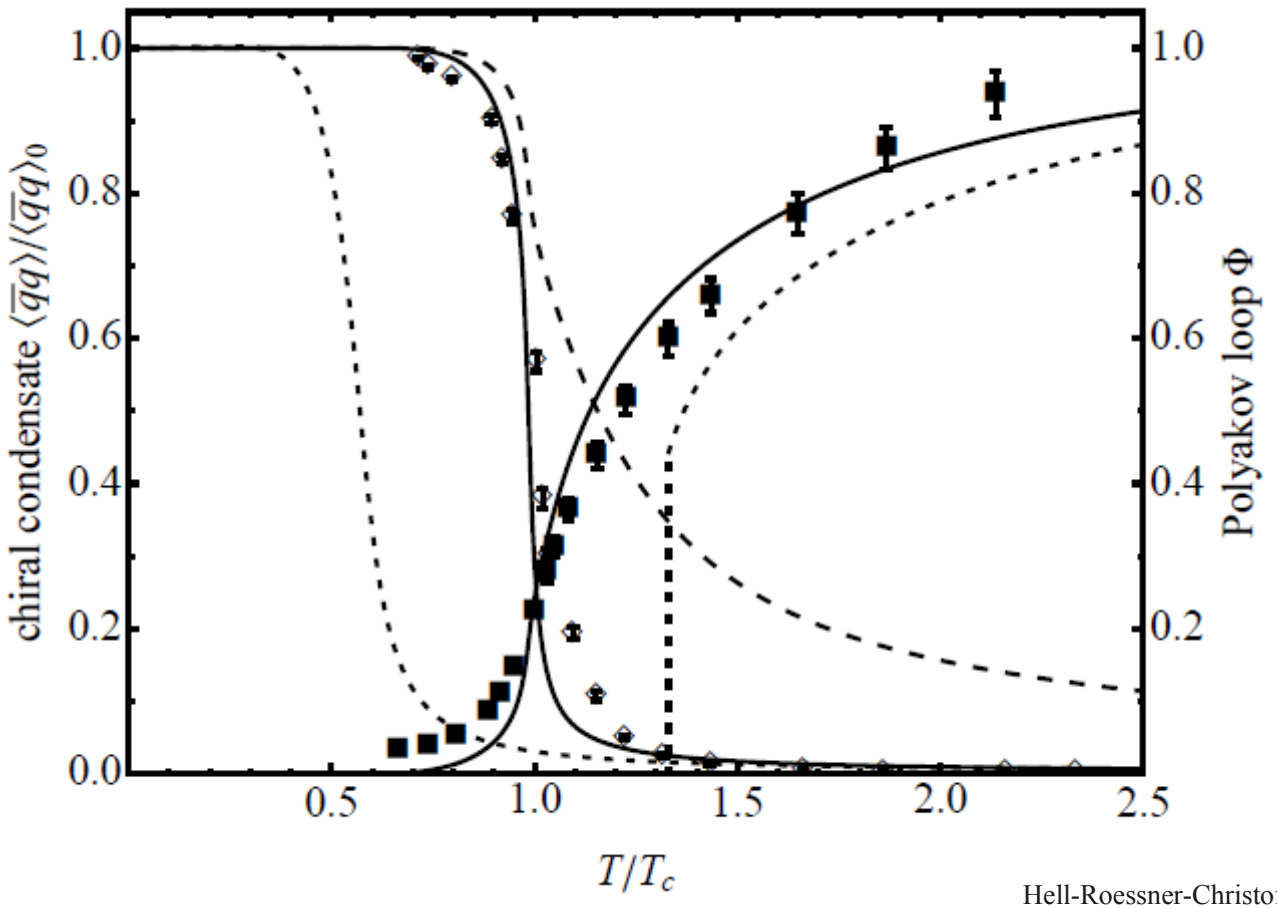
	Low T	High T
$m_q = \infty$ Deconfine	Center Symmetric (Disordered)	Center Broken (Ordered)
$m_q = 0$ Chiral	Chiral Broken (Ordered)	Chiral Symmetric (Disordered)

QCD Phase Transition

Two Order Parameters and Model Study

Polyakov-loop coupled chiral effective models

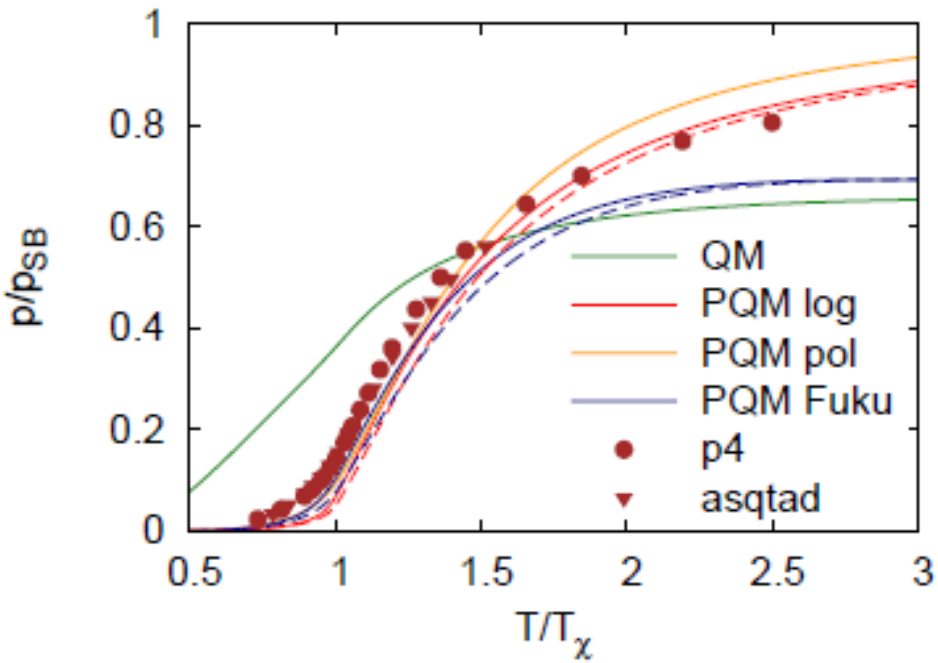
Fukushima, Weise, Ratti, Roessner...

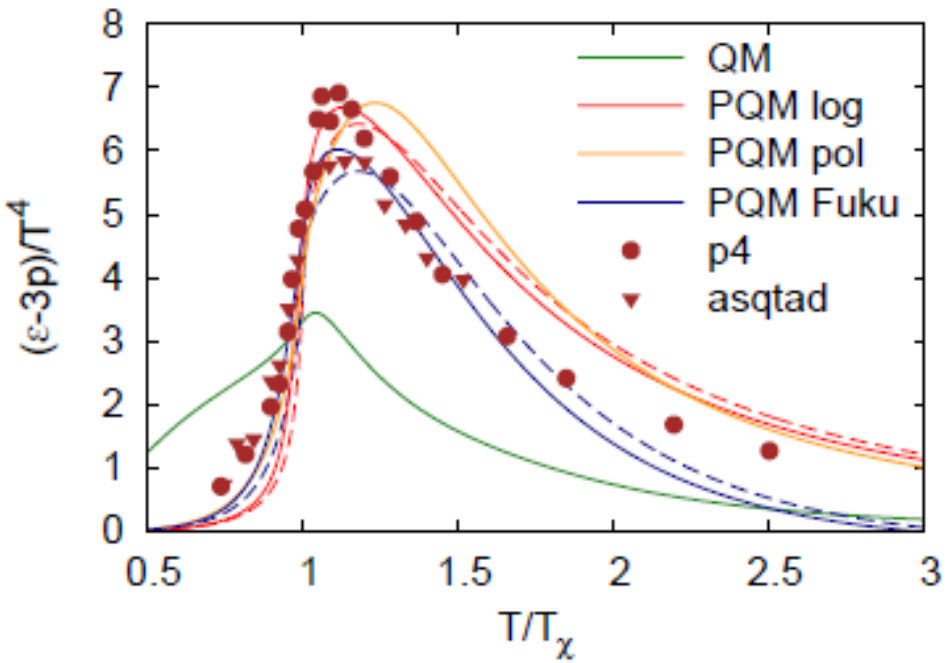


Hell-Roessner-Christoforetti-Weise (2009)

Model Approach to Thermodynamics

Wambach-Schaefer-Wagner (2009)



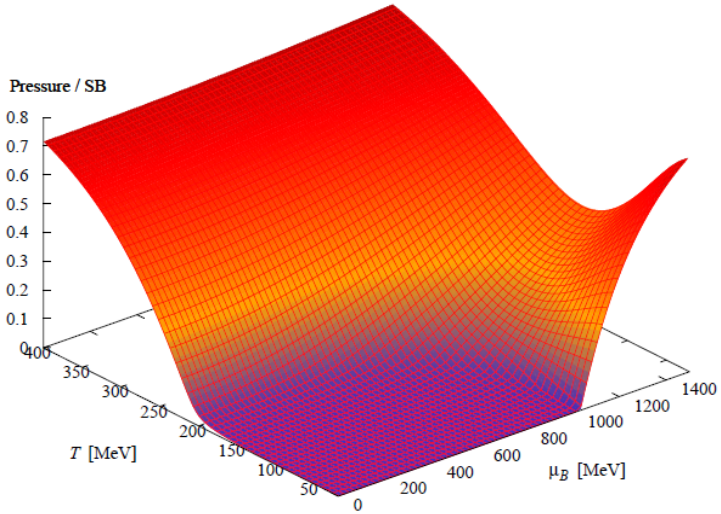


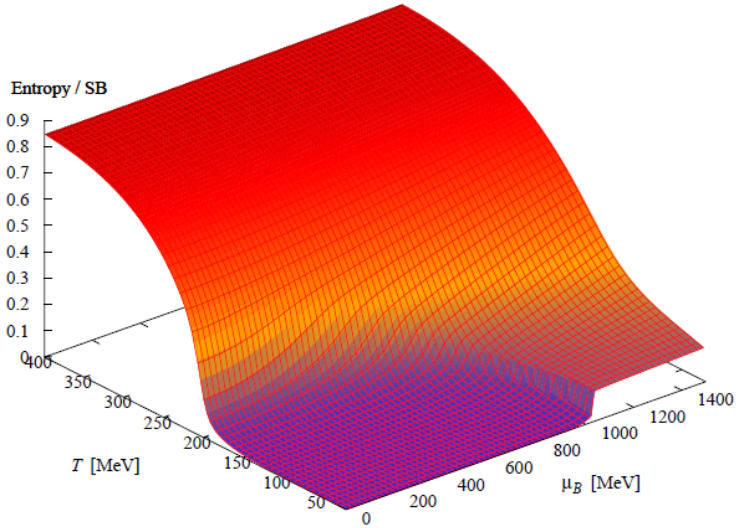
Pressure in unit of the Stefan-Boltzmann counts the number of liberated degrees of freedom (quarks and gluons)

Trace of the energy-momentum tensor measures the interaction strength (deviation from free massless gas)

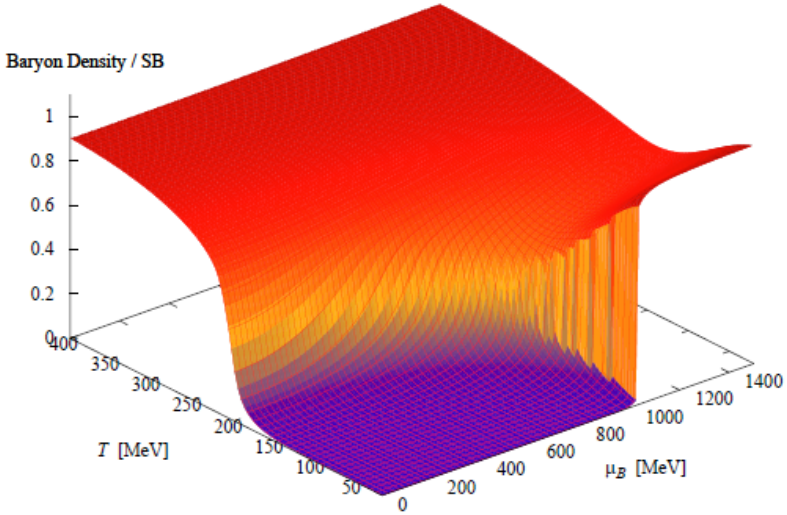
Extrapolation to Finite Density

3D Plots – Pressure and Entropy (density) /SB



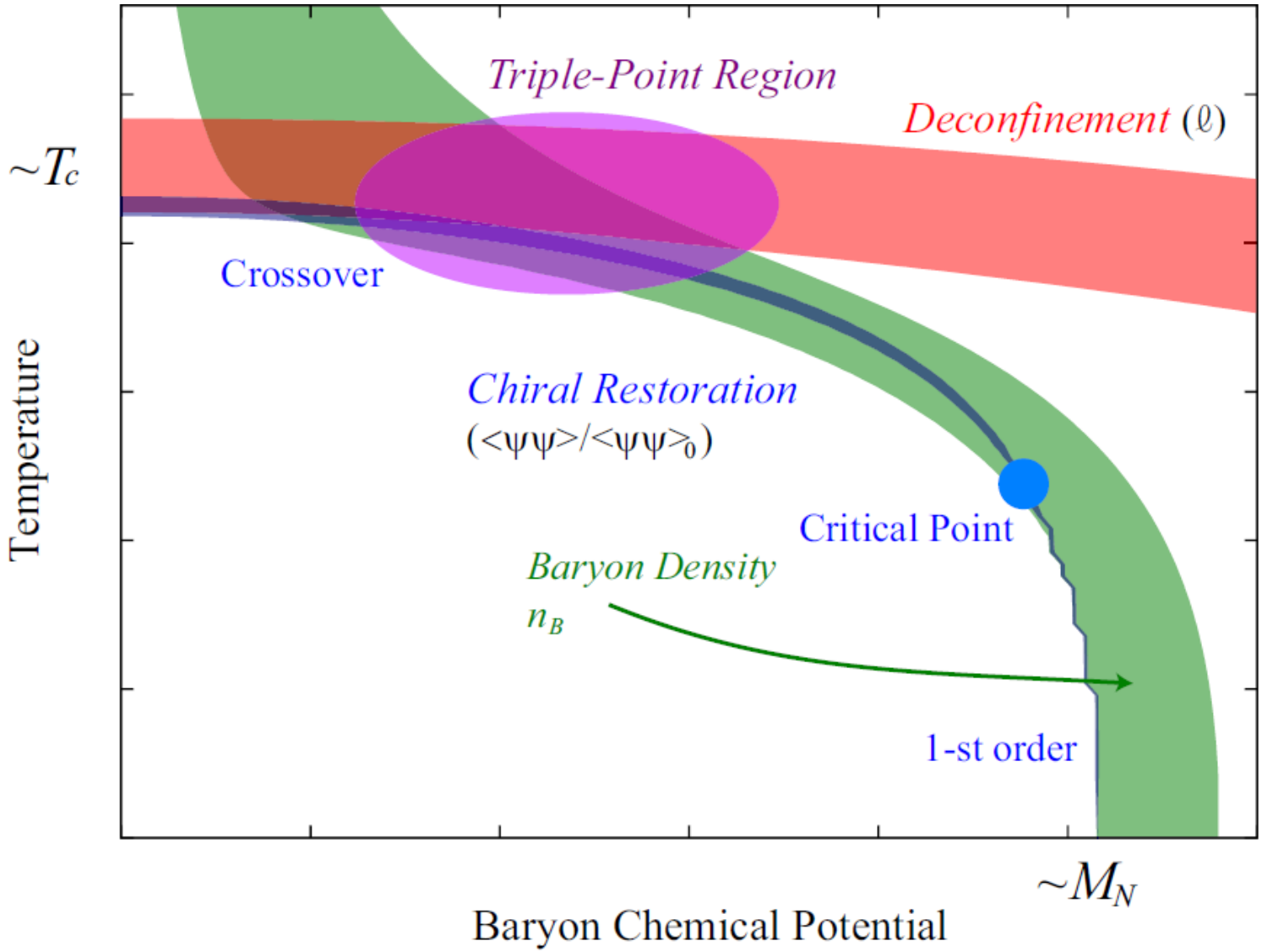


Quark Density /SB



Triple-Point-Like Region

Phase Diagram from PNJL Model



Experimental Efforts

Relativistic Heavy-Ion Collider (RHIC)

Google map

Relativistic Heavy-Ion Collision

Picture by T.D. Lee

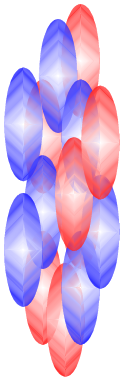
對撞生新態

核如重如牛

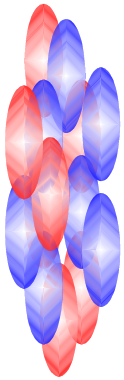
Nuclei as heavy as bulls
Through collision
Generate new states of matter

How Much Energy?

Gold-Gold (Au) Collision
Top energy per *N*-*N* collision
~ 200GeV $\gamma \sim 100$



$A \sim 197$

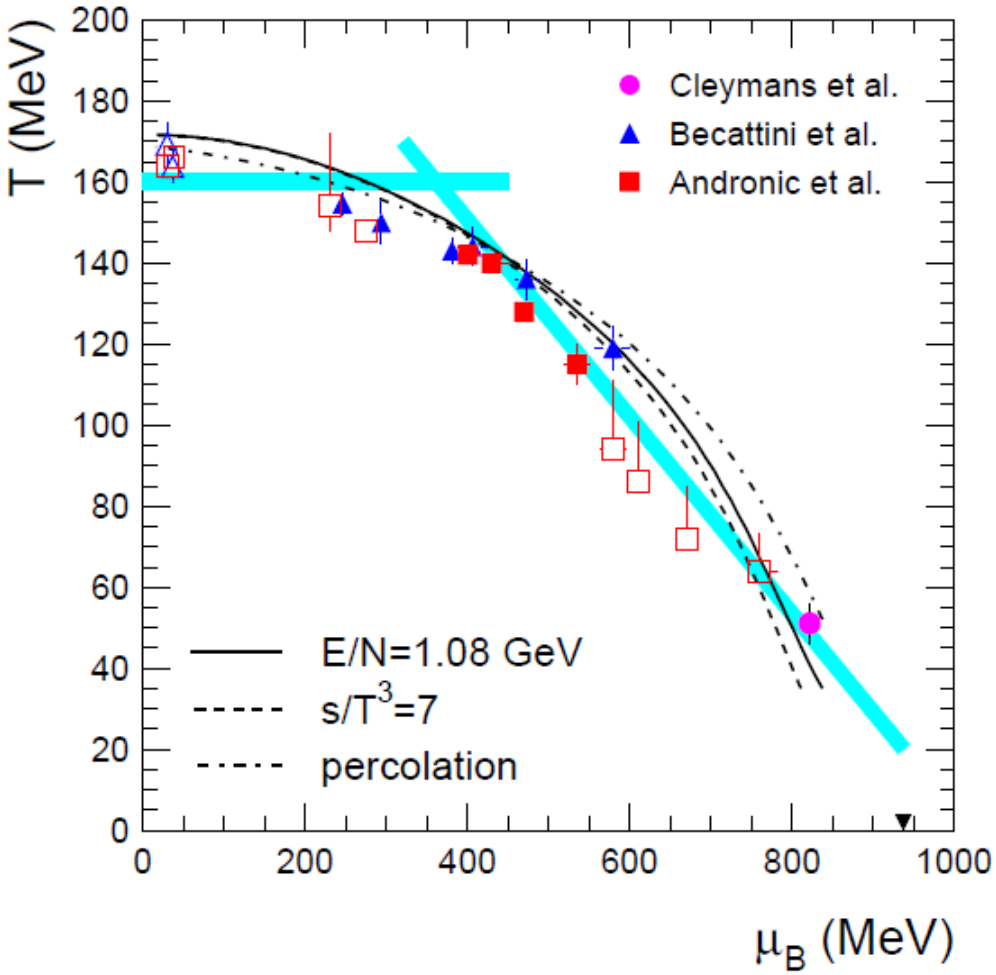


Comparable to the kinetic energy when two mosquitoes collide!

Not bulls...

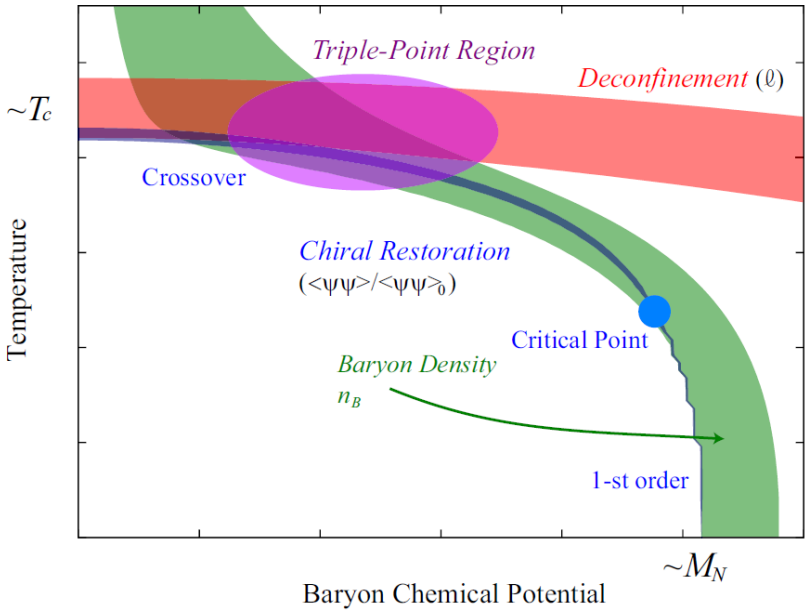
Chemical Freezeout Curve

Interpretation of Data by Statistical Model



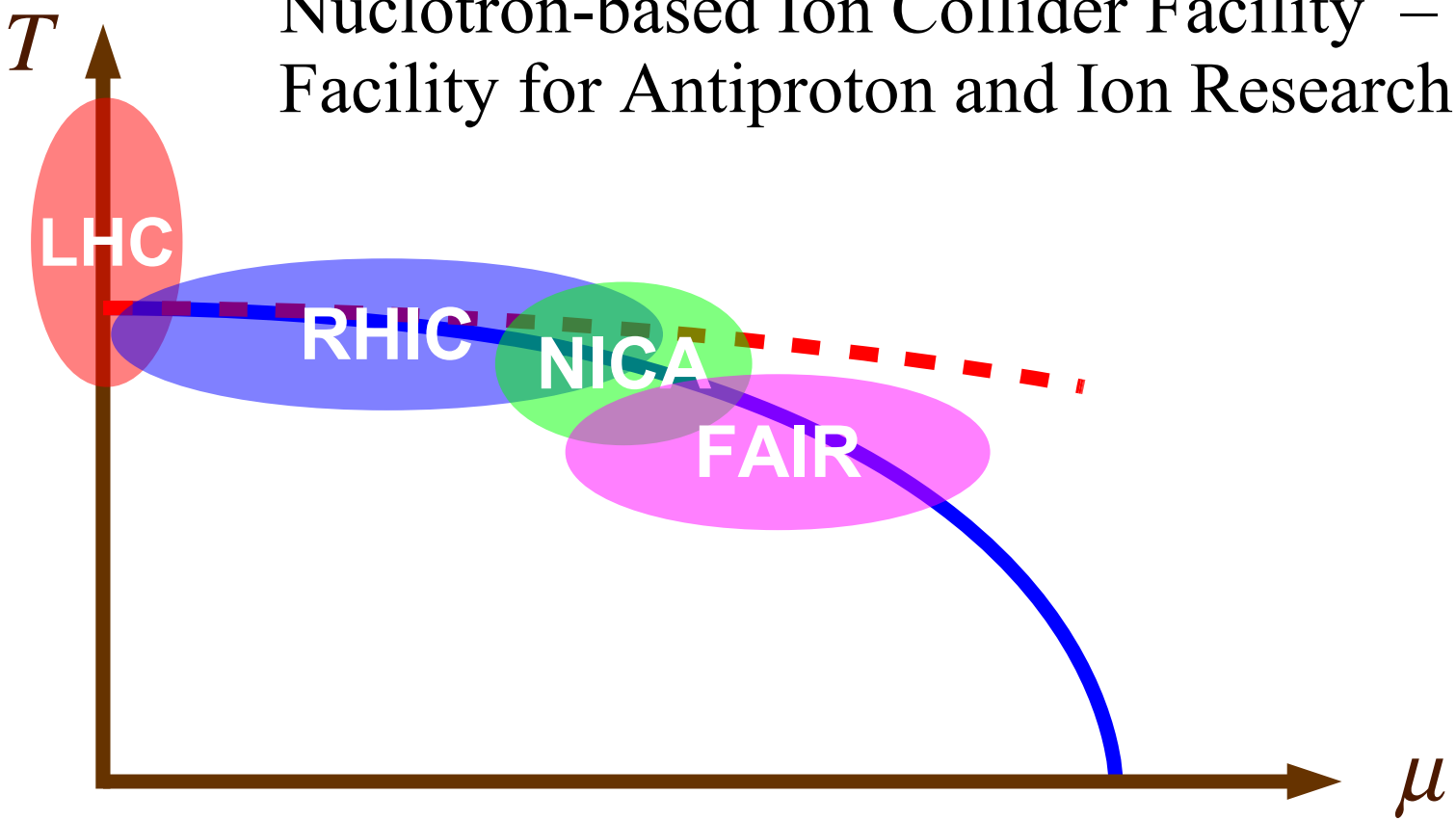
Andronic-Blaschk- Braun-Munnzinger-Cleymans-
-Fukushima-McLerran-Oeschler-Pisarski-Redlich-
-Sasaki-Satz-Stachel (2009)

c.f. Phase Diagram



Future Projects

Large Hadron Collider – CERN
Relativistic Heavy-Ion Collider – BNL
Nuclotron-based Ion Collider Facility – JINR
Facility for Antiproton and Ion Research – GSI



Topological $\mathcal{P}$ and $\mathcal{CP}$ Violation

Gauge Action

- Ordinary Action of Yang-Mills Theories

$$F_{\mu\nu}F^{\mu\nu} = 2F_{0i}F^{0i} + F_{ij}F^{ij}$$

Even w.r.t. spatial and temporal indices

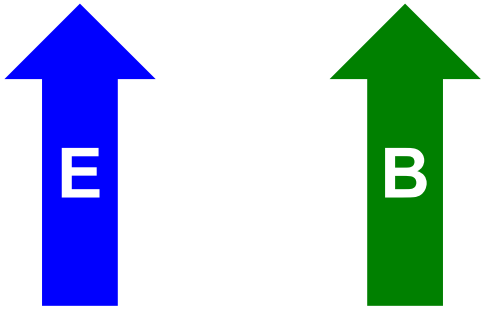
- Topological Contribution **$\mathcal{P}$− and $\mathcal{CP}$− odd terms**

$$F_{\mu\nu}\widetilde{F}^{\mu\nu} = 2F_{01}F^{23} + 2F_{02}F^{31} + 2F_{03}F^{12}$$

Odd w.r.t. spatial and temporal indices

Parallel ***E*** and ***B***

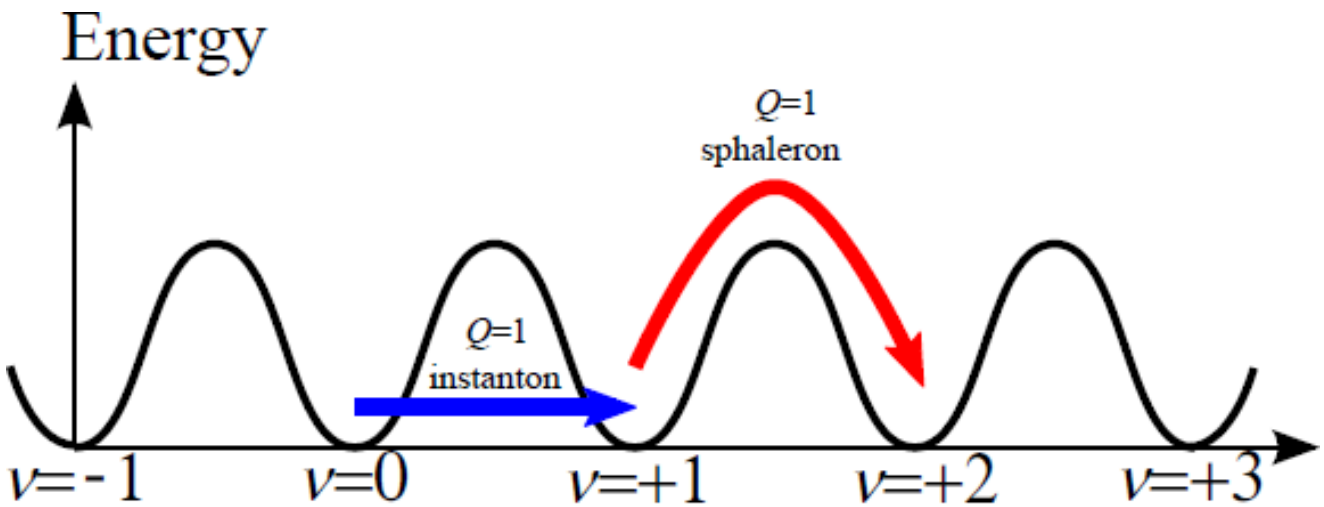
$$F_{\mu\nu}\widetilde{F}^{\mu\nu} = 2\boldsymbol{E}\cdot\boldsymbol{B}$$



θ-Vacuum and Strong CP Problem

Topological Excitations and θ-vacuum

Faddeev-Jackiw-Rebbi



Chern-Simons number

Strong CP Problem

$$S_{\mathrm{QCD}} = -\frac{1}{2}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu} + \theta\frac{g^2}{16\pi^2}\,\mathrm{tr}\,F_{\mu\nu}\tilde{F}^{\mu\nu}$$

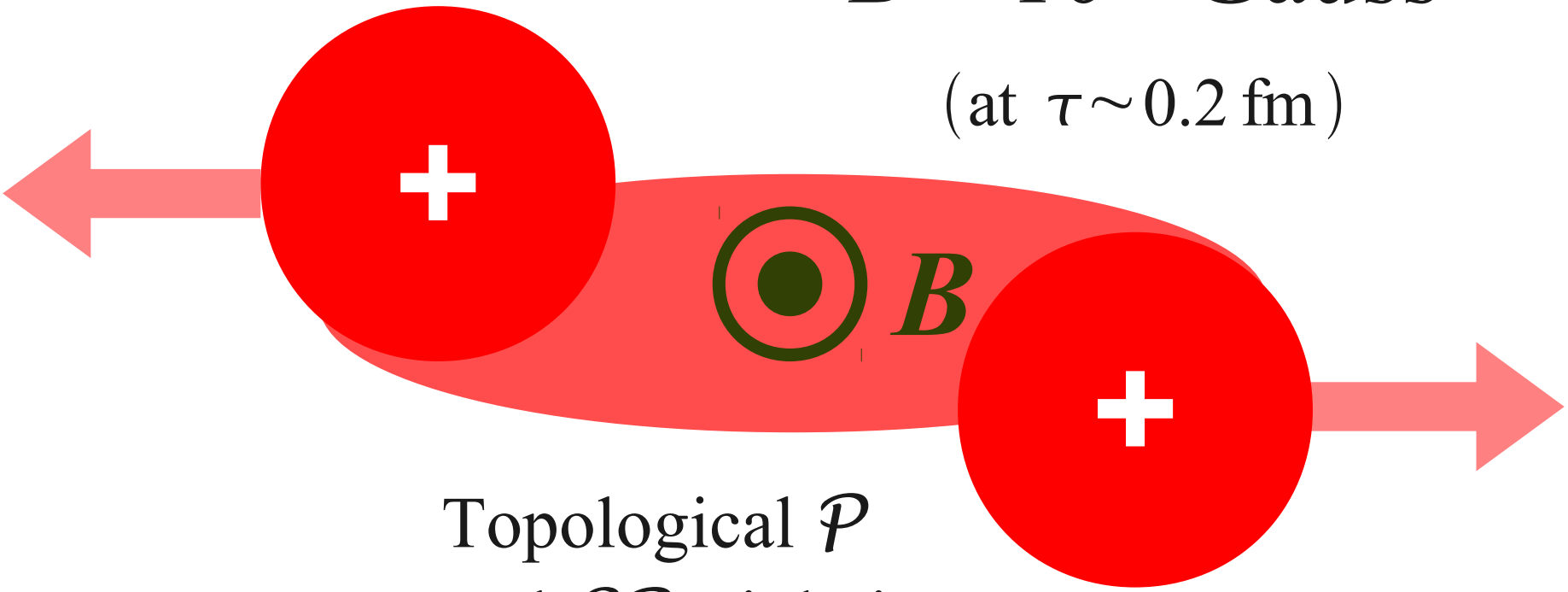
$$|\theta| < 0.7\times 10^{-11}$$

How to detect topological excitations?

Non-central Collision

$$B \sim 10^{18}\,\text{Gauss}$$

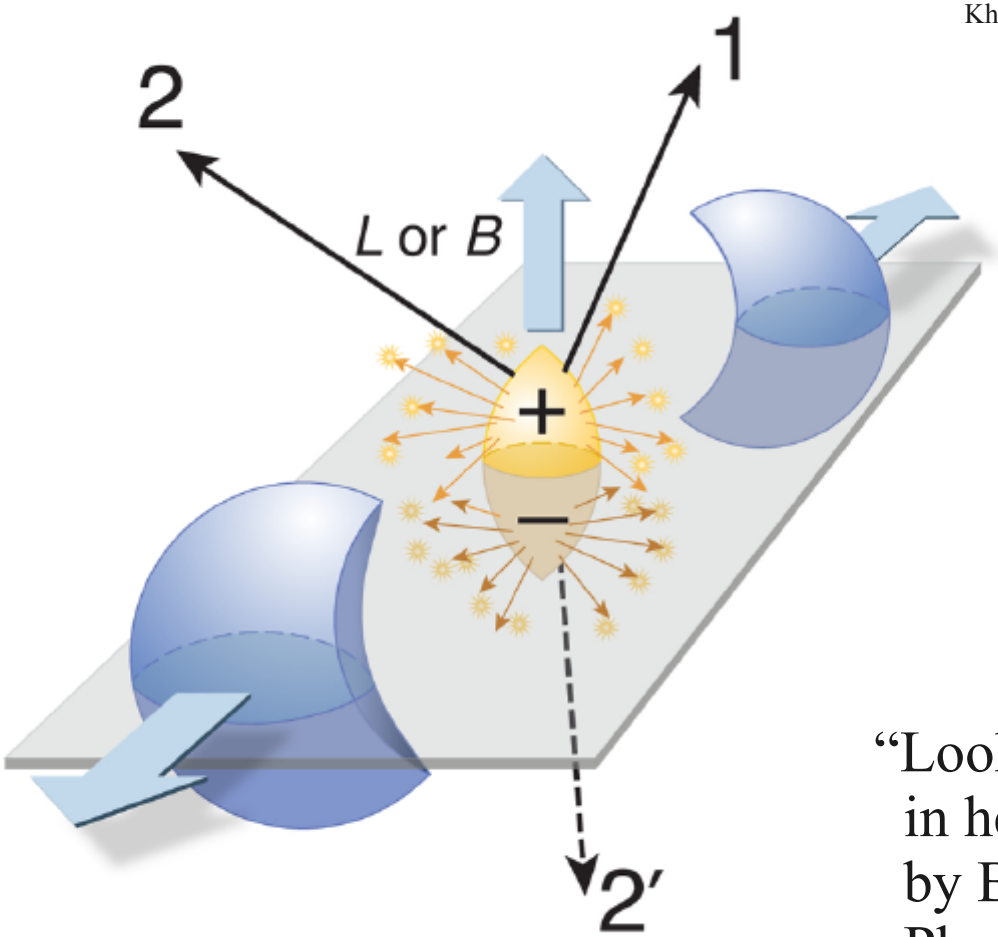
$$(\text{at } \tau \sim 0.2\,\text{fm})$$



Topological $\mathcal{P}$
and $\mathcal{CP}$ Violation
Finite θ ?

Charge Asymmetry

Kharzeev-McLerran-Warringa (2007)



"Looking for parity violation in heavy-ion collisions"
by Berndt Müller
Physics 2, 104 (2009)

Anomaly Relations

Induced N_5 by Non-Abelian Topological Effects

$$\frac{dN_5}{dt} = -\frac{g^2 N_f}{8\pi^2}\int d^3x \ \mathrm{tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Exact Anomaly Relation

Introduce μ_5 to describe induced N_5

Induced J by the presence of N_5 and B

$$\boldsymbol{j} = \frac{e^2 \mu_5}{2\pi^2}\boldsymbol{B}$$

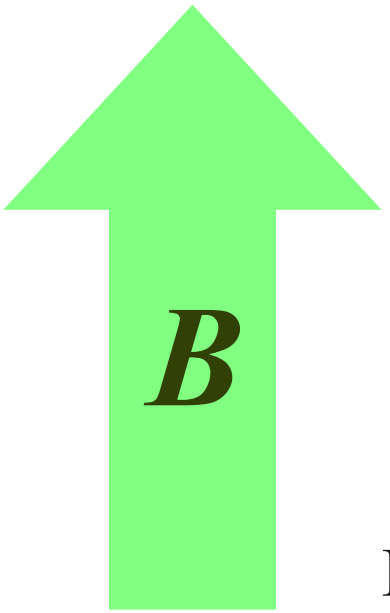
Exact Anomaly Relation

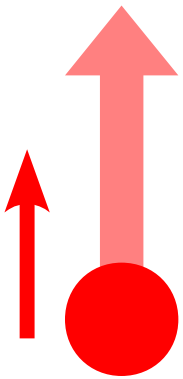
$$\left(\boldsymbol{j} = \sum_{i=\mathrm{flavor}} \frac{q_i^2 \mu_5}{2\pi^2}\boldsymbol{B} \quad \text{in QCD}\right)$$

Metlitski-Zhitnitsky (2005)
Fukushima-Kharzeev-Warringa (2008)

Chiral Magnetic Effect

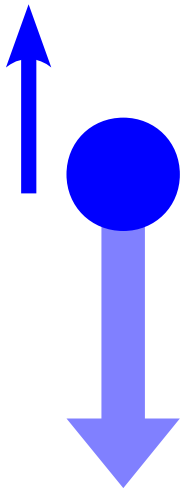
Classical Picture





Right-handed Quark
= momentum parallel to spin

Left-handed Quark
= momentum parallel to spin



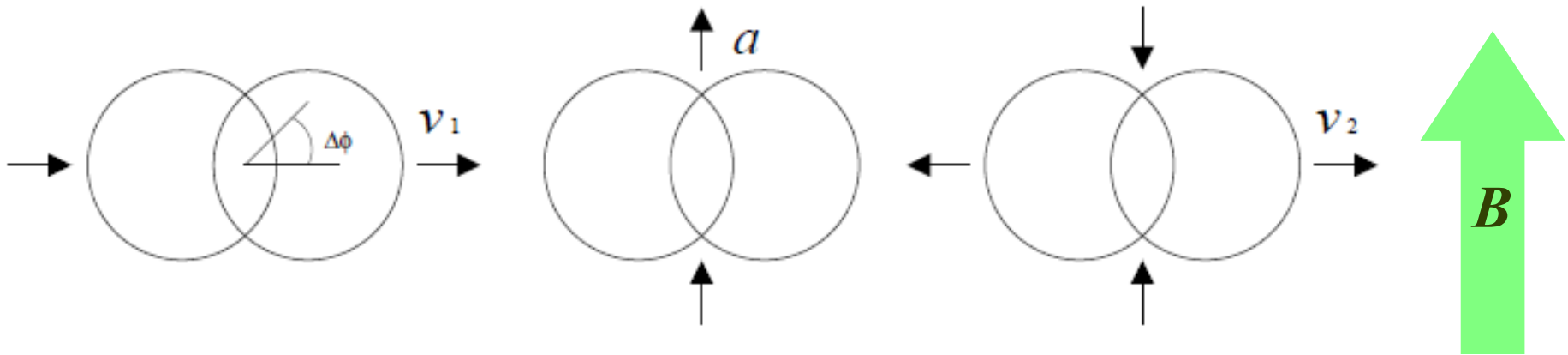
$$\boldsymbol{J} \neq 0 \quad \text{if} \quad N_5 = N_R - N_L \neq 0$$

Kharzeev-McLerran-Warringa (2007)
Fukushima-Kharzeev-Warringa (2008)

Observable

Measured Multiplicity

$$\frac{\mathrm{d}N_\pm}{\mathrm{d}\phi} \propto 1 + 2v_{1\pm}\cos(\Delta\phi) + 2a_\pm \sin(\Delta\phi) + 2v_{2\pm}\cos(2\Delta\phi) + \cdots$$



ϕ : Azimuthal angle

v_1 : Directed flow

v_2 : Elliptic flow

$$a \sim J$$

Experimental Data

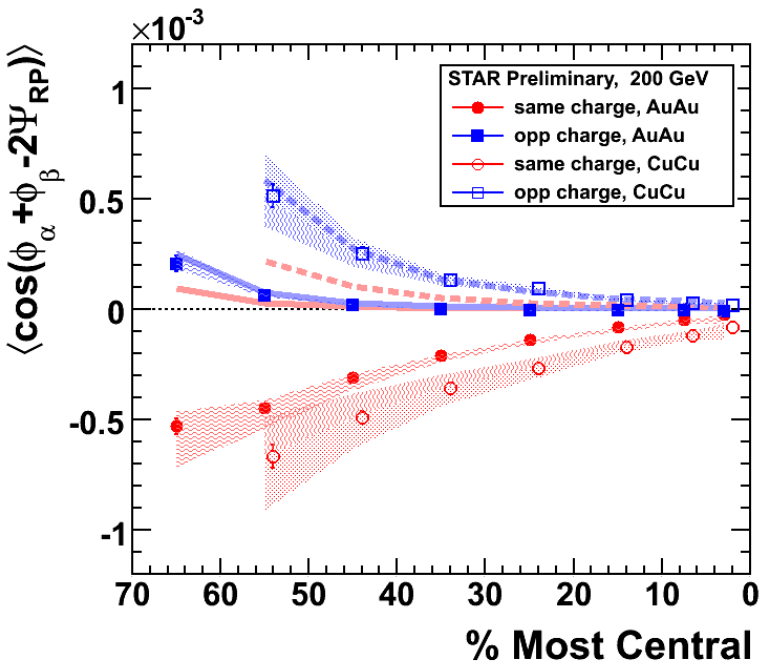
Observable (fluctuation measurement)

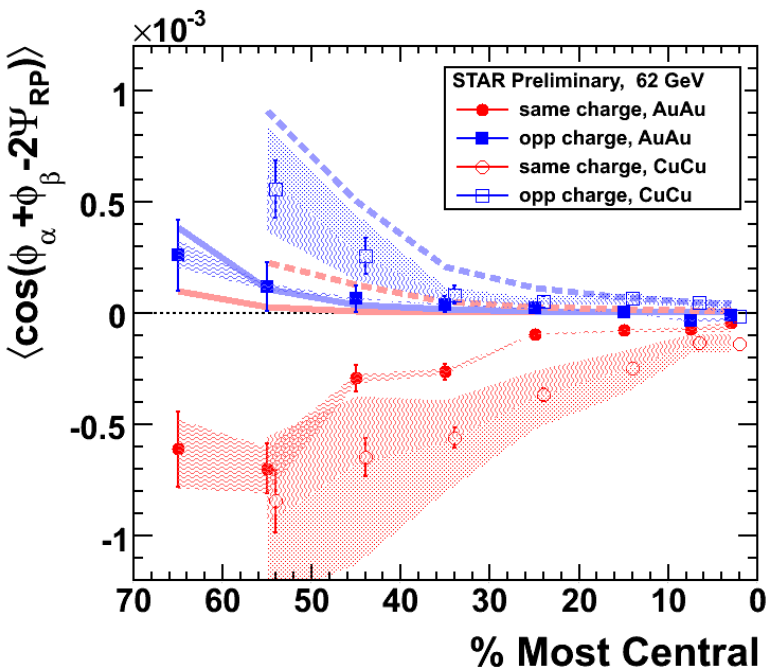
$$\langle\langle\cos(\Delta\phi_\alpha+\Delta\phi_\beta)\rangle\rangle \equiv \left\langle\!\!\left\langle \frac{1}{N_\alpha N_\beta}\sum_{i=1}^{N_\alpha}\sum_{j=1}^{N_\beta}\cos(\Delta\phi_{\alpha,i}+\Delta\phi_{\beta,j})\right\rangle\!\!\right\rangle$$

$$= \langle\langle\cos\Delta\phi_\alpha\cos\Delta\phi_\beta\rangle\rangle - \langle\langle\sin\Delta\phi_\alpha\sin\Delta\phi_\beta\rangle\rangle$$

$$= \left(\langle\langle v_{1,\alpha}v_{1,\beta}\rangle\rangle + B^{\rm in}_{\alpha\beta}\right) - \left(\langle\langle a_\alpha a_\beta\rangle\rangle + B^{\rm out}_{\alpha\beta}\right).$$

Exp. Results





Star Collaboration
PRL103, 251601 (2009)

Electric-current Susceptibility

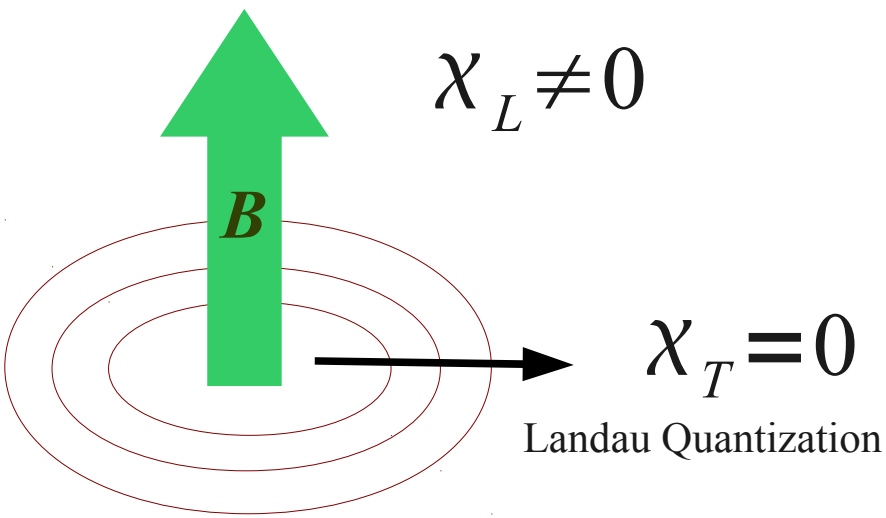
Charge Asymmetry Measurement

$$\sim\langle J_L^2 - J_T^2\rangle = \langle J_L\rangle^2 + (\chi_L - \chi_T)$$

$$\chi_L - \chi_T = VT\, N_c \sum_f \frac{q_f^2 |q_f B|}{(2\pi)^2}$$

Constrained by Anomaly

Fukushima-Kharzeev-Warringa (2009)



Summary

- Many theoretical and experimental activities toward phase structure of dense QCD.
  - Triple-point-like region involving Quarkyonic Matter
  - More experimental data coming from various facilities all over the world

- The idea of the Chiral Magnetic Effect has opened a new research field in hot and dense QCD.
  - Detailed information on topological fluctuations
  - Topological $\mathcal{P}$- and $\mathcal{CP}$-odd effects found in experiments