12/02/13

Einstein-Podolsky-Rosen-like correlation on a coherent-state basis and Continuous-Variable entanglement

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arXiv:1109.0349

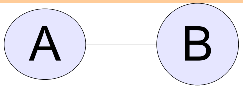
Quantum Entanglement

- Foundation of Quantum mechanics:
  Einstein-Podolsky-Rosen paradox; Bell's theorem.

•Experimental goal: Generation of entangled states and Observation of Quantum correlation

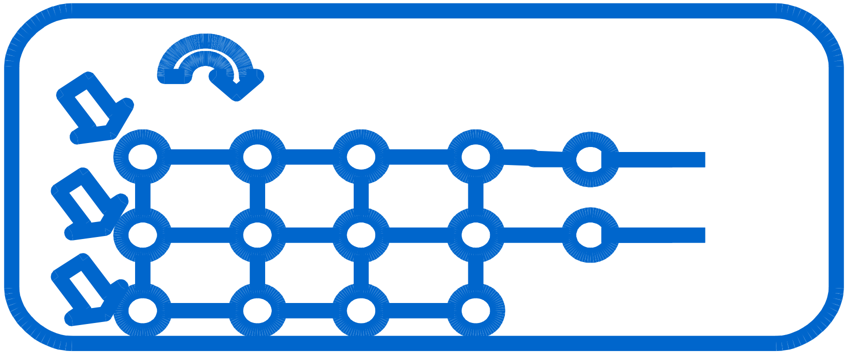
•Resource of communication and computation in quantum information processing

Quantum key distribution



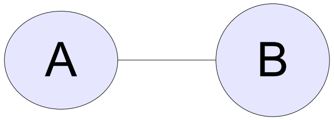
Measurement-based one way quantum computation

Entanglement +
Local measurement



Universal quantum computer
= Simulation of any physical system

Continuous-variable systems in Quantum Optics



- Bosonic fields described by a set of canonical variables $[\hat{x}_A, \hat{p}_A] = [\hat{x}_B, \hat{p}_B] = \cdots = i$

$$\langle \Delta^2 \hat{x} \rangle \langle \Delta^2 \hat{p} \rangle \geqslant 1/4$$

- Gaussian states

 -Minimum uncertainty states

 Coherent states/ Squeezed states /...

- Wave function is Gaussian

 -Any property is determined by the second moments:

$$\langle \Delta^2 \hat{x}_A \rangle, \langle \Delta^2 \hat{p}_A \rangle, \langle \Delta^2 \hat{x}_B \rangle, \langle \Delta^2 \hat{p}_B \rangle, \langle (\Delta \hat{x}_A)(\Delta \hat{x}_B) \rangle, \langle (\Delta \hat{p}_A)(\Delta \hat{p}_B) \rangle,$$

$$\langle (\Delta \hat{x}_A)(\Delta \hat{p}_B) \rangle, \langle (\Delta \hat{p}_A)(\Delta \hat{x}_B) \rangle, \cdots$$

 Measurable by optical homodyne detection

EPR correlation

Einstein-Podolsky-Rosen (EPR) state

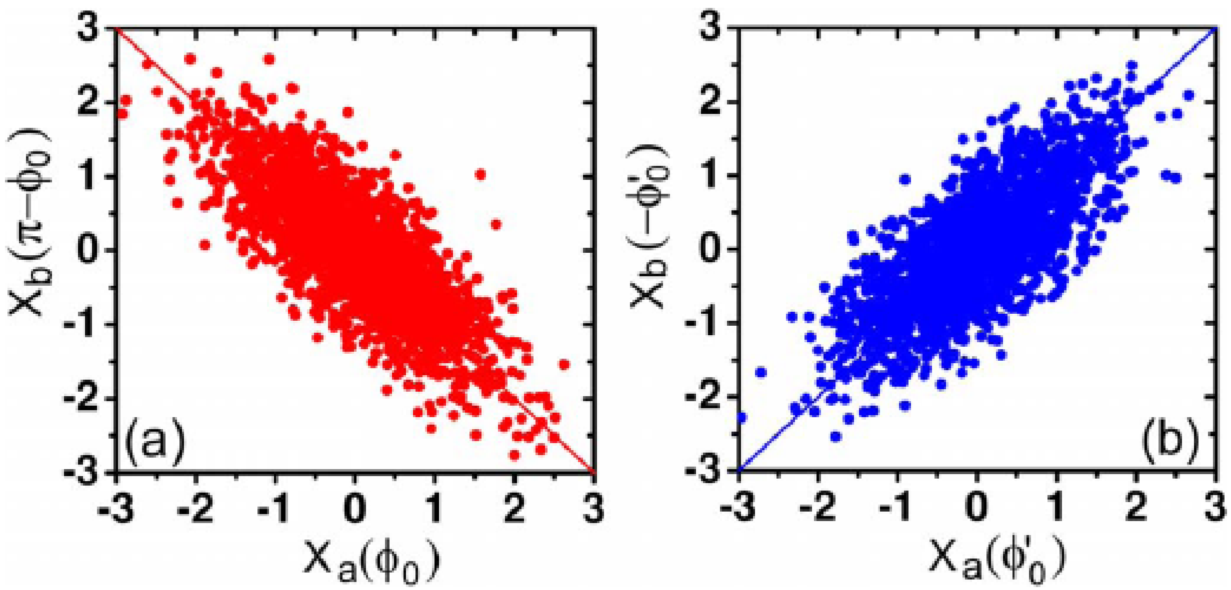
$$|\psi\rangle_{AB} := \int dx\, |x\rangle_A |x\rangle_B / \sqrt{2\pi}$$

A B

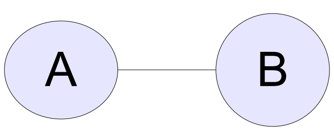
$$|x_1\rangle \cdots\cdots\cdots |x_1\rangle$$
$$|x_2\rangle \cdots\cdots\cdots |x_2\rangle$$
$$|x_3\rangle \cdots\cdots\cdots |x_3\rangle$$
$$\vdots$$
$$|p_1\rangle \cdots\cdots\cdots |-p_1\rangle$$
$$|p_2\rangle \cdots\cdots\cdots |-p_2\rangle$$
$$|p_3\rangle \cdots\cdots\cdots |-p_3\rangle$$
$$\vdots$$

Positions are correlated and
Momentums are anti-correlated

$$\langle \Delta^2(\hat{p}_A + \hat{p}_B)\rangle \sim 0 ; \langle \Delta^2(\hat{x}_A - \hat{x}_B)\rangle \sim 0$$



ETO *et al.* PHYSICAL REVIEW A 79, 050302(R) (2009)



Stronger Simultaneous correlation can be a signature of entanglement

EPR-like uncertainties and entanglement

Product criterion for entanglement

$$C := \|[x, p]\|/2$$

$$\langle \Delta^2 (u\hat{x}_A - v\hat{x}_B) \rangle \langle \Delta^2 (u\hat{p}_A + v\hat{p}_B) \rangle < C^2$$

⇨ the state is entangled.

V. Giovannetti et al., Phys. Rev. A 67, 022320 (2003)

EPR correlation

$$\langle \Delta^2 (\hat{x}_A - \hat{x}_B) \rangle \sim 0; \langle \Delta^2 (\hat{p}_A + \hat{p}_B) \rangle \sim 0$$

EPR-like operators

$$u\hat{x}_A - v\hat{x}_B; \quad u\hat{p}_A + v\hat{p}_B \qquad u^2 + v^2 = 1$$

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$$\left\langle \Delta^2 (u\,\hat{x}_A - v\,\hat{x}_B)\right\rangle \left\langle \Delta^2 (u\,\hat{p}_A - v\,\hat{p}_B)\right\rangle \geqslant C^2$$

Uncertainty relation!

V. Giovannetti et al., Phys. Rev. A 67, 022320 (2003)

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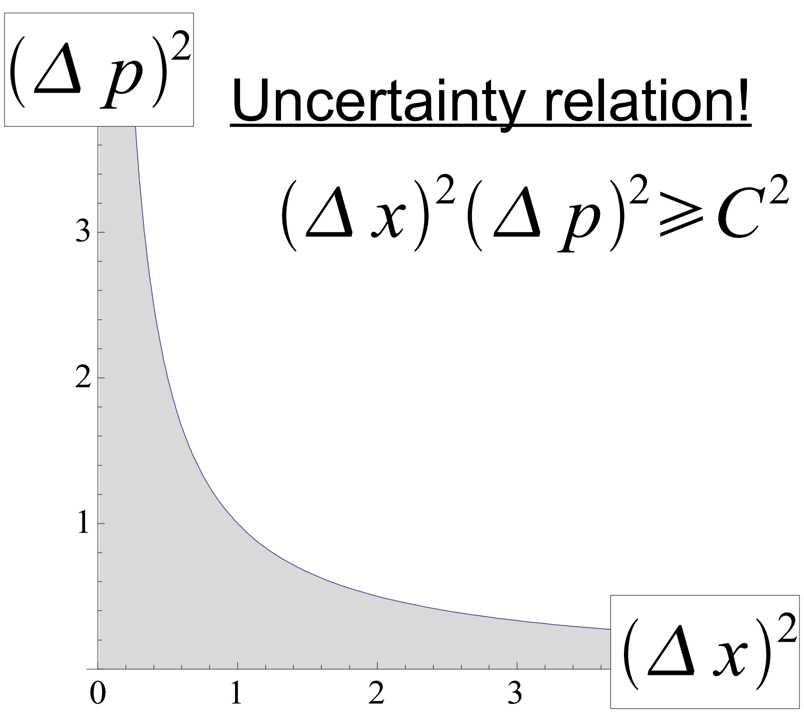
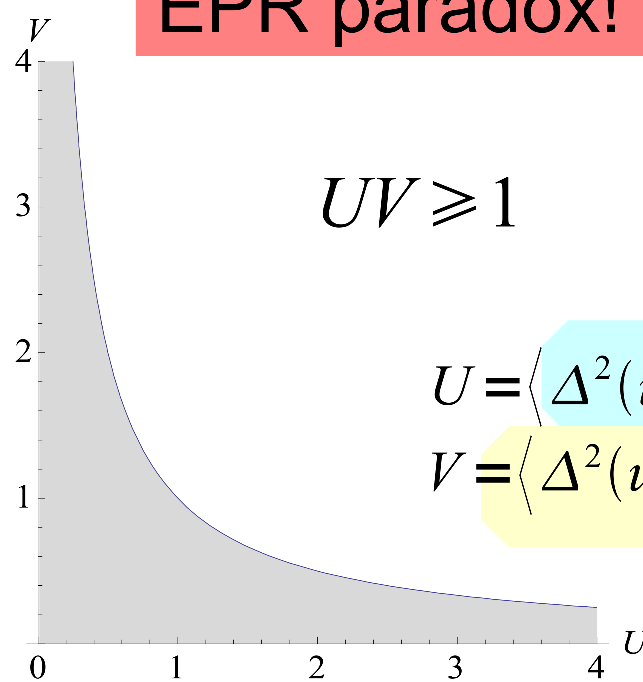
Uncertainty relation!

$$(\Delta x)^2 (\Delta p)^2 \geqslant C^2$$



EPR paradox!

$$UV \geqslant 1$$



$$U = \left\langle \Delta^2(u\hat{x}_A - v\hat{x}_B)\right\rangle / C$$

$$V = \left\langle \Delta^2(u\hat{p}_A + v\hat{p}_B)\right\rangle / C$$

EPR-like uncertainties and entanglement

Product criterion for entanglement

$$C := \| [x, p] \| / 2$$

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⇨ the state is entangled.

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Outline of outline of proof.

1. Assume separable form

$$\rho_{AB} = \sum_i p_i (\rho_i)_A \otimes (\sigma_i)_B$$

2. Prove the inequality

$$\langle \Delta^2 U \rangle \langle \Delta^2 V \rangle \geqslant 1$$

$$U = \langle \Delta^2 (u \hat{x}_A - v \hat{x}_B) \rangle / C$$

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Violation of the inequality

$$U = \langle \Delta^2 (u\,\hat{x}_A - v\,\hat{x}_B) \rangle / C$$

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$$\rho_{AB} = \sum_i p_i (\rho_i)_A \otimes (\sigma_i)_B$$

2. Prove the inequality

$$\langle \Delta^2 U \rangle \langle \Delta^2 V \rangle \geqslant 1$$

$$\rho_{AB} \neq \sum_i p_i (\rho_i)_A \otimes (\sigma_i)_B$$

⇦ Violation of the inequality

Definition of entangled states.

$$U = \langle \Delta^2 (u \hat{x}_A - v \hat{x}_B) \rangle / C$$

$$V = \langle \Delta^2 (u \hat{p}_A + v \hat{p}_B) \rangle / C$$

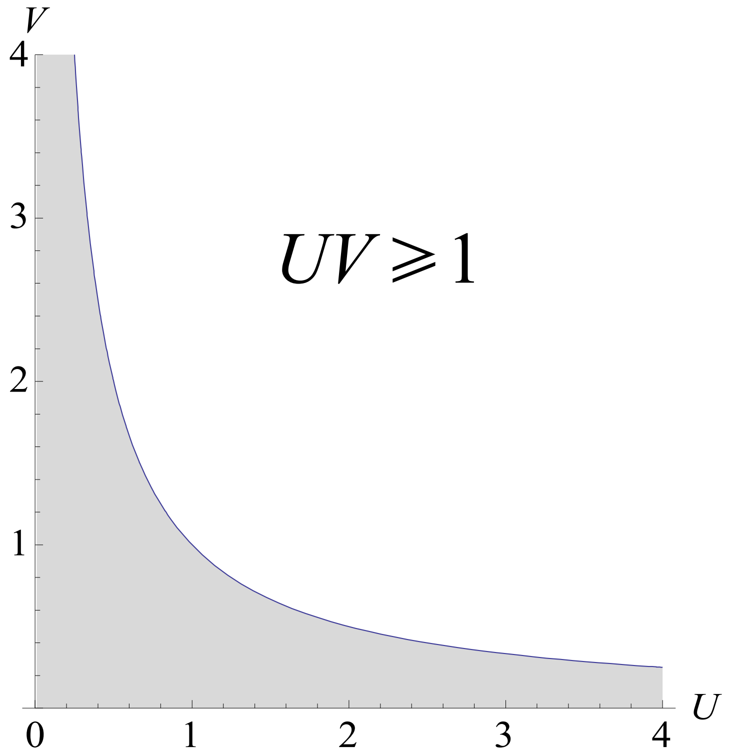
Product criterion and Sum criterion

- Product criterion

  Entanglement is signified by the EPR paradox.

- Sum criterion

  Sufficient for the detection of Gaussian entanglement (with an optimization of local Gaussian unitary operation)



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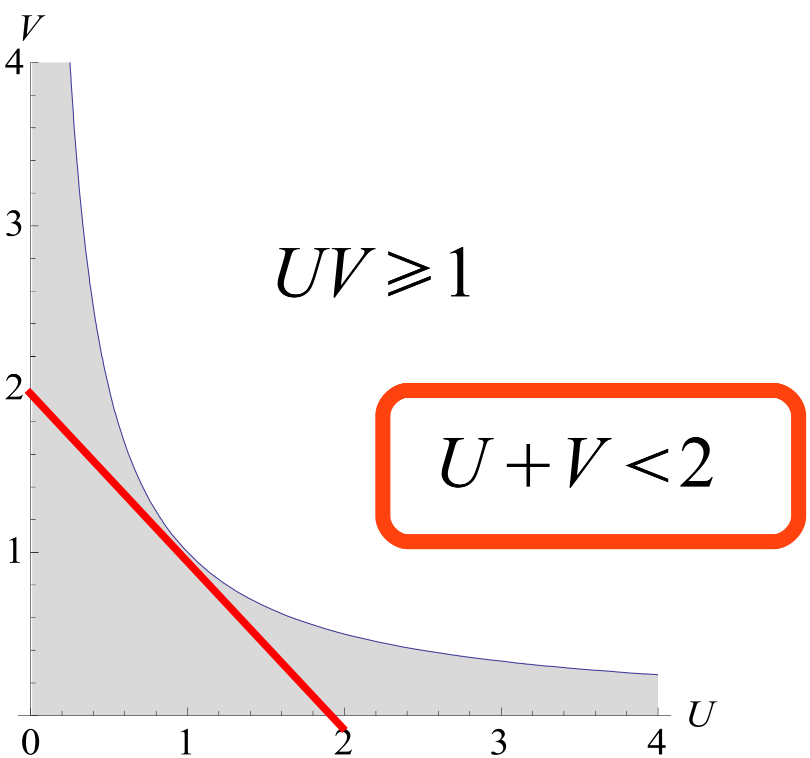

Duan et al, Phys. Rev. Lett. 84 2722 (2000)

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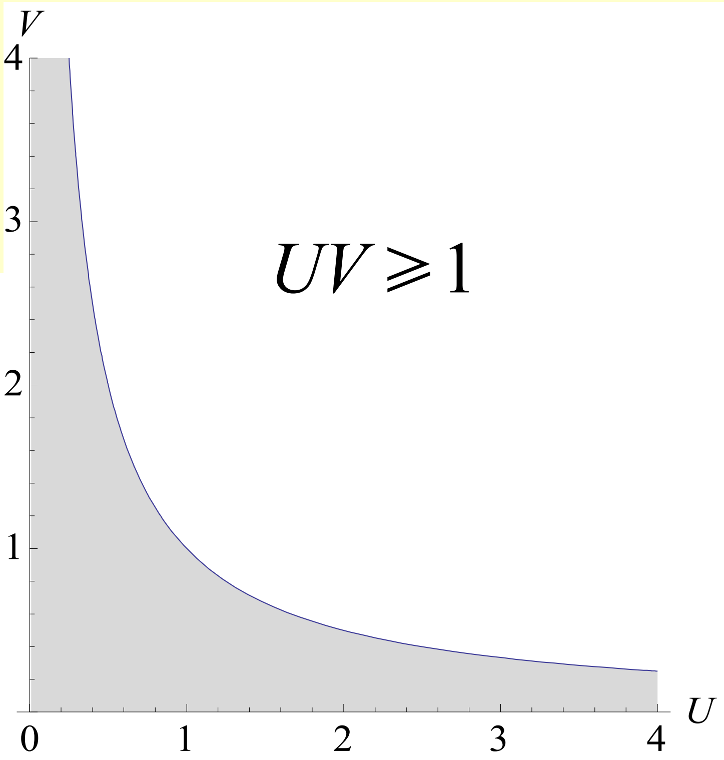
Quantum noise & entanglement

We can say basically,

- Entanglement = EPR-like correlation
- Entanglement detection = Observation of EPR-paradox
- Homodyne detection in Experiments

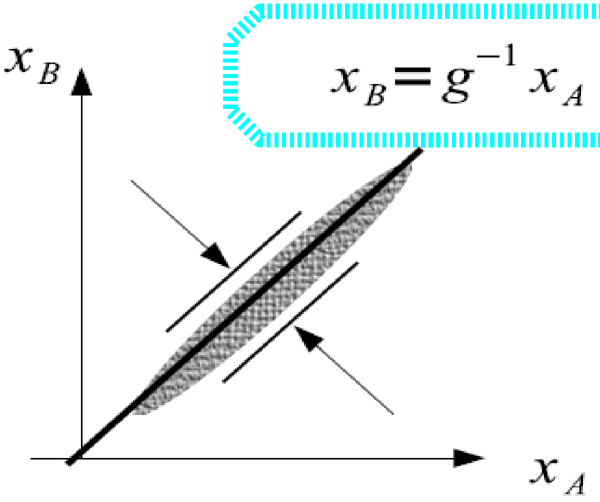
$$\langle \Delta^2 \hat{x} \rangle \langle \Delta^2 \hat{p} \rangle \geqslant 1/4$$



Another measure of the correlation.

. Variances (Uncertainties) $\left\langle \Delta^2 (\hat{x}_A - \hat{x}_B) \right\rangle$

. Intensities of the distributions $P(x_A, x_B)$



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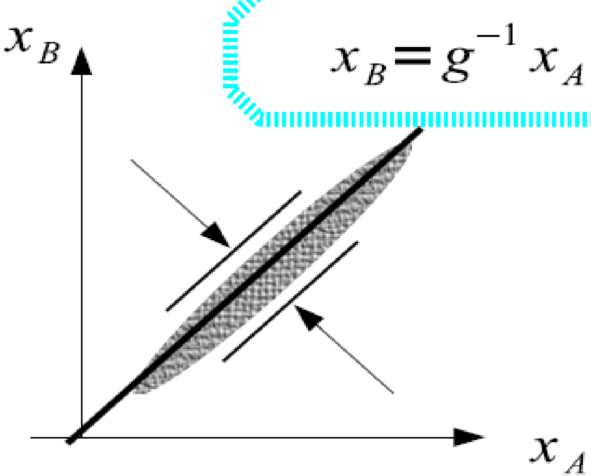
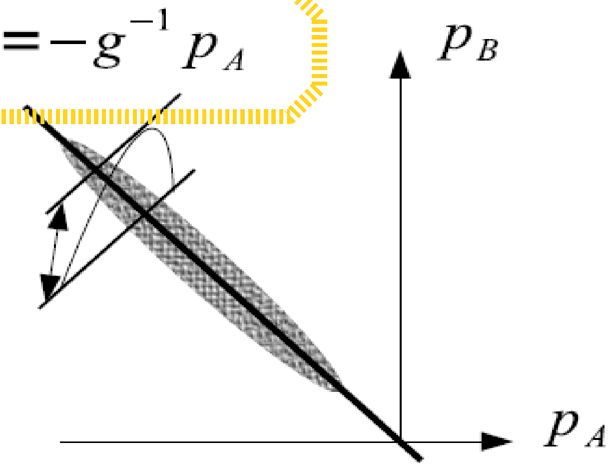
. Variances (Uncertainties) $\left\langle \Delta^2(\hat{x}_A - \hat{x}_B) \right\rangle$
. Intensities of the distributions $P(x_A, x_B)$

$x_B = g^{-1} x_A$



$p_B = -g^{-1} p_A$



$\alpha_A = g\alpha_B^*$

$\alpha_A = x_A + i\, p_A$

$\alpha_B = x_B + i\, p_B$

<u>Conjugate-amplitude pair of coherent states</u> $|\alpha\rangle |g\alpha^*\rangle$

Another measure of the correlation.

. Variances (Uncertainties) $\langle \Delta^2(\hat{x}_A - \hat{x}_B) \rangle$

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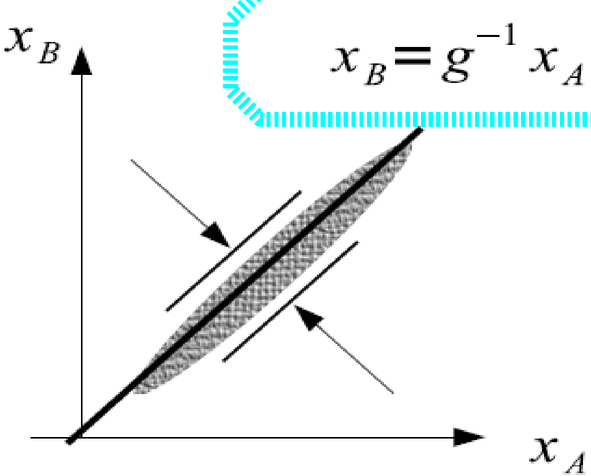

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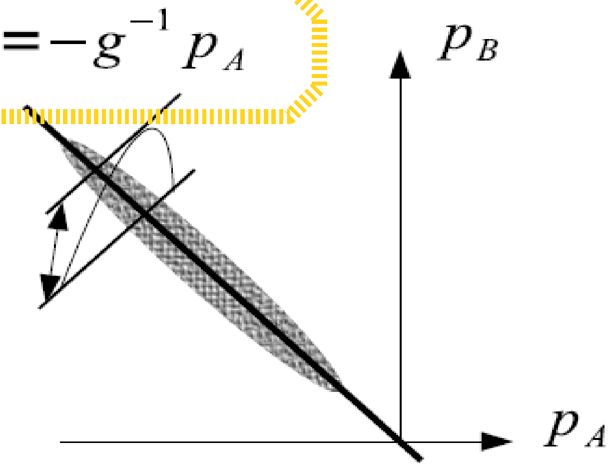

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<u>Conjugate-amplitude pair of coherent states</u> $|\alpha\rangle |g\alpha^*\rangle$

- Coherent states $|\alpha\rangle$ $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ $\alpha \in \mathbb{C}$
- Husimi -Q function $Q(\alpha) = \langle \alpha|\rho|\alpha\rangle / \pi$

Heterodyne measurement

New criterion with the EPR-like correlation on a coherent-state basis

Strength of the correlation for entanglement

$$\left\langle |v\alpha\rangle_A\langle v\alpha| \otimes |u\alpha^*\rangle_B\langle u\alpha^*| \right\rangle$$

Projection to the pair of coherent states

$$u^2+v^2=1$$

uncertainty relation

$$\Delta x \Delta p \geq |[x,p]|/2$$

A sort of uncertainty relation

$$\langle \rho_{th} \rangle_\rho \leqslant \frac{\lambda}{1+\lambda}$$

EPR paradox!

$$\rho_{th} = \frac{\lambda}{\pi}\int e^{-\lambda|\alpha|^2} |\alpha\rangle\langle\alpha| d^2\alpha$$

$$= \frac{\lambda}{1+\lambda}\sum_{n=0}^{\infty}\left(\frac{1}{1+\lambda}\right)^n |n\rangle\langle n|$$

R. Namiki, arXiv:1109.0349

New criterion with the EPR-like correlation on a coherent-state basis

Strength of the correlation for entanglement

$$\left\langle \int d^2\alpha\, p_\lambda(\alpha) \left| v\alpha \right\rangle_A \left\langle v\alpha \right| \otimes \left| u\alpha^* \right\rangle_B \left\langle u\alpha^* \right| \right\rangle > \frac{\lambda}{1+\lambda}$$

⇨ the state is entangled.

$$u^2 + v^2 = 1$$

$$p_\lambda(\alpha) := \frac{\lambda}{\pi} e^{-\lambda|\alpha|^2}$$

R. Namiki, PRA 83, 042323 (2011)

uncertainty relation

$$\Delta x \Delta p \geq |[x, p]|/2$$

A sort of uncertainty relation

$$\langle \rho_{th} \rangle_\rho \leq \frac{\lambda}{1+\lambda}$$

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New criterion with the EPR-like correlation on a coherent-state basis

Strength of the correlation for entanglement

$$\left\langle \int d^2\alpha\, p_\lambda(\alpha) |v\alpha\rangle_A \langle v\alpha| \otimes |u\alpha^*\rangle_B \langle u\alpha^*| \right\rangle > \frac{\lambda}{1+\lambda}$$

$$\alpha = x \pm i\,p$$

$$\left\langle \int d^2\alpha\, p_\lambda(\alpha) |v\alpha\rangle_A \langle v\alpha| \otimes |u\alpha\rangle_B \langle u\alpha| \right\rangle \leqslant \frac{\lambda}{1+\lambda}$$

$$p_\lambda(\alpha) := \frac{\lambda}{\pi} e^{-\lambda|\alpha|^2}$$

Universal relation

uncertainty relation

$$\Delta x \Delta p \geq |[x, p]|/2$$

A sort of uncertainty relation

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Performance of new criterion

Strength of the correlation for entanglement

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For a standard form of the Gaussian state & λ → 0

$$\tfrac{1}{4}(U+1)(V+1) < 1$$

$$U := \left\langle \Delta^2 (u\hat{x}_A - v\hat{x}_B) \right\rangle / C$$

$$V := \left\langle \Delta^2 (u\hat{p}_A + v\hat{p}_B) \right\rangle / C$$

EPR-like uncertainties!

Performance of new criterion

Strength of the correlation for entanglement

$$\left\langle \int d^2\alpha\, p_\lambda(\alpha) |v\alpha\rangle_A \langle v\alpha| \otimes |u\alpha^*\rangle_B \langle u\alpha^*| \right\rangle > \frac{\lambda}{1+\lambda}$$

For a standard form of the Gaussian state & $\lambda \to 0$

$$\tfrac{1}{4}(U+1)(V+1) < 1$$

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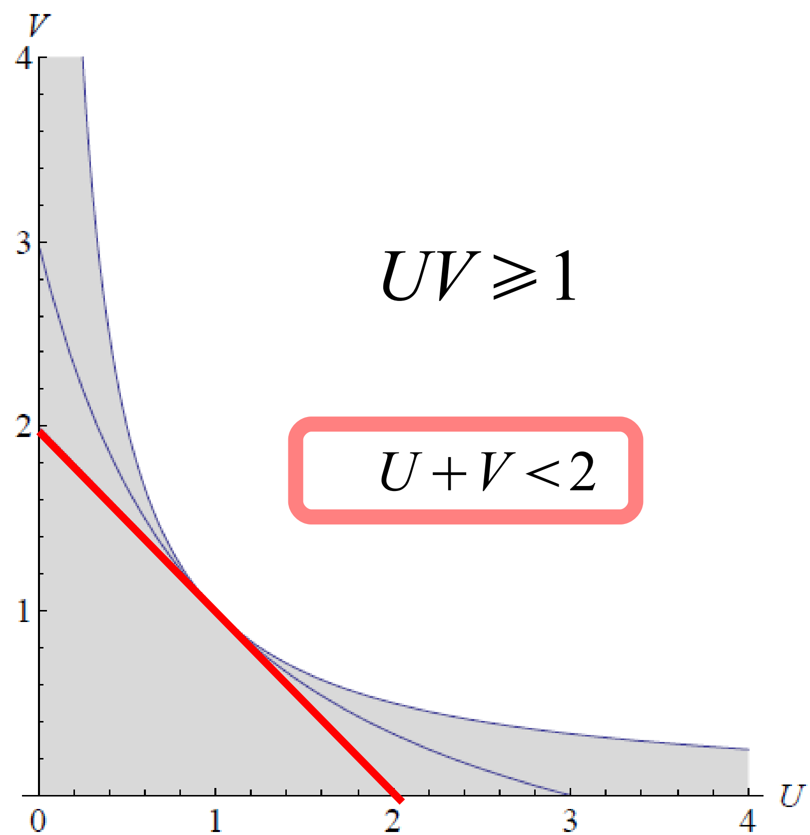
EPR-like uncertainties!

New criterion covers Gaussian entanglement!



Summary

$$\langle \Delta^2 \hat{x} \rangle \langle \Delta^2 \hat{p} \rangle \geqslant 1/4$$

EPR-like correlation

- EPR-paradox signifies the entanglement
- Detection of Gaussian entanglement

Coherent-state-based EPR-like correlation

- New criterion for entanglement
- Detection of Gaussian entanglement
- Simple interrelation to the standard method
- Probability distribution (c.f. Quantum noise) Heterodyne measurements
- Novel basis to comprehend quantum entanglement

R. Namiki, arXiv:1109.0349

- Benchmark for genuine quantum memories and gates

Poster #80 R. Namiki and Y. Tokunaga, PRA 85, 010305(R) (2012)