The gravitational wave from an EMRI binary

Influence of the beyond the test particle limit

Soichiro Isoyama

The Next Generation of Physics, Spun from Universality & Emergence
Developing Independent Researchers to Explore New Frontiers

YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

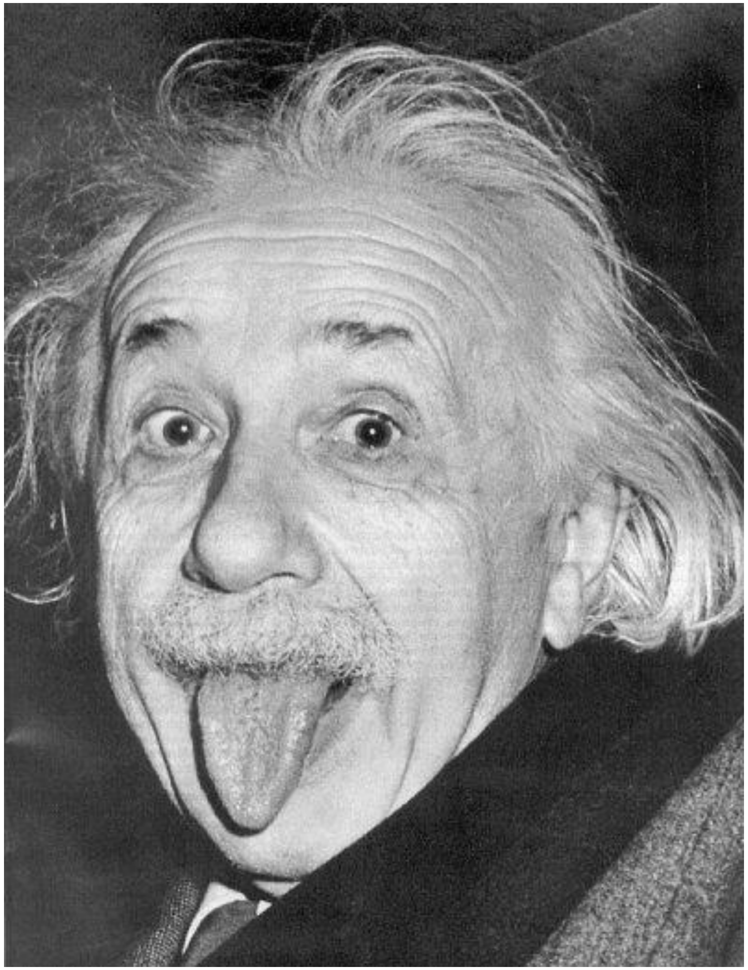
Collaborators : Norichika Sago, Ryuichi Fujita, and Takahiro Tanaka

Testing general relativity

Well confirmed as the correct gravitational theory in our solar system by various experiments

(e.g. light bending by the sun)

How about in a **very strong** gravitational field such as **a black hole** ?




"Massive Spinning black Hole ?"
Testing this conjecture with gravitational waves

What’s gravitational wave ?

Einstein equation :

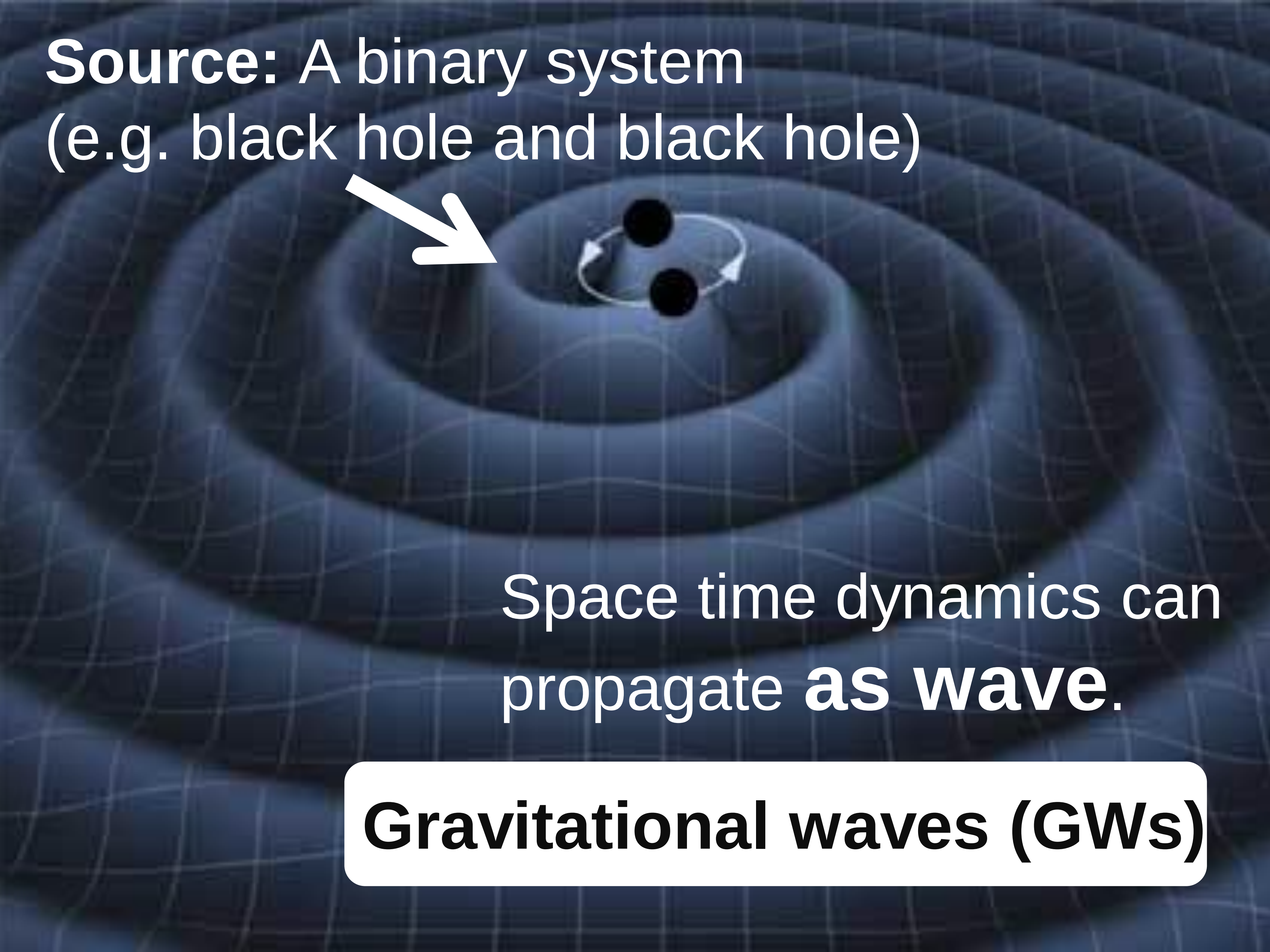
$$G_{\mu\nu}[g_{\alpha\beta}] = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Determine
space time structure

Source of gravity
(e.g. moving star)

Space time is also dynamical

The source is dynamical


Source: A binary system
(e.g. black hole and black hole)
Space time dynamics can
propagate as wave.
Gravitational waves (GWs)

Why gravitational waves ?

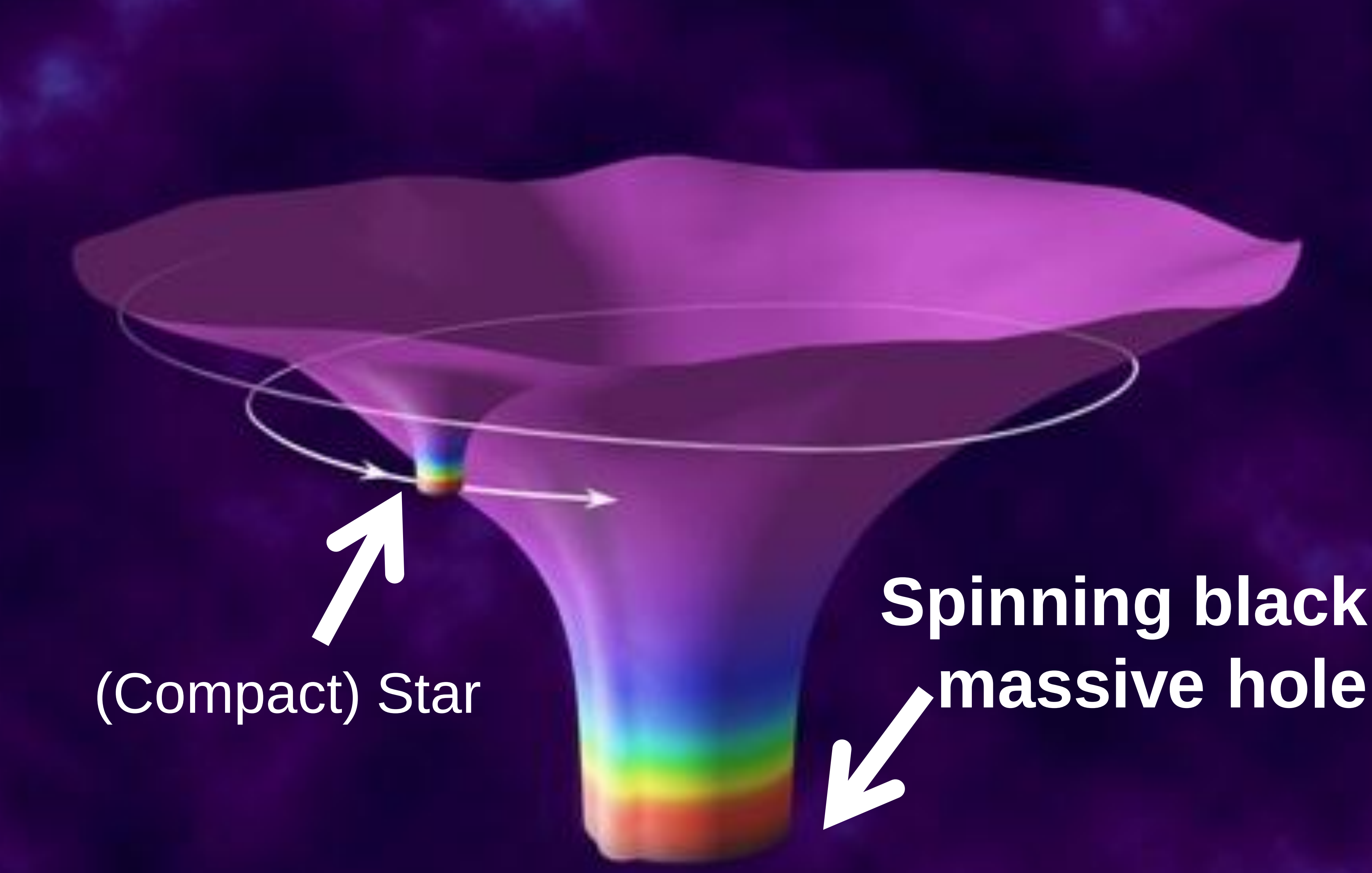
Directly tell us about the spacetime structure of the source

- The brand new window for our universe

New observational method always reveal yet unknown side of our universe.

- Very transparent

Hard to sealed off in propagating our universe



Extreme Mass Ratio Inspirals *EMRI*

A compact star plunge into a super massive black hole at the center of galaxy.

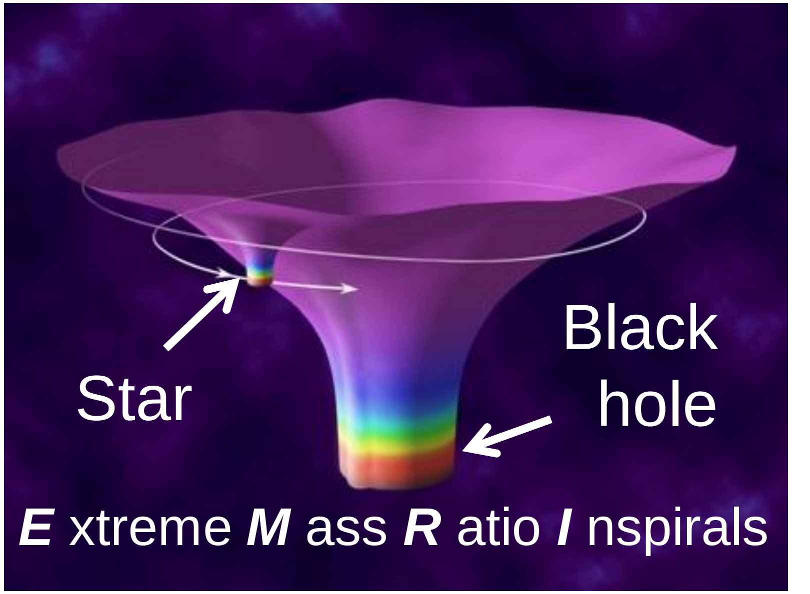
Typical parameters of EMRI

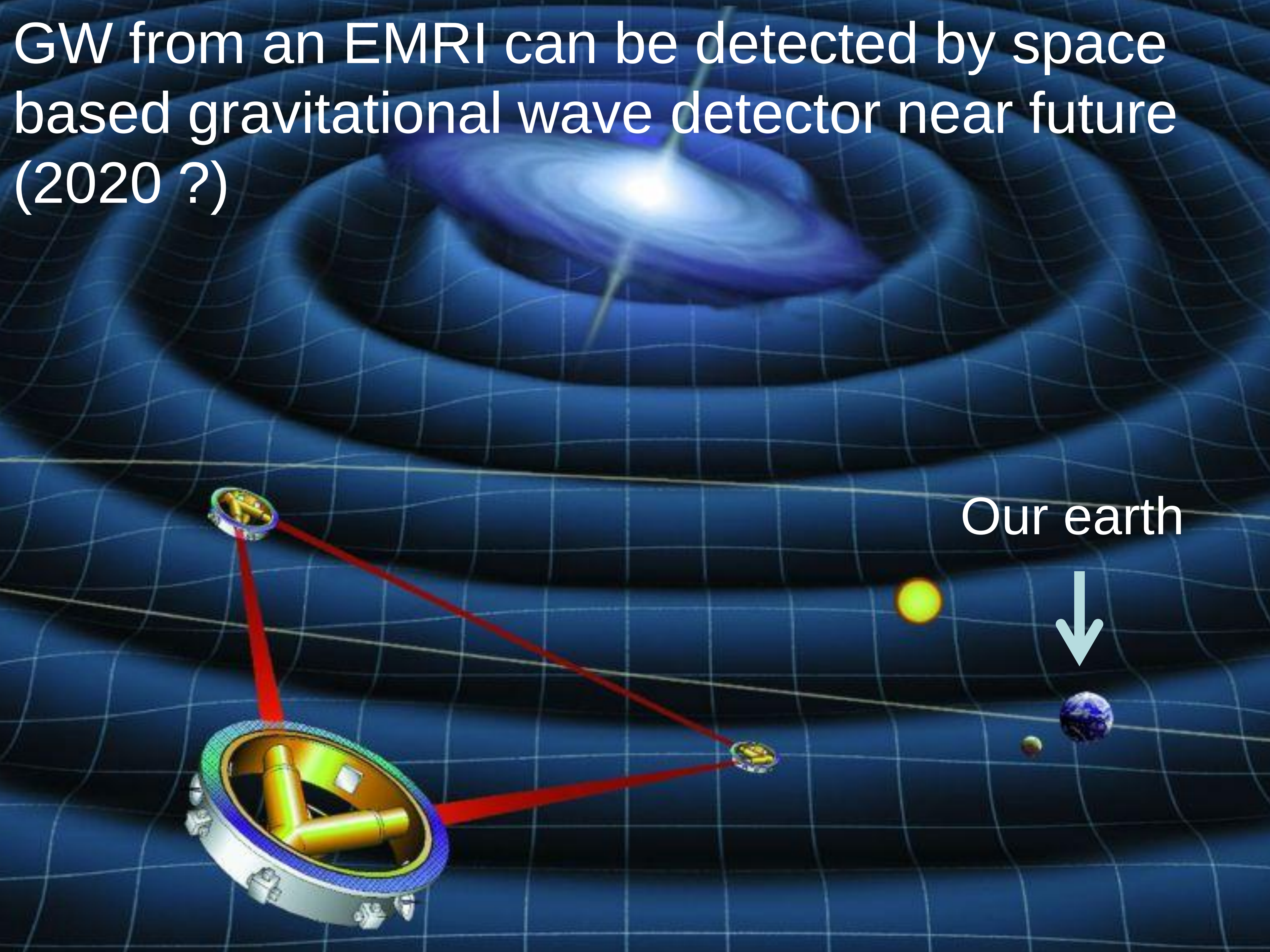
$$\mathrm{Star} \sim 10 M_{\odot},\ \ \mathrm{Black\ hole} \sim 10^{6\sim 8} M_{\odot}$$

The solar mass : $1 M_{\odot} \sim 2.0 \times 10^{30} \mathrm{kg}$

The velocity of the star : $v \approx c$

Before absorbed by a black hole, a star runs around the black hole million times per a year.




GW from an EMRI can be detected by space based gravitational wave detector near future (2020 ?)
Our earth

Accumulate the information of the black hole into **gravitational waves**

We can do **the very precise test** of the general relativity near a black hole.

The amplitude of GW is quite small.

- Coming from cosmological distance
- Smallness of the gravitational constant

Required resolution (example)

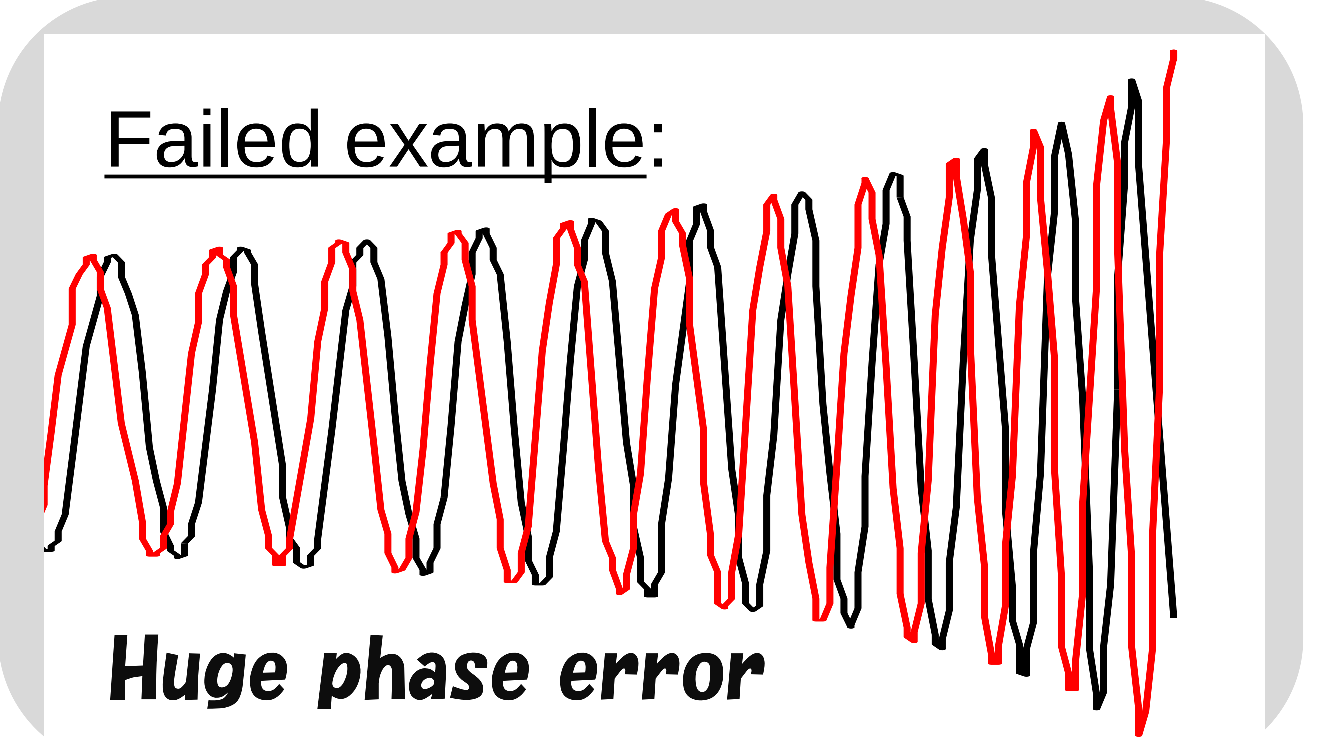
V.S.

1.0 km

10^{-16} km

The GW signal is hidden deep inside of noises

Correlate with **the wave form predicated theoretically** to extract information



Need **very accurate** theoretical prediction

Predict GWs of an EMRI

Einstein equation is hard to be solved

$$\frac{\text{mass of star}}{\text{mass of B.H.}} \approx O(10^{-4}) \sim O(10^{-8})$$

→ Small expansion parameter

GWs treated as small perturbations on a black hole

"Black hole perturbation"

$$g_{\alpha\beta} = \underbrace{g^{(0)}_{\alpha\beta}}_{\text{Spinning black hole}} + \underbrace{h^{(1)}_{\alpha\beta} + h^{(2)}_{\alpha\beta} + \dots}_{\text{Small Perturbations (GW)}}$$

Status of black hole perturbation

$$g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + \boxed{h^{(1)}_{\alpha\beta}} + \boxed{h^{(2)}_{\alpha\beta}} + \cdots$$

Regge-Wheeler (1957)
Zerilli (1970)
Teukolsky+ (1973)

Leading order (test particle limit)

The solutions (GWs) are well understood

Next leading order (beyond test particle limit)

Partially known, **far from full solution**, however.

Q. Precise GR test needs next leading order ?

Accumulated phase of the GW

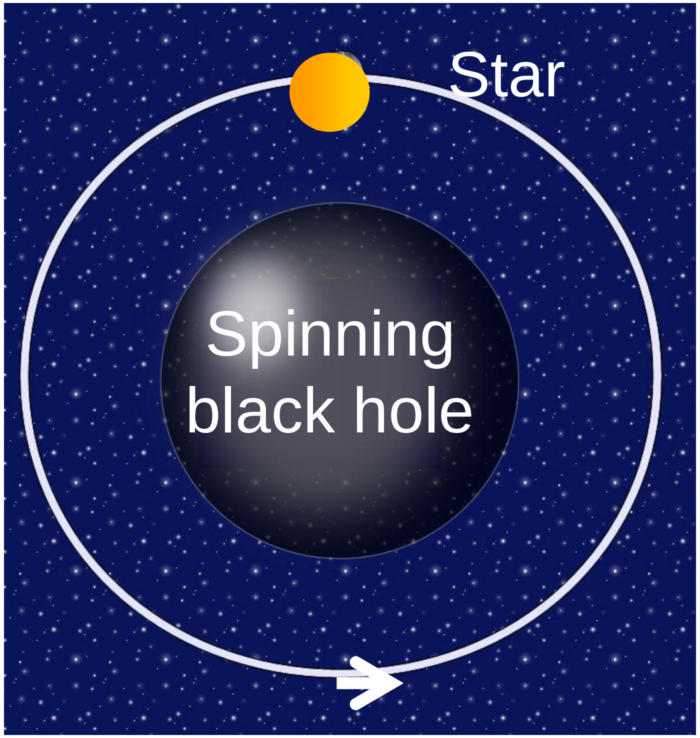
The Considered EMRI

- A spinning black hole with angular momentum

$$a := J/M_{\mathrm{B.H.}}$$

- A star in **qusi-**circular orbit (The same direction of B.H. spin)

⇒ **Shrink** due to GW emission



Estimate the phase correction from the next leading order in the black hole perturbation

EMRI in a circular orbit

GWs' phase = the star's number of orbiting around the spinning black hole : ***N***

$$N := \int_{R_f}^{R_i} dR \, \Omega_\phi \frac{(dE/dR)}{|dE/dt|}$$

E	Star's kinetic energy and rest mass
Ω_ϕ	Star's angular velocity
dE/dt	Energy loss due to GW emission

Further approximation : $|\vec{v}| \ll c$

Assume the velocity of the star is **small**

➡ GW's phase is **analytically** expressed as series expansion (to some truncated order.)

$$N := \int_{R_f}^{R_i} dR\, \Omega_\phi \frac{(dE/dR)}{|dE/dt|}$$

e.g. Star's energy loss due to GW emission

$$\left\langle \frac{dE}{dt}(v) \right\rangle = \frac{32}{5}\nu^2 v^{10} \left[\sum_{n=0}^{n=6} A_n(a) v^n + O(v^7) \right]$$

Leading

$$+ \frac{32}{5}\nu^3 v^{10} \left[\sum_{n=1}^{n=6} B_n(a) v^n + O(v^7) \right]$$

Next leading

$a := J/M_{\mathrm{B.H.}}$ $\quad \nu$ the mass ratio of the EMRI

GW's phase correction

Estimate the phase correction from next leading order via **extrapolation**

	Small velocity approximation	Full black hole perturbation
Leading	**A** : known	**B** : known
Next leading	**α** : known	**UNKNOWN**
⋮	⋮	⋮

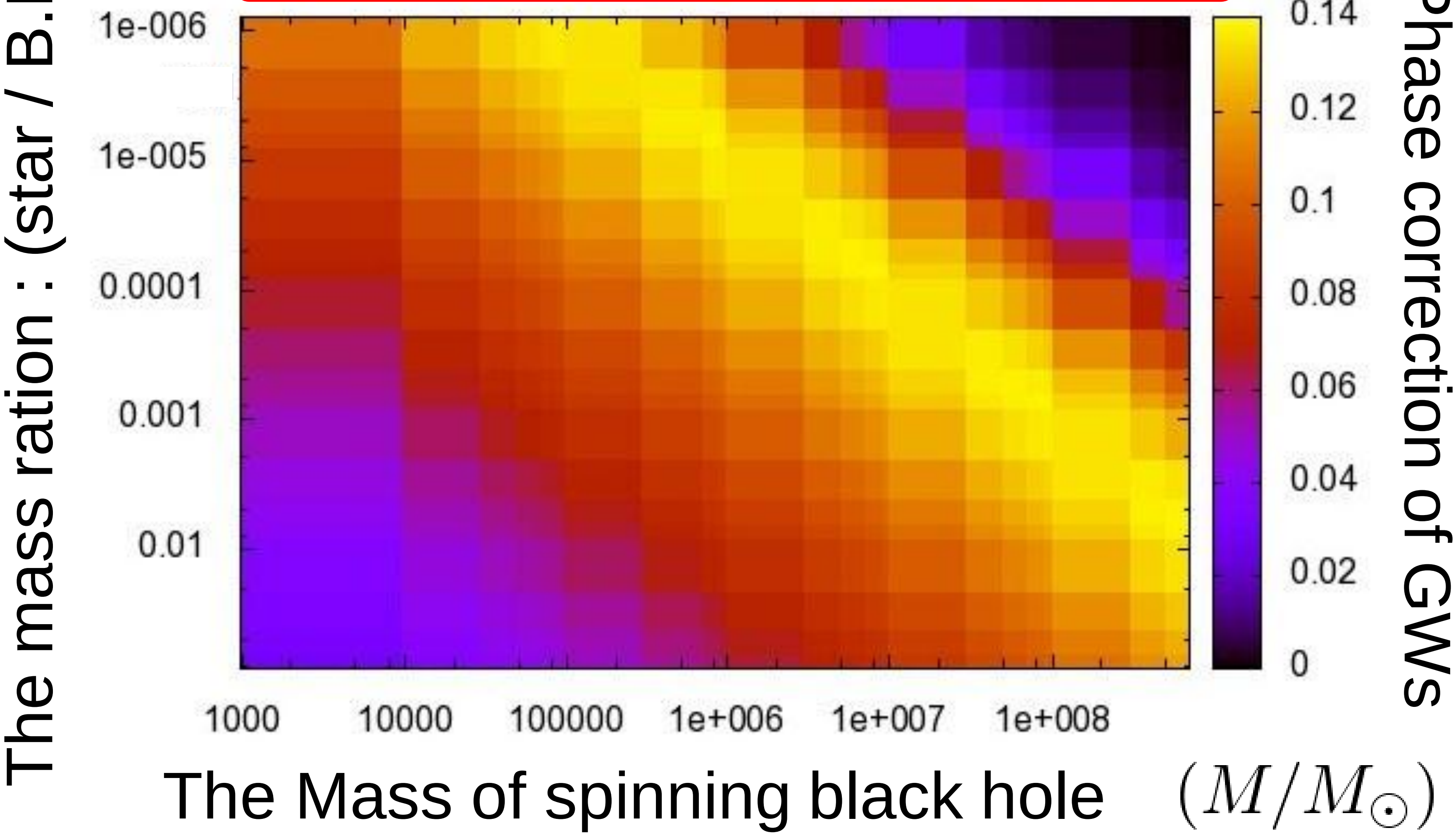
UNKNOWN next leading correction to the GWs' phase

$$\approx (B/A) \times \alpha$$

Results

EMRI GWs phase correction from the next leading order for 1 year observation before plunge into B.H.

$$a = J/M_{\mathrm{B.H.}} = 0.9, \ (a_{\mathrm{MAX}} = 1)$$



Well suppressed

(c.f. Leading order : **million** phase)

Summary of the talk

Black hole perturbation

In a circular orbit, the next leading order correction to the phase of gravitational waves from an EMRI binary

⇒ **Well suppressed due to the extreme mass ratio in the binary**

"We are ready to testing GR near black holes"

A future direction

The same conclusion holds for an EMRI in **non-circular orbit**?

The end of planned talk

Thank you.

補遺