Weak measurement and quantum interference

— from a geometrical point of view

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Outline

- The basics of geometric phases
  - Berry's phase (1984) — cyclic adiabatic change
    A neglected player in 60 year history of quantum theory
  - Pancharatnam's phase (1956) — polarization optics
  - Geometric phase is now recognized as an important concept in quantum and classical physics. — Now in textbooks of QM!
- Geometric phase in quantum eraser
  - Geometric phase as fringe shift
  - Nonlinear behavior of fringe shift
- Weak measurement
  - Quantum eraser and weak measurement
  - Geometric phase in weak measurement
  - Anomaly of weak values

Collaborators: H. Kobayashi, S. Tamate, Y. Ikeda, K. Ogawa, and T. Nakanishi

Absolute phase of wavefunction

- Freedom of choice of the phase of wavefunction
  — choice of gauge

$$|\psi'\rangle = \mathrm{e}^{\mathrm{i}\phi}|\psi\rangle$$

- $|\psi'\rangle$ and $|\psi\rangle$ represent a same physical state.
- Gauge-independent quantities (N.B. $\langle\psi'| = \mathrm{e}^{-\mathrm{i}\phi}\langle\psi|$)
  - Probability

$$P = \langle\psi'|\psi'\rangle = \langle\psi|\psi\rangle$$

  - Expectation value of an operator A

$$\langle\hat{A}\rangle = \langle\psi'|\hat{A}|\psi'\rangle = \langle\psi|\hat{A}|\psi\rangle$$

  - Density matrix operator

$$\hat{\rho} = |\psi'\rangle\langle\psi'| = |\psi\rangle\langle\psi|$$

Relative phase

- Superposition of two states $|\psi_1\rangle$ and $|\psi_2\rangle$

$$|\psi(\alpha)\rangle := |\psi_1\rangle + \mathrm{e}^{\mathrm{i}\alpha}|\psi_2\rangle$$

  α: relative phase for superposition

- Probability $P(\alpha) = \langle\psi(\alpha)|\psi(\alpha)\rangle$ gives interference pattern.

$$P(\alpha) = \langle\psi_1|\psi_1\rangle + \langle\psi_2|\psi_2\rangle + \mathrm{e}^{\mathrm{i}\alpha}\langle\psi_1|\psi_2\rangle + \text{c.c.}$$

- Gauge dependence of the interference terms

$$\mathrm{e}^{\mathrm{i}\alpha}\langle\psi_1|\psi_2\rangle = \mathrm{e}^{\mathrm{i}(\alpha+\phi_1-\phi_2)}\langle\psi_1'|\psi_2'\rangle$$

- The relative phase α is gauge-dependent and depends on the difference of absolute phases.

$$\alpha' = \alpha + (\phi_1 - \phi_2)$$

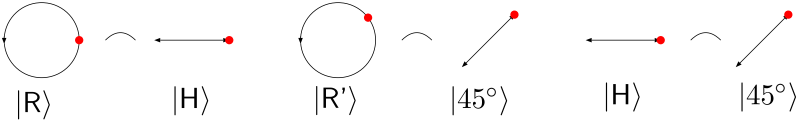
- We can adjust the phase difference $(\phi_1 - \phi_2)$ so that $\langle \psi_2' | \psi_1' \rangle$ becomes real, i.e.,

$$0 = \arg\langle \psi_1' | \psi_2' \rangle = \arg\langle \psi_1 | \psi_2 \rangle - \phi_1 + \phi_2$$

- Such two (non-orthogonal) states $|\psi_1'\rangle$ and $|\psi_2'\rangle$ are considered *in-phase* and written as

$$|\psi_1'\rangle \frown |\psi_2\rangle$$

- Example — In-phase relations between photon polarizations



Non transitivity of in-phase relation

Pancharatnam, Proc. Indian Acad. Sci. A **44**, 247 (1956)

- In-phase relation is not transitive;

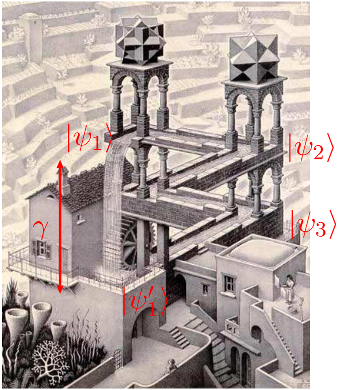
$$|\psi_1\rangle \frown |\psi_2\rangle \quad \text{and} \quad |\psi_2\rangle \frown |\psi_3\rangle$$
$$\text{does not imply} \quad |\psi_1\rangle \frown |\psi_3\rangle$$

- Comparing $|\psi_1'\rangle$ ($\frown |\psi_3\rangle$) and $|\psi_1\rangle$, we have

$$|\psi_1'\rangle = e^{i\gamma}|\psi_1\rangle$$

where

$$\gamma = \arg\langle\psi_1|\psi_3\rangle\langle\psi_3|\psi_2\rangle\langle\psi_2|\psi_1\rangle$$



γ is *gauge-independently* determined by the three states $|\psi_1\rangle$, $|\psi_2\rangle$, and $|\psi_3\rangle$. — Geometric (Berry, Pancharatnam) phase

Movie

In-phase relations

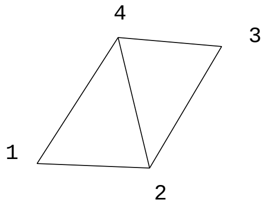
Threeness

- $\langle 1|3\rangle\langle 3|2\rangle\langle 2|1\rangle$ is the minimal gauge-independent complex number

$\langle 1\|1\rangle$	real
$\langle 1\|2\rangle\langle 2\|1\rangle$	real
$\langle 1\|3\rangle\langle 3\|2\rangle\langle 2\|1\rangle$	complex

- $\gamma = \arg\langle 1|3\rangle\langle 3|2\rangle\langle 2|1\rangle$ is the minimal non-trivial phase
- Cases of higher number are reduced to the case of three

$$\begin{aligned} &\arg\langle 1|4\rangle\langle 4|3\rangle\langle 3|2\rangle\langle 2|1\rangle \\ &= \arg\langle 1|4\rangle\langle 4|2\rangle\langle 2|1\rangle + \arg\langle 2|4\rangle\langle 4|3\rangle\langle 3|2\rangle \end{aligned}$$



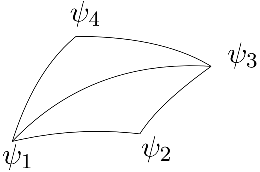
Integration of geometric phase

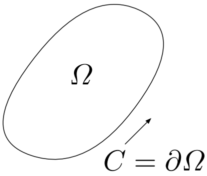
- Geometric phase for four states (geodesic quadrangle)

$$\gamma(\psi_1, \psi_2, \psi_3, \psi_4) = \gamma(\psi_1, \psi_2, \psi_3) + \gamma(\psi_1, \psi_3, \psi_4)$$

- Geometric phase for continuous closed loop C

$$\gamma(C) = \oint_C \gamma = \int_\Omega \mathrm{d}\gamma, \quad C = \partial\Omega$$





$\gamma(\psi_1, \psi_2, \psi_3)$ is a building block of geometric phase.

Two-level system — Poincaré sphere

- Two-state system (basis kets: $|e_1\rangle$, $|e_2\rangle$)

$$|\psi\rangle = a_1|e_1\rangle + a_2|e_2\rangle \quad \in \mathcal{H} = \mathbb{C}^2$$

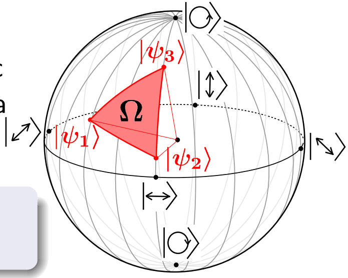
- One-to-one correspondence between states ρ and 3-D unit vectors $\boldsymbol{s}$

$$\rho = \frac{1}{2}(\hat{1} + \boldsymbol{s}\cdot\boldsymbol{\sigma}), \quad \boldsymbol{\sigma} : \text{Pauli matrices}$$

— Poincaré sphere

- It is easy to show that the geometric phase γ is related to the surface area Ω.

$$\gamma(\psi_1, \psi_2, \psi_3) = -\frac{\Omega}{2}$$



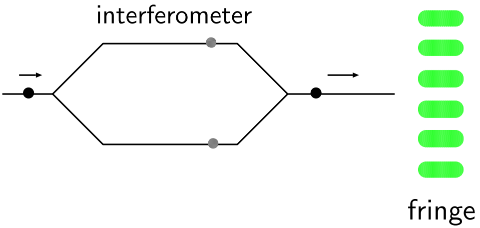
Quantum eraser

Scully and Drühl: Phys. Rev. **25**, 2208 (1982)

- Complementarity (wave-particle duality)

  Interference fringe (wave) $\leftrightarrow$ Which-path information (particle)

- In interferometry, an attempt to obtain the which-path information by marking (or whatever) destroys the interference fringe.
- However, if the information is lost somehow, the fringe is recovered. — Quantum eraser (due to post selection)



(cf) Wheeler's delayed choice experiment

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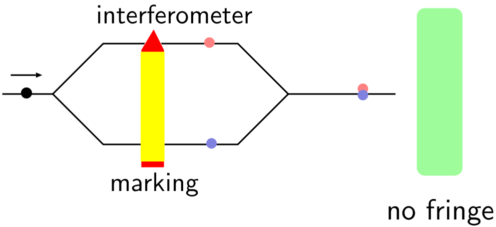

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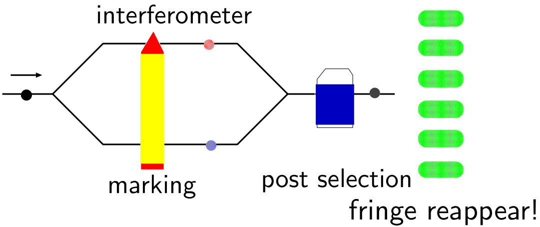
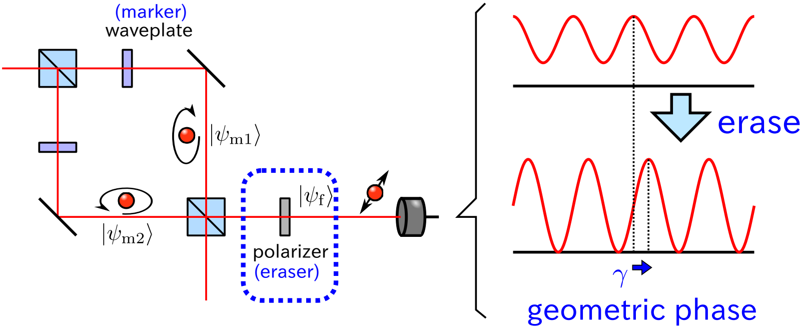


(cf) Wheeler's delayed choice experiment

Geometric phase in quantum eraser

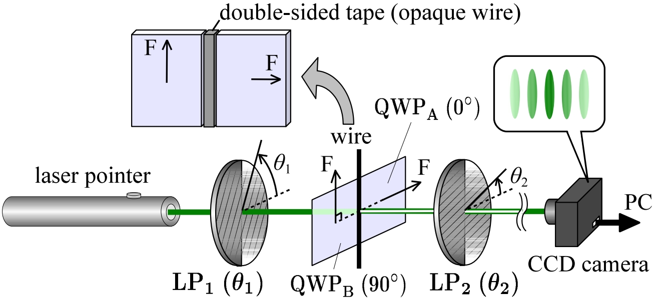


- Polarization marking: $|\psi_{m1}\rangle$, $|\psi_{m2}\rangle$
- Erasing marker with $|\psi\rangle_f \Rightarrow$ geometric phase:

$$\gamma = \arg\langle\psi_{m1}|\psi_f\rangle\langle\psi_f|\psi_{m2}\rangle\langle\psi_{m2}|\psi_{m1}\rangle$$

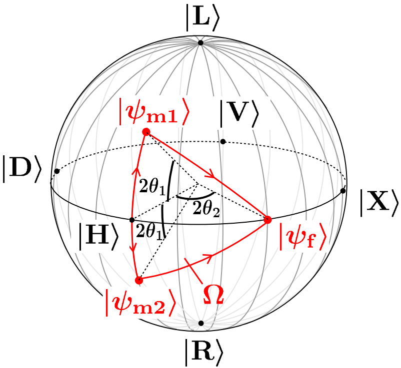
Experimental demonstration

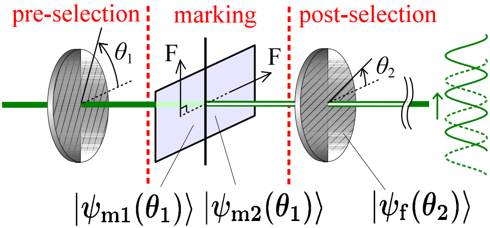
- Measurement of geometric phase in quantum eraser



- Very easy setup
- H. Kobayashi, MK *et al.*, J. Phys. Soc. Jpn. **80**, 034401 (2011)

State change on the Bloch sphere





- State parameter : $\theta_1, \ \theta_2$
- Polarization states : $|\psi_{\mathrm{m1}}(\theta_1)\rangle, \ |\psi_{\mathrm{m2}}(\theta_1)\rangle, \ |\psi_{\mathrm{f}}(\theta_2)\rangle$

Nonlinear geometric phases

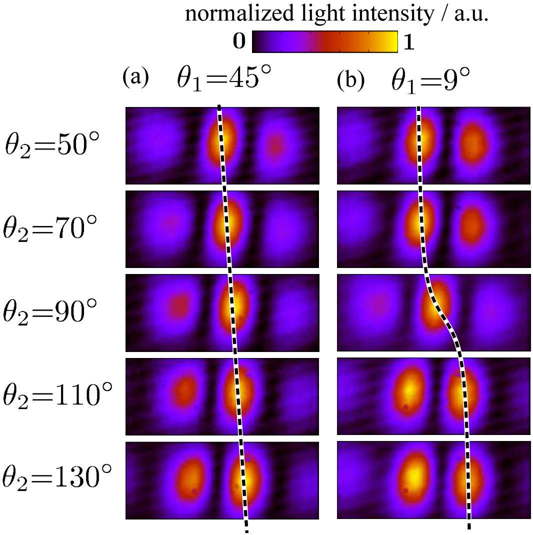
- For $\theta_1 = \pi/4$

$$\Omega \propto \theta_2$$

- For $\theta_1 \ll 1$

Ω varies rapidly around $\theta_2 = \pi/2$

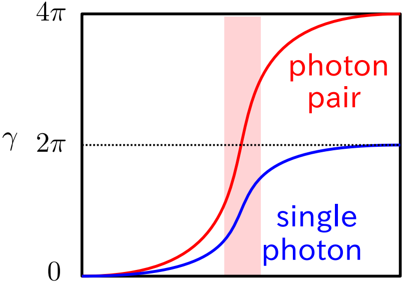
Interference images on CCD

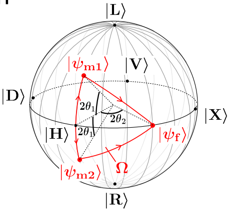


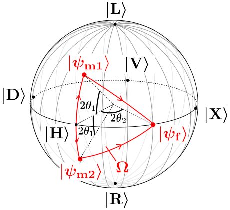
- Identically polarized Photon pairs ⇒ Twice the geometric phase
- $|\Psi_i\rangle = |\psi_i\rangle|\psi_i\rangle \quad (i = 1, 2, 3)$

$$\arg\langle\Psi_1|\Psi_3\rangle\langle\Psi_3|\Psi_2\rangle\langle\Psi_2|\Psi_1\rangle = 2\arg\langle\psi_1|\psi_3\rangle\langle\psi_3|\psi_2\rangle\langle\psi_2|\psi_1\rangle$$

- Twice the amplification for same bandwidth

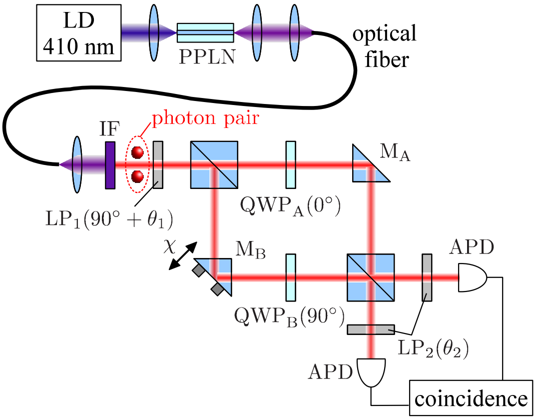






Observation of two-photon geometric phases

- H. Kobayashi, MK *et al.*, Phys. Rev. A **83**, 063808 (2011)

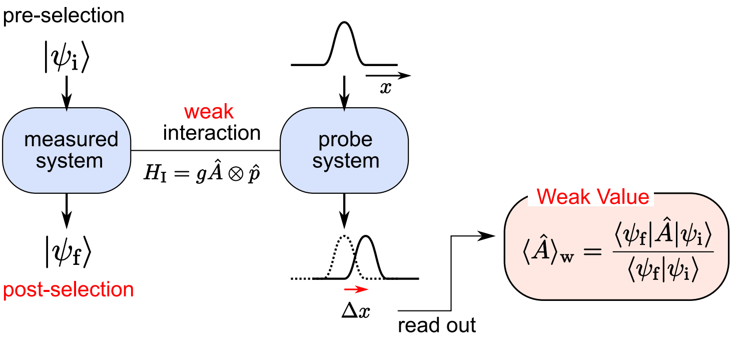


- Photon pairs are generated by parametric down conversion
- Geometric phases are observed in two-photon interference

Weak measurement

Aharonov *et al.*, PRL **60**, 1351 (1988)

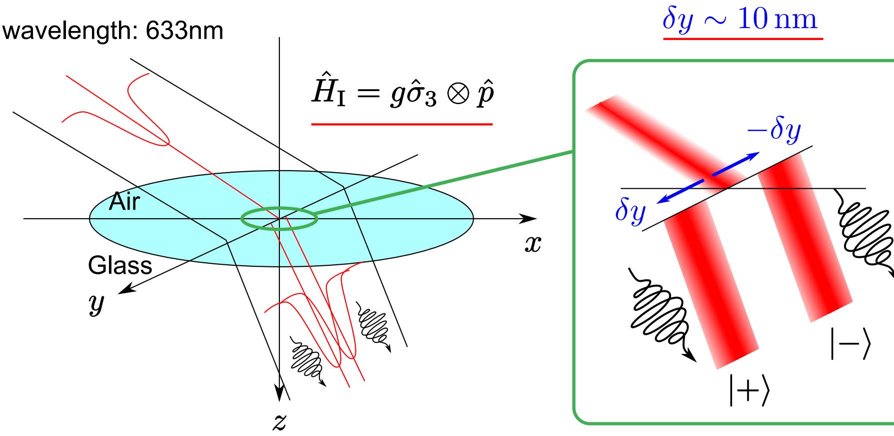
- Quantum measurement with *weak interaction* and *post selection*
- The measured value could be much larger than in the ordinary measurement — weak value
- Probability of success is sacrificed.



Application of weak measurement

Hosten *et al.*, Science **319**, 787 (2008).

- Detection of optical spin Hall effect



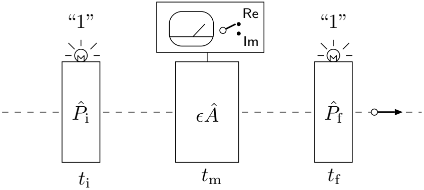
- Tiny amount of beam shifts $\sim 0.1\,\mathrm{nm}$ can be measured.
  Enhancement factor: $\sim 10^4$

Weak value and probe shift

- Utilizing the indirect measurement model, we can easily find the probe shift as

$$\langle \Delta x \rangle \propto \epsilon \, \mathrm{Re} \langle \hat{A} \rangle_{\mathsf{w}} = \epsilon \, \mathrm{Re} \frac{\langle \psi_{\mathsf{f}} | \hat{A} | \psi_{\mathsf{i}} \rangle}{\langle \psi_{\mathsf{f}} | \psi_{\mathsf{i}} \rangle}$$

  for the interaction Hamiltonian $\hat{H}_{\mathsf{I}} = \epsilon \hat{A} \otimes \hat{p}$
- The coupling ϵ must be small (weak) but by setting $\langle \psi_{\mathsf{f}} | \psi_{\mathsf{i}} \rangle$ small enough, we can have a large probe shift.
- We can measure the imaginary part by using different interaction Hamiltonian.



Relationship between quantum eraser and weak measurement

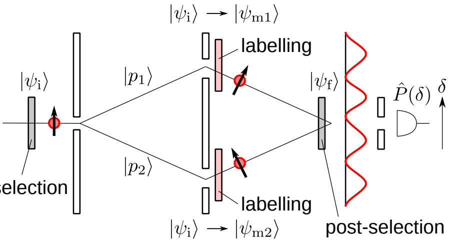
- S. Tamate, MK *et al.*, *New J. Phys.* **11**, 093025 (2009)

quantum eraser

$$\hat{P}(\delta) = |\phi(\delta)\rangle\langle\phi(\delta)|$$

$$|\phi(\delta)\rangle = \frac{1}{\sqrt{2}}(|p_1\rangle + e^{i\delta}|p_2\rangle)$$

measurement of phase difference

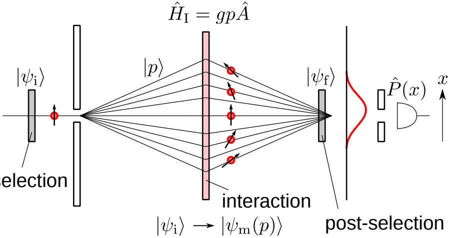


weak measurement

$$\hat{P}(x) = |x\rangle\langle x|$$

$$|x\rangle = \frac{1}{\sqrt{2\pi\hbar}}\int e^{-ixp/\hbar}|p\rangle dp$$

mesurement of phase gradient



Weak operator — two-state operator

- Initial state $|\,\mathsf{i}\,\rangle$ (Pre-selection), Final state $|\,\mathsf{f}\,\rangle$ (Post-selection)
- Weak operator (for $\langle\,\mathsf{i}\,|\,\mathsf{f}\,\rangle \neq 0$)

$$\hat{W} := \frac{|\,\mathsf{i}\,\rangle\langle\,\mathsf{f}\,|}{\langle\,\mathsf{f}\,|\,\mathsf{i}\,\rangle} \quad (= \hat{W}_{\mathsf{if}})$$

Gauge independent (cf. $|\,\mathsf{i}\,\rangle\langle\,\mathsf{f}\,|$: gauge dependent)

- $\hat{W}$ characterizes the pre- and post-selected ensemble
- For $|\,\mathsf{i}\,\rangle = |\,\mathsf{f}\,\rangle$, $\hat{W}$ reduces to the density operator $\hat{\rho} = |\,\mathsf{i}\,\rangle\langle\,\mathsf{i}\,|$.
- Normalized, $\mathrm{Tr}\,\hat{W} = 1$.

Weak value by weak operator

- With weak operator $\hat{W}$, the weak value for a quantity $\hat{A}$ is represented as

$$\langle \hat{A} \rangle_{\mathsf{w}} = \mathrm{Tr}\, \hat{A}\hat{W} = \frac{\langle\, \mathsf{f}\,|\hat{A}|\, \mathsf{i}\,\rangle}{\langle\, \mathsf{f}\,|\, \mathsf{i}\,\rangle} \quad (= \langle \hat{A} \rangle_{\mathsf{w;if}})$$

  (cf) $\langle \hat{A} \rangle = \mathrm{Tr}\, \hat{A}\hat{\rho} = \langle\, \mathsf{i}\,|\hat{A}|\, \mathsf{i}\,\rangle$

- Unlike density operators, weak operators are not Hermite nor normal; $\hat{W}^\dagger \neq \hat{W}$, $[\hat{W}^\dagger, \hat{W}] \neq 0$.
  — No spectral decomposition

- Real and imaginary parts (Hermite):

$$\hat{W}_{\mathsf{R}} := (\hat{W} + \hat{W}^\dagger)/2$$
$$\hat{W}_{\mathsf{I}} := (\hat{W} - \hat{W}^\dagger)/2\mathrm{i}$$

Physical meaning of the pre- and post selected ensemble

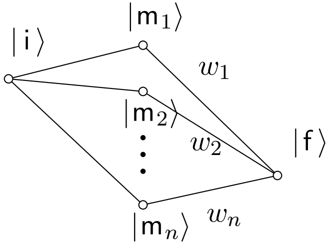
- Density operator ρ_{i}: preselected ensemble
- Weak operator $\hat{W}_{\mathsf{if}}$: ensemble filtered with preselection (i) and postselection (f).
- Analogy — a group of student of high school (i) who successfully entered university (f).
  - The ensemble is determined after high school.
  - The measurements have to be done in the high school.
  - The measurements have to be weak. (Hard tests destroy students!)

Weak values for projection operators

- The weak values w_k for projection operators $\hat{P}_k = |\mathsf{m}_k\rangle\langle\mathsf{m}_k|$ for an orthonormal basis $|\mathsf{m}_k\rangle$ $(k = 1, 2, \ldots, n)$.

$$w_k = \langle\hat{P}_k\rangle_{\mathsf{w}} = \mathrm{Tr}\,\hat{P}_k\hat{W} = \frac{\langle\mathsf{m}_k|\,\mathsf{i}\,\rangle\langle\,\mathsf{f}\,|\mathsf{m}_k\rangle}{\langle\,\mathsf{f}\,|\,\mathsf{i}\,\rangle}$$

- Due to the completeness $\hat{P}_1 + \hat{P}_2 + \cdots + \hat{P}_n = \hat{1}$, we have $w_1 + w_2 + \cdots + w_n = 1$.



Complex probability measure

- w_k seems to represent the probability that the initial state $|\,\mathrm{i}\,\rangle$ is transfered to the final state $|\,\mathrm{f}\,\rangle$. through the intermediate states $|\mathrm{m}_k\rangle$.
- However, the *probabilities* w_k take complex values.
- Despite of the fact that the eigenvalue of $\hat{P}_k$ are in $\{0, 1\}$, the weak value $w_k = \langle \hat{P}_k \rangle_{\mathrm{w}}$ can take a value outside of $[0, 1]$ and even be complex.
- When you think of a probability for quantum paths, you must allow it to be negative, larger than one, or even complex.

R.P. Feynman: "QED — The strange theory of light and matter", Princeton University Press (1985).

Three-box paradox

- An example of negative probability
  Aharonov and Vaidman: *Lecture Notes in Physics* **M72**, 369 (2002)
- Three-state system $\{|\mathsf{m}_1\rangle, |\mathsf{m}_2\rangle, |\mathsf{m}_3\rangle\}$.

$$|\,\mathsf{i}\,\rangle = (|\mathsf{m}_1\rangle + (|\mathsf{m}_2\rangle + |\mathsf{m}_3\rangle))/\sqrt{3} = (|\mathsf{m}_2\rangle + (|\mathsf{m}_1\rangle + |\mathsf{m}_3\rangle))/\sqrt{3}$$
$$|\,\mathsf{f}\,\rangle = (|\mathsf{m}_1\rangle + (|\mathsf{m}_2\rangle - |\mathsf{m}_3\rangle))/\sqrt{3} = (|\mathsf{m}_2\rangle + (|\mathsf{m}_1\rangle - |\mathsf{m}_3\rangle))/\sqrt{3}$$

- The probabilities (weak values) are

$$w_1 = 1, \quad , w_2 = 1, \quad w_3 = -1$$
$$w(\{1,2\}) = 2, \quad w(\{2,3\}) = 0, \quad w(\{3,1\}) = 0$$
$$w(\{1,2,3\}) = 1$$

Post-selected sub-ensemble

- For the (pre- and post-selected) sub-ensemble $\hat{W}_{\mathsf{if}}$, the probabilites for paths $\hat{P}_{\mathsf{i}} = |\,\mathsf{i}\,\rangle\langle\,\mathsf{i}\,|$ and $\hat{P}_{\mathsf{f}} = |\,\mathsf{f}\,\rangle\langle\,\mathsf{f}\,|$ are

$$\langle |\,\mathsf{i}\,\rangle\langle\,\mathsf{i}\,|\rangle_{\mathsf{w}} = \langle |\,\mathsf{f}\,\rangle\langle\,\mathsf{f}\,|\rangle_{\mathsf{w}} = 1$$

  where $\langle\,\mathsf{i}\,|\,\mathsf{f}\,\rangle \neq 0$.
- A system that belongs to this sub-ensemble considered to have been in the state $|\,\mathsf{i}\,\rangle$ for sure and in the state $|\,\mathsf{f}\,\rangle$ for sure as well.
- Even though $[\hat{P}_{\mathsf{i}}, \hat{P}_{\mathsf{f}}] \neq 0$ (incompatible), each of $\hat{P}_{\mathsf{i}}$ and $\hat{P}_{\mathsf{f}}$ have a definite value for the sub-ensemble $\hat{W}_{\mathsf{if}}$.

Weak value and geometric phase

- For the spectral decomposition $\hat{A} = \sum_k a_k \hat{P}_k$ of a physical quantity $\hat{A}$, its weak value is

$$\langle \hat{A} \rangle_{\mathsf{w}} = \sum_k a_k w_k$$

- Anomaly of weak value has root in the complex probability measure $\{w_k\}$.
- The phase of w_k is

$$\begin{aligned} \arg\, w_k &= \arg \frac{\langle \mathsf{f} \,|\, \mathsf{m}_k \rangle \langle \mathsf{m}_k \,|\, \mathsf{i} \rangle}{\langle \mathsf{f} \,|\, \mathsf{i} \rangle} \\ &= \arg \langle \mathsf{i} \,|\, \mathsf{f} \rangle \langle \mathsf{f} \,|\, \mathsf{m}_k \rangle \langle \mathsf{m}_k \,|\, \mathsf{i} \rangle = \gamma_3(\mathsf{i}, \mathsf{m}_k, \mathsf{f}) \end{aligned}$$

$=$ the geometric phase determined by $|\,\mathsf{i}\,\rangle$, $|\,\mathsf{m}_k\rangle$, $|\,\mathsf{f}\,\rangle$

Weak value and geometric phase (2)

- In typical situation of weak measurements, $|\,\mathrm{i}\,\rangle$ and $|\,\mathrm{f}\,\rangle$ are almost orthogonal.
- On the Poincare sphere, they are located almost conjugate positions. The phase (the area of spherical triangle) is strongly dependent on the choice of the intermediate state $|\mathrm{m}_k\rangle$.
- In cases of $|\,\mathrm{i}\,\rangle = |\,\mathrm{f}\,\rangle$, the geometric phase vanishes and w_k becomes real:

$$w_k = \frac{\langle\,\mathrm{i}\,|\mathrm{m}_k\rangle\langle\mathrm{m}_k|\,\mathrm{i}\,\rangle}{\langle\,\mathrm{i}\,|\,\mathrm{i}\,\rangle} = |\langle\,\mathrm{i}\,|\mathrm{m}_k\rangle|^2$$

which reduces to the normal probalibity of finding.

The weak value, determined by three states, is closely related to the geometric phase.

Pre and post selection with mixed states

- The weak operator for pre- and post-selection with mixed state can be defined as

$$\hat{W} = \frac{\hat{\rho}_{\mathsf{i}}\hat{\rho}_{\mathsf{f}}}{\mathrm{Tr}\,\hat{\rho}_{\mathsf{f}}\hat{\rho}_{\mathsf{i}}}$$

  where $\hat{\rho}_{\mathsf{i}}$ and $\hat{\rho}_{\mathsf{f}}$ are the density operators of pre- and post-selection states.
- The pure state case can be recovered by $\hat{\rho}_{\mathsf{f}} = |\,\mathsf{f}\,\rangle\langle\,\mathsf{f}\,|$, $\hat{\rho}_{\mathsf{i}} = |\,\mathsf{i}\,\rangle\langle\,\mathsf{i}\,|$.
- The post-selection with completely mixed state, $\hat{\rho}_{\mathsf{f}} = \hat{1}/n$ yields

$$\hat{W} = \hat{\rho}_{\mathsf{i}}, \quad \langle\hat{A}\rangle_{\mathsf{w}} = \mathrm{Tr}\,\hat{A}\hat{\rho}_{\mathsf{i}} = \langle\hat{A}\rangle,$$

  which corresponds to no post-selection cases.

Weighted sum — flagility of weak value

- Post-selection with the mixed state composed of two states $|\mathrm{f}_1\rangle$, $|\mathrm{f}_2\rangle$ weighted by p_1, p_2 (≥ 0), $p_1 + p_2 = 1$;

$$\hat{\rho}_{\mathsf{f}} = p_1|\mathrm{f}_1\rangle\langle\mathrm{f}_1| + p_2|\mathrm{f}_2\rangle\langle\mathrm{f}_2|$$

- Weak value for $|\,\mathsf{i}\,\rangle$ and ρ_{f}:

$$\langle\hat{A}\rangle_{\mathsf{w}} = \frac{p_1|\langle\,\mathsf{i}\,|\mathrm{f}_1\rangle|^2\langle\hat{A}\rangle_{\mathsf{w1}} + p_2|\langle\,\mathsf{i}\,|\mathrm{f}_2\rangle|^2\langle\hat{A}\rangle_{\mathsf{w2}}}{p_1|\langle\,\mathsf{i}\,|\mathrm{f}_1\rangle|^2 + p_2|\langle\,\mathsf{i}\,|\mathrm{f}_2\rangle|^2}$$

  where $\langle\hat{A}\rangle_{\mathrm{w}j} = \langle\,\mathsf{i}\,|\hat{A}|\,\mathsf{f}_j\rangle/\langle\,\mathsf{i}\,|\,\mathsf{f}_j\rangle$ $(j = 1, 2)$.
- Weak values with large absolute value (or small $|\langle\,\mathsf{i}\,|\mathrm{f}_j\rangle|$) are added with small weights.
- Large weak values are fragile against mixing.

Determination of wavefunction with weak measurement

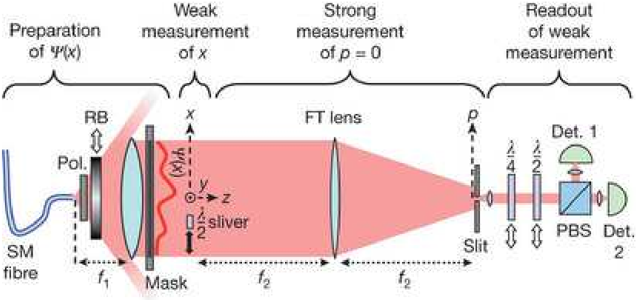
- An orthonormal basis vectors: $|\mathsf{m}_k\rangle$ $(k = 1, 2, \ldots, n)$
- An arbitrary initial state $|\psi\rangle = |\,\mathsf{i}\,\rangle$.
  The state for the post selection: $|\,\mathsf{f}\,\rangle = (1/\sqrt{n}) \sum_{k=1}^{n} |\mathsf{m}_k\rangle$.
- The weak value for the projector $\hat{P}_k = |\mathsf{m}_k\rangle\langle\mathsf{m}_k|$ is

$$\langle \hat{P}_k \rangle_{\mathsf{w}} = \psi_k \Big/ \sum_{k=1}^{n} \psi_k$$

  where $\psi_k = \langle \mathsf{m}_k | \psi \rangle$ is the probability amplitude or the wave function.
- By repeating weak measurement for all k, we can measure the wave function ψ_k.
- The gauge is fixed with the choice of phases of basis vectors $|\mathsf{m}_k\rangle$.
- It is believed that the wavefunction cannot be measured because it is complex and guage-dependent.

Experimental determination of wavefunction

- Continuous 1D basis: $\{|x\rangle \mid -\infty < x < \infty\}$
- Photon beam spread 1-dimensionally $\Psi(x)$



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Summary

- Simple experiment for measuring geometric phases in quantum eraser
  Kobayashi *et al.*, J. Phys. Soc. Jpn. **80**, 034401 (2011)
- Relationship between nonlinear geometric phases and weak value
  Tamate *et al.*, *New J. Phys.* **11**, 093025 (2009)
- Observation of two-photon geometric phase and its nonlinearity
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