

Computational Approach for Dynamics of Many-Fermion Systems

- from Nuclear Physics to Optical Science -

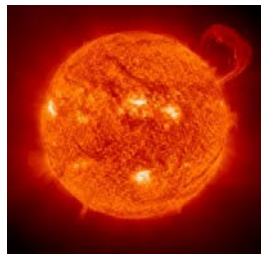
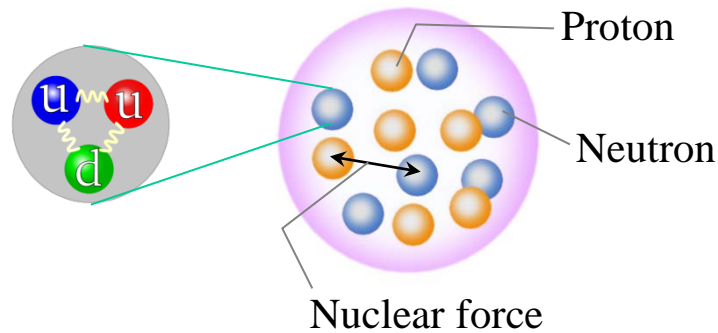
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University of Tsukuba

Links among Hierarchies in Nuclear Physics

Combining particle physics,
nuclear physics,
and astrophysics by computation

- Origin of elements
- Evolution of the universe



Fuel of Stars

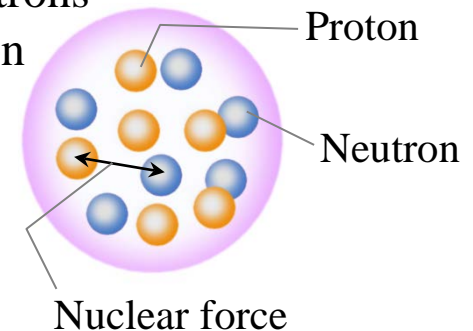


Origin of Elements

Quantum dynamics of
many-fermion systems

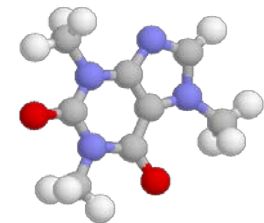
Nuclei

protons and neutrons
strong interaction



Atoms, molecules, solids

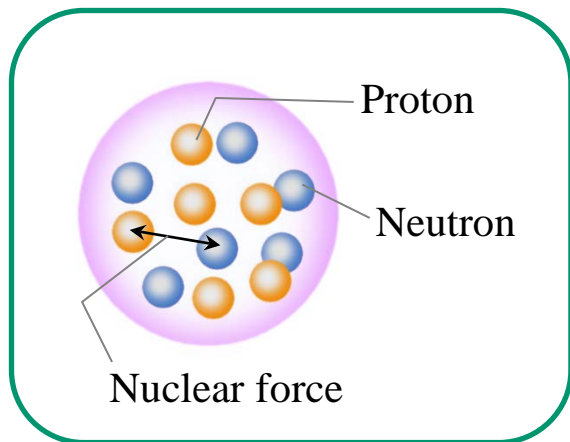
electron many-body system
Coulomb interaction



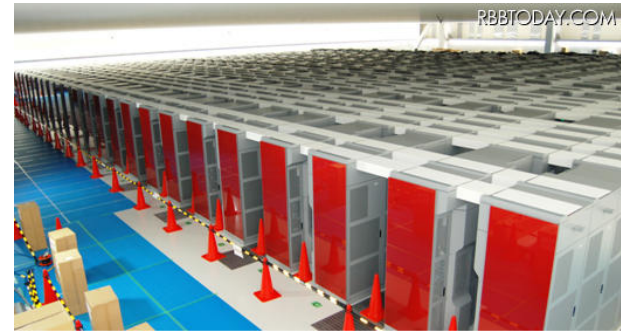
Computational Approach for Dynamics of Many-Fermion Systems:

- from Nuclear Physics to Optical Science -

A number of concepts and methods on finite, quantum, fermion systems have been developed in nuclear physics.



Large-scale computing has a high potential to link different hierarchies.



K-computer@Kobe



Optical sciences developing as multi-disciplinary fields. In particular, nonlinear electron dynamics by intense and ultrashort laser pulses.

Let me start to talk on

How I come to work on electron dynamics.

Quantum Simulation for Many-Fermion Systems: Nuclear Collision

Time-dependent Hartree-Fock theory.
 3D time-dependent Schroedinger equation is solved with real-time and real-space method.

H. Flocard, S.E. Koonin, M.S. Weiss,
 Phys. Rev. 17(1978)1682.

Time evolution of proton and neutron orbitals.

$$i\hbar \frac{\partial}{\partial t} \psi_i(\vec{r}, t) = h[n(\vec{r}, t)]\psi_i(\vec{r}, t)$$

$$n(\vec{r}, t) = \sum_i |\psi_i(\vec{r}, t)|^2$$

$$\psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N, t) = A \{ \psi_1(\vec{r}_1, t) \psi_2(\vec{r}_2, t) \dots \psi_N(\vec{r}_N, t) \}$$

17

THREE-DIMENSIONAL TIME-DEPENDENT HARTREE-FOCK...

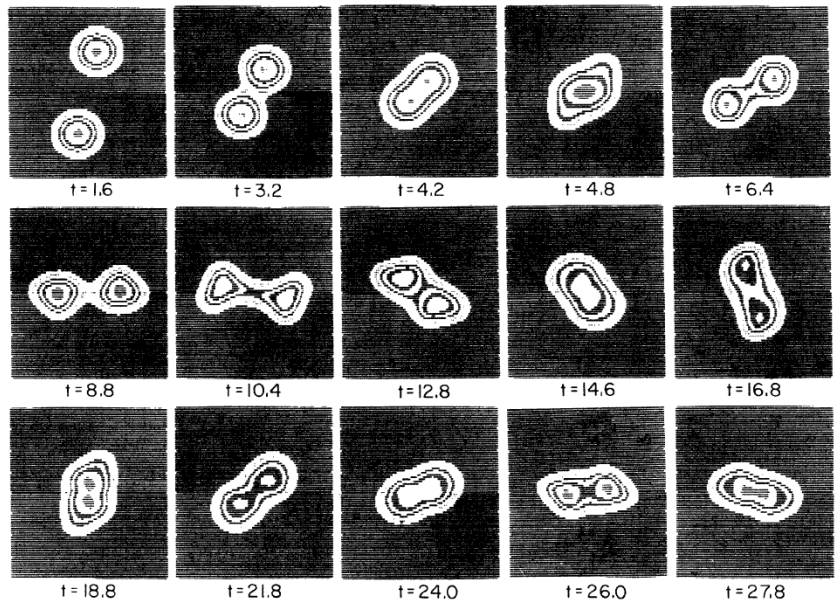
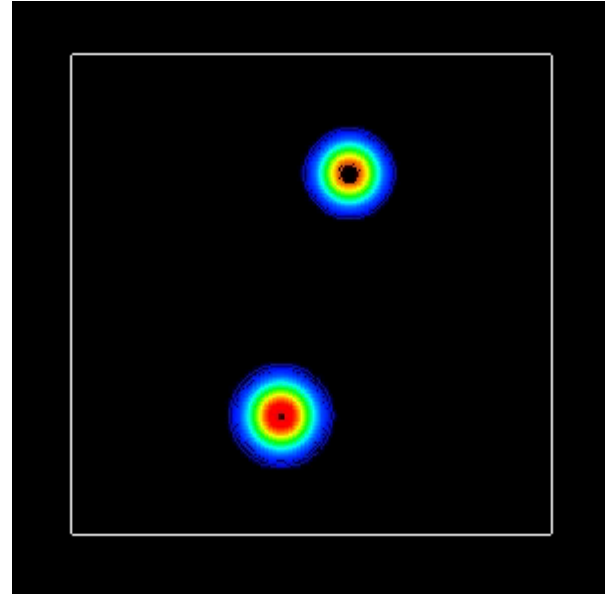


FIG. 2. Contour lines of the density integrated over the coordinate normal to the scattering plane for an $^{16}\text{O} + ^{16}\text{O}$ collision at $E_{\text{lab}} = 105 \text{ MeV}$ and incident angular momentum $L = 13\hbar$. The times t are given in units of 10^{-22} sec .

Spatial grid $30 \times 28 \times 16$, time step 4×10^2

$^{24}\text{O} - ^{16}\text{O}$ collision at $E = 16 \text{ MeV}$



Time: 10^{-22} s , Length: 10^{-14} m

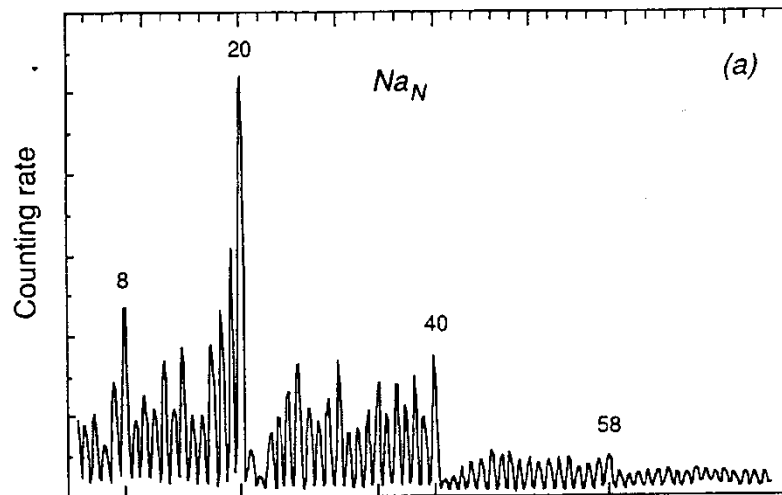
Crossover between nuclear physics and nanoscience: Atomic clusters

Synthesis of atomic clusters in cluster beam in the vacuum

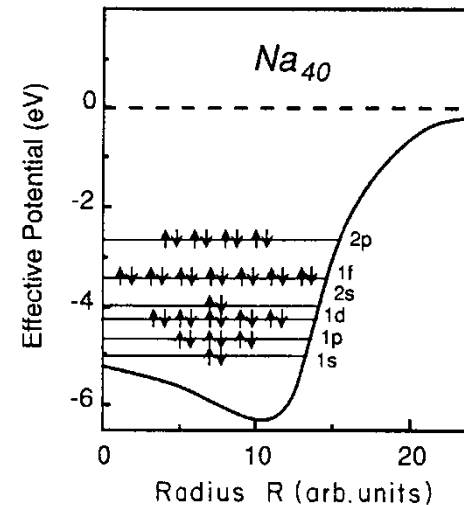
- **1984**: Discovery of magic numbers in metallic clusters
Prediction of giant dipole resonance (soon observed)
- **1985**: Discovery of fullerene C_{60} in cluster beam

(I was in graduate school during 1982-1987)

Abundance spectrum of Alkali metal clusters, electronic mean field and magic number



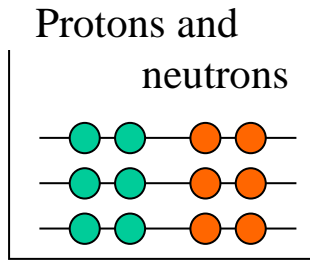
Magic number in Na clusters
Knight et.al.(1984)



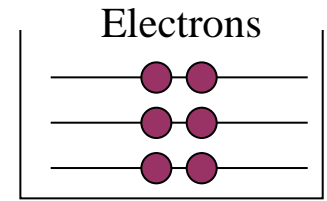
Metallic clusters: 2,8,20,40,58,92,138, ...

Atomic nuclei: 2,8,20,28,50,82,126, ...

Finite Fermion Systems confined in spherical potential



Nuclei

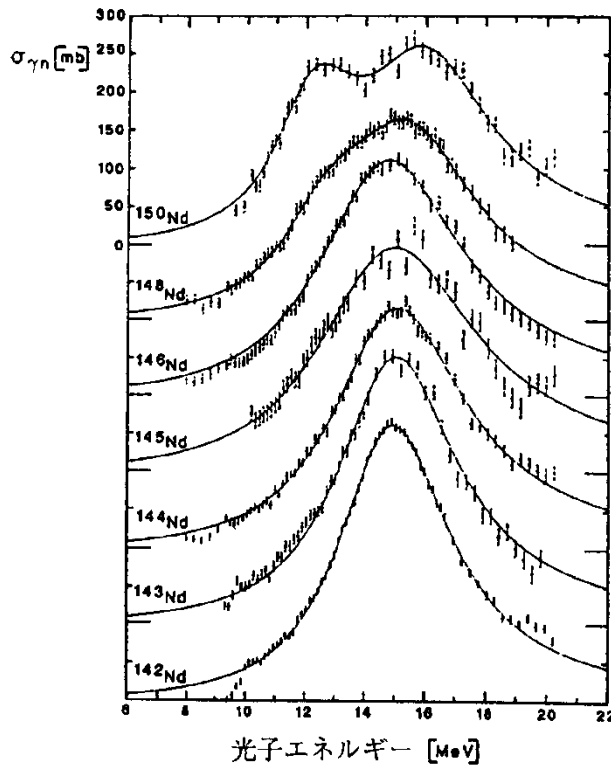


Metallic clusters

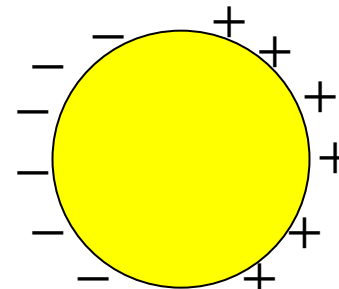
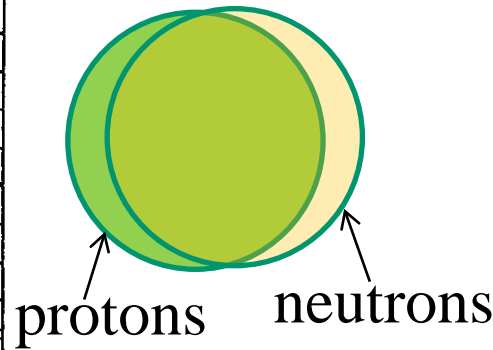
Common properties

- Magic number
- Deformation
- Collective excitation

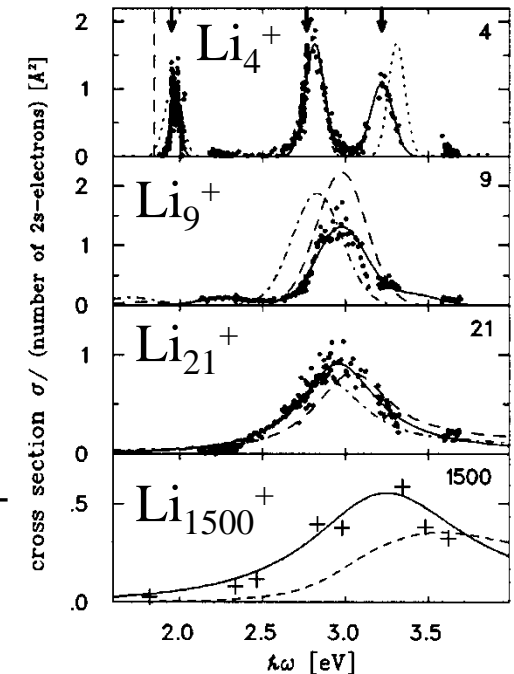
Giant Dipole Resonance



Optical absorption of atomic nuclei



Mie plasmon, a surface plasmon of spherical particle



Optical absorption spectrum of Li clusters

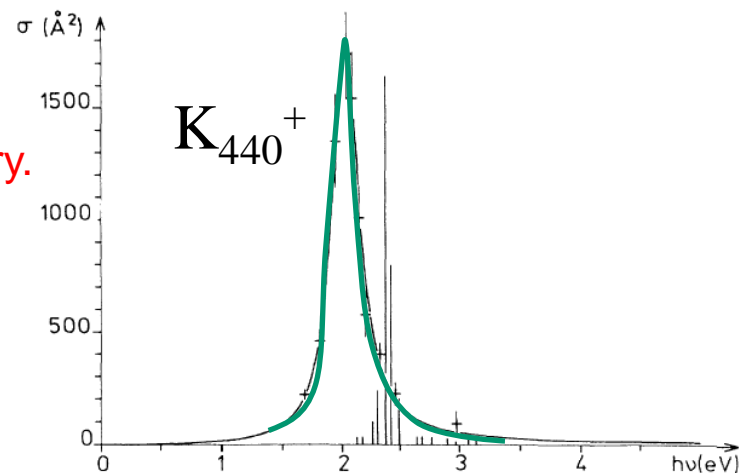
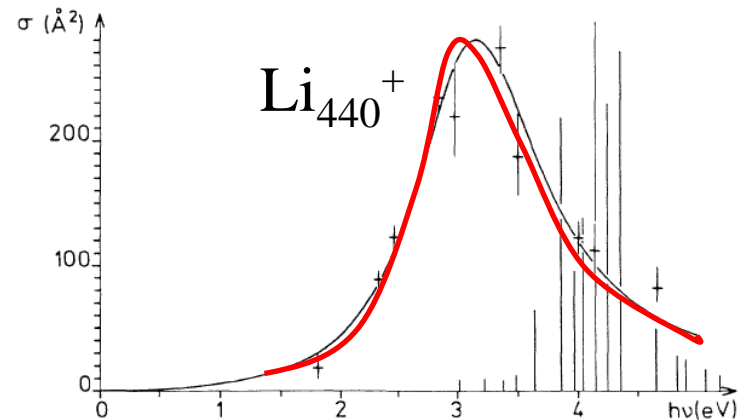
I participated in atomic cluster physics in 1992.

There are a lot of interesting but “complex and difficult” problems.

- Different shape of absorption profiles among alkali metals, Li, Na, K. Why?
- Bulk concepts (e.g. dielectric function) effective for small systems?

How to describe optical response?

- Hundred of electrons.
- Roughly spherical but precisely having no symmetry.



Absorption spectrum (Mie plasmon) of Alkali-metal clusters, Li and K

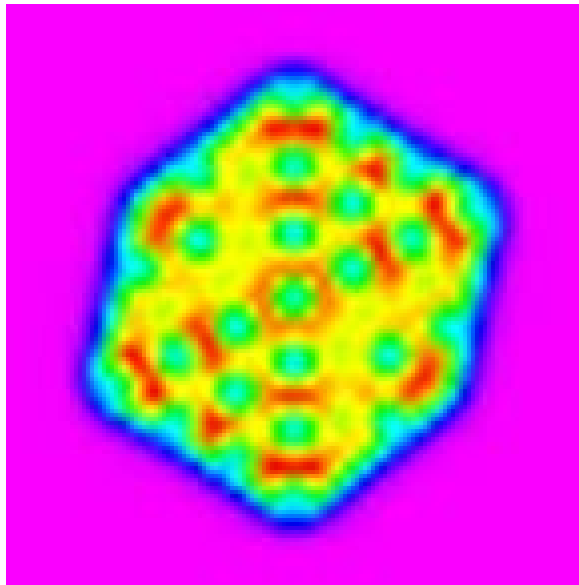
My answer was to combine - Nuclear time-dependent mean-field method –
 and - First-principles density-functional theory in condensed matter physics -

$$i\hbar \frac{\partial}{\partial t} \psi_i(t) = \{h_{KS}[\rho(t)] + V_{ext}(t)\} \psi_i(t)$$

$$\rho(t) = \sum_i |\psi_i(t)|^2$$

Electron density change from that in the ground state.

Na₁₄₇⁺



Assume
 icosahedral
 geometry

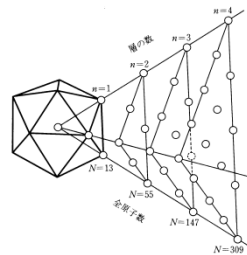
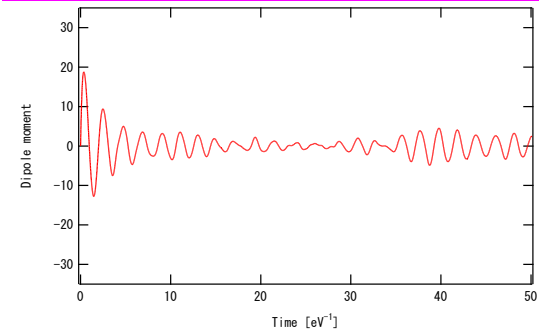
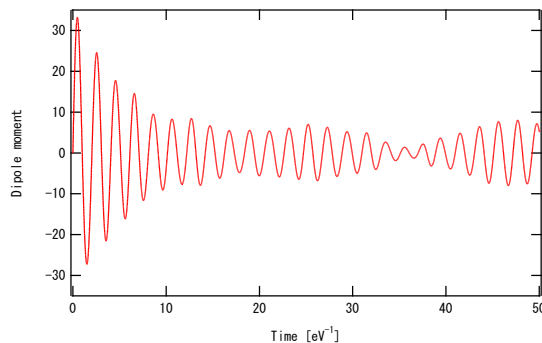
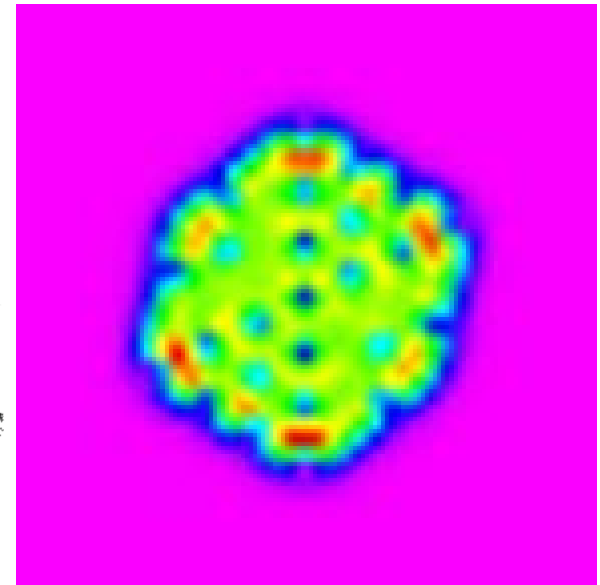


図 5・61 多層正二十面体構造(MIC)の概念図⁹⁰⁾
 nはMICの層の数であり、Nは全原子数である。1層構造(n=1)では原子数N=13に対応し、2層構造(n=2)ではN=55、3層構造(n=3)ではN=147…となる。

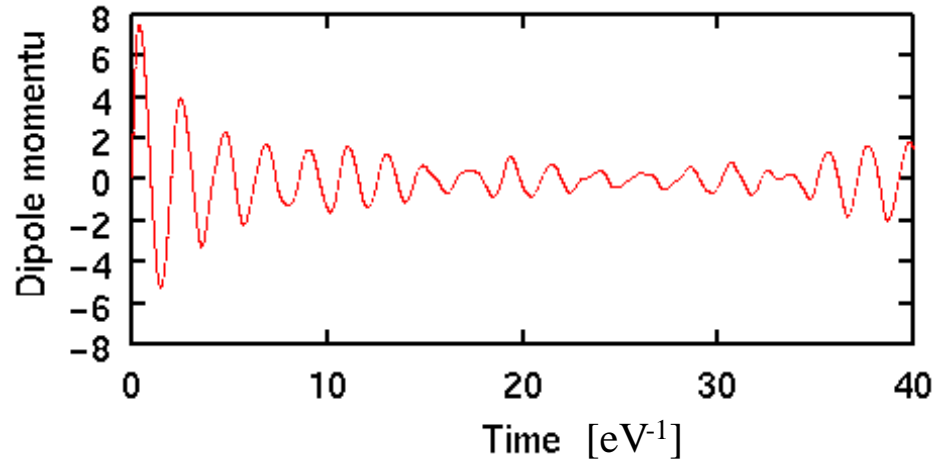
Li₁₄₇⁺



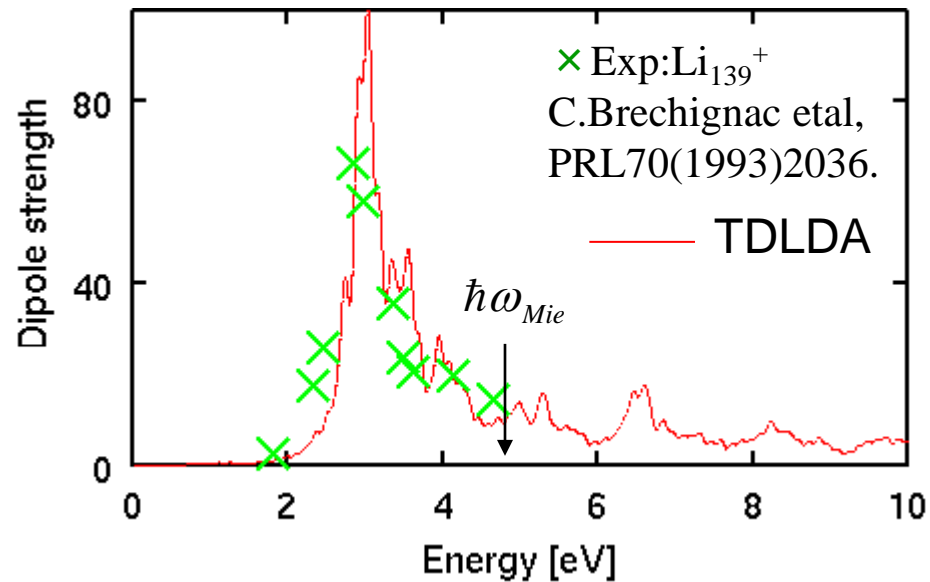
Optical absorption spectrum of Li_{147}^+

K. Y, G.F. Bertsch, Phys. Rev. B54, 4484 (1996).

$$\alpha(t) = \frac{e^2}{\hbar k} \int d\vec{r} z \rho(\vec{r}, t)$$



$$\alpha(\omega) = \int dt e^{-i\omega t} \alpha(t)$$



The width comes from electron-atom elastic scattering (Landau damping)

Time-dependent mean-field theory

$$i\hbar \frac{\partial}{\partial t} \psi_i(t) = \{h_{KS}[\rho(t)] + V_{ext}(t)\} \psi_i(t)$$

$$\rho(t) = \sum_i |\psi_i(t)|^2$$

Single-particle dynamics in mean-field potential

- Protons and neutrons in nuclei
- Electrons in atoms, molecules, and solids

TDHF (Time-dependent Hartree-Fock)

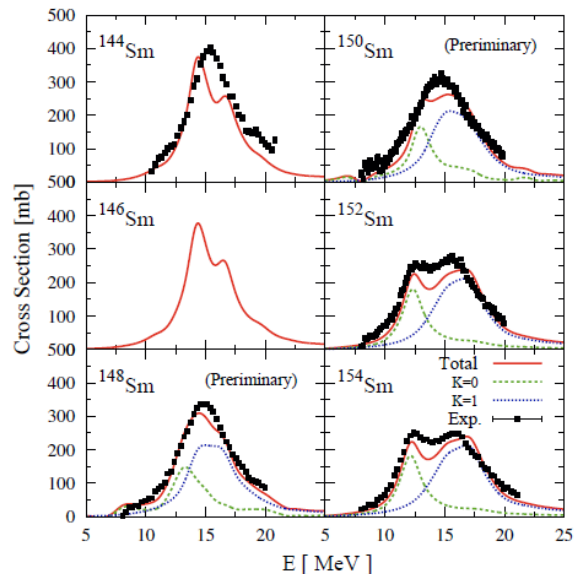
TDDFT (Time-dependent density-functional theory)

Universal tool for photoabsorption

Nuclei

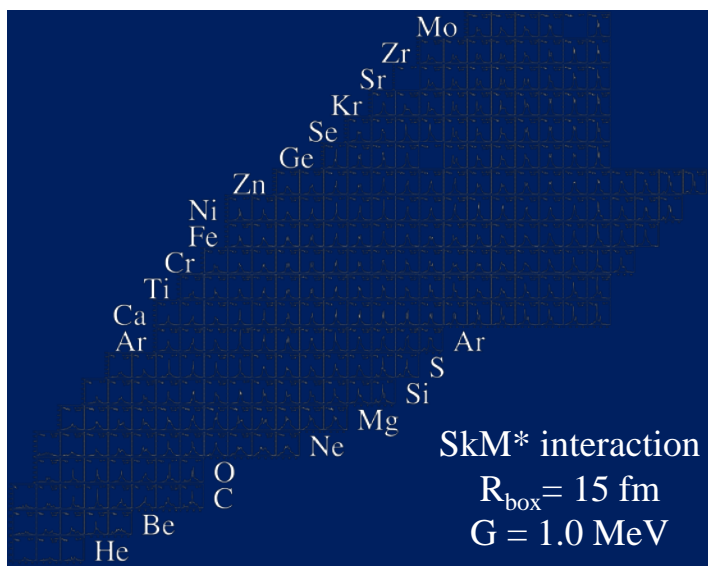
Molecules

Real-time TDHF+BCS, S. Ebata Ph.D thesis (2010)



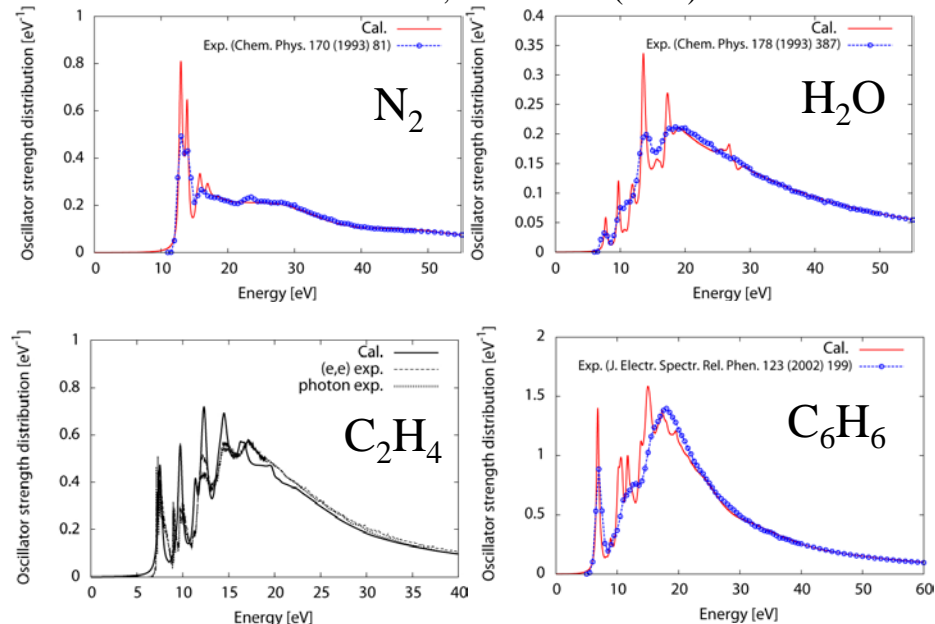
Attempt for systematic calculations

T. Inakura, T. Nakatsukasa, K.Y. Phys. Rev. C80, 044301 (2009)

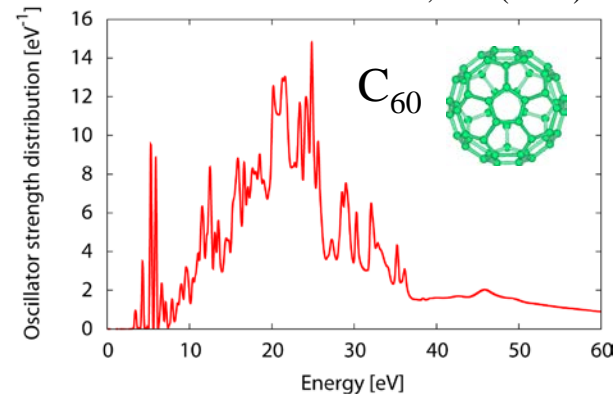


Absorption spectra of molecules by real-time TDDFT

Y. Kawashita, Ph.D thesis (2009)



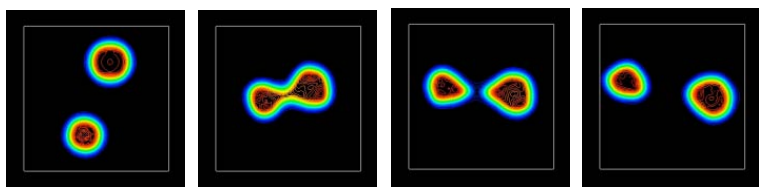
J. Mol. Struct. THEOCHEM 914, 130 (2009).



Optical absorption spectrum is a linear response property
(within perturbation theory).

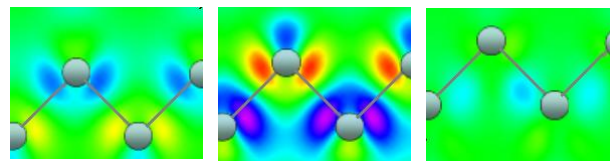
Next, on nonlinear fermion dynamics.

In low-energy nuclear physics,
heavy-ion collision has been
the major phenomena of
nonlinear nuclear dynamics.



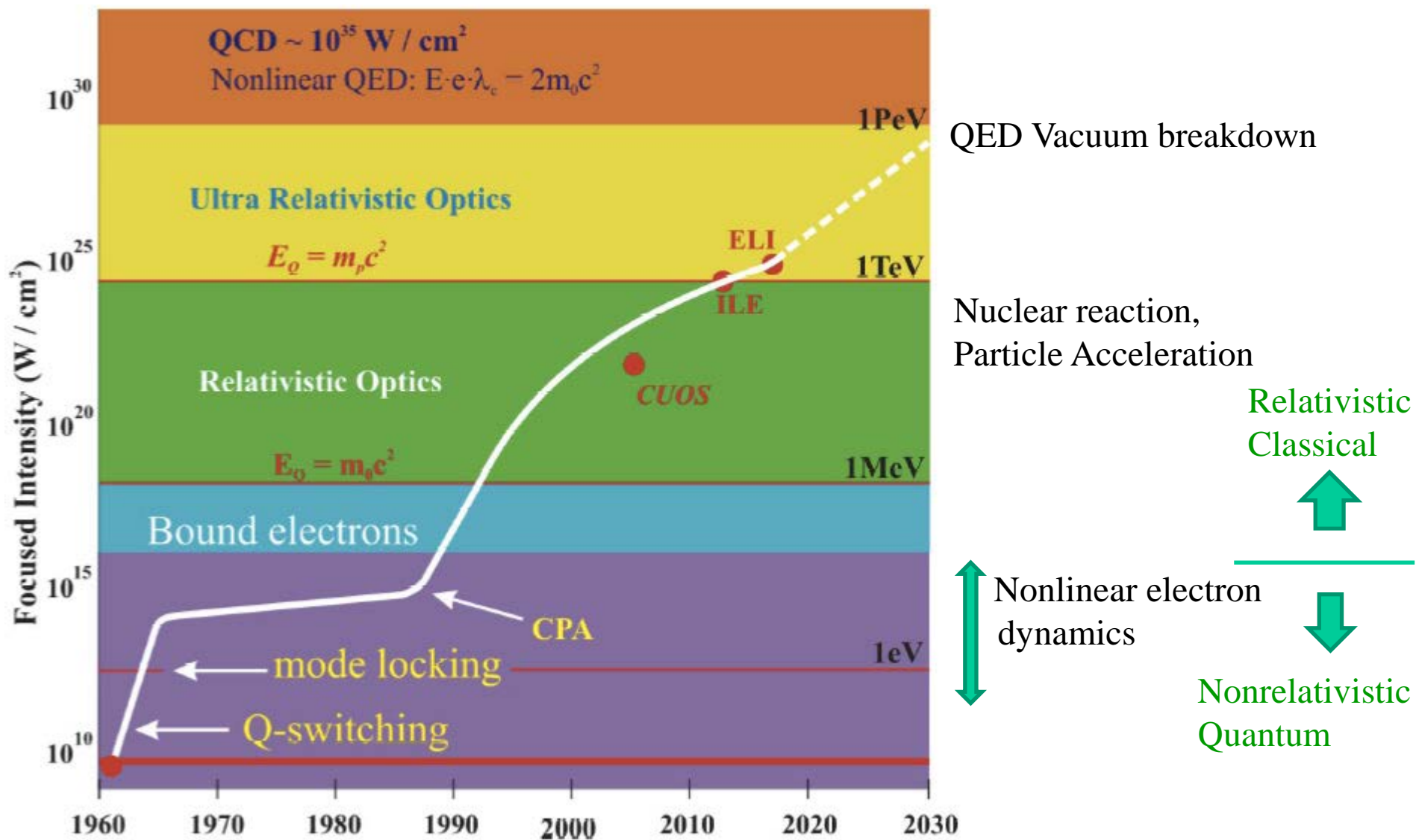
Nuclear TDHF simulation

In optical sciences,
intense laser pulse induces
a variety of phenomena reflecting
nonlinear electron dynamics.

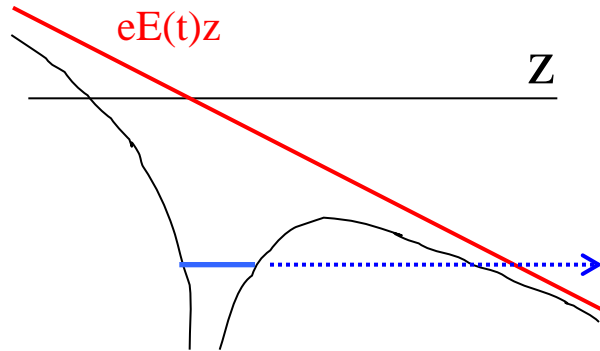


Electron dynamics in solid, TDDFT simulation

Intense Laser Pulses: G. Mourou @ PIF2010

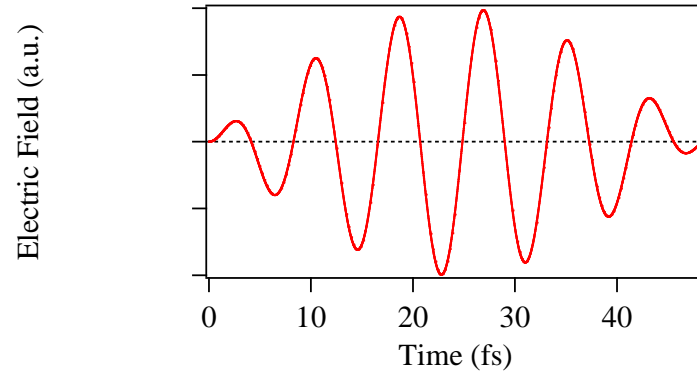


Intense and Ultrashort



10^{13} - 10^{15} W/cm²

Intensity of applied laser pulse is comparable to electric field in matter which binds electrons.



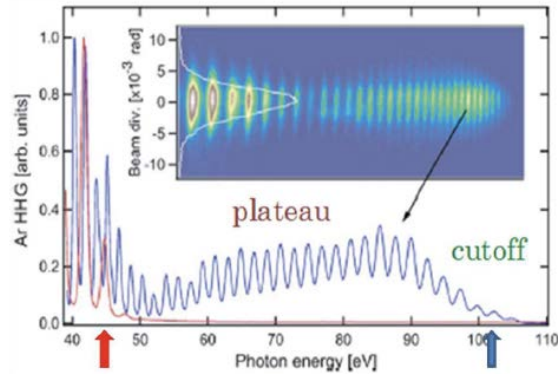
10^{-15} s (1 femto sec)

Pulse duration shorter than molecular vibration comparable to electron dynamics.

Nonlinear Electron Dynamics induced by intense and ultrashort laser pulse

Atoms:

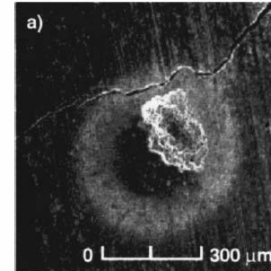
High Harmonic
Generation and
Attosecond pulse
generation



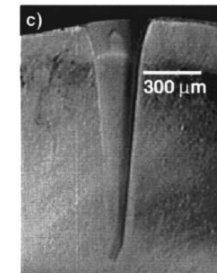
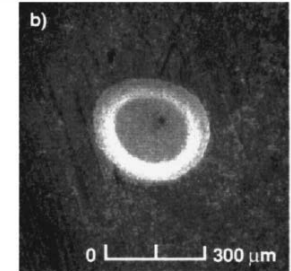
Solids:

Nonthermal laser machining,
formation of electron-hole plasma
by intense and ultrashort pulse

1.4 ns



350 fs



Molecules:

Tomographic imaging of
molecular orbital
by electron rescattering

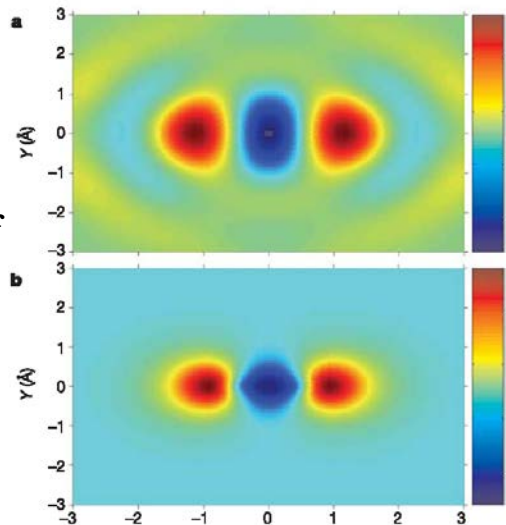


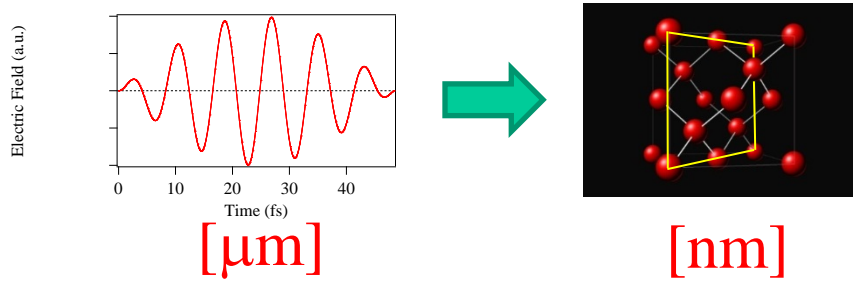
FIG. 1. (a) Drilling of enamel (tooth) with conventional 1053 nm, nanosecond pulses (ablation threshold=30 J/cm² for $\tau_p=1.4$ ns). (b) Same as in (a) but with the pulse duration reduced to the ultrashort regime (ablation threshold=3 J/cm² for $\tau_p=350$ fs). In both cases, the laser spot size was 300 μ m. (c) cross section of hole made with 350 fs pulses.

Intense laser pulse propagation in solid
requires

undivided treatment of quantum mechanics and electromagnetism


Different length scales of light wavelength and atomic size
requires

multi-scale description



Theoretical description of light propagation in matter

Electromagnetism and Quantum Mechanics

Maxwell equation	Constitutive relations	Schroedinger equation
$\vec{\nabla} \cdot \vec{B} = 0$	$\vec{D} = \vec{D}[\vec{E}, \vec{H}] \neq \epsilon \vec{E}$	$i\hbar \frac{\partial}{\partial t} \psi_i = \frac{1}{2m} \left(-i\hbar \vec{\nabla} + \frac{e}{c} \vec{A} \right)^2 \psi_i - e\phi \psi_i$
$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$	$\vec{B} = \vec{B}[\vec{E}, \vec{H}] \neq \mu \vec{H}$	$\vec{\nabla}^2 \phi = -4\pi \{en_{ion} - en_e\}$
$\vec{\nabla} \cdot \vec{D} = \rho$		<div style="text-align: center;">  <p style="margin: 0;">Linear response (perturbation) theory</p> </div>
$\vec{\nabla} \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{j}$		$\epsilon(\omega), \mu(\omega)$

For extremely intense field,
we need to solve coupled Maxwell + Schroedinger equations

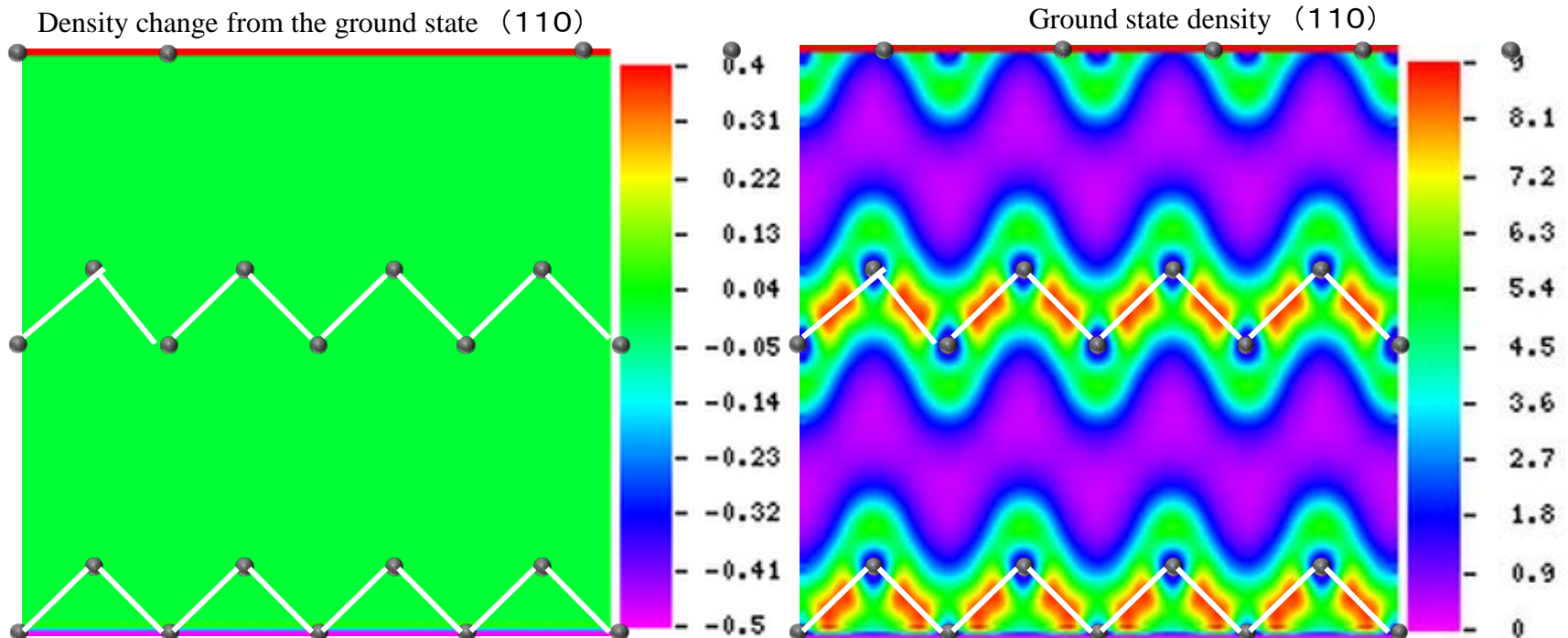
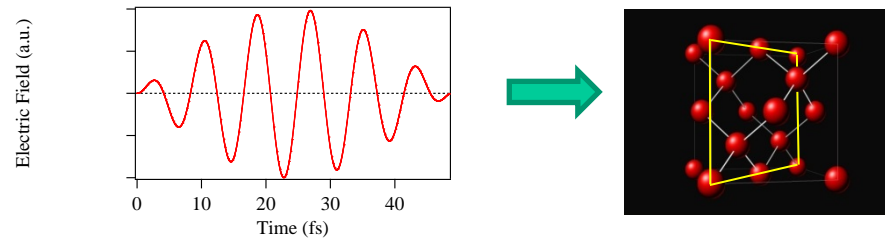
Two spatial scales, light wavelength [μm] and electron dynamics [nm]

Microscopic Electron dynamics in crystalline Si under spatially uniform, time-dependent electric field

Time-dependent Bloch orbitals,
an extension of band-theory for electron dynamics

G.F. Bertsch, J.-I. Iwata, A. Rubio, K. Y., Phys. Rev. B62, 7998 (2000).

$I = 3.5 \times 10^{14} \text{ W/cm}^2$,
 $T = 50 \text{ fs}$, $\hbar\omega = 0.5 \text{ eV}$



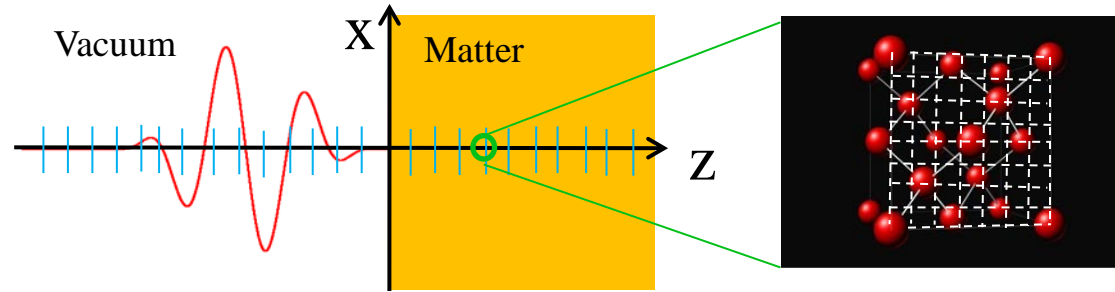
Coupled Maxwell + TDDFT multi-scale simulation

K. Y., T. Sugiyama, Y. Shinohara, T. Otobe, G.F. Bertsch, Phys. Rev. B85, 045134 (2012).

Two coordinates: Macroscopic and microscopic

Macroscopic grids for Z (μm scale) to calculate propagation of vector potential

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} A_Z(t) - \frac{\partial^2}{\partial Z^2} A_Z(t) = \frac{4\pi}{c} J_Z(t)$$



Electron orbitals: $\psi_{i,Z}(\vec{r}, t)$

Coupled by macroscopic vector potential and current averaged over unit cell volume

$J_Z(t)$

$A_Z(t)$

$$J(Z, t) = \int_{\Omega} d\vec{r} \vec{j}_{e,Z}$$

$$\vec{j}_{e,Z} = \frac{\hbar}{2mi} \sum_i (\psi_{i,Z}^* \vec{\nabla} \psi_{i,Z} - \psi_{i,Z} \vec{\nabla} \psi_{i,Z}^*) - \frac{e}{4\pi c} n_{e,Z} \vec{A}$$

At each macroscopic grid point Z, we consider Kohn-Sham orbital with microscopic grids (nm) to describe electron dynamics

$$i\hbar \frac{\partial}{\partial t} \psi_{i,Z} = \frac{1}{2m} \left(-i\hbar \vec{\nabla} + \frac{e}{c} \vec{A}_Z(t) \right)^2 \psi_{i,Z} - e\phi_Z \psi_{i,Z} + \frac{\delta E_{xc}}{\delta n} \psi_{i,Z}$$

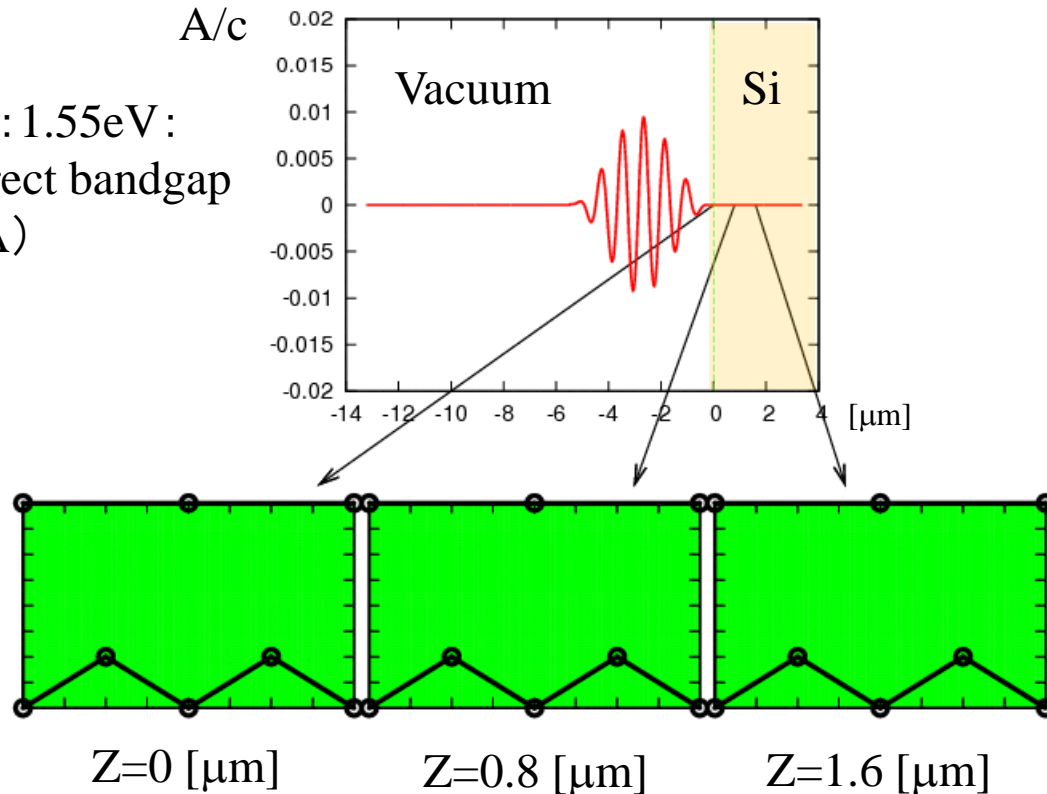
$$\vec{\nabla}^2 \phi_Z = -4\pi \{ en_{ion} - en_{e,Z} \}$$

Propagation of weak pulse

(Linear response regime, separate dynamics of electrons and E-M wave)

$$I=10^{10}\text{W}/\text{cm}^2$$

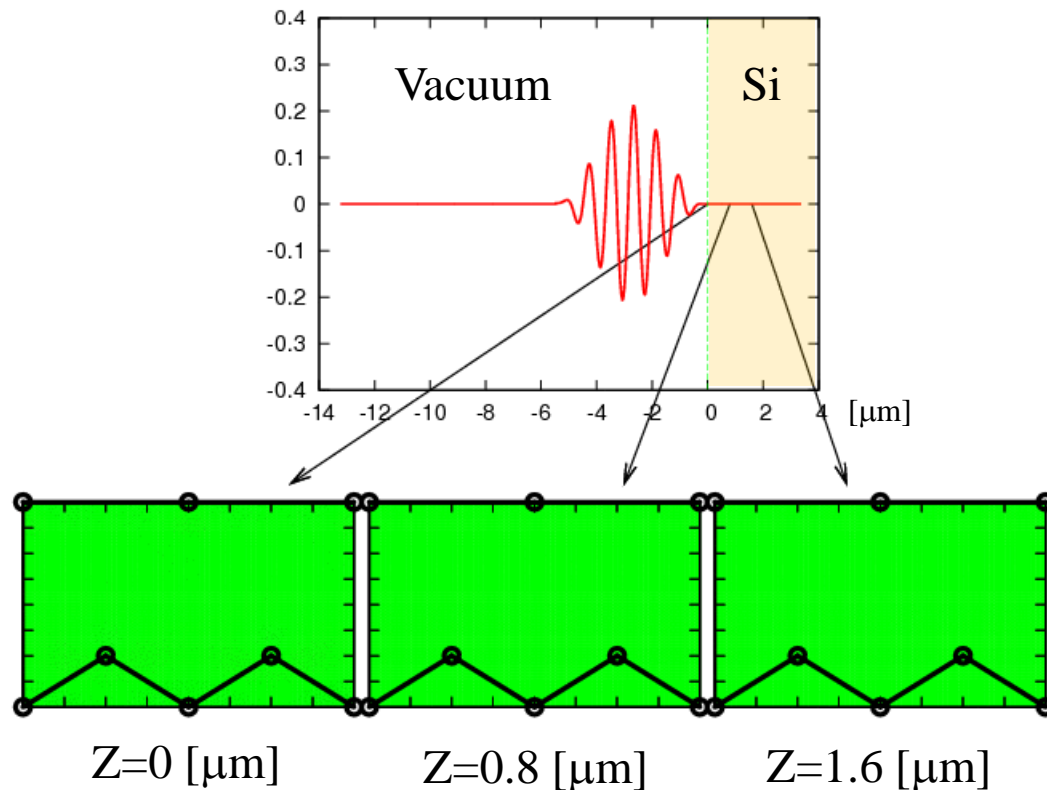
Laser frequency : 1.55eV:
lower than direct bandgap
2.4eV(LDA)



More intense laser pulse

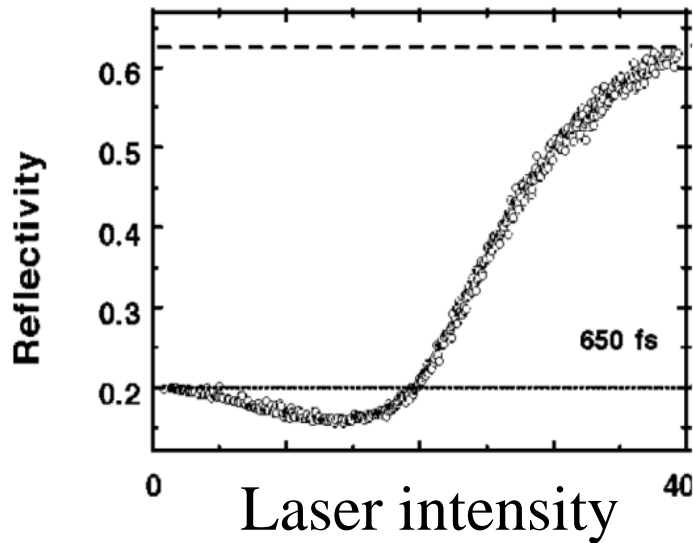
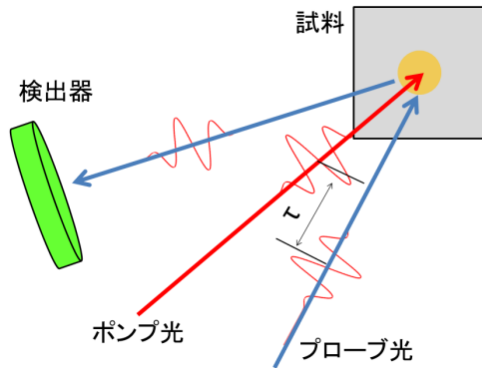
Maxwell and TDKS equations no more separate.

$$I = 5 \times 10^{12} \text{W/cm}^2$$

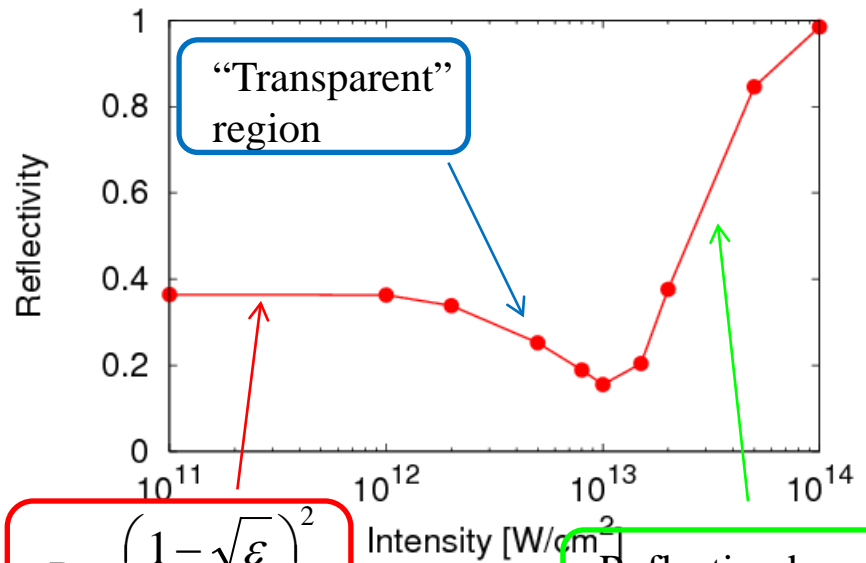
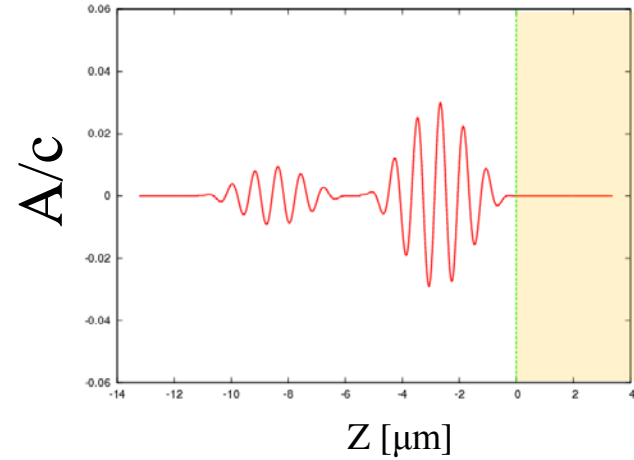


Reflectivity depends on laser intensity.

Pump-probe measurement



Pump-probe simulation



$$R = \left(\frac{1 - \sqrt{\epsilon}}{1 + \sqrt{\epsilon}} \right)^2$$

Reflection by electron-hole plasma

Generation of dense electron-hole plasmas in silicon
K. Sokolowski-Tinten and
D. von der Linde, Phys. Rev. B **61**, 2643 (2000)

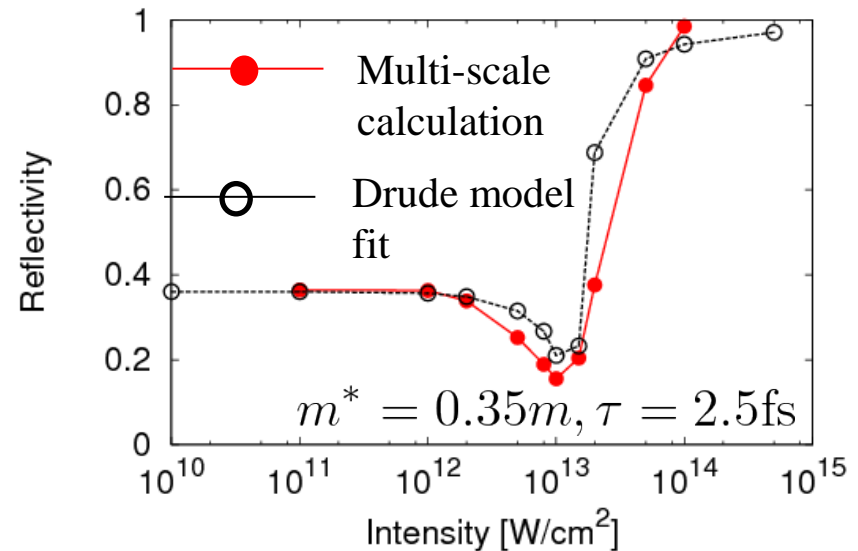
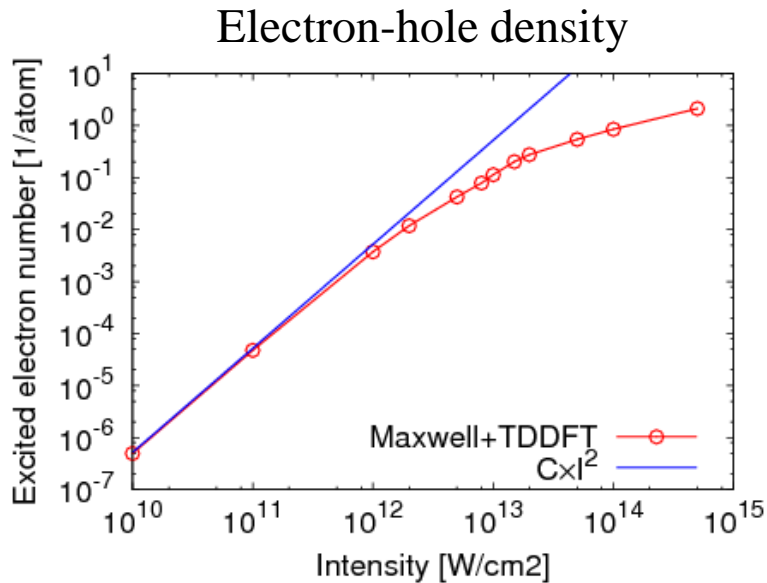
Interpretation by Drude model

Laser pulse excites electrons.
Excited electrons behave metallic.

$$\varepsilon(n_{ph}) = \varepsilon_{GS} - \frac{4\pi e^2 n_{ph}}{m^*} \frac{1}{\omega(\omega + \frac{i}{\tau})}$$

n_{ph} electron-hole density

τ collision time



Time-dependent mean-field calculation includes collision effect !?

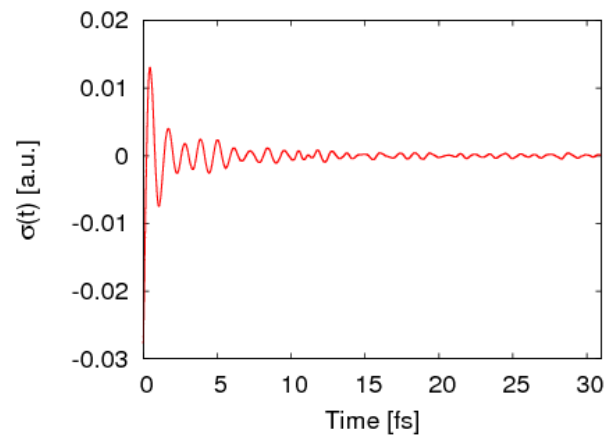
Summary

- Nuclear physics (has been and will) provide useful concepts/methods for new emergences in a variety of fields.
- Large scale computing has a high potential to link different hierarchies.
- I showed an example, the time-dependent mean-field theory
 - Universal tool for linear optical response
 - Maxwell / Schroedinger description for nonlinear electromagnetism (regarded as a new light source by computation after the laser, synchrotron radiation, XFEL, ...)

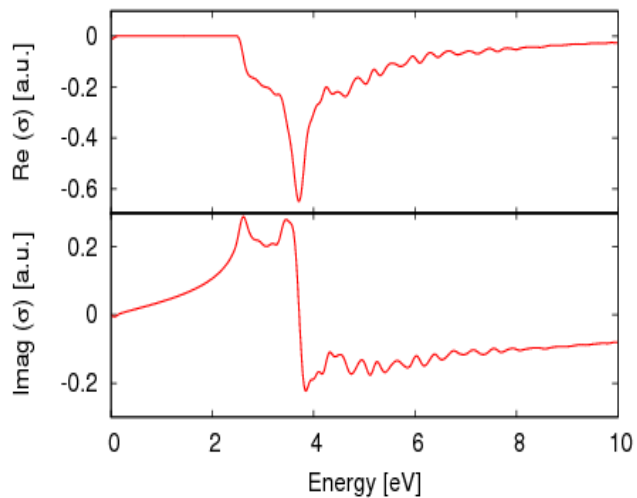
線形応答

$$j(t) = \int^t dt' \sigma(t - t') E(t')$$

$\sigma(t)$



$\sigma(\omega)$



$\epsilon(\omega)$

$$\epsilon(\omega) = 1 - \frac{4\pi i}{\hbar} \sigma(\omega)$$

