

2012 Feb 13, GCOEシンポジウム「階層の連結」, Kyoto University

# Rotating FFLO Superfluid in cold atom gases



Niigata University, Youichi Yanase  
Tomohiro Yoshida

Group member (from 2009 Oct.)



Tomohiro Yoshida

“FFLO Superfluid”

“Spin triplet  
Superconductivity”



Daisuke Maruyama

“Non-centrosymmetric  
Superconductivity”



Shuheii Takamatsu



Shunsuke Kawabe

# Introduction to FFLO state

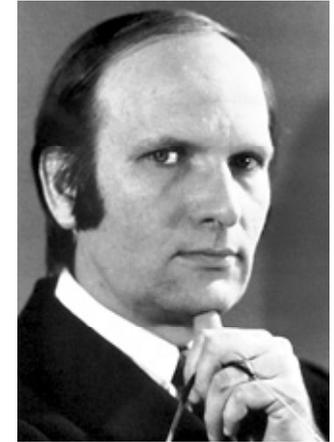
# Standard theory of Superfluidity/SC



J. Bardeen



L. N. Cooper



J. R. Schrieffer

## BCS Theory (1957)

Fermions + Attractive interaction

 Cooper pairs with  $q=0$

Basic Assumption: Total momentum of Cooper pair is zero.

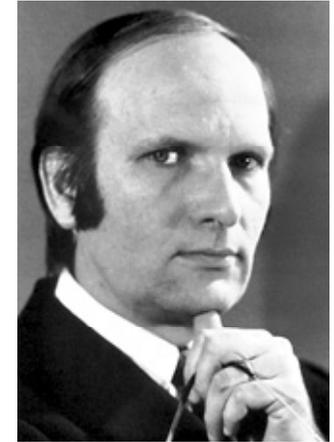
# Standard theory of Superfluidity/SC



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## BCS Theory (1957)

Fermions + Attractive interaction

→ Cooper pairs with  $q=0$

**FFLO state** Condensate of Cooper pairs with  $q \neq 0$

Theory: Fulde-Ferrell (1964), Larkin-Ovchinnikov (1964)

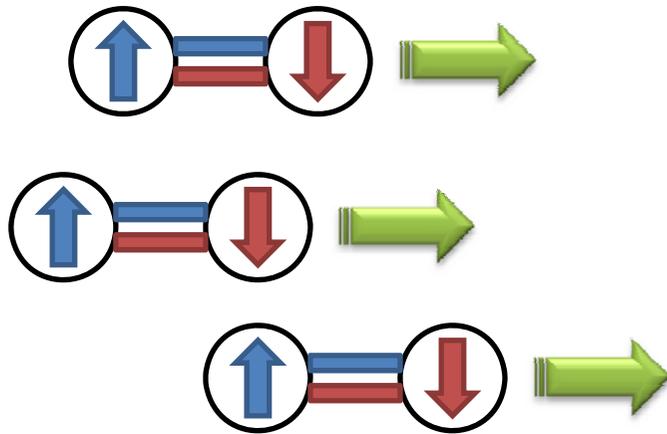
# FFLO Superfluidity/Superconductivity

Condensate of Cooper pairs with finite total momentum  $\pm q$

➡ Multi-component superconductor

Fulde-Ferrell state

$$\Delta(r) = \Delta \exp(iqx)$$

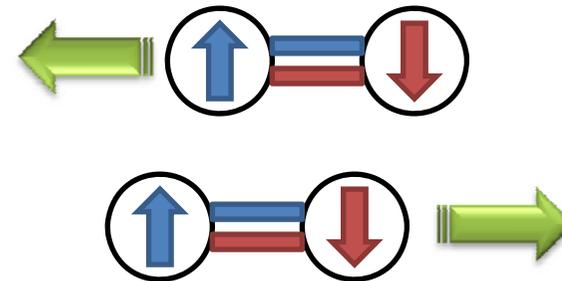


Under current, Non-centro. SC

Broken inversion symmetry

Larkin-Ovchinnikov state

$$\Delta(r) = \Delta \cos(qx)$$



$^3\text{He}$  thin film, **Superconductor**

Broken translation symmetry

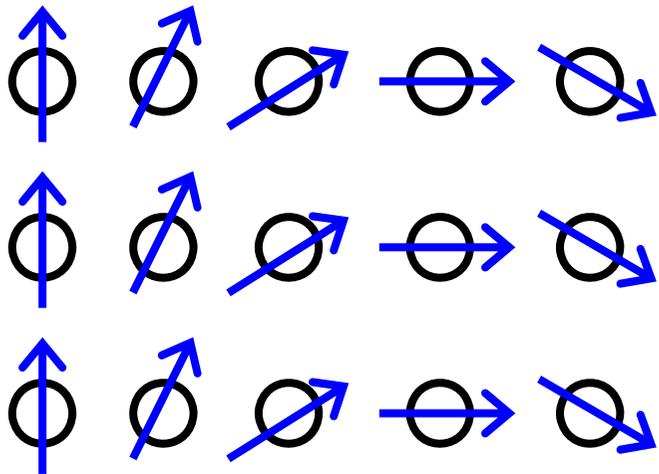
# FFLO Superfluidity/Superconductivity

Condensate of Cooper pairs with finite total momentum  $\pm q$

 Multi-component superconductor

Fulde-Ferrell state

$$\Delta(r) = \Delta \exp(iqx)$$

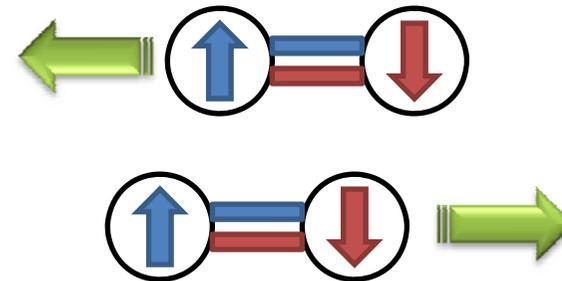


Under current, Non-centro. SC

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# FFLO Superfluidity/Superconductivity

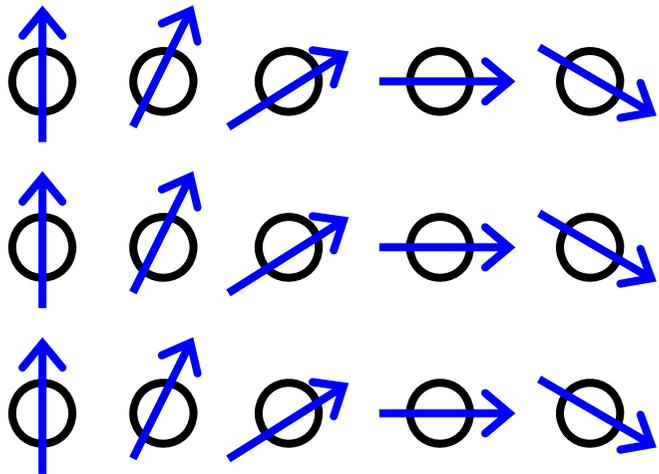
Condensate of Cooper pairs with finite total momentum  $\pm q$



Multi-component superconductor

Fulde-Ferrell state

$$\Delta(r) = \Delta \exp(iqx)$$

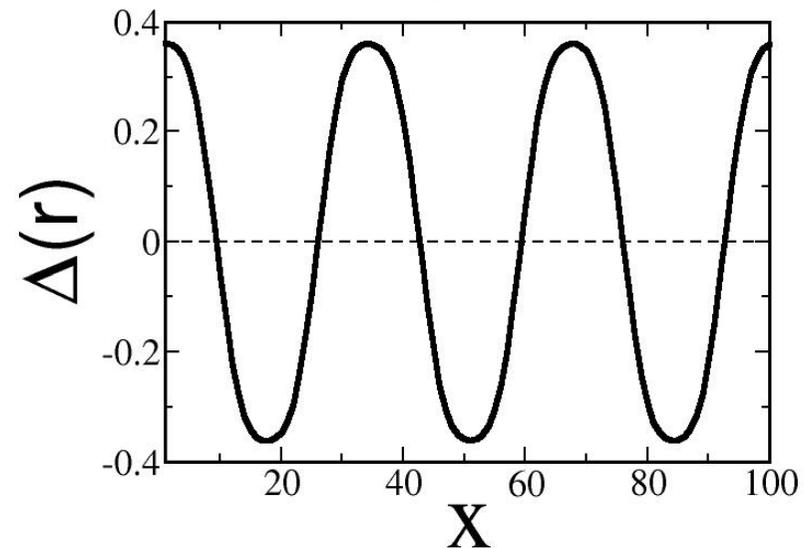


Under current, Non-centro. SC

Broken inversion symmetry

Larkin-Ovchinnikov state

$$\Delta(r) = \Delta \cos(qx)$$



$^3\text{He}$  thin film, **Superconductor**

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# FFLO Superfluidity/Superconductivity

Condensate of Cooper pairs with finite total momentum  $\pm q$

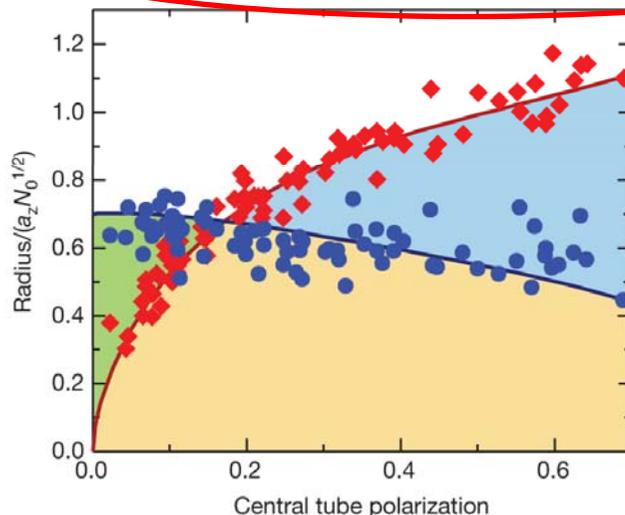
Theory: Fulde-Ferrell (1964), Larkin-Ovchinnikov (1964)

## Candidates

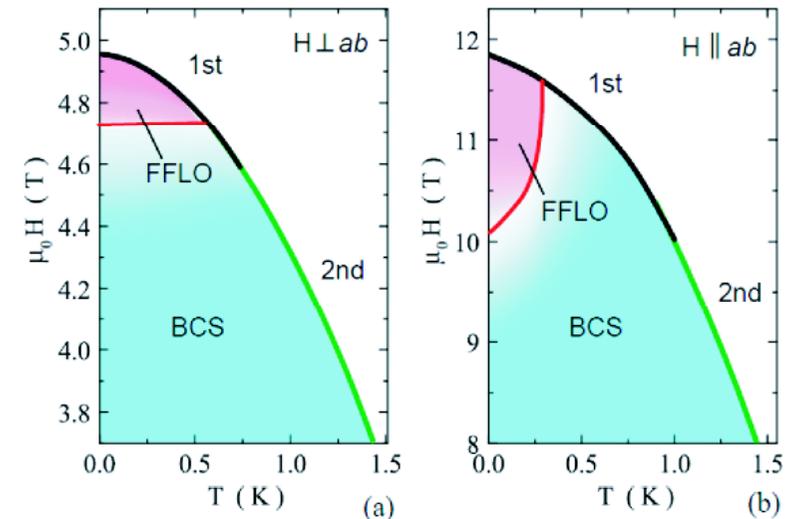
(1) Superconductors in magnetic field

CeCoIn<sub>5</sub>, (TMTSF)<sub>2</sub>X .....

(2) Imbalanced cold Fermi gases



MIT, Rice, Paris (2006-)



(3) High density quark matter  
(Color superconductivity)

(4) <sup>3</sup>He thin film

(5) Stripe phase in Cuprates

# Why we couldn't realize FFLO phase in SC for 40 years ?

FFLO phase is suppressed by

(1) Disorder  We need a high quality single crystal

(2) Orbital pair-breaking effect  Low dimension,  
Heavy effective mass

Strongly correlated electron systems

(3) Fermi liquid correction ( $F_0^a < 0$ )

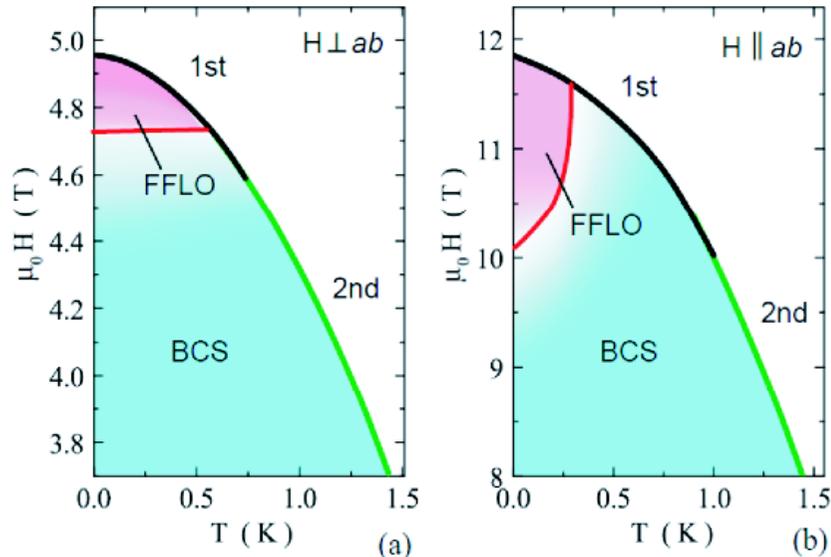


Strongly correlated electron systems ???

# Why we could find the FFLO phase ?

Yanase (2008)

## (1) CeCoIn<sub>5</sub>



Matsuda-Shimahara (2007)

Radovan et al. (2003)

Bianchi et al. (2003)

(2) (TMTSF)<sub>2</sub>X Yonezawa et al. (2008)

(3) λ-(BETS)<sub>2</sub>X Uji et al. (2006)

(4) κ-(ET)<sub>2</sub>X Lortz et al. (2007)

Clean and Pauli-limited SC

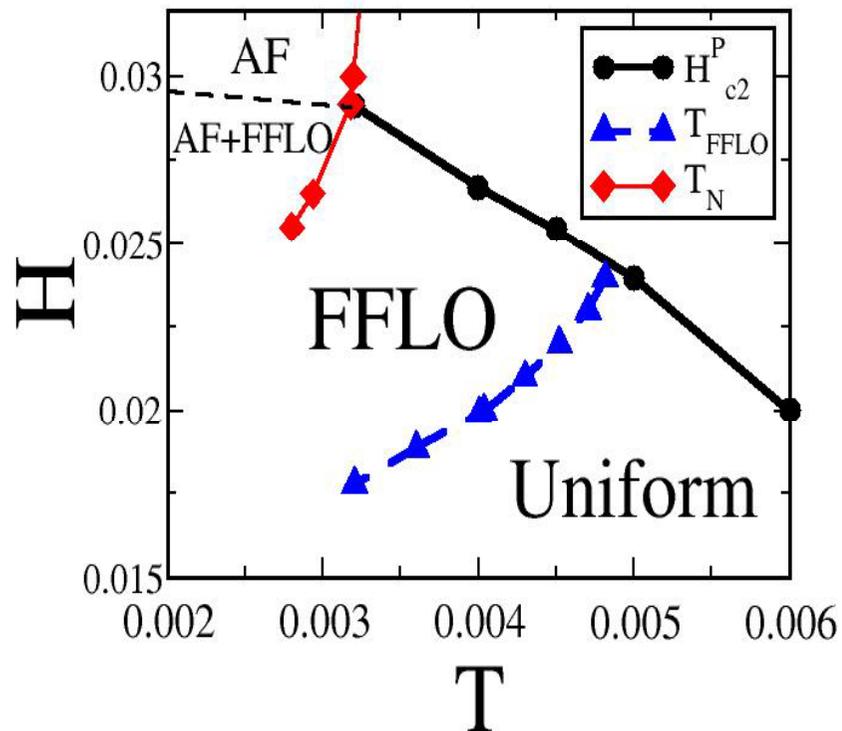
All of these compounds  
are close to AFQCP

Fermi liquid correction, retardation effect, parity mixing ...

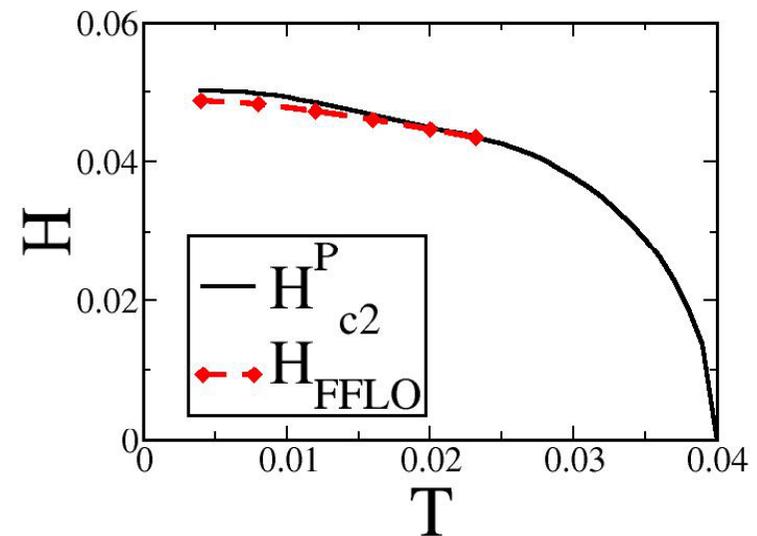
# FFLO superconductivity near AFQCP

Yanase (2008)

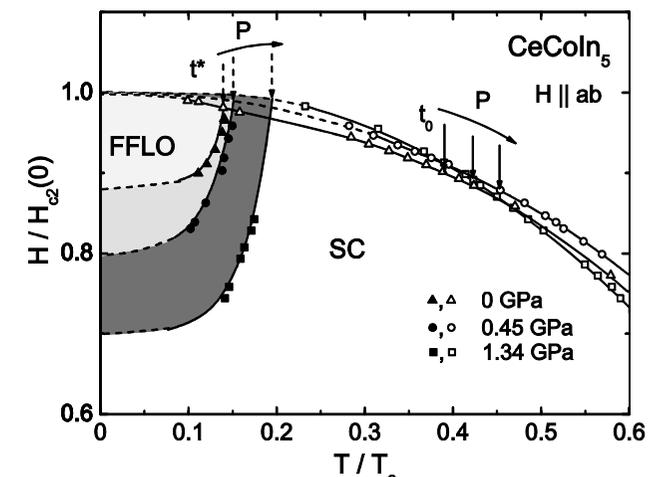
## 2D Hubbard model + FLEX



## BCS theory



## CeCoIn<sub>5</sub> Miclea et al. (2006)



Antiferromagnetic QCP



Stable FFLO phase

# FFLO phase in cold Fermi gases

Y. Y. PRB (2009)

T. Yoshida and Y. Y. PRA (2011)

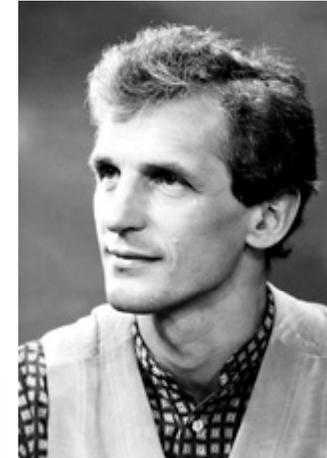
# Superfluid in cold atom gases

BEC in dilute Bose gas

Superfluidity in Fermi gas



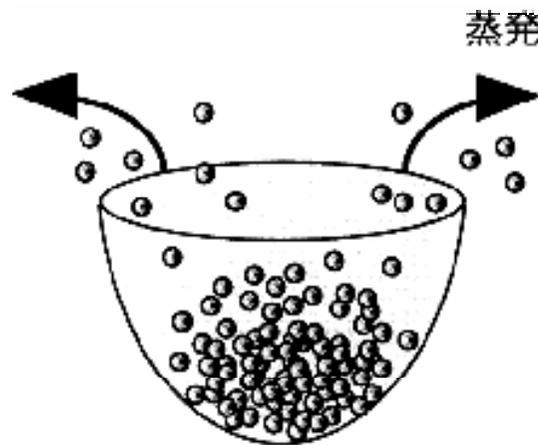
E. A. Cornell



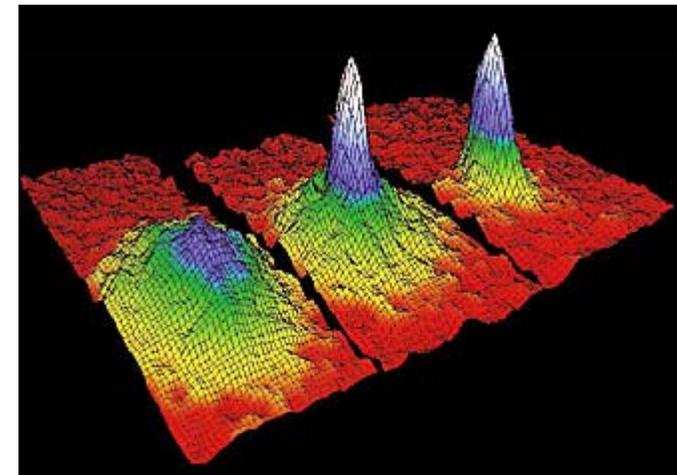
W. Ketterle



C. E. Wieman



Reduced temperature



BEC in  $^{87}\text{Rb}$  atoms

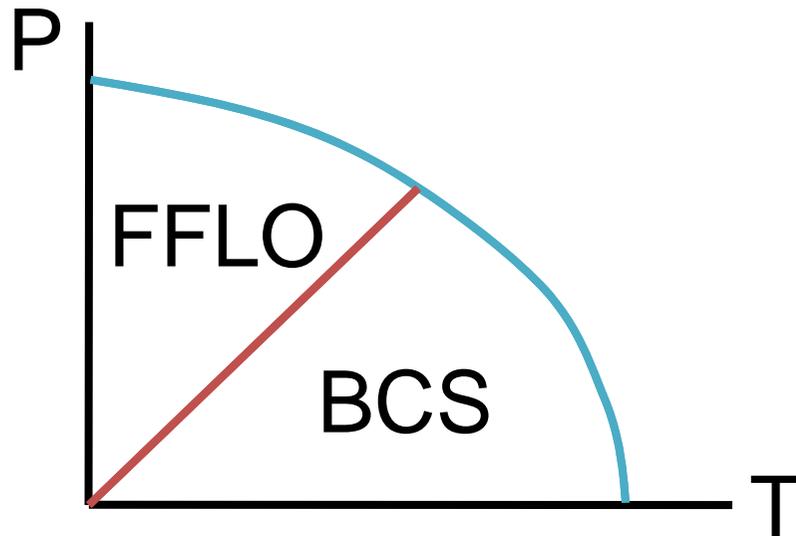
# Advantages of cold Fermi gases

(1) Disorder free

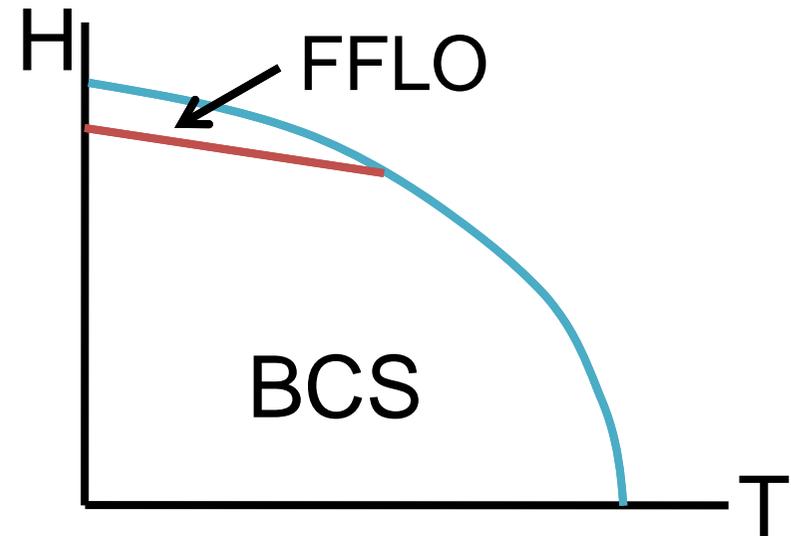
(2) No orbital pair-breaking effect → Pure FFLO state without vortex

(3) Attractive interaction → Fermi liquid correction stabilizes FFLO

Cold Fermi gases



Superconductors



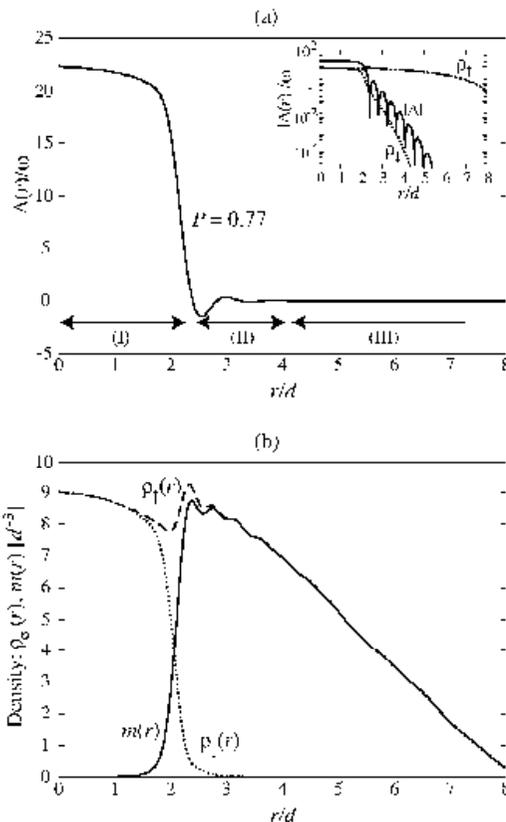
Discussion with Machida and Mizushima

# Disadvantage ? : Trap potential

Harmonic trap  $W(r) = \frac{1}{2}\omega_h^2 r^2$

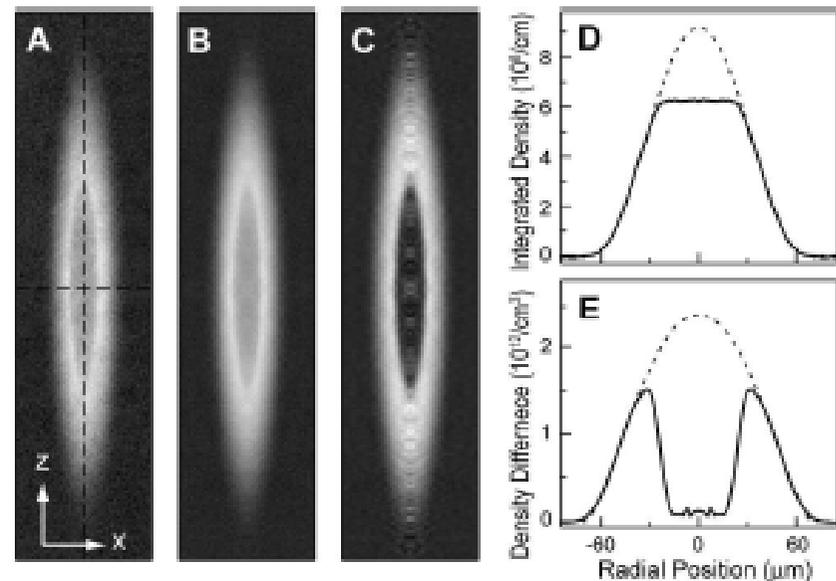
No translation symmetry !!

We cannot distinguish the FFLO state from phase separation



Theory:  
Mizushima et al.

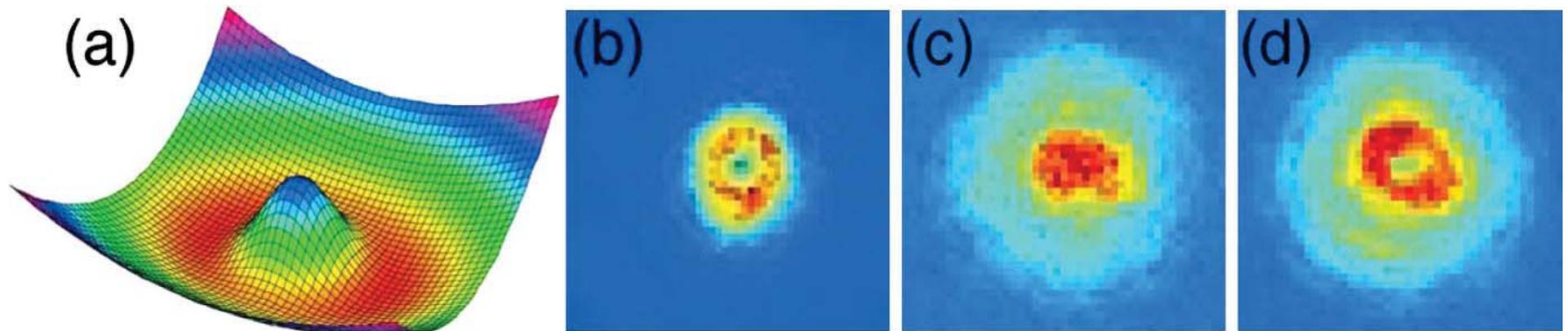
Experiment: Shin et al. (2006)



# Toroidal trap

$$W(r) = \frac{1}{2}\omega_h^2 r^2 + \omega_t \exp(-r^2/\xi^2)$$

C. Ryu et al. (2007) in NIST



No translation symmetry  
but, rotation symmetry !

Our idea

FFLO superfluid phase in the toroidal trap

## Model

Lattice model in a trap potential

$$H = t \sum_{\langle i,j \rangle, \sigma} c_{i,\sigma}^\dagger c_{j,\sigma} + \sum_i (W_i - \mu) n_i - 2H \sum_i S_i^z + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Low density limit  Continuum gas

## Trap potential

Harmonic potential  $W(r) = \frac{1}{2} \omega_h^2 r^2$

Toroidal potential  $W(r) = \frac{1}{2} \omega_h^2 r^2 + \omega_t \exp(-r^2 / \xi^2)$

## Method

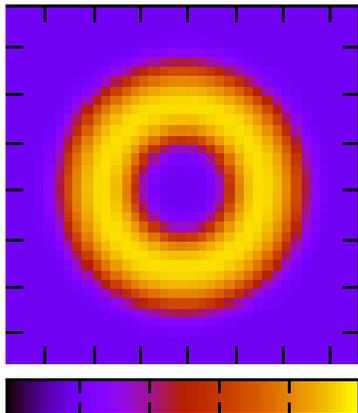
BdG (M.F.A.) , RSTA (Self-consistent 1-loop)

# Mean field theory (BdG equation)

Order parameter

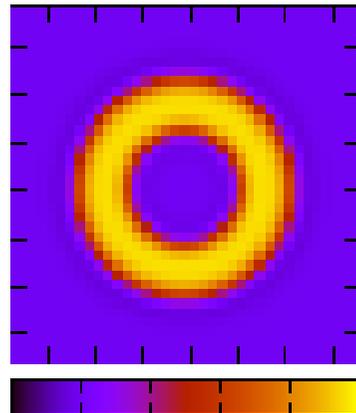
Imbalance:  $P = (n_1 - n_2)/(n_1 + n_2)$

(a)  $P=0$



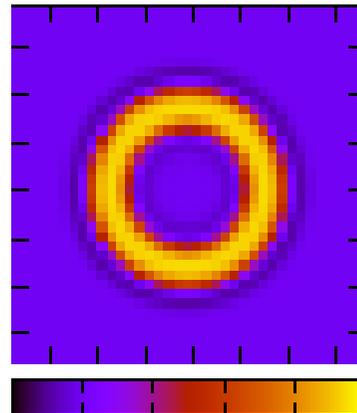
-0.4 0 0.4 0.8 1.2 1.6

(b)  $P=0.21$



-0.4 0 0.4 0.8 1.2 1.6

(c)  $P=0.39$

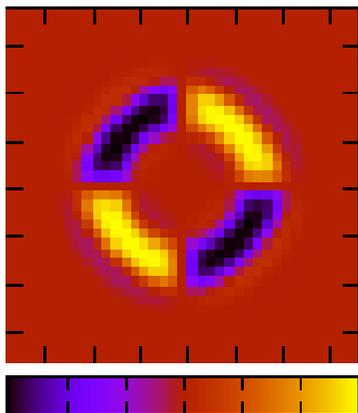


-0.4 0 0.4 0.8 1.2 1.6

Radial-FFLO

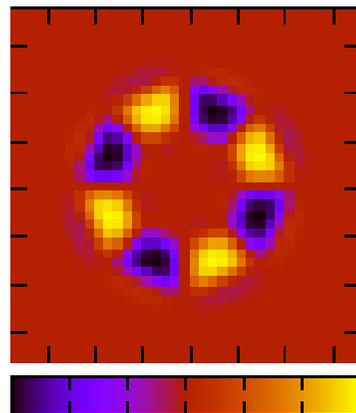
✗ No symmetry breaking

(d)  $P=0.44$



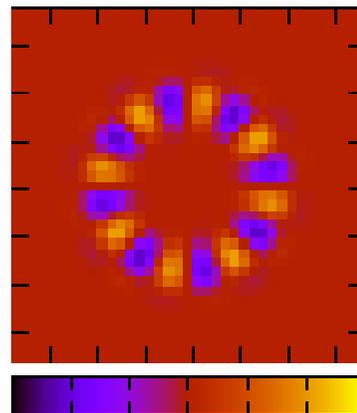
-1.5 -1 -0.5 0 0.5 1 1.5

(e)  $P=0.49$



-1.5 -1 -0.5 0 0.5 1 1.5

(f)  $P=0.69$



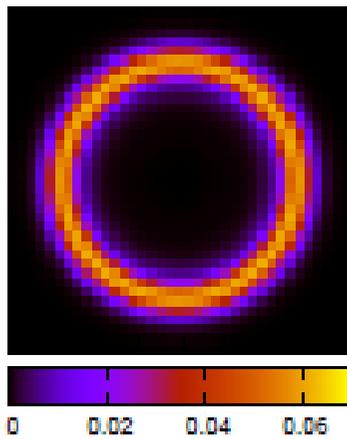
-1.5 -1 -0.5 0 0.5 1 1.5

Angular-FFLO

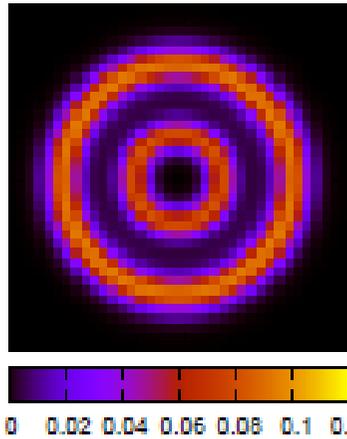
Rotation symmetry breaking !!

# Local Population imbalance

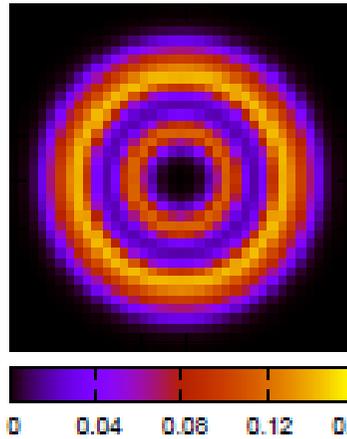
(a)  $P=0.1$



(b)  $P=0.21$



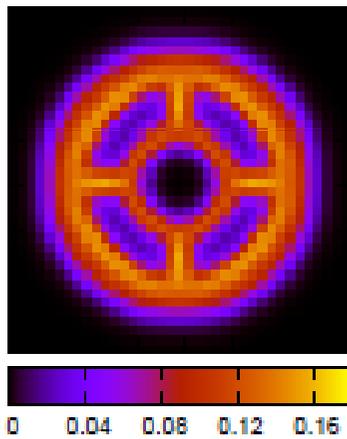
(c)  $P=0.39$



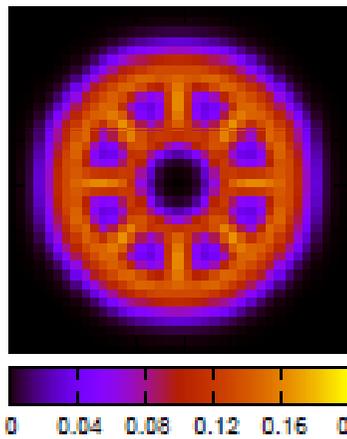
Radial-FFLO

✗ No symmetry breaking

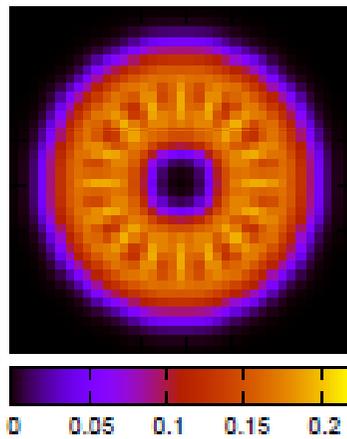
(d)  $P=0.44$



(e)  $P=0.49$



(f)  $P=0.69$

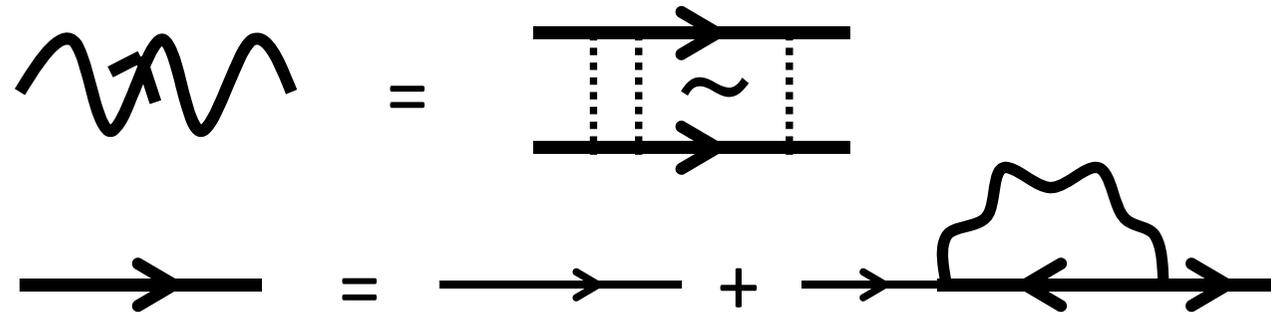


Angular-FFLO

distinguished from phase separation !!

# Beyond mean field theory (RSTA)

RSTA= Real Space Self-consistent T-matrix Approximation



Inhomogeneity: Exact  $\longrightarrow$  Mesoscopic fluctuation

fluctuation: 1-loop  $\longrightarrow$  Thermal fluctuation

History

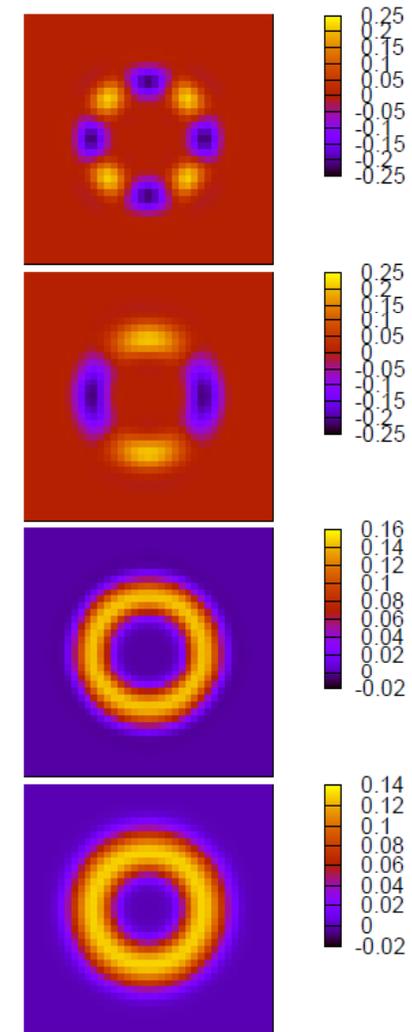
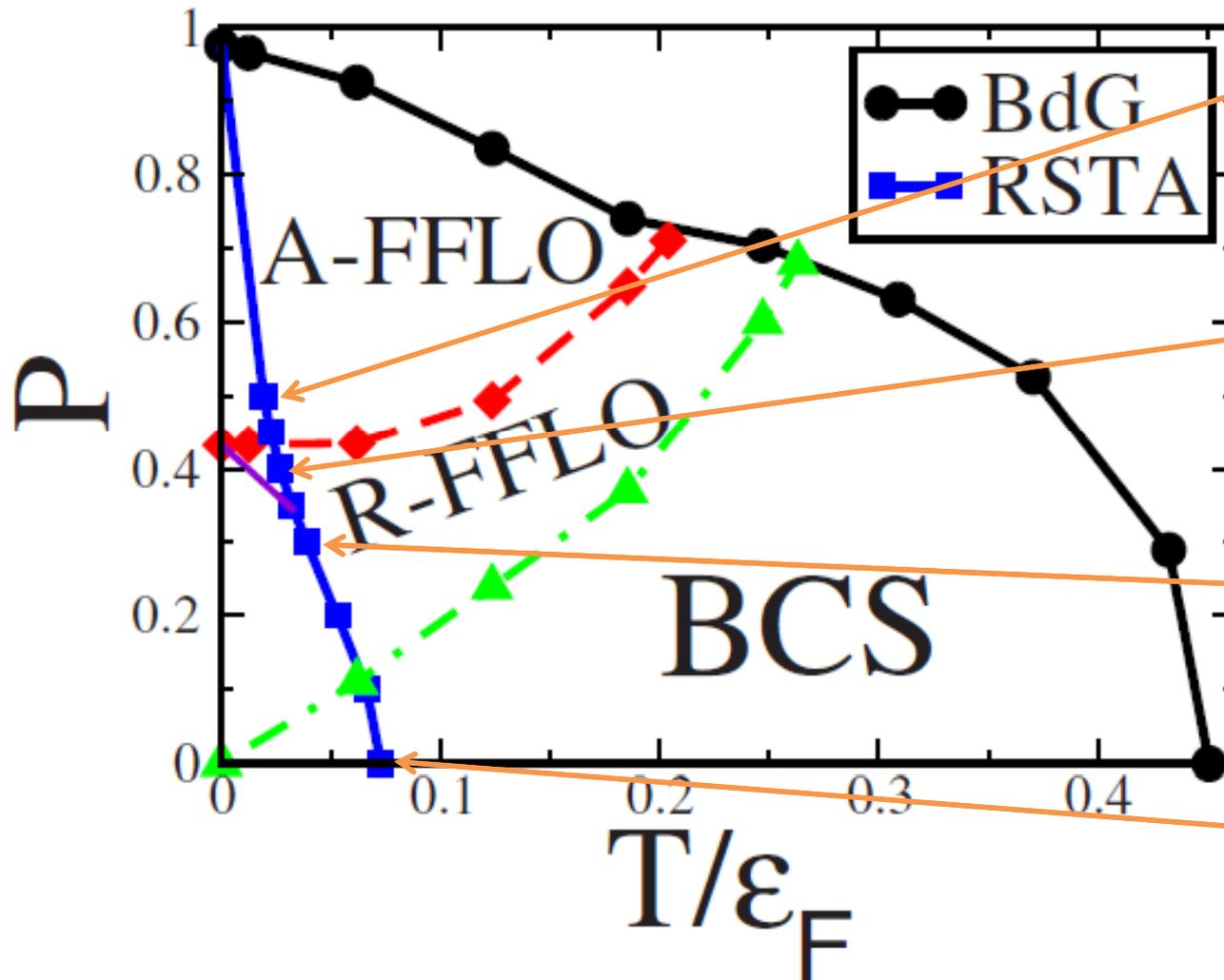
(1) Pseudogap in disordered high- $T_c$  cuprates (2006)

(2) Superconductor-Insulator transition in Diamond (2008)

# Phase diagram in RSTA

BCS side near crossover

$$e_b/\epsilon_F \sim 0.43$$



Angular-FFLO state is stable near BCS-BEC crossover !!

# Rotating FFLO phases

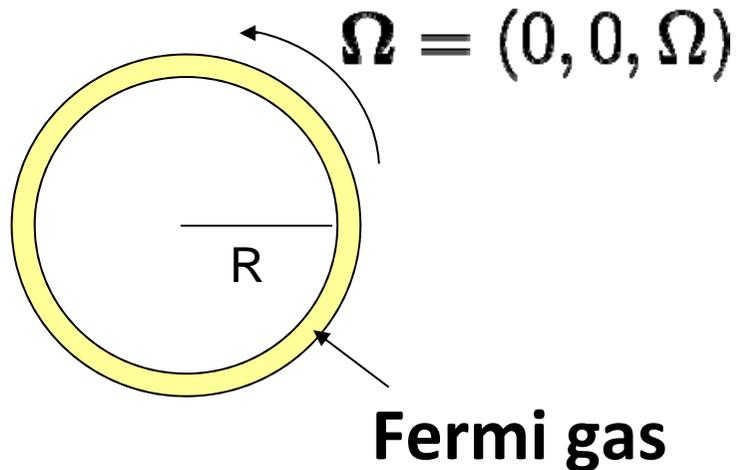
T. Yoshida and Y. Y. (2011)

Spontaneous rotation symmetry breaking



Intriguing effect of rotation

# Rotating FFLO phase in the toroidal trap



Rotation



Phase shift

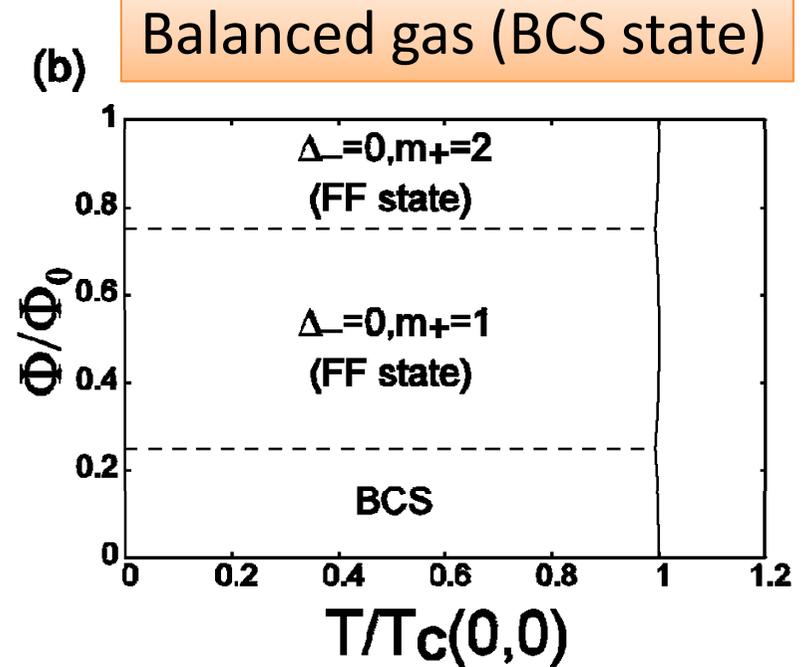
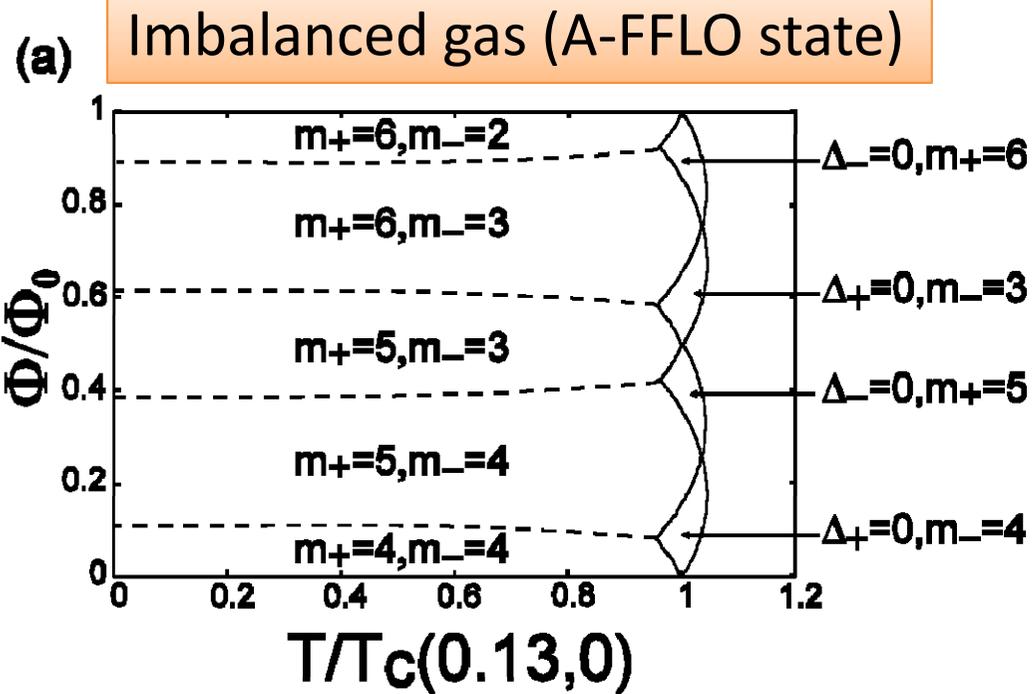
One dimensional attractive Hubbard model

$$H = -t \sum_{i,\sigma} (e^{i\Phi} \hat{c}_{i+1\sigma}^\dagger \hat{c}_{i\sigma} + e^{-i\Phi} \hat{c}_{i-1\sigma}^\dagger \hat{c}_{i\sigma}) - |U| \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} - \sum_{i,\sigma} \mu_\sigma \hat{n}_{i\sigma}$$

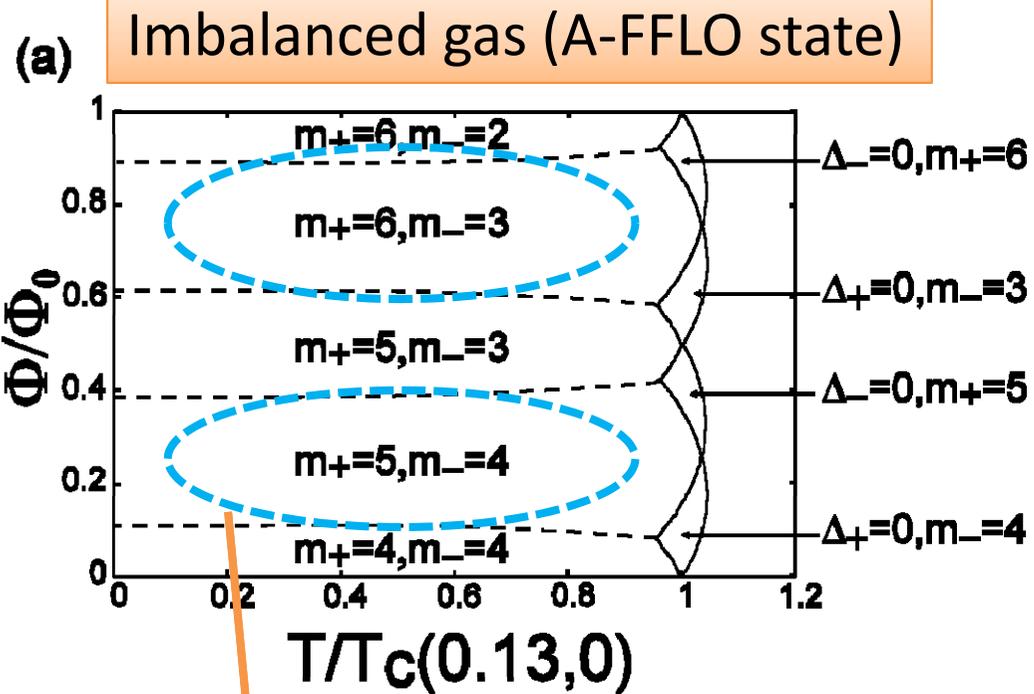
$$t = 1/2m, \quad \Phi = R\Omega/2t$$

quasi-1D model in BCS region  BdG equation

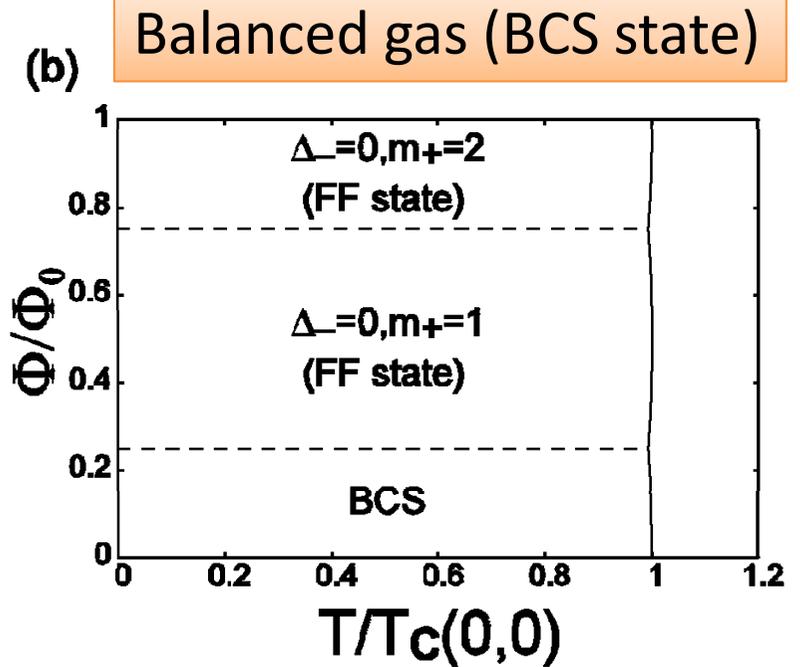
# T- $\Phi$ phase diagram



# Half quantum vortex state

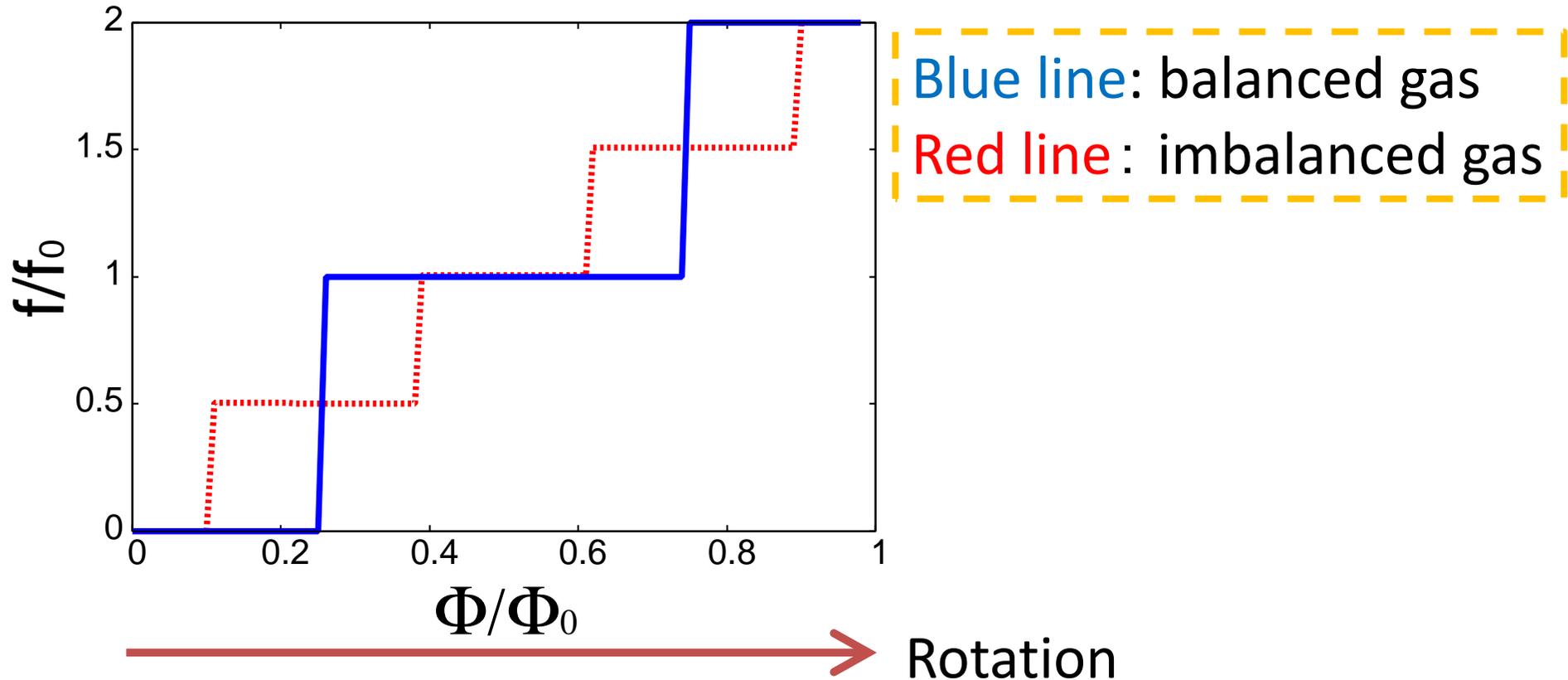


Half quantum vortex



Half period of Little-Parks oscillation

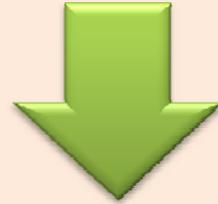
# Half quantized mass current



The mass current is half quantized in the A-FFLO superfluid state

## Summary

Toroidal trap + Feshbach resonance



We can produce and observe the Angular-FFLO state!!



Rotation

Half quantum vortex state

Study in future: (A) Trap geometry  
(B) Optical lattice  
(C) Dipole interaction

## Summary

Toroidal trap + Feshbach resonance



We can produce and observe the Angular-FFLO state!!



Rotation

Half quantum vortex state

Thank you very much for your attentions